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Non-additivity in accounting valuation: Internally generated goodwill as an aggregation of interacting assets

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ABSTRACT: In this paper we propose a new method to explain the creation and measure the value of internally generated goodwill (IGG). Our method is based on the idea that firm value is affected by interactions between assets used in combination to conduct business. This novel approach contrasts with the traditional additive approaches to valuing IGG, which assume assets are independent. We use Choquet capacities, i.e., non-additive aggregation operators, to explain the creation of IGG, and demonstrate from a sample of U.S. high technology sector firms that this model performs better than the traditional additive Ohlson model on accuracy in forecast enterprise value.

Keywords: Goodwill – Internally generated goodwill – Accounting valuation model – Synergy – Choquet integral – Residual income model

I. INTRODUCTION

Prior literature suggests that there is no reason why the fair value of a set of assets should equal the sum of the fair values of each asset: fair value measure is not additive (McKeown 1971; Ijiri 1975). A firm composed of several interdependent assets cannot therefore be appropriately valued by summing the individual fair values of its assets. Internally generated goodwill (IGG) is the excess value of a firm over the sum of the fair values of its identifiable assets. As Lee (1971, 323) explains, “the problem of accounting for the value of goodwill reflects, therefore, a much greater valuation problem, involving all the resources contributing to business profits.” This is the most general expression of the additivity issue in accounting.

Discussion of additivity in valuation is not limited to past literature. Even recent accounting standards raise the issue. In practice, firms are required to compute the total fair value of each reporting unit¹ every year, in order to test the goodwill allocated to this reporting unit for impairment (see paragraphs 350-20-35-4 to 35-15 of the ASC).² U.S. accounting standards (ASC 350-20-35-22 to 35-24) add that computing the overall fair value of a reporting unit simply by aggregating the values of the assets making up the reporting unit is not an appropriate valuation method, as the overall value of a reporting unit can differ from (and often exceed) the sum of the fair values of its components. In the absence of a quoted market price, ASC 350-20-35-24 recommends the use of an overall approach to compute the total value of the reporting unit (e.g., multiples or discounted cash flow models). This requirement to use an overall method to compute the value of a set of assets for impairment tests illustrates at the reporting unit level the additivity issue identified by Ijiri (1975).

In this context, IGG (also called “going-concern goodwill”) represents “the ability [of a firm] as a stand-alone business to earn a higher rate of return on an organized collection of net assets than would be expected if those assets had to be acquired separately [...]” (Arnold et al. 1994, 19; Johnson and Petrone 1998, 296). As we will show, IGG arises from synergies between assets in place, organized into a specific system. The term “synergy” is often used to refer to the synergistic effect resulting from combination of the acquirer and the target firm in an acquisition

¹ According to paragraph 350-20-35-34 of the Accounting Standards Codification (ASC), “A component of an operating segment is a reporting unit if the component constitutes a business for which discrete financial information is available and segment management, as that term is defined in paragraph 280-10-50-7, regularly reviews the operating results of that component.”

² We refer to U.S. accounting standards (FAS 141, FAS 142) using the new Accounting Standards Codification.

(Ma and Hopkins 1988, 77; Johnson and Petrone 1998; Henning et al. 2000). In this paper, we only use the term “synergy” in relation to interaction between assets existing within a given firm, independently of a business combination. In the above definition of IGG, measurement of the sum of the fair values of a firm’s identifiable assets is a key issue.

Prior literature has proposed two methods to value IGG. (1) The “direct” method, where the unrecorded goodwill equals the expected present value of abnormal earnings under the residual income formula as expressed in Ohlson (1991; 1995; see also Schultze and Weiler 2010), and implemented by Dechow et al. (1999) for instance. (2) The “indirect” method, which consists of subtracting the fair value of a firm’s assets³ from its enterprise value in the case of a business combination. The concept of “enterprise value” is traditionally defined as market value plus total debt minus cash, which corresponds to an “equity side” approach in the context of a theoretical takeover. In this paper, we implement an “asset side” approach focusing on the economic value of the assets of the firm, defined as the fair value of total assets plus the internally generated goodwill.⁴ This indirect approach has been possible in practice since introduction of the purchase method by FAS 141 (FASB 2001, 2007), which requires the acquirer to estimate and disclose the fair value of assets and liabilities acquired in a business combination.⁵

Both of these approaches have several drawbacks. They focus on outflows and do not explain how IGG is created. The nature of IGG remains largely a mystery, as no reconciliation between individual asset values and the firm’s total value is provided. The non-additive nature of a set of interacting assets is overlooked due to the focus on the economic consequences of IGG, i.e., abnormal earnings or excess enterprise value over fair value of assets. More specifically, the second method links computation of IGG to a business combination: as Zanoni (2009, 1) highlights, “goodwill is the part of the enterprise value that does not appear in financial statements but that emerges only when acquired” – through a business combination, for instance. It should not be forgotten that the economic nature of IGG is completely independent of a business combination, even though a business combination provides the fair values necessary to implement the computations required by the model proposed in this paper.

³ Assuming the fair value of the firm’s assets is known (see below).

⁴ We do not restate cash because we make no assumptions regarding the interactions between cash and other classes of assets. Cash is therefore part of current assets.

⁵ Paragraphs 805-10, 805-20, 805-30, 805-740, and 805-40 of the ASC.

In this paper we focus on the valuation and explanation of IGG. We propose a valuation method that reconciles the overall value of a set of assets with the individual fair values of the assets. This method can calculate and explain an overall valuation for a set of interacting assets by using a non-additive aggregation approach able to measure the value of the assets' structure, i.e., the interaction value of a set of assets. In relaxing the additive postulate underlying financial valuation, the proposed reconciliation between the overall fair value of a firm and the individual fair values of its assets can explain creation of IGG.

Goodwill emerges from an “inadequate theory of aggregation of assets” (Miller 1973, 280). Hence in valuation, “the choice of the aggregation function to be used is far from being arbitrary and should be based upon properties dictated by the framework in which the aggregation is performed” (Grabisch et al. 2009, 11). Our proposed approach consists of an aggregation method, recognizing that using an asset in combination with other assets leads to interaction that affects firm value, i.e., a structured set of assets may increase the overall value of a firm. Importing the concept of Choquet capacities (Choquet 1953), which are non-additive measures, from other areas of literature (see, e.g., the field of expected utilities without additivity: Gilboa 1988; Schmeidler 1989; Wu and Gonzalez 1999), we are able to model the value of different combinations of assets and assess how much they contribute to enterprise value.

We use a sample of U.S. high-technology firm acquisitions to test our model's accuracy relative to the observed values with out-of-sample predictions. Next, we compare the accuracy of our model to the accuracy of the traditional additive Ohlson model. The results show that our model outperforms the standard residual income model as regards accuracy in forecasting the enterprise value of these high-technology firms, producing smaller forecasting errors. For enterprise value predictions, relaxing the additive postulate in aggregation brings about a clear improvement.

We make several contributions to the literature through this paper. First, from a theoretical perspective, we show that the additivity postulate may be inappropriate because of the properties of the measured object. Second, we propose an alternative aggregation function consistent with non-additivity. Third, we suggest an empirical implementation of this new method to measure IGG, which is the accounting expression of the non-additivity of fair values. Our approach explains the creation of IGG by identifying specific synergies and inhibitions between a set of assets. Finally, from a practical perspective, once the synergies and inhibitions are estimated

based on previous business combinations in a given industry, the method developed in this paper could be used to value a firm in that industry solely on the basis of the individual fair values of its assets, independently of any business combination. This last contribution is particularly relevant for goodwill impairment tests, as the value of a reporting unit composed of interacting assets must be assessed at least annually. In goodwill impairment testing, the asset-based method developed in this paper could be used to value IGG instead of the overall approach required by the ASC (paragraphs 350-35-4 to 35-19).

The remainder of this paper is organized as follows. Section II provides background on IGG and existing valuation methods, as well as the research objectives. Section III describes the research design, demonstrates how Choquet capacities solve the aggregation issue and highlights the similarities and differences between the residual income model and our approach. Section IV describes the data and sample used to implement our model. Section V presents the empirical results and a performance comparison between the proposed valuation method and the residual income model in predicting enterprise value. Section VI concludes this study.

II. NATURE OF GOODWILL AND MEASUREMENT THEORY

IGG does not arise from an acquisition, but exists *per se*. However, we use acquisitions to collect the data (assets and liabilities at fair value) and implement the method that is the subject of this paper. The different concepts referred to in this paper and the difference between IGG and other forms of goodwill are presented below.

Internally Generated Goodwill

IGG is generally not recorded in the accounting system and exists independently of any business combination. However, it becomes part of the recognized accounting goodwill when the firm is acquired. The reporting requirements for an acquisition provide useful data. The price paid by the acquirer often exceeds the book value of net identifiable assets of the target. Using a bottom-up perspective (Johnson and Petrone 1998), i.e., starting from the book value, in accordance with Henning et al. (2000, 376), the resources acquired that were not reflected in the target's accounts but certainly have some value for the acquirer (see Figure 1) include:

1. Excess of the fair values over the book values of the target's recognized net assets, and fair values of other net assets not recognized by the target (Johnson and Petrone 1998), in practice generally resulting from "revaluation" (❶ in Figure 1).

2. Fair value of the “going concern” element of the target’s existing business, also called “internally generated goodwill” (Ma and Hopkins 1988, 77) or “going-concern goodwill” (Johnson and Petrone 1998; Henning et al. 2000) (❷ in Figure 1).
3. Fair value of synergies arising from combining the acquirer’s and target’s businesses and net assets. This is often called “combination goodwill” (Johnson and Petrone 1998, 296) or “synergy goodwill” (Henning et al. 2000) (❸ in Figure 1).⁶
4. Overvaluation of the consideration paid by the acquirer and overpayment (or underpayment) by the acquirer (Johnson and Petrone 1998). This last component is referred to as “residual goodwill” by Henning et al. (2000) (❹ in Figure 1).

This article focuses on component ❷ of Figure 1, which can be considered as pre-existing goodwill that was internally generated by the target. The value of this goodwill is “entirely dependent on the business as a going concern, with all of its assets interacting and combining with one another to earn the overall profit” (Lee 1971, 319).

Figure 1 summarizes the breakdown of the purchase price paid by the acquirer. The sum of components ❷, ❸ and ❹ represents the total goodwill, also called purchased goodwill or accounting goodwill (component ❺ in Figure 1). Figure 1 also presents computation of the main concepts used in this article (purchase price, fair value of assets, fair value of liabilities, enterprise value and IGG) and cites our data sources.

Insert Figure 1 About Here

Measurement Theory and Research Objectives

Figure 1 illustrates a computation method for IGG (“Enterprise value minus fair value of assets”, or “Market value of equity plus fair value of liabilities minus fair value of assets”) but does not explain the nature of IGG. The role of synergy between assets in formation of IGG raises some concerns about the additivity of fair values.

Discussion on the additivity of fair values is not new. Ijiri (1975, 93) comments that “the fair value measure is not additive. In the case of historical cost, the historical cost of resources *A* and *B* together is by definition the sum of the historical cost of *A* and the historical cost of *B*.”

⁶ See an earlier comment on the use of the term “synergy” in this paper. This “combination goodwill” relates to the synergistic effect of combining two firms.

Generally, we do not have this additivity in fair value. (...) The fair value of an entity is (...) known to be quite different from the sum of the fair values of the resources it owns”.

Ijiri (1975, 93) also observes that “goodwill presents a serious aggregation problem because the value of the whole is not necessarily equal to the sum of the values of its parts”, adding that this has been one of the oldest issues in accounting, discussed by Yang (1927), Canning (1929) and Paton and Littleton (1940), and in later periods by Gynther (1969) and Miller (1973).

The organization itself is non-additive in nature, as managers combine assets into firms to save transaction costs by organizing resources in a more efficient manner than the market (Coase 1937; Williamson 1983). The additive valuation of assets structured into a going concern cannot properly reflect the synergistic effect of this efficient organization of resources. A non-additive approach therefore seems more appropriate.

Basu and Waymire (2008, 171) enlighten understanding of these issues. They argue that “economic intangibles are cumulative, synergistic, and frequently inseparable from other tangible assets and/or economic intangibles”, and add that “it is usually futile to estimate a separate accounting value for individual intangibles.” It would therefore seem logical to appraise IGG as the value of the interactions between existing assets within a firm.

As IGG mainly arises from synergy between assets, our paper has three objectives: (1) From a theoretical perspective, to solve the problem of fair value’s non-additivity in valuing a firm; (2) To implement such a solution empirically; (3) To compare how a model based on non-additivity performs against a traditional additive model (e.g., Ohlson 1995).

III. RESEARCH DESIGN

Relaxing the Additivity Postulate

The Measurement Process

The object of measurement is to convey information about objects in a way that reflects economic reality. Let P be the set of principals p observed in reality, let S be the set of surrogates s intended to represent the principal. The function $m: P \rightarrow S$ is called the measurement process. From a normative standpoint, one desirable property of the function m is that it preserves the structure of the principals represented in the surrogates.⁷ Determining the nature of the object to be measured is thus critical, as it will determine the “minimal set of properties a function should

⁷ Ijiri calls such a measure *consistent* or *perfect* (1975, 42).

fulfill to be an aggregation function” (Grabisch et al. 2009, 1). To preserve the non-additive structure of the economic reality, other specifications of measures with other properties must be used (relaxing additivity), and a less restrictive property must be applied, as explained below (see also section II above).

Unsuitability of the Traditional Additivity Concept

Relaxing the additivity postulate is necessary to solve the aggregation issue mentioned by Miller (1973). The financial accounting system relies on this additivity concept. Asset valuation methods assume, for the sake of convenience, that the value of a set of N assets is equal to the sum of the values of its N components i.e., the overall value of the set equals the sum of the individual value of each asset.⁸

This standard financial arithmetic uses the mathematical notion of “measure”. In its general definition, this concept is based on several properties (see Appendix 1A),⁹ one of which is additivity: for all sets A (that are) disjoint from B , $m(A \cup B) = m(A) + m(B)$. This condition is a very strict constraint. It is one of the main assumptions underlying standard aggregation operators such as the Riemann (1857) and Lebesgue (1918, 1928) integrals.

The notion of “measure” is widely used in financial accounting. This approach to valuation places highly specific constraints on the view of the organization, and assumes there is no interaction between economic resources (assets). For instance, in the case of a firm using three assets A, B, and C, the measurement process in financial accounting is represented by the right-hand side of Figure 2 (ignoring the left-hand side example for the moment).

Insert Figure 2 About Here

In the numerical representation, it is implicitly assumed that assets A, B, and C interact with each other. As a result, this particular setting hypothesizes that the sum of the fair values of each asset is equal to the overall fair value of all assets taken together. To preserve the structure of economic reality, namely by representing interaction between assets, other specifications of measures must be used with other properties. This is achieved in particular by relaxing the additivity property, and using a monotonicity property. This leads us to use of Choquet capacities, which we describe below.

⁸ $V(\sum_{i=1}^N A_i) = \sum_{i=1}^N V(A_i)$.

⁹ In order not to interrupt the flow of the paper, we present all the mathematical calculations in Appendix 1.

The Enterprise as a Structured Set of Assets: Relaxing the Additivity Postulate

The additivity property, based on the hypothesis that the monetary value of the different items is interchangeable, seems intuitively justified. However, it proves particularly inappropriate in the case of the structured, specific-purpose set of assets that make up an organization (Casta and Bry 2003). “The optimal combination of assets (for example: brands, distribution networks, production capacities, etc.) is a question of managerial know-how and a key factor in the creation of intangible assets [like goodwill]. This is why the importance of a particular item in a set may vary depending on its position in the structure.” (p. 169). Its interaction with the other items may even be the source of value creation, such that the overall value of a set of assets may exceed the sum of the individual assets’ values.

To reconcile individual fair values with the overall value of the firm, it is necessary to use other “measures” that model interactions between the assets of a firm and measure the intensity of the relationship between sub-sets of assets. Consider a firm having a structured set X of three assets A, B, and C. We can graphically represent $P(X)$, the power set of X , in order to analyze the potential interactions (neutrality, synergy, inhibition) between the three assets (see the right-hand side of Figure 3, ignoring the left-hand side for the moment).

Insert Figure 3 About Here

As the real structure of the economic reality is unknown, the most general form is assumed in the numerical representation, which exhibits every possible interaction between the three assets A, B, and C. At each node of the lattice, the interaction between two or three components can be reflected through the non-additive measure μ (defined below). The function μ captures the nature of the interaction between two or more assets. In other words, this function must offer special attributes for modelling (1) neutrality, (2) synergy, and (3) inhibition between assets.

For the goal of non-additive firm valuation, Casta and Bry (1998, 2003) suggest using Choquet capacities (Choquet 1953) instead of m as the non-additive measure μ . Choquet capacities (see Appendix 1B) are a generalization of the measure concept. They allow non-additive aggregation (for a review of this approach in the field of multi-criteria analysis, see Grabisch and Labreuche 2010).

Choquet capacities respect a property called “monotonicity”, meaning that “adding a new item to a combination cannot decrease its importance” (Marichal 2002, 3). This is a less constraining property than the additivity presented above. Following from the monotonicity

property, for two disjoint sets A and B , Choquet capacities can behave as follows, depending on the modelling requirement:

- Additive: $\mu(A \cup B) = \mu(A) + \mu(B)$ (neutrality between assets)
- Over-additive: $\mu(A \cup B) > \mu(A) + \mu(B)$ (synergy between assets)
- Under-additive: $\mu(A \cup B) < \mu(A) + \mu(B)$ (inhibition between assets).

The definition of Choquet capacities requires the measures of all subsets of X to be specified, that is to say $2^n - 1$ capacities to be estimated.¹⁰ This makes it possible to identify all the interactions between a set of assets.

Non-Additive Aggregation and Firm Valuation

Non-additive, i.e., over (or under)-additive, aggregation appears to offer a relevant approach to assess the synergistic effect between assets that is the source of IGG. However, using the mathematical concept of non-additive aggregation for firm valuation requires certain definitions of the mathematical tools that will be implemented.

The Concept of Non-Additive Aggregation

The Choquet integral stems directly from the capacities presented above. It is a generalization of the integral to non-additive measures. According to Grabisch et al. (2009), the Choquet integral generalizes the Lebesgue integral (Lebesgue 1918, 1928) in the sense that the Choquet integral equals the Lebesgue integral when the capacities are additive (Marichal 2002) (see Appendix 1C).

As a result of monotonicity, the Choquet integral is increasing with respect to both the measure and the integrand. Hence, it can be used as an aggregation operator. This integral allows non-additive aggregation of a set of assets where interactions between sub-sets of assets create (or destroy) value.

Firm Valuation under a Non-Additive Approach

The following graphic illustration inspired by Murofushi and Sugeno (2000) provides a detailed presentation of the firm valuation method using the Choquet integral.

Graphic illustration of the additive approach. A firm has three assets A, B, and C ranked by order of increasing value. The fair values of these three assets are 100, 150, and 250

¹⁰ The number of items in $P(X)$ is equal to 2^n if X consists of n items.

respectively.¹¹ They can be represented by an *increasing simple function* f . The shaded area represents the overall fair value of the firm's assets under the additive approach (see the left-hand side of Figure 2).

The standard valuation approach requires computation of the area below the curve. The Lebesgue integral¹² (equation (1)) of this *simple function* of assets is:

$$\begin{aligned} V_L = L_{(f)} &= \sum_{i=0}^{n-1} (a_{i+1} - a_i) m(A_{i+1}) \\ &= (100 - 0) * m(A, B, C) + (150 - 100) * m(B, C) + (250 - 150) * m(C) \quad (1) \\ &= (100 - 0) * 3 + (150 - 100) * 2 + (250 - 150) * 1 = 500 \end{aligned}$$

where V_L represents the overall value of the assets based on the Lebesgue integral, $A_{i+1} = \{x \in X : f(x) \geq a_{i+1}\}$, and $m(A_i)$ is the Lebesgue measure of A_i , representing the length of the intervals.¹³

Graphic illustration of the Choquet capacity-based non-additive approach (see Figure 3, left-hand side). We now want to value the same firm, taking into account interactions between the assets. Choquet capacities can achieve this. A learning method for estimating the capacities is presented and implemented in the next paragraph, but first let us assume that we know the capacities (i.e., each μ) for this set of assets.

The Choquet capacities of the set of assets A, B, and C in this simplified example are:

- $\mu(A) = \mu(B) = \mu(C) = 1$;
- $\mu(A, B) = 2$; i.e., neutrality between assets A and B (because $\mu(A, B) = \mu(A) + \mu(B)$);
- $\mu(A, C) = 2$; i.e., neutrality between assets A and C (because $\mu(A, C) = \mu(A) + \mu(C)$);
- $\mu(B, C) = 1.5$; i.e., **25% inhibition** between assets B and C (because $\mu(B, C) < \mu(B) + \mu(C)$, (i.e., $\mu(B, C)/[\mu(B)+\mu(C)] = 1.5/2 = 0.75$, hence 25% of inhibition).

¹¹ $a_1 = 100, a_2 = 150, a_3 = 250. \Omega = \{A, B, C\}; f : \Omega \rightarrow [0, +\infty]$ ($f(A) = 100, f(B) = 150, f(C) = 250$).

¹² Equation A1 in Appendix 1C.

¹³ The common Riemann integral of that function gives the same result: $V_R = R_{(f)} = \sum_{i=1}^3 f(x_i) = 100 + 150 + 250 = 500$ where V_R represents the overall value of the assets based on the Riemann integral, and $x_i = A, B, C$, respectively.

- $\mu(A,B,C) = 4$; i.e., **33% synergy** between assets A, B, and C (because $\mu(A,B,C) > \mu(A) + \mu(B) + \mu(C)$, i.e., $\mu(A,B,C)/[\mu(A)+\mu(B)+\mu(C)] = 4/3 = 1.33$, hence 33% of synergy).

For a firm having three assets with a given ranking of fair values, only three Choquet capacities will be used to compute the overall firm value, because only three kinds of interaction are possible, as seen below. Three other interaction coefficients could be used for another firm with another ranking of assets. Consequently, considering many firms and every possibility, seven capacities are possible with three assets (see section V below). For example, if $f(A) < f(B) < f(C)$, $\mu(A,B,C)$, $\mu(B,C)$, and $\mu(C)$ are required. If $f(C) < f(A) < f(B)$, $\mu(C,A,B)$, $\mu(A,B)$, and $\mu(B)$ are required, etc.

Compared to the traditional additive approach, the non-additive method can be seen as an extension (in the case of synergy) or contraction (in the case of inhibition) of the x -axis length of the area associated with every fair value difference below the curve.

To represent the value of this structured set of assets graphically, the interaction value between assets A, B, and C (i.e., the synergy of 33%) is expressed by an extension of the x -axis length of the area associated with the 33% fair value difference of the three assets (i.e., from 3 to 4) resulting in the hatched area below the curve. Furthermore, the inhibition between assets B and C is represented by a contraction of the x -axis length of the area associated with the 25% fair value difference between assets B and C (i.e., from 2 to 1.5) resulting in the dotted area below the curve. Hence, the new curve is distorted by synergies and inhibitions between assets as expressed in Figure 3 (left side).

In short, the overall value of the firm calculated by a non-additive aggregation operator is equal to the shaded and hatched area below the new curve, i.e., the value of the Choquet integral. The capacities weigh the fair value differences for each combination of assets. The value of the Choquet integral¹⁴ (equation (2)) relative to the capacities for this set of assets is:

¹⁴ Equation A2 in Appendix 1C.

$$\begin{aligned}
V_C = C_{(f)} &= \sum_{i=0}^{n-1} (a_{i+1} - a_i) \mu(A_{i+1}) \\
&= (100 - 0) * \mu(A, B, C) + (150 - 100) * \mu(B, C) + (250 - 150) * \mu(C) \quad (2) \\
&= (100 - 0) * 4 + (150 - 100) * 1.5 + (250 - 150) * 1 \\
&= 400 + 75 + 100 = 575
\end{aligned}$$

where V_C represents the valuation based on the Choquet integral.

In this illustration, both the synergies between the assets A, B, and C, and the inhibition between assets B and C are recognized. This leads to a new value of 575 for the firm (compared to only 500 with the additive approach).

Choquet Capacities Learning Method¹⁵

As explained by Casta and Bry (2003, 172), “modelling through a Choquet integral requires construction of a measure which is relevant to the semantic of the problem”. Since the measure is theoretically non-divisible, it becomes necessary to define the value of $2^n - 1$ coefficients $\mu(A)$ where $A \in P(X)$. Similar to Grabisch (2008), we suggest an indirect econometric method based on a regression model to estimate the coefficients. In cases where the structure of the interaction can be defined approximately, it is also possible to reduce the combinatory part of the problem by restricting analysis of the synergy to the items contained in the useful subsets (see Casta and Bry 1998). Determining the Choquet capacities (that is to say $2^n - 1$ coefficients) involves a well-known problem for which many methods have been elaborated (Grabisch et al. 2008). We propose a specific indirect estimation method using a learning sample made up of firms for which we know the firm’s overall value and the individual value of each item in the set of assets.

Let us consider I firms described by their overall value V and a set X of J real variables x^j representing the individual value of each item in the assets. Let f_i be the function assigning every variable x^j its value for firm i $f_i : x^j \rightarrow x_i^j$. The aim is to determine a set of Choquet capacities μ in order to come as close as possible to the following relationship:

$$\forall i : C_{(f_i)} = EV_i \quad (3)$$

where EV_i is the Enterprise Value for firm i .

Let A be an element of $P(X)$ and $g_A(f_i)$ be the function called *generator relative to A* and defined for firm i as:¹⁶

¹⁵ This subsection is partly based on Casta and Bry (2003) and includes several new developments.

$$i \rightarrow g_A(f_i) = \int 1_{\{x: f_i(x) > y\}}(A) dy \quad (4)$$

The Lebesgue and Choquet integrals can then be written as in equations (5) and (6)¹⁷:

$$L_{(f)} = \sum_{A \in P(X)} g_A(f) * m(A) \quad (5)$$

$$C_{(f)} = \sum_{A \in P(X)} g_A(f) * \mu(A) \quad (6)$$

Thus, according to equations (3) and (6) we can write the following *econometric* model:

$$\forall i \ EV_i = \sum_{A \in P(X)} \mu(A) * g_A(f_i) + \varepsilon_i \quad (7)$$

where $\mu(A)$ is now a parameter and ε_i is a residual which must be minimized in the adjustment. It is possible to model this residual as a random variable or, more simply, to restrict calculations to an empirical minimization of the ordinary least squares type. The model given below is linear with $2^J - 1$ parameters: the $\mu(A)$ for all the subsets A of $P(X)$. The dependent variable is the enterprise value EV . The explanatory variables are the generators corresponding to the subsets of $P(X)$. A standard multiple regression provides the estimation of these parameters, that is to say the required set of capacities.

For each A item $P(X)$, we compute the corresponding generator under equation (4). In the discrete case, the generator functions are the difference between the value of assets $i+1$ and i . It should be noted that the suggested model is linear with respect to the generators, but obviously non-linear in the variables x^j .

The Residual Income Model

The objective of this and the next paragraph is to compare the residual income model, which will be used as a benchmark in the empirical section, and the synergy model based on non-additivity (“synergy model” in the rest of the article). Using the well-known dividend discount valuation model and clean surplus relation, the residual income model (e.g., Ohlson 1995) states the following relationship between market value of equity, book value of equity and expected abnormal earnings:

¹⁶ This expression represents the difference in fair values of ranked assets and corresponds to the figures (100 – 0), (150 – 100) and (250 – 150) in the example of equation 2.

¹⁷ See proof in Appendix 1D.

$$MV_t = BV_t + \sum_{\tau=1}^{+\infty} R^{-\tau} E_t[x_{t+\tau}^a] = BV_t + GW_t \quad (8)$$

Where:

MV_t : Market value at time t ;

BV_t : Book value at time t ;

R : 1 + cost of equity capital;

x_t^a : abnormal earnings in t defined as $x_t - (R - 1) * BV_{t-1}$

x_t : reported earnings in t ;

$E_t[.]$: the expectation operator in date t .

The market value of equity equals the firm's book value plus the present value of expected abnormal earnings. In this model the unrecorded goodwill appears as expressed in equation (8).¹⁸ To obtain the enterprise value (EV_t), the market value of debt can be added on both sides of equation (8) leading to equation (9):

$$MV_t + D_t = EV_t = (BV_t + D_t) + \sum_{\tau=1}^{+\infty} R^{-\tau} E_t[x_{t+\tau}^a] = TA_t + GW_t \quad (9)$$

where TA_t are total assets at fair value.

The Non-Additivity-based Synergy Model

The synergy model states that the enterprise value can be computed via the Choquet integral of the firm's assets using the appropriate set of capacities. For $A \in P(X)$, X being a set of assets, μ a set of Choquet capacities over $P(X)$, and $g_A(f)$ the generator relative to A , we have relationship (10):

$$C_{(f)} = EV_t = \sum_{A \in P(X)} \mu(A) * g_A(f) \quad (10)$$

Similarities and Differences between the Residual Income and Synergy Models

The non-additive approach can comprise similar relationships to the residual income model, though with the emphasis on interactions between assets instead of expectations of abnormal earnings as the source for IGG. As explained below, the residual income and synergy models express enterprise value as the sum of two components: the value of total assets and the value of

¹⁸ In a context of unbiased accounting (assets are recorded at fair value), the unrecorded goodwill equals the IGG.

the IGG. In the residual income model, IGG is measured based on the effect of interactions between assets on expected abnormal earnings. Conversely, in the synergy model, IGG is determined directly from the interactions between assets through the Choquet capacities.

Principles

We can also write equation (10) adding and subtracting a Lebesgue integral in the right-hand side, leading to equation (11) developed in equation (12):

$$EV_t = L_{(f)} + [C_{(f)} - L_{(f)}] \quad (11)$$

$$EV_t = \sum_{A \in P(X)} m(A) * g_A(f) + \sum_{A \in P(X)} [\mu(A) - m(A)] * g_A(f) \quad (12)$$

The first term represents the additive value of assets, whereas the second one indicates the value of the combination of assets. It can be positive (synergies generate value) or negative (inhibitions destroy value). This equation allows for differentiation between two components of enterprise value:

$$EV_t = L_{(f)} + [C_{(f)} - L_{(f)}] = TA_t + IV_t \quad (13)$$

In equation (13), the first term represents the **additive value of total assets** (TA_t), and the second term the **value of interactions between assets** (IV_t). Hence, the same relations as in the Ohlson model can be expressed:

$$EV_t = MV_t + D_t = TA_t + IV_t \quad (14)$$

$$MV_t = TA_t - D_t + IV_t \quad (15)$$

$$MV_t = BV_t + IV_t = BV_t + GW_t \quad (16)$$

The fair equity market value of the firm is its equity book value plus the value of interactions generated by combination of assets at time t . In the above equation (16), the goodwill emerges formally, as it does in the residual income formula, but with an essential difference: the value of the goodwill is directly generated by interaction between assets, not by expected discounted abnormal earnings. Table 1 summarizes the similarities and differences between the residual income and synergy models.¹⁹

Insert Table 1 About Here

¹⁹ For a numerical example, see Appendix 2.

IV. SAMPLE AND DATA

Our valuation model is based on the concept of interactions between assets. As interaction between assets can vary from one sector to another, we decided to focus on a specific economic sector where the role of synergies between assets can be assumed to be important in the value creation process. We obtained our sample from the deals analysis database of Thomson One Banker covering the period 2002-2009 with the following criteria: (1) The deal has a value of at least \$100 million;²⁰ (2) Both the target and the acquirer are listed U.S. firms; (3) The deal has been completed; (4) The target macro-industry is high technology.²¹ We chose to study the high technology sector (macro-industry: HT in Thomson One Banker), because this sector had the highest number of acquisition deals after the financial industry. The financial industry was not considered due to standard finance theory assumptions that the benefits from interactions between assets are already priced (a diversification effect) (Markowitz 1952; Sharpe 1964). This is also consistent with the assumption in Feltham and Ohlson (1995, 694) with regards to financial assets: “their book and market values coincide to equal fa_t [financial assets]”. The healthcare industry was also studied and results are qualitatively similar,²² but other industries were not studied because the number of acquisitions was too small.

180 business combinations between 2002 and 2009 met these criteria. Acquirers’ 10-Q or 10-K reports (depending on the date of acquisition), available from the SEC EDGAR database, were used to obtain the purchase price allocations of these business combinations. The purchase price is allocated between current, tangible, and identifiable intangible assets, with the level of detail varying from one firm to another. The advantage of using assets’ fair values as estimated in purchase price allocations is that some intangible assets are identified only in business combinations, leading to more extensive recognition of intangible assets. Due to insufficient and missing disclosures in 10-Q and 10-K reports, the final sample comprised 101 high technology sector firms for which fair values of assets and liabilities were available.

²⁰ A purchase price in excess of \$100 million increases the likelihood of finding relevant data in the acquirer’s 10-K/Q.

²¹ Acquisitions meeting these criteria were distributed between the different macro-industries as follows: finance (223), high technology (180), healthcare (133), energy and power (61), industrials (56), materials (48), consumer products and services (43), telecommunications (42), real estate (37), media and entertainment (36), consumer staples (30), retail (29), and government and agencies (1).

²² Results are available from the authors upon request.

As explained in section III, the Choquet capacities learning method requires a known fair enterprise value in order to infer a set of capacities. In practice, we need the fair values of assets and debts, and the market value of equity. Fair values of assets and debts are not generally observable for every firm due to historical cost accounting. As Figure 1 shows, outside the context of an acquisition (business combination), only the book values of assets and liabilities (components ⑥ and ⑥') are available from the published annual report. The market value of equity as a stand-alone entity (component ⑤) can be considered equivalent to the market value of the firm (Johnson and Petrone 1998, 296). This value is available if the firm is listed on a stock market. To avoid including any market control premium, we collected the market values of the target firms' equity seven trading days prior to the acquisition announcement (Henning et al. 2000), as stated in Datastream.

The other components of Figure 1 are known if the studied firm (the target) has been acquired. In that case, the price paid (component ④), or "purchase price" (Henning et al. 2000), becomes available in the acquirer's annual report. Since Financial Accounting Standard 141 (FASB 2001) was released, U.S. acquiring entities have been required to allocate the price of an acquired entity to the assets acquired and liabilities assumed based on their estimated fair values at the date of acquisition, and to disclose this "purchase price allocation" (PPA) in the notes to their financial statements. This obligation was unchanged in the revised version of FAS 141 (FASB 2007) and is now included in the Accounting Standards Codification (ASC) as paragraphs 805-10-50 and 805-30-50 (see Appendix 3). The PPA states the fair values of identifiable assets (③) and liabilities (②), including intangible assets. In conjunction with the market value of equity (⑤), the IGG can be deduced ($② = ⑤ + ② - ③$). Following Henning et al. (2000), we used business combination fair value estimates at the acquisition date to obtain the fair value of the firm's assets and liabilities.²³

To implement the model, we decided to regroup the fair values of identified assets into three broad categories of assets: current assets (accounts receivable, cash or cash equivalents, other

²³ We are aware that Purchase Price Allocations and disclosures may be subject to managerial discretion and may constitute a bias estimate of assets' fair values (Shalev 2009). However, we believe that this methodology leads to better estimates and recognition of the fair value of assets and liabilities than the use of book values.

current assets, tax assets), tangible assets (PP&E, other non-current assets) and intangible assets (completed technologies, customer relationships and trade names and trademarks). In-process research and development (IPR&D) was excluded from intangible assets because of the change in accounting treatment (from expensing to capitalization of IPR&D) resulting from the revision of SFAS 141 during our period of investigation. However, as a robustness check, we reran all statistical treatments including IPR&D in intangible assets. Results are qualitatively similar. Table 2, Panels A and B, presents the descriptive statistics of our sample.

Insert Table 2 About Here

The IGG is the most important component of enterprise value in our sample, accounting for a mean of 34.3% of total enterprise value (median of nearly 33.8%). Current assets represent the second-largest component of enterprise value, accounting for a mean of 31.5% (29.8%). Predictably for the high-technology sector, the mean value of identified intangible assets represents a significant percentage of enterprise value: 25.3% (24.2%). Finally, tangible assets represent 9.0% of enterprise value (4.7%).

The model theoretically requires that each asset's individual fair value should be known. However, even when assets are restated to fair value, the gap between the overall fair value of total assets and the enterprise value is still large (around 35% of enterprise value). This suggests that part of the enterprise value stems from unexplained factors which we believe could arise from synergies between assets.

In section V, we use a residual income model as a benchmark to test the relative performance of the synergy model. To compute the variables needed to implement the model, following Dechow et al. (1999), we require the book value of equity, cost of capital, earnings, and analysts' estimates of earnings. Book values and earnings before extraordinary items were collected from Compustat annual data, market values from Datastream, and earnings forecasts from I/B/E/S. Costs of capital were obtained using a CAPM approach, with a 5-year beta and the implied equity premium available from Damodaran²⁴ for the U.S. market. Table 2, Panel C, summarizes these panel data variables. No analyst coverage was found for 16 firms, reducing the size of the sample for the model including that variable.

²⁴<http://pages.stern.nyu.edu/~adamodar/>.

V. EMPIRICAL RESULTS

Descriptive Statistics of Explanatory Variables: Generator Functions

We compute generator functions under equation (4) for each of the 101 firms. This is simply a different way of describing a set of assets for a firm that is practical for estimating the capacities. As this equation is important, let us consider one firm in our sample: DataDomain Inc, having the following set of assets (in millions \$): tangible assets (40.46); current assets (81.73) and intangible assets (357.90). Figure 4 represents this set of assets graphically:

Insert Figure 4 About Here

Hence, according to equation (4), we derive the value of the generators for DataDomain Inc. as:

$$g_{TA,CA,IA} = \int \mathbf{1}_{\{x:f(x)>y\}}(TA, CA, IA)dy = \int_0^{40.46} \mathbf{1}_{\{x:f(x)>0\}}(TA, CA, IA) = 40.46$$

$$g_{CA,IA} = \int \mathbf{1}_{\{x:f(x)>y\}}(CA, IA)dy = \int_{40.46}^{81.73} \mathbf{1}_{\{x:f(x)>40.46\}}(CA, IA) = 81.73 - 40.46 = 41.27$$

$$g_{IA} = \int \mathbf{1}_{\{x:f(x)>y\}}(IA)dy = \int_{81.73}^{357.90} \mathbf{1}_{\{x:f(x)>81.73\}}(IA) = 357.90 - 81.73 = 276.17$$

the other generators being equal to 0.²⁵

Table 3 reports descriptive statistics of generator functions, calculated in the same way for the total sample, then scaled by total assets to standardize generators and avoid them being driven by large firms alone.

Insert Table 3 About Here

Estimation of Choquet Capacities

Under the learning procedure presented in section III and equation (7), the following model is applied to the sample in order to estimate the Choquet capacities:

$$EV_i = \mu_1 g_{TAi} + \mu_2 g_{CAi} + \mu_3 g_{IAi} + \mu_4 g_{TA,CAi} + \mu_5 g_{CA,IAi} + \mu_6 g_{TA,IAi} + \mu_7 g_{TA,CA,IAi} + \varepsilon_i \quad (17)$$

Estimation of the set of Choquet capacities for the sample is reported in Table 4.

Insert Table 4 About Here

Table 4 shows the values of every sub-set of assets in the structure.²⁶ We do not report the standard error and therefore p-value of the estimate for the capacity $\mu(TA,CA)$ as the

²⁵ Only three generators and three capacities are computed for one firm, since there are only three classes of assets. In the example of DataDomain, $TA < CA < IA$, hence the computation of $g_{TA,CA,IA}$, $g_{CA,IA}$ and g_{IA} .

²⁶ The adjusted R^2 is reported although it has no relevance in a regression without an intercept.

monotonicity constraint is binding for this capacity (see note below table 4). We interpret the estimated capacities in the next paragraph.

Interpretation of Results: Effect of Interactions Between Assets in the HT Sector

Let $g(A_i)$ be the \$ fair value of a generator related to a set of assets (one or more assets) recorded in the accounting system, i.e., the \$ amount of the combination of assets in the balance sheet, and $\mu(A_i)$ the appropriate capacity for this combination. The enterprise value is equal to the Choquet integral described in equation (7).

With three assets (TA , CA , and IA), remembering that a capacity in a decision-making context is a “weight related to a subset of criteria” (Marichal 2000), we can explain the economic meaning of a capacity in the following way:²⁷

- $\mu(TA) = 0.713$ means that \$1 of tangible assets recorded in the accounting system (alone) contributes 71.3 cents of enterprise value;
- $\mu(CA) = 2.097$ means that \$1 of current assets recorded in the accounting system (alone) contributes \$2.097 of enterprise value;
- $\mu(IA) = 2.303$ means that \$1 of intangible assets recorded in the accounting system (alone) contributes \$2.303 of enterprise value;
- $\mu(TA, CA) = 2.097$ means that \$2 of tangible assets combined with current assets recorded in the accounting system contribute \$2.097 of enterprise value;
- $\mu(CA, IA) = 3.262$ means that \$2 of current assets combined with intangible assets recorded in the accounting system contribute \$3.262 of enterprise value;
- $\mu(TA, IA) = 2.543$ means that \$2 of tangible assets combined with intangible assets recorded in the accounting system contribute \$2.543 of enterprise value;
- $\mu(TA, CA, IA) = 4.845$ means that \$3 of tangible assets combined with current assets and intangibles assets recorded in the accounting system contribute \$4.845 of enterprise value.

The estimated capacities can also be considered in terms of the marginal contributions of a particular subset to enterprise value. For two companies with the following asset structure (TA , CA , IA):

²⁷ For a subset $A \in P(X)$ the associated Choquet capacity can be written: $\mu(A) = \int 1_A \mu(A)$. In other words, it provides the value of the integral when the subset equals 1 and all the other subsets equal 0.

- Firm 1: (80, 200, 500)
- Firm 2: (80, 200, 501)

The contribution of the extra dollar invested in intangible assets for firm 2 as compared to firm 1 is:

$$EV_2(80, 200, 501) - EV_1(80, 200, 500) = \mu(IA) = \$2.303$$

The same analysis can be performed for all the other combinations (e.g. $\mu(TA, CA, IA) = 4.845$ means that the marginal contribution of investing \$3 in a combination of *TA*, *CA* and *IA* is \$4.845, everything else being equal).

The interpretation of the estimated capacities for single assets must be carefully considered when judging the contributions of asset classes to enterprise value, since single assets interact at higher levels. If the capacity of a set of assets consisting of only one asset is below one, that does not necessarily mean that the asset does not contribute significantly to enterprise value. It is possible that the asset contributes to enterprise value when combined with other classes of assets. An asset can have a low contribution to enterprise value alone, but be very valuable in combination with other assets.

Table 5 presents an interpretation of the capacities in terms of synergies/interactions among asset classes.

Insert Table 5 About Here

In Table 5, the signs of interpretation displayed in the right-hand column show that no synergy appears at the dual combination level. Synergies are only generated between all three categories of assets: between [tangible assets and current assets] and intangible assets, between [current assets and intangible assets] and tangible assets and also between [tangible assets and intangible assets] and current assets. Inhibition exists between all three categories of assets, considered separately. The synergies outweigh inhibitions because their size, as measured by the product of the capacities (see Table 4) and the corresponding generators (summarized in Table 3), outweighs the size of inhibitions (measured in the same way). On average, they generate an overall positive value (i.e., IGG).

Performance of the Model

Cohen et al. (2009) argue that asset-pricing models should be evaluated by their ability to provide estimates close to the current stock price. Hence, inspired by Barth et al. (2005), who implement by-industry out-of-sample predictions of the residual income model, we decided to

focus on the accuracy of the synergy model in predicting out-of-sample enterprise value given a set of Choquet capacities and the fair asset values of firms. The following jackknifing procedure was implemented to generate contemporaneous out-of-sample enterprise value predictions for each firm without using that firm's data to generate its predicted equity value.

- (i) Model (17) is estimated on (N-1) firms, to generate a set of Choquet capacities;
- (ii) The firm's enterprise value not used in the sample for the learning procedure is estimated on the basis of the fair values of the assets and the set of capacities estimated in step (i) with model (17);
- (iii) We compare the enterprise value estimation with the actual enterprise value;
- (iv) We repeat this procedure for the N firms in the sample.

The prediction error metric employed is the absolute percentage error (AE):

$$AE = \text{abs}(EV_{it} - \text{predicted } EV_{it})/EV_{it} \quad (18)$$

The performance of the model, as tested by this procedure, is reported in the first line of Table 6, Panel A.

Insert Table 6 About Here

The synergy model diverges from the true enterprise value by a median value of 27%. The mean prediction error (31%) is slightly higher. To the best of our knowledge, there is no literature on out-of-sample prediction of enterprise value. However, a comparable error level was noted on out-of-sample predictions of equity values by Barth et al. (2005, 331-332) with a residual income model at the industry level, similar to those obtained by Nekrasov and Schroff (2009, 1997) at the industry level for their fundamental risk-adjusted residual income model.²⁸ To test the superiority of the synergy model, we run a comparative procedure below.

Relative Performance of the Synergy Model Compared to the Residual Income Benchmark

In order to estimate the predictive power of the synergy valuation model, similar to Barth et al. (2005) and Nekrasov and Schroff (2009), we decided to benchmark our model with the residual income valuation model, because of its accounting-based nature and the two models' similar IGG valuation (although deriving from different methods as explained in section III). Dechow et al. (1999) provide an empirical implementation of this class of model based on the

²⁸ We compare our results to the findings of these two papers because the authors provide out-of-sample prediction errors of different versions of the Ohlson model. However, our approach focuses on enterprise value that merely corresponds to the sum of equity value and total debt, whereas Barth et al. (2005) and Nekrasov and Schroff (2009) focus solely on equity value.

Ohlson (1995) model. Using three assumptions: the dividend-discount model (19), the clean surplus relation (20), and the abnormal earnings dynamics (21 and 22), Dechow et al. (1999) derive equation (23):

$$P_t = \sum_{\tau=1}^{\infty} R^{-\tau} E_t[d_{t+\tau}] \quad (19)$$

$$BV_t = BV_{t-1} + x_t - d_t \quad (20)$$

$$x_{t+1}^a = \omega x_t^a + v_t + \varepsilon_{1,t+1} \quad (21)$$

$$v_{t+1} = \gamma v_t + \varepsilon_{2,t+1} \quad (22)$$

$$MV_t = \alpha_0 + \alpha_1 BV_t + \alpha_2 x_t^a + \alpha_3 v_t + \varepsilon_t \quad (23)$$

Where:

d_t : Dividend flow at time t ;

R : 1 + cost of equity capital;

x_t : Earnings at time t ;

x_t^a : Residual income at time t ;

BV_t : Book value at time t ;

v_t : Other information at time t ;

ω : Auto-regressive coefficient of abnormal earnings dynamics, $\omega \in [0;1[$;

γ : Auto-regressive coefficient of other information dynamics, $\gamma \in [0;1[$;

To compute v_t , following Dechow et al. (1999), we use the difference between expected abnormal earnings for period $t + 1$ and expected abnormal earnings based only on current period abnormal earnings:

$$v_t = E_t[x_{t+1}^a] - \omega x_t^a \quad (24)$$

The period t conditional expectation of period $t + 1$ earnings can be measured using the median consensus analyst forecast of period $t + 1$ earnings, denoted f_t . This gives:

$$E_t[x_{t+1}^a] = f_t^a = f_t - (R - 1) * b_t \quad (25)$$

The other information can thus be measured as:

$$v_t = f_t^a - \omega x_t^a \quad (26)$$

Values for R , ω must be established. For the discount factor R , we used the CAPM model to determine the appropriate cost of capital for firms (Sharpe 1964), under the traditional formula:

$$k_{i,t} = r_{f,t} + \beta_i(r_{prem,t}) \quad (27)$$

The U.S. annual equity market premium was provided by Damodaran and the risk-free rate was proxied by the T-bond rate. This gave a specific cost of equity capital for each firm and each year.

We determined ω value as the first order autoregression coefficient for abnormal earnings, estimated using a pooled time-series cross-sectional regression from 1975 for the earliest data to 2009 in our sample. The persistence coefficient was estimated at 0.425 (p-value of 0.000). The remaining variables are then easily computed using the equations presented above.

Equation (23) is estimated on the exact same sample as the synergy model. To construct the “other information” variable, two consecutive analyst forecasts are necessary, reducing the size of the sample for that variable from 85 to 79. Consequently, 22 firms (with only one or no analyst forecasts) from the initial sample were not included in the regression integrating the “other information” variable. We also run the model without this variable on the entire sample, to observe the relative impact of reduction of the sample compared to the new right hand-side variable.

The accuracy of the Ohlson model was thus tested and compared to the synergy model using the jackknifing procedure presented in section V. Table 6 reports the performances of the Ohlson and synergy models in predicting enterprise values. As the residual income model predicts equity values, the total value of debts is added to the estimated values of equity. To be able to compare the models, the out-of-sample percentages of error of the two models were computed with expression (18) on the exact same samples for which all data were available. To assess the statistical significance of differences in prediction errors, we compared the mean and median for absolute percentage error (*AE*). For tests comparing means we used a standard paired t-test, and for medians, we used the Wilcoxon matched-pairs signed-rank test.

The synergy model clearly outperforms the Ohlson model in terms of central predictions (mean and median). The differences are statistically significant both in terms of mean and median.²⁹

The predictive power of the Ohlson model integrating the “other information” variable slightly improves the model by a mean 2% (43% vs. 41%). The loss in the sample size is

²⁹ In the healthcare industry, results are qualitatively similar.

outweighed by the predictive power of the other information variable. Inclusion of this last variable also greatly reduces dispersion of the predictions (with standard deviation dropping from 43% to 27%). Table 6 also clearly indicates that the predictive power of the synergy model is higher in terms of central predictions of enterprise value as judged by mean and median errors.

VI. DISCUSSION AND CONCLUSION

IGG arises from interactions between assets generating synergies that create abnormal profitability for the firm. Existing valuation methods compute the present value of abnormal earnings (i.e., residual income models) or measure this value indirectly by subtracting the fair value of assets identified in a business combination from the enterprise value. The drawback of these approaches is that they do not explain how goodwill is created. As a result a paradox emerges: *internally* generated goodwill is evaluated with measures that focus on *external flows* (i.e., abnormal flows). This paradox is consistent with the aggregation issue identified by Miller (1973): goodwill emerges from an “inappropriate theory of aggregation of assets.” Prior literature focuses on external flows because additive measurement is appropriate in that case, yet application of this approach to assets is impossible as the fair value of a set of assets is not additive. This paper sets out to solve this paradox.

Valuation of IGG based on synergies between assets identified by Choquet capacities offers an interesting approach that can solve this aggregation issue. This method is consistent with the fact that the fair value of a set of assets is not additive, and goodwill results from positive interactions between assets. To assess the validity of this approach, we use the Ohlson model as a benchmark. We compare the accuracy in predicting enterprise value and conclude that the synergy model outperforms the Ohlson model.

Using a non-additive aggregation method based on Choquet capacities opens up an interesting field where interactions between assets can be modeled as well as the effect of the structure on firm value. However, the method presented and implemented in this study suffers from certain limitations.

Even in a business combination (the source of our data), some intangible assets may still remain unidentified. As we value IGG only by interactions between assets, the role of synergies may be overstated if there are unidentified intangible assets or, more generally, understated assets. However, this under-estimation of assets will also distort the estimated coefficients in the Ohlson model.

Implementing Choquet capacities requires specification of every interaction between sub-samples of a set of assets, that is to say $2^n - 1$ interaction coefficients. This can be complex to implement and interpret. However, it is possible to group the assets into major classes (assuming no interaction within each of these classes) and some methods exist to limit the order of interactions (i.e., using 2-additive capacities instead of k-additive capacities, see Miranda et al. (2005)).

The additivity assumption is a stabilized implicit hypothesis that is often unintentionally accepted in financial accounting. It provides an “invisible” management instrument (Hatchuel and Weil 1995), bounding the representation of organizations in a specific view. By relaxing the additive postulate, this article not only proposes a new way to measure IGG but also opens the debate on the role of additivity in management.

APPENDIX 1

Mathematical Developments

1A.1 – Definition of a (probability) measure

Given a measure space (Ω, Z) where Ω is a set and Z a sigma-algebra.³⁰ The function $m: Z \rightarrow [0, +\infty]$ is called a measure if it satisfies the following properties:

- (i) null empty set: $m(\emptyset) = 0$;
- (ii) if $(A_n)_n \in Z$ pairwise disjoint, then $m(\bigcup_n A_n) = \sum_n m(A_n)$

If $m(\Omega) = 1$, then m is called a probability measure.

1A.2 Example

To illustrate the notion of a measure as defined above in 1A.1, let us take the example of a firm with three different assets A, B, C (in a mathematical context, these three assets are assumed to be pairwise disjoint). In this particular case the basic set Ω consists of the assets A, B, C , that is $\Omega = \{A, B, C\}$. Let the sigma-algebra Z be the power set of Ω , i.e.

$$Z = \{\{A\}, \{B\}, \{C\}, \{A, B\}, \{B, C\}, \{A, C\}, \{A, B, C\}, \emptyset\}$$

Z illustrates how any asset can be combined with any other asset of Ω . Furthermore, we assume the standard additivity property normally applied in accounting theory. This additivity property is represented by a measure as defined above. Therefore, let

$m: Z \rightarrow [0, +\infty]$ be such a measure on Ω satisfying properties (i) and (ii) as defined in 2A.1. Since $\{A\}, \{B\}, \{C\}$ are three pairwise disjoint assets, property (ii) gives: $m(\bigcup_n A_n) = m(A \cup B \cup C) = m(\Omega) = m(A) + m(B) + m(C)$.

³⁰ A sigma-algebra Z is a subset of the power set of Ω , $P(\Omega)$, with the following properties:

- (i) $\Omega \in Z$
- (ii) If $A \in Z$, then $A^c (= \Omega \setminus A) \in Z$
- (iii) If $(A_n)_n \in Z$, then $\bigcup_n A_n \in Z$.

This demonstrates that using standard (Lebesgue) measures results in the standard additivity property assumed in accounting, i.e., the value of a sum of assets equals the sum of the values of each asset.

1B.1 – Definition of Choquet capacities

Given a measure space (Ω, Z) and letting Z be $P(\Omega)$, the function $\mu: Z \rightarrow [0, +\infty]$ is called *Choquet capacity* if it satisfies the following properties (see the formalization in Grabisch et al. 2008; Grabisch et al. 2009, 172-177):

- (i) null empty set: $\mu(\emptyset) = 0$;
- (ii) monotonicity: For all $A, B \in Z$ with $A \subset B$: $\mu(A) \leq \mu(B)$.

1B.2 – Example

To illustrate the properties of a Choquet capacity, let us take the same example as in 1A.2. Given three disjoint assets A , B and C and a Choquet capacity instead of a Lebesgue measure, property (ii) in 1B.1 gives the following:

$$\mu\left(\bigcup_n A_n\right) = \mu(A \cup B \cup C) = \mu(\Omega) \leq \mu(A) + \mu(B) + \mu(C).$$

Hence, the value of a sum of assets is not necessarily the sum of the values of each asset (additivity). Synergies or inhibitions can thus be modeled between a combination of assets.

1C.1 – Definition of the Lebesgue integral in the finite case

Given a measure space (Ω, Z) and assuming that $\Omega = \{w_1, \dots, w_n\}$ ($w_i < w_j, \forall i < j$), let $f: \Omega \rightarrow [0, +\infty]$ be a simple function³¹ taking values a_1, \dots, a_N with $(a_i < a_j, \forall i < j, f(w_i) = a_i)$ and $A_n = \{w \in \Omega : f(w) = a_n\}$. Furthermore, let m be the Lebesgue measure. m satisfies all properties of a measure as defined in 2A.1, i.e.,

$$m(\{w \in \Omega^n : a_k \leq w_k < b_k, k = 1, \dots, N\}) = (b_1 - a_1) * \dots * (b_N - a_N)$$

The Lebesgue integral of f is thus defined as:

³¹ Let $f: \Omega \rightarrow [0, +\infty]$ be a simple function, i.e., f can be written as $f(x) = \sum a_n \mathbf{1}_{A_n}(x)$ whereas $\mathbf{1}_A \{x\} = 1$ if $x \in A$ and $\mathbf{1}_A \{x\} = 0$ if $x \notin A$. $\mathbf{1}$ is called the “indicator function” and a_n ($n = 1, \dots, N$) are increasing distinct values $\in R^+$ and $A_n = \{w \in \Omega : f(w) = a_n\}$. Representing a set of assets by a simple function makes it possible to compute the value of a firm as the area under the curve, as represented in section III.

$$L_{(f)} = \int_{\Omega} f dm = \sum_{k=0}^{N-1} (a_{k+1} - a_k) m(A_{k+1}) \quad (\text{A1})$$

with $A_{k+1} = \{w \in \Omega : f(w) \geq a_{k+1}\}$ and $a_0 = f(w_0) = 0$

1C.2 Definition of the Choquet integral in the finite case

Given a measure space (Ω, Z) with $\Omega = \{w_1, \dots, w_n\}$ ($w_i < w_j, \forall i < j$), let $f : \Omega \rightarrow [0, +\infty]$ be a simple function as above. Furthermore, let μ be a set of Choquet capacities (on (Ω, Z)). The Choquet integral of f with respect to μ is thus defined as:

$$C_{(f)} = \int_{\Omega} f d\mu = \sum_{k=0}^{N-1} [a_{k+1} - a_k] \mu(A_{k+1}) \quad (\text{A2})$$

whereas $A_{k+1} = \{w_{k+1}, \dots, w_N\}$ and $a_0 = f(w_0) = 0$.

In the finite case presented above, it is easy to see that the Choquet integral is a generalization of the standard Lebesgue integral. Besides, if capacity μ is additive, then the Choquet integral reduces to a Lebesgue integral (Marichal 2002). The Choquet integral extends the Lebesgue integral to possibly non-additive measures.

1D – Proof of Equations (5) and (6)

Let $\mathbf{1}_A(B)$ be the indicator function which takes value 1 if $B \in A$ and 0 otherwise. We can thus rewrite equations (1) and (3) as in equation (A3) and (A4):

$$L_{(f)} = \int \left(\sum_{A \in P(X)} \mathbf{1}_{(A=\{x: f(x) > y\})} \right) dy * m(A) \quad (\text{A3})$$

$$C_{(f)} = \int \left(\sum_{A \in P(X)} \mathbf{1}_{(A=\{x: f(x) > y\})} \right) dy * \mu(A) \quad (\text{A4})$$

The expression of the Lebesgue integral (A3) and the Choquet integral (A4) are equivalent to equation (A5) and (A6) respectively:

$$L_{(f)} = \sum_{A \in P(X)} \left(\int \mathbf{1}_{(A=\{x: f(x) > y\})} dy \right) * m(A) \quad (\text{A5})$$

$$C_{(f)} = \sum_{A \in P(X)} \left(\int \mathbf{1}_{(A=\{x: f(x) > y\})} dy \right) * \mu(A) \quad (\text{A6})$$

If we denote $g_A(f)$ as the value of the expression $\int \mathbf{1}_{(A=\{x, f(x)>y\})} dy$, the Lebesgue integral may be expressed as stated in equation (A7) and the Choquet integral as in equation (A8):

$$L_{(f)} = \sum_{A \in P(X)} g_A(f) * m(A) \quad (\text{A7})$$

$$C_{(f)} = \sum_{A \in P(X)} g_A(f) * \mu(A) \quad (\text{A8})$$

APPENDIX 2

Residual Income Model and Synergy Model: A Numerical Example

Let us take the same example firm as in section III, with total asset fair values of 500, equity book value of 300, and market fair value of debts of 200. We also assume that the fair enterprise value of this firm is 575. Finally, the fair market values of the three assets (A, B and C) of this firm, as above are 100, 150, and 250 respectively.

Residual Income Model. This model, applied to this firm, leads to the formula expressed in equation (9) (see above). Assuming that the market is efficient, the present value of expected abnormal earnings will equal the excess fair enterprise value over the total fair asset value: 75. This is also the value of IGG. We thus have the following relationship:

$$EV_t = 300 + 200 + 75 = 575$$

Notice that if there were no reference to the fair enterprise value, assumptions on the expected abnormal earnings dynamics would have been required to compute IGG (expected present value of abnormal earnings), whereas in the synergy model, we only require the fair market values of assets and the appropriate Choquet capacities.

Synergy Model. This model, applied to this firm, will give the following results, using the Choquet integral (equation (2)) with the same set of capacities as in the example above. As expressed in equation (13), we can split enterprise value between the book value of assets and the interaction value as an expression of Choquet and Lebesgue integrals.

Taking into account the value of the generators, the total value of the assets TA_t is:

$$TA_t = L_{(f)} = \sum_{A \in P(X)} m(A) * g_A(f) \quad (A9)$$

$$\begin{aligned} TA_t &= m(A,B,C) * g_{A,B,C} + m(B,C) * g_{B,C} + m(C) * g_C \\ &= [3] * 100 + [2] * 50 + [1] * 100 = 300 + 100 + 100 = 500 \end{aligned}$$

And the value of the interaction between assets IV_t is:

$$IV_t = \sum_{A \in P(X)} [\mu(A) - m(A)] * g_A(f) \quad (A10)$$

$$IV_t = [4 - 3] * 100 + [1.5 - 2] * 50 + [1 - 1] * 100 = 75$$

Hence, equation (13) results in the following expression:

$$EV_t = TA_t + IV_t = 500 + 75 = 575$$

This expression can be also rewritten: $EV_t = BV_t + D_t + IV_t = 300 + 200 + 75 = 575$.

As in the residual income model, the goodwill equals 75.

APPENDIX 3
Disclosure Requirements (FASB)

Paragraph 805-30-50 of the Accounting Standards Codification (ASC) states that the “acquirer shall disclose (...) the acquisition-date fair value of the total consideration transferred and the acquisition-date fair value of each major class of consideration, such as the following:

1. Cash
2. Other tangible or intangible assets, including a business or subsidiary of the acquirer
3. Liabilities incurred, for example, a liability for contingent consideration
4. Equity interests of the acquirer, including the number of instruments or interests issued or issuable and the method of determining the fair value of those instruments or interests” (§ 805-30-50-1).

We provide below an excerpt from an illustration given by the FASB in paragraphs 805-10-55-37 to 41.

Paragraph 805-10-55-38, Accounting Standards Classification (FASB)

On June 30, 20X0, Acquirer acquired 15 percent of the outstanding common shares of Target. On June 30, 20X2, Acquirer acquired 60 percent of the outstanding common shares of Target. Target is a provider of data networking products and services in Canada and Mexico. As a result of the acquisition, Acquirer is expected to be the leading provider of data networking products and services in those markets. It also expects to reduce costs through economies of scale.

Paragraph 805-10-55-39, ASC

The goodwill of \$2,500 arising from the acquisition consists largely of the synergies and economies of scale expected from combining the operations of Acquirer and Target. All of the goodwill was assigned to Acquirer's network segment.

Paragraph 805-10-55-41, ASC

At June 30, 20X2	\$
Consideration	
Cash	5,000
Equity instruments	4,000
Contingent consideration arrangement	<u>1,000</u>
Fair value of total consideration transferred	10,000
Fair value of acquirer's equity interest in Target held before the business combination	<u>2,000</u>
	<u>12,000</u>
Acquisition-related costs (including in selling, general, and administrative expenses in Acquirer's income statement for the year ending December 31, 20X2)	1,250
Recognized amounts of identifiable assets acquired and liabilities assumed	
Financial assets	3,500
Inventory	1,000
Property, plant, and equipment	10,000
Identifiable intangible assets	3,300
Financial liabilities	(4,000)
Liability arising from a contingency	<u>(1,000)</u>
Total identifiable net assets	12,800
Noncontrolling interest in Target	(3,300)
Goodwill	<u>2,500</u>
	<u>12,000</u>

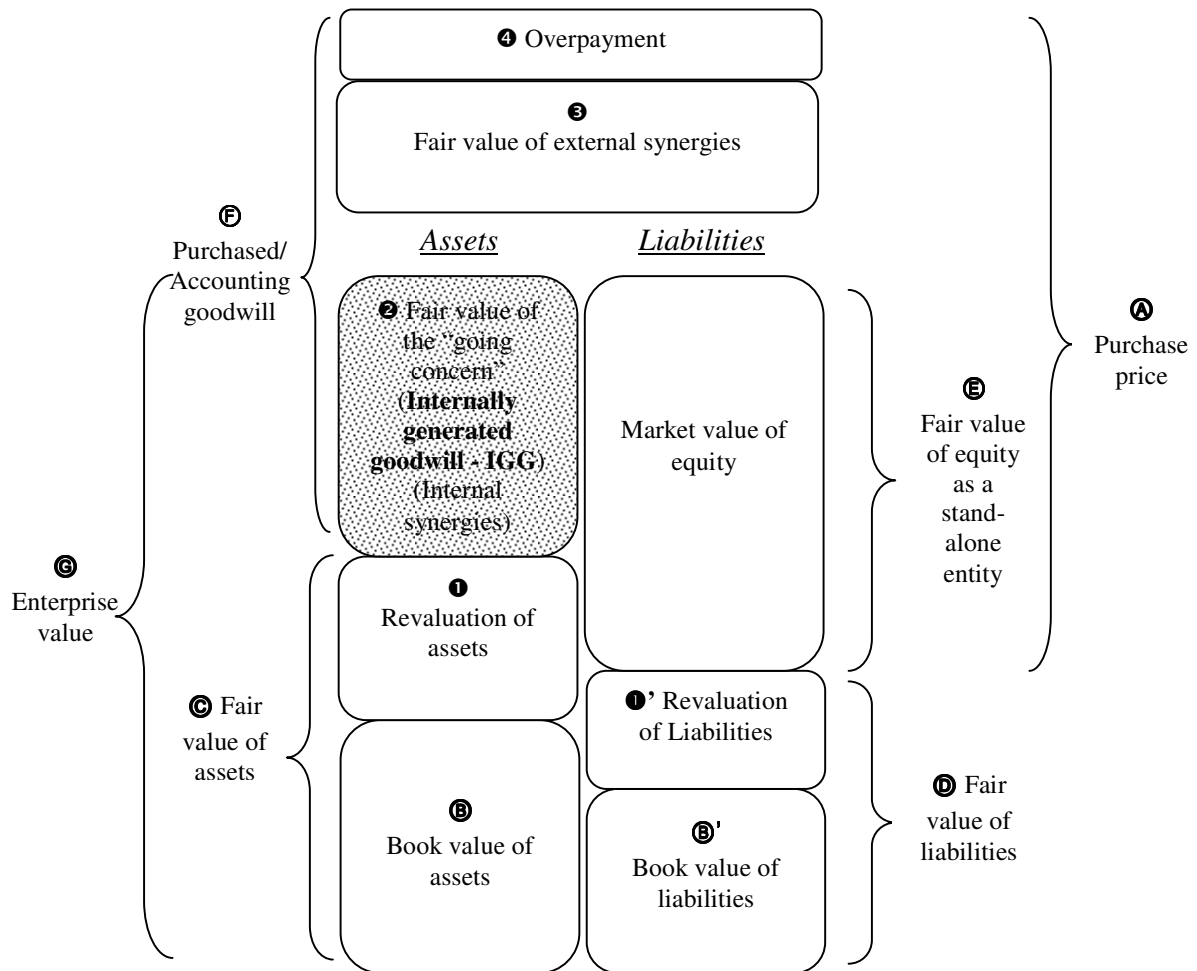
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Figure 1 Purchase Price Breakdown³²



Purchase price = $\text{A} = \text{C} + \text{F} - \text{D}$

Fair value of assets = $\text{C} = (\text{1} + \text{B})$

Fair value of liabilities = $\text{D} = (\text{1}' + \text{B}')$

Enterprise value = $\text{C} = \text{E} + \text{D}$ [Equity side] = $\text{2} + \text{C}$ [Asset side]

Internally generated goodwill (IGG) = $\text{2} = \text{C} - \text{C}$

- Sources of data:
- Annual report (Purchase price allocation [PPA]): **A, C, D, F**
 - Annual report (Balance sheet): **B, B'**
 - Market capitalization: **E**.

³² For the sake of simplicity, Figure 1 represents a 100% acquisition with no non-controlling interests to take into consideration.

FIGURE 2

Additive Valuation of a Set of Assets

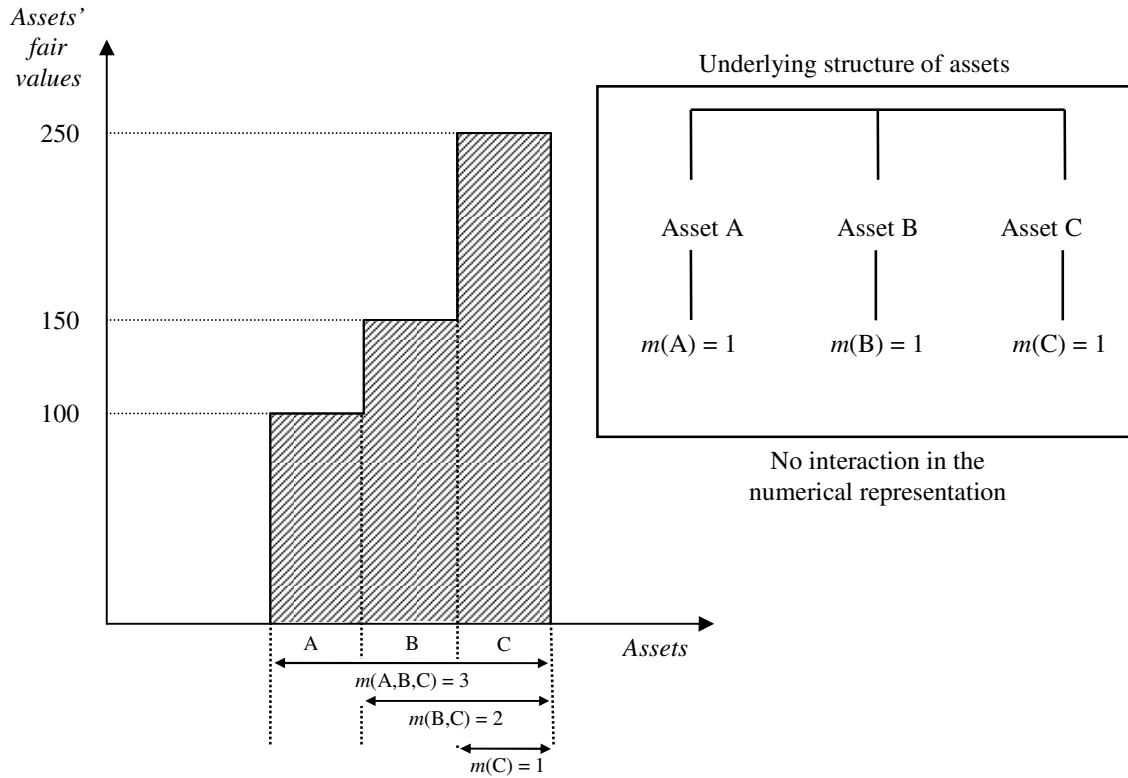


FIGURE 3

Valuation of a Set of Assets using the Choquet integral for Synergy and Inhibition Effects

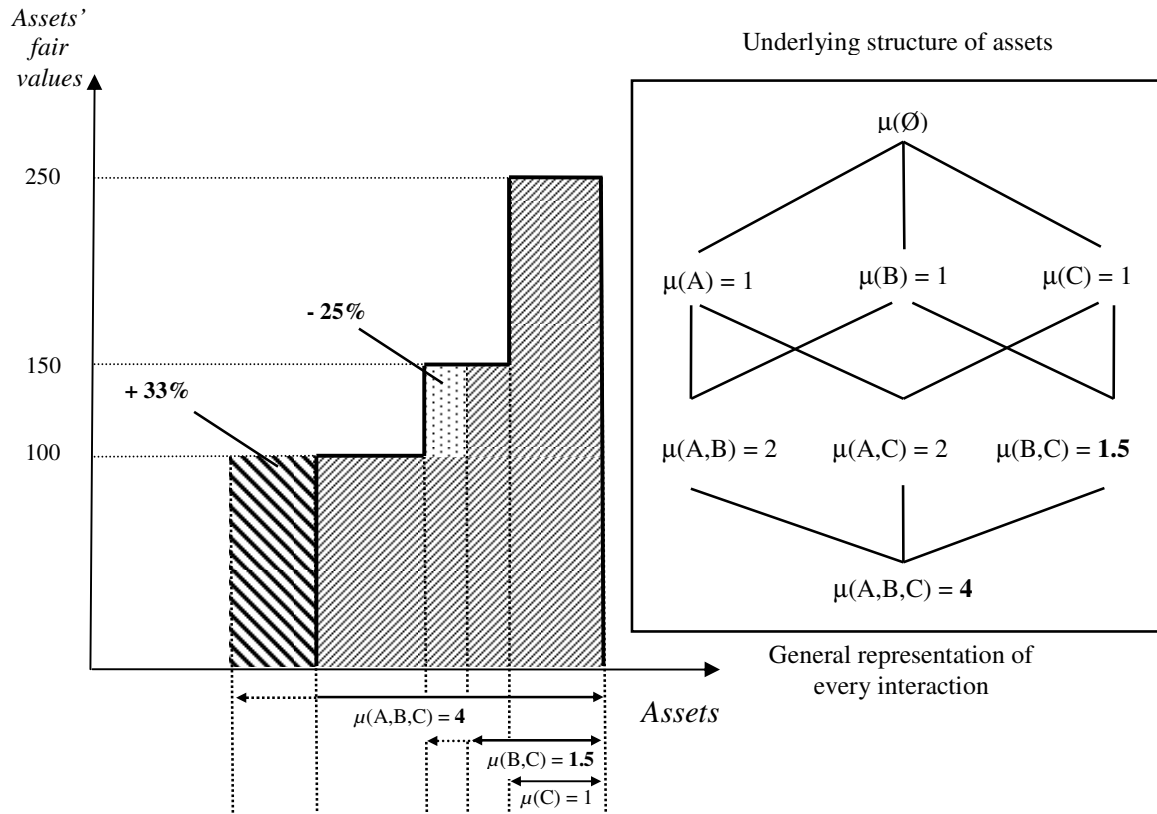
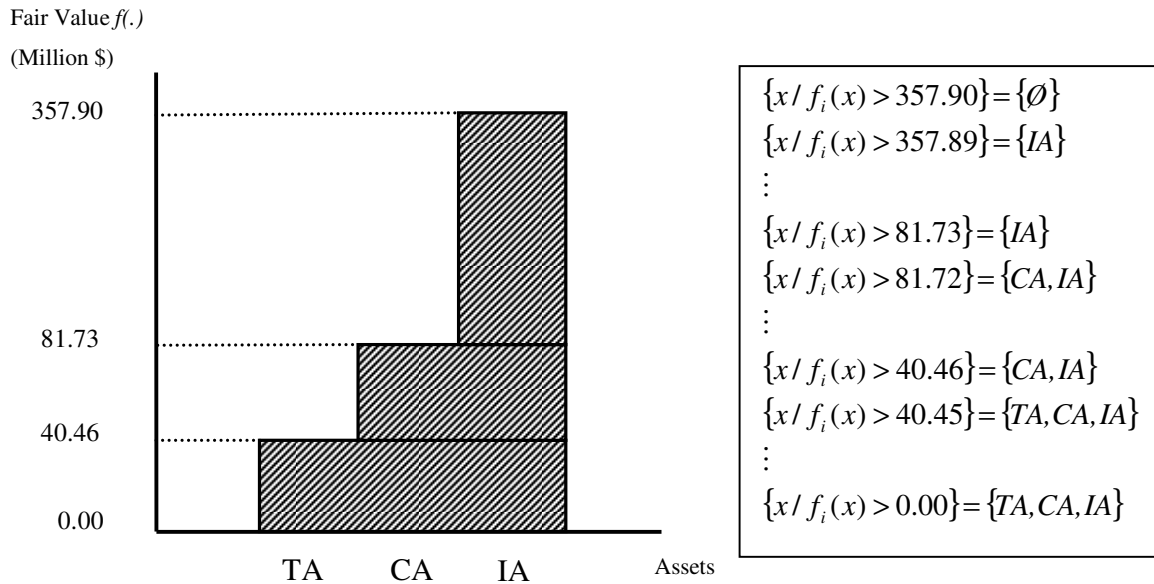


FIGURE 4

Graphical Representation of the Set of Assets of DataDomain Inc.



Generators are computed in the right-hand box of the figure for X , the set of assets of DataDomain Inc., and $x \subseteq P(X)$. For the sake of simplicity, they are computed in levels. In a second step, they are scaled by total assets.

TABLE 1
Synergy Model – Ohlson Model Comparison

	Synergy model	Residual income model
Enterprise value		$TA_t + GW_t$
Origin of goodwill	Interactions between assets, positive synergies, value of the structure	Output flows, abnormal earnings
Measure	Non-additive, Combinatorial	Additive
Expression of goodwill	$IV_t = C_{(f)} - L_{(f)}$	$\sum_{\tau=1}^{\infty} R^{-\tau} E[x_{t+\tau}^a]$

TABLE 2
Descriptive Statistics of the Sample

Panel A (in million \$ and percentage of Enterprise value)	N	Mean	St. dev.	1 st quartile	Median	3 rd quartile	
Tangible assets (TA)	101	129.728	633.971	7.100	18.773	73.039	
Tangible assets (% of EV)	101	9.0%	14.3%	2.3%	4.7%	9.3%	
Current assets (CA)	101	350.485	748.210	52.212	96.613	251.569	
Current assets (% of EV)	101	31.5%	15.9%	20.7%	29.8%	42.6%	
Intangible assets (IA)	101	332.411	718.091	38.700	90.800	234.070	
Intangible assets (% of EV)	101	25.3%	15.6%	12.8%	24.2%	34.1%	
Internally generated goodwill	101	408.692	861.897	48.637	120.593	366.527	
Internally generated goodwill (% of EV)	101	34.3%	22.6%	19.9%	33.8%	50.6%	
Enterprise value (MV + D)	101	1,221.317	2,693.103	201.890	373.505	1,010.860	
Panel B (in percentage of total assets)	N	Mean	St. dev.	1 st quartile	Median	3 rd quartile	
Tangible assets (TA)	101	12.2%	14.4%	3.5%	8.4%	15.3%	
Current assets (CA)	101	48.6%	20.2%	34.7%	50.2%	62.6%	
Intangible assets (IA)	101	39.2%	20.0%	24.2%	35.6%	52.6%	
Internally generated goodwill	101	74.9%	74.6%	24.8%	51.0%	102.5%	
Enterprise value	101	174.9%	74.6%	124.8%	151.0%	202.5%	
Panel C (in million \$ unless otherwise mentioned)	N	Mean T (Panel data)	Mean	St. dev.	1 st quartile	Median	3 rd quartile
Book value	101	10.50	282.242	812.752	17.378	68.770	207.862
Market values	101	5.78	1,397.743	3,511.482	151.672	369.557	1,052.535
Earnings	101	10.48	2.287	195.007	-9.358	0.645	10.235
Cost of capital (in %)	101	5.78	0.146	0.069	0.093	0.129	0.186
Earnings forecasts	85	3.98	40.378	119.231	1.200	10.260	37.280

Notes. EV stands for Enterprise Value, MV for Market Value, D for Debts. Every asset is stated at estimated fair value at the date of acquisition. The Internally generated goodwill is estimated as Market value of equity (€ in Figure 1) + Debts (€) – fair value of assets (€).

TABLE 3**Summary Statistics of the Generator Function (scaled by total assets)**

	N	Mean	St. dev.	1 st quartile	Median	3 rd quartile
g_{TA}	101	0.017	0.095	0.000	0.000	0.000
g_{CA}	101	0.191	0.229	0.000	0.107	0.333
g_{IA}	101	0.104	0.192	0.000	0.000	0.142
$g_{TA,CA}$	101	0.013	0.042	0.000	0.000	0.000
$g_{CA,IA}$	101	0.197	0.140	0.072	0.208	0.301
$g_{TA,IA}$	101	0.006	0.033	0.000	0.000	0.000
$g_{TA,CA,IA}$	101	0.085	0.069	0.034	0.069	0.128

TABLE 4
Estimations of Choquet Capacities

	Coefficient (μ)	Std. Error	p-value
g_{TA}	0.713	0.311	0.024
g_{CA}	2.097	0.244	0.000
g_{IA}	2.303	0.479	0.000
$g_{TA,CA}^*$	2.097	na.	na.
$g_{CA,IA}$	3.262	0.395	0.000
$g_{TA,IA}$	2.543	1.232	0.042
$g_{TA,CA,IA}$	4.845	0.632	0.000
N	101		
R-square	0.859		
Adj.R-square	0.850		

* Consistent with the properties of the Choquet capacities presented in section 3.5.3, the equation is estimated under the constraint of monotonicity (i.e. $\forall A \subset B, \mu(A) \leq \mu(B)$). This constraint is binding for $\mu(TA, CA)$. Therefore we have: $\{CA\} \subset \{TA, CA\}, \mu(CA) = \mu(TA, CA) = 2.097$.

TABLE 5
Synergies and Inhibitions in the HT Sector

Additive value (1)	Interaction value (2)		(2) – (1)	Sign of interpretation
$\mu(TA) + \mu(CA)$	2.810	$\mu(TA, CA)$	2.097	-0.713 (-)
$\mu(CA) + \mu(IA)$	4.400	$\mu(CA, IA)$	3.262	-1.137 (-)
$\mu(TA) + \mu(IA)$	3.015	$\mu(TA, IA)$	2.543	-0.472 (-)
$\mu(TA, CA) + \mu(IA)$	4.400	$\mu(TA, CA, IA)$	4.845	0.445 (+)
$\mu(CA, IA) + \mu(TA)$	3.975	$\mu(TA, CA, IA)$	4.845	0.870 (+)
$\mu(TA, IA) + \mu(CA)$	4.640	$\mu(TA, CA, IA)$	4.845	0.204 (+)
$\mu(TA) + \mu(CA) + \mu(IA)$	5.113	$\mu(TA, CA, IA)$	4.845	-0.268 (-)

TABLE 6
Out-of-Sample Prediction Errors of the Synergy and Ohlson Models

Model	N	Mean	St. dev.	1 st quartile	Median	3 rd quartile
Panel A: Full sample						
Synergy	101	31%	24%	16%	27%	43%
Ohlson (no v_t)	101	43%	43%	19%	34%	52%
Equality test p-value		0.015			0.014	
Panel B: Reduced sample						
Synergy	79	28%	23%	14%	22%	39%
Ohlson (with v_t)	79	41%	27%	20%	37%	53%
Equality test p-value		0.001			0.000	

Notes. Panel A represents the full sample. To construct the “other information” variable, analyst forecasts are required, reducing the size of the sample for that variable from 101 to 85; when forecasts are available, two consecutive analyst forecasts are necessary, further reducing the size of the sample from 85 to 79. Consequently, 22 firms from the initial sample were not used in the regression integrating the “other information” variable. Panel B presents this reduced sample.