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**Fractional integration and cointegration in  
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**DT-GREQAM**

# Fractional integration and cointegration in stock prices and exchange rates\*

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## Abstract

This paper examines the relationships between the CAC40 index, the Dow Jones index and the Euro/USD exchange rate using daily data over the period 1999-2008. We find that these variables are I(1) nonstationary series, but they are fractionally cointegrated: equilibrium errors exhibit slow mean reversion, responding slowly to shocks. Therefore, with regard to the recent empirical cointegration literature, taking into account fractional cointegration techniques appears as a promising way to study the long-run relationships between stock prices and exchange rates.

**Keywords:** fractional cointegration, long memory, stock prices, exchange rates

**JEL classification:** C12, C22, F31, F37, G15.

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## 1. Introduction

Since the Euro's introduction in 1999, fluctuations of the Euro against Dollar have been surprisingly high with a first period of Euro depreciation followed by a lasting Euro appreciation. As a result, the Euro/Dollar constitutes an interesting example to investigate the long-run relationship between stock prices and exchange rates. Then, we explore the relationship between the French stock market, the US stock market and the Euro/Dollar exchange rate.

This paper can be related to two strands of the literature on stock prices. The first one investigates the international connection between stock markets. If stock prices reflect fundamentals – and they certainly should except in episodes of bubbles – they are closely related to expectations about future real economic activity (Binswanger (2000)). International trade and capital inflows have among other factors increased comovements (Otto et al. (2001)) of outputs. If stock market prices reflect these outputs comovements, they should be internationally connected as well. The second one focuses on the relationship between exchange rates and stock prices. There is no real consensus on this question yet. On the one hand, macroeconomic models predict a negative relationship with a causality that goes from exchange rates to stock prices<sup>1</sup> - an appreciation deteriorates competitiveness, profits and decreases stock prices; on the other hand, finance literature through the portfolio model highlights a positive relationship from stock prices to exchange rates: an increase in stock prices may attract capital flows and cause an exchange rate appreciation (Phylaktis and Ravazzolo (2005)).

Based on recent advanced econometric methods (Caporale and Gil-Alana (2004a), Gil-Alana (2007)) we explore the long-run relationship between the Euro/Dollar exchange rate and French and US stock markets on daily data. We provide evidence that the error correction term follows a fractionally integrated process with long memory. This means that we find a cointegration relationship with a degree of integration lower than unity: this result contrasts with previous studies since they only used standard cointegration techniques. Table 1 lists some papers dealing with the long-run relationships between international stock prices, and some others focusing on the long-run relationship between stock prices and exchange rates. These studies show clearly that there is a lack of consensus about the long run relationships between stock prices and exchange rates.

### Table 1. Literature about the long-run relationship

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<sup>1</sup>The relationship is negative if we consider that a rise in the exchange rate is an appreciation.

### between stock prices and exchange rate

	Period (Frequency)	Countries	Method	Cointeg. between EM <sup>1</sup>	Cointeg. EM and ER <sup>2</sup>
Bahmani-Oskooee and Sohrabian (1992)	07/1973-12/1988 (Monthly)	US	Engle-Granger (1987)	...	No
Granger, Huang and Chin-Wei (2000)	03/01/86-16/06/98 (Daily)	Japan, Taiwan Singapore	Gregory and Hansen (1996)	...	No
Huang, Chin-Wei and Hu (2000)	01/10/92-30/06/97 (Daily)	Taiwan, Japan US, China Hong Kong	Gregory and Hansen (1996)	Yes <sup>3</sup>	...
Kanas (1998)	03/01/83-29/11/96 (Daily)	US, UK, Italy Germany, France Switzerland Netherlands	Johansen (1995) Bierens (1997)	No	...
Phylaktis and Ravazzolo (2005)	01/1980-12/1998 (Monthly)	Hong Kong Malaysia, Singapore Thailand, Philippines	Johansen (1995)	Yes <sup>6</sup>	Yes <sup>6</sup>
Nieh and Lee (2001)	01/10/93-15/02/96 (Daily)	G7 countries	Engle-Granger (1987) Johansen (1995)	...	No
Smyth and Nandha (2003)	02/01/95-23/11/01 (Daily)	Bangladesh, India Pakistan, Sri-Lanka	Engle-Granger (1987) Johansen (1995)	...	No
Staverek (2005)	01/93-12/03 (Monthly)	Austria, France Czech Rep. Germany, Hungary Poland, Slovakia UK, US	Johansen (1995)	...	Yes <sup>4</sup>
Wu (2000)	1990-2000 (Weekly)	Singapore, US	Johansen (1995)	Yes <sup>5</sup>	Yes

**Table 1 (continued).**

	Period (Frequency)	Countries	Method	Cointeg. between EM <sup>1</sup>	Cointeg. EM and ER <sup>2</sup>
Ajayi and Mougoue (1996)	1985-1991 (Daily)	Canada, France Germany, Italy Netherlands Japan, UK, US	Engle-Granger (1987)	...	Yes
Pan Fok and Liu (2007)	1988-1998 (Daily)	Hong Kong Japan, Taiwan Thailand	Johansen (1995)	...	Yes

Notes: <sup>1</sup>: EM: equity markets, <sup>2</sup>: ER: exchange rates, <sup>3</sup>: Only for Shanghai and Shenzhen markets, <sup>4</sup>: Except for France, Czech Republic and Slovakia, <sup>5</sup>: Exchange rates data are 4 bilateral exchange rates vis-à-vis the Singapore \$ (Malaysian Ringgit, Indonesian Rupiah, US \$, Jap Yen), <sup>6</sup>: Significant relation only for the post-liberalisation period.

Some reasons could be given to explain this lack of consensus. For instance, Pan, Fok and Liu (2007) suggest that countries using a managed floating system do not show generally cointegration; following these authors, another reason could be the noise from using daily data (see also Granger, Huang and Chin-Wei (2000)). Chamberlain, Howe and Popper (1997), Griffin, Nardari, and Stulz (2004) and Pan, Fok and Liu (2007) note that the interactions between exchange rates and stock prices are more significant using higher frequency data; one of the reasons is that "equity flows toward a country are mainly driven by the previous day's return in that country's equity market" (Griffin, Nardari, and Stulz (2004)). Another explanation is based on the testing procedures: if there is a possible nonlinear cointegration - i.e. if there exists a dynamic nonlinear relationship between these financial variables - or a possible fractional cointegration, the co-movements cannot be detected by standard linear cointegration techniques. The method below explores the possibility of a long term fractional relationship.

The rest of the paper is organized as follows. We present the econometric method in Section 2 and the empirical analysis in Section 3. Finally, some concluding remarks are given in Section 4.

## 2. The econometric approach: fractional integration and cointegration

We know that a time series  $y_t$  follows an *ARFIMA*( $p, d, q$ ) (autoregressive fractionally integrated moving average) process if

$$\Phi(L)(1 - L)^d y_t = \mu + \Theta(L)\varepsilon_t$$

with  $\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ ,  $\Theta(L) = 1 + \theta_1 L + \dots + \theta_q L^q$ ,  $L$  is the Backward shift operator i.e.  $Ly_t = y_{t-1}$  and  $\varepsilon_t \sim iid(0, \sigma^2)$ . Different cases are possible, depending on the value of the long memory parameter  $d$ ; for example,  $y_t$  is stationary and possesses shocks that disappear hyperbolically when  $0 < d < 1/2$ , but is non-stationary and mean reverting for  $1/2 \leq d < 1$ .

Moreover, fractional cointegration can be defined as follows. Let us consider two time series  $y_t$  and  $x_t$  that are both  $I(d)$ , where  $d$  is not necessarily an integer;  $y_t$  and  $x_t$  are fractionally cointegrated when the residuals, defined by  $u_t = y_t - \beta x_t$ , are  $I(d - b)$  with  $b > 0$ , where  $b$  is also not necessarily an integer.

There is some recent literature dealing with fractional cointegration (for instance, see among others Baillie and Bollerslev (1994), Caporale and Gil-Alana (2004a and b), Christensen and Nielson (2006), Davidson (2005), Dittman (2001), Gil-Alana (2007), Hassler, Marmol and Velasco (2006), Kellard, Dunis and Sarantis (2007), Kim and Phillips (2001), Marinucci and Robinson (2001), Nielsen (2007), Robinson and Marinucci (2003), Robinson and Yajima (2002), Tse, Anh and Tieng (1999), Velasco (2003)).

We use here the methodology elaborated by Robinson (1994) for testing unit root and other nonstationary hypotheses. Let us consider the null hypothesis defined by

$$H_0 : \theta = 0 \tag{1}$$

in the model given by:

$$y_t = \beta' z_t + x_t \tag{2}$$

and

$$(1 - L)^{d+\theta} x_t = u_t \tag{3}$$

for  $t = 1, 2, \dots$ , where  $y_t$  is the observed time series,  $z_t$  is a  $k \times 1$  vector of deterministic regressors,  $u_t$  is a (possibly weakly autocorrelated)  $I(0)$  process, and  $d$  is a real parameter. The Lagrange Multiplier (LM) statistic proposed by Robinson (1994), called  $\hat{r}^2$ , has a standard asymptotic distribution under some regularity conditions:

$$\hat{r} \xrightarrow[d]{} N(0, 1) \quad \text{as } T \longrightarrow \infty.$$

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<sup>2</sup>See the Appendix for the exact definition of  $\hat{r}$ .

Thus, it is a one-sided test of  $H_0 : \theta = 0$  : we reject  $H_0$  against  $H_1 : \theta > 0$  if  $\hat{r} > z_\alpha$  and against  $H_1 : \theta < 0$  if  $\hat{r} < -z_\alpha$ , where the probability that a standard normal variate exceeds  $z_\alpha$  is  $\alpha$ . This Robinson (1994)'s test has been used in several papers in order to detect fractional integration (Caporale and Gil-Alana (2007a, b and c), Gil-Alana and Nazarski (2007)), fractional integration with nonlinear models (Gil-Alana and Caporale (2006), Cunado, Gil-Alana and Perez de Gracia (2007)), fractional integration with structural breaks (Caporale, Cunado and Gil-Alana (2007), Gil-Alana (2008)), fractional cointegration (Caporale and Gil-Alana (2004a and b)) and seasonal fractional cointegration (Gil-Alana (2007)).

In order to detect the cointegration, we adopt the two-step strategy of Caporale and Gil-Alana (2004a and b) and Gil-Alana (2007), based on the Robinson (1994) test: in the first step, we test the order of integration of each series, and if they are of the same order, we test, in the second step, the order of integration of the estimated residuals of the cointegration relationship<sup>3</sup>. Let us call  $e_t$ , the estimated equilibrium errors between two series  $X_{1t}$  and  $X_{2t}$  (this can be easily generalized to more series):

$$e_t = X_{1t} - \hat{\alpha}X_{2t}$$

where  $\hat{\alpha}$  is the OLS estimator of the cointegrating parameter. Let us consider the model:

$$(1 - L)^{d+\theta}e_t = u_t \tag{4}$$

where  $u_t$  is a  $I(0)$  process ; we applied the Robinson (1994)'s testing procedure in order to test the null hypothesis  $H_0 : \theta = 0$  against the alternative  $H_1 : \theta < 0$ . If the null hypothesis is rejected, it implies that the equilibrium error exhibits a smaller degree of integration than the original series:  $X_{1t}$  and  $X_{2t}$  are thus fractionally cointegrated. On the opposite, if the null hypothesis is not rejected, the series are not cointegrated because the order of integration of  $e_t$  is the same as the order of the original series.

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<sup>3</sup>Caporale and Gil-Alana (2004a) note that the ordinary least squares (OLS) estimation of the equilibrium error can produce an estimator which may suffer from second-order bias in small samples, but they choose to use it on the grounds of simplicity; in this paper, the sample sizes are enough large to neglect this problem.



### 3. Integration and cointegration analysis

#### 3.1 The data

The data consist of 2342 observations of log-transformed daily closing stock market indices ( $CAC_t$ , the CAC40 index,  $DJ_t$ , the Dow Jones Industrial Average) and the Dollar-Euro exchange rate  $S_t$  (a rise in  $S_t$  represents an appreciation of Euro against Dollar), over the period January 4, 1999 - January 22, 2008, i.e. since the start of the EMU and the Euro ( $T = 2342$ , where  $T$  is the number of observations).

#### 3.2 Empirical results

##### 3.2.1 Integration analysis of individual series

The first step in the empirical analysis is to investigate the order of integration of the three individual series. We perform the Kwiatowski et al. (1992) (KPSS) test for unit root, where the null hypothesis is the stationarity, on the raw series and on the first differenced series. The results are reported in Table 2 and clearly show the rejection of the null hypothesis by the KPSS test on the raw series and the non-rejection of the null hypothesis on the differenced series, which means that the three raw series contain a unit root.

**Table 2. KPSS test on each individual series**

	$KPSS(0)$	$KPSS(6)$		$KPSS(0)$	$KPSS(6)$
$CAC_t$	41.594	5.965	$\Delta CAC_t$	0.125	0.148
$DJ_t$	40.779	5.887	$\Delta DJ_t$	0.046	0.050
$S_t$	23.910	3.449	$\Delta S_t$	0.138	0.148

Note:  $KPSS(i)$  is the  $\tau$ -statistic of order  $i$  of the Kwiatowski, Phillips, Schmidt and Shin (1992) test; the critical values are: 0.119 (10%), 0.146 (5%) and 0.216 (1%).

This is confirmed by the Table 3 showing the results of the statistic  $\hat{r}$  of the Robinson (1994)'s tests applied to each individual series. Different values of  $d$  are considered, thus testing for a unit root ( $d = 1$ ) but also other fractional possibilities. We indicate in bold the minimum of the absolute values of the Robinson (1994) test statistics, and by "\*", the nonrejection values of the null hypothesis  $H_0 : \theta = 0$  in the model (3) at the 95% significance level. In this table, we can observe that the minimum of the absolute values of the Robinson (1994) test statistic occur always when  $d = 0.95$  or 1. This permits to conclude that all the series may contain a unit root or are together close to the unit root case.

**Table 3. Testing the order of integration of each individual series with the test of Robinson (1994)**

$CAC_t$				$DJ_t$			
$d$	—	1	(1, $t$ )	$d$	—	1	(1, $t$ )
0.90	7.26	4.87	4.87	0.90	7.42	4.65	4.65
0.95	3.12	<b>1.10*</b>	<b>1.10*</b>	0.95	3.25	<b>1.21*</b>	<b>1.21*</b>
1.00	<b>-0.21*</b>	-1.84	-1.84	1.00	<b>-0.12*</b>	-1.63*	-1.63*
1.05	-2.94	-4.22	-4.22	1.05	-2.89	-4.02	-4.02
1.10	-5.20	-6.18	-6.18	1.10	-5.17	-6.04	-6.04

$S_t$			
$d$	—	1	(1, $t$ )
0.90	5.66	7.37	7.26
0.95	1.71	2.83	2.80
1.00	<b>-1.41*</b>	<b>-0.68*</b>	<b>-0.69*</b>
1.05	-3.94	-3.47	-3.46
1.10	-6.02	-5.72	-5.71

Note: We consider only the test where  $u_t$  is assumed to be white noise, with different specifications: with no regressors (—), with an intercept (1) and with a linear trend ((1,  $t$ )). In bold: the absolute value of the minimum of the Robinson (1994) test statistic. \*: nonrejection values of the null hypothesis  $H_0 : \theta = 0$  at the 95% significance level (the critical value is 1.65 in absolute value).

### 3.2.2 Cointegration analysis

Following the second step of the strategy of Caporale and Gil-Alana (2004a and b) and Gil-Alana (2007), we consider now the possibility of the series being fractionally cointegrated. In Table 4, we can observe some estimation results of the cointegrating equation:

$$e_t = CAC_t - \hat{\alpha}_1 - \hat{\alpha}_2 DJ_t - \hat{\alpha}_3 S_t \quad (5)$$

where  $e_t$  is the estimated equilibrium error. The  $\hat{\alpha}_i$  are the OLS estimated values of the coefficients of this regression; the corresponding standard errors are in parenthesis. The results of the KPSS test show the rejection of the null hypothesis of stationarity of the estimated equilibrium errors and thus that the series are not cointegrated.

**Table 4. Some cointegration results**

$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$KPSS(0)$	$KPSS(2)$
-4.194	1.961	-0.868	12.063	4.098
(0.07)	(0.01)	(0.01)		

Note: The  $\hat{\alpha}_i$  are the estimated values of the coefficients in the cointegrating regression  $CAC_t = \alpha_1 + \alpha_2 DJ_t + \alpha_3 S_t + noise$ ; the estimation method is the ordinary least squares and the standard errors are in parenthesis.  $KPSS(i)$  is the statistic of order  $i$  of the Kwiatowski, Phillips, Schmidt and Shin (1992) test; the critical values are given in Shin (1994).

Table 5 shows the results of one-sided tests of Robinson (1994) on the estimated residuals  $e_t$  from the cointegrating regression (5): we calculate  $\hat{r}$ , the Robinson (1994) LM statistic, testing  $H_0 : \theta = 0$  against the alternative  $H_1 : \theta < 0$  in a model given by (4).

**Table 5. Robinson (1994)'s tests on the estimated residuals from the cointegrating regression**

$d$	—	1	(1, $t$ )
0.750	5.31	5.48	5.58
0.775	3.08	3.16	3.23
0.800	1.07*	1.08*	1.13*
0.825	<b>-0.72*</b>	<b>-0.77*</b>	<b>-0.74*</b>
0.850	-2.34	-2.44	-2.42
0.875	-3.80	-3.94	-3.93
0.900	-5.11	-5.28	-5.28

Note: We consider only the test where  $u_t$  is assumed to be white noise, with different specifications: with no regressors (—), with an intercept (1) and with a linear trend ((1,  $t$ )). In bold: the absolute value of the minimum of the Robinson (1994) test statistic. \*: nonrejection values of the null hypothesis  $H_0 : \theta = 0$  at the 95% significance level (the critical value is 1.65 in absolute value).

We can observe that the nonrejection values of  $H_0 : \theta = 0$  occur always for values of  $d \in (0.800, 0.825)$ , for any type of regressor. This implies that the estimated residuals from the cointegrating regression are of an inferior order of integration than that of the individual series; this shows that a long-run equilibrium relationship may exist. We observe that the minimum of the absolute values of the Robinson (1994) test statistic corresponds to the value of the long memory parameter  $d$  equal to 0.825; this means that the

fractional cointegration specification is accepted. Thus, a fractionally cointegrated relationship does exist between  $CAC_t$ ,  $DJ_t$  and  $S_t$ : the equilibrium error exhibits slow mean reversion; it responds very slowly to shocks, i.e. deviations from equilibrium are persistent, although there exists a long-run relationship.

We have shown that a fractionally cointegrated relationship exists between  $CAC_t$ ,  $DJ_t$  and  $S_t$  over the period considered. This result suggests output comovements between US and France. They may be due to intensive international trade between those countries or to the fact that stock prices reflect fundamentals -and fundamentals between US and France are connected.

#### 4. Concluding remarks

Adopting the two-step strategy of Caporale and Gil-Alana (2004a, 2004b) and Gil-Alana (2007), based on the Robinson (1994) test, we show that the CAC40 index is fractionally cointegrated with the Dow Jones index and the Dollar/Euro exchange rate: the equilibrium errors exhibit slow mean reversion, responding very slowly to shocks: the effects of shocks disappear in the long run. The original contribution of this article is to show that taking into account fractional cointegration techniques appears as a promising way to study the long-run relationships between stock prices and exchange rates.

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## Appendix

The Lagrange Multiplier (LM) statistic proposed by Robinson (1994), called  $\hat{r}$ , is given by:

$$\hat{r} = \left( \frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2} \quad (6)$$

where  $T$  is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j, \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{\sigma}^2 = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j, \hat{\tau})^{-1} I(\lambda_j),$$

$$\begin{aligned} \hat{A} = & \frac{2}{T} \left[ \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right. \\ & \left. \times \left( \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right], \end{aligned}$$

$$\hat{\tau} = \arg \min_{\tau \in T^*} \hat{\sigma}^2, \quad \psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|, \quad \lambda_j = \frac{2\pi j}{T}$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j, \hat{\tau}), \quad g(\lambda, \tau) = \frac{2\pi}{\sigma^2} f(\lambda, \tau, \sigma^2);$$

$f$  is the spectral density of  $u_t$ ,  $T^*$  is a suitable set of  $\mathbb{R}^k$  and  $I(\lambda_j)$  is the periodogram of

$$\hat{u}_t = (1 - L)^d y_t - \hat{\beta}' w_t \quad (7)$$

evaluated at  $\lambda_j$  with

$$w_t = (1 - L)^d z_t$$

and

$$\hat{\beta} = \left( \sum_{t=1}^T w_t w_t' \right)^{-1} \sum_{t=1}^T w_t (1 - L)^d y_t.$$

Note that  $\sigma^2$  is generally no longer the variance of  $u_t$ , but rather the variance of the innovation sequence in a normalized Wold representation of  $u_t$ . Robinson (1994) shows that  $\hat{r}$  has a standard asymptotic distribution under some regularity conditions:

$$\hat{r} \xrightarrow{d} N(0, 1) \quad \text{as } T \longrightarrow \infty.$$