HAL open science

On the long-run evolution of inheritance: France 1820-2050

Thomas Piketty

To cite this version:

Thomas Piketty. On the long-run evolution of inheritance: France 1820-2050. 2010. halshs-00564853v2

HAL Id: halshs-00564853

https://shs.hal.science/halshs-00564853v2

Preprint submitted on 11 Feb 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

PARIS SCHOOL OF ECONOMICS
ÉCOLE D'ÉCONOMIE DE PARIS

WORKING PAPER N° 2010 - 12

On the long-run evolution of inheritance: France 1820-2050

Thomas Piketty

JEL Codes: E10, E20, H0
Keywords: inheritance, bequest, wealth, capital

PARIS-JOURDAN SCIENCES ECONOMIQUES
48, BD JOURDAN – E.N.S. – 75014 PARIS
TÉL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10
www.pse.ens.fr

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE – ECOLE DES HAUTES ETUDES EN SCIENCES SOCIALES
ÉCOLE DES PONTS PARISTECH – ECOLE NORMALE SUPÉRIEURE – INSTITUT NATIONAL DE LA RECHERCHE AGRONOMIQUE

On the Long-Run Evolution of Inheritance: France 1820-2050

Thomas Piketty

Paris School of Economics *

Working Paper

First version: November 13th, 2009

This version: May 16th, 2010**

Abstract: This paper attempts to document and account for the long run evolution of inheritance. We find that in a country like France the annual flow of inheritance was about 20%-25% of national income between 1820 and 1910, down to less than 5% in 1950, and back up to about 15% by 2010. A simple theoretical model of wealth accumulation, growth and inheritance can fully account for the observed U-shaped pattern and levels. Using this model, we find that under plausible assumptions the annual bequest flow might reach about 20%-25% of national income by 2050. This corresponds to a capitalized bequest share in total wealth accumulation well above 100%. Our findings illustrate the fact that when the growth rate g is small, and when the rate of return to private wealth r is permanently and substantially larger than the growth rate (say, r=4%-5% vs. g=1%-2%), which was the case in the 19th century and early 20th century and is likely to happen again in the 21st century, then past wealth and inheritance are bound to play a key role for aggregate wealth accumulation and the structure of lifetime inequality. Contrarily to a widely spread view, modern economic growth did not kill inheritance.

* Professor of Economics at the Paris School of Economics (PSE) & Directeur d'études at the Ecole des Hautes Etudes en Sciences Sociales (EHESS)

** I am grateful to seminar participants at the Paris School of Economics, Universitat Pompeu Fabra (Barcelona), the Massachussetts Institute of Technology, Harvard University, New York University, Boston University and the University of Chicago for helpful reactions. All comments are welcome (piketty@ens.fr).

A detailed data appendix supplementing the present working paper is available on-line at www.jourdan.ens.fr/piketty/inheritance/ .

1. Introduction ……... 1

2. Related literature ……....7

3. Data sources & methodology ……...12
3.1. Basic accounting equation: $B_t/Y_t = \mu_t\, m_t\, W_t/Y_t$ ……12
3.2. National income & wealth accounts: Y_t and W_t ……..17
3.3. Estate tax data: B_t^f, μ_t and v_t ..……..23

4. The U-shaped pattern of inheritance: a simple decomposition ……27
4.1. The aggregate wealth-income ratio effect W_t/Y_t ……..…27
4.2. The mortality rate effect m_t ……..…27
4.3. The μ_t* ratio effect ……..28

5. Wealth accumulation, growth & inheritance: a steady-state model ……32
5.1. Notations & definitions ……33
5.2. Steady-state inheritance flows in the exogenous saving model ……..37
5.3. Steady-state inheritance flows in the dynastic model ……47
5.4. Steady-state inheritance flows in the wealth-in-the-utility model ……53

6. Simulations …… 56
6.1. Simulating the 1820-1913 quasi-steady-state ……58
6.2. Simulating the 20th century chaotic U-shaped pattern ……...60
6.3. Simulating the 21st century: towards a new steady-state? ……61

7. Applications & directions for future research …….. 64
7.1. The share of inheritance in total lifetime resources ……....64
7.2. Labor-based vs inheritance-based inequality ……..69
7.3. The share of inheritance in total wealth accumulation ……...72

8. Concluding comments ……78

1. Introduction

There are basically two ways to become rich: either through one's own work, or through inheritance. In Ancien Regime societies, as well as during the 19th century and early 20th century, it was self-evident to everybody that the inheritance channel was an important one. For instance, 19th century and early 20th century novels are full of stories where ambitious young men have to choose between becoming rich through their own work or by marrying a bride with large inherited wealth – and often opt for the second strategy. However, in the late 20th century and early 21st century, most observers seem to believe that this belongs to the past. That is, most observers – novelists, economists and laymen alike – tend to assume that labor income is now playing a much bigger role than inherited wealth in shaping people's lives, and that human capital and hard work have become the key to personal material well-being. Although this is rarely formulated explicitly, the implicit assumption seems to be that the structure of modern economic growth has led to the rise of human capital, the decline of inheritance, and the triumph of meritocracy.

This paper asks a simple question: is this optimistic view of economic development justified empirically and well-grounded theoretically? Our simple answer is "no". Our empirical and theoretical findings suggest that inherited wealth will most likely play as big a role in 21st century capitalism as it did in 19th century capitalism – at least from an aggregate viewpoint.

This paper makes two contributions. First, by combining various data sources in a systematic manner, we document and establish a simple – but striking – fact: the aggregate inheritance flow has been following a very pronounced U-shaped pattern in France since the 19th century. To our knowledge, this is the first time that such long-run, homogenous inheritance series are constructed for any country.

More precisely, we define the annual inheritance flow as the total market value of all assets (tangible and financial assets, net of financial liabilities) transmitted at death or through inter-vivos gifts during a given year.[1] We find that the annual inheritance flow was about 20%-25% of national income around 1900-1910. It then gradually fell to less than

[1] It is critical to include both bequests (wealth transmitted at death) and gifts (wealth transmitted inter vivos) in our definition of inheritance, first because gifts have always represented a large fraction of total wealth transmission, and next because this fraction has changed a lot over time. Throughout the paper, the words "inheritance" or "bequest" or "estate" will refer to the sum of bequests and gifts, unless otherwise noted.

10% in the 1920s-1930s, and to less than 5% in the 1950s. It has been rising regularly since then, with an acceleration of the trend during the past 30 years, and according to our latest data point (2008), it is now close to 15% (see Figure 1).

If we take a longer run perspective, then the 20th century U-shaped pattern looks even more spectacular. The inheritance flow was relatively stable around 20%-25% of national income throughout the 1820-1910 period (with a slight upward trend), before being divided by a factor of about 5-6 between 1910 and the 1950s, and then multiplied by a factor of about 3-4 between the 1950s and the 2000s (see Figure 2).

These are truly enormous historical variations – but they appear to be well founded empirically. In particular, we find similar patterns with our two fully independent estimates of the inheritance flow. The gap between our "economic flow" series (computed from national wealth estimates, mortality tables and observed age-wealth profiles) and our "fiscal flow" series (computed from observed bequest and gift tax data) can be interpreted as a measure of tax evasion and other measurement errors. This gap appears to approximately constant over time, and relatively small, so that our two series deliver fairly consistent long run patterns (see Figures 1 & 2).

If we use personal disposable income (national income minus taxes plus cash transfers) rather than national income as the denominator, then we find that the inheritance flow observed in the early 21st century is back to about 20%, i.e. approximately the same level as that observed one century ago (see Figure 3). This simply comes from the fact that disposable income was as high as 90%-95% of national income during the 19th century and early 20th century (when taxes and transfers were almost non existent), while it is now about 70% of national income. Though we prefer to use the national income denominator (both for conceptual and empirical reasons),[2] this is an important fact to keep in mind when studying these issues. An annual inheritance flow around 20% of disposable income

[2] Whether one should use national or disposable income as denominator is a matter of perspective. If one assumes that government expenditures are useless, and that the rise of government during the 20th century has limited the ability of private individuals to save, accumulate and transmit private wealth, then one should use disposable income. But to the extent that government expenditures are mostly useful (e.g. assuming that in the absence of public spending in health and education, then individuals would have to had to pay at least as much to buy similar services on the market), it seems more justified to use national income. One additional advantage of using national income is that it tends to be better measured. Disposable income can display large time-series and cross-country variations for purely definitional reasons. E.g. in France, disposable income would jump from 70% to about 80% of national income if one includes in-kind health transfers (such as insurance reimbursements), and to about 90% of national income if one includes all in-kind transfers (education, housing, etc.). See Appendix A.

is a very large flow. It is typically larger than the annual flow of new savings, and almost as big as the annual flow of capital income. As we shall see, it corresponds to a cumulated, capitalized bequest share in aggregate wealth accumulation well above 100%.

The second – and most important – contribution of this paper is to account for these facts, and to draw lessons for other countries and for the future. We show that a simple theoretical model of wealth accumulation, growth and inheritance can easily explain why the French inheritance flow seems to return to a high steady-state value around 20% of national income. Consider first a dynastic model where all savings come from capital income. Wealth holders save a fraction g/r of their asset returns, so that aggregate private wealth W_t and national income Y_t grow at the same rate g, and the wealth-income ratio $\beta=W_t/Y_t$ is stationary. It is straightforward to prove that the steady-state inheritance flow-national income ratio is equal to $b_y=\beta/H$, where H is generation length (average age at parenthood). If $\beta=600\%$ and $H=30$, then $b_y=20\%$. We show that this intuition can be generalized to more general saving models. Namely, as long as the (real) growth rate g is sufficiently small and the (real) rate of return on private wealth r is sufficiently large – say, g=1%-2% vs. r=4%-5% –, then steady-state b_y is close to the class saving level β/H.

The key intuition boils down to a simple r>g logic. In countries with large growth, such as France during the 1950s-1970s, then wealth coming from the past (i.e. accumulated or received by one's parents or grand-parents, who were relatively poor as compared to today's incomes) does not matter too much. What counts is new wealth accumulated out of current income. Inheritance flows are bound to be a small fraction of national income. But in countries with low growth, such as France in the 19th century and since the 1970s, the logic is reversed. With low growth, successors simply need to save a small fraction of their asset returns in order to ensure that their inherited wealth grows at least as fast as national income. In effect, g small and r>g imply that wealth coming from the past is being capitalized at a faster rate than national income. So past wealth tends to dominate new wealth, rentiers tend to dominate labor income earners, and inheritance flows are large relative to national income. As $g\rightarrow 0$, then $b_y \rightarrow \beta/H$ – irrespective of saving behavior.

The r>g logic is simple, but powerful. We simulate a full-fledged, out-of-steady-state version of this model, using observed macroeconomic and demographic shocks. We are able to reproduce remarkably well the observed evolution of inheritance flows in France over almost two centuries. The 1820-1913 period looks like a prototype low-growth,

rentier-friendly quasi-steady-state. The growth rate was very small: g=1.0%. The wealth-income ratio β was 600%-700%, the capital share α was 30%-40%, so the average rate of return on private wealth was as large as r=α/β=5%-6%. Taxes at that time were very low, so after-tax returns were almost as high as pre-tax returns. It was sufficient for successors to save about 20% of their asset returns to ensure that their wealth grows as fast as national income (or actually slightly faster). The inheritance flow was close to its steady-state value b_y=β/H=20%-25%. The 1914-1945 capital shocks (involving war destructions, and most importantly a prolonged fall in asset prices) clearly dismantled this steady-state. It took a long time for inheritance flows to recover, especially given the exceptionally high growth rates observed during the 1950s-1970s (g=5.2% between 1949 and 1979). The recovery accelerated since the late 1970s, both because of low growth (g=1.7% between 1979 and 2009), and because of the long term recovery of asset prices and of the wealth-income ratio (β=500%-600% in 2008-9). As predicted by the theoretical model, the inheritance flow is now close to its steady-state value b_y=β/H=15%-20%.

We then use this model to predict the future. According to our benchmark scenario, based upon current growth rates and rates of returns, the inheritance flow will stabilize around 16% of national income by 2040, i.e. at a lower level than the 19th century steady-state. This is due both to higher projected growth rates (1.7% rather than 1.0%) and to lower projected after-tax rates of return (3.0% rather than 5.3%). In case growth slows down to 1.0% after 2010, and after-tax returns rise to 5.0% (which corresponds to the suppression of all capital taxes, and/or to a combination of capital tax cuts and a rising global capital share), then the model predicts that the inheritance flow will keep rising and converge towards 22%-23% after 2050. In all plausible scenarios, the inheritance-income ratio in the coming decades will be at least 15%-20%, i.e. closer to the 19th century levels than to the exceptionally low levels prevailing during the 1950s-1970s. A come-back to postwar levels would require pretty extreme assumptions, such as the combination of high growth rates (above 5%) and a prolonged fall in asset prices and aggregate wealth-income ratios.

Now, the fact that aggregate inheritance flows return to 19th century levels obviously does not imply that the concentration of inheritance and wealth will return to 19th century levels. On distributional issues, this macro paper has very little to say. We view the present research mostly as a positive exercise in aggregate accounting of wealth, income and inheritance, and as a building block for future work on inequality. One should however bear in mind that the historical decline of wealth concentration in developed societies has

been quantitatively less important than some observers tend to imagine. E.g. according to the latest SCF (Survey of consumer finances), the top 10% owns 72% of U.S. aggregate wealth in 2007, while the middle 40% owns 26% and the bottom 50% owns 2%.[3] In a country like France, the top 10% currently owns about 60% of aggregate wealth, and the bottom 50% owns around 5%. These 60%-70% top decile wealth shares are certainly lower than the 90% top decile wealth shares observed in developed countries around 1900-1910, when there was basically no middle class at all.[4] But they are not that much lower. It has also been known for a long time that these high levels of wealth concentration have little to do with the life cycle: top wealth shares are almost as large within each age group.[5] The bottom line is that the historical decline in intra-cohort inequality of inherited wealth has been less important quantitatively than the long term changes in the aggregate inheritance-income ratio. So aggregate evolutions matter a lot for the study of inequality.

In order to illustrate this point, we provide applications of our aggregate findings to the measurement of two-dimensional inequality in lifetime resources (labor income vs inheritance) by cohort. By making approximate assumptions on intra cohort distributions, we compute simple two-dimensional inequality indicators, and we find that they have changed a lot over the past two centuries. In the 19th century, top successors vastly dominated top labor earners (not to mention bottom labor earners) in terms of total lifetime resources. Cohorts born in the 1900s-1950s faced very different life opportunities. For the first time maybe in history, high labor income was the key for high material well-being. According to our computations, cohorts born in the 1970s and after will fall somewhere in between the “rentier society” of the 19th century and the “meritocratic society” of the 20th century – and in many ways will be closer to the former than to the latter.

Do our findings also apply to other countries? We certainly do not pretend that the fairly specific U-shaped pattern of aggregate inheritance flows found for France applies everywhere as a universal law. It probably also applies to Continental European countries that were hit by similar growth and capital shocks. For countries like the U.S. and the U.K., which were little hit by war destructions, but suffered from the same mid-century fall in asset prices, the long-run U-shaped pattern of aggregate inheritance flows was possibly

[3] Here we simply report raw wealth shares from the 2007 SCF (see Kennickell (2009, Table 4)), with no correction whatsoever. Kennickell later compares the top wealth levels reported in the SCF with other sources (such as Forbes 500 rankings), and finds that the SCF understates top wealth shares.
[4] See Piketty, Postel-Vinay and Rosenthal (2006).
[5] See e.g. Atkinson (1983, p.176, table 7.4) for U.K. top wealth shares broken down by age groups.

somewhat less pronounced.[6] In fact, we do not really know. We tried to construct similar series for other countries. But unfortunately there does not seem to exist any other country with estate tax data that is as long run and as comprehensive as the French data.

In any case, even though we cannot make detailed cross country comparisons at this stage, the economic mechanisms revealed by the analysis of the French historical experience certainly apply to other countries as well. In particular, the r>g logic applies everywhere, and has important implications. For instance, it implies that in countries with very large economic and/or demographic growth rates, such as China or India, inheritance flows must be a relatively small fraction of national income. Conversely, in countries with low economic growth and projected negative population growth, such as Spain, Italy or Germany, then inheritance is bound to matter a lot during the 21st century. Aggregate inheritance flows will probably reach higher levels than in France. More generally, a major difference between the U.S. and Europe (taken as a whole) from the viewpoint of inheritance might well be that demographic (and to a lesser extent economic) growth rates have been historically larger in the U.S., thereby making inheritance flows relatively less important. This has little to do with cultural differences. This is just the mechanical impact of growth rates and of the r>g logic. And this may not last forever. If we take a very long run, global perspective, and make the assumption that economic and demographic growth rates will eventually be relatively small everywhere (say, g=1%-2%), then the conclusion follows mechanically: inheritance will matter a lot pretty much everywhere.

The rest of this paper is organized as follows. In section 2, we relate this work to the existing literature. In section 3, we describe our methodology and data sources. In section 4, we present a decomposition of the U-shaped pattern into three components: an aggregate wealth-income effect, a mortality effect, and a relative wealth effect. In section 5, we provide theoretical results on steady-state inheritance flows. In section 6, we report simulation results based upon a full fledged version of this model. In section 7, we present applications of our results to the structure of lifetime inequality and to the share of inheritance in aggregate wealth. Section 8 offers concluding comments.

[6] See section 3.2 below.

2. Related literature

2.1. Literature on top incomes

This paper is related to several literatures. First, this work represents in our view the logical continuation of the recent literature on the long run evolution of top income and top wealth shares initiated by Piketty (2001, 2003), Atkinson (2005) and Piketty and Saez (2003). In this collective research project, we constructed homogenous, long run series on the share of top decile and top percentile income groups in national income, using income tax return data. The resulting data base now includes annual series for over 20 countries, including most developed economies over most of the 20th century.[7] One the main findings is that the historical decline in top income shares that occurred in most countries during the first half of the 20th century was largely due to the fall of top capital incomes, which apparently never fully recovered from the 1914-1945 shocks, possibly because of the rise of progressive income and estate taxes (the "fall of rentiers"). Another important finding is that the large rise in top income shares that occurred in the U.S. (and, to a lesser extent, in other anglo-saxon countries) since the 1970s seem to be mostly due to the unprecedented rise of very top labor incomes (the "rise of working rich").

One important limitation of this literature, however, is that although we did emphasize the distinction between top labor vs. top capital incomes, we did not go all the way towards a satisfactory decomposition of inequality between a labor income component and an inherited wealth component. First, due to various legal exemptions, a growing fraction of capital income has gradually escaped from the income tax base (which in several countries has almost become a labor income tax in recent decades), and we did not seriously attempt to impute full economic capital income (as measured by national accounts) back into our income-tax-returns-based series.[8] This might seriously affect some of our conclusions (e.g. about working rich vs rentiers),[9] and is likely to become

[7] See Atkinson and Piketty (2007, 2010) for the complete set of country studies, and Atkinson, Piketty and Saez (2010) for a recent survey. To a large extent, this project is a simple extension of Kuznets (1953) pioneering and innovative work. Kuznets was the first researcher to combine income tax return data with national income accounts data in order to compute top income shares series, using U.S. data over the 1913-1948 period. In a way, what we do in the present paper is also following Kuznets: we attempt to integrate national income and wealth accounts with income and estate tax data.

[8] Partial corrections were made for a number of countries, but there was no systematic attempt to develop an imputation method. One should be aware of the fact that for most countries (including France, the U.K. and the U.S.), our series measure the share of top reported incomes (rather than top economic incomes).

[9] Wolff and Zacharias (2009) attempt to combine income and wealth data from the Survey of consumer finances (SCF) in order to obtain more comprehensive measures of top capital income flows in the US during

increasingly problematic in the coming decades. So it is important to develop ways to correct for this. Next, even if we were able to observe (or impute) full economic capital income, this would not tell us anything about the share of capital income coming from one's own savings and the share originating from inherited wealth. In income tax returns, one does not observe where wealth comes from. For a small numbers of countries, long run series on top wealth shares (generally based upon estate tax returns) have recently been constructed.[10] These studies confirm that there was a significant decline in wealth concentration during the 1914-1945 period, apparently with no recovery so far.[11] But they do not attempt to break down wealth into an inherited component and a life-cycle or self-made component: these works use estate tax data to obtain information about the distribution of wealth among the living (using mortality multiplier techniques), but not to study the level of inheritance flows per se.[12]

This paper attempts to bridge this gap, by making use of the exceptionally high quality of French estate tax data. We felt that it was necessary to start by trying to reach a better empirical and theoretical understanding of the aggregate evolution of the inheritance-income ratio, which to us was very obscure when we started this research. However the next step is obviously to close this detour via macroeconomics and to integrate endogenous distributions back into the general picture.

2.2. Literature on intergenerational transfers and aggregate wealth accumulation

The present paper is also very much related to the literature on intergenerational transfers and aggregate wealth accumulation. However as far we know our paper is the first attempt to account for the observed historical evolution of inheritance, and to take a long run perspective on these issues. Although the perception of a long term decline of inheritance relatively to labor income seems to be relatively widespread, to our knowledge there are

---

the 1980s-1990s. As they rightly point out, it is not so much that the "working rich" have replaced "coupon-clipping rentiers", but rather that "the two groups now appear to co-habitate at the top end of the distribution".

[10] See Kopczuk and Saez (2004) for the U.S., Piketty, Postel-Vinay and Rosenthal (2006) for France, and Roine and Waldenstrom (2009) for Sweden. These studies follow the pioneering work by Lampman (1962) and Atkinson and Harrison (1978), who respectively use U.S. 1922-1956 estate tax tabulations and U.K. 1923-1972 estate tax tabulations in order to compute top wealth share series.

[11] Given the relatively low quality of available wealth data for the recent period, especially regarding top global wealth holders, one should be modest and cautious about this conclusion.

[12] One exception is Edlund and Kopczuk (2009), who use the fraction of women in top estate brackets as a proxy for the relative importance of inherited vs self-made wealth. This is a relatively indirect way to study inheritance, however, and it ought to be supplemented by direct measures.

very few papers which formulate this perception explicitly.[13] For instance, in their famous controversy about the share of inheritance in U.S. aggregate wealth accumulation, both Kotlikoff and Summers (1981) and Modigliani (1986) were using a single – and relatively ancient and fragile – data point for the U.S. aggregate inheritance flow (namely, for year 1962). In addition to their definitional conflict, we believe that the lack of proper data contributes to explain the intensity of the dispute, which the subsequent literature did not fully resolve.[14] We return to this controversy when we use our aggregate inheritance flows series to compute inheritance shares in the total stock of wealth. The bottom line is that with steady-state inheritance flows around 20% of national income, the cumulated, capitalized bequest share in aggregate wealth accumulation is bound to be well above 100% - which in a way corroborates the Kotlikoff-Summers viewpoint. We hope that our findings contribute to clarify this long standing dispute.

2.3. Literature on calibrated models of wealth distributions

Our work is also related to the recent literature attempting to use calibrated general equilibrium models in order to replicate observed wealth inequality. Several authors have recently introduced new ingredients into calibrated models, such as large uninsured idiosyncratic shocks to labor earnings, tastes for savings and bequests, and/or asset returns.[15] In addition to the variance and functional form of these shocks, one key driving force in these models is naturally the macroeconomic importance of inheritance flows: other things equal, larger inheritance flows tend to lead to more persistent inequalities and higher steady-state levels of wealth concentration. However this key parameter tends to be imprecisely calibrated in this literature, and is generally underestimated: it is often based upon relatively ancient data (typically dating back to the KSM controversy and using data from the 1960s-1970s) and frequently ignores inter vivos gifts. We hope that our findings can contribute to offer a stronger empirical basis for these calibrations.

---

[13] E.g. Galor and Moav (2006) take as granted the "demise of capitalist class structure", but are not fully explicit about what they mean by this. It is unclear whether this is supposed to be an aggregate phenomenon (involving a general rise of labor income relatively to capital income and/or inheritance, as suggested by their informal discussion of the "rise in human capital") or a purely distributional phenomenon (involving a compression of the wealth distribution, for given aggregate wealth-income and inheritance-income ratios, as suggested by their theoretical model). De Long (2003) takes an explicitly long term perspective on inheritance and informally discusses the main effects at play. However his intuition according to which the rise of life expectancy per se should lead to a decline in the importance of inheritance relatively to labor income turns out to be wrong, as we show in this paper.

[14] See e.g. Kessler and Masson (1989), Gale and Scholz (1994), Gokhale et al (2001).

[15] See e.g. Castaneda, Dias-Gimenes and Rios-Rull (2003), DeNardi (2004), Nirei and Souma (2007), Benhabib and Bisin (2009), Benhabib and Zhu (2009), Fiaschi and Marsili (2009) and Zhu (2010). See Cagetti and De Nardi (2008) for a recent survey of this literature.

2.4. Literature on estate multipliers

Finally, our paper is closely related to the late 19th century and early 20th century literature on national wealth and the so-called "estate multiplier". At that time, many economists were computing estimates of national wealth, especially in France and in the UK. In their view, it was obvious that most wealth derives from inheritance. They were satisfied to find that their national wealth estimates W_t (obtained from direct wealth census methods) were always approximately equal to 30-35 times the inheritance flow B_t (obtained from tax data). They interpreted 30-35 as generation length H, and they viewed the estate multiplier formula $e_t=W_t/B_t=H$ as self-evident.[16] In fact, it is not self-evident. This formula is not an accounting equation, and strictly speaking it is valid only under fairly specific models of saving behaviour and wealth accumulation, such as the class saving model. It is difficult to know exactly what model the economists of the time had in mind. From their informal discussions, one can infer that it was close to a stationary model with zero growth and zero saving (in which case $e_t=H$ is indeed self-evident), or maybe a model with small growth originating from slow capital accumulation and a gradual rise of the wealth-income ratio. Of course we now know that capital accumulation alone cannot generate positive self-sustained growth: one needs positive rates of productivity growth $g>0$. Economists writing in the 19th and early 20th centuries were not fully aware of this, and they faced major difficulties with the modelling of steady-state, positive self-sustained growth. This is probably the reason why they were unable to formulate an explicit dynamic, non-stationary model explaining where the estate multiplier formula comes from.

The estate multiplier literature disappeared during the interwar period, when economists realized that the formula was not working any more, or more precisely when they realized that it was necessary to raise the multiplier e_t to as much as 50 or 60 in order to make it work (in spite of the observed constancy of H around 30).[17] Shortly before World War 1, a number of British and French economists also started realizing on purely logical grounds that the formula was too simplistic. They started looking carefully at age-wealth profiles, and developed the so-called "mortality multiplier" literature, whereby wealth-at-death data is being re-weighted by the inverse morality rate of the given age group in order to

[16] For standard references on the "estate multiplier" formula, see Foville (1893), Colson (1903) and Levasseur (1907). The approach was also largely used by British economists (see e.g. Giffen (1878)), though less frequently than in France, probably because French estate tax data was more universal and easily accessible, while the British could use the income flow data from the schedular income tax system.
[17] See e.g. Colson (1927), Danysz (1934) and Fouquet (1982).

generate estimates for the distribution of wealth among the living (irrespective of whether this wealth comes from inheritance or not).[18] Unlike the estate multiplier formula, the mortality multiplier formula is indeed a pure accounting equation, and makes no assumption on saving behaviour. The price to pay for this is that the mortality multiplier approach does not say anything about where wealth comes from: this is simply a statistical technique to recover the cross-sectional distribution of wealth among the living.[19]

In the 1950s-1960s, economists then started developing the life cycle approach to wealth accumulation.[20] This was in many ways the complete opposite extreme to the estate multiplier approach. In the life cycle model, inheritance plays no role at all, individuals die with zero wealth (or little wealth), and the estate multiplier $e_t=W_t/B_t$ is infinite (or very large, say 100 or more). It is interesting to note that this theory was formulated precisely at the time when inheritance was at its historical nadir. According to our series, the inheritance flows were about 4% of national income in the 1950s-1960s, vs. as much as 20%-25% at the time of estate multiplier economists (see Figure 2). Presumably, economists were in both cases very much influenced by the wealth accumulation and inheritance patterns prevailing at the time they wrote.

Our advantage over both estate-multiplier and life-cycle economists is that we have more years of data. Our two-century-long perspective allows us to clarify these issues and to reconcile the various approaches in a unified framework (or so we hope). The lifecycle motive for saving is logically plausible. But it clearly cohabits with many other motives for wealth accumulation (bequest, security, prestige and social status, etc.). Most importantly, we show that with low growth rates and high rates of return, past wealth naturally tends to dominate new wealth, and inheritance flows naturally tend to converge towards levels that are not too far from those posited by the estate multiplier formula, whatever the exact combination of these saving motives might be.

[18] See Mallet (1908), Séailles (1910), Strutt (1910), Mallet and Strutt (1915) and Stamp (1919). This other way to use estate tax data was followed by Lampan (1962), Atkinson and Harrison (1978), and more recent authors (see above). See also Shorrocks (1975).

[19] The accounting equation given in section 3 below ($e_t=W_t/B_t=1/\mu_t m_t$) is of course identical to the mortality multiplier formula, except that we use it the other way around: we use it to compute inheritance flows from the wealth stock, while it has generally been used to compute the wealth of living from decedents' wealth.

[20] See e.g. Brumberg and Modigliani (1954), Ando and Modigliani (1963) and Modigliani (1986).

3. Data sources and methodology

The two main data sources used in this paper are national income and wealth accounts on the one hand, and estate tax data on the other hand. Before we present these two data sources in a more detailed way, it is useful to describe the basic accounting equation that we will be using throughout the paper in order to relate national accounts and inheritance flows. In particular, this is the accounting equation that we used to compute our "economic inheritance flow" series.

3.1. Basic accounting equation: $B_t/Y_t = \mu_t\, m_t\, W_t/Y_t$

If there was no inter vivos gift, i.e. if all wealth transmission occurred at death, then in principle one would not need in any estate tax data in order to compute the inheritance flow. One would simply need to apply the following equation:

$$\mathbf{B_t/Y_t = \mu_t\ m_t\ W_t/Y_t}$$

$$\textbf{I.e.} \qquad \mathbf{b_{yt} = \mu_t\ m_t\ \beta_t} \qquad \textbf{(3.1)}$$

With: B_t = annual inheritance flow

Y_t = national income

W_t = aggregate private wealth

m_t = annual mortality rate = (total number of decedents)/(total living population)

μ_t = ratio between average wealth of the deceased and average wealth of the living

$b_{yt} = B_t/Y_t$ = aggregate inheritance flow-national income ratio

$\beta_t = W_t/Y_t$ = aggregate private wealth-national income ratio

Alternatively, equation (3.1) can be written in per capita terms:

$$b_t/y_t = \mu_t\, w_t/y_t = \mu_t\, \beta_t \qquad (3.2)$$

With: b_t = average inheritance per decedent

y_t = average national income per living individual

w_t = average private wealth per living individual

Equation (3.1) is a pure accounting equation: it does not make any assumption about behaviour or about anything. For instance, if the aggregate wealth-income ratio β_t is equal to 600%, if the annual mortality rate m_t is equal to 2%, and if people who die have the same average wealth as the living (μ_t=100%), then the annual inheritance flow b_{yt} has to be equal to 12% of national income. In case old-age individuals massively dissave in order to finance retirement consumption, or annuitize their assets so as to die with zero wealth, as predicted by the pure life-cycle model, then μ_t=0% and b_{yt}=0%. I.e. there is no inheritance at all, no matter how large β_t and m_t might be. Conversely, in case people who die are on average twice as rich as the living (μ_t=200%), then for β_t=600% and m_t=2%, the annual inheritance flow has to be equal to 24% of national income.

If we express the inheritance flow B_t as a fraction of aggregate private wealth W_t, rather than as a fraction of national income Y_t, then the formula is even simpler:

$$b_{wt} = B_t/W_t = \mu_t\, m_t \qquad (3.3)$$

I.e. the inheritance-wealth ratio b_{wt} is equal to the mortality rate multiplied by the μ_t ratio. In case μ_t=100%, e.g. if the age-wealth profile is flat, then b_{wt} is equal to the mortality rate. The estate multiplier $e_t=W_t/B_t$ is simply the inverse of b_{wt}. We will return to the evolution of the inheritance-wealth ratio b_{wt} later in this paper. But for the most part we choose to focus the attention upon the inheritance-income ratio b_{yt} and accounting equation (3.1), first because the evolution of the wealth-income ratio $\beta_t=W_t/Y_t$ involves economic processes that are interesting per se (and interact with the inheritance process); and next because national wealth data is missing in a number of countries, so that for future comparison purposes we find it useful to emphasize b_{yt} ratios, which are easier to compute (if one has fiscal data). Also, b_{yt} has arguably greater intuitive economic appeal than b_{wt}. E.g. it can easily be compared to other flow ratios such as the capital share α_t or the saving rate s_t.

An example with real numbers might be useful here. In 2008, per adult national income was about 35,000€ in France. Per adult private wealth was about 200,000€. That is, $\beta_t=W_t/Y_t=w_t/y_t$=560%. The mortality rate m_t was equal to 1.2%, and we estimate that μ_t was approximately 220%.[21] It follows from equations (3.1) and (3.3) that the inheritance-

[21] In 2008, French national income Y_t was about 1,700 billions €, aggregate private wealth W_t was about 9,500 billions €, and adult population was about 47 millions, so y_t≈35,000€ and w_t≈200,000€. The number of adult decedents was about 540,000, so the mortality rate m_t≈1.2%. Here we give round up numbers to simplify exposition. For μ_t we actually report the gift-corrected ratio μ_t* (see below), so "average inheritance

income ratio b_{yt} was 14.7% and that the inheritance-wealth ratio b_{yt} was 2.6%. It also follows from equation (3.2) that average inheritance per decedent b_t was about 440,000€, i.e. about 12.5 years of average income y_t (μ_t x β_t = 12.5). One can then introduce distributional issues: about half of decedents have virtually no wealth, the other half owns about twice the average (i.e. about 25 years of average income); and so on.[22]

For the time being, however, we concentrate on b_{yt} and equation (3.1), which is more suitable for the macro level analysis of inheritance. But it is important to keep in mind that the three accounting equations (3.1), (3.2) and (3.3) are by construction fully equivalent.

What kind of data do we need in order to compute equation (3.1)? First, we need data on the wealth-income ratio $\beta_t=W_t/Y_t$. To a large extent, this is given by existing national accounts data, as described below. It is conceptually important to use private wealth as the numerator (i.e. the sum of all tangible and financial assets owned by private individuals, minus their financial liabilities) rather than national wealth (i.e. the sum of private wealth and government wealth). Private wealth can be transmitted at death, while government wealth cannot. Practically, however, this does not make a big difference, since private wealth usually represents over 90% of national wealth (i.e. government net wealth is typically positive but small). The choice of the income denominator is unimportant, as long as one uses the same denominator on both sides of the equation. For reasons explained in the introduction, we choose to use national income (rather for instance than personal disposable income) as the denominator.

Next, we need data on the mortality rate m_t. This is the easiest part: demographic data is plentiful and easily accessible.[23] In practice, children usually own very little wealth and receive very little income. In order to abstract from the large historical variations in infant mortality, and in order to make the quantitative values of the m_t and μ_t parameters easier to interpret, we define them over the adult population. That is, we define the mortality rate m_t as the adult mortality rate, i.e. the ratio between the number of decedents aged 20-year-old and over and the number of living individuals aged 20-year-old and over.

---

per decedent" corresponds to "total bequests and gifts divided by number of decedents". For complete computations and exact values, see Appendix A, Table A2, line 2008.

[22] See section 7 below.

[23] All detailed demographic series and references are given in Appendix C.

Similarly, we define μ_t as the ratio between the average wealth of decedents aged 20-year-old and over and the average wealth of living individuals aged 20-year-old and over.[24]

Finally, we need data to compute the μ_t ratio. This is the most challenging part, and also the most interesting part from an economic viewpoint. In order to compute μ_t we need two different kinds of data. First, we need data on the cross-sectional age-wealth profile. The more steeply rising the age-wealth profile, the higher the μ_t ratio. Conversely, if the age-wealth profile is strongly hump-shaped, then μ_t will be smaller. Next, we need data on differential mortality. For a given age-wealth profile, the fact that the poor tend to have higher mortality rates than the rich implies a lower μ_t ratio. In the extreme case where only the poor (say, zero-wealth individuals) die, and the rich never die, then the μ_t ratio will be permanently equal to 0% (even with a steeply rising cross-sectional age-wealth profile), and there will be no inheritance. There exists a large research literature on differential mortality. We simply borrow the best available estimates from this literature. We checked that these differential mortality factors are consistent with the age-at-death differential between wealthy decedents and poor decedents, as measured by estate tax data and demographic data; they are consistent.[25]

Regarding the age-wealth profile, one would ideally like to use exhaustive, administrative data on the wealth of the living, such as wealth tax data. However such data generally does not exist for long time periods, and/or only covers relatively small segments of the population. Wealth surveys do cover the entire population, but they are not fully reliable (especially for top wealth holders, which might bias estimated age-wealth profiles), and in any case they are not available for long time periods. The only data source offering long-run, reliable raw data on age-wealth profiles appears to be the estate tax itself.[26] This is wealth-at-death data, so one needs to use the differential mortality factors to convert them back into wealth-of-the-living age-wealth profiles.[27] This data source combines many advantages: it covers the entire population (nearly everybody has to file an estate tax

[24] Throughout the paper, "adult" means "20-year-old and over". In practice, children wealth is small but positive (parents sometime die early). In our estimates, we do take into account children wealth, i.e. we add a (small) correcting factor to the μ_t ratio in order to correct for the fact that the share of adult wealth in total wealth (both among the deceased and among the living) is slightly smaller than 100%. See Appendix B2.

[25] See Appendix B2. We use the mortality rates differentials broken down by wealth quartiles and age groups estimated by Attanasio and Hoynes (2000). If anything, we probably over-estimate differential mortality a little bit. Consequently, our resulting μ_t series and inheritance series are probably (slightly) under-estimated.

[26] The fact that we use estate tax data to compute our economic inheritance flow series does not affect the independence between the economic and fiscal series, because for the economic flow computation we only use the relative age-wealth profile observed in estate tax returns (not the absolute levels).

[27] Whether one starts from wealth-of-the-living or wealth-at-death raw age-wealth profiles, one needs to use differential mortality factors in one way or another in order to compute the μ_t ratio.

return in France), and it is available on a continuous and homogenous basis since the beginning of the 19th century. We checked that the resulting age-wealth profiles are consistent with those obtained with wealth tax data and (corrected) wealth survey data for the recent period (1990s-2000s); they are consistent.[28]

We have now described how we proceed in order to compute our "economic inheritance flow" series using equation (3.1). There is however one important term that needs to be added to the computation in order to obtain meaningful results. In the real world, inter vivos gifts do exist and play an important role in the process of intergenerational wealth transmission and in shaping the age-wealth profile. In France, gifts have always represented a large fraction of total wealth transmission (around 20%-30%), and moreover this fraction has changed a lot over time (currently it is almost 50%). Not taking them into account would bias the results in important ways. The simplest way to take gifts into account is to correct equation (3.1) in the following way:

$$\mathbf{B_t/Y_t = \mu_t^* \, m_t \; W_t/Y_t} \qquad \mathbf{(3.1')}$$

With: $\mu_t^* = (1+v_t)\,\mu_t$ = gift-corrected ratio between decedents wealth and wealth of the living
$v_t = V_t^{f0}/B_t^{f0}$ = observed fiscal gift-bequest ratio
B_t^{f0} = raw fiscal bequest flow (total value of bequests left by decedents during year t)
V_t^{f0} = raw fiscal gift flow (total value of inter vivos gifts made during year t)

Equation (3.1') simply uses the observed, fiscal gift-bequest ratio during year t and upgrades the economic inheritance flow accordingly. Intuitively, the gift-corrected ratio μ_t^* attempts to correct for the fact that the raw μ_t under-estimates the true relative importance of decedents' wealth (decedents have already given away part of their wealth before they die, so that their wealth-at-death looks artificially low), and attempts to compute what the μ_t ratio would have been in the absence of inter-vivos gifts. Of course, this simple way to proceed is not fully satisfactory, since the individuals who make gifts during year t are usually not the same as the individuals who die during year t (on average gifts are made about 7-8 years before the time of death). In the simulated model, we re-attribute gifts to the proper generation of decedents, and re-simulate the entire age-wealth profile dynamics in the absence of gifts. We show that this creates time lags, but does not significantly affect long-run levels and patterns of the inheritance-income ratio.

[28] See Appendix B2 and section 4.3 below.

Before we present and analyse the results of these computations, we give more details about our two main data sources: national accounts data and estate tax data. Readers who feel uninterested by these details might want to go directly to section 4.

3.2. National income and wealth accounts: Y_t and W_t

National income and wealth accounts have a long tradition in France, and available historical series are of reasonably high quality.[29] In particular, the national statistical institute (Insee) has been compiling official national accounts series since 1949. Homogenous, updated national income accounts series covering the entire 1949-2008 period and following the latest international guidelines were recently released by Insee. These are the series we use in this paper for the post-1949 period, with no adjustment whatsoever. National income Y_t and its components are defined according to the standard international definitions: national income equals gross domestic product minus capital depreciation plus net foreign factor income, etc.

Prior to 1949, there exists no official national accounts series in France. However a very complete set of retrospective, annual income accounts series covering the 1896-1949 period was compiled and published by Villa (1994). These series use the concepts of modern national accounts and are based upon a systematic comparison of raw output, expenditure and income series constructed by many authors. Villa also made new computations based upon raw statistical material. Although some of year-to-year variations in this data base are probably fragile, there are good reasons to view these annual series as globally reliable.[30] These are the series we use for the 1896-1949 period, with minor adjustments, so as to ensure continuity in 1949. Regarding the 1820-1900 period, though a number of authors have produced annual national income series, we are not sure that the limited raw statistical material available for the 19th century makes such an exercise really meaningful. Moreover we do not really need annual series for our purposes. So for the 19th century, we use decennial-averages estimates of national income (these

[29] All national accounts series, references and computations are described in a detailed manner in Appendix A. Here we simply present the main data sources and conceptual issues.

[30] In particular, the factor income decompositions (wages, profits, rents, business income, etc.) series released by Villa (1994) rely primarily on the original series constructed by Dugé de Bernonville (1933-1939), who described very precisely all his raw data sources and computations. For more detailed technical descriptions of the Dugé and Villa series, see Piketty (2001, pp.693-720).

decennial averages are almost identical across the different authors and data sources), and we assume fixed growth rates, saving rates and factor shares within each decade.[31]

The national wealth part of our macro data base requires more care than the national income part. It is only in 1970 that Insee started producing official, annual national wealth estimates in addition to the standard national income estimates. For the post-1970 period, the wealth and income sides of French national accounts are fully integrated and consistent. That is, the balance sheets of the personal sector, the government sector, the corporate sector, and the rest of the world, estimated at asset market prices on January 1st of each year, are fully consistent with the corresponding balance sheets estimated on the previous January 1st and the income and savings accounts of each sector during the previous year, and the recorded changes in asset prices.[32] We use these official Insee balance sheets for the 1970-2009 period, with no adjustment whatsoever. We define private wealth W_t as the net wealth (tangible assets, in particular real estate, plus financial assets, minus financial liabilities) of the personal sector. W_t is estimated at current asset market prices (real estate assets are estimated at current real estate prices, equity assets are estimated at current stock market prices, etc.). This is exactly what we want, since our objective is to relate aggregate private wealth to the inheritance flow, and since – according to estate tax law – the value of bequests is always estimated at the market prices of the day of death (or on the day the gift is made). Although this is of no use for our purposes, one can also define government wealth W_{gt} as the net wealth of the government sector, and national wealth $W_{nt} = W_t+W_{gt}$. According to the Insee estimates, private wealth during the 1990s-2000s has always represented around 90%-95% of national wealth. I.e. government wealth is positive but small: government tangible and financial assets only slightly exceed the value of government debt. During the 1970s-1980s, private wealth was equal to about 85%-90% of national wealth. Government net wealth was somewhat bigger than it is today, both because public debt was smaller and because the government owned more tangible and financial assets (the public sector was bigger at that time).[33]

Prior to 1970, we have to use various non-official, national wealth estimates. For the 1820-1913 period, national wealth estimates are plentiful and relatively reliable. This was a time of almost zero inflation (0.5% per year on average during the 1820-1913 period), so there

[31] We used the 19th century series due to Bourguignon and Lévy-Leboyer (1985) and Toutain (1987).
[32] The concepts and methods used in Insee-Banque de France balance sheets are broadly similar to the flows-of-funds and tangible-assets series released by the U.S. Federal Reserve and Bureau of Commerce.
[33] More details on these issues are provided in Appendix A4.

was no big problem with asset prices. Most importantly, the economists of the time were literally obsessed with national wealth (which they found to be much more interesting than national income), and many of them produced relatively sophisticated national wealth estimates. They used the decennial censuses of tangible assets organized by the tax administration (the tax system of the time relied extensively on the property values of real estate, land and business assets, so such censuses played a critical role). They took into account the growing stock and bond market capitalisation and the booming foreign assets, and they explained in a precise and careful manner how they made all the necessary corrections in order to avoid all forms of double counting. We certainly do not pretend that these national wealth estimates are perfectly comparable to the modern, official balance sheets. In particular, these estimates are never available on an annual basis, and they certainly cannot be used to do short run business cycle analysis. But as far as decennial averages are concerned, we consider that the margin of error on these estimates does not exceed 5%-10%. As compared to the enormous historical variations in aggregate wealth-income ratios and in the inheritance-income ratio, in which we are primarily interested in, such margins of errors are negligible. According to these national wealth estimates, private wealth at that time accounted for as much as 97%-98% of national wealth, i.e. net government wealth was slightly positive but negligible.

The period 1914-1969 is the time period for which French national wealth estimates are the most problematic. This was a chaotic time for wealth, both because of war destructions and because of large inflation and wide variations in the relative price of the various assets. Very few economists compiled detailed, reliable national balance sheets for this time period. We proceed as follows. We use only two data points, namely the national wealth estimate for year 1925 due to Colson (1927), and the national wealth estimate for year 1954 due to Divisia, Dupin and Roy (1956). These are the two most sophisticated estimates available for this time period. They both rely on a direct wealth census method, and they both attempt to estimate assets and liabilities at asset market prices prevailing in 1925 and 1954, which is what we want. Moreover, Colson is the author of some of the most sophisticated pre-World War 1 national wealth estimates (we used his estimates for 1896 and 1913), and his 1925 computations are based on the same methods and sources as those used for 1896 and 1913. Divisia and his co-authors view the Colson 1896-1913-1925 estimates as their model, and they also attempt to follow the same methodology. To the extent that national wealth can be estimated during such a chaotic time period, this is probably the best one can find.

For the missing years, we estimate private wealth W_t by using a simple wealth accumulation equation, based upon the private saving flows S_t coming from national income accounts. Generally speaking, year-to-year variations in private wealth W_t can be due either to volume effects (savings) or to price effects (asset prices might rise or fall relatively to consumer prices). That is, the accumulation equation for private wealth can be written as follows:

$$\mathbf{W_{t+1} = (1+q_{t+1})\ (1+p_{t+1})\ (W_t + S_t) \quad (3.4)}$$

In equation (3.4), p_{t+1} is consumer price inflation between year t and year t+1, and q_{t+1} is the real rate of capital gain (or capital loss) between year t and year t+1, which we define as the excess of asset price inflation over consumer price inflation. For the 1970-2009 period, since French national income and wealth accounts are fully integrated, q_t can indeed be interpreted as the real rate of capital gains. For the pre-1970 period, q_t is better interpreted as a residual error term: it includes real asset price inflation, but it also includes all the variations in private wealth that cannot be accounted for by saving flows. For simplicity, we assume a fixed q_t factor during the 1954-1970 period (i.e. we compute the implicit average q_t factor needed to account for 1970 private wealth, given 1954 private wealth and 1954-1969 private savings flows). We do the same for the 1925-1954 period, the 1913-1925 period, and for each decade of the 1820-1913 period. The resulting decennial averages for the private wealth-national income ratio $\beta_t = W_t/Y_t$ are plotted on Figure 4. Summary statistics on the accumulation of private wealth in France over the entire 1820-2009 period are given on Table 1.

Again, we do not pretend that the resulting annual series are fully satisfactory, and we certainly do not recommend that one uses them for short run business cycle analysis, especially for the 1913-1925 and 1925-1954 sub-periods, for which the simplifying assumption of a fixed capital gain effect makes little sense. However we believe that the resulting decennial averages are relatively precise. In particular, it is re-insuring to see that most of wealth accumulation in the medium and long run seems to be well accounted for by savings. This suggests that saving rates are reasonably well measured by our national accounts series, and that in the long run there exists no major divergence between asset prices and consumer prices. The fact that our private wealth series delivers economic

inheritance flow estimates that are reasonably well in line with the observed fiscal flow also gives us confidence about our wealth estimates.

A few additional points about the long-run evolution of the wealth-income ratio β_t might be worth noting here.[34] Consider first the 1820-1913 period. We find that β_t gradually rose from about 550%-600% around 1820 to about 650%-700% around 1900-1910 (see Figure 4). The real growth rate g of national income was 1.0%.[35] The savings rate s was about 8%-9%, so that the average savings-induced wealth growth rate $g_{ws}=s/\beta$ was 1.4%. I.e. it was larger than g. This explains why the wealth-income ratio was rising during the 19th century: savings were slightly higher than the level required for a steady-state growth path (i.e. the savings rate was slightly higher than $s^*=\beta g$=6%-7%). The observed real growth rate of private wealth g_w was actually 1.3%, i.e. slightly below g_{ws}. In our accounting framework, we attribute the differential to changes in the relative price of assets, and we find a modest negative q effect (-0.1%) (see Table 1). Of course, it could just be that we slightly overestimate 19th century saving rates, or that we slightly underestimate the 19th century rise in the wealth-income ratio, or both. But the important point is that our stock and flow series are broadly consistent. Although the data is imperfect, it is also well established that a very substantial fraction of the 19th century rise in the wealth-income ratio (and possibly all of it) went though the accumulation of large foreign assets.[36]

Consider now the 1913-2009 period. The real growth rate g of national income was 2.6%, thanks to the high growth postwar decades. The real growth rate of private wealth g_w was 2.4%. Given observed saving flows (and taking into account wartime capital destructions, which we include in volume effects), private wealth should have grown slightly faster, i.e. we find that the saving-induced wealth growth rate g_{ws} was 2.9%. We again attribute the differential to real capital gains, and we find a modest negative q effect (-0.4%) (see Table 1). Taken literally, this would mean that the 1949-2009 gradual rise in the relative price of assets has not yet fully compensated the 1913-1949 fall, and that asset prices are currently about 30% lower than what they were at the eve of World War 1. Again, it could also be that we slightly overestimate 20th century saving flows, or underestimate end-of-

[34] For a more detailed technical analysis of the series, see Appendix A3 and A4.
[35] All "real" growth rates (either for national income or for private wealth) and "real" rates or return referred to in this paper are defined relatively to consumer price inflation. Any CPI mismeasurement would translate into similar changes for the various rates without affecting the differentials and the ratios.
[36] Net foreign assets gradually rose from about 2% of private wealth in 1820 to about 15% around 1900-1910, i.e. from about 10% of national income to about 100% of national income. See Appendix A, Table A16.

period wealth stocks.[37] The important point is that our stock and flow data sources are mutually consistent. In the long run, the bulk of wealth accumulation is well accounted for by savings, both during the 19th and the 20th century. As a first approximation, the 1913-1949 fall in the relative price of assets seems to have been almost exactly compensated by the 1949-2009 rise, so that the total 1913-2009 net effect is close to zero.

The other important finding is that the 1913-1949 fall in the aggregate wealth-income ratio was not due – for the most part – to the physical destructions of the capital stock that took place during the wars. We find that β_t dropped from about 600%-650% in 1913 to about 200%-250% in 1949. Physical capital destructions per se seem to account for little more than 10% of the total fall. On the basis of physical destructions and the observed saving response (saving flows were fairly large in the 1920s and late 1940s), we find that private wealth should have grown at g_{ws}=0.9% per year between 1913 and 1949, i.e. almost as fast as national income (g=1.3%). However the market value of private wealth fell dramatically (g_w=-1.7%), which we attribute to a large negative q effect (q=-2.6%). This large real rate of capital loss can be broken down into a variety of factors: holders of nominal assets (public and private bonds, domestic and foreign) were literally expropriated by inflation; real estate prices fell sharply relatively to consumer prices (probably largely due to sharp rent control policies enacted in the 1920s and late 1940s); and stock prices also fell to historical lows in 1945 (probably reflecting the dramatic loss of faith in capital markets, as well as the large nationalization policies and capital taxes enacted in the aftermath of World War 2). In effect, the 1914-1945 political and military shocks generated an unprecedented wave of anti-capital policies, which had a much larger impact on private wealth than the wars themselves.

This asset price effect explains why the wealth-income ratio also seems to have fallen substantially in countries whose territories were not directly hit by the wars. In the U.K., the private wealth-national income ratio was apparently as large as 650%-750% in the late 19th and early 20th century, down to 350%-400% in the 1950s-1970s, up to about 450%-

[37] In the benchmark estimates reported on Table 1, private saving flows are defined as the sum of personal savings and net corporate retained earnings (our preferred definition). If we instead use personal saving flows, we find a lower g_{ws} (2.0%) and a modest positive q effect (+0.4%). Taken literally, this would mean that asset prices are currently about 40% higher than what they were in 1913, but that if we deduct the cumulated value of corporate retained earnings, then they are actually 30% smaller. Within our accounting framework, retained earnings account for about a third of total real capital gains during the 1949-2009 period, which seems reasonable. For detailed results, see Appendix A5, Table A19, from which Table 1 is extracted.

550% in the 1990s-2000s.[38] In the U.S., it seems to have declined from about 550%-600% in the early 20th century and in the interwar period to about 350%-400% in the 1950s-1970s, up to 450%-500% in the 1990s-2000s.[39] This suggests that the U.K. and the U.S. have gone through the same U-shaped pattern as France – albeit in a somewhat less pronounced manner, which seems consistent with the above observations. We stress however that these U.K.-U.S. series are not fully homogenous over time; nor are they fully comparable to our French series. We report them for illustrative purposes only. The U-shaped pattern is probably robust, but the exact levels should be interpreted with caution.

Finally, it is worth noting that if we use disposable income rather than national income as the denominator, then the wealth-income ratios reached in France in the 2000s (750%-800%) appear to be slightly higher than the levels observed in the 19th and early 20th century, rather than slightly smaller (see Figure 5). We feel that it is more justified to look at the wealth-national income ratios, but this is a matter of perspective.

3.3. Estate tax data: B_t^f, μ_t and v_t

Estate tax data is the other key data source used in this paper.[40] It plays an essential role for several reasons. First, because of various data imperfections (e.g. regarding national wealth estimates), we thought that it was important to compute two independent measures of inheritance flows: one "economic flow" indirect measure (based upon national wealth estimates and mortality tables, as described above) and one "fiscal flow" direct measure. The fiscal flow is a direct measure in the sense that it was obtained simply by dividing the observed aggregate bequest and gift flow reported to the tax administration (with a few corrections, see below) by national income, and therefore makes no use at all of national wealth estimates. Next, we need estate tax data in order to compute the gift-bequest ratio $v_t = V_t^{f0}/B_t^{f0}$, and in order to obtain reliable, long-run data on the age-wealth profile and to

[38] Here we piece together the following data sources: for the late 19th century and early 20th century, we use the private wealth and national income estimates of the authors of the time (see e.g. Giffen (1878) and Bowley (1920)); for the period going from the 1920s to the 1970s, we use the series reported by Atkinson and Harrison (1978); for the 1990s-2000s we use the official personal wealth series released on hmrc.gov.uk. See also Solomou and Weale (1997, p.316), whose 1920-1995 UK wealth-income ratio series display a similar U-shaped pattern (from 600% in the interwar down to less than 400% in the 1950s-1970s, up to 500%-600% in the 1980s-1990s).

[39] Here we use for the post-1952 period the net worth series (household and non-profit sectors) released by the Federal Reserve (see e.g. *Statistical Abstract of the U.S. 2010*, Table 706), and for the pre-1952 period the personal wealth series computed by Kopczuk and Saez (2004, Table A) and Wolff (1989). .

[40] All estate tax series, references and computations are described in a detailed manner in Appendix B. Here we simply present the main data sources and conceptual issues.

compute the μ_t ratio. Finally, we also use estate tax data in order to know the age structure of decedents, heirs, donors and donees, which we need for our simulations.

French estate tax data is exceptionally good, for one simple reason. As early as 1791, shortly after the abolition of the tax privileges of the aristocracy, the French National Assembly introduced a universal estate tax, which has remained in force since then. This estate tax was universal because it applied both to bequests and to inter-vivos gifts, at any level of wealth, and for nearly all types of property (both tangible and financial assets). The key characteristic of the tax is that the successors of all decedents with positive wealth, as well as all donees receiving a positive gift, have always been required to file a return, no matter how small the estate was, and no matter whether the heirs and donees actually ended up paying a tax or not. This followed from the fact that the tax was thought more as a registration duty than as a tax: filling a return has always been the way to register the fact that a given property has changed hands and to secure one's property rights.[41]

Between 1791 and 1901, the estate tax was strictly proportional. The tax rate did vary with the identity of the heir or donee (children and surviving spouses have always faced much lower tax rates than other successors in the French system), but not with the wealth level. The proportional tax rates were fairly small (generally 1%-2% for children and spouses), so there was really very little incentive to cheat. The estate tax was made progressive in 1901. In the 1920s, tax rates were sharply increased for large estates. In 1901, the top marginal rate applying to children heirs was as small as 5%; by the mid 1930s it was 35%; it is currently 40%. Throughout the 20th century these high top rates were only applied to small segments of the population and assets. So the aggregate effective tax rate on estates has actually been relatively stable around 5% over the past century in France.[42] Most importantly, the introduction of tax progressivity did not significantly affect the universal legal requirement to fill a return, no matters how small the bequest or gift.

There is ample evidence that this legal requirement has been applied relatively strictly, both before and after the 1901 reform. In particular, the number of estate tax returns filled

[41] This is reflected in the official name of the tax, which since 1791 has always been "droits d'enregistrement" (more specifically, "droits d'enregistrement sur les mutations à titre gratuity" (DMTG)), rather than "impôt sur les successions et les donations". In the U.S., the estate tax is simply called the "estate tax".

[42] See Appendix A, Table A9, col. (15). This low aggregate effective tax rate reflects the fact that top rates only apply to relatively high wealth levels (e.g. the top 40% marginal rate currently applies to per children, per parent bequests above 1.8 millions euros), and the fact that tax exempt assets and tax rebates for inter vivos gifts have become increasingly important over time. See Appendix B for more details.

each year has generally been around 65% of the total number of adult decedents (about 350,000 yearly returns for 500,000 adult decedents, both in the 1900s and in the 2000s). This is a very large number, given that the bottom 50% of the population hardly owns any wealth at all. We do upgrade the raw fiscal flow in order to take non-filers into account, but the point is that the corresponding correction is small (generally around 5%-10%).

The other good news for scholars is that the raw tax material has been well archived. Since the beginning of the 19th century, the tax authorities transcribed individual returns in registers that have been preserved. In a previous paper we used these registers to collect large micro samples of Paris decedents every five year between 1807 and 1902, which allowed us to study the changing concentration of wealth and the evolution of age-wealth profiles.[43] Ideally one would like to collect micro samples for the whole of France over the two-century period, but this has proved to be too costly so far.

So in this paper we rely mostly on aggregate national data collected by the tax administration. For the 1826-1964 period, we use the estate tax tabulations published on a quasi-annual basis by the French Ministry of Finance. For the whole period, these tables indicate the aggregate value of bequests and gifts reported in estate tax returns, which is the basic information that we need. Starting in 1902, these annual publications also include detailed tabulations on the number and value of bequests and gifts broken down by size of estate and age of decedent or donor. These tabulations were abandoned in the 1960s-1970s, when the tax administration started compiling electronic files with nationally representative samples of bequest and gift tax returns. We use these so-called “DMTG” micro files for years 1977, 1984, 1987, 1994, 2000 and 2006. The data is not annual, but it is very detailed. Each micro-file includes all variables reported in tax returns, including the value of the various types of assets, total estate value, the share going each heir or donee, and the demographic characteristics of decedents, heirs, donors and donees.

We proceed as follows. We start from the raw fiscal bequest flow B_t^{f0}, i.e. the aggregate net wealth transmitted at death, as reported to tax authorities by heirs, whoever they are. In particular, we do not exclude the estate share going to surviving spouses, first because it has always been relatively small (about 10%),[44] and next because we choose in the

[43] See Piketty, Postel-Vinay and Rosenthal (2006).
[44] The spouse share has always been about 10% of the aggregate estate flow, vs. 70% for children and 20% for non-spouse, non-children heirs, typically siblings and nephews/nieces (see Appendix C2). It is unclear why one should exclude the spouse share and not the latter. In any case, this would make little difference.

present paper to adopt a gender-free, individual-centred approach to inheritance. So we ignore marriage and gender issues altogether, which given our aggregate perspective seems to be the most appropriate option.[45]

We first make an upward correction to B_t^{f0} for non filers (see above), and we then make another upward correction for tax exempt assets. When the estate tax was first created, the major exception to the universal tax base was government bonds, which benefited from a general estate tax exemption until 1850. Between 1850 and World War 1, very few assets were exempted (except fairly specific assets like forests). Shortly after World War 1, and again after World War 2, temporary exemptions were introduced for particular types of government bonds. In order to foster reconstruction, new real estate property built between 1947 and 1973 also benefited from a temporary exemption. Most importantly, a general exemption for life insurance assets was introduced in 1930. It became very popular in recent decades. Life insurances assets were about 2% of aggregate wealth in the 1970s and grew to about 15% in the 2000s. Using various sources, we estimate that the total fraction of tax exempt assets in aggregate private wealth gradually rose from less than 10% around 1900 to 20% in the interwar period, 20%-25% in the 1950s-1970s and 30%-35% in the 1990s-2000s. We upgrade the raw fiscal bequest flow accordingly.

We apply the same upward corrections to inter vivos gifts, leaving the gift-bequest ratio v_t unaffected. To the extent that gifts are less well reported to tax authorities than bequests, this implies that we probably under-estimate their true economic importance. Also, in this paper we entirely ignore informal monetary and in-kind transfers between households, as well as parental transfers to children taking the form of educational investments, tuition fees and other non-taxable gifts (which ideally should all be included in the analysis, in one way or another).[46] That is, we only consider formal, potentially taxable gifts.

[45] Gender-based wealth inequality is an important issue. On average, however, women have been almost as rich as men in France ever since the early 19th century (with aggregate women-men wealth ratios usually in the 80%-90% range; this is largely due to the gender neutrality of the 1804 Civil Code; see Piketty et al (2006)). So the aggregate consequences of ignoring gender issues cannot be very large.

[46] Parental transfers to non-adult children and educational investments raise complicated empirical and conceptual issues, however. One would need to look at the financing of education as a whole.

4. The U-shaped pattern of inheritance: a simple decomposition

The accounting equation $B_t/Y_t = \mu_t^* \; m_t \; W_t/Y_t$ allows for a simple and transparent decomposition of changes in the aggregate inheritance flow. Here the important finding is that the long-run U-shaped pattern of B_t/Y_t is the product of three U-shaped curves, which explains why it was so pronounced. We take these three effects in turn: the aggregate wealth-income effect W_t/Y_t, the mortality rate effect m_t, and the μ_t^* ratio effect.

4.1. The aggregate wealth-income ratio effect W_t/Y_t

We already described the U-shaped pattern the aggregate wealth-income ratio β_t (see Figure 4). By comparing this pattern with that of the inheritance flow b_{yt} (see Figure 2), one can see that the 1913-1949 decline in the aggregate wealth-income ratio explains about half of the decline in the inheritance-income ratio. Between 1913 and 1949, β_t dropped from 650%-700% to 200%-250%. I.e. it was divided by a factor of about 2.5-3. In the meantime, b_{yt} dropped from 20%-25% to 4%. I.e. it was divided by a factor of about 5-6.

4.2. The mortality rate effect m_t

Where does the other half of the decline in the inheritance-income ratio come from? By construction, it comes from a combination of μ_t^* and m_t effects. The simplest term to analyze is the mortality rate m_t. The demographic history of France since 1820 is simple. Population was growing at a small rate during the 19th century (less than 0.5% per year), and was quasi-stationary around 1900 (0.1%). The only time of sustained population growth during the past two centuries was due to the well-known postwar baby-boom, with population growth rates around 1% in the 1950s-1960s. Population growth has been declining since then, and in the 1990s-2000s it was approximately 0.5% per year (about a third of which comes from net migration flows). According to official projections, population growth will be less than 0.1% by 2040-2050, with a quasi-stationary population after 2050. Adult population was about 20 millions in the 1820s, 30 millions in the 1950s, 50 millions in the 2010s, and is projected to stabilize below 60 millions.

The evolution of mortality rates follows directly from this and from the evolution of life expectancy. Between 1820 and 1910, the mortality rate was relatively stable around 2.2%-2.3% per year (see Figure 6). This corresponds to the fact that the population was growing

at a very small rate, and that life expectancy was stable around 60, with a slight upward trend (see Figure 7). In a world with a fully stationary population and a fixed adult life expectancy equal to 60, then the adult mortality rate (i.e. the mortality rate for individuals aged 20-year-old and above) should indeed be exactly equal to 1/40 = 2.5%. Since population was rising a little bit, the mortality rate was a bit below that.

There was a purely temporary rise in mortality rates in the 1910s and 1940s due to the wars. Ignoring this, we have a regular downward trend in the mortality rate during the 20th century, with a decline from about 2.2%-2.3% in 1910 to about 1.6% in the 1950s-1960s and 1.1%-1.2% in the 2000s. According to official projections, this downward trend is now over, and the mortality rate is bound to rise in the coming decades, and to stabilize around 1.4%-1.5% after 2050 (see Figure 6). This corresponds to the fact that the French population is expected to stabilize by 2050, with an age expectancy of about 85, which implies a stationary mortality rate equal to 1/65 = 1.5%. The reason why the mortality rate is currently much below this steady-state level is because the large baby-boom cohorts are not dead yet. When they die, i.e. around 2020-2030, then the mortality rate will mechanically increase, and so will the inheritance flow. This simple demographic arithmetic is obvious, but important. In the coming decades, this is likely to be a very big effect in countries with negative projected population growth (Spain, Italy, Germany). In the extreme case where each couple has only one kid, the new cohorts are twice as small as the dying cohorts, and inheritance flows can mechanically become very large.

However the large inheritance flows observed in the 2000s are not due to a mortally rate effect. The U-shaped mortality effect will start operating only in future decades. The 2000-2010 period actually corresponds to the lowest historical mortality ever observed, with mortality rates as low as 1.1%-1.2%. On the basis of mortality rates alone, the inheritance flow in the 1990s-2000s should have been much smaller than what we actually observe.

4.3. The μ_t* ratio effect

So why has there been such a strong recovery in the inheritance flow since the 1950s-1960s, and why is the inheritance flow so large in the 1990s-2000s? We now come to the most interesting part, namely the μ_t* ratio effect. Here it is important to distinguish between the raw ratio μ_t and the gift-corrected ratio $\mu_t^* = (1+v_t)\ \mu_t$. We plot on Figure 8 the historical evolution of the μ_t and μ_t* ratios, as estimated using observed age-wealth-at-death profiles

and differential mortality parameters. We plot on Figure 9 the inheritance flow-private wealth ratio $b_{wt}= m_t\ \mu_t^*$. We also show on Table 2 some of the raw wealth-at-death profiles that we used for the computation of our μ_t series.

Between 1820 and 1910, the μ_t ratio was around 130%. I.e. on average decedents' wealth was about 30% bigger than the average wealth of the living. There was actually a slight upward trend, from about 120% in the 1820s to about 130%-140% in 1900-1910. But this upward trend disappears once one takes inter vivos gifts into account: the gift-bequest ratio v_t was as high as 30%-40% during the 1820s-1850s, and then gradually declined, before stabilizing at about 20% between the 1870s and 1900-1910.[47] When we add this gift effect, i.e. when we take into account the fact that decedents have already given away about 30%-40% of their wealth when they die in the 1820s-1840s, and about 20% of their wealth when they die in the 1870s-1910s, then we find that the gift-corrected μ_t^* ratio was stable at about 160% during the 1820-1913 period (see Figure 8).

During this entire period, cross-sectional age-wealth profiles were steeply increasing up until the very old, and were becoming more and more steeply increasing over time (see Table 2).[48] Here we report and use profiles for the all of France. In Paris, where many of the top wealth holders lived, age-wealth profiles were even more steeply increasing.[49]

The 1913-1949 capital shocks clearly had a strong disturbing impact on age-wealth profiles. Observed profiles gradually become less and less steeply-increasing at old age after World War 1, and shortly become hump-shaped in the aftermath of World War 2 (see Table 2).[50] Consequently, our μ_t ratio estimates declined from about 140% at the eve of World War 1 to about 90% in the 1940s (see Figure 8). The gift-bequest ratio was stable around 20% throughout this period, so the μ_t^* went through a similar evolution.

One possible explanation for this change in pattern is that it was too late for the elderly to recover from the capital shocks (war destruction, capital losses), while active and younger

[47] We know little as to why inter vivos gifts were so high in the early 19th century. This seems to correspond to the fact that dowries (i.e. large inter-vivos gifts at the time of wedding) were more common at that time.
[48] The raw profiles reported on Table 2 do not take into account differential mortality. If one were to use them to compute μ_t ratios without making any correction, then one would find substantially bigger for values for μ_t than those reported on Figure 8, e.g. 171% instead of 135% for 1912. Estimated age-wealth profiles for the living (i.e. after taking into account the differential mortality correction) are also steeply increasing up until the very old. See Appendix B2, Tables B3-B5 for detailed computations and results.
[49] See Piketty, Postel-Vinay and Rosenthal (2006).
[50] The differential-mortality-corrected profiles look even more hump-shaped (see Appendix B2).

cohorts could earn labour income and accumulate new wealth. It could also be that elderly wealth holders were hit by proportionally larger shocks, e.g. because they held a larger fraction of their assets in nominal assets such as public bonds.

The most interesting fact is the strong recovery of the μ_t and μ_t* ratios which took place since the 1950s. The raw age-wealth-at-death profiles gradually became upward sloping again. In the 1900s-2000s, decedents aged 70 and over are about 20%-30% richer than the 50-to-59-year-old decedents (see Table 2).[51] As a consequence, the μ_t ratio gradually rose from about 90% in the 1940s-1950s to over 120% in the 2000s (see Figure 8).

Next, and most importantly, the gift-bequest ratio v_t rose enormously since the 1950s. The gift-bequest ratio was about 20%-30% in the 1950s-1960s, and then gradually increased to about 40% in the 1980s, 60% in the 1990s and over 80% in the 2000s. This is by far the highest historical level ever observed. Gifts currently represent almost 50% of total wealth transmission (bequests plus gifts) in France. That is, when we observe wealth at death, or wealth among the elderly, we are actually observing the wealth of individuals who have already given away almost half of their wealth. So it would make little sense to study age-wealth profiles without taking gifts into account. Gifts are probably less well reported than bequests to the tax administration, so it is hard to see how our tax-data-measured v_t ratio can be over-estimated. If anything, we probably underestimate the gift effect. We do not know whether such as large rise in gifts also occurred in other countries.[52]

The age differential between decedents and donors has remained relatively stable around 7-8 years throughout the 20th century, and in particular during the past few decades. On average, people have always made gifts about 7-8 years before they die. So the impact of gifts on the average age at which individuals receive wealth transfers has been relatively limited. We compute the evolution of the average age of “receivers” (by weighting average age of heirs and average age of donees by the relevant amounts), and we find that the rise

[51] Differential-mortality-corrected profiles are basically flat above age 50 (see Appendix B2). Using the 1998 and 2004 Insee wealth surveys, we find age-wealth profiles which are slightly declining after age 50 (the 70-to-79 and 80-to-89-year-old own about 90% of the 50-to-59-year-old level). However this seems to be largely due to top-wealth under-reporting in surveys. Using wealth tax data (see Zucman (2008, p.68)), we find that the fraction of the 70-to-79 and 80-to-89-year-old subject to the wealth tax (i.e. with wealth above 1 million €) is around 200%-250% of the corresponding fraction for the 50-to-59-year-old (average taxpayers wealth is similar for all age groups). This steeply rising profile does not show up at all in wealth surveys, and might also be under-estimated in estate tax data (e.g. because the elderly hold more estate-tax-exempt assets).

[52] However the upward trend in gifts clearly started before new tax incentives were put in place in the late 1990s and 2000s, so it is hard to identify the the tax effect per se. For additional details, see Appendix B.

of gifts since the 1980s merely led to a pause in the historical rise in the average age of receivers (currently about 45-year-old), but not to an absolute decline.[53]

The most plausible interpretation for this large increase in gifts is the rise in life expectancy: wealthy parents realize that they are not going to die very soon, and decide that they should help their children to buy an apartment or start a business before they die. Tax incentives might also have played a role.[54] There is an issue as to whether such a high gift-bequest ratio is sustainable in the long run, which we address in the simulations.

For the time being, it is legitimate to add the gift flow to the bequest flow, especially given the relatively small and stable age differential between decedents and donors. Consequently, we find that the gift-corrected μ_t* ratio has increased enormously since World War 2, from about 120% in the 1940s-1950s to over 180% in the 1990s and over 220% in the 2000s (see Figure 8).

To summarize: the long run decline in the mortality m_t seems to have been (partially) compensated by a long run increase in the μ_t* ratio. Consequently, the product of two, i.e. the inheritance-wealth ratio $b_{wt}= m_t\ \mu_t$*, declined much less than the mortality rate: b_{wt} was about 3.3%-3.5% in the 19th century (the estate multiplier $e_t=1/b_{wt}$ was about 30), and it is above 2.5% in the 2000s (the estate multiplier is about 40). One obvious explanation as to why wealth tends to get older when age expectancy increases is because individuals wait longer before they inherit. However there are many other effects going on, so it is useful to clarify this simple effect with a stylized model, before moving on to full-fledged simulations.

[53] See Appendix C, Table C8. The slight decline in average age of heirs plotted on Figure 7 for the post-2040 period corresponds to another effect, namely a slight projected rise in average age at parenthood.

[54] According to on-line IRS data, the U.S. gift-bequest ratio is about 20% in 2008 (45 billions $ in gifts and 230 billions $ in bequests were reported to the IRS). Unfortunately, the bequest data relates to less than 2% of U.S. decedents (less than 40,000 decedents, out of a total of 2.5 millions), and we do not really know what fraction of gifts were actually reported to the IRS. On-line IRS tables also indicate steeply rising age-wealth-at-death profiles. This is consistent with the findings of Kopczuk (2007) and Kopczuk and Luton (2007).

5. Wealth accumulation, inheritance & growth: a simple steady-state model

Why is it that the long-run decline in mortality rate m_t seems to be compensated by a corresponding increase in the μ_t ratio? I.e. why does the relative wealth of the old seem to rise with life expectancy? More generally, what are the economic forces that seem to be pushing for a constant inheritance-income steady-state ratio b_{yt} (around 20% of national income), independently from life expectancy and other parameters?

In order to highlight the key effects at play, we develop in this section a stylized theoretical model of wealth accumulation, inheritance and growth. We use various savings models (exogenous savings model, dynastic model, and wealth-in-the-utility model) and derive simple steady-state formulas for the μ_t ratio and the inheritance-income and inheritance-wealth ratios $b_{yt}=\mu_t m_t \beta_t$ and $b_{wt}=\mu_t m_t$. We prove two main results.

First, we show that with pure class savings, then the m_t and μ_t effects exactly compensate one another, so that the steady-state ratios b_{wt} and b_{yt} are simply equal to 1/H and β/H, where H is generation length (age at parenthood) and β is the aggregate wealth income ratio. That is, inheritance ratios do not depend at all on life expectancy or the growth rate.

Next, we show that this result extends to other saving models, assuming that growth rates are relatively low. Typically, for g=1% or g=2%, inheritance ratios are almost exclusively determined by the age at parenthood H – and not so much on life expectancy and other parameters (thereby providing an explanation for the 20% magic number). More generally, we find that the steady-state ratios μ, b_y and b_w always tend to be decreasing functions of the growth rate g and increasing functions of the rate of return r. That is, higher growth and/or lower rates of return reduce the relative importance of inheritance.

The steady-state inheritance formulas developed in this section are simple and can be used with real numbers so as to better understand the quantitative importance of each effect. However they naturally rely on strong demographic and macroeconomic steady-state assumptions, which are at odds with the real world. In section 6, we present simulation results based upon a full-fledged, out-of-steady-state version of this model, using observed demographic and macro shocks, and show that the basic intuitions and results obtained in the stylized steady-state model are robust.

5.1. Notations and definitions

5.1.1. Demography.

We use a relatively standard, Solow-type model of capital accumulation and growth, but with a specific demographic structure. In order to obtain meaningful theoretical formulas for inheritance flows (i.e. formulas that can be used with real numbers), we need a dynamic model with a realistic demographic structure. Models with infinitely lived agents or perpetual youth models will not do, and standard two-period or three-period overlapping generations models will not do either.

To keep notations simple, we consider a continuous-time OLG model with the following deterministic, stationary demographic structure (see Figure 9).[55] Everybody becomes adult at age a=A, has exactly one kid at age a=H>A, and dies at age D>H. As a consequence everybody inherits at age a=I=D-H. This is a gender free population. There is no inter vivos gift: all wealth is transmitted at death. Cohort x is defined as the set of individuals born at time x. Each cohort size N^x is normalized to 1 and includes a continuum [0;1] of agents. So at any time t, the living population includes a mass $N_t(a)=1$ of adult individuals of age a ($A \leq a \leq D$). Total adult population N_t is permanently equal to D-A. The adult mortality rate m_t is also stationary and is given by:

$$\mathbf{m_t = m^* = \frac{1}{D-A}} \qquad \textbf{(5.1)}$$

Example. Around 1900, we have A=20, H=30 and D=60, so that people inherit at age I=D-H=30, and m*=1/(D-A)=1/40=2.5%. Around 2020, we have A=20, H=30 and D=80, so that people inherit at age I=D-H=50, and m*=1/(D-A)=1/60=1.7%.

5.1.2. Production.

We assume a standard two-factor production function, with exogenous productivity growth:

$$\mathbf{Y_t = F(K_t,H_t) = F(K_t,e^{gt}L_t)} \qquad \textbf{(5.2)}$$

[55] All results can be extended to the case with a non-stationary population N_t growing at a fixed rate n (generally by replacing g by g+n in the formulas and results). Below we also relax the deterministic mortality assumption and introduce demographic noise, i.e. the fact that different individuals die at different ages (or have children at different ages), and therefore that different individuals inherit at different ages.

With: Y_t = national income

K_t = physical (non-human) capital

H_t = human capital = efficient labor supply = $e^{gt} L_t$

L_t = labor supply

g = exogenous labor productivity growth rate

Since population N_t=D-A is stationary, so is labor supply L_t. We assume that all adults inelastically supply one unit of labor each year from age a=A until some exogenous retirement age a=R≤D (say, A=20 and R=60), so that aggregate labor supply L_t=R-A.

So in the steady state of our model, everything will grow at some exogenous growth rate g≥0. One might want to plug in endogenous growth models into this setting. By doing so, one could generate interesting two-way interactions between growth and inheritance.[56] However in order to simplify the analysis, and also because growth depends on so many factors on which we know relatively little, we choose in this paper to take the growth rate g as given and to study its one-way impact on aggregate inheritance flows.

For notational simplicity, we also assume away government debt (and government assets). In the closed economy case (no foreign assets),[57] private wealth W_t is exactly equal to the domestic capital stock K_t, and the aggregate private wealth-national income ratio is exactly equal to the domestic capital-output ratio:

$$\boldsymbol{\beta_t = W_t/Y_t = K_t/Y_t \qquad (5.3)}$$

We note $Y_t=Y_{Kt}+Y_{Lt}$ the functional distribution of income, with Y_{Kt}= capital income and Y_{Lt}= labor income. We note $\alpha_t=Y_{Kt}/Y_t$ the capital share, $1-\alpha_t=Y_{Lt}/Y_t$ the labor share, and r_t the average rate of return to private wealth: $r_t = Y_{Kt}/W_t = \alpha_t/\beta_t$.

Example. Typically the wealth-income income ratio β_t = 600%, the capital share α_t = 30%, and the average rate of return to private wealth r_t = 5%.

[56] E.g. with credit constraints high inheritance flows can have a negative impact on growth-inducing investments (high-inheritance low-talent agents cannot easily lend money to low-inheritance high-talent agents). So high inheritance could lead to lower growth, which itself tends to reinforce high inheritance, as we see below. This two-way process can naturally generate multiple growth paths (with a high inheritance, low mobility, low growth path, and conversely). See Piketty (1997) for a similar steady-state multiplicity.

[57] Below we also consider the open economy case. See Appendix E for the corresponding equations.

For simplicity, we assume a Cobb-Douglas production function $F(K,H)=K^{\alpha} H^{1-\alpha}$, which as a first approximation seems to be a reasonably good description of the real world in the long-run.[58] Together with the competitive factor markets assumption and the closed economy assumption, the Cobb-Douglas specification implies that the capital share α_t is permanently equal to α, so that the rate of return is simply an inverse function of the wealth-income ratio: $r_t = \alpha/\beta_t$.

We note y_t, w_t, y_{Kt}, y_{Lt} the per adult averages of all aggregate variables: $y_t=Y_t/N_t=y_{Lt}+r_tw_t$, $w_t=W_t/N_t$, $y_{Kt}=Y_{Kt}/N_t=r_tw_t$, and $y_{Lt}=Y_{Lt}/N_t$. We use subscripts for time t, parentheses for age a and superscripts for cohort x. E.g. $y_t(a)$ is the average income at time t of individuals aged a-year-old at that time (they belong to cohort x=t-a), y_t^x is the average income at time t of individuals who were born at time x (they have age a=t-x), and $y^x(a)$ is the average income at age a of individuals who were born at time x (this happens at time t=x+a).

The aggregate cross-sectional age-wealth and age-labor income profiles at time t are noted $w_t(a)$ and $y_{Lt}(a)$, and the aggregate longitudinal age-wealth and age-labor income profiles followed by cohort x are noted $w^x(a)$ and $y_L^x(a)$.

When we refer to particular individuals we use subscripts i. E.g. y_{Lti} (resp. w_{ti}) is the labor income (resp. the wealth) at time t of a given individual i. In this paper, we are primarily interested in the evolution of aggregate ratios. We use linear saving models, which allow us to solve for aggregate evolutions without keeping track of the intra-cohort distributions of labor income and wealth. So we will mostly concentrate upon per adult averages y_t, w_t, y_{Lt} and age-level averages $y_t(a)$, $w_t(a)$, $y_{Lt}(a)$. But it is worth noting that our results hold not only for the representative-agent interpretation of the model (zero intra cohort inequality), but also for any given level of permanent, intra-cohort labor income inequality stemming

[58] All results below can easily be extended to CES production functions of the form $F(K,H) = [a K^{(\gamma-1)/\gamma} + (1-a) H^{(\gamma-1)/\gamma}]^{\gamma/(\gamma-1)}$, where γ is the constant elasticity of substitution between K and H (γ=1 corresponds to Cobb-Douglas, γ=0 to putty-clay, and γ=∞ to a linear production function). In competitive equilibrium the capital share α is then given by $\alpha=Y_K/Y=r\beta= a\beta^{1-1/\gamma}$. I.e. the capital share α is an increasing function of the wealth-income ratio β if and only if γ>1. The fact that capital shares were slightly below normal levels in historical periods when the wealth-income ratio was below normal levels (e.g. in the 1950s) tends to suggest that γ is somewhat bigger than 1. However the assumption of competitive factor markets is quite heroic, especially during those periods (e.g. rent control and other policies influencing factor prices certainly played a big role at mid-20th century), and so is this inference process. See Appendix A for detailed factor shares series.

from some exogenous heterogeneity in skills or luck or taste or effort, as well as for various structures of idiosyncratic shocks on saving behavior.[59]

5.1.3. Age-labor income profile and pension system

For simplicity, we assume that the cross-sectional age-labor income profile $y_{Lt}(a)$ is flat among adult workers, i.e. all age groups between age a=A and age a=R have the same average labor income at any time t.[60] This assumption of a flat cross-sectional age-labor income profile obviously does not apply to longitudinal profiles: with positive productivity growth g>0, average labor income y_{Lt} grows at rate g in steady-state, i.e. longitudinal age-labor income profiles are upward sloping.

We take as given the existence of an unfunded, pay-as-you-go pension system financed by a flat payroll tax rate τ_ρ on all adult workers and offering a flat replacement rate $\rho \leq 1$ to adults older than retirement age R.[61] To simplify notations, we integrate pension income into labor income $y_{Lt}(a)$. That is, $y_{Lt}(a)$ is equal to "augmented labor income", which we define as net-of-pension-tax labor income for working adults (A<a<R) and pension income for retired adults (R<a<D).[62] In effect we assume a simple two-tier cross-sectional age-labor income profile $y_{Lt}(a)$ among adults (see Figure 10):

If a ☻ [A,R[, $y_{Lt}(a) = (1-\tau_\rho)\,\hat{y}_{Lt}$

If a ☻ [R,D], $y_{Lt}(a) = \rho\,(1-\tau_\rho)\,\hat{y}_{Lt}$

With: $\hat{y}_{Lt} = \dfrac{D-A}{R-A}\, y_{Lt}$ = average pre-tax labor income of adult workers at time t

τ_ρ = budget-balanced pension tax rate = $\dfrac{\rho(D-R)}{R-A+\rho(D-R)}$

[59] We will make this clear in the context of each specific saving model as we go along.

[60] In simulations we use observed, non-flat age-labor income profiles. In France, the current profile $y_{Lt}(a)$ is moderately upward sloping: the average labor income of adults aged 20-to-29 and 30-to-39 is about 70%-80% of the average labor income of those aged 40-to-49 and 50-to-59. See Appendix D, Table D4.

[61] In a world with r>g, it is unclear why people would want to have pay-as-you-go pension systems (whose internal rate of return is by definition equal to g). In order to do a proper welfare analysis, one would need to introduce into the model the reasons why pay-as-you-go systems were introduced in the first place (and why they remain popular in most developed countries), i.e. uninsurable uncertainty about the rate of return on private wealth r. This is in turn would have an impact on the welfare analysis of inheritance and on the structure of optimal taxes. We leave these difficult normative issues to future research.

[62] In what follows we often omit to specify that labor income y_{Lt} actually refers to "augmented" labor income.

In order to offer 100% replacement rates, the pension tax τ_ρ must by definition be equal to the share of pensioners in total adult population, which in the absence of population growth is simply equal to retirement length D-R divided by total adult life length D-A.

In practice, pay-as-you-go pension systems offer significant replacement rates ρ in most developed countries (usually over 50%). In France, ρ is currently about 70%-80%.[63] In the theoretical results below, we use ρ as a free parameter of the model. When we set ρ to 100%, we effectively shut down the life-cycle saving motive. When we reduce ρ, we gradually make lifecycle wealth accumulation more important. This allows us to investigate the quantitative interaction between private wealth accumulation, inheritance flows and the generosity of pay-as-you-go pension systems.

5.1.4. Aggregate inheritance flow

Since each cohort size N^x is normalized to 1, the aggregate inheritance flow B_t is equal to per decedent inheritance b_t. Since everybody dies at age a=D, per decedent inheritance b_t is equal to the average wealth $w_t(D)$ of D-year-old individuals.[64] So the ratio $\mu_t = b_t/w_t$ between average wealth of decedents and average wealth of the living is given by:

$$\boldsymbol{\mu_t} = \frac{b_t}{w_t} = \frac{w_t(D)}{w_t} \qquad \textbf{(5.4)}$$

In order to compute the value of μ_t, we simply need to study the dynamics of the age-wealth profile $w_t(a)$. The inheritance-income ratio $b_{yt} = B_t/Y_t = m_t\, \mu_t\, \beta_t$ and the inheritance-wealth ratio $b_{wt} = B_t/W_t = m_t\, \mu_t$ are then given by applying accounting equations (3.1)-(3.3).

5.2. Steady-state inheritance flow in the exogenous savings model

We start by solving the model with exogenous saving rates. That is, we assume that the average saving rates out of labor income s_L and out of capital income s_K are the same for

[63] That is, the average (augmented) labor income of adults aged 60-to-69, 70-to-79 and 80-and-over is about 70%-80% of the average labor income of those aged 50-to-59. See Appendix D, Table D4. In principle, with A=20, R=60, D=80, the pension tax rate should be τ_ρ=33% for ρ=100% and τ_ρ =26% for ρ=70%. In practice, the French pension tax rate is a bit smaller (it is closer to 20%), thanks to the fact that retired cohorts are somewhat smaller than working cohorts (i.e. population growth n is small but >0).

[64] For simplicity, here we ignore differential mortality (i.e. we implicitly assume uniform mortality rates for the poor and the rich). Of course we do take into account differential mortality in the simulations. In effect, this simply introduces a uniform downward correction factor on b_t. See Appendix B, section B2.

all age groups (in particular there is no dissaving at old age). We take s_L and s_K as given and constant over time.[65]

A special case of this formulation is uniform savings: $s_L=s_K=s$. Another special case is so called "class savings" ($s_L=0$ and $s_K>0$), whereby savings come solely from capital income. In the general case ($s_L\geq 0$, $s_K\geq 0$), the aggregate savings rate s is given by $s=\alpha s_K+(1-\alpha)s_L$, where α is the Cobb-Douglas capital share.

The exogenous savings model is obviously not very satisfactory from an intellectual viewpoint. We later move to more micro founded models.[66] However it provides a useful benchmark and helps to clarify some of the key intuitions. Also note that real world aggregate saving rates happen to be relatively flat with respect to age (consumption tends to track down income pretty closely), or at least much less age-dependant that what most micro models would tend to predict.[67] Whatever the exact explanation for this fact might be (imperfect capital markets, imperfect foresight, etc.), it is useful to know what the implications are for the long dynamics of age-wealth profiles and inheritance flows.[68]

5.2.1. Steady-state wealth-income ratio and rate of return

A well know property of our Solow-type wealth accumulation model is that the long-run wealth-income ratio $\beta_t=W_t/Y_t$ and rate of return r_t are uniquely determined:[69]

[65] I.e. we assume $s_t(a) = s_L y_{Lt}(a) + s_K r_t w_t(a)$ (with $s_t(a)$ = average savings of a-year-old individuals).

[66] One possible micro rationale for class saving behaviour is the dynastic model (see below). The more traditional, Kaldor-Pasinetti-type justification involves income effects: zero-wealth workers have wages below or around subsistence consumption and save little or not at all; while high-wealth capitalists are far above subsistence and save a large fraction of their capital income. One needs however to assume that the subsistence consumption level grows at rate g (maybe because of reference group effects). Our formulation for the general case is closer to Kaldor (1966) than to Pasinetti (1962), as the different savings propensities attach to types of income rather than to classes of people (i.e. once workers have started accumulated wealth from their labor income, their saving rate out of capital income is the same as the capitalists' saving rate; Kaldor's justification for this is corporate savings; but this is not really a micro founded explanation).

[67] See section 6 below and Antonin (2009) for recent estimates using French expenditure surveys.

[68] The flat saving rates that we assume here could come from any micro model, and do not need to be the same for all individuals. Because of linearity, all results below hold for any distribution of savings rates s_{Lti} and s_{Kti} (with both permanent intra-cohort heterogeneity and idiosyncratic within-lifetime shocks), as long as average savings rates s_L and s_K are the same for all age groups. More generally, all results obtained under the exogenous savings model also hold for any distribution of labor income y_{Lti} and rate of return r_{ti}, as long as age-level averages are the same. The exact structure of shocks matters for the steady-state wealth distribution, but has no impact on steady-state aggregate ratios.

[69] This simply comes from the wealth accumulation equation $dW_t/dt = sY_t$, i.e. $d\beta_t/dt = s-g\beta_t=0$ iff $\beta^*=s/g$. If population N_t grows at rate n>0 (or n<0), then one simply needs to replace g by g+n.

<u>Proposition 1</u> (exogenous savings, closed economy)

Assume exogenous saving rates $s_L \geq 0$, $s_K \geq 0$. Note $s=\alpha s_K+(1-\alpha)s_L$ (aggregate savings rate). As $t \to +\infty$, the wealth-income ratio $\beta_t=W_t/Y_t \to \beta^*$ and the rate of return $r_t \to r^*$.

Steady-state β^* and r^* are uniquely determined by: $\boldsymbol{\beta^*= s/g}$ and $\boldsymbol{r^*= \alpha/\beta^*= \alpha g/s}$

Example: If the savings rate s=10% and the growth rate g=2%, then the long-run wealth-income ratio β^*=500%. If the Cobb-Douglas capital share α=30%, then this corresponds to a long-run rate of return r^*=6%.

The Harrod-Domar-Solow formula $\beta^*=s/g$ is a pure accounting equation. It necessarily holds in steady-state, whatever the production function or the savings model might be. If the long run savings rate is equal to s, then in the long run β^* converges toward s/g.[70]

In the Cobb-Douglas specification, the long run rate of return $r^*=\alpha g/s$ can in principle be larger or smaller than the growth rate g, depending on whether on the capital share α is larger or smaller than the savings rate s. In practice however, α is usually much larger s in real world economies, so steady-state r^* is larger than g.[71] In any case, the rate of return r^* is always an increasing function of g. For a given saving rate, higher growth makes capital relatively scarcer, and therefore marginally more productive.

The rate of wealth reproduction $s_K r^*$ is by construction always less than g in steady-state. Otherwise this would not be a steady-state: with $s_K r^*>g$, wealth holders accumulate new wealth at a faster rate than national income growth (even in the absence of any labor income), and the wealth-income ratio rises indefinitely. So $s_K r^* \leq g$. As long as $s_L>0$, one can see that $g-s_K r^*=g(1-\alpha)s_L/s$ is strictly positive: it is equal to the growth rate times the share of labor income savings in total savings. With uniform savings, it is simply equal to $(1-\alpha)g$. The equality $s_K r^*=g$ corresponds to the class savings case $s_L=0$ and $s_K>0$.

[70] The formula $\beta^*=s/g$ was first derived by Harrod (1939) and Domar (1947) using fixed-coefficient production functions, in which case β^* is entirely given by technology, hence the knife-edge growth conclusion (Harrod emphasized the inherent instability of the growth process; Domar stressed the possibility that β^* and s can adjust in case the natural growth rate g+n differs from s/β^*). The classic derivation of the formula with a production function Y=F(K,L) involving capital-labor substitution, thereby making balanced growth path possible, is due to Solow (1956). Authors of the time had limited national accounts at their disposal to estimate the parameters of the formula. In numerical illustrations they typically took β^*=400%, g=2%, s=8%.

[71] Note also that in micro founded models $\alpha<s$ and $r^*<g$ lead to dynamic inconsistencies: the present value of future resources is infinite, so agents should be willing to borrow, not to save. See the dynastic model below.

Example: With s_L=0%, s_K=20%, g=1% and α=30%, then the aggregate savings rate $s=\alpha s_k$=6%. So the long-run wealth-income ratio β*=s/g=600%, and the long-run rate of return r*=α/β*=5%. Wealth holders get a 5% return, consume 80% of it and save 20%, so that their wealth grows at 1%, just like national income. This is a steady-state.

5.2.2. Steady-state inheritance flows: class savings case

How do the steady-state age-wealth profile $w_t(a)$, the μ_t ratio and the inheritance flow ratios b_{yt} and b_{wt} look like in this well-known model? Consider first the class savings case: s_L=0, s_K>0. Then the steady-state age-wealth profile $w_t(a)$ takes a simple form (see Figure 11):

If a ☻ [A,I[, then $w_t(a) = 0$
If a ☻ [I,D], then $w_t(a) = \overline{w}_t$

Since s_L=0, young individuals have zero wealth until the time they inherit. Then, at age a=I, everybody inherits (some inherit very little or nothing at all, some inherit a lot, depending on the cross-sectional distribution, and on average they inherit $b_t=w_t(I)=w_t(D)$), so that average wealth $w_t(a)$ jumps to some positive level $\overline{w}_t = b_t$. Now, the interesting point is that in the cross-section all age groups with age a between I and D has the same average wealth $w_t(a)=\overline{w}_t$. This is because in steady-state the growth effect and the saving effect exactly compensate each other. Take the group of individuals with age a>I at time t. They inherited a-I years ago, at time s=t-a+I. They received average bequests $b_s=w_s(I)$ that are smaller than the average bequests $b_t=w_t(I)$ inherited at time t by the I-year-old. Since everything grows at rate g in steady-state, we simply have: $b_s= e^{-g(a-I)} b_t$. But although they received smaller bequests, they saved a fraction s_K=g/r* of the corresponding return, so at time t their inherited wealth is now equal to: $w_t(a) = e^{s_K r^*(a-I)} e^{-g(a-I)} b_t = b_t = w_t(I) = \overline{w}_t$.

Given this age-wealth profile, the average wealth w_t over all age groups a ☻ [I,D] is given by: $w_t=(D-I)\overline{w}_t/(D-A)=H\overline{w}_t/(D-A)$. It follows that the steady-state ratio $\mu^*=w_t(D)/w_t=\overline{w}_t/w_t$ is entirely determined by demographic parameters:

$$\mu^* = \frac{w_t(D)}{w_t} = \frac{D-A}{H} \qquad \textbf{(5.5)}$$

Once we know μ^*, we can easily compute steady-state inheritance flow ratios $b_w^*=m^*\mu^*$ and $b_y^*=m^*\mu^*\beta^*$. Here the important point is that since the mortality rate $m^*=1/(D-A)$, the product $m^*\mu^*$ is simply equal to one divided by generation length H, and does not depend on adult life length D-A. We summarize these observations in the following proposition:

<u>Proposition 2</u> (class savings, closed economy)

Assume pure class savings: $s_L=0$ & $s_K>0$. As $t\rightarrow+\infty$, $\mu_t\rightarrow\mu^*$, $b_{wt}\rightarrow b_w^*$ and $b_{yt}\rightarrow b_y^*$.

Steady-state ratios μ^*, b_w^* and b_y^* are uniquely determined as follows:

(1) The ratio μ^* between average wealth of decedents and average adult wealth depends solely on demographic parameters: $\boldsymbol{\mu^* = \grave{\mu} = (D-A)/H}$ (>1).

(2) The inheritance flow-private wealth ratio $b_w^*=\mu^*m^*$ and the estate multiplier $e^*=1/b_w^*$ depend solely on generation length H: $\boldsymbol{b_w^* = \grave{b}_w = 1/H}$ and $\boldsymbol{e^* = \grave{e} = H}$

(3) The inheritance flow-national income ratio $b_y^*=\mu^*m^*\beta^*$ depends solely on the aggregate wealth-income ratio β^* and on generation length H: $\boldsymbol{b_y^*= \grave{b}_y = \beta^*/H}$

Proposition 2 is simple, but powerful. It holds for any growth rate g, saving rate s_K, and life expectancy D. It says that societies with a higher life expectancy D will have both lower mortality rates m_t and higher μ_t ratios, and that in steady state both effects will exactly compensate each other, so that the product of the two does not depend on life expectancy. The product $b_{wt}=m_t\mu_t$ will only depend on generation length H, i.e. the average age at which people have children – a parameter which has been relatively constant over the development process (around H=30). If we assume that the wealth-income ratio β^* also tends to be constant in the long run (around $\beta^*=600\%$), then we have a simple explanation as to why the aggregate inheritance flow $b_y^*=\beta^*/H$ always seems to return to approximately 20% of national income.

The intuition is the following: in aging societies with higher life expectancy, people die less often, but they die with higher relative wealth, so that the aggregate inheritance flow is unchanged. In effect, the entire wealth profile is simply shifted towards older age groups: one has to wait longer before inheritance, but one inherits larger amounts, so that from a lifetime perspective inheritance is just as important as before.[72]

Example. Assume $\beta^* = 600\%$ and H=30. Then $b_w^* = 1/H = 3.3\%$ and $b_y^*= \beta^*/H = 20\%$.

[72] In section 7 below, we translate these results expressed in cross-sectional macroeconomic flows into results expressed in longitudinal lifetime resources.

I.e. the aggregate inheritance flow equals 20% of national income, irrespective of other parameter values, and in particular irrespective of life expectancy D.

- Around 1900, we have A=20, H=30 and D=60, so that people inherit at age I=D-H=30. In steady-state, $m^*=1/(D-A)=2.5\%$ and $\mu^*=(D-A)/H=133\%$. Then $b_w^*=m^*\mu^*$ equals 3.3% of private wealth and $b_y^*=m^*\mu^*\beta^*$ equals 20% of national income.
- Around 2020, we have A=20, H=30 and D=80, so that people inherit at age I=D-H=50. In steady-state, $m^*=1/(D-A)=1.7\%$, $\mu^*=(D-A)/H=200\%$. Then $b_w^*=m^*\mu^*$ again equals 3.3% of private wealth and $b_y^*=m^*\mu^*\beta^*$ again equals 20% of national income.

Although this is a very crude model, we believe that this simple result provides the right intuition as to why the historical decline in mortality rates was to a large extent compensated by an historical rise in the relative wealth of decedents. Moreover, as we see below, this intuition obtained in the class savings model generalizes to more general savings behaviour, assuming that the growth rate g is relatively small.

The discontinuous age-wealth profile obtained in this model (see Figure 11) is obviously an artefact due to the deterministic demographic structure, and would immediately disappear once one introduces demographic noise (as there is in the real world), without affecting the results. E.g. assume that individuals, instead of dying with certainty at age a=D, die at any age on the interval [D-d;D+d], with uniform distribution. Then individuals will inherit at any age on the interval [I-d;I+d]. To fix ideas, say that A=20, H=30, D=70 and d=10, i.e. individuals die at any age between 60 and 80, with uniform probability, and therefore inherit at any age between 30 and 50, with uniform probability. Then one can show that the steady-state age-wealth profile has a simple linear shape (see Figure 12), and that the theoretical results of proposition 2 are wholly unaffected.[73]

In the real world, there are several other types of demographic noise (age at parenthood is not the same everybody, fathers and mothers usually do not die at the same time, there is differential mortality, there are inter vivos gifts, etc.), and we take all of these into account in the full fledged simulated model. The important point, however, is that the basic intuition provided by proposition 2 is essentially unaffected by demographic noise.

[73] If $A \le a \le I-d$, then nobody has inherited, so $w_t(a)=0$. If $I-d \le a \le I+d$, then a fraction $(a-I+d)/2d$ has already inherited, and for those individuals the growth and capitalization effects again cancel each other, so that $w_t(a)$ is a linear fraction of age: $w_t(a)=(a-I+d)\,\overline{w}_t/2d$. If $a \ge I+d$, then everybody has inherited, so the age-wealth profile is flat: $w_t(a)=\overline{w}_t$. Average wealth $w_t=[2d\,\overline{w}_t/2+(D-I-d)\,\overline{w}_t]/(D-A)=(D-I)\,\overline{w}_t/(D-A)$ remains the same as before, and so do all other results of proposition 2.

5.2.4. Steady-state inheritance flow: general case

In the general case ($s_L \geq 0$ & $s_K \geq 0$), one can show that steady-state inheritance ratios depend negatively on the growth rate, and converge towards class saving levels as $g \to 0$:

Proposition 3 (exogenous savings model, closed economy)
Assume exogenous saving rates $s_L > 0$, $s_K \geq 0$. As $t \to +\infty$, $\mu_t \to \mu^* = \mu(g) < \grave{\mu}$
Higher growth reduces inheritance: $\boldsymbol{\mu'(g) < 0}$
With low growth, inheritance ratios converge to class saving levels: $\boldsymbol{\lim_{g \to 0} \mu(g) = \grave{\mu}}$

Proposition 3 is a generalization of Proposition 2 and includes it as a special case. The general formula for steady-state $\mu^* = \mu(g)$ turns out to be reasonably simple:

$$\mu(g) = \frac{1 - e^{-(g - s_K r^*)(D-A)}}{1 - e^{-(g - s_K r^*)H}} \qquad \textbf{(5.6)}$$

With $s_L > 0$, the steady-state rate of wealth reproduction $s_K r^*$ is strictly less than the growth rate g, and $g - s_K r^* = g(1-\alpha)s_L/s > 0$. If $s_L \to 0$, then $g - s_K r^* \to 0$. Simple first order approximation using the formula $\mu(g)$ shows that steady-state μ^* then tends toward $\grave{\mu} = (D-A)/H$.[74] This is just a continuity result: as we get closer to class savings, we converge toward the same age-wealth profile and inheritance ratios, whatever the growth rate might be.

The more interesting part of Proposition 3 is that for any saving behaviour ($s_L > 0$, $s_K \geq 0$), steady-state μ^* also tends toward the same class-saving level $\grave{\mu}$ when the growth rate g tends toward 0. In the uniform savings case ($s_L = s_K = s$), $g - s_K r^* = (1-\alpha)g$, so we simply have:

$$\mu(g) = \frac{1 - e^{-(1-\alpha)g(D-A)}}{1 - e^{-(1-\alpha)gH}} \qquad \textbf{(5.7)}$$

First-order approximations again show that $\mu(g) \to \grave{\mu}$ as $g \to 0$. Steady-state inheritance ratios b_w^* and b_y^* also tend toward their class saving levels $\grave{b}_w = 1/H$ and $\grave{b}_y = \beta^*/H$ when growth rates go to zero. Conversely, the higher the growth rate g, the lower the steady-state inheritance ratios $\mu^* = \mu(g)$, b_w^* and b_y^*.

---

[74] For $g - s_K r^*$ small, $\mu(g) \approx \grave{\mu}\,[1 - (g - s_K r^*)(D-A-H)/2]$.

The intuition is the following. With $s_L>0$, the age-wealth profile is less extreme than the class saving profile depicted on Figure 11. Young workers now accumulate positive wealth before they inherit (and accumulate positive wealth even if they never inherit). So the relative wealth of the elderly μ_t will always be lower than under class savings. Since labor income grows at rate g, this effect will be stronger for higher growth rates. With large growth, young workers earn a lot more than their parents. This reduces the importance of inheritance. But with low growth, the inheritance effect increasingly dominates, and the steady-state age-wealth profile looks closer and closer to the class saving profile. So inheritance flows converge towards class saving levels, irrespective of saving behavior.[75]

Next, and most importantly, formulas (5.6)-(5.7) can be used to quantify the magnitude of the effects at play. The point is that convergence towards class saving levels happens very fast. That is, for low but realistic growth rates (typically, g=1% or g=2%), we find that $\mu(g)$ is already very close to $\grave{\mu}$. That is, inheritance-wise, a growth rate of g=1% or g=2% is not very different from a growth rate g=0%.

Example. Assume g=1% and uniform savings ($s=s_K=s_L$). Then for A=20, H=30, D=60, i.e. I=D-H=30, we have $\mu(g)$=129%. This is lower than $\grave{\mu}$=(D-A)/H=133% obtained under class savings, but not very much lower. With β^*=600%, this corresponds to b_y^*=19% instead of b_y^*=20% under class savings. With A=20, H=30, D=80, i.e. I=D-H=50, we get $\mu(g)$=181% under uniform savings instead of $\grave{\mu}$=200% with class savings, and again b_y^*=19% instead of b_y^*=20%. Assuming g=2%, we still get b_y^*=19% with D=60, and b_y^*=17% with D=80, instead of b_y^*=20% in both cases under class savings.[76]

Assume for instance that there was a major structural shift from class saving behaviour in the 19th century to uniform savings in the 20th century (or even to reverse class saving, where all savings come from labor income), for instance because of a structural decline in wealth concentration. This will tend to make wealth less persistent over time, and therefore to reduce the steady-state magnitude of inheritance flows. However with growth rates

[75] See Appendix E, Figures E1-E2. For a given saving rate s, steady-state β^* (and not only μ^*) rises as g decreases, which also pushes towards higher b_y^*. If $s\to 0$ as $g\to 0$, so as to keep $\beta^*=s/g$ and $r^*=\alpha/\beta^*$ constant, then in effect $g/r^*\to 0$ as $g\to 0$, i.e. with low growth the capitalization effect is infinitely large as compared to the growth effect. The extreme case g=0 is indeterminate in the exogenous savings model: if g=0 and s>0, then as $t\to+\infty$, $\beta_t\to+\infty$ and $r_t\to 0$; if g=0 and s=0, then β^* and r^* are entirely determined by initial conditions; in both cases our key result still holds: $\mu_t\to\grave{\mu}$ as $t\to+\infty$.

[76] See Appendix E, Table E1 for detailed computations using formulas (5.6)-(5.7).

around g=1%-2% the effect will be quantitatively extremely modest: the annual inheritance flow will be 17%-19% of national income instead of 20%.

In order to obtain more substantial declines in μ^* and b_y^*, one needs to assume much larger growth rates. E.g. with g=5%, then one gets b_y^*=17% with D=60, and b_y^*=13% with D=80 (again for β^*=600%). This can contribute to explain why inheritance flows remained low during the 1950s-1970s period, when growth rates were indeed exceptionally high.[77]

As $g \to +\infty$, then $\mu^*=\mu(g) \to 1$, $b_w^* \to 1/(D-A)$ and $b_y^* \to \beta^*/(D-A)$. Assume D=80, so that adult life length D-A=60 is twice as long as generation length H=30. Then infinite growth leads to a doubling of the estate multiplier e^* (from e^*=30 to e^*=60), and a division by two of the inheritance flow b_y^* (from b_y^*=20% to b_y^*=10%, for given β^*=600%). In case life expectancy rises to D=110, then the inheritance flow is divided by three. With infinite growth, $b_w^* \to 0$ and $b_y^* \to 0$ as $D \to +\infty$. That is, societies where people die later and later resemble societies where one never dies, and inheritance effectively vanishes. The key point, however, is that this naive intuition only applies to the case with infinite growth. With plausible growth rates, then the inheritance flow b_y^* depends almost exclusively on generation length H, and is little affected by the rise of life expectancy D.

The generosity of the pay-as-you-go pension system also has a limited impact on inheritance flows in this model. Under class savings, the replacement rate ρ has no impact at all, since there is no saving from labor income. With s_L>0, the replacement rate has an effect going in the expected direction: lower pensions make the elderly relatively poorer, thereby reducing μ^* and b_y^*. But for small growth rates, the quantitative impact is again limited. E.g. with g=1% and D=80, then going from ρ=100% to ρ=0% (no pension at all) makes b_y^* go from 18% to 17%.[78] However this is partly an artefact due to the exogenous saving modelling. Here the impact of the pension system is by construction solely due to the mechanical effect going through age-income profiles (for fixed saving rates). In the real world, if there was no pension system at all, at least some individuals would presumably react by saving more while active and by dissaving while retired, i.e. they would adopt non-flat age-saving rates profiles and accumulate lifecycle wealth. In order to address this issue, one needs to use endogenous saving models, which we do below.

[77] Of course the other part of the explanation is that β_t was much smaller than 600% at that time. Here we report the findings obtained for b_y^* under the assumption of a fixed β^*=600%, so as to isolate the effect going through age-wealth profiles and the resulting μ^* ratio.

[78] See Appendix E, Table E1. In Appendix E we provide a closed-form formula for $\mu(g,\rho)$ extending the above formula and show that our key result still holds: ¬ $\rho \leq 1$, $\mu(g,\rho) \to$ Ù as $g \to 0$.

5.2.4. Open economy

So far we could not study separately the effect of g and r on steady-state inheritance flows, since r=r* was entirely determined by g in the long run. This followed from the closed economy assumption. So consider now the opposite extreme case of a small open economy taking as given the world rate of return r>0. We have the following result:

Proposition 4 (exogenous savings model, open economy).
Assume exogenous saving rates $s_L \geq 0$ & $s_K \geq 0$, and a given world rate of return $r \geq 0$.
As $t \to +\infty$, $\mu_t \to \mu^* = \mu(g,r)$. If $r > \theta = g/s_K$, then $\mu(g,r) = \bar{\mu}$. If $r < \theta$, then $\mu(g,r) < \bar{\mu}$.
Lower growth and/or higher rates of return raise inheritance: $\mu'(g) < 0$, $\mu'(r) > 0$.
With low growth and/or high rates of return, inheritance ratios converge to class saving levels: $\lim_{g \to 0} \mu(g,r) = \lim_{r \to \theta} \mu(g,r) = \bar{\mu}$

Strictly speaking, the case $r > \theta$ cannot be a long run outcome. With $s_K r > g$, wealth holders in our small open economy accumulate an infinite quantity of foreign assets (relatively to domestic output and domestic assets) and eventually become the owners of the entire world. This in principle should push downwards the world rate of return r. But it takes a long time to own the entire world. So such a process can apply during many decades. This model is useful to understand the wealth accumulation patterns prevailing in France (and other European countries such as the U.K.) in the 19th century and early 20th century. The French β_t rose from 550%-600% in 1820 to 650%-700% in 1900-1910, and most of the rise came from the accumulation of foreign assets. The case $r > \theta$ is indeed particularly likely to prevail in environments with low growth and high wealth concentration (so that wealth holders can afford re-investing a large fraction s_K of their asset returns). E.g. with g=1% and s_K=25%, the world rate of return r simply needs to be larger than $\theta = g/s_K$ =4%. So if r=5%, then $s_K r$=1.25%, i.e. private wealth grows 25% faster than domestic output, which over a few decades makes a big difference.

What we add to these well-know open economy insights is the inheritance dimension. The interesting result here is that in case $r > \theta$ then μ_t always converges towards its maximum class-saving level $\bar{\mu}$, whatever the growth rate g and the labor saving rate s_L. That is, g or s_L do not need to be infinitely small: they just need to be such that $r > \theta$. Intuitively, labor income as a whole matters less and less along such explosive paths, and the age-wealth profile becomes almost exclusively determined by inheritance receipts.

The case r<θ corresponds to balanced open-economy development paths. In steady-state the stock of net foreign assets (positive or negative) is a constant fraction of domestic output and assets. Here the steady-state μ*=μ(g,r) is determined by the same formula as in the closed economy case, except that now both g and r are free parameters:

$$\mu(g,r) = \frac{1-e^{-(g-s_K r)(D-A)}}{1-e^{-(g-s_K r)H}} \qquad \textbf{(5.8)}$$

The intuition for μ'(g)<0 is the same as before: higher growth raises the relative wealth of the young and reduces the relative wealth of elderly (and therefore the relative importance of inheritance). The intuition for μ'(r)>0 is the opposite: a higher rate of return gives more weight to past inheritance and raises the relative wealth of the elderly.

In the same way as in the closed economy case, the important point about this formula is that it converges very fast to class saving levels as g→0 and/or as r→ θ.

The formula μ(g,r) also shows that the g effect is quantitatively larger than the r effect, because the r effect is multiplied by s_K<1. That is, the absolute growth rate g matters, and not only the differential r-g. For given r-g, the steady-state μ*=μ(g,r) and the corresponding b_y* will be lower for higher g.

Example. Assume r-g=3%, D=80, s_K=20%. If g=1% and r=4%, then we obtain μ*=μ(g,r)=194% and b_y*=19% (i.e. almost as much as the class saving levels μ=200% and b_y*=20%). But if g=5% and r=8%, then we get μ*=μ(g,r)=136% and b_y*=14%.[79]

5.3. Steady-state inheritance flow in the dynastic model

We now move to the infinite-horizon, dynastic model. Each dynasty i is assumed to maximize a utility function of the following form:

$$\mathbf{U_i = \int_{t\geq s} e^{-\theta t}\, u(c_{ti})\, dt} \qquad \textbf{(5.9)}$$

[79] See Appendix E, Table E2 for detailed computations using the μ(g,r) formula.

Where θ is the rate of time preference, c_{ti} is the consumption flow of dynasty i at time t, and $u(c) = c^{1-\sigma}/(1-\sigma)$ is a standard utility function with constant intertemporal elasticity of substitution (IES). The constant IES is equal to 1/σ. Realistic values for the IES are usually considered to be relatively small (typically between 0.2 and 0.5), and in any case smaller than one, i.e. σ is a parameter that is typically bigger than one.

As is well known, the closed-economy steady-state rate of return r* in dynastic models is uniquely determined by the modified Ramsey-Cass golden rule of capital accumulation:[80]

$$\mathbf{r^* = \theta + \sigma g} \qquad \mathbf{(5.13)}$$

The special case g=0 implies r*=θ. More generally, for g≥0, the steady-state rate of return r* is always larger than the growth rate g in the dynastic model.[81] In the same way as in the exogenous saving model, r* is also an increasing function of g.[82]

Once r* is uniquely determined, other aggregates follow. With Cobb-Douglas production, the steady-state wealth-income ratio $\beta_t=W_t/Y_t$ is uniquely determined by: $\beta^*= \alpha/r^*$.

Example: If θ=1%, σ=2, g=2%, then r*=5%. If α=30%, then β*=600%.

It is also well known that any wealth distribution such that the aggregate wealth-income ratio is equal to β* is a steady-state of the dynastic model.[83] Because of the OLG

---

[80] This follows directly from the first-order condition describing the optimal consumption path: $dc_t/dt=(r-\theta)c_t/\sigma$ i.e. utility-maximizing agents want their consumption path to grow at rate $g_c=(r-\theta)/\sigma$. This is a steady-state iff $g_c=g$, i.e. r=r*=θ+σg. If r>r* they accumulate indefinitely, and if r<r* they borrow indefinitely.

[81] Since σ is typically >1, one can be sure that r*=θ+σg>g. In the (unplausible) case where σ<1, then in theory one could have r*<g. However this would then violate the transversality condition, so this would not be a steady-state (the net present value of future income flows would be infinite, and everybody would like to borrow infinite amounts against future resources, thereby pushing r upwards).

[82] The fact that the equilibrium, aggregate rate of return on assets r*(g) is always higher than g and an increasing function of g in standard models (r*=αg/s with exogenous savings, r*=θ+σg with dynastic savings) is well known to macroeconomists (see e.g. Baker et al (2005) for an application to pension projections).

[83] See e.g. Bertola et al (2006, chapter 3). Note however that the steady-state equation r*=θ+σg only applies to the case of perfect capital markets, and in particular in the absence of uninsurable idiosyncratic shocks. So in order to keep the model simple we need to assume that all generations of a given dynasty i have the same labor productivity parameter $\bar{y}_{Lti}=y_{Lti}/y_{Lt}$, i.e. the same relative position in the distribution of labor income of their time. In the absence of any other shock, all generations of a given dynasty i also have the same relative position $\bar{b}_{ti}=b_{ti}/b_t$ in the wealth-at-death distribution of their time. Relative positions for labor income and inherited wealth do not need to be perfectly correlated across dynasties: any stationary joint distribution $G(\bar{y}_{Lti}, \bar{b}_{ti})$ is a steady-state of the dynastic model, as long as the aggregate $\beta_t=w_t/y_t$ is equal to β*. Shocks on dynastic productivity parameters would generate precautionary savings (i.e. high productivity parents would accumulate extra wealth in order to protect their children against the risk of having a low

demographic structure, we need to specify how various generations of the same dynasty act with one another when they are alive at the same time. Consider a dynasty i with successive generations born at time x_i, x_i+H, x_i+2H, etc. At time t∈[x_i+H,x_i+D], generation x_i has age a∈[H,D], and his child born at time x_i+H is also alive and has age a∈[0,I]. We do not want to enter into the modelling of inter vivos gifts, so for simplicity we assume that parents start to care about their children's consumption level only after they die. That is, we assume that generation x_i is maximizing $U_i = \int_{t\geq s} e^{-\theta t}u(c_{ti})dt$ with s=x_i+A, and where c_{ti} denotes the consumption path followed by generation x_i for t∈[x_i+A,x_i+D], by generation x_i+H for t∈[x_i+D,x_i+D+H], and so on. We start with the simplest case:

<u>Proposition 5</u>: (dynastic model, closed economy)

Assume ρ=1, and no borrowing against future inheritance.

Then inheritance ratios in the dynastic model are the same as in the class saving model.

As t→+∞, $\mu_t \rightarrow \mu = (D-A)/H$, $b_{wt} \rightarrow \overline{b}_w = 1/H$ and $b_{yt} \rightarrow \overline{b}_y = \beta^*/H$

The intuition for this result is the following. Since the pay-as-you-go pension system offers 100% replacement rates (ρ=1), there is no need at all for lifecycle saving. We also assume that young age agents cannot borrow against their future inheritance, which in the real world is indeed difficult, if not impossible.[84] With these two assumptions, the consumption and wealth profiles look exactly the same as in the class saving model (see Figure 11):

If a∈[A,I[, $c_t(a)= y_{Lt}(a)$ and $w_t(a)=0$

If a∈[I,D], $c_t(a)= y_{Lt}(a) + (r^*-g)\overline{w}_t$ and $w_t(a)=\overline{w}_t$

That is, until the time they inherit, young workers simply consume their labor income (s_L=0) and accumulate no wealth. Then they inherit. They still consume their full labor income, and in addition they can now consume a fraction 1-s_K of the return to their inherited wealth, and save the rest, with s_K=g/r*. The growth and saving effects again cancel out, so that everybody above inheritance age has the same wealth $\overline{w}_t$. The reason why dynastic agents behave in the same way as in the exogenous class saving model (with a marginal propensity to save out of labor income s_L=0, and a marginal propensity to save out of

productivity). This would lead to higher aggregate wealth accumulation and push steady-state r<r*. This would complicate the resolution of the model, without offering substantial additional insights.

[84] In particular, there is uncertainty about time of parental death, future parental wealth, etc. Also parents may not like to see their children borrow against their own death, and can threaten to disinherit them.

capital income $s_K=g/r^*$) can be phrased as follows. In this deterministic model, the consumption path of every dynasty (poor or rich) grows at rate g in steady-state. Since labor income naturally grows at rate g, zero-wealth dynasties do not need to save out of labor income. However wealth does not naturally grows at rate g. So if wealthy dynasties do not save, and instead consume the full return to their inherited wealth, then their future consumption will not grow. In order to make sure that their wealth and future capital income grows at rate g, they need to save a fraction $s_K=g/r^*$. Now, because $r^*>g$, $s_K=g/r^*<100\%$: wealthy dynasties consume a positive fraction $1-g/r^*$ of the return to their inherited wealth and save the rest.

Therefore the relative wealth of decedents μ_t converges towards class saving level $\grave{\mu}$, and the inheritance ratios b_{wt} and b_{yt} converge toward $1/H$ and β^*/H. So if $H=30$ and $\beta^*=600\%$, then the dynastic model predicts that the inheritance flow should be equal to 20% of national income, whatever the growth rate g and life expectancy D might be.

If we allow for borrowing against future inheritance, and we solve for the time-consistent steady-state (whereby parents anticipate that their children borrow against future inheritance and adjust their saving behaviour accordingly), we find that inheritance ratios will be even larger than the class saving levels. Intuitively, parents leave larger bequests so as to compensate for the fact that children consume part of it before they die:

<u>Proposition 6</u>: (dynastic model, closed economy)
Assume $\rho=1$, and borrowing against future inheritance.
Then inheritance ratios in the dynastic model are larger than in the class saving model.
As $t\rightarrow+\infty$, $\mu_t\rightarrow\mu^*>\grave{\mu}$, $b_{wt}\rightarrow b_w^*>1/H$ and $b_{yt}\rightarrow b_y^*> \beta^*/H$

The borrowing effect can be very large (b_y^* can be well above 20%).[85] But we are not sure that the full borrowing case is empirically relevant. The results obtained with $\rho<1$ and no-borrowing are probably more relevant:

<u>Proposition 7</u>: (dynastic model, closed economy)
Assume $\rho<1$, and no borrowing against future inheritance.
Then inheritance ratios in the dynastic model are smaller than in the class saving model.

[85] See Appendix E for numerical illustrations and Figure E3 for the age-wealth profiles.

As $t \to +\infty$, $\mu_t \to \mu^* = \rho\,[1 - \frac{(1-\alpha)\beta_L}{\beta^*}]$ (with β_L= lifecycle wealth in years of labor income)

If the pay-as-you-go pension system offers replacement rates below 100%, then utility maximizing agents will start accumulating lifecycle wealth $w_{Lt}(a)$, i.e. hump-shaped wealth with maximal value at retirement age a=R and going to zero as a→D. Typically this takes the form of private pension funds. In the context of the dynastic model, this extra form of wealth accumulation will entirely crowd out other forms of wealth (since the aggregate wealth-income ratio $\beta^*=\alpha/r^*$ is fixed). So for instance if lifecycle wealth represents half of aggregate wealth, then steady-state inheritance ratios will be divided by two.

One can show that steady-state lifecycle wealth $\beta_L=w_{Lt}/y_{Lt}$ in this model is given by a relatively simple closed form formula, which we calibrate in order to estimate the size of this effect. If there is no capitalization effect (r-g=0), and no pension system at all (ρ=0), then β_L is given by the standard Modigliani triangle formula. β_L is negligible when retirement length D-R is low, e.g. in the 19th century and early 20th century (indeed it is null if R=D=60). But for late 20th century and early 21st century parameters (say, R=60 and D=70 or D=80), then β_L can be as large as 400%-600%. In theory it can therefore absorb a very large fraction of β*. With r-g>0, β_L is reduced significantly: lifecycle savers benefit from a capitalization effect, so they do not need to save and accumulate as much. Most importantly, pay-as-you go pension systems are an important feature of the real world, and they reduce drastically the need for lifecycle wealth accumulation.

Example. Assume r*=θ+σg=5% and α=30%, so that β*=600%. Assume that the pension system offers a replacement rate ρ=80%, which is roughly the case in France. Then we find that utility maximizing agents will accumulate lifecycle wealth $(1-\alpha)\beta_L$ around 40% of national income for D=70 and 80% of national income for D=80. That is, lifecycle wealth is predicted to represent around 10% of aggregate wealth accumulation.[86] Consequently, the theoretical formulas indicate that the inheritance flow b_y* should be about 18% of national income (rather than 20% in the case ρ=1). The crowding out impact of lifecycle wealth is only slightly larger with higher growth rates. In case ρ=50%, then the crowding out effect is

[86] This is larger than the estimated share of annuitized wealth in French aggregate private wealth, which appears to be less than 5% (mostly through the annuitized fraction of life insurance assets). See Appendix A.

much larger: with g=2%, b_y* falls to 16% if D=70 and to 13% if D=80. With ρ=0% (i.e. no pension system at all), b_y* falls to 12% if D=70 and to 7% if D=80.[87]

These results suggest that the generosity of public pension system can be an important determinant of inheritance flows in advanced economies – together with the growth rate. The dynastic model predicts that steady-state inheritance flows would be approximately divided by two if the pay-as-you-go pension system was abolished. With more realistic pension reforms, the effects are less spectacular, but still significant. E.g. the dynastic model predicts that for a given growth rate (say, g=2%), countries with replacement rates around 70%-80% (such as France or Germany) should have inheritance flows b_y* around 18% of national income, while countries with replacement rates around 50% (such as the U.K. or the U.S) should have inheritance flows around 14%-15% of national income.

These illustrative computations should however be viewed as upper bound estimates of the likely negative impact of lifecycle wealth on inheritance flows. We assume that young workers have perfect foresight and save whatever it takes in order to suffer zero consumption loss at retirement, which might not hold empirically.[88] Most importantly, the 100% crowding out property of the dynastic model is rather extreme, and seems at odd with available evidence.[89] Other models, such as the wealth-in-the-utility model, have less extreme implications. When the lifecycle saving motive becomes more important, then steady-state aggregate wealth accumulation β* also rises (and the rate of return r* declines), so that crowding out is only partial. The fact that r* and β* are entirely pinned down by preference and technology parameters in the dynastic model ($r^*=\theta+\sigma g$, $\beta^*=\alpha/r^*$) has extreme implications regarding other policy issues, which also seem to be at odd with empirical evidence. E.g. the dynastic model implies that when the capital income tax rate τ_K rises from 0% to 30%-40% (which is roughly what happened during the 20th century), then β* should also decline by 30%-40%, so that the after-tax rate of return $(1-\tau_K)r^*$ remains the same as before. Prima facie, the long run β* appears to have been relatively stable around 600%, and after-tax returns seem to have declined accordingly.[90]

---

[87] See Appendix E, Tables E3-E4 and Figures E5-E8 for detailed computations and age-wealth profiles.
[88] We also assume that young workers cannot borrow from future inheritance, and that expected inheritance does not reduce lifecycle saving. This is more realistic than full borrowing, but probably too extreme.
[89] According to Blau (2009), empirical estimates of the crowding-out effect of pension wealth on total household wealth are much closer to zero than to -1. From a different angle, Poterba (2001) finds that there seems to be little effect of demographic changes and retirement patterns on observed asset returns.
[90] For national accounts based series on capital and labor tax rates τ_K and τ_L in France, see Appendix A. With taxes, the dynastic steady-state conditions are $(1-\tau_K)r^*=\theta+\sigma g$, and $\beta^*=\alpha/r^*=(1-\tau_K)\alpha/(\theta+\sigma g)$.

Another reason why lifecycle wealth might not fully crowd out other forms of wealth is the fact that pension funds can be invested in foreign assets – thereby raising the economy's β^* without affecting the rate of return. But again this cannot be addressed in the context of the dynastic model, since opening up the economy leads to degenerate outcomes. If the world rate of return r is above $r^*=\theta+\sigma g$, then the economy accumulates an infinite quantity of foreign assets and eventually owns the rest of the world. And if r is below r*, then the economy borrows indefinitely and is eventually owned by the rest of the world. In order to study open economy issues, one needs to use less extreme models.

5.4. Steady-state inheritance flow in the wealth-in-the-utility model

We now move to the finite-horizon, wealth-in-the-utility model. Each agent i is assumed to maximize a utility function of the following form:

$$\mathbf{V_i = V(\, U_{Ci}\, ,\, w_i(D)\,)} \qquad \mathbf{(5.14)}$$

With: $U_{Ci} = [\, \int_{A\le a\le D} e^{-\theta(a-A)} c_i(a)^{1-\sigma} da\,]^{\frac{1}{1-\sigma}}$

$V(U,w) = (1-s_B)\log(U)+s_B\log(w)$

s_B = share of lifetime resources devoted to end-of-life wealth $w_i(D)$

$1-s_B$ = share of lifetime consumption flow $c_i(a)$ ($a \in [A,D]$)

θ = rate of time preference, $1/\sigma$ = intertemporal elasticity of substitution

One standard interpretation for this formulation is that agents care directly about the bequest $b_i=w_i(D)$ which they leave to the next generation. It could also be that they care about what wealth brings to them. In the presence of uninsurable lifetime shocks (income, health, time of death), people might like the security that goes with wealth. So this utility function can be interpreted as a reduced form for precautionary savings.[91] People might also derive direct utility from the prestige, power and social status conferred by wealth.[92] Presumably the exact combination of these saving motives varies a lot across individuals, just like other tastes.[93] Whatever the interpretation, we have the following results:

[91] See e.g. Dynan, Skinner and Zeldes (2002) for a calibrated model illustrating how plausible uncertainty about end of life health spendings can generate substantial savings and wealth accumulation

[92] See e.g. Carroll (2000), who argues that this wealth-loving model is the best explanation as to why saving rates increase so much with the level of lifetime income. See also Dynan et al (2004) and Kpoczuk (2007).

[93] See Kopczuk and Lupton (2007).

<u>Proposition 8</u> (wealth-in-the-utility model, closed economy)

As $t \to +\infty$, $\mu_t \to \mu^* = \mu(g)$, $b_{wt} \to b_w^* = \mu^* m^*$, and $b_{yt} \to b_y^* = \mu^* m^* \beta^*$

Higher growth reduces inheritance: $\boldsymbol{\mu'(g)<0}$

For reasonable parameter values, and low growth, inheritance ratios are very close to class saving levels: **μ^* close to $\hat{\mu}$ and b_y^* close to β^*/H**

The generosity of the pension system ($\rho \leq 1$) has a small impact on b_y

<u>Proposition 9</u> (wealth-in-the-utility model, open economy).

As $t \to +\infty$, $\mu_t \to \mu^* = \mu(g,r)$, $b_{wt} \to b_w^* = \mu^* m^*$, and $b_{yt} \to b_y^* = \mu^* m^* \beta^*$

Lower growth and/or higher rates of return raise inheritance: $\boldsymbol{\mu'(g)<0, \mu'(r)>0}$.

With reasonable parameter values, and low growth and/or high rates of return, inheritance ratios are very close to class saving levels: **μ^* close to $\hat{\mu}$ and b_y^* close to β/H**

The generosity of the pension system ($\rho \leq 1$) has no impact on b_y.

So we obtain the same general results as with the exogenous saving model and the dynastic model. That is, steady-state inheritance ratios are a decreasing function of the growth rate and an increasing function of the rate or return; for low growth, they are almost exclusively determined by generation length H. Technically, one important difference is that the formulas for steady-state β^*, r^* and μ^* are more complicated in the wealth-in-the-utility model than in other models. In the closed economy case can be solved only by numerical methods. On the other hand the wealth-in-the-utility-function model implies a very simple closed-form formula for the steady-state inheritance flow b_y^*:

$$b_y^* = b_y(g,r) = \frac{s_B \lambda (1-\alpha) e^{(r-g)H}}{1 - s_B e^{(r-g)H}} \qquad (5.15)$$

This formula follows directly from the fact that agents devote a fraction s_B of their lifetime resources (labor income and inherited wealth) to their end-of-life wealth.[94] It holds both in the closed and open economy cases, and for any structure of intra-cohort labor income or preference shocks.[95] The intuition as to why the inheritance-income ratio b_y^* is a rising

[94] This formula applies to the inheritance-domestic income ratio; the formula for the inheritance-national income ratio is more complicated. See Appendix E1. The factor λ corrects for the differences between the lifetime profile of labor income and inheritance flows, and is typically close to 1. See section 7 below.

[95] Agents in this model do not care directly about the welfare of the next generation (but simply about their end-of-life wealth), so we do not need to assume any longer that all generations of a given dynasty i have the same labor parameter. Any shock structure will do, both for the distribution of labor income y_{Li} and of preference parameters s_{Bi}. By linearity, the aggregate steady-state only depends on the average s_B.

function of r-g is straightforward. The excess of the rate of return over the growth rate exactly measures the extent to which wealth coming from the past is being capitalized at a faster pace than the growth rate of current income.

Numerical solutions to the theoretical formulas yield the following results.[96] Assume A=20, H=30, D=80, s_B=10%, and g=1%. Then in the closed-economy case we get r*=4% and b_y*=22%. If life expectancy was instead D=60, we would get instead b_y*=21%. I.e. inheritance ratios are almost exclusively determined by generation length H, and depend very little on life expectancy. With g=2%, we get r*=5% and b_y*=18% (both for D=60 and D=80). One needs to assume much larger growth rates to obtain more significant declines. In the open-economy case, inheritance can reach higher levels. E.g. with D=80, s_B=10%, g=1% and r=5%, then b_y*=30%.

Also, note that in the closed-economy case, the generosity of pay-as-you-go pension system has a smaller impact than in the dynastic model (because crowding out is only partial, which seems more realistic). In the open-economy case, the generosity of the pay-as-you-go pension system has no impact at all: in effect, additional pension wealth is entirely invested abroad, so b_y* is wholly unaffected. Though full international capital mobility is a somewhat extreme assumption, this result seems to capture a plausible intuition explaining why inherited wealth and pension wealth are to a large extent disconnected issues in today's global economy. That is, with high capital mobility, the fact that the generosity of pay-as-you-pension systems varies across countries should have little impact the magnitude of inheritance flows in the various countries.

[96] See Appendix E, Tables E5-E11 for detailed results. Here we report partial results from Table E11 (closed economy case) and Table E7 (open economy case).

6. Simulations

We now come to the full fledged simulations. Our simulated model is conceptually simple. We start from observed demographic data. We also take as given observed national-accounts aggregate values for all macroeconomic variables (growth rates, factor shares, tax rates, rates of return, saving rates). We then make different assumptions about saving behaviour in order to see whether we can replicate observed age-wealth profiles, μ_t ratios and the resulting inheritance flows.

More precisely, we constructed an exhaustive, annual demographic data base on the age structure of the living population and of decedents, heirs, donors and donees in France over the 1820-2008 period. In practice, bequest and gift flows accrue to individuals in several different payments during their lifetime: usually both parents do not die in the same year, sometime individuals receive gifts from their parents, and sometime they receive bequests and gifts from individuals other than their parents. We use the estate tax returns micro-files available since the 1970s (and the historical tabulations broken by decedent and donor age group available for the earlier period), as well as historical demographic data on age at parenthood, in order to compute the exact fraction of bequest and gift flow accruing to each cohort and transmitted by each cohort during each year of the 1820-2008 period. In the simulated model, the value of bequests is endogenous: it depends on the wealth at death of the relevant cohorts, as determined by the endogenous dynamics of the age-wealth profile. But the fraction of the aggregate bequest flow going to each cohort is taken from observed data. Regarding gifts, in some variants we take the observed gift-bequet ratio v_t as given, and in some other variants we assume other gift-bequest ratios (so as to check whether long run patterns are affected by v_t). In all variants, the age structure of donors and donees is exogenously given by our demographic data base.

Regarding the economic side of the model, we proceed as follows. We start from observed factor shares in national income, as measured by national accounts: $Y_t = Y_{Kt} + Y_{Lt}$. We use national accounts tax and transfer series to compute aggregate, net-of-tax labor and pension income $(1-\tau_{Lt})Y_{Lt}$ (where τ_{Lt} is the aggregate labor tax rate, excluding pension payroll taxes). We use income tax data to estimate the age-labor income profile (including pension income) $Y_{Lt}(a)$ throughout the period, which we take as given. On this basis we attribute an average net-of-tax labor and pension income $y_{Lt}(a)$ to each cohort for each

year of the 1820-2008 period. Because we use liner saving models, we do not attempt to model intra-cohort inequality labor income or wealth.

We also take as given the average pre-tax rate of return r_t, which we compute by dividing capital income Y_{Kt} by aggregate private wealth W_t, and the average after-tax rate of return $r_{dt}=(1-\tau_{Kt})r_t$ (where τ_{Kt} is the aggregate capital tax rate). We assume that wealth holders from all age groups get the same average after-tax rate of return r_{dt} on their wealth $W_t(a)$. This is very much a simplifying assumption. In the real world, rates of return vary widely across assets: typically, returns on stock and real estate are much larger than returns on bonds.[97] This might possibly entail systematic differences across age groups.[98] However we know very little on such systematic variations, so as a first approximation attributing the same average return to all age groups seems like the most reasonable assumption.

Our national-accounts approach to average rates of return r_t and r_{dt} also appears to be the most appropriate option. To the extent that national accounts correctly measure annual flows of capital income Y_{Kt} (rental income, interest, dividend, etc.), then r_t and r_{dt} indeed measure the true average rate of return received by holders of private wealth W_t in France over the past two centuries. National accounts are not perfect. But this is arguably the most comprehensive data source we have, and one ought to start from there.

We present two main series of simulations: one for the 1820-1913 quasi-steady-state period, and one for the 1900-2008 U-shaped period (which was then extended to the future). In the first one, we start from the observed age-wealth profile in 1820, and attempt to simulate the evolution of the profile during the 1820-1913 period. In the second one, we start from the observed age-wealth profile in 1900, and attempt to simulate the evolution of the age-wealth profile during the 1900-2008 period. In both cases, the cohort level transition equation for wealth is the following:[99]

$$\mathbf{W_{t+1}(a+1) = (1+q_{t+1})\,[W_t(a) + s_{Lt}Y_{Lt}(a) + s_{Kt}r_{dt}W_t(a)]} \qquad \mathbf{(6.1)}$$

(+ bequests and gifts received – bequests and gifts transmitted)

[97] E.g. according to Barro (2009, Table 1), the average real rate of return on stocks has been as large as 7.5% over the 1880-2005 period, vs. 1.0% for bonds (averages over 11 Oecd countries).
[98] Thanks to linear savings, and because we focus on age-level averages, we do not need to assume that all individuals get the same return: we are just assuming that average returns are the same for all age groups.
[99] The full transition equations, and detailed simulation results, are given in Appendix D.

The real rates of capital gains q_t come from our aggregate wealth accumulation equation.[100] The only parameters on which we need to make assumptions are the labor-income and capital-income savings rates s_{Lt} and s_{Kt}. We make various assumptions on these and analyze the extent to which we replicate observed age-wealth profiles, μ_t ratios and resulting inheritance flows. In all simulations we make sure that the aggregate savings $s_t=(1-\alpha_t)s_{Lt}+\alpha_t s_{Kt}$ (where α_t is the observed, after-tax capital share) is equal to the observed private savings rate s_t, which according to national accounts data has been relatively stable around 8%-10% in France in the long run (see Figure 14).

By construction, the simulated model always perfectly reproduces the aggregate wealth-income ratio $\beta_t=W_t/Y_t$. The name of the game is the following: what assumptions on saving behaviour also allow us to reproduce the observed dynamics of age-wealth profiles, the μ_t ratio and the inheritance flow-national income ratio b_{yt}?

Our main conclusion is summarized on Figures 15a-15b. By making simple assumptions on savings behaviour (namely, class saving for the 1820-1913 period, and uniform saving for the 1913-2008 period), we are able to reproduce remarkably well the observed evolution of the aggregate inheritance flow over almost two centuries. If we then use the model to predict the future, we find that the inheritance flow should stabilize or keep rising, depending on the future evolutions of growth rates and after-tax rates of return.

6.1. Simulating the 1820-1913 quasi-steady-state

The most interesting period to simulate and investigate is maybe the 1820-1913 period. As was already stressed, this is because this time period looks very close to the theoretical steady-state associated to the class saving model, with s_K close to g/r, and s_L close to 0.

The first thing to notice is that the 1820-1913 period was a time when the rate of return to private wealth r was much bigger than the growth rate g. Generally speaking, factor shares appear to have been relatively stable in France over the past two centuries, with a capital share generally around 30% (see Figure 16). Note however that according to the best available data, the capital share during the 19th century was somewhat higher than during the 20th century (30%-40%, vs 20%-30%). Dividing capital shares by aggregate wealth-

[100] See section 3.2 above and Appendix A5.

income ratios, we get average rates of returns to private wealth r_t of about 5%-6% in 1820-1913, much larger than the growth rate, which on average was only 1.0% (see Figure 17).

We run several simulations. If we assume uniform saving rates, then we under-predict somewhat the aggregate evolution of inheritance. Most importantly, we predict an age-wealth profile in 1900-1910 that is flat after age 60 (or even slightly declining after age 70), while the observed profile is steeply increasing, including for the very old. This has a limited impact on the aggregate μ_t and b_{yt} ratios, because at that time few people died after age 70. But this is an important part of the observed data. This shows that uniform saving is an inadequate description of actual savings behaviour at that time. If we assume that all savings came from capital income, which implies $s_K \approx 25\%$-30% and $s_L \approx 0\%$ (instead of $s_K = s_L \approx 8\%$-10%), then we can predict adequately both the evolution of the inheritance-income ratio b_{yt} and the evolution of the age-wealth profiles $w_t(a)$.

Given the very large wealth concentration prevailing at that time, class saving behavior seems highly plausible. The income levels and living standards attained by wealth holders were so much higher than those of the rest of the population that is was not too difficult for them to save 25%-30% of their capital income annually. In order to fully account for the steepness of the age-wealth profile around 1900-1910, one would actually need to assume not only that (most) savings come from capital income, but also that the average saving rate $s_K(a)$ actually rises with age. This could be explained by a simple consumption satiation effect among elderly wealth holders. To properly study this issue, one would need however to explicitly introduce distributional issues and to use micro data.

We also did various sensitivity checks by varying the gift-bequest ratio v_t. In particular, in one variant, we set v_t=0% for the entire 1820-1913 period, i.e. 19th century wealth holders were assumed to make no inter vivos gifts and to hold on their wealth until the die. Of course, this leads us to under-predict the observed inheritance (bequests plus gifts) flow at the beginning of the period. The interesting finding, however, is that we get approximately the same inheritance-income ratio at the end of the period (about 20%) than the observed ratio with gifts (but with an even more steeply increasing age-wealth profile). This validates our methodological choice of adding gifts to bequests. The existence of inter-vivos gifts has an impact on the timing of inheritance receipts, but very little impact on the long run aggregate flow of aggregate wealth transmission.

6.2. Simulating the 20th century chaotic U-shaped pattern

We proceed in the same way for the 20th century. Whether we assume uniform savings or class savings, the model predicts a decline in the μ_t ratio during the 1913-1949 period. The channel through which this effect operates is the one that we already described, i.e. it was too late for the elderly to start re-accumulating wealth again after the shocks. However we get a significantly better fit by assuming that aggregate savings behaviour has shifted from class savings to uniform savings during the 1913-1949 period. For instance, if we look at the inheritance-income ratio at its lowest point, i.e. during the 1950s (4.3%), we predict 5.3% with uniform saving and 6.0% with class saving.

Intuitively, this structural change in saving behaviour could come from the large decline in wealth concentration that occurred during that time: top wealth holders were much less prosperous than they used to be, and they were not able to save as much. It could even be that they saved even less than labor earners, for instance if they tried to maintain their living standards for too long. The other possible interpretation as to why we slightly over predict the observed 1950s inheritance flow (even with uniform saving) is because the capital shocks of the 1913-1949 disproportionally hit elderly wealth holders, e.g. because they held a larger fraction of their wealth in bonds and other nominal assets. In the simulated model, we assume that the shocks (both the destruction shocks and the capital losses) hit all wealth holders in a proportional manner. Finally, it is possible that the gradual rise in age expectancy that occurred during this period led to a rise in lifecycle savings out of labor income. The data we use in this paper is insufficient to settle this issue. Our aggregate approach allows us to adequately reproduce the general pattern over a two century period. But in order to better understand the micro processes at work, one would clearly need to model explicitly distributional issues and to use micro data.

The post 1949 simulations confirm the view that a structural shift from class saving to uniform saving occurred during the 20th century. All saving models predict a strong recovery of μ_t and b_{yt} between the 1950s and the 2000s (especially since the 1970s, due to lower growth rates, see below). But class saving would lead us to over predict the recovery, with an inheritance flow of 16.8% in 2010, vs 14.4% with uniform savings, vs 13.8% with reverse class savings (i.e. zero saving from capital income), vs 14.5% in the observed data. We interpret this as evidence in favour of the uniform saving assumption as an adequate way to describe postwar aggregate savings behaviour (as a first

approximation). This interpretation seems to be consistent with micro evidence from French household budget surveys: aggregate age-saving rates profiles have been quasi-flat during the 1978-2006 period, and do not appear to vary systematically with factor income composition.[101] This is imperfect data, however, and this issue would need to be better addressed in future research, by introducing explicitly distributional effects.

The simulations as a whole also confirm the critical importance of the r>g logic. Also, as predicted by the theoretical formulas, the absolute level of g appears to have a stronger quantitative impact than the differential r-g. This is exemplified by the 1949-1979 period. Growth rates were above 5%, which slowed down considerably the rise of the μ_t ratio. During the 1979-2009 period, growth slowed down to 1%-2%, the rise of the μ_t ratio was more rapid, and so was the recovery of the inheritance-income ratio b_{yt}. This simple growth effect plays a much bigger role than saving behaviour, as predicted by the theory.

Finally, capital taxes play an important role in our simulations. The average rate of return on private wealth $r_t=\alpha_t/\beta_t$ has always been much larger than the growth rate g_t in France, both during the 19th and the 20th centuries (see Table 3). The major change is that the effective capital tax rate τ_{Kt} was less than 10% prior to World War 1, then rose to about 20% in the interwar period, and finally grew to 30%-40% in the postwar period.[102] This had a large impact on the differential between $r_{dt}=(1-\tau_{Kt})r_t$ and g_t. In particular, capital taxes largely explain why the differential was relatively small (but still positive) during the 1949-1979 period, in spite of positive capital gains. In our simulations, this differential has a smaller impact on μ_t and b_{yt} than the absolute growth rate level, but the effect is still significant. We further investigate this issue with 21st century simulations.

6.3. Simulating the 21st century: towards a new steady-state?

In our baseline scenario, we assume that growth rates in 2010-2100 will be the same as the 1979-2009 average (1.7%), that the aggregate saving rate will be the same as the

[101] Using Insee household budget surveys for 1978, 1984, 1989, 1994, 2000 and 2006, one finds aggregate age-saving rates profiles that are rising somewhat until age 40-49, and almost flat above age 40-49: sligltly declining in 1978-1984-1989, flat in 1994-2000, slightly rising in 2006. In any case, these variations across age groups are always very small as compared to variations over permanent income quartiles. To the extent that wealth and capital income are adequately measured in such surveys, average savings rates also seem to vary little with respect to factor income shares. See Antonin (2009).

[102] Inheritance taxes are included, but have always been a small fraction of the total capital taxes, which mostly consist of flow taxes such as the corporate tax, personal capital income taxes, and housing taxes. See Appendix A, Tables A9-A11 for detailed series. There are approximate estimates, based on simplifying assumptions (especially regarding product taxes incidence). But the orders of magnitude seem to be right.

1979-2009 average (9.4%), and that the capital share will be the same as the 2008 value (26%).[103] On the basis of the historical evolutions described in section 3.2 above, we assume that asset prices remain the same (relatively to consumer prices) after 2010.

In this scenario, we predict that the inheritance-income ratio b_{yt} will keep increasing somewhat after 2010, but will soon stabilize at about 16% (see Figure 15a). There are several reasons why this new steady-state level is substantially below the 20%-25% quasi-steady-state level prevailing in 1820-1913. First, our projected growth rate (1.7%) is small, but bigger than the 19th century growth rate (1.0%). Next, our projected after-tax rate of return (3.0%) is substantially smaller than the 19th century level (5.3%).

We then consider an alternative scenario with a growth slowdown after 2010 (1.0%), and a rise of the after-tax rate of return to 5.0%. This could be due either to a large rise in the capital share (say, because of increased international competition to attract capital), or to a complete elimination of capital taxes (which could also be triggered by international competition), or to a combination of the two. Under these assumptions, the inheritance-income ratio converges towards a new steady-state around 22%-23% by 2050-2060, i.e. approximately the same level as that prevailing in the early 20th century (see Figure 15b).

This finding confirms that the rise in life expectancy has little effect on the long run level of inheritance. With low growth and high returns, the inheritance-income ratio depends almost exclusively on generation length H. Detailed results also show that the largest part of the effect (about two thirds) comes from the growth slowdown, versus about one third for the rise in the net-of-tax rate of return. This decomposition is relatively sensitive to assumptions about savings behaviour, however.

We also explored various alternative scenarios. With a 5% growth rate after 2010, and a rise in saving rate to 25%, so as to preserve a plausible aggregate wealth income ratio, inheritance flows converge towards about 12% of national income by 2050-2060. With no rise in savings, inheritance flows converge to about 5%-6% of national income (i.e. approximately the same level as in the 1950s-1960s). But this is largely due to the fall in the aggregate wealth-income ratio. Another equivalent scenario would involve large scale capital shocks similar to the 1913-1949 period, with capital destructions, and/or a

[103] The capital share that has been approximately constant since the late 1980s, but is significantly larger than the level observed in the late 1970s-early 1980s.

prolonged fall in asset prices, due to rent control, nationalization, high capital taxes or other anti-capital policies. Given the chaotic 20th century record, one certainly cannot exclude such a radical scenario. The bottom line, however, is that a return to the exceptionally low inheritance flows of the 1950s-1960s can occur only under fairly extreme assumptions. One needs a combination of exceptionally high growth rates during several decades and a large fall in aggregate wealth-income ratio.

Finally, we made simulations assuming that the gift-bequest ratio v_t did not rise after 1980. This is an important sensitivity check, because the large rise in gifts in recent decades played an important role in the overall analysis. We find a predicted inheritance-income ratio of 15% by 2050, instead of 16% in the baseline scenario. This suggests that the current gift levels are almost fully sustainable. We also simulated the entire 1900-2100 period assuming there was no gift at all. In the same way as for the 1820-1913 period, this has little effect on long run patterns, which again validates the way we treated gifts.

7. Applications & directions for future research

7.1. The share of inheritance in total lifetime resources by cohort

In this paper, we mostly focused on the cross-sectional inheritance flow-national income ratio $b_{yt}=B_t/Y_t$. However this ratio is closely related to another ratio: namely the share of inheritance in the lifetime resources of the currently inheriting cohort, which we note $\hat{\alpha}_t$.

To see why, consider again the deterministic, stationary demographic structure introduced in section 5. Everybody becomes adult at age A, has one kid at age H, inherits at age I=D-H, and dies at age D. Each cohort size is normalized to 1, so that total (adult) population N_t is equal to (adult) life length D-A. Per decedent inheritance $b_t=B_t=b_{yt}Y_t$ and per adult income $y_t=Y_t/(D-A)$. At time t, the cohort receiving average inheritance b_t is the cohort born at time x=t-I. We note $\tilde{y}_t = b_t e^{rH} + \tilde{y}_{Lt}$ the total lifetime resources received by cohort x, where $b_t e^{rH}$ is the end-of-life capitalized value of their inheritance resources, and $\tilde{y}_{Lt}$ is the end-of-life capitalized value of their labor income resources. We define $\hat{\alpha}_t = b_t e^{rH}/\tilde{y}_t$ the share of inheritance in total lifetime resources of this cohort. Assuming flat age-labor income profile $y_{Lt}(a)=y_{Lt}$ (i.e. full replacement rate ρ=1), we have:

$$\tilde{y}_{Lt} = \int_{A\le a\le D} e^{r(D-a)} y_L{}^x(a)\, da = \int_{A\le a\le D} e^{r(D-a)} y_{Lt}\, e^{g(a-I)}\, da$$

$$\text{I.e.} \quad \tilde{y}_{Lt} = \lambda(D-A) y_{Lt}\, e^{rH} = \lambda Y_{Lt}\, e^{rH} = \lambda(1-\alpha) Y_t\, e^{rH}$$

$$\text{With:} \quad \lambda = \frac{e^{(r-g)(I-A)} - e^{-(r-g)(D-I)}}{(r-g)(D-A)} \qquad (7.1)$$

We therefore have a simple formula for $\hat{\alpha}_t$ as a function of b_{yt}:

Proposition 10. Define $\hat{\alpha}_t$ the share of inheritance in the total lifetime resources of the cohort inheriting at time t. Then we have: $\hat{\alpha}_t = \dfrac{b_{yt}}{b_{yt} + \lambda(1-\alpha)}$ (7.2)

With: b_{yt} = inheritance flow-national income ratio

1-α = labor share in national income

λ = correcting factor given by equation (7.1)

The inheritance share $\hat{\alpha}_t$ can be viewed as an indicator of the functional distribution of resources accruing to individuals. During their lifetime, individuals from cohort x receive on average a fraction $\hat{\alpha}_t$ of their resources through inheritance, and a fraction 1-$\hat{\alpha}_t$ through their labor income. $\hat{\alpha}_t$ is simply related to the standard cross-sectional capital share α. If λ≈1, which as we see below is typically the case, then $\hat{\alpha}_t > \alpha$ iff $b_{yt} > \alpha$. That is, the share of inheritance in lifetime resources is larger than the capital share in national income if and only if the inheritance flow is larger than the capital share. In general, both cases can happen: there are societies where the capital share is large but the inheritance share is low (say, because most wealth comes from lifecycle accumulation), and conversely there are societies where the inheritance is large but where the capital share is low (say, because capital serves mostly as storage of value and produces little flow returns).

It is interesting to see that in practice the inheritance share $\hat{\alpha}$ and the capital share α happen to have the same order of magnitude (typically around 20%-30%) – mostly by coincidence, as far as we can see. Proposition 10 is pure accounting, and it holds for any saving model, both in and out of steady-state (one simply needs to use time-varying g_t and r_t to compute λ). If we now apply Proposition 10 to the steady-state models analyzed in section 5, then we just need to replace b_{yt} by the relevant steady-state value b_y. So for instance in the class saving model or in the dynastic model, we have $b_y=\beta/H$, so that:

$$\hat{\alpha} = \frac{b_y}{b_y + \lambda(1-\alpha)} = \frac{\beta}{\beta + \lambda(1-\alpha)H} \qquad (7.3)$$

Example. With benchmark values β=600%, H=30, α=30%, λ=1, we have b_y=20%, and $\hat{\alpha}=b_y/(b_y+1-\alpha)$=22%. That is, in steady-state each cohort derives $\hat{\alpha}$=22% of its lifetime resources through inheritance, and 1-$\hat{\alpha}$=78% through labor. To put it differently, inheritance resources represents $\gamma=b_y/(1-\alpha)$=29% of their labor resources.

We now come to the correcting factor λ. Intuitively, λ corrects for differences between the lifetime profile of labor income flows and the lifetime profile of inheritance flows. That is, λ measures the relative capitalized value of 1€ in labor resources vs 1€ in inheritance resources, given the differences in lifetime profile between both flows of resources.

In the stylized model with deterministic demographic structure, all inheritance flows come at age a=I, while labor income flows come from age a=A until age a=D. The flows received before age a=I are smaller in size but needs to be capitalized; the flows received after age a=I are larger in size but needs to be discounted. In case r-g=0%, then the growth and capitalization effects cancel each other, so λ is exactly equal to 100%. Simple first order approximations using the λ formula (equation (7.1) above) also show that if inheritance happens around mid-life (say, A=20, H=30, D=80, I=D-H=50), then λ will tend to be close to 100% even if r-g>0.[104] When inheritance happens early in adult life (say, A=20, H=30, D=80, I=D-H=30), then λ is below 100%. Flows of resources accruing earlier in life are worth more from a lifetime, capitalized value perspective. Since inheritance flows were received relatively earlier in life one century ago, this effect implies that – other things equal – the relative importance of labour income should have increased over time.

Example. Assume r-g=3% (say, g=2%, r=5%). With A=20, H=30, D=80, then λ=114%. With A=20, H=30, D=60, then λ=79%.[105]

In practice, however, there are several other counteracting effects. In the real world, individuals receive bequests and inter vivos gifts at different point in their life (and not only at age a=I), and the gift-bequest ratio has risen over time. Also the cross-sectional age-labor income profile is not flat: young and old individuals receive smaller average labor income and middle age individuals.

So we use our simulated model, based upon observed and simulated data on the complete age profiles of bequest, gift and labor income receipts, in order to compute the correcting factor λ^x for all cohorts born in France between x=1800 and x=2030. We find that λ^x has been remarkably constant around 90%-110% over two centuries, with no long run trend. Since we observe bequest and gift flows until 2008, the latest cohorts for which we have complete (or near complete) observed data are those born in 1950s-1960s, for whom the λ^x factor is about 100%-110%. For cohorts born in the 1970s and later, our computations increasingly rely on our simulations on future inheritance flows, i.e. on our assumptions about 2010-2100 growth rates and rates of return. Under the benchmark scenario (g=1.7%, $(1-\tau_K)r$=3.0%), we find that λ^x will be stable around 100%-110% for

[104] $\lambda = [e^{(r-g)(I-A)} - e^{(r-g)(I-D)}]/(r-g)(D-A) = 1 + (r-g)(2I-A-D)$. With I=(A+D)/2, the first-order term disappears.
[105] See Appendix E, Table E5 for illustrative computations using the λ formula.

cohorts 1970-2030. Under the growth slowdown-rising wealth returns scenario (g=1.0%, $(1-\tau_K)r$=5.0%), we find that λ^x will be rising to about 110%-120% for cohorts 1970-2030.[106]

We also use our simulated model in order to compute the capitalized value of lifetime resources $\tilde{y}^x = \tilde{b}^x + \tilde{y}_L{}^x$ for all French cohorts born between x=1800 and x=2030. Unsurprisingly, we find that the inheritance share in lifetime resources $\hat{\alpha}^x = \tilde{b}^x / \tilde{y}^x$ has been following a marked U-shaped pattern: $\hat{\alpha}^x$ was about 20%-25% for 19th century cohorts, fell to less than 10% for cohorts born in the 1900s-1930s, then gradually rose to 15%-20% for cohorts born in the 1950s-1960s, and is expected to stabilitize around 20%-25% for cohorts 1970-2030 (benchmark scenario). If we instead plot the ratio $\gamma^x = \tilde{b}^x / \tilde{y}_L{}^x$ between average inheritance resources and average labor resources ($\gamma^x = \hat{\alpha}^x/(1-\hat{\alpha}^x)$), then all levels are simply shifted upwards. I.e. 19th century cohorts received in inheritance the equivalent of about 30% of their lifetime labor income; this figure declined to about 12% for cohorts 1900-1930, and is projected to be about 30% for cohorts 1970-2030 (see Figure 19a).

Here it might be useful to give some orders of magnitude. Consider the cohorts born in the 1960s, who have already received a large fraction of their gifts and bequests in the 1990s-2000s. We find that their average lifetime resources, capitalized at age 50 (in the 2010s), are about 1.78 millions €, out of which about 320,000€ come from inheritance, and about 1.46 millions € come from labor income.[107] So we have: $\hat{\alpha}^x$=18% and γ^x=22%. Given that λ is close to 1, these average labor income resources roughly correspond to the product of average per adult labor income (currently about 25,000€ in France) by average adult life length (about 60 years). With the cohorts born in the 1970s, we find 2.02 millions €, 440,000€ and 1.58 millions €. So $\hat{\alpha}^x$=22% and γ^x=28%. On Figure 19a we therefore plot γ^x=22% for the 1960s and γ^x=28% for the 1970s.

As predicted by the simplified theoretical model (Proposition 10), the historical evolution of the cohort-level inheritance-labor income ratio γ^x (Figure 19a) is the mirror image of the pattern found for the cross-sectional inheritance flow-national income ratio b_{yt} (Figure 15a). There are two interesting differences, however.

[106] See Appendix D, Tables D7-D8 for detailed simulation results.
[107] See Appendix D, Table D7. Values are expressed in 2009 euros. As far as the shares are concerned, it is of course irrelevant at what age we capitalize lifetime resources (as long as we use the same age for inheritance and labor resources, and a common rate of return).

First, the U-shaped pattern is less marked for γ^x than for b_{yt}. At its lowest point, i.e. in the 1950s, the inheritance flow b_{yt} was less than 5% of national income. In comparison, the lowest point of γ^x, which was attained for cohorts born in the 1900s-1930s, is somewhat above 10%. This is because all members of a given cohort do not inherit exactly at the same time. E.g. cohorts born in the 1900s-1930s inherited everywhere between the 1940s and 1970s. So when we compute cohort level averages of inheritance resources, we tend to smooth cross-sectional evolutions of the inheritance flow-national income ratio. The cohort level pattern is nevertheless quite spectacular. Cohorts born in the 19th century were used to receive by inheritance the equivalent of about 30% of their lifetime labor income. This figure suddenly fell to little more than 10% for cohorts born in the 1900s-1930s, and it took several decades before returning to 19th century levels. The point is that cohorts born in the 1900s-1930s (and to a lesser extent those born in the 1940s-1950s) had to rely mostly on themselves in order to accumulate wealth. Maybe it is not too surprising if they happen to be strong believers in lifecycle theory.

Next, it is striking to see that in our benchmark simulations $\hat{\alpha}^x$ and γ^x attain approximately the same levels for cohorts born in the 1970s and after as for 19th century cohorts ($\hat{\alpha}^x \approx$20%-25%, $\gamma^x \approx$30%), in spite of the fact that we project b_{yt} to stabilize below 19th century levels (15%-16% instead of 20%-25%). This is due to a differential tax effect. $\hat{\alpha}^x$ and γ^x were computed from the simulated model, which uses observed after-tax resources, so these are effectively after-tax ratios. The aggregate labor income tax rate τ_L rose from less than 10% in the 19th century-early 20th century to about 30% in the late 20th century-early 21st century.[108] The aggregate inheritance tax rate has remained relatively small throughout the 19th-20th centuries (about 5%, with no trend).[109] This mechanically raises the after-tax value of inheritance resources relatively to labor resources. Since modern fiscal systems tax labor much more heavily than inherited wealth, the inheritance flow-national income ratio does not need to be as large as during the 19th century in order to generate the same share of inheritance in disposable lifetime resources.[110]

[108] See Appendix A, Table A11, col.(11). Here we exclude pension-related payroll taxes from labor income taxes (otherwise the aggregate labor tax rate would exceed 50%, see col.(9)). This follows from the fact that we treat pensions as replacement income, i.e. as part of (augmented) labor income.

[109] See Appendix A, Table A9, col.(15). Inheritance taxes were included in capital income flow taxes τ_K, which can be questioned. Given their low level, however, a direct imputation method would not make a big difference to our α^{x*} and γ^x estimates. For a discussion of tax incidence issues, see Appendix A2.

[110] One might argue that the rise of taxes allowed for the rise of government services (e.g. education, health), and that this should be added to income. However these services are generally open to everybody, irrespective of whether one lives off labor income or inheritance. So as a first approximation α^{x*} and γ^x appear to be consistent measures of the aggregate share of inheritance in disposable lifetime resources.

For illustrative purposes, we did the same computations with the growth slowdown-rising wealth returns scenario (g=1.0%, $(1-\tau_K)r$=5.0%), under which b_{yt} is projected to return to the 19th century levels (see Figure 15b). Because of the differential tax effect, we project that $\hat{\alpha}^x$ will be about 25%-30% for cohorts born in the 1970s-1980s, and as large as 35%-40% for cohorts born in the 2010s-2020s, which corresponds to an inheritance-labor ratio γ^x over 60%.[111] That is, we project that cohorts born in the coming years will receive in inheritance the equivalent of over 60% of what they will receive in labor income during their entire lifetime, far above 19th century levels (see Figure 19b). This shows that taxes can have a strong impact on the balance between inheritance and labor resources.

7.2. Labor-based vs inheritance-based inequality

Now that we have computed the inheritance share in average lifetime resources, we are in a position to put inequality back into the picture. Our objective is to illustrate that changes in aggregate ratios $\hat{\alpha}^x$ and γ^x matter a great deal for the study of inequality. We do this by making simple assumptions about the intra-cohort distributions of labor income and inheritance, taken from the recent literature on top income and top wealth shares.

Our distributional assumptions are summarized on Table 4. The inequality of labor income has been relatively stable in France throughout the 20th century. So we assume constant shares for the bottom 50%, the middle 40%, and the top 10% of the intra-cohort distribution of labor income for all cohorts born in 1820-2020. Wealth concentration has always been much larger than that of labor income. It was particularly high during the 1820-1913 period, when the top 10% (the “upper class”) owned over 90% of aggregate wealth, with little left for the middle 40% (the “middle class”) and the bottom 50% (the “poor”). Basically, there was no middle class. Today, the poor still own less than 5% of aggregate wealth. But the middle class share rose from 5% to 35%, while the upper class share dropped from 90% to 60%. Wealth concentration declined mostly during the 1914-1945 period, and seems to have stabilized since the 1950s-1960s (as a first approximation).[112] For simplicity, we apply 1910 inherited wealth shares by fractiles to all cohorts born in 1820-1870, we apply 2010 shares to all cohorts born in 1920-2020, and we assume linear changes in shares for cohorts born between 1870 and 1920.

[111] See Appendix D, Table D8 for detailed simulation results.
[112] For a detailed analysis of historical changes in wealth concentration in France, see Piketty et al (2006).

By applying these assumptions to the lifetime inheritance-labor income resources ratio γ^x plotted on Figure 19a, we obtain the inequality indicators plotted on Figures 20a-23a (benchmark scenario). Consider first the ratio between the lifetime resources available for the top 50% successors and those available for the bottom 50% labor earners. Since the top 50% wealth share has been stable at 95%, and the bottom 50% labor income share has been stable 30%, this ratio follows exactly the same U-shaped pattern as the aggregate γ^x, with levels multiplied by about three. In the 19th century, the top 50% successors received in inheritance about 100% of what the bottom 50% labor earners received in labor income throughout their lifetime. Then this ratio dropped to 30%-40% for cohorts born in the 1900s-1930s. According to our computations, this ratio has now well recovered, and is be about 90% for cohorts born in the 1970s-1980s (see Figure 20a).

Take again the example of the cohorts born in the 1970s. On average they will receive 440,000€ in inheritance. But the bottom half will receive almost no inheritance (40,000€), while the upper half will receive almost twice this amount (840,000€). This is roughly what the bottom 50% labor earners will receive in labor income (950,000€). So we get the ratio of 88% plotted for the 1970s on Figure 20a. On average, the bottom 50% labor earners earn little more than the minimum wage: their lifetime labor income roughly corresponds to the product of about 15,000€ by adult life length (about 60 years). For the sake of concreteness they can be thought of as minimum wage workers.

Consider now the ratios between what top 10% and top 1% successors receive in inheritance and what minimum wage workers receive in labor income. Due to the decline in wealth concentration, these inequality indicators are still lower for current generations than what they used to be in the 19th century. But they are much higher than what they used for cohorts born in 1900-1940, in spite of the fact that intra cohort distributions have remained the same. This illustrates the importance of changes in the aggregate ratio γ^x.

For cohorts born between the 1900s and the 1950s, it was almost impossible to become rich through inheritance. Even if you belong to the top 10% or top 1% successors, or if you marry with such a person, the corresponding lifetime resources would be a lot smaller than those you can attain by making your way to the top 10% or top 1% of the labor income hierarchy of your time (see Figures 21a-22a). This is what most people would describe as

a “meritocratic society”. Material well-being required high labor income. For the first time maybe in history, it was difficult to live as well by simply receiving inheritance.

In the 19th century, the world looked very different. Top 10% inheritance resources were roughly equivalent to top 10% labor resources. Top 1% inheritance resources were almost three times as large as top 1% labor resources. I.e. top rentiers vastly dominated top labor earners. If you want to attain high living standards in the 19th century, then inheriting from your parents or your spouse’s family is a much better strategy than work. This looks very much like a “rentier society”.

Life opportunities open to today’s generations are intermediate between the meritocratic society of the 1900-1950 cohorts and the rentier society of the 19th century. For cohorts born in the 1970s, we find that the lifetime resources attained by the 1% successors and top 1% labor earners will be roughly equivalent. I.e. finding a top 1% job or a top 1% spouse will get you to the same living standards: you obtain about 10 millions € in both cases (see Table 5). In the 19th century, the spouse strategy was three times more profitable. For early 20th century cohorts, the job strategy was twice more profitable.

The decline in wealth concentration makes it less likely to inherit sufficiently large amounts to sustain high living standards with zero labor income. But it makes it more likely – for a given aggregate inheritance-labor ratio γ^x – to receive amounts which are not enough to be a rentier, but which still make a big difference in life, at least as compared to what most people earn. Using standard Pareto assumptions on the shape of the intra cohort distribution of inherited wealth, we find that the cohort fraction inheriting more than minimum wage lifetime income (about 950,000€ for 1970s cohorts) was less than 10% in the 19th century, and will be as large as 12%-14% for cohorts born in the 1970s-2000s. Among cohorts born in the 1900s-1930s, this almost never happened: only 2%-3% of each cohort inherited that much (see Figure 23a).

We did the same computations under the low-growth, high-return scenario (see Figures 20b-23b). Unsurprisingly, given that we project the aggregate inheritance-labor ratio γ^x to rise well above 19th century values, we also find that our lifetime inequality indicators reach unheard of levels. At the top 1% level, the spouse strategy again becomes almost three times more profitable: the aggregate effect entirely compensates the distribution effect.

These computations should be viewed as illustrative and exploratory. They ought to be improved in many ways. First, progressive taxation of inheritance and labor income can obviously have a strong impact on such inequality indicators, both in the short run (mechanical effect) and in the long run (endogenous intra-cohort distribution effect). Here we ignored progressive taxes altogether. I.e. in our aggregate computations we simply assumed that inheritance and labor income taxes were purely proportional.

Next, we made no assumption about the individual-level rank correlation between the intra-cohort distributions of inheritance and labor income. Our inequality indicators hold for any joint distribution $G(\tilde{b}_i^x, \tilde{y}_{Li}^x)$. In practice, $\text{corr}(\tilde{b}_i^x, \tilde{y}_{Li}^x)$ might be endogenous. With publicly financed education and the lessening of credit constraints, one might expect the correlation to decline over time. But this could be counterbalanced by the fact that top heirs now need to work in order to reach the same relative living standards as in the past. So the correlation might have increased. It could also be that the moral value attached to work has risen somewhat, so that top successors work more than they used to. Or maybe they have always worked. We do not know of any evidence on this interesting issue.

Finally, we looked at a country with a relatively stable distribution of labor income. So for simplicity we assumed full stability, including at the top. In practice, the top 1% share actually rose a little bit in France in the late 1990s-early 2000s (from about 6% to 7%-8% of aggregate labor income). This is too small a trend to make a significant difference so far. But if we were to make the same computations for the U.S., where the top 1% share rose from 6%-7% in the 1970s to 15%-20% in the 2000s, this would have strong and contradictory impacts on our inheritance-labor inequality indicators. The rise of the working rich reduces the inequality between top successors and top labor earners. But it increases the inequality between the working poor and successors as a whole. It also has dynamic effects on the future intra-cohort distributions of inherited wealth.

7.3. The share of inheritance in aggregate wealth accumulation

The inheritance flow-national income ratio $b_{yt}=B_t/Y_t$ analyzed in this paper is also closely related to the share of inheritance in aggregate wealth accumulation, which we note φ_t.

There are two competing definitions of φ_t in the economics literature. Modigliani (1986, 1988) define φ_t^M as the share of non-capitalized past bequests in total wealth, while Kotlikoff and Summers (1981, 1988) use the share of capitalized past bequests φ_t^{KS} :

$$\varphi_t^M = \hat{B}_t/W_t \text{ , with: } \hat{B}_t = \int_{s\leq t} B_{st}\, ds \qquad (7.4)$$

$$\varphi_t^{KS} = \tilde{B}_t/W_t \text{ , with: } \tilde{B}_t = \int_{s\leq t} B_{st}\, e^{r_{st}}\, ds \qquad (7.5)$$

With: B_{st} = aggregate bequests received at time s by individuals who are still alive at time t
r_{st} = cumulated return to wealth between time s and time t

Consider again the deterministic, stationary demographic structure introduced in section 5. Everybody becomes adult at age A, has one kid at age H, inherits at age I=D-H, and dies at age D. Each cohort size is normalized to 1, so that total (adult) population N_t is equal to life length D-A. Along a steady-state path with constant growth rate g, rate of return r, wealth-income ratio $\beta=W_t/Y_t$ and inheritance flow-income ratio $b_y=B_t/Y_t$, we have:[113]

$$\varphi_t^M = \int_{t-H\leq s\leq t} \frac{b_y}{\beta}\, e^{-g(t-s)}\, ds = \frac{b_y}{\beta}\,\frac{1-e^{-gH}}{g} \qquad (7.6)$$

$$\varphi_t^{KS} = \int_{t-H\leq s\leq t} \frac{b_y}{\beta}\, e^{(r-g)(t-s)}\, ds = \frac{b_y}{\beta}\,\frac{e^{(r-g)H}-1}{r-g} \qquad (7.7)$$

Proposition 11. Define φ_t^M the non-capitalized bequest share in aggregate wealth and φ_t^{KS} the capitalized bequest share. In steady-state: $\varphi^M=\frac{b_y}{\beta}\,\frac{1-e^{-gH}}{g}$ and $\varphi^{KS}=\frac{b_y}{\beta}\,\frac{e^{(r-g)H}-1}{r-g}$

Equations (7.6)-(7.7) are again pure accounting equations. They hold for any saving model. If we now apply them to the saving models analyzed in section 5, then we just need to replace b_y by the relevant steady-state value. So for instance in the class saving model or in the dynastic model, we have $b_y=\beta/H$. Therefore:

$$\varphi^M = \frac{1-e^{-gH}}{gH} \quad \text{and} \quad \varphi^{KS} = \frac{e^{(r-g)H}-1}{(r-g)H}$$

[113] Alternatively one can replace b_y/β by b_w ($=B_t/W_t$) in equations (7.6)-(7.7).

It immediately follows that ┐ g>0, φ^M<1, and ┐ r-g>0, φ^{KS}>1.

Example: With H=30, then φ^M=75% if g=2% and φ^M=52% if g=5%.
If r-g=3%, then φ^{KS}=162%. If r-g=5%, then φ^{KS}=232%.[114]

More generally, if steady-state b_y is close to β/H, or not too much below, which as we saw in section 5 is generally the case with low growth and/or high returns, the same properties hold. That is, φ^M is structurally below 100%, while φ^{KS} is structurally above 100%.

The Modigliani definition φ^M is particularly problematic, since it fails to recognize that inherited wealth produces flow returns. This mechanically leads to artificially low values for the inheritance share φ^M in aggregate wealth accumulation. It is particularly puzzling to see that φ^M can be equal to 75% or 52% in the class saving model – a model where by construction 100% of wealth comes from inheritance, and where successors are just consuming part of the return to their inheritance and saving the rest. As was pointed out by Blinder (1988), a Rockefeller with zero lifetime labor income would appear to be a life-cycle saver in Modigliani's definition, as long he does not consume the full return to his inherited wealth. In effect, Modigliani defines saving as labor income plus capital income minus consumption (and then defines lifecycle wealth as the capitalized value of past savings, and inherited wealth as aggregate wealth minus lifecycle wealth), while Kotlikoff-Summers define saving as labor income minus consumption. Given that the capital share is generally larger than the saving rate, this of course makes a big difference.

The Kotlikoff-Summers definition is conceptually more consistent. But in a way it suffers from the opposite drawback. For reasonable parameter values, φ^M is bound to be larger than 100% (or close to 100%). It is also extremely sensitive to the exact value of r-g.

By applying both definitions φ_t^M and φ_t^{KS} (out-of-steady-state equations (7.4)-(7.5)) to our simulated model based upon two-century-long observed French data, we find the following results (see Figures 24a-b and 25a-b). The uncapitalized inheritance share was about 80% of aggregate wealth during the 19th century and until World War 1. It then dropped to 50%-60% in the 1930s-1950s, and to 40% in the 1960s-1980s. It is interesting to note that the historical nadir happens rather late for φ_t^M (in the 1970s), much later than the historical

[114] See Appendix E, Table E12 for illustrative computations using these formulas.

nadir for b_{yt} (which occurred in the 1950s). This time lag simply stems from the fact that φ_t^M is based upon the cumulated value of b_{yt} of the previous decades. In the benchmark scenario, we find that φ_t^M will be above 60% in the 2010s and should stabilize above 70% after 2040 (see Figure 24a). In the low-growth, high-return scenario, we find that φ_t^M stabilizes above 80% during the 21st century – as in the 19th century (see Figure 24b).

When we capitalize past bequests, we find that φ_t^{KS} is always above 100%, including during the low-inheritance postwar period, and that it is vey sensitive to r-g. In the 19th century, r was so large (5%-6%) and g so low (1%) that we mechanically find extremly high φ_t^{KS} (as large as 450% in the 1870s-1880s).[115] The capitalized bequest share φ_t^{KS} was about 250%-300% around 1900-1910, then gradually dropped to about 150% in the 1960s-1980s. Again the nadir happens very late, due to the same time lags as above, and to the decennial variations in growth and asset returns (e.g. returns were low in the 1970s). In the benchmark scenario, we find that φ_t^{KS} stabilizes around 150% in the 21st century, due to the relatively low projected r-g (see Figure 25a). In the low-growth, high-return scenario, we find that φ_t^M stabilizes above 250%-300% (the same level as in 1900-1910), due to the much larger r-g (see Figure 25b).

We conclude from these computations that φ_t^M and φ_t^{KS} are fragile concepts. First, it is apparent from our French findings that the study of wealth accumulation and inheritance requires long term perspectives and adequate data sources. One should be careful when computing φ_t^M and φ_t^{KS} from one data point and steady-state assumptions. In the KSM controversy, both sides used single-data-point estimates of the U.S. inheritance flow b_{yt}, and applied steady-state formulas similar to equations (7.6)-(7.7) in order to compute φ_t^M and φ_t^{KS}. Due to the limitations of U.S. estate tax data (which only covers the very top), they did not have direct measures of the fiscal inheritance flow. So they computed b_{yt} by using national wealth estimates and age-wealth profiles for year 1962 (using the 1962 Survey of consumer finances). Kotlikoff and Summers (1981) applied the capitalized definition, and found that φ_t^{KS} was about 80% (and possibly larger than 100%) in the U.S. in the 1960s-1970s. By using essentially the same data, Modigliani (1986) concluded that

[115] See Appendix D5, Table D9 for detailed results.

φ_t^M was as low as 20%-30% in the U.S. in the 1960s-1970s.[116] Using SCF data from the 1980s, Gale and Scholz (1994) found that φ_t^M was closer to 40%.[117]

These U.S. estimates (say, $\varphi_t^M \approx$20%-40%, $\varphi_t^{KS} \approx$80%-100%) are somewhat lower than our French estimates for the 1960s-1980s. It could well be that inheritance flows are indeed somewhat lower in the U.S., due to higher economic and (especially) demographic growth, and/or to the crowding out effect of funded pension wealth. However, U.S. estimates are based upon relatively fragile data, so it could also be that they understate true economic inheritance flows. In particular, they tend to rely on relatively low gift-bequest ratios v_t (and sometime ignore gifts altogether) – a parameter which is hard to estimate in the absence of good fiscal data. This probably contributes to explain why the U.S. literature tends to adopt relatively low inheritance flow-aggregate wealth ratios b_{wt}, typically as low as 1%-1.5%, while we always find ratios above 2% in France.[118]

In any case, inheritance flows have probably changed a lot in the U.S. since the 1970s-1980s. In order to settle the issue, it would be necessary to construct homogenous, yearly (or decennial) U.S. series on β_t, μ_t, b_{wt} and b_{yt} up until the present day, as we have done for France. Given U.S. data limitations, one way to proceed would be to use the retrospective information on bequests and gifts available in SCF questionnaires. One needs however to find ways to adequately upgrade these self-reported bequest and gift flows, which in French wealth surveys appear to be far below fiscal flows.[119]

Next, and most importantly, even in a steady-state world with perfect data, none of the definitions φ_t^M and φ_t^{KS} would be really satisfactory. On the one hand, the Modigliani definition ignores the fact that inheritance produces flow returns, which amounts to

[116] In addition to the estimate of the 1962 inheritance flow, both Kotlikoff-Summers and Modigliani used data on age-income and consumption profiles in the U.S. during the 1950s-1970s. Both sides were essentially applying different definitions to the same raw data (with a few differences, generally reinforcing each side).
[117] Using a 1975 French wealth survey, Kessler and Masson (1989) also find φ_t^M around 40%.
[118] E.g. Gokhale et al (2001) simulate the transmission of inequality via bequests by assuming inheritance flows around 6% of aggregate labor income and 1% of aggregate wealth, which seems very small to us. These flow ratios are taken from Auerbarch et al (1995, p.25). They are based upon relatively ancient age-wealth profiles (taken from 1962 and 1983 SCF) and seem to wholly ignore inter vivos gifts.
[119] See Wolff (2002) for an attempt to use retrospective information on bequests and gifts reported in the 1989-1998 SCF (with no upward correction). We tried to use the retrospective questionnaires of the French wealth surveys conducted in 1992, 1998 and 2004, but found that self-reported bequest and gift flows were less than 50% of the fiscal flows (a lower bound of the true economic flows, given tax exempt assets). This is not due to imperfect recall: we also found this low ratio by comparing self-reported and fiscal flows for the past few years before each survey. We see no reason why reporting rates should be higher in similar wealth surveys in other countries, such as the SCF in the US. Reporting rates might also be biased, e.g. people who have consumed most of their inherited wealth might be particularly reluctant to report wealth transfers.

assuming away the existence of rentiers (this should be part of the empirical demonstration, not of the assumptions). On the other hand, the Kotlikoff-Summers definition φ_t^{KS} is mostly a measure of the magnitude of the capitalized resources available for consumption by successors. It does not really say anything about the relative importance of inherited vs self-made wealth. For instance, in case successors entirely consume their bequest the day they receive it, then φ_t^{KS} would still be far above 100%, even though 0% of aggregate wealth belongs to successors, and 100% belongs to self-made individuals who received zero bequest.

The problem with both definitions is that they are based upon a representative-agent approach. In practice, the wealth accumulation process always involves two different kinds of people and wealth trajectories. In every economy, there are inheritors or “rentiers” (people who typically consume part the return to their inherited wealth, and during the course of their lifetime consume more than their labor income), and there are savers or “self-made men” (people who do not inherit much but do accumulate wealth through labor income savings, so that their capitalized consumption is less than their capitalized labor income). A natural way to proceed would be to distinguish explicitly between these two groups, and to define φ_t as the wealth share of the second group. The downside is that this definition is more data demanding. While φ_t^M and φ_t^{KS} can be computed using aggregate data, φ_t requires micro level data on the joint distribution $H_t(w_{ti}, \tilde{b}_{ti})$ of current wealth and capitalized inherited wealth.

8. Concluding comments

What have we learned from this paper? In our view, the main contribution of this paper is to demonstrate empirically and theoretically that there is nothing inherent in the structure of modern economic growth that should lead a long run decline of inherited (non-human) wealth relatively to labor income.

The fact that the "rise of human capital" is to a large extent an illusion should not come as a surprise to macroeconomists. With stable capital shares and wealth-income ratios, the simple arithmetic of growth and wealth accumulation is likely to operate pretty much in the same way in the future as it did in the past. In particular, the r>g logic implies that past wealth and inheritance are bound to play a key role in the future.

As we have shown, there is no reason to expect demographic changes per se to lead to a decline in the relative importance of inheritance. Rising life expectancy implies that heirs inherit later in life. But this is compensated by the rise of inter vivos gifts, and by the fact that wealth also tends to get older in aging societies – so that heirs inherit bigger amounts.

Now, does this mean that the rise of human capital did not happen at all? No. It did happen, in the sense that human capital is what made long run productivity growth and self sustained economic growth possible. We know from the works of Solow and the modern endogenous growth literature that (non-human) capital accumulation alone cannot deliver self-sustained growth, and that human capital is what made g>0. The point, however, is that a world with g positive but small (say, g=1%-2%) is not very different from a world with g=0%.

If the world rates of productivity and demographic growth are small in the very long run (say, by 2050-2100), then the r>g logic implies that inheritance will eventually matter a lot pretty much everywhere – as it did in ancient societies. Past wealth will tend to dominate new wealth, and successors will tend to dominate labor income earners. This is less apocalyptic than Karl Marx: with g=0%, the wealth-income ratio rises indefinitely, leading either to a rising capital share, or to a fall in the rate of return, and in any case to non sustainable economic and political outcomes. With g>0, at least we have a steady-state. But this is a rather gloom steady-state.

The main limitation of this paper is that we did not attempt to analyze socially optimal tax policy. We have seen in our simulations that 20th century capital taxes, by reducing the differential between $(1-\tau_K)r$ and g, can and did have a significant impact on the steady-state magnitude of inheritance flows, i.e. on the extent to which wealth perpetuates itself over time and across generations. In order to properly address these issues, one would need however to explicitly introduce inequality and normative concerns into the model, which we did not do in this paper, and which we plan to do in future research. We hope that our results will be useful for scholars interested in capital and inheritance taxation.

The other important – and closely related – limitation of this paper is that we constantly assumed a common rate of return r on private wealth for all individuals. In the real world, the average r is larger than g, but the effective r varies enormously across individuals, over time and over assets. Available data and anecdotal evidence suggest that higher wealth individuals tend to get higher average returns (e.g. because of fixed costs in portfolio management, or risk aversion effects, or both).[120] By assuming a common rate of return, we almost certainly underestimate the inheritance share and overestimate the labor share in capitalized lifetime resources – possibly by large amounts.

In some cases, inherited wealth might also require human skills and effort in order to deliver high returns. That is, it sometime takes labor input to get high capital income. If anything, the empirical relevance of the theoretical distinction between labor and capital income has probably increased over the development process, following the rise of financial intermediation and the separation of ownership and control. I.e. with perfect capital markets, any dull successor should be able to get a high return. But the hererogeneity and potential endogeneity of asset returns are important issues which should be taken into account in a unified positive and normative analysis of inheritance. This raises major conceptual and empirical challenges for future research.

[120] See e.g. Calvet, Campbell and Sodini (2009).

References

Note: this list of references includes all publications quoted in the working paper and in the data appendices, with the exception of unsigned administrative publications (typically, statistical publications), the references of which are given when they are quoted (generally in the data appendices).

J. Accardo & P. Monteil, “Le patrimoine au décès en 1988”, *INSEE-Résultats*, 1995, n°390 (série Consommation-Modes de vie n°71), 117p.

A. Ando & F. Modigliani, “The Life-Cycle Hypothesis of Saving: Aggregate Implications and Tests”, *American Economic Review*, 1963, n°63

C. Antonin, « Age, revenu et comportements d’épargne des ménages. Une analyse théorique et empirique sur la période 1978-2006 », Master thesis, PSE, 2009

L. Arrondel & A. Laferrère, "Les partages inégaux de succession entre frères et sœurs", *Economie et Statistique*, 1992, n°256, pp.29-42.

L. Arrondel & A. Laferrère, "La transmission des grandes fortunes : profil des riches défunts en France", *Economie et Statistique*, 1994, n°273, pp. 41-52.

L. Arrondel & A. Laferrère, "Taxation and Wealth Transmission in France", *Journal of Public Economics*, 2001, n°79, pp.3-33.

L. Arrondel & A. Masson, “L’impôt successoral a-t-il un impact sur les transferts entre generations ?”, 2006, *Informations sociales*, 8 p.

A. Atkinson, *Unequal Shares – Wealth in Britain*, London: Allen Lane, 1972, 322p.

A. Atkinson & A.J. Harrison, *Distribution of Personal Wealth in Britain, 1923-1972*, Cambridge University Press, 1978, 330p.

A. Atkinson, *The Economics of Inequality*, Clarendon Press, 1983, 330p.

A. Atkinson, “Top Incomes in the UK over the Twentieth Century”, *Journal of the Royal Statistical Society*, 2005, n°168(2), pp.325-343

A. Atkinson & T. Piketty (eds.), *Top Incomes Over the Twentieth Century*, vol.1, Oxford University Press, 2007, 585p.

A. Atkinson & T. Piketty (eds.), *Top Incomes – A Global Perspective*, vol.2, Oxford University Press, 2010, 776p.

A. Atkinson, T. Piketty & E. Saez, “Top Incomes in the Long-Run of History”, *Journal of Economic Literature*, 2010, forthcoming

O. Attanasio & H. Hoynes, "Differential Mortality and Wealth Accumulation", *Journal of Human Resources*, 2000, n°35, pp.1-29

A. Auerbach, J. Gokhale, L. Kotlikoff, J. Sabelhaus & D. Weil, “The Annuitization of Americans’ Resources: A Cohort Analysis”, NBER Working Paper 5089, 1995

A. Babeau, “Le rapport macroéconomique du patrimoine au revenu des ménages”, *Revue Economique*, 1983, n°34(1), pp.64-123

D. Baker, J.B. DeLong & P. Krugman, “Asset Returns and Economic Growth”, *Brookings Papers on Economic Activity*, 2005, n°1, pp.289-315

R.J. Barro, “Rare Disasters, Asset Prices and Welfare Costs”, *American Economic Review*, 2009, n°99, pp.243-264

J. Benhabib & A. Bisin, “The Distribution of Wealth and Fiscal Policy in Economies with Finitely Lived Agents”, mimeo, NYU, 2009

J. Benhabib & S. Zhu, “Age, Luck and Inheritance”, mimeo, NYU, 2009

G. Bertola, R. Foellmi & J. Zweimuller, *Income Distribution in Macroeconomic Models*, Cambridge University Press, 2006, 417p.

D. Blau, “How Do Pensions Affect Household Wealth Accumulation?”, mimeo, Ohio State University, 2009

A. Blinder, « Comments on Modigliani and Kotlikoff-Summers », in *Modelling the Accumulation and Distribution of Wealth*, D. Kessler & A. Masson eds., pp.68-76, Oxford University Press, 1988

J. Bourdieu, G. Postel-Vinay & A. Suwa-Eisenmann, « Pourquoi la richesse ne s'est-elle pas diffusée avec la croissance ? Le degré zéro de l'inégalité et son évolution en France : 1800-1940 », *Histoire et mesure*, 2003, vol. 23, no 1-2, pp. 147-198.

J. Bourdieu, G. Postel-Vinay & A. Suwa-Eisenmann, « Défense et illustration de l’enquête 3000 familles », *Annales de démographie historique*, 2004, n°1, p. 19-52

F. Bourguignon & L. Lévy-Leboyer, *L’économie française au 19ème siècle – Analyse macroéconomique*, Economica, 1985, 362p.

J. Bouvier, F. Furet & M. Gillet, *Le mouvement du profit en France au 19ème siècle – Matériaux et études*, Paris : Mouton, 465p.

A. Bozio, “Réformes des retraites: estimations sur données françaises”, PhD thesis, PSE, 2006, 414p.

R. Brumberg & F. Modigliani, « Utility Analysis and the Consumption Function : An Interpretation of Cross-Section Data », in *Post Keynesian Economics*, K. Kurihara ed., Rutgers University Press, 1954

A. Bowley, *The Change in the Distribution of National Income, 1880-1913*, Clarendon Press, 1920, 36p.

M. Cagetti & M. De Nardi, “Wealth inequality: data and models”, *Macroeconomic Dynamics*, 2008, vol. 12 (sup. S2), pp. 285-313

L. Calvet, J. Campbell & P. Sodini, “Fight Or Flight? Portfolio Rebalancing by Individual Investors”, *Quarterly Journal of Economics*, 2009, n°124, pp.301–348.

N. Campion, « Nouvelle évaluation de la fortune des ménages (1959-1967) », *Consommation* (Credoc), 1971, vol.17, n°1, pp.35-72

G. Canceill, “Héritages et donations immobilières”, *Economie et statistiques*, 1979, n°114, pp.95-102

P. Cornut, *Répartition de la fortune privée en France par département et par nature de biens au cours de la première moitié du 20ème siècle*, Armand Colin, 1963, 656p.

J.J. Carré, P. Dubois & E. Malinvaud, *La croissance française – Un essai d’analyse économique causale de l’après-guerre*, Editions du Seuil, 1972, 710p.

C.D. Carroll, « Why Do the Rich Save So Much? », in J. Slemrod ed., *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, Harvard Universty Press, 2000

A. Castaneda, J. Dias-Gimenes & V. Rios-Rull, “Accounting for the U.S. Earnings and Wealth Inequality”, *Journal of Political Economy*, 2003, 111(4), 818-857

C. Colson, *Cours d’économie politique*, Paris : Gauthier-Villars, 1903, 1918, 1927 (several editions), 774p.

E.S. Danysz, « Contribution à l’étude des fortunes privées d’après les déclarations de successions », *Bulletin de la Statistique Générale de France*, 1934, pp.5-171

J. B. DeLong, "Bequests: An Historical Perspective," in A. Munnell, ed., *The Role and Impact of Gifts and Estates*, Brookings Institution, 2003

M. DeNardi, « Wealth Inequality and Intergenerational Links », *Review of Economic Studies*, 2004, 71(), 743-768

F. Divisia, J. Dupin and R. Roy, *A la recherche du franc perdu* (volume 3 : *Fortune de la France*), Paris : Serp, 1956, 122p.

E. Domar, « Expension and Employment », *American Economic Review*, 1947, n°37(1), pp.34-55

L. Dugé de Bernonville, « Les revenus privés », *Revue d’Economie Politique*, 1933-1939 (yearly publication ; see Piketty, 2001, p.781 for full references)

K. Dynan, J. Skinner & S. Zeldes, “The Importance of Bequests and Lifecycle Savings in Capital Accumulation: A New Answer”, *American Economic Review*, 2002, n°92(2), pp.274-278

K. Dynan, J. Skinner & S. Zeldes, “Do the Rich Save More?”, *Journal of Political Economy*, 2004, n°112(2), pp.397-444

L. Edlund & W. Kopczuk, "Women, Wealth and Mobility", *American Economic Review*, 2009, n°99(1), pp.146-78

D. Fiaschi & M. Marsili, “Distribution of Wealth and Incomplete Markets: Theory and Empirical Evidence”, mimeo, Pisa, 2009

L. Fontvieille, “Evolution et croissance de l’Etat Français: 1815-1969”, *Economie et Sociétés* (Cahiers de l’ISMEA, série « Histoire quantitative de l’économie française »), 1976, n°13, pp.1655-2144

A. Fouquet, « Les comptes de patrimoines : quelques aspects méthodologiques », in *Accumulation et répartition des patrimoines*, D. Kessler, A. Masson & D. Strauss-Kahn eds., pp.97-116, Economica, 1982

A. Fouquet & M. Meron, “Héritages et donations”, *Economie et statistiques*, 1982, n°145, pp.83-98

A. Foville, « The Wealth of France and of Other Countries », *Journal of the Royal Statistical Society*, 1893, n°56(4), pp. 597-626

J. Friggit, “Long Term (1800-2005) Investment in Gold, Bonds, Stocks and Housing in France – with Insights into the U.S.A. and the U.K.: a Few Regularities”, CGPC Working Paper, 2007, 55p.

W. Gale & J. Scholtz, “Intergenerational Transfers and the Accumulation of Wealth”, *Journal of Economic Perspectives*, 1994, n°8(4), pp.145-160

O. Galor & O. Moav, “Das Human Kapital: A Theory of the Demise of the Class Structure”, *Review of Economic Studies*, 2006, n°73, pp.85-117

R. Giffen, “Recent Accumulations of Capital in the United Kingdom”, *Journal of the Royal Statistical Society*, 1878, n°41(1), pp. 1-39

J. Gokhale, L. Kotlikoff, J. Sefton & M. Weale, “Simulating the Transmission of Wealth Inequality via Bequests”, *Journal of Public Economics*, 2001, n°79, pp.93-128

M.J. Gordon, “Dividends, Earnings and Stock Prices”, *Review of Economics and Statistics*, 1959, 41(2), pp.99-105

R.F. Harrod, “An Essay in Dynamic Theory”, *Economic Journal*, 1939, n°49(193), pp.14-33

N. Kaldor, “Marginal Productivity and the Macro-Economic Theories of Distribution”, *Review of Economic Studies*, 1966, n°33(4), pp.309-319

A.B. Kennickell, “Ponds and Streams: Wealth and Income in the U.S., 1989-2007”, Federal Reserve Board, Discussion Paper 2009-13, 87p.

D. Kessler & A. Masson, “Bequest and Wealth Accumulation: Are Some Pieces of the Puzzle Missing?”, *Journal of Economic Perspectives*, 1989, 3(3), pp.141-152

W. Kopczuk, "Bequest and Tax Planning: Evidence from Estate Tax Returns", *Quarterly Journal of Economics*, 2007, n°122(4), pp.1801-1854

W. Kopczuk & J. Lupton, "To Leave or Not to Leave: The Distribution of Bequest Motives", *Review of Economic Studies*, 2007, n°74(1), pp.207-235.

W. Kopczuk & E. Saez, “Top Wealth Shares in the United States, 1916-2000: Evidence from Estate Tax Returns", *National Tax Journal*, 2004, n°57(2), pp.445-487

L. Kotlikoff, “Intergenerational Transfers and Savings”, *Journal of Economic Perspectives*, 1988, n°2(2), pp.41-58

L. Kotlikoff & L. Summers, “The Role of Intergenerational Transfers in Aggregate Capital Accumulation”, *Journal of Political Economy*, 1981, n°89, pp.706-732

S. Kuznets, *Shares of Upper Income Groups in Income and Savings, 1913-1948*, National Bureau of Economic Research, 1953, 707p.

A. Laferrère, « Successions et héritiers », *INSEE-Cadrage*, 1990, n°4, 49p.

A. Laferrère & P. Monteil, « Successions et héritiers en 1987 », *Document de travail INSEE-DSDS*, 1992, n°F9210, 67p.

A. Laferrère & P. Monteil, “Le patrimoine au décès en 1988”, *Document de travail INSEE-DSDS*, 1994, n°F9410, 177p.

R.J. Lampman, *The share of top wealth-holders in national wealth 1922-1956*, Princeton University Press, 1962

E. Levasseur, *Questions ouvrières et industrielles en France sous la Troisième République*, Paris : Arthur Rousseau, 1907, 968p.

M. Lévy-Leboyer, “L’étude du capital français au 19ème siècle: données et lacunes”, in *Pour une histoire de la statistique*, INSEE, 1977, volume 1, pp.393-416

S. Lollivier, « Les mutations immobilières de 1962 à 1979 », *Economie et statistiques*, 1986, n°185, pp.35-46

M. Malissen, *L’autofinancement des sociétés en France et aux Etats-Unis*, Dalloz, 1953, 246p.

B. Mallet, « A Method of Estimating Capital Wealth from the Estate Duty Statistics », *Journal of the Royal Statistical Society*, 1908, n°71(1), pp.65-101

B. Mallet & H.C. Strutt, “The Multiplier and Capital Wealth”, *Journal of the Royal Statistical Society*, 1915, n°78(4), pp.555-599

A. Masson & D. Strauss-Kahn, « Croissance et inégalité des fortunes de 1949 à 1975 », *Economie et Statistiques*, 1978, n°98, pp.31-49

A. Masson, “A Cohort Analysis of Wealth-Age Profiles Generated by a Simulation Model in France (1949-1975)”, *The Economic Journal*, 1986, n°96, pp.175-190

A. Masson, “Permanent Income, Age and the Distribution of Wealth”, *Annales d’Economie et de Statistique*, 1988, n°9, pp.227-256

F. Modigliani, “Life Cycle, Individual Thrift and the Wealth of Nations”, *American Economic Review*, 1986, n°76(3), pp.297-313

F. Modigliani, « The Role of Intergenerational Transfers and Lifecyle Savings in the Accumulation of Wealth », *Journal of Economic Perspectives*, 1988, n°2(2), pp.15-40

M. Nirei & W. Souma, “A Two Factor Model of Income Distribution Dynamics”, *Review of Income and Wealth*, 2007, n°53(3), pp.440-459

L. Pasinetti, “Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth”, *Review of Economic Studies*, 1962, n°29(4), pp.267-279

T. Piketty, « The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing », *Review of Economic Studies*, 1997, n°64, pp.173-189

T. Piketty, *Les hauts revenus en France au 20ème siècle – Inégalités et redistributions, 1901-1998*, Paris : Grasset, 2001, 807p.

T. Piketty, « Income Inequality in France, 1901-1998 », *Journal of Political Economy*, 2003, n°111(5), pp.1004-1042

T. Piketty & E. Saez, “Income Inequality in the United States, 1913-1998”, *Quarterly Journal of Economics*, 2003, n°118(1), pp.1-39

T. Piketty, G. Postel-Vinay & J.L. Rosenthal, “Wealth Concentration in a Developing Economy: Paris and France, 1807-1994”, *American Economic Review*, 2006, n°96(1), pp.236-256

J. Poterba, “Demographic Structure and Asset Returns”, *Review of Economics and Statistics*, 2001, n°83, pp.565-584

J. Roine and D. Waldenstrom, “Wealth Concentration over the Path of Development: Sweden, 1873-2006”, *Scandinavian Journal of Economics*, 2009, n°111, pp.151-187

A. Sauvy, *Histoire économique de la France entre les deux guerres*, volumes 1, 2 & 3, Economica, 1984, 422p., 439p & 476p..

J. Séaillès, *La répartition des fortunes en France*, Editions Felix Alcan, 1910, 143p.

A.F. Shorrocks, « The Age-Wealth Relationship : A Cross-Sectional and Cohort Analysis », *Review of Economics and Statistics*, 1975, n°57(2), pp.155-163

S. Solomou & M. Weale, « Personal Sector Wealth in the United Kingdom, 1922-1956 », *Review of Income and Wealth*, 1997, n°43(3), pp.297-318

J.C. Stamp, "The Wealth and Income of the Chief Powers", *Journal of the Royal Statistical Society*, 1919, n°82(4), pp.441-507

R. Solow, "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, 1956, 70(1), pp.65-94

H.C. Strutt, "Notes on the Distribution of Estates in France and the United Kingdom", *Journal of the Royal Statistical Society*, 1910, n°73(6), pp.634-644

J.C. Toutain, « Le produit intérieur brut de la France de 1789 à 1990 », *Economie et Sociétés* (Cahiers de l'ISMEA, série « Histoire économique quantitative »), 1997, n°1, pp.5-187

P. Villa, « Un siècle de données macroéconomiques », *INSEE-Résultats*, 1994, n°303-304 (série Economie générale n°86-87), 266p.

E. Wolff, « Trends in Aggregate Household Wealth in the U.S. 1900-1983 », *Review of Income and Wealth*, 1989, n°34(3), pp.1-29

E. Wolff, "Inheritances and Wealth Inequality, 1989-1998", *American Economic Review*, n°92(2), pp.260-264

E. Wolff & A. Zacharias, « Household Wealth and the Measurement of Economic Well-Being in the U.S. », *Journal of Economic Inequality*, 2009, n°7, pp.83-115

S. Wright, « Measures of Stock Market Value and Returns for the US Nonfinancial Corporate Sector, 1900-2002 », *Review of Income and Wealth*, 2004, n°50(4), pp.561-584

J. Zhu, "Wealth Distribution under Idiosyncratic Investment Risk", mimeo, NYU, 2010

G. Zucman, « Les hauts patrimoines fuient-ils l'ISF ? Une estimation sur la période 1995-2006 », Master thesis, PSE, 2008

Figure 1: Annual inheritance flow as a fraction of national income, France 1900-2008

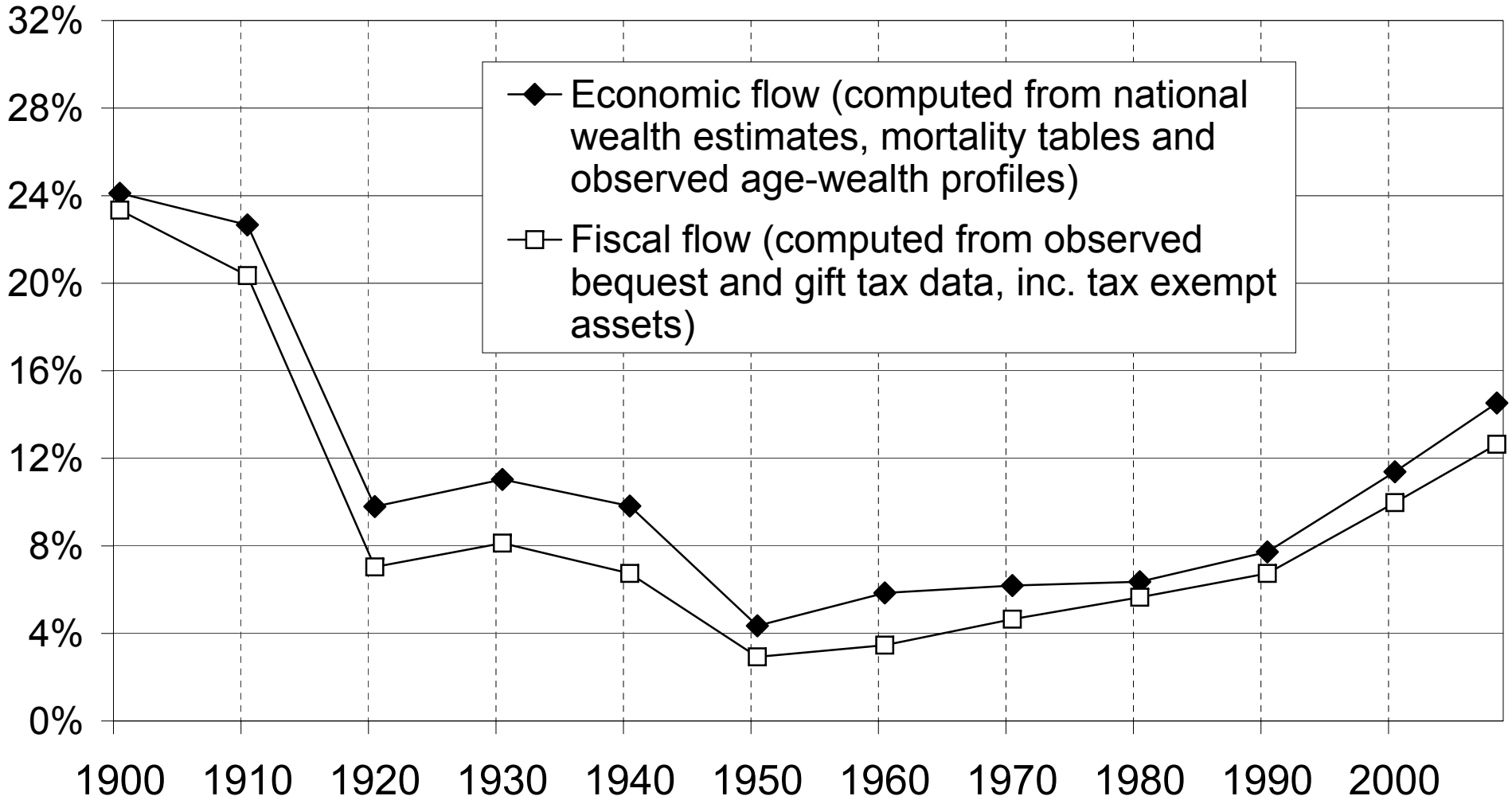

Figure 2: Annual inheritance flow as a fraction of national income, France 1820-2008

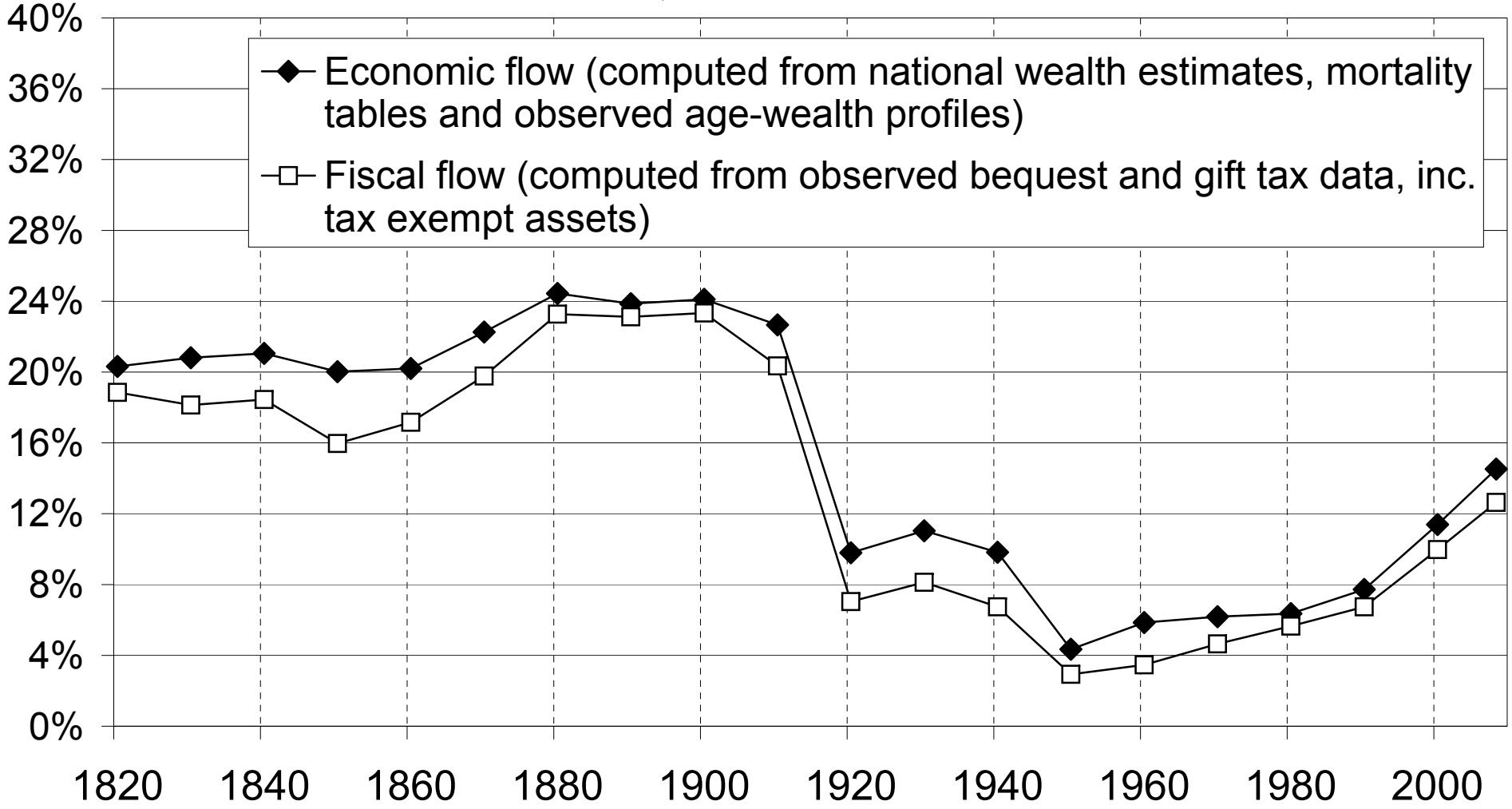

Figure 3: Annual inheritance flow as a fraction of disposable income, France 1820-2008

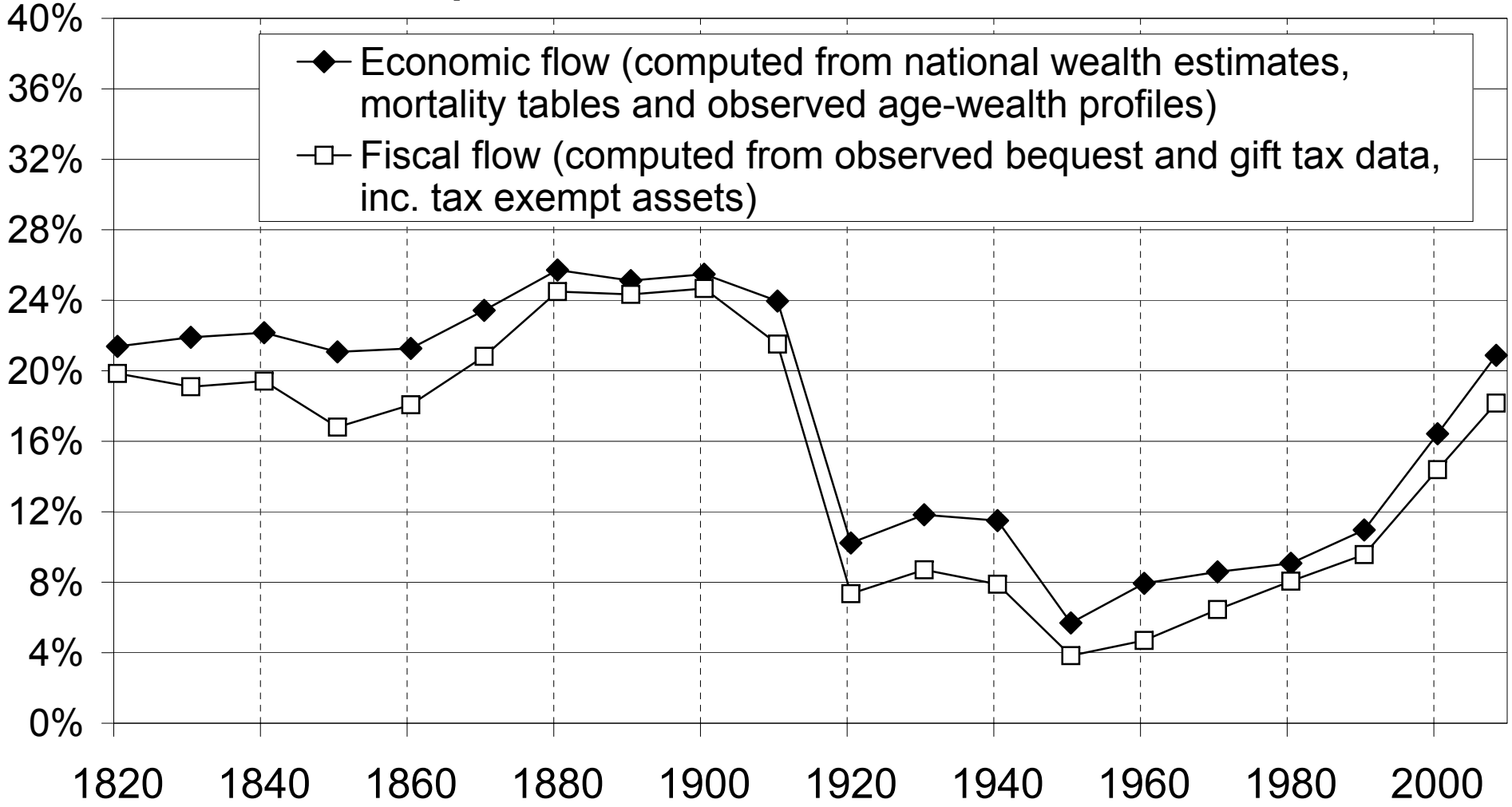

Figure 4: Wealth-income ratio in France 1820-2008

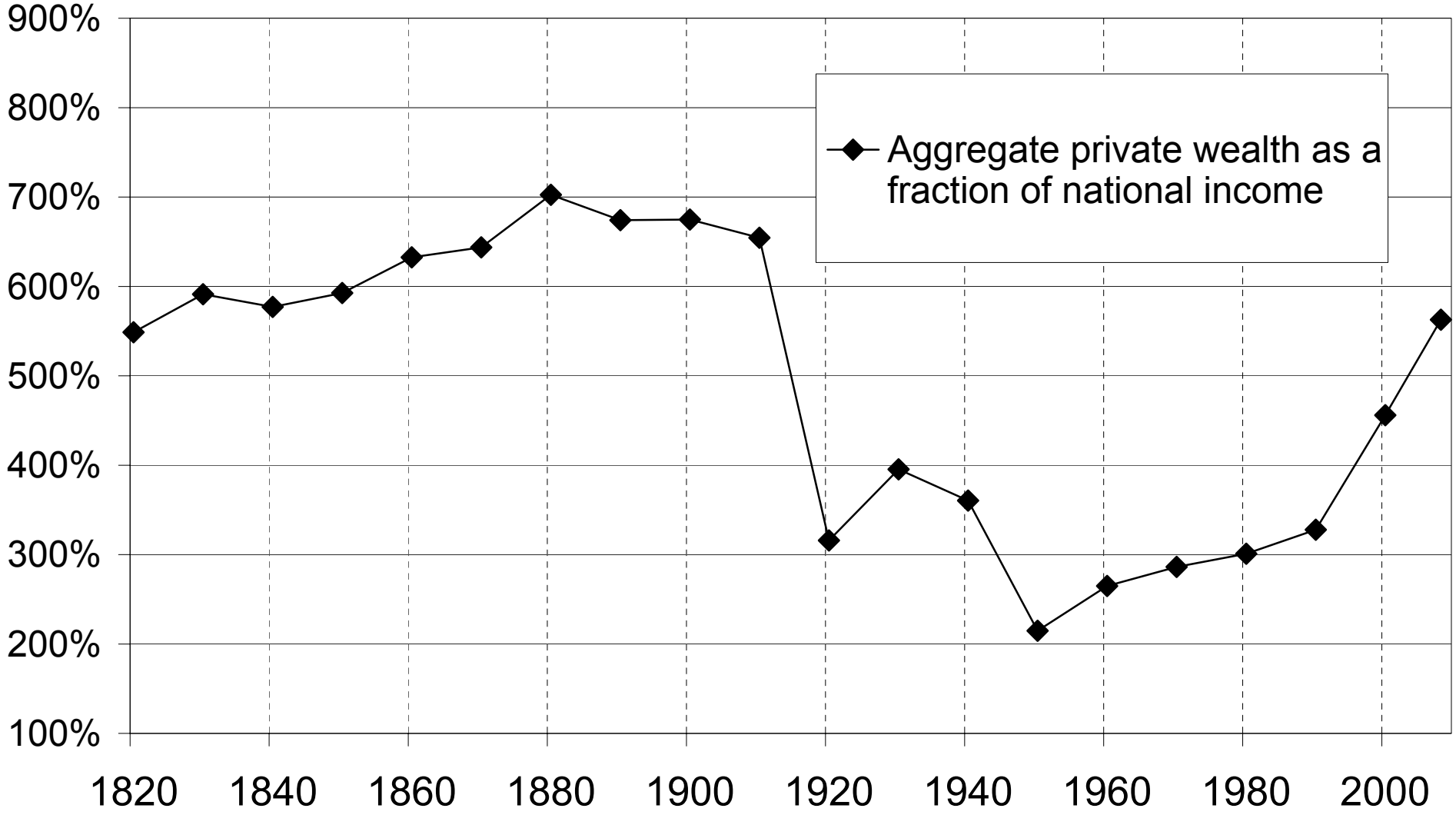

Figure 5: Wealth-disposable income ratio in France 1820-2008

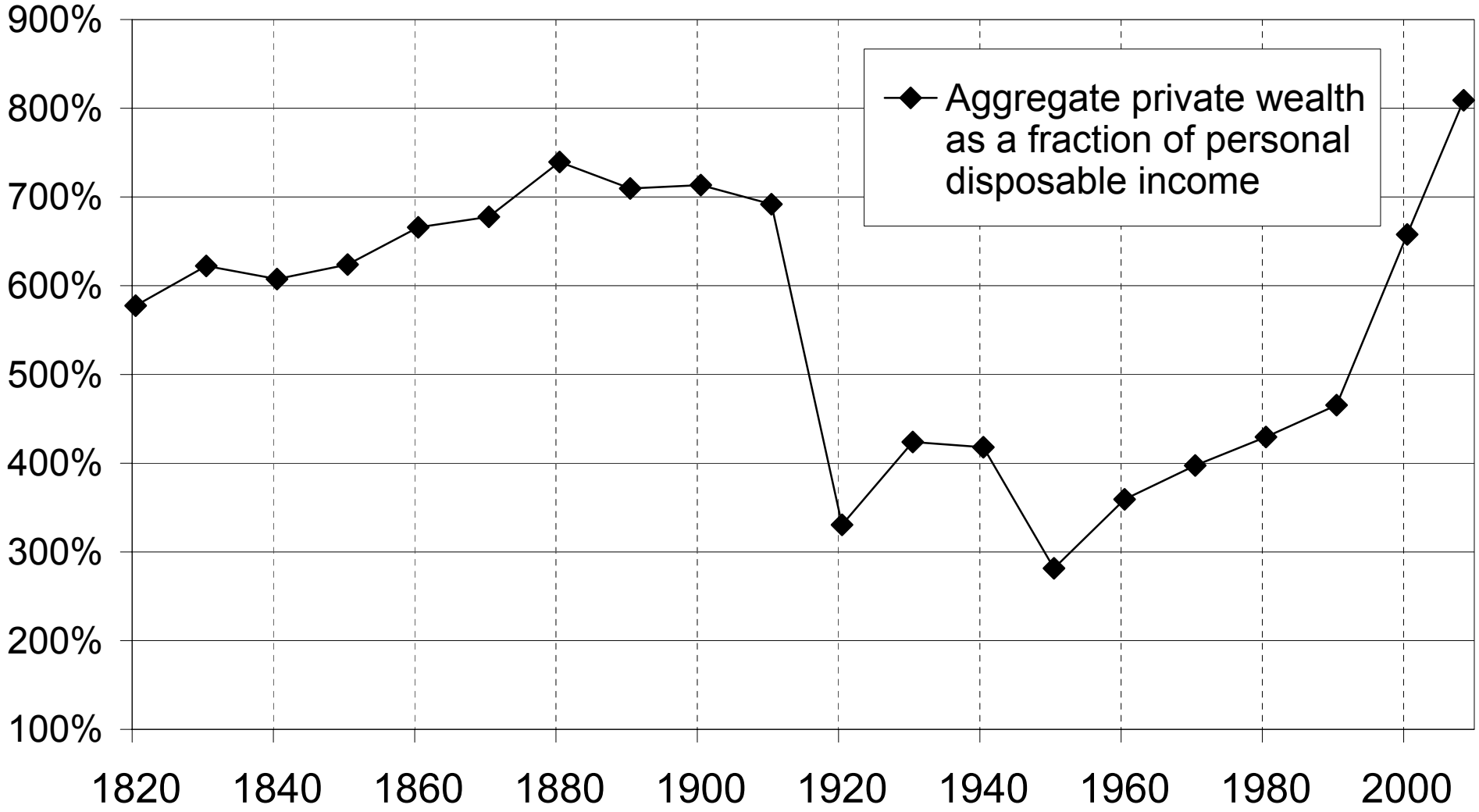

Figure 6: Mortality rate in France, 1820-2100

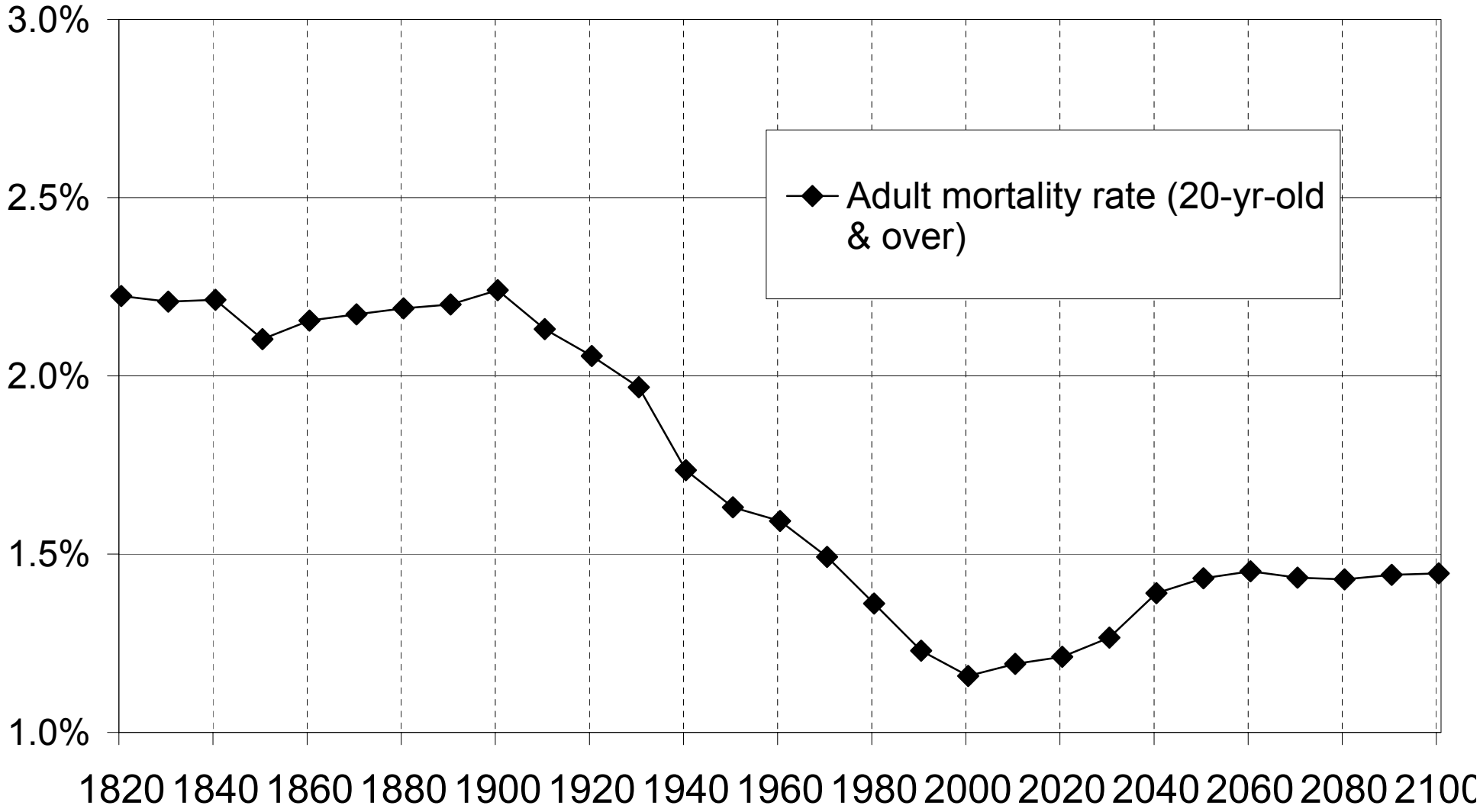

Figure 7: Age of decedents & heirs in France, 1820-2100

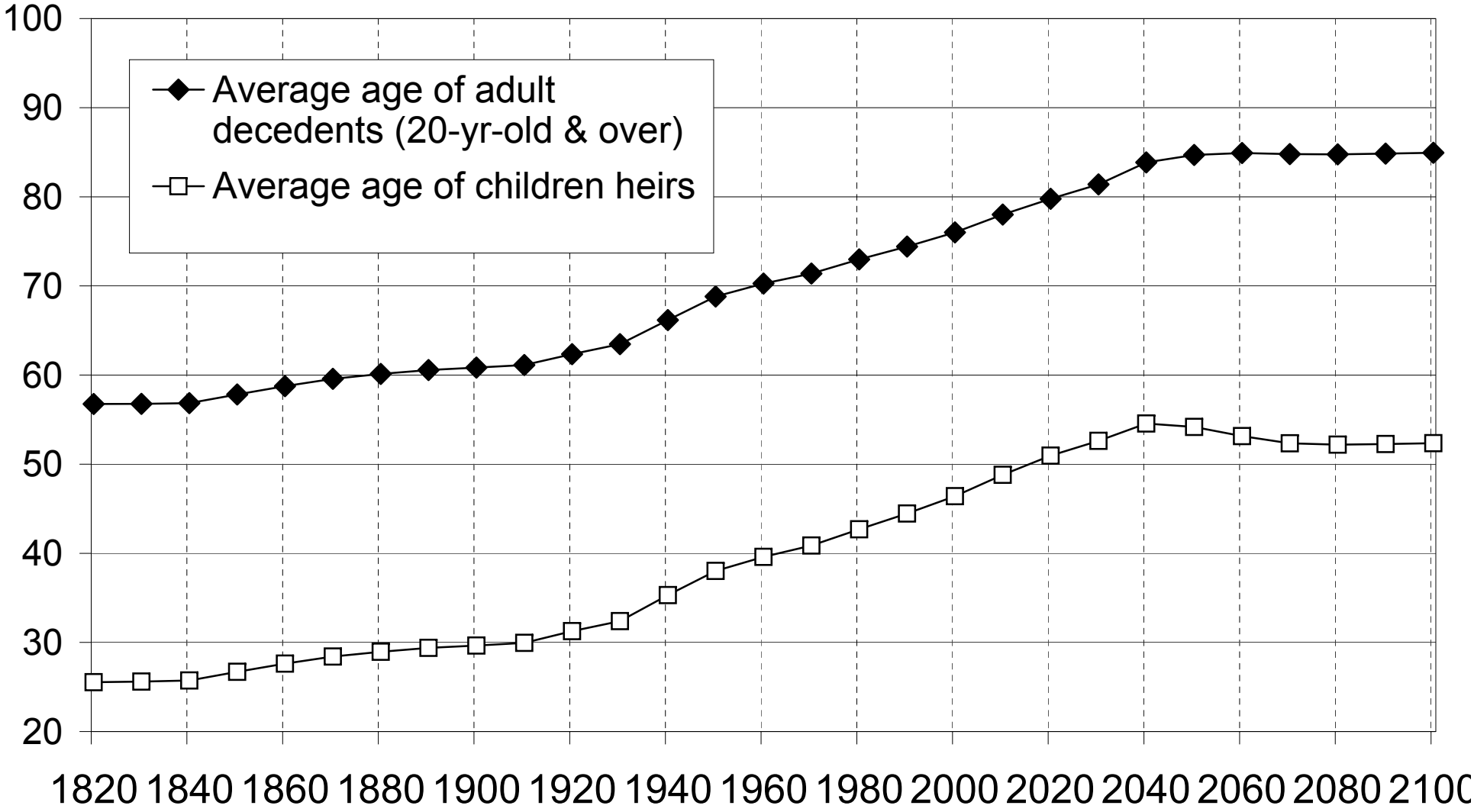

Figure 8: The ratio between average wealth of decedents and average wealth of the living France 1820-2008

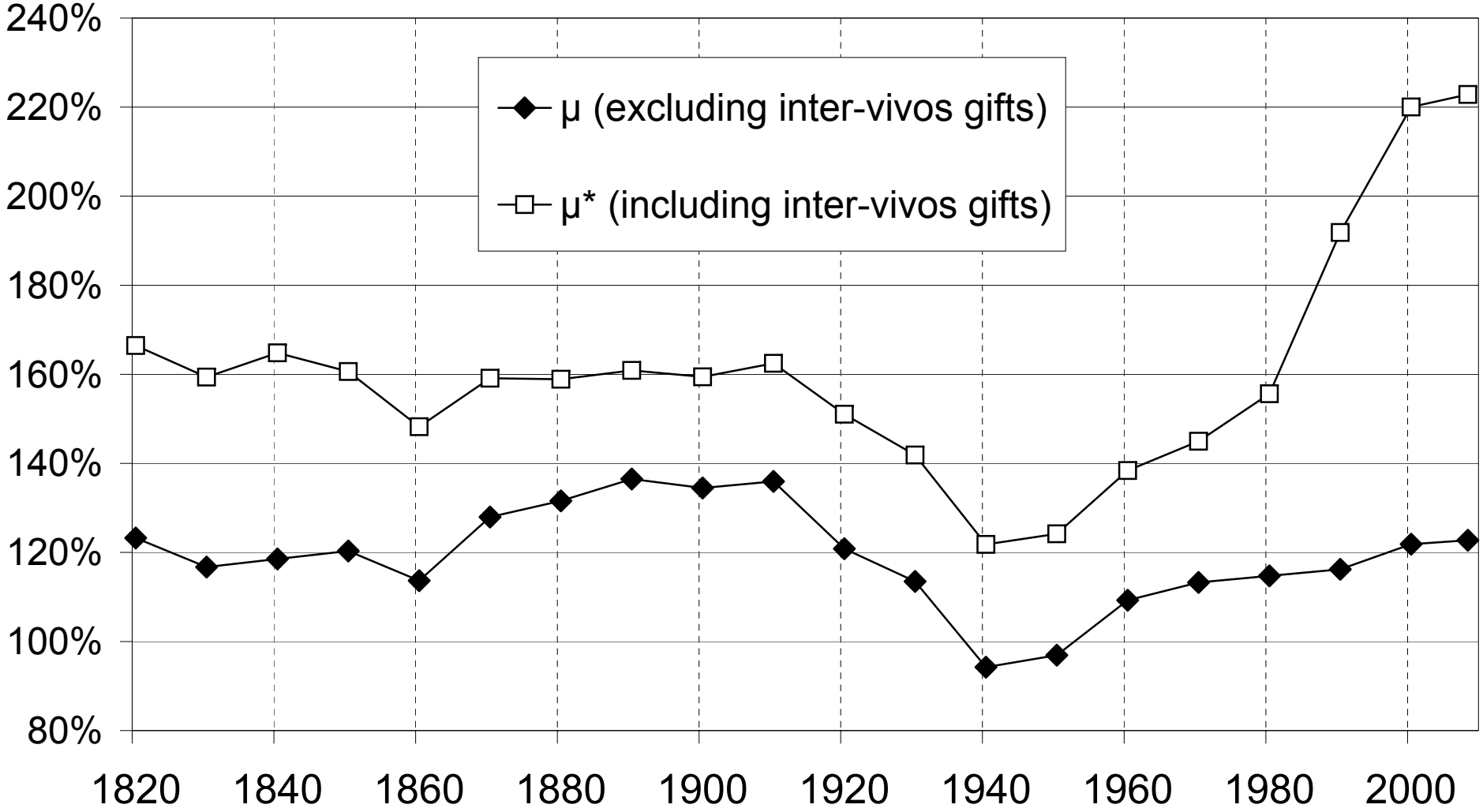

Figure 9: Inheritance flow vs mortality rate in France, 1820-2008

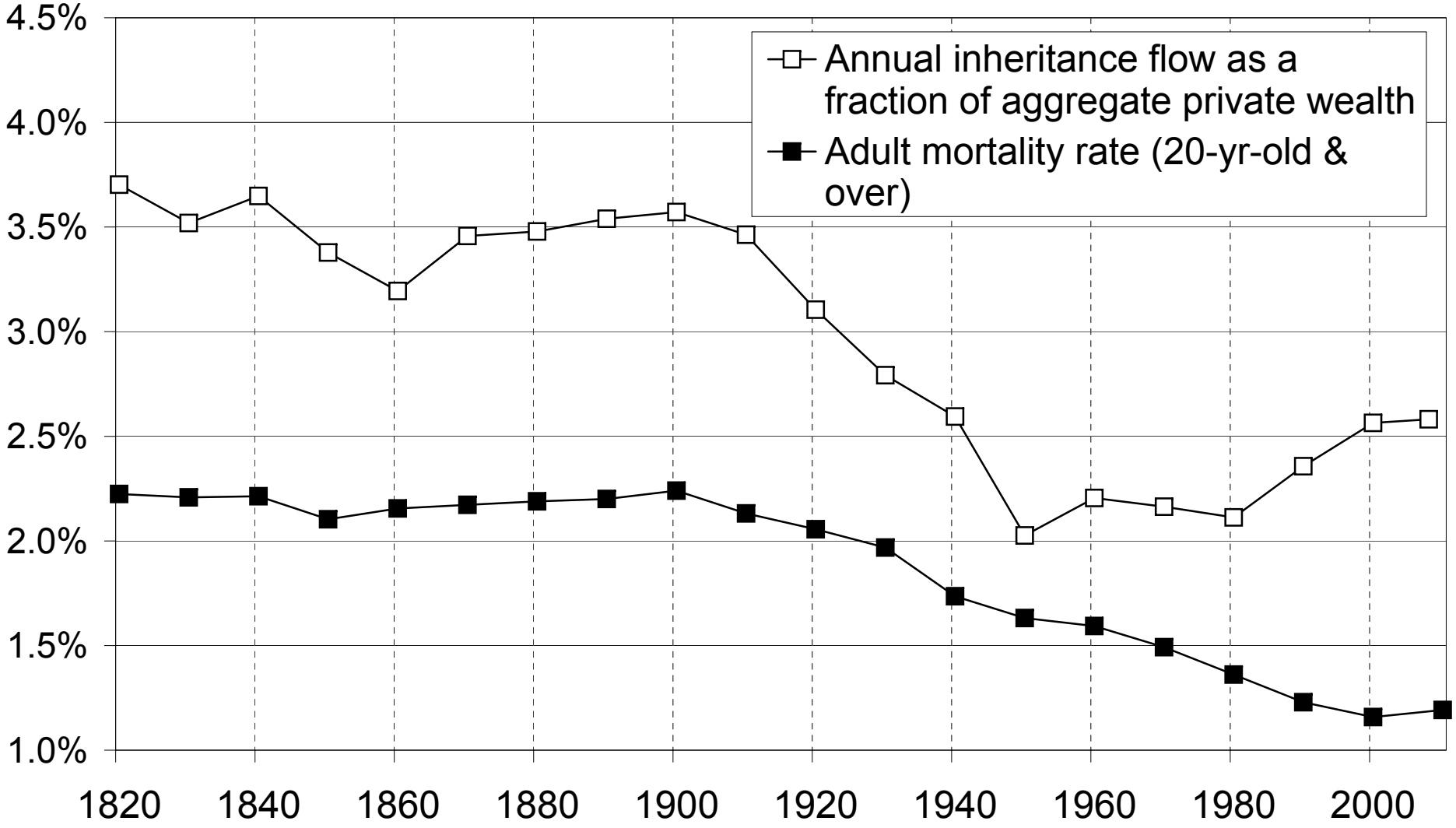

Figure 10: Demographic structure of the model (continuous time)

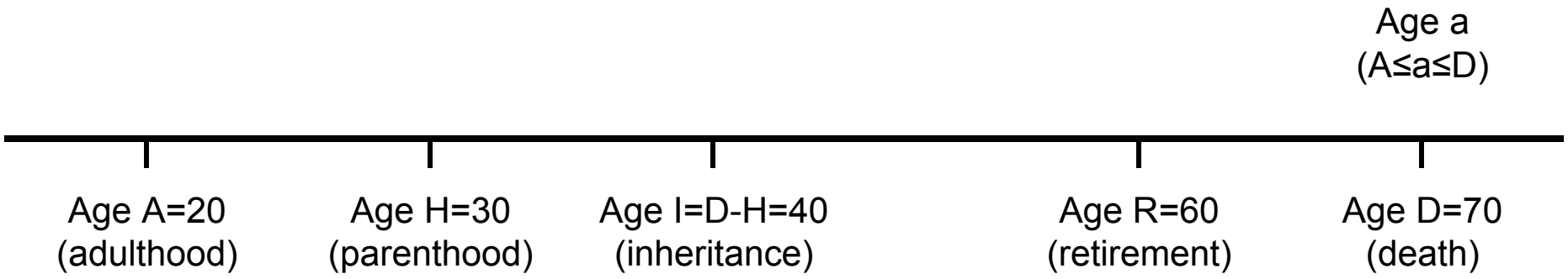

Figure 11: Cross-sectional age-labor income profile $y_{Lt}(a)$

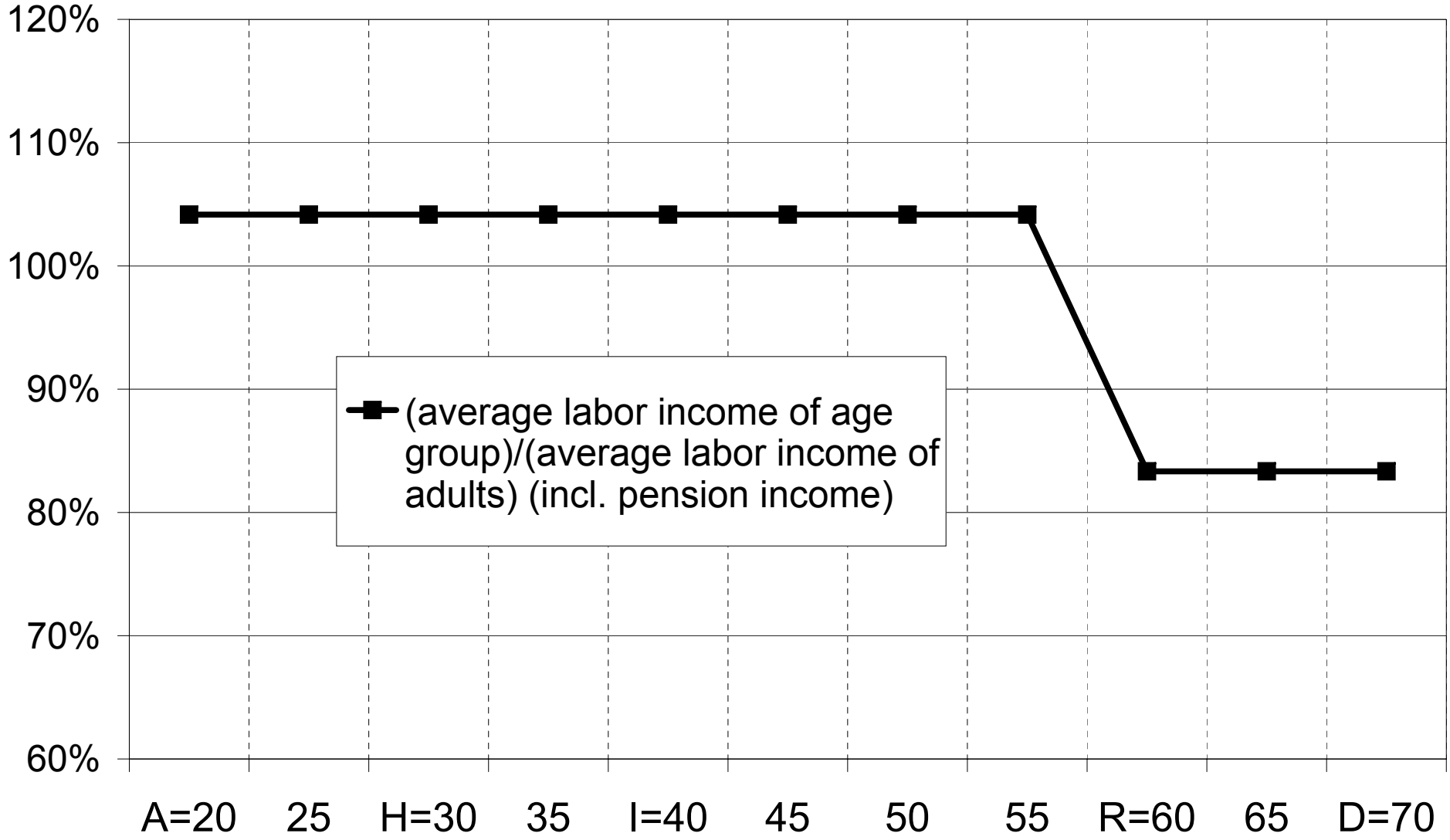

Figure 12: Steady-state cross-sectional age-wealth profile in the class savings model (s_L=0, s_K>0)

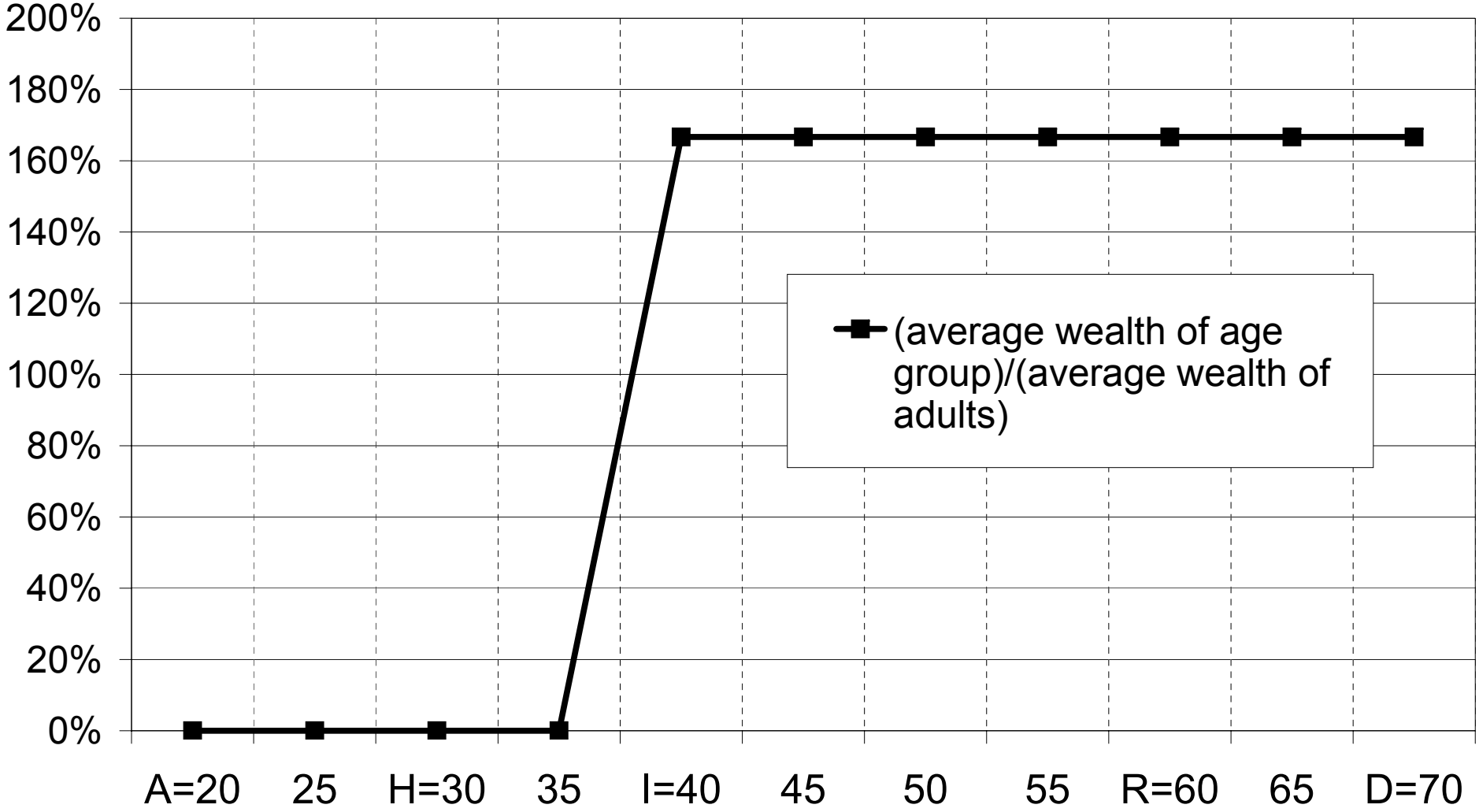

Figure 13: Steady-state cross-sectional age-wealth profile in the class savings model with demographic noise

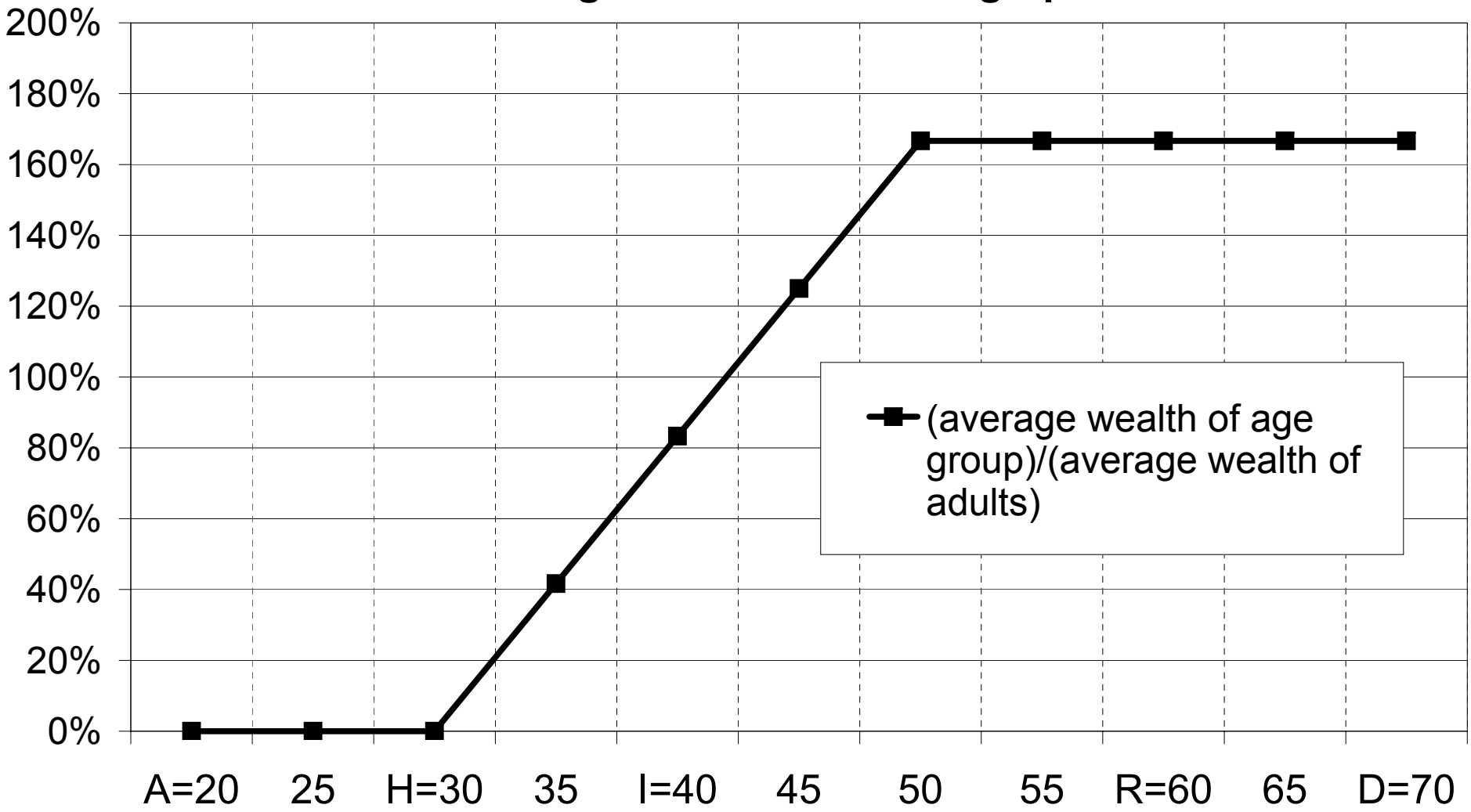

Figure 14: Private savings rate in France 1820-2008

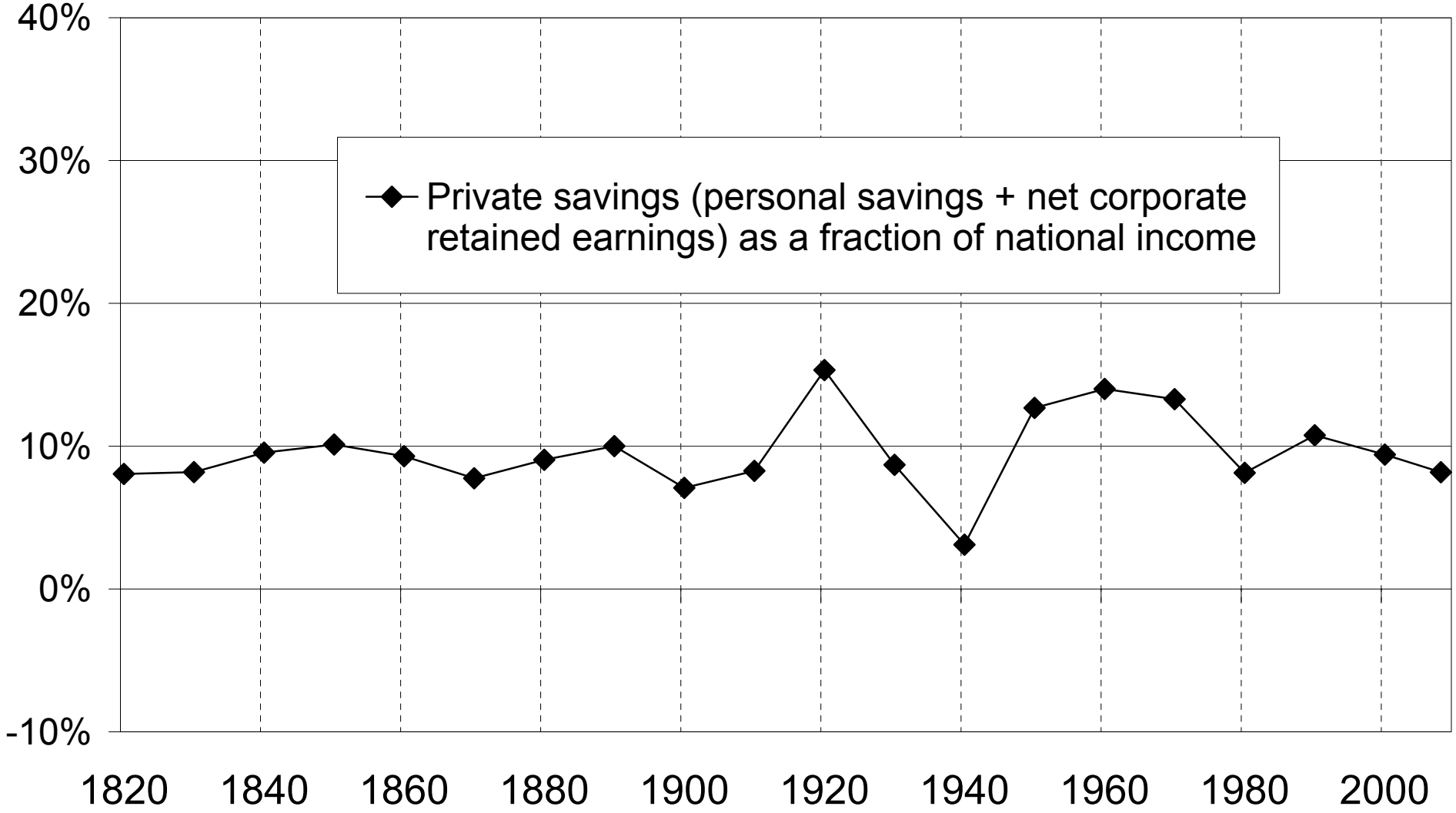

Figure 15a: Observed vs simulated inheritance flow B/Y, France 1820-2100

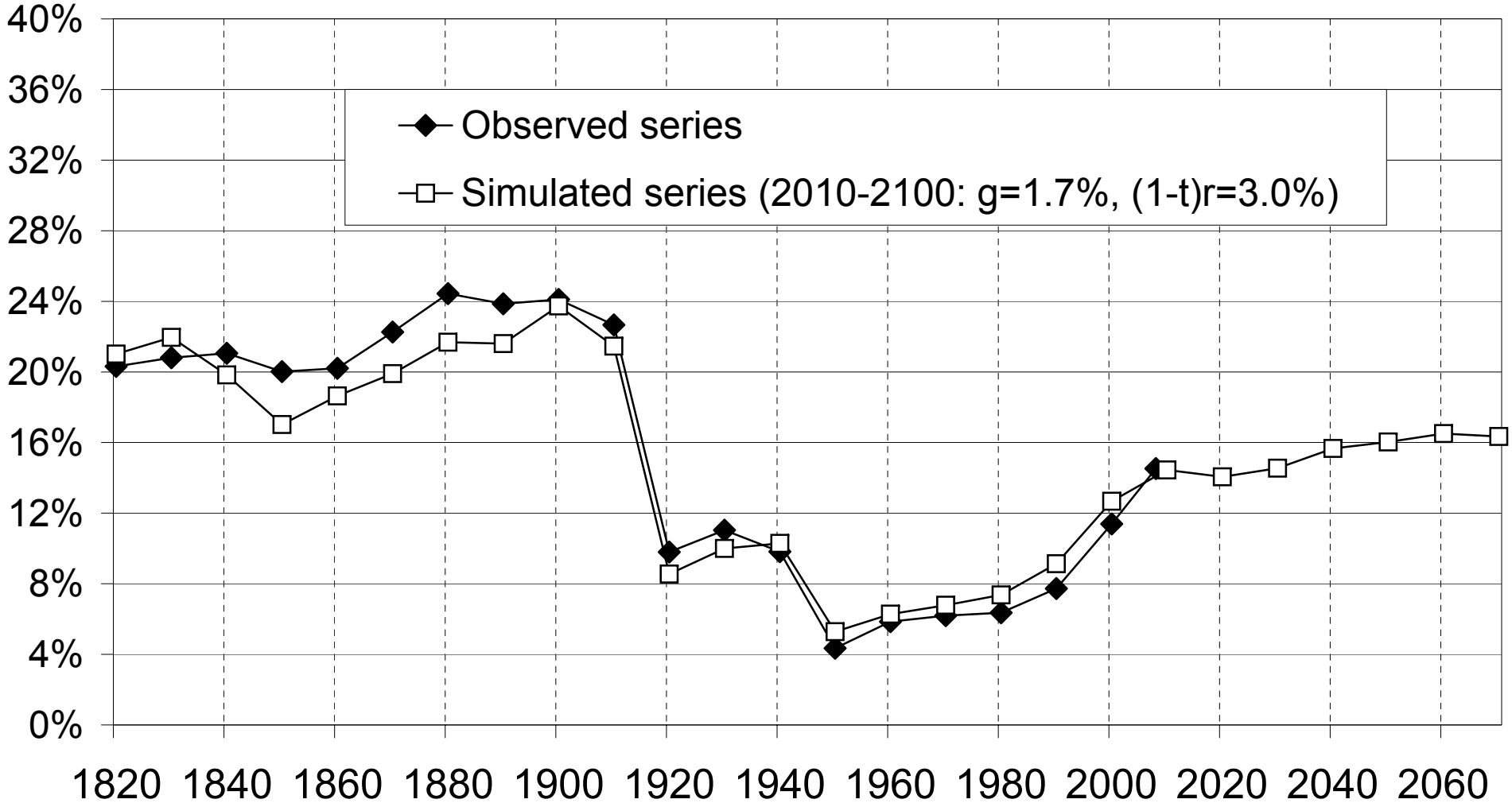

Figure 15b: Observed vs simulated inheritance flow B/Y, France 1820-2100

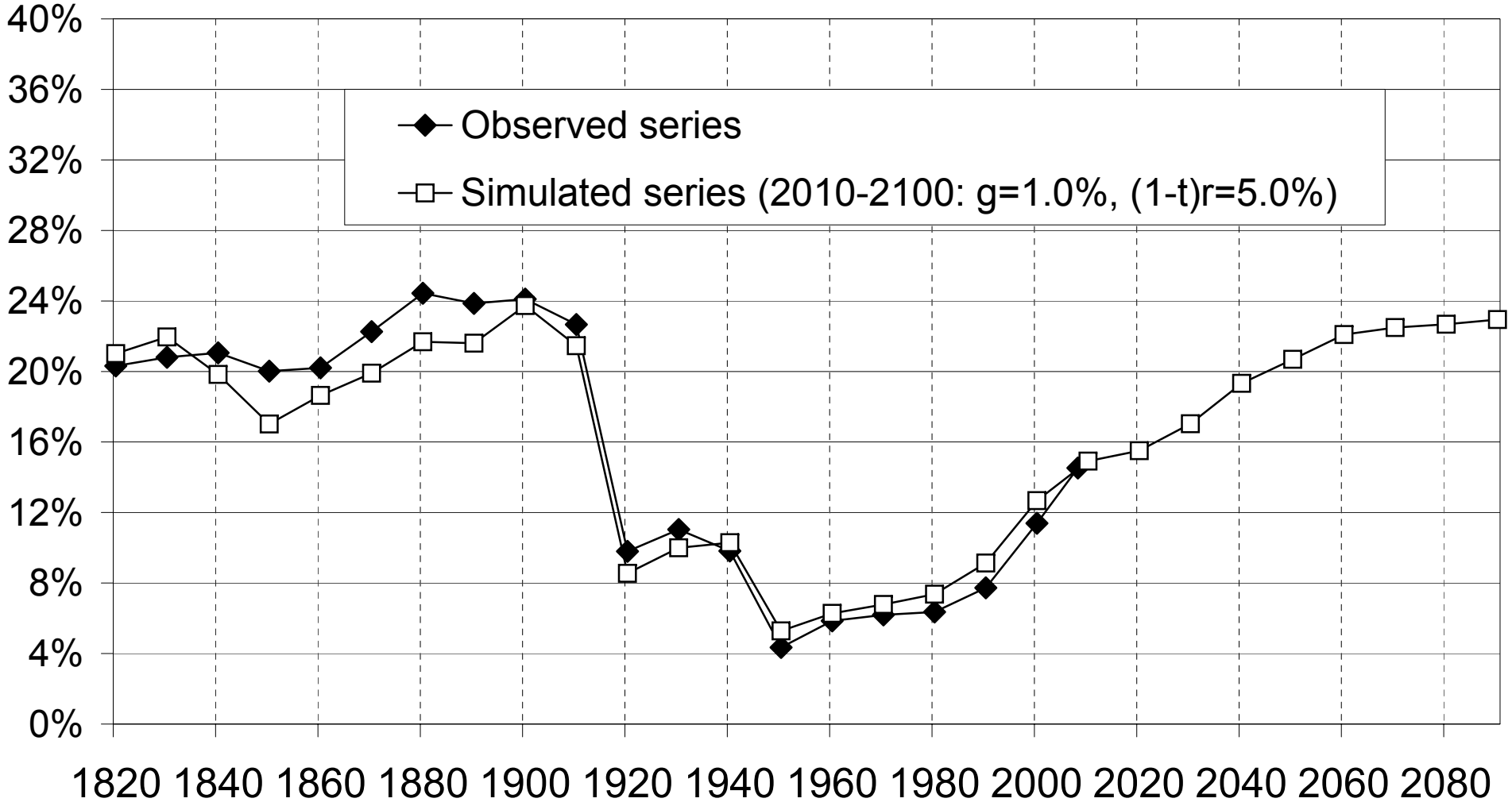

Figure 16: Labor & capital shares in national income, France 1820-2008

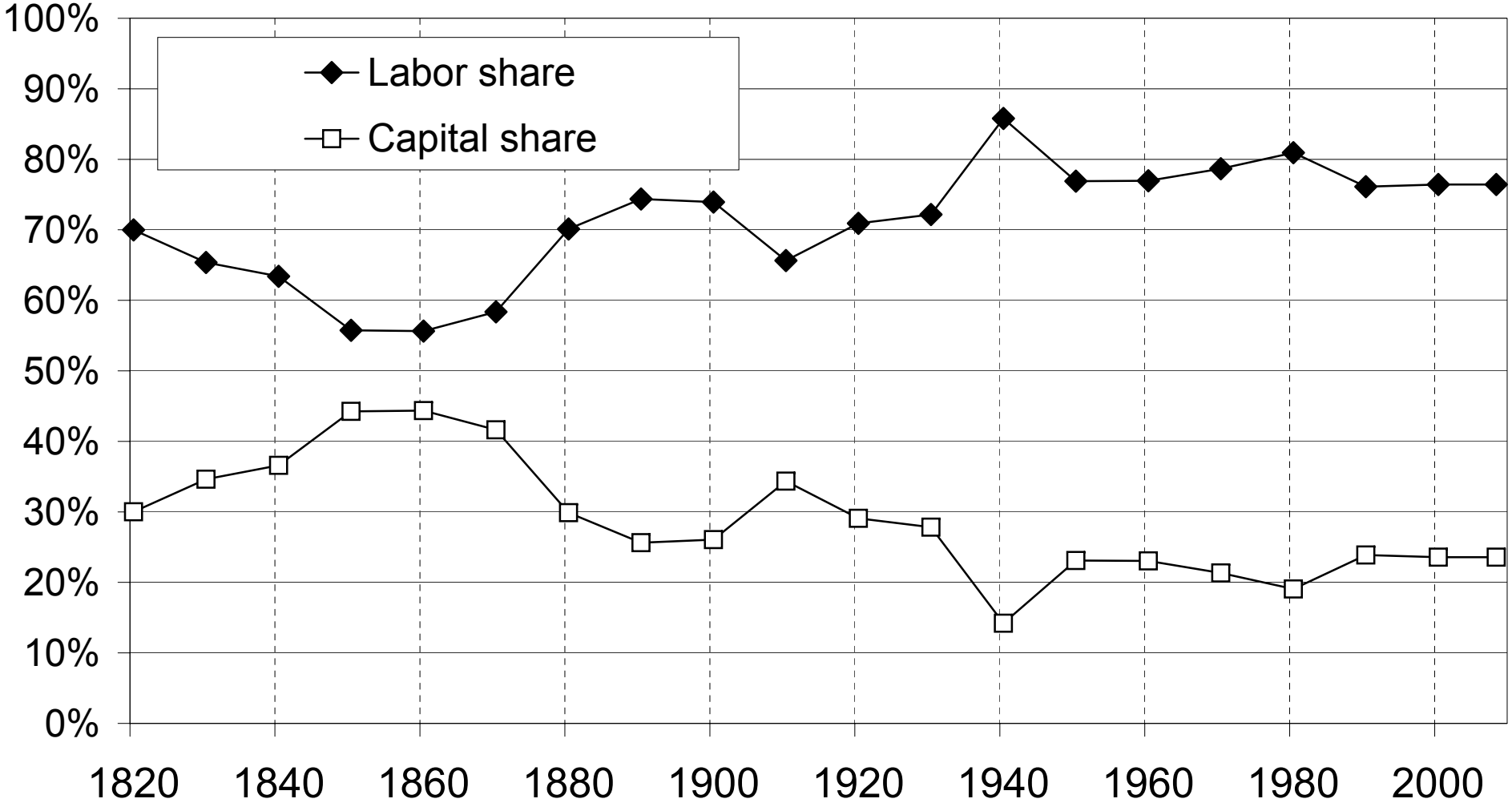

Figure 17: Rate of return vs growth rate France 1820-1913

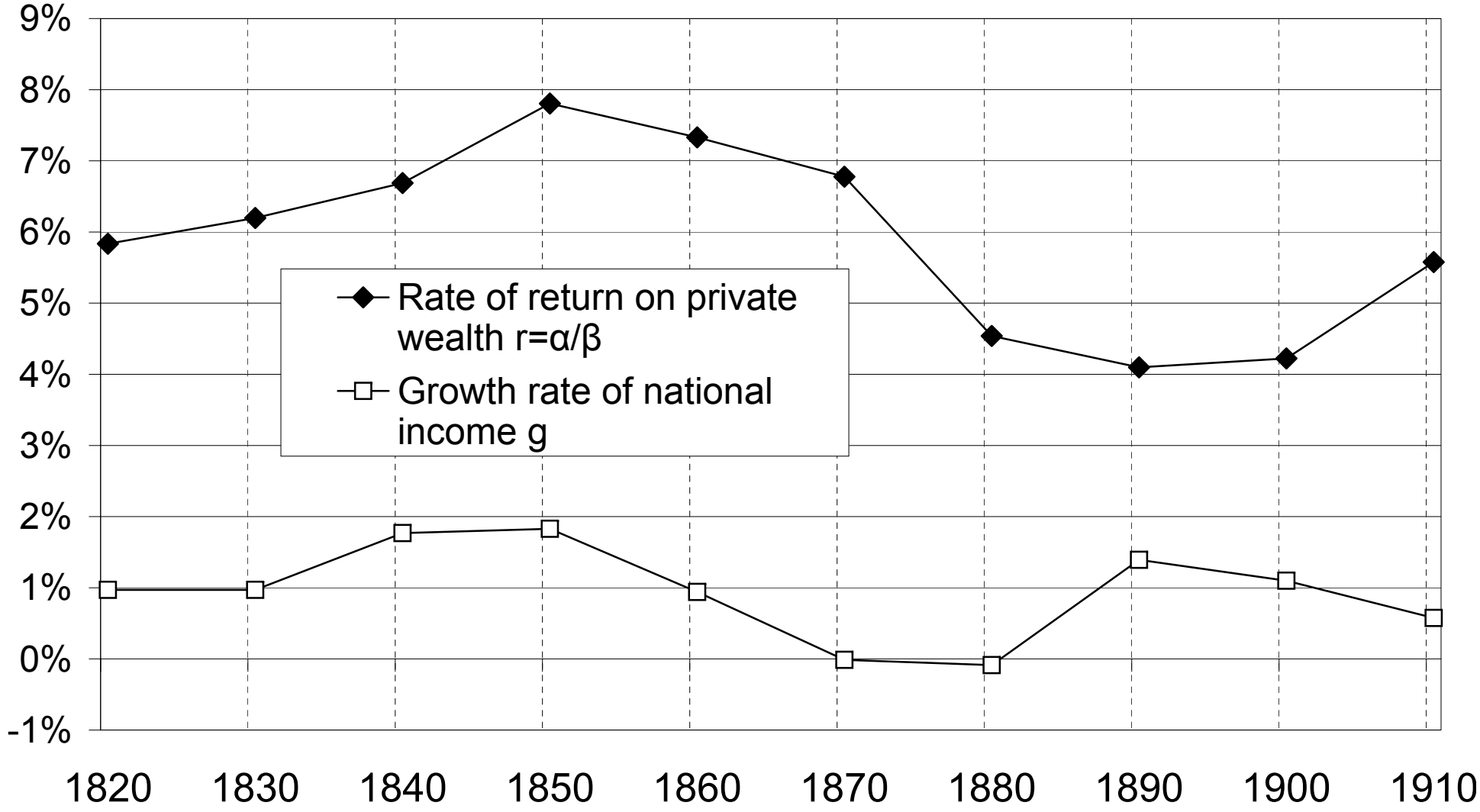

Figure 18: Capital share vs savings rate France 1820-1913

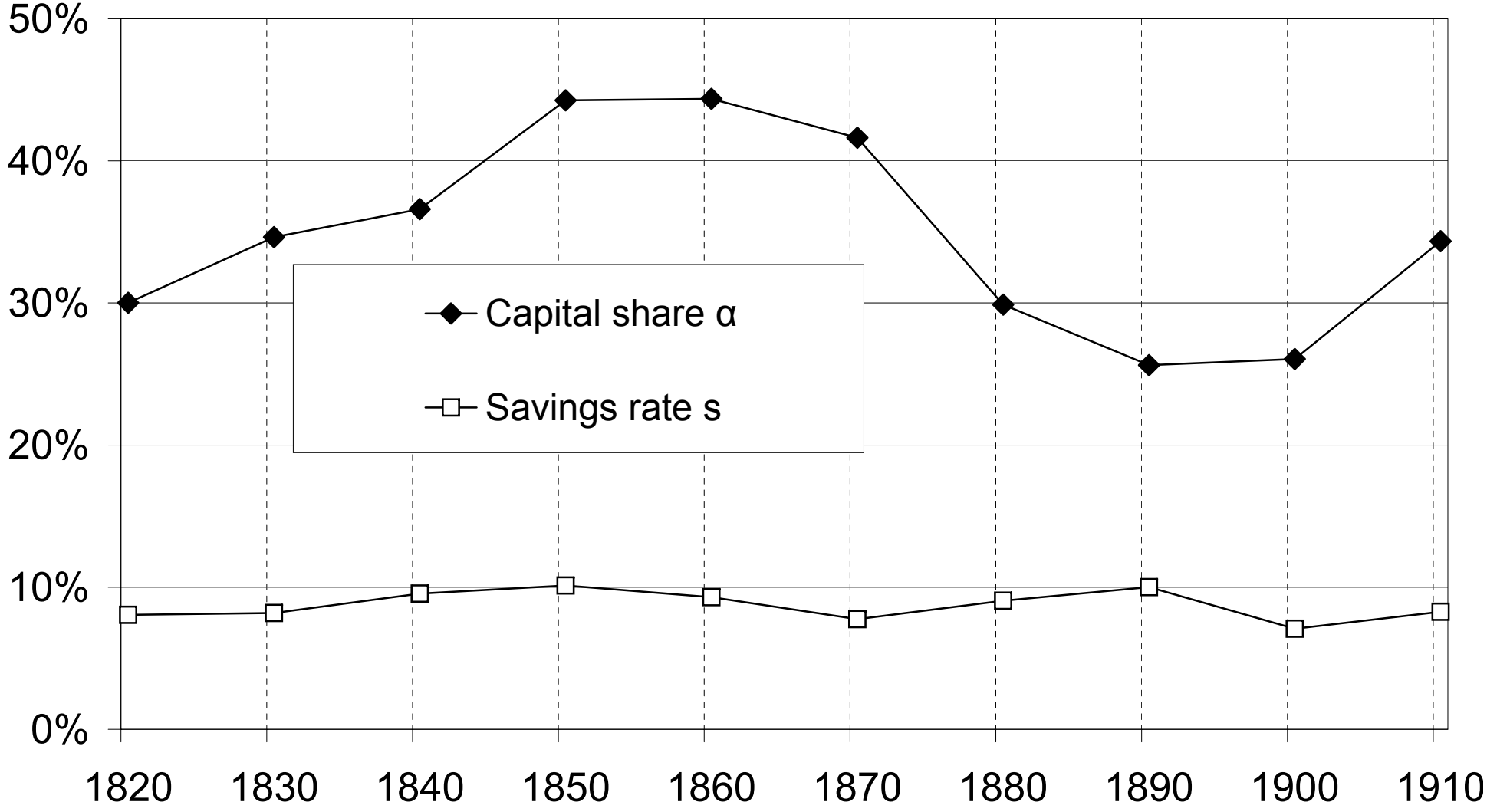

Figure 19a: The share of inheritance in lifetime ressources received by cohorts born in 1820-2020

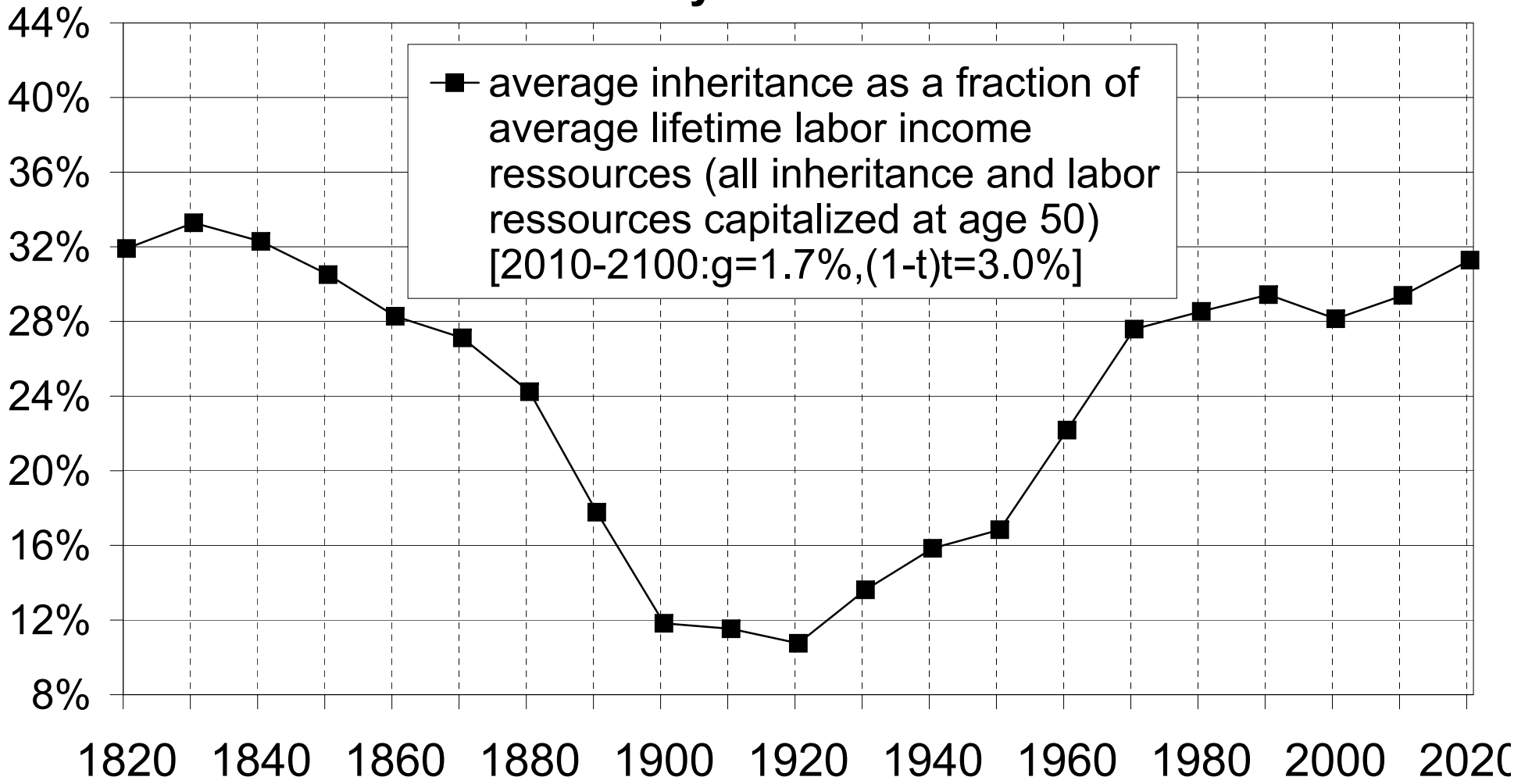

Figure 20a: Top 50% successors vs bottom 50% labor income earners (cohorts born in 1820-2020)

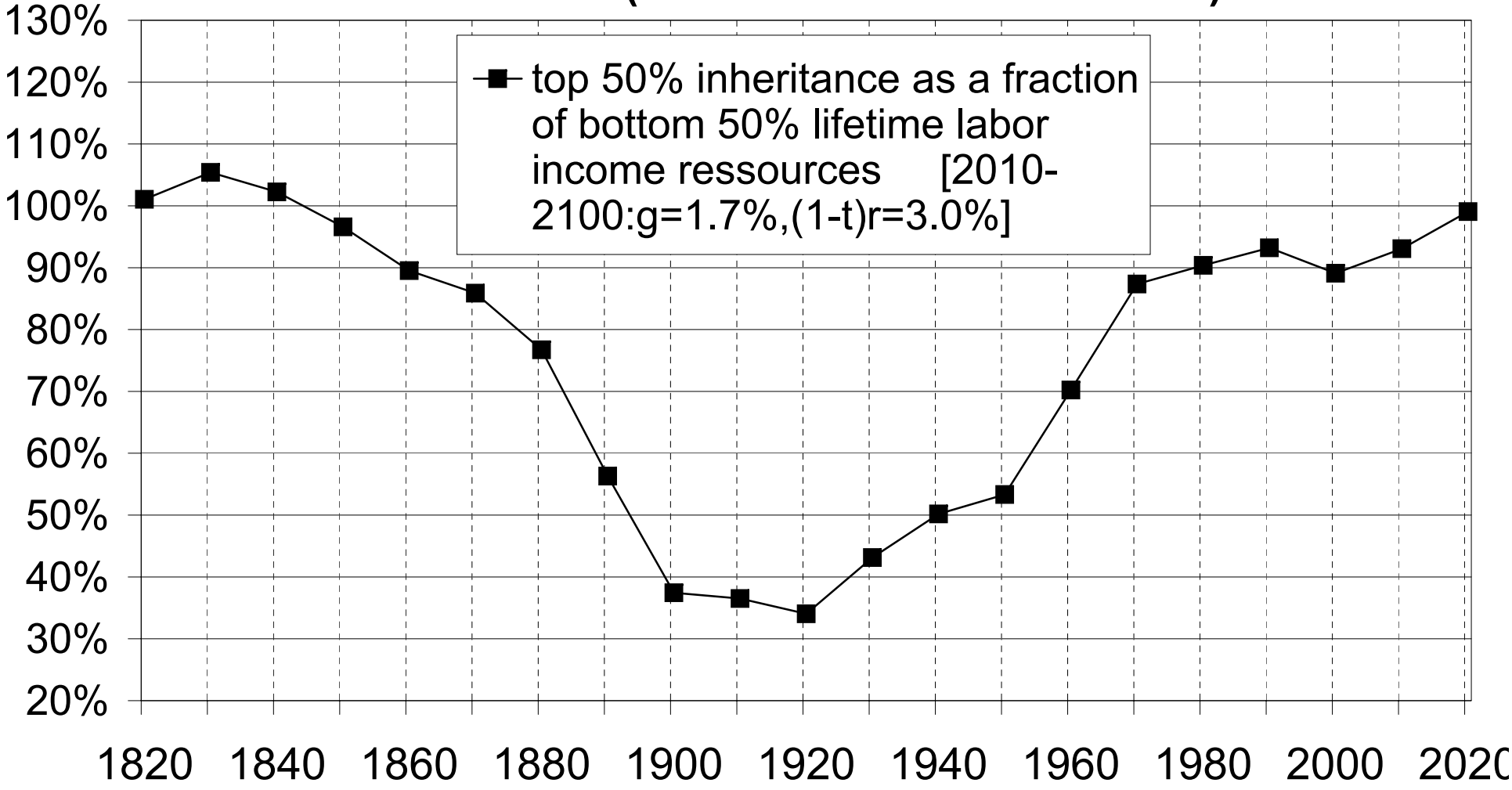

Figure 21a: Top 10% successors vs bottom 50% labor income earners (cohorts born in 1820-2020)

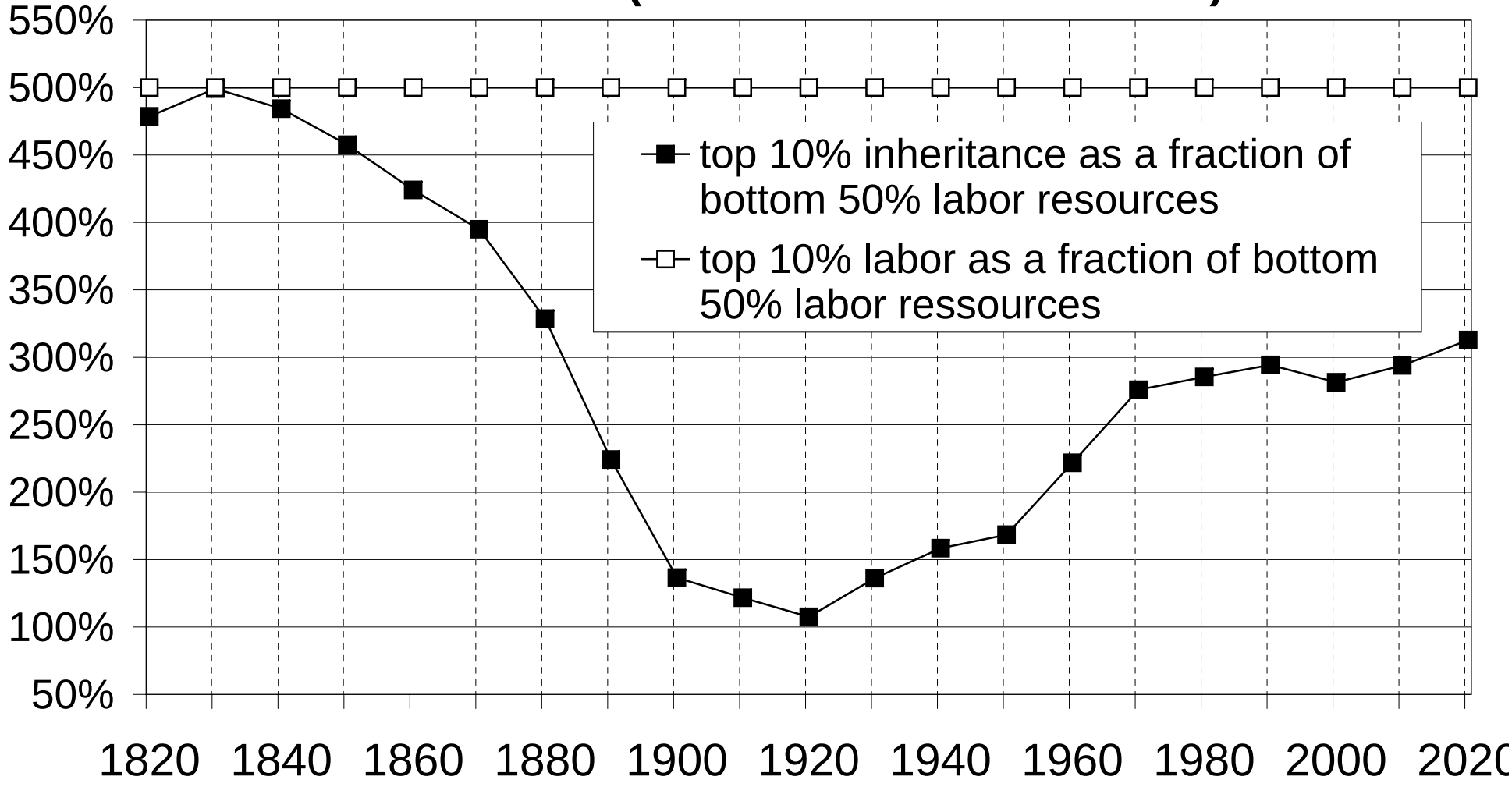

Figure 22a: Top 1% successors vs bottom 50% labor income earners (cohorts born in 1820-2020)

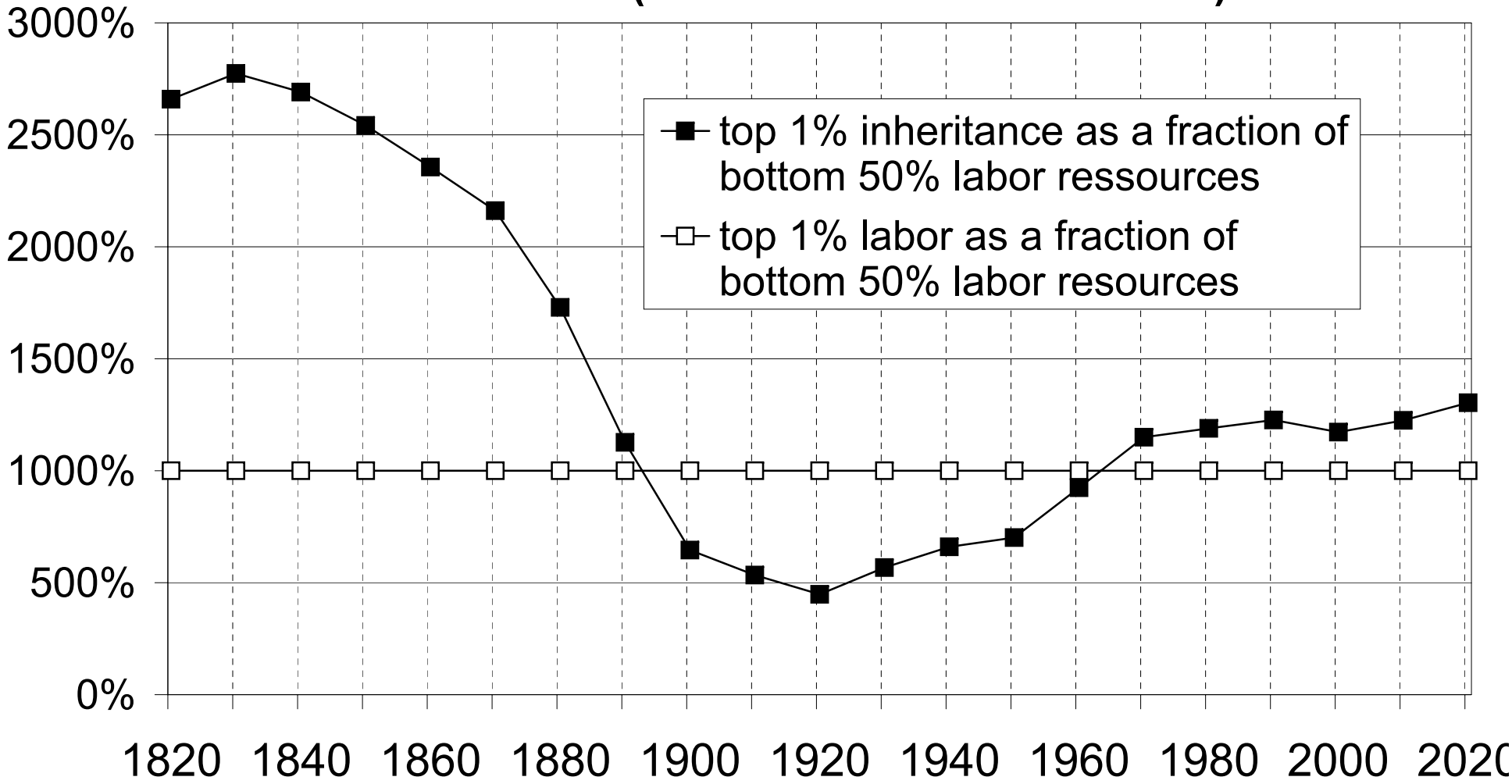

Figure 23a: Cohort fraction inheriting more than bottom 50% labor income (cohorts born in 1820-2020)

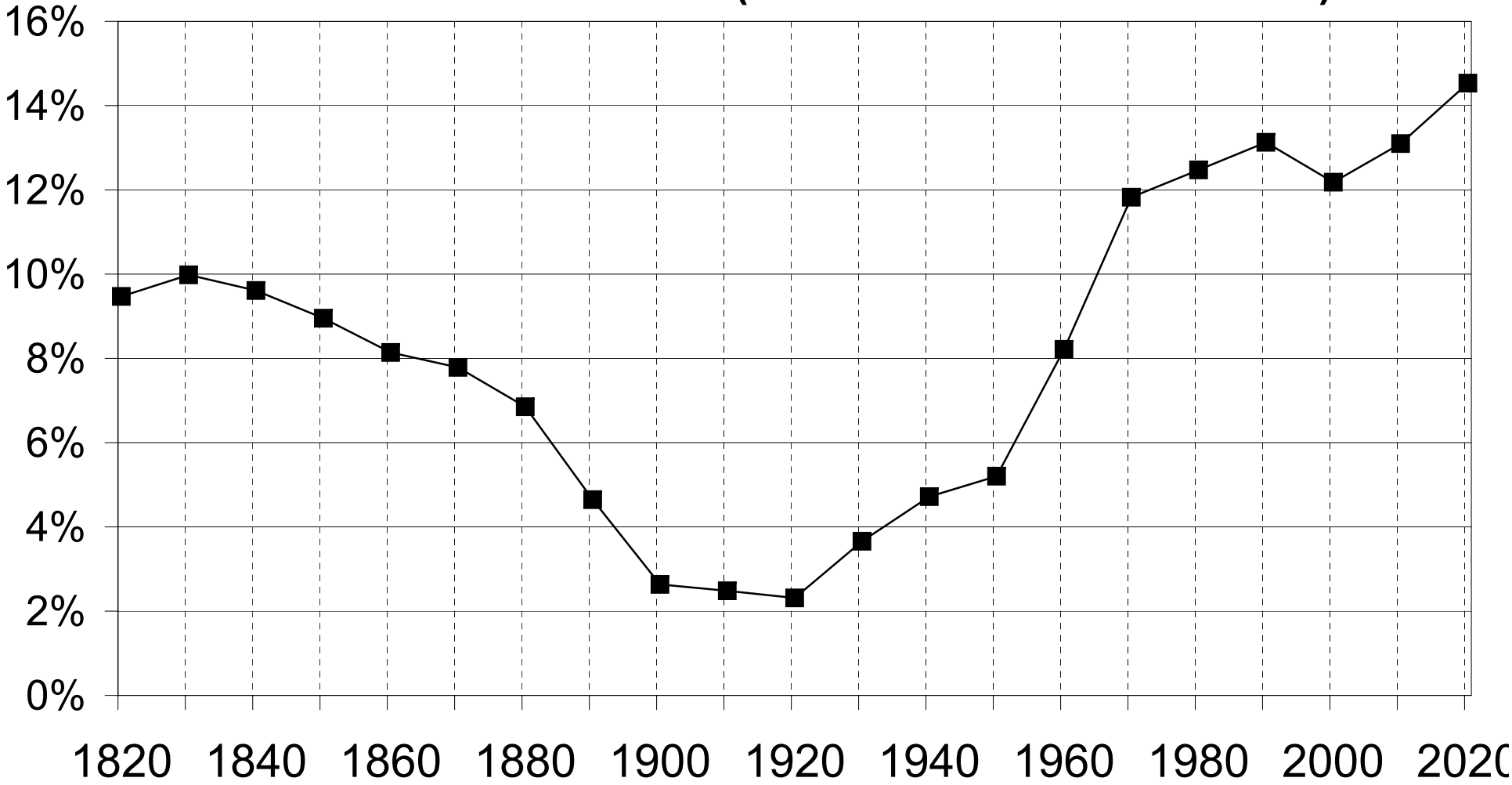

Figure 19b: The share of inheritance in lifetime ressources received by cohorts born in 1820-2020

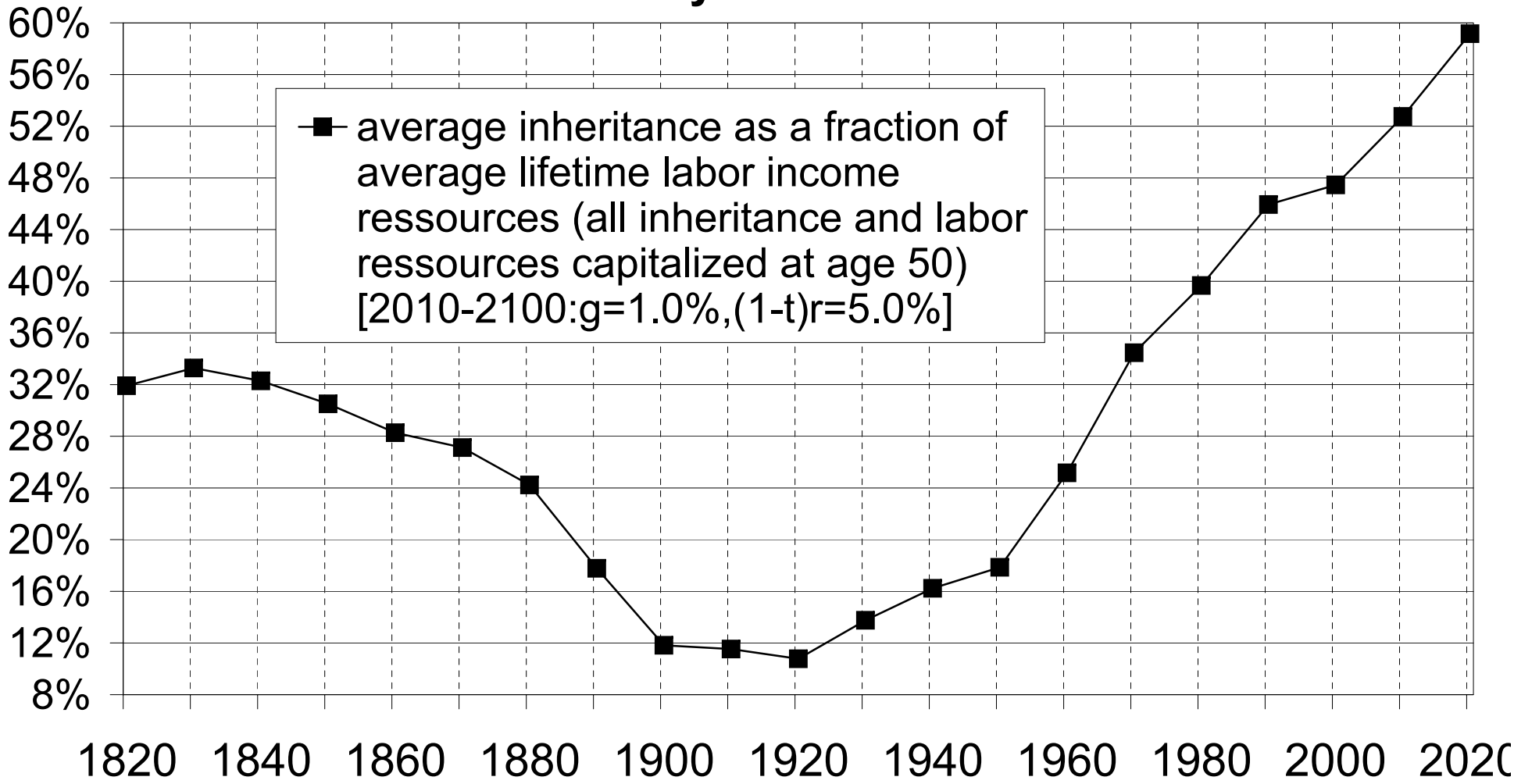

Figure 20b: Top 50% successors vs bottom 50% labor income earners (cohorts born in 1820-2020)

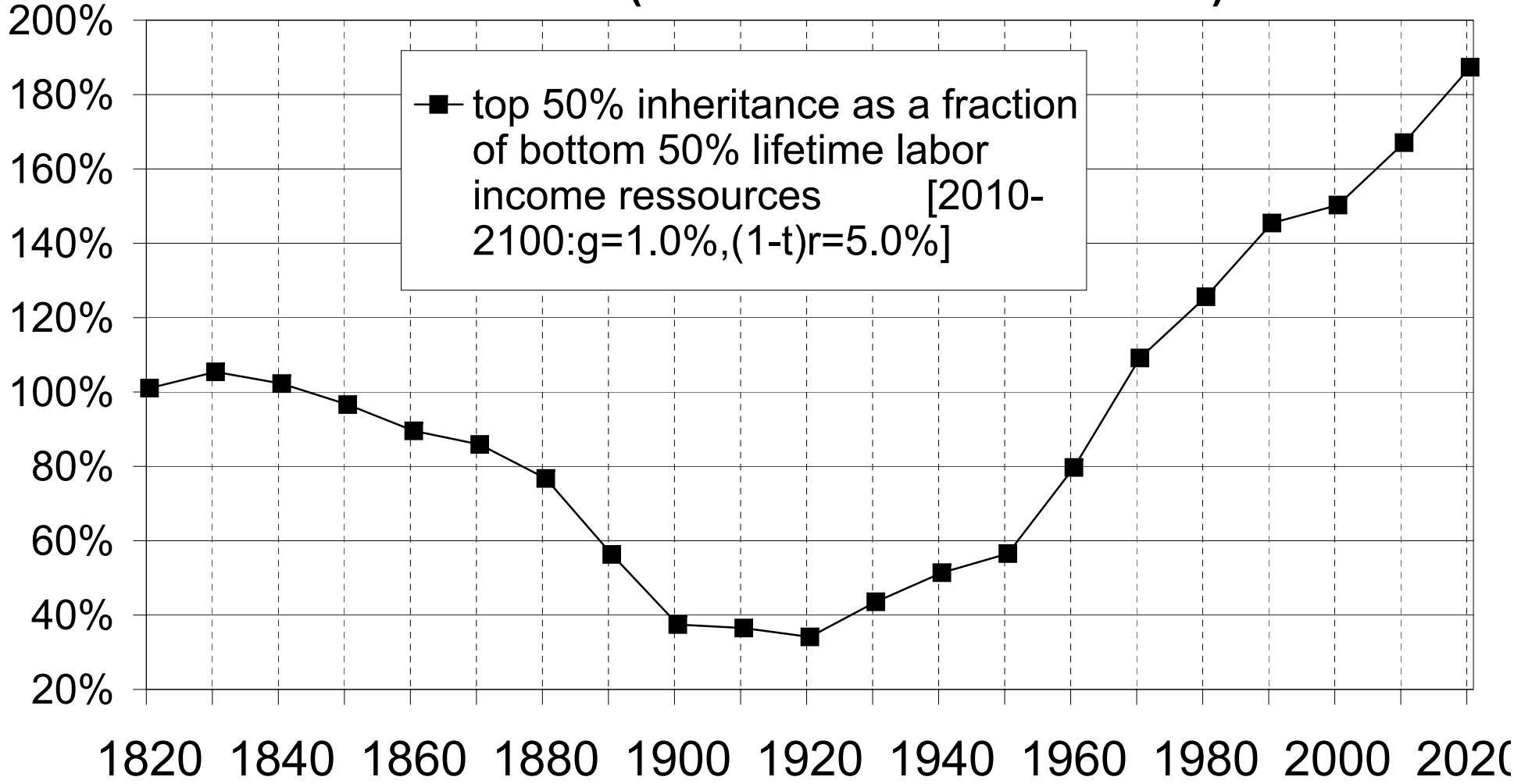

Figure 21b: Top 10% successors vs bottom 50% labor income earners (cohorts born in 1820-2020)

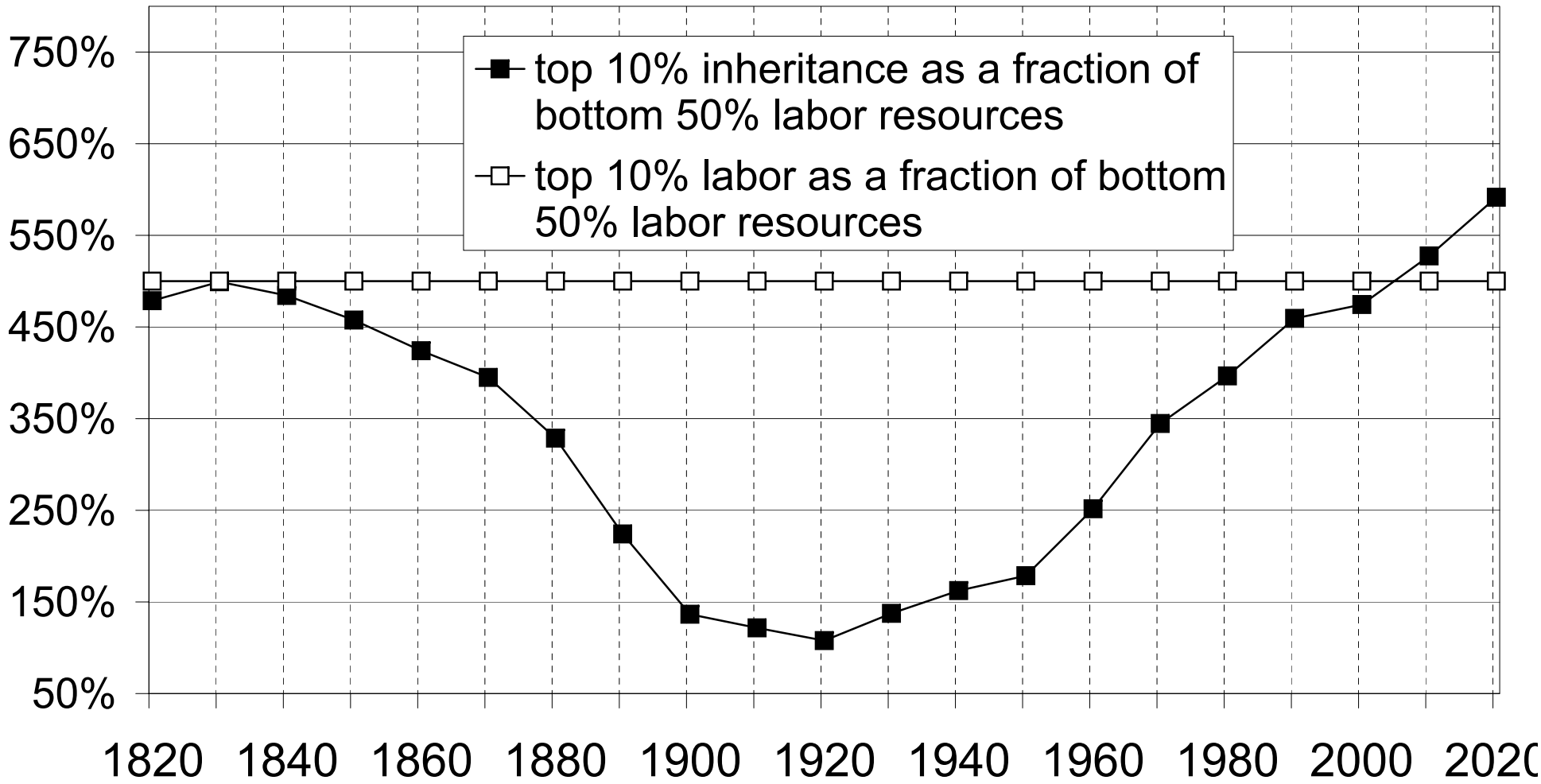

Figure 22b: Top 1% successors vs bottom 50% labor income earners (cohorts born in 1820-2020)

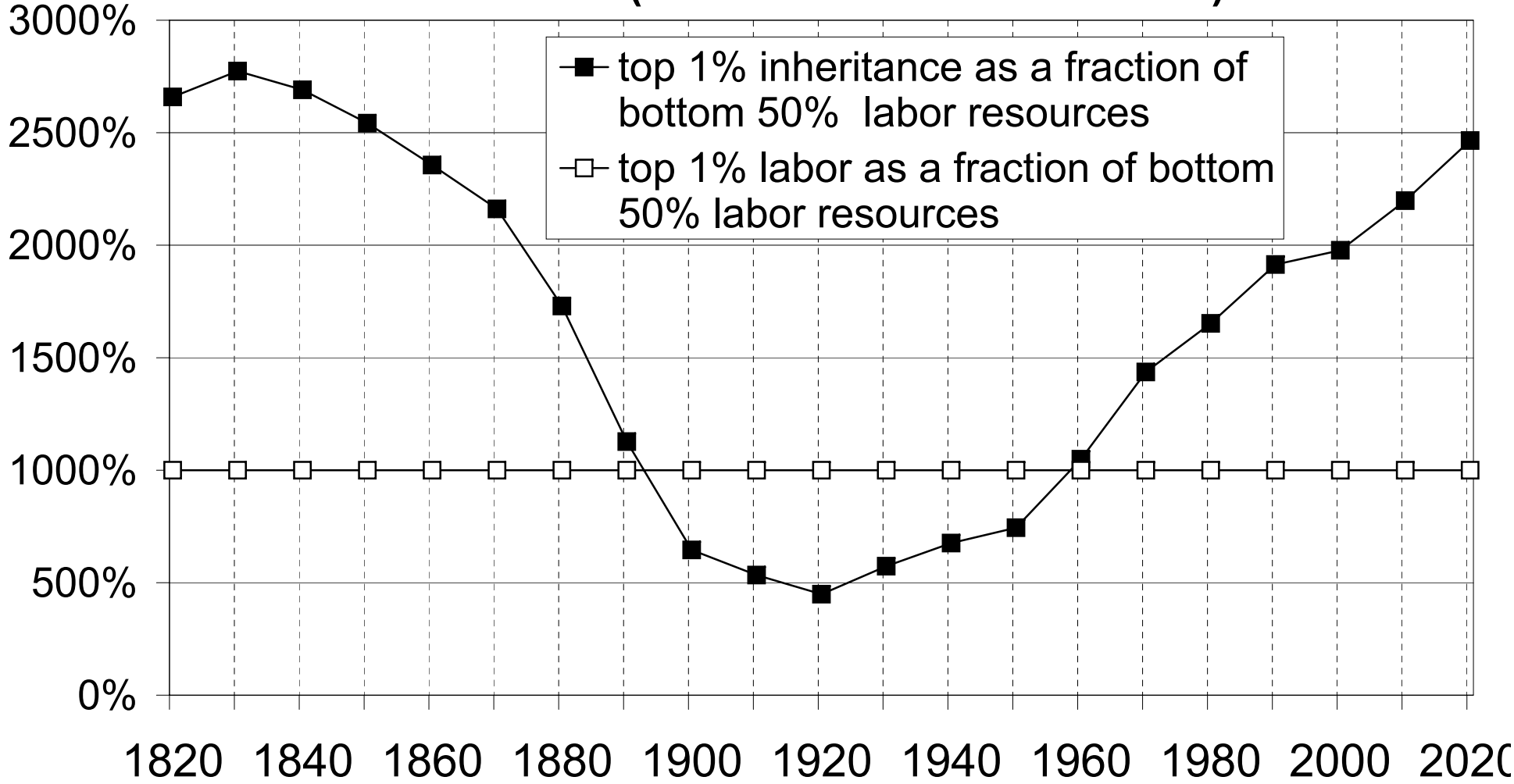

Figure 23b: Cohort fraction inheriting more than bottom 50% labor income (cohorts born in 1820-2020)

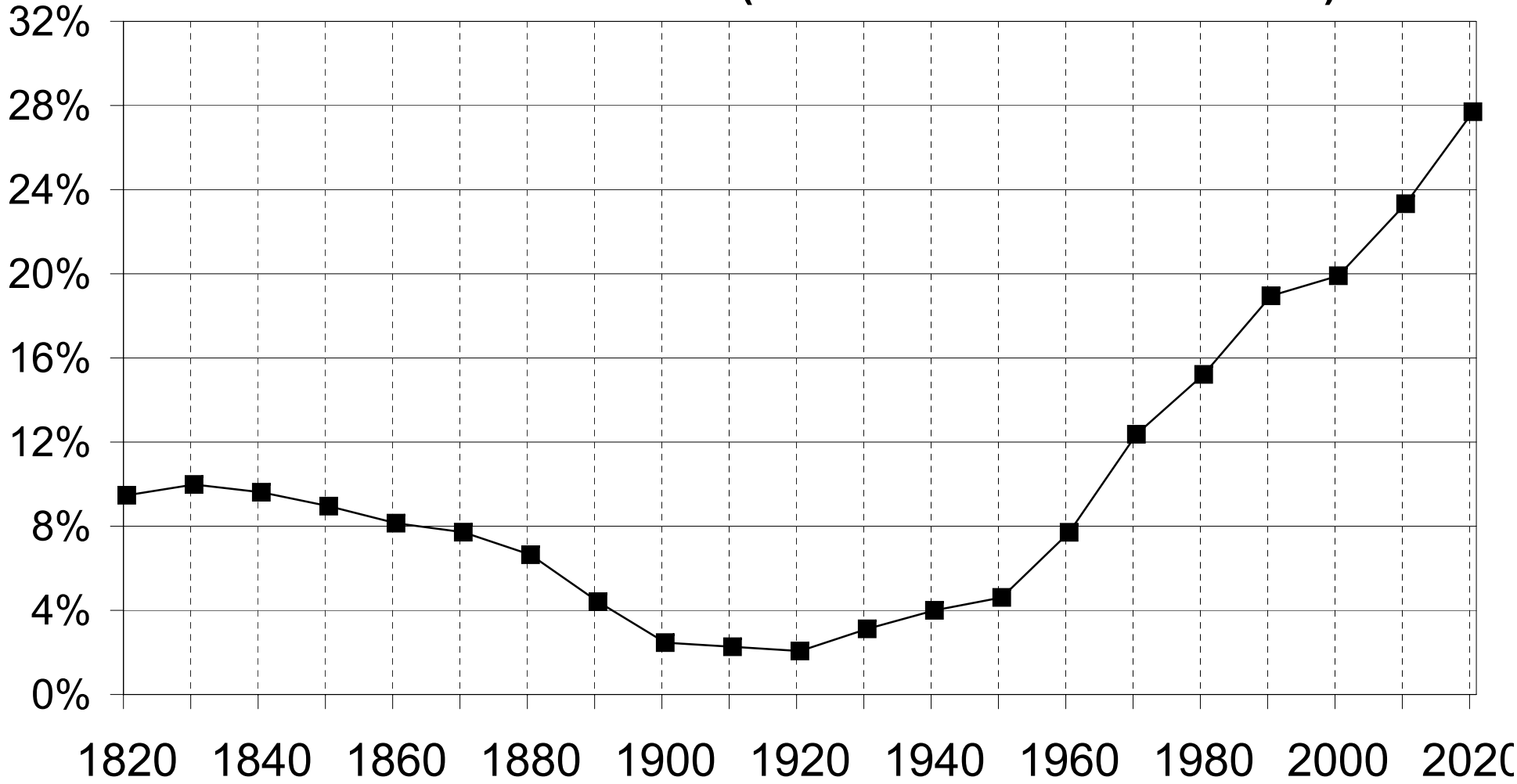

Figure 24a: The share of non-capitalized inheritance in aggregate wealth accumulation , France 1850-2100

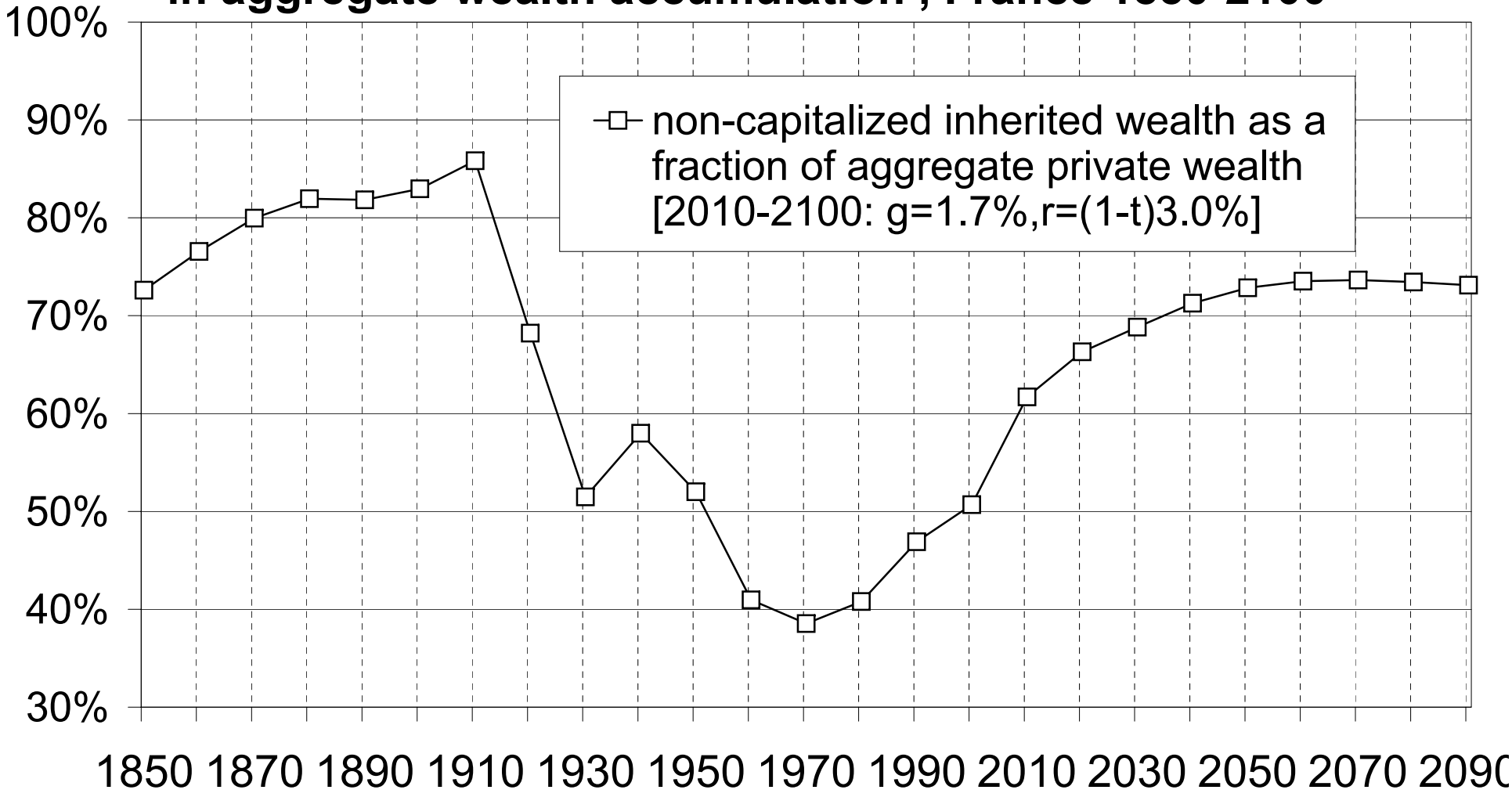

Figure 25a: The share of capitalized inheritance in aggregate wealth accumulation , France 1900-2100

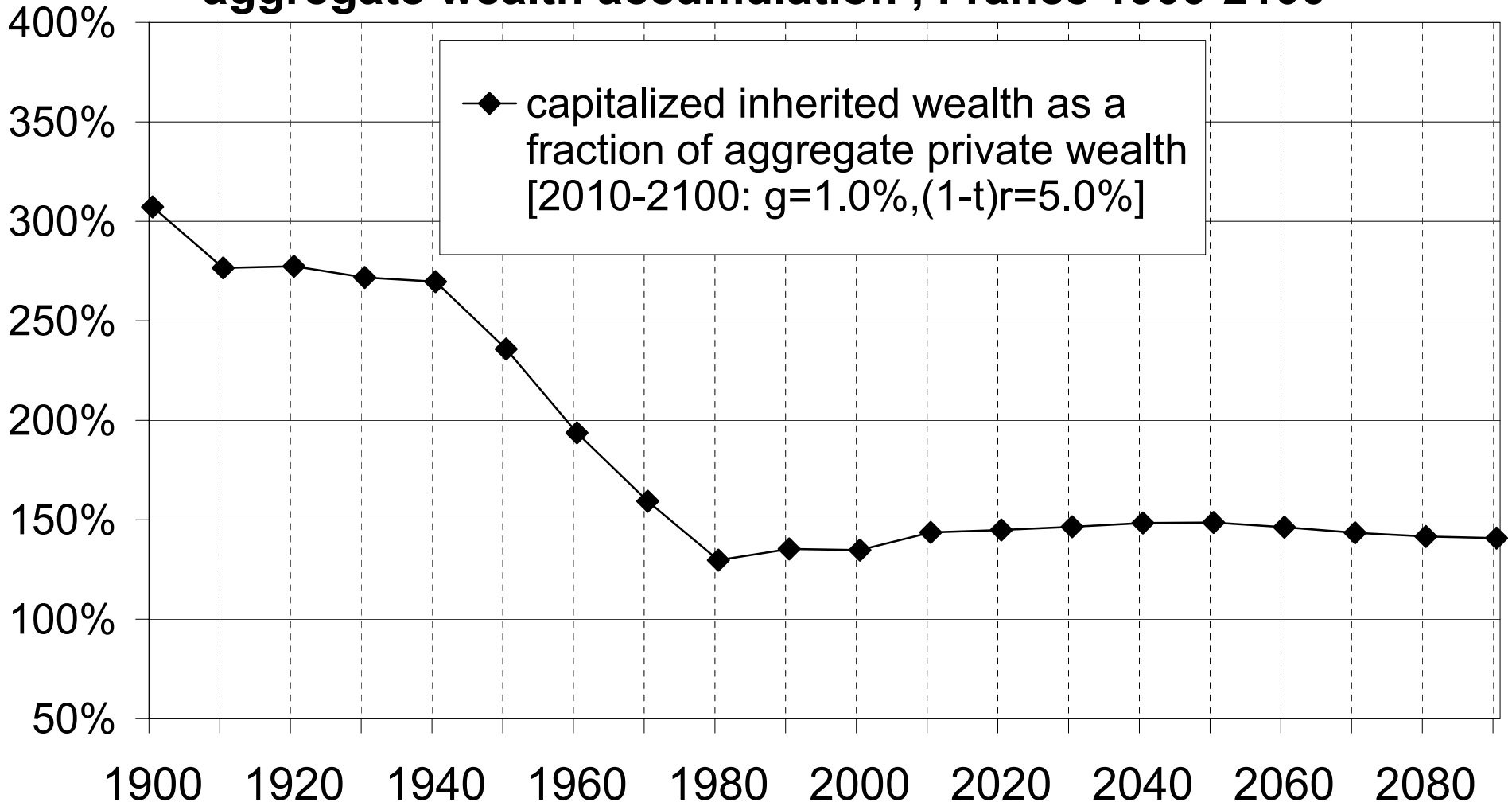

Figure 24b: The share of non-capitalized inheritance in aggregate wealth accumulation , France 1850-2100

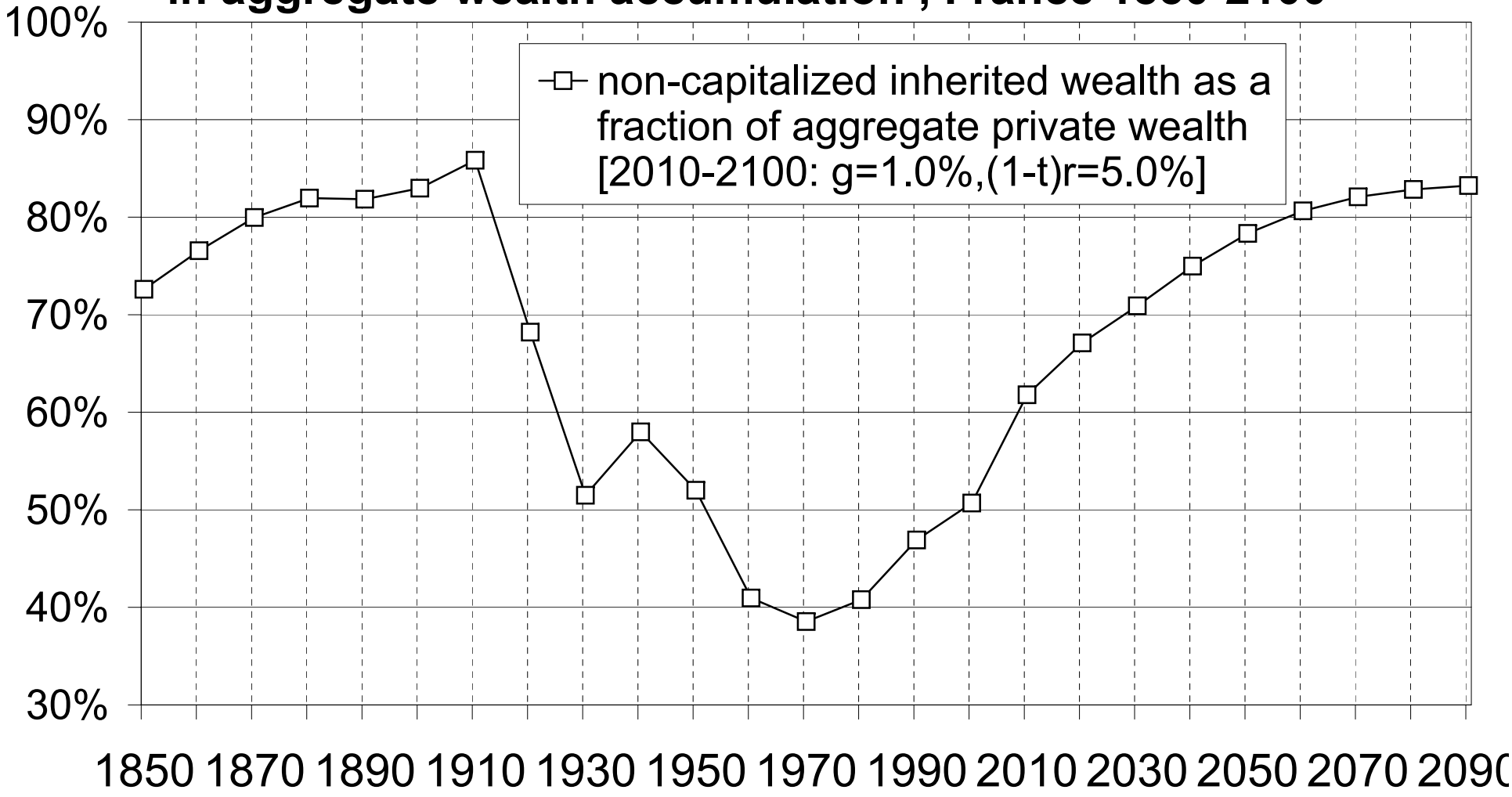

Figure 25b: The share of capitalized inheritance in aggregate wealth accumulation , France 1900-2100

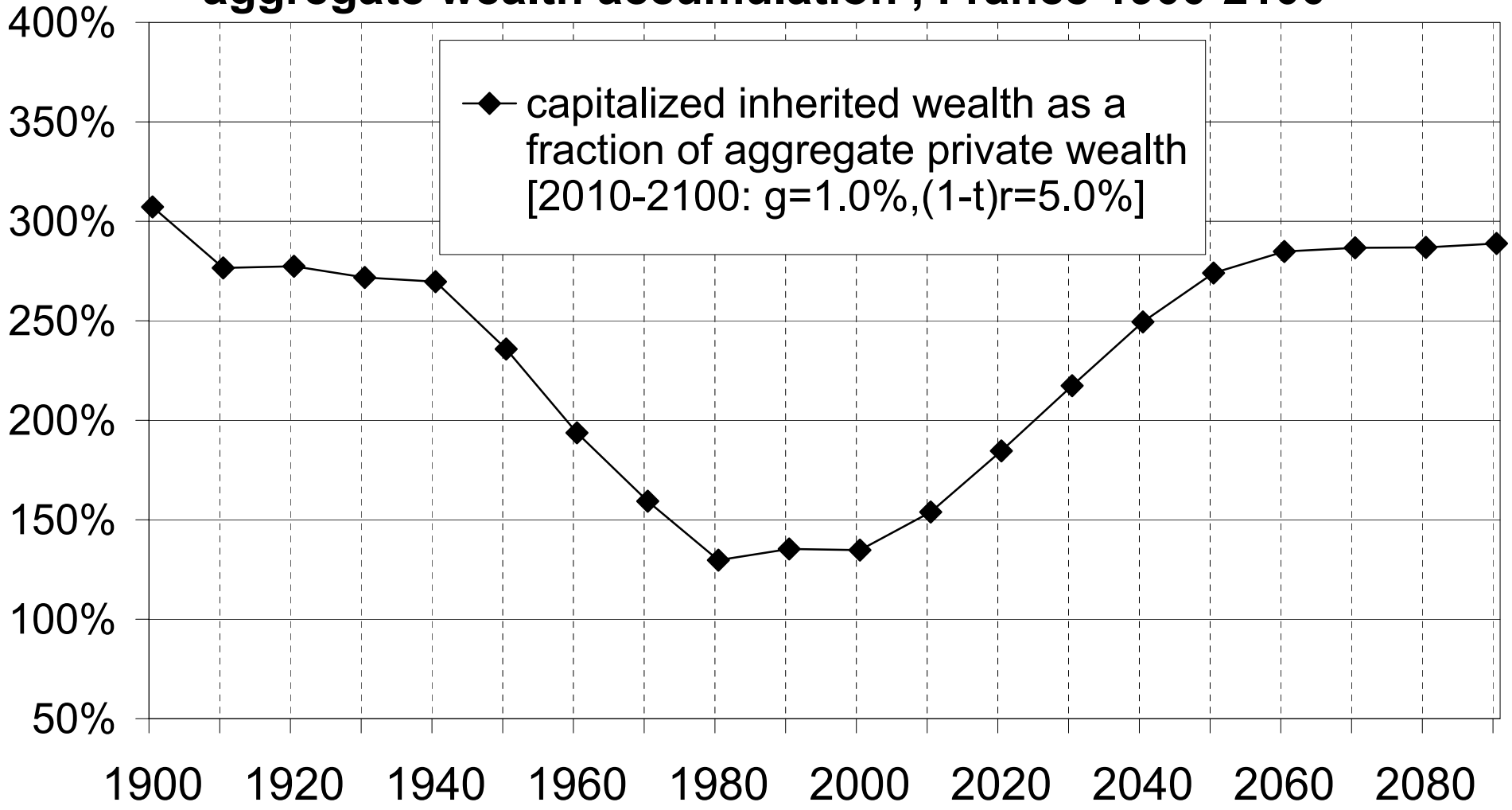

Table 1: Accumulation of private wealth in France, 1820-2009

	Real growth rate of national income g	Real growth rate of private wealth g_w	Savings-induced wealth growth rate $g_{ws} = s/\beta$	Capital-gains-induced wealth growth rate q	*Memo: Consumer price inflation* *p*
1820-2009	1.8%	1.8%	2.1%	-0.3%	*4.4%*
1820-1913	1.0%	1.3%	1.4%	-0.1%	*0.5%*
1913-2009	2.6%	2.4%	2.9%	-0.4%	*8.3%*
1913-1949	1.3%	-1.7%	0.9%	-2.6%	*13.9%*
1949-1979	5.2%	6.2%	5.4%	0.8%	*6.4%*
1979-2009	1.7%	3.8%	2.8%	1.0%	*3.6%*

Table 2: Raw age-wealth-at-death profiles in France, 1820-2008

	20-29	30-39	40-49	50-59	60-69	70-79	80+
1820	29%	37%	47%	100%	134%	148%	153%
1850	28%	37%	52%	100%	128%	144%	142%
1880	30%	39%	61%	100%	148%	166%	220%
1902	26%	57%	65%	100%	172%	176%	238%
1912	23%	54%	72%	100%	158%	178%	257%
1931	22%	59%	77%	100%	123%	137%	143%
1947	23%	52%	77%	100%	99%	76%	62%
1960	28%	52%	74%	100%	110%	101%	87%
1984	19%	55%	83%	100%	118%	113%	105%
2000	19%	46%	66%	100%	122%	121%	118%
2006	25%	42%	74%	100%	111%	106%	134%

Table 3: Rates of return vs growth rates in France, 1820-2009

	Growth rate of national income g	Rate of return on private wealth $r = \alpha/\beta$	Capital tax rate τ_K	After-tax rate of return $r_d = (1-\tau_K)\alpha/\beta$	Real rate of capital gains q	Rate of capital destruct. (wars) d	After-tax real rate of return (incl. k gains & losses) $r_d = (1-\tau_K)\alpha/\beta + q + d$
1820-2009	**1.8%**	**6.8%**	19%	**5.4%**	-0.1%	-0.3%	**5.0%**
1820-1913	**1.0%**	**5.9%**	8%	**5.4%**	-0.1%	0.0%	**5.3%**
1913-2009	**2.6%**	**7.8%**	31%	**5.4%**	-0.1%	-0.7%	**4.6%**
1913-1949	**1.3%**	**7.9%**	21%	**6.4%**	-2.6%	-2.0%	**1.8%**
1949-1979	**5.2%**	**9.0%**	34%	**6.0%**	0.8%	0.0%	**6.8%**
1979-2009	**1.7%**	**6.9%**	39%	**4.3%**	1.0%	0.0%	**5.3%**

Table 4: Intra-cohort distributions of labor income and inheritance, France, 1910 vs 2010

Shares in aggregate labor income or inherited wealth	**Labor income 1910-2010**	**Inherited wealth**	
		1910	**2010**
Top 10% "Upper Class"	**30%**	**90%**	**60%**
incl. Top 1% "Very Rich"	*6%*	*50%*	*25%*
incl. Other 9% "Rich"	*24%*	*40%*	*35%*
Middle 40% "Middle Class"	**40%**	**5%**	**35%**
Bottom 50% "Poor"	**30%**	**5%**	**5%**

Table 5: Lifetime inequality: illustration with cohorts born in the 1970s

Lifetime ressources capitalized at age 50	**Labor income**	**Inherited wealth**	*Inherited wealth with 1910 distribution*
Top 10% "Upper Class"	**4 740 000 €**	**2 640 000 €**	3 960 000 €
incl. Top 1% "Very Rich"	*9 480 000 €*	*11 000 000 €*	*22 000 000 €*
incl. Other 9% "Rich"	*4 210 000 €*	*1 710 000 €*	*1 960 000 €*
Middle 40% "Middle Class"	**1 580 000 €**	**390 000 €**	60 000 €
Bottom 50% "Poor"	**950 000 €**	**40 000 €**	40 000 €
Cohorts averages (€ 2009)	1 580 000 €	440 000 €	440 000 €