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**February 2011**

**DT-GREQAM**

# Long-run relationships between international stock prices: further evidence from fractional cointegration tests

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## Abstract

The recent empirical literature supports the view that most of the international stock prices are not pairwise cointegrated. However, by using fractional cointegration techniques, this paper shows that France, Germany, Hong Kong, and Japan stock prices indices are pairwise fractionally cointegrated with US stock prices. Equilibrium errors are mean reverting with half-life lying between 2 and 12 days. It is worthwhile noting that emerging markets like Brazil and Argentina are not pairwise cointegrated with the US stock market. These new results have important implications for asset pricing and international portfolio strategy.

**Keywords:** equity markets, fractional cointegration, long memory

**JEL classification:** C12, C22, F31, F37, G15.

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# 1 Introduction

A great number of papers have used cointegration techniques to examine the long-run relationships between international stock prices, motivated by the fact that cointegration between stock prices has several important implications for asset pricing. Firstly, cointegration between prices of some national stock markets implies that these markets share a common stochastic trend. As a consequence, potential benefits from long run diversification will be reduced since deviations of one market away from the equilibrium relationship can be expected to reverse geometrically over the long run. Secondly, as stated by Granger (1986), evidence of cointegration among world capital markets may lead to the rejection of the efficient markets hypothesis since cointegration induces short run predictability of prices via the error correction mechanism; however, Richards (1995), among others, argues that cointegration among stock prices may not necessarily imply violation of market efficiency. Thirdly, some researchers have documented the long-run predictability of prices through the Winner–Loser reversal effect (Richards, 1995) which states that markets that have experienced superior performance can be expected to underperform over the longer term, and vice versa.

On the empirical side, the literature analyzing the long-run relationships between international stock markets has produced mixed results. Some papers (Kasa, 1992, Corhay et al., 1993, Dunis and Shannon, 2005; Fraser and Oyefeso, 2005; Diamandis, 2009) found at least one common stochastic trend between international stock indices using Johansen’s (1988) multivariate linear cointegration tests. However, the recent literature generally supports the view that most of the international stocks are not linearly pairwise cointegrated (Chan et al., 1997; Kanas, 1998; Pynnönen and Knif, 1998; Huang and Fok, 2001; Davies, 2006; Li, 2006; Olusi and Abdul-Majid, 2008) or that the evidence for multivariate cointegration is weak<sup>1</sup> (Ahlgren and Antell, 2002). Focusing on UK (Taylor and Tonks, 1989), European stock markets (Rangvid, 2001; Garcia Pascual, 2003; Bley, 2009) or Pacific-Basin countries (Phylaktis and Ravazzolo, 2005), some papers suggest moreover that the integration process of financial markets may be time and/or country dependent.

An important reason for these mixed results is that usual linear testing techniques may be inadequate in presence of non-standard dynamics, such as nonlinearity or structural change. Therefore, Li (2006) applies rank test for non linear cointegration while Davies (2006) uses regime switching cointe-

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<sup>1</sup>Ahlgren and Antell (2002) and Richards (1995) point out that some of the previous empirical results can be explained by the small-sample bias and size distortion of Johansen’s LR tests for cointegration. Moreover, they underline the fact that Johansen’s tests appear to be sensitive to the lag length specification in the VAR model.

gration techniques and Huang and Fok (2001) suggest that markets may be temporally cointegrated by using tests related to the stochastic permanent breaks model.

The aim of this paper is to provide further evidence on the pairwise linkages between the US and some of the major foreign equity markets by taking into account the fractional cointegration hypothesis<sup>2</sup>. According to this hypothesis, the cointegration errors tend to revert back hyperbolically (and not geometrically) to some mean (or deterministic trend): as in the standard cointegration framework, fractional cointegration introduces arbitrage opportunities, since there is some predictability of prices in the long-run (Winner-Loser effect) as well as in the short-run (through the fractionally error-correction mechanism suggested by Granger, 1986). However, in a strategic (or static in the sense of Lucas) asset allocation perspective, fractional cointegration reduces the gain of portfolio diversification.

In this paper we consider France, Germany, UK and Japan equity markets and some emerging countries' equity markets like Argentina, Brazil and Hong Kong. We will proceed in three steps. The first step consists in investigating the order of integration of each national stock index. In a second step, we use the strategy of Gil-Alana (2003) and Caporale and Gil-Alana (2004a, 2004b) to consider the possibility of the series being pairwise fractionally cointegrated. The third step consists in measuring the persistence of shocks for countries whose stock markets are related to the US one in the long-run. We confirm that all stock index series we consider are non-stationary  $I(1)$ . Except for Germany, we find no standard cointegration between US stock market and the foreign stock markets into consideration, as already stated by Kanas (1998) for the case of France and UK. However, we conclude that the US stock market is fractionally cointegrated with the French, German, Japanese, UK and Hong Kong stock market indices. Conversely, the relationship is neither significant for the US stock market and the Argentinian index, nor for the US stock market and the Brazilian index. The persistence is measured by the half-lives which lie between 1.88 and 11.88 days, depending on the country considered. The highest half-life is the Japanese one, the lowest being the French one.

The rest of the paper is organized as follows. The next section presents the econometric method for detecting fractional integration and cointegration. The empirical application is carried out in Section 3 while Section 4 contains some concluding comments.

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<sup>2</sup>The paper of Pynnönen and Knif (1998) constitutes a first attempt to apply the fractional cointegration hypothesis in the case of stock markets. Using the Cheung and Lai's (1993) fractional cointegration test in the case of two scandinavian stock markets, the authors found no evidence of fractional cointegration.

## 2 The econometric approach

### 2.1 Fractional integration and cointegration

A time series  $y_t$  follows an  $ARFIMA(p, d, q)$  (autoregressive fractionally integrated moving average) process if

$$\Phi(L)(1 - L)^d y_t = \mu + \Theta(L)\varepsilon_t,$$

with

$$\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p, \Theta(L) = 1 + \theta_1 L + \dots + \theta_q L^q,$$

$L$  is the Backward shift operator i.e.  $Ly_t = y_{t-1}$  and  $\varepsilon_t \sim iid(0, \sigma^2)$ . Different cases are possible, depending on the value of the long memory parameter  $d$ ; for example,  $y_t$  is stationary and possesses shocks that disappear hyperbolically when  $0 < d < 1/2$ , but is non-stationary and mean reverting for  $1/2 \leq d < 1$ . Moreover, fractional cointegration can be defined as follows. Let us consider two time series  $y_t$  and  $x_t$  that are both  $I(d)$ , where  $d$  is not necessarily an integer;  $y_t$  and  $x_t$  are fractionally cointegrated when the residuals, defined by  $e_t = y_t - \beta x_t$ , are  $I(d - b)$  with  $b > 0$ , where  $b$  is also not necessarily an integer. There is a growing literature dealing with fractional cointegration<sup>3</sup>.

We use here the methodology elaborated by Robinson (1994) for testing unit root and other nonstationary hypotheses. Let us consider the null hypothesis defined by  $H_0 : \theta = 0$  in the model given by  $y_t = \beta' X_t + e_t$  and  $(1 - L)^{d+\theta} e_t = u_t$ , for  $t = 1, 2, \dots$ , where  $y_t$  is the observed time series,  $X_t$  is a  $k \times 1$  vector of deterministic regressors,  $u_t$  is a (possibly weakly autocorrelated)  $I(0)$  process, and  $d$  is a real parameter. The Lagrange Multiplier (LM) statistic proposed by Robinson (1994), called  $\hat{r}$  (see Appendix for details) has a standard asymptotic distribution under some regularity conditions:  $\hat{r} \xrightarrow{d} N(0, 1)$  as  $T \rightarrow \infty$ . Thus, it is a one-sided test of  $H_0 : \theta = 0$  : we reject  $H_0$  against  $H_1 : \theta > 0$  if  $\hat{r} > z_\alpha$  and against  $H_1 : \theta < 0$  if  $\hat{r} < -z_\alpha$ , where the probability that a standard normal variate exceeds  $z_\alpha$  is  $\alpha$ . This Robinson (1994)'s test has been used in several papers in order to detect fractional integration<sup>4</sup>.

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<sup>3</sup>For instance, see among others Baillie and Bollerslev (1994), Caporale and Gil-Alana (2004a and b), Cheung and Lai (1993), Christensen and Nielson (2006), Davidson (2005), Dittman (2001), Hassler, Marmol and Velasco (2006), Kim and Phillips (2001), Marinucci and Robinson (2001), Nielsen (2006), Robinson and Marinucci (2003), Robinson and Yajima (2002), Tse, Anh and Tieng (1999), Velasco (2003).

<sup>4</sup>Among others: Caporale and Gil-Alana (2004a and b, 2007a and b), Gil-Alana (2003), and Gil-Alana and Nazarski (2007).

In order to detect the cointegration, we adopt the two-step strategy of Gil-Alana (2003) and Caporale and Gil-Alana (2004a and b) based on the Robinson (1994)<sup>5</sup> test: in the first step, we test the order of integration of each series, and if they are of the same order, we test, in the second step, the order of integration of the estimated residuals of the cointegration relationship. Gil-Alana (2003) and Caporale and Gil-Alana (2004a) note that the ordinary least squares (OLS) estimation of the equilibrium error can produce an estimator which may suffer from second-order bias in small samples, but they choose to use it on the grounds of simplicity; in this paper, the sample sizes are enough large to neglect this problem. Let us call  $e_t$ , the estimated equilibrium errors between two series  $y_t$  and  $x_t$  (this can be easily generalized to more series):  $e_t = y_t - \widehat{\beta}x_t$  where  $\widehat{\beta}$  is the OLS estimator of the cointegrating parameter. Let us consider the model:  $(1 - L)^{d+\theta}e_t = u_t$  where  $u_t$  is a  $I(0)$  process ; we applied the Robinson (1994)'s testing procedure in order to test the null hypothesis  $H_0 : \theta = 0$  against the alternative  $H_1 : \theta < 0$ . If the null hypothesis is rejected, it implies that the equilibrium error exhibits a smaller degree of integration than the original series:  $y_t$  and  $x_t$  are thus fractionally cointegrated. On the opposite, if the null hypothesis is not rejected, we will admit that the series are not cointegrated because the order of integration of  $e_t$  is the same as the order of the original series.

## 2.2 Half-life analysis

One way to estimate the persistence of the estimated residuals from the cointegration regression is to fit an ARFIMA model to  $e_t$  and then estimate its impulse response function. By allowing the long memory parameter  $d$  to take non-integer values, the fractional model accommodates a broader range of low-frequency, mean-reverting dynamics than do standard time series models.

The mean-reverting property holds if  $d < 1$  whereas the impact of a shock is known to persist forever in case of a unit-root process:  $d = 1$ . This can be

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<sup>5</sup>Gil-Alana (2003) conducts Monte-Carlo experiments in order to examine the size and power properties of the Robinson's (1994) test relative to the usual Engle-Granger's ADF and the Geweke & Porter-Hudak tests, the later being used by Cheung and Lai (1993) in order to test fractional cointegration.

These experiments show that Robinson's (1994) tests perform better than the ADF and GPH tests both in term of power and size. As stated by Gil-Alana (2003), "the difference in power between Robinson's (1994) and the ADF and GPH tests for cointegration should not be surprising given that the ADF test assumes a strict  $I(0)$  and  $I(1)$  distinction and the GPH test requires estimation of the differencing parameter, whereas Robinson (1994) tests do allow fractional differencing and do not require estimation of the fractional differencing parameter".

seen from the moving average representation for  $(1 - L)e_t = A(L)\varepsilon_t$  where

$$A(L) = (1 - L)^{1-d}\Psi(L) = 1 + a_1L + a_2L^2 + \dots$$

$$\Psi(L) = 1 + \psi_1L + \psi_2L^2 + \dots$$

The moving average coefficients  $a_j$ ,  $j = 1, \dots$ , are referred to as the impulse responses and can be computed as follows:

$$a_j = \sum_{k=0}^j \frac{\Gamma(k + d - 1)}{\Gamma(d - 1)\Gamma(k + 1)} \psi_{j-k},$$

where the  $(\psi_j)$  can be computed recursively:

$$\psi_0 = 1, \psi_j = \theta_j + \sum_{i=1}^{\min(j,p)} \phi_i \psi_{j-i} \quad \text{if } 1 \leq j \leq q$$

and

$$\psi_j = \sum_{i=1}^{\min(j,p)} \phi_i \psi_{j-i} \quad \text{if } j \geq q + 1.$$

The cumulative impulse response function over  $j$  periods of time is given by  $C_j = 1 + a_1 + \dots + a_j$  and it tracks the impact of a unit innovation at time  $t$  on the long run equilibrium relationship at time  $t + j$ . As  $j \rightarrow \infty$   $C_\infty = A(1)$ , measuring the long-run impact of the innovation (Campbell and Mankiw, 1987). Cheung and Lai (1993) show that for  $d < 1$ ,  $C_\infty = 0$ , implying shock-dissipating behavior. Conversely for  $d \geq 1$ ,  $C_\infty \neq 0$ , the effect of a shock will not die out. Mean reversion (i.e.  $C_\infty = 0$ ) occurs as long as  $d < 1$ . A measure of persistence usually considered in the literature is the half-life, which indicates how long it takes after a unit shock to dissipate by half on the long-run equilibrium. The half-life can be computed from the  $C_j$  function as  $t = h$  at where  $C_h = 0.5$ . For ARMA models, an analytical expression for the half-life can be derived; for example, it is well known that the half-life of the AR(1) model  $e_t = \phi e_{t-1} + \varepsilon_t$  is given by  $h = -\log(2)/\log(\phi)$ . However, for the ARFIMA model, the half-life remains difficult to compute. This problem can be solved plotting the impulse response function and using linear interpolation.

## 3 Empirical analysis

### 3.1 The data

The different series are the daily closing values for the following stock indices: *BOV* (Bovespa, Brazil), *CAC* (CAC 40, France), *DAX* (Dax, Germany),



*FTSE* (FTSE 100, UK), *HS* (Hang Seng, Hong Kong), *NK* (Nikkei 225, Japan), *MERV* (Merval, Argentina) and *SP* (Standard and Poor's 500, USA). We consider the log-transformed daily data over the period January 4, 1999 - March 6, 2008 ( $T = 2389$  where  $T$  is the sample size). The log-transformed daily series over the whole period are plotted in Figure 1 (the descriptive statistics of the returns are given in Table 1).

[Insert Table 1 here]  
[Insert Figure 1 here]

## 3.2 Empirical results

### 3.2.1 Integration analysis of individual series

The first step in the empirical analysis is to investigate the order of integration of the individual series. We first perform the Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS) test for unit root, where the null hypothesis is the stationarity, on the raw series and on the first differenced series. The results are reported in Table 2 and clearly show the rejection of the null hypothesis by the KPSS test on the raw series and the non-rejection of the null hypothesis on the differenced series, which means that all the log-transformed daily series contain a unit root.

[Insert Table 2 here]

Table 3 summarizes the results of the FELW estimation procedure (Shimotsu (2006)) of the long memory parameter  $d$ , for the different series;  $\hat{d}$  are the estimators of  $d$  and  $\hat{\sigma}$  are the estimated standard-errors;  $d_l$  and  $d_u$  are the lower and upper bounds of the confidence intervals, respectively  $\hat{d} - 1.96\hat{\sigma}$  and  $\hat{d} + 1.96\hat{\sigma}$ . The orders of integration lies between 0.964 and 1.080, and the unit value lies always in the 95% confidence intervals, whatever the series; it confirms again that the series contain a unit root, i.e., the null  $d = 1$  cannot be rejected.

[Insert Table 3 here]

Table 4 <sup>6</sup> shows the results of the statistic  $\hat{r}$  of the Robinson (1994)'s tests applied to each individual series. Different values of  $d$  are considered, thus testing for a unit root ( $d = 1$ ) but also other fractional possibilities. We

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<sup>6</sup>**Acknowledgements:** The authors thank Luis A. Gil-Alana for providing various FORTRAN programs for the Robinson (1994)'s test, that they translated in GAUSS.

can observe that the minimum of the absolute values of the Robinson (1994) test statistic occurs always when  $d = 0.95$  or  $1$  (and  $1.05$  for *MERV*). This permits to conclude that all the series may contain a unit root or are close to the unit root case.

[Insert Table 4 here]

### 3.2.2 Cointegration analysis

In the second step of the strategy of Gil-Alana (2003) and Caporale and Gil-Alana (2004a and b), we consider now the possibility of the series being cointegrated. We consider first the classical standard cointegration. In Table 5, we can observe some results of the OLS regression of the pairwise (bivariate) cointegrating regression:  $e_t = y_t - \hat{\alpha} - \hat{\beta}SP_t$  where  $e_t$  is the estimated equilibrium error and  $y_t$  is the foreign stock index. The results of the KPSS test show the rejection of the null hypothesis of stationarity of the estimated equilibrium errors and thus that the series are not cointegrated. This suggests that the US equity market and the equity markets in the UK, Germany, France, Brazil, Hong Kong, Argentina and Japan are not pairwise cointegrated during the considered period. The same results are found by Kanas (1998) for the period 03/01/83 - 29/11/96, in the case of UK, Germany and France<sup>7</sup>. The Johansen's tests exhibit mostly the same results, except for the case of Germany which appears pairwise cointegrated with the US stock market at the 5% level.

[Insert Table 5 here]

Concerning the fractional cointegration, Table 6 shows the results of one-sided tests of Robinson (1994) on the estimated residuals  $e_t$  from the cointegrating regression: we compute the statistic  $\hat{r}$ , testing  $H_0 : \theta = 0$  against the alternative  $H_1 : \theta < 0$  in the model  $(1 - L)^{d+\theta}e_t = u_t$ ; as noted in Caporale and Gil-Alana (2004a and b), we can use the asymptotic Normal distribution because of the consistency of the cointegrating parameters and the desirable properties of Robinson's (1994) tests. Two cases are considered.

[Insert Table 6 here]

**Case 1.** No fractionally pairwise cointegration with the Standard & Poor's index: Bovespa and Merval.

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<sup>7</sup>Kanas (1998) uses the Dow Jones index for US stock market whereas we use the Standard and Poor's.

In this case, the estimated residuals from the cointegrating regression are of a higher order of integration than that of the individual series; therefore, there does not exist a long run equilibrium relationship.

**Case 2.** Fractionally pairwise cointegration with the Standard & Poor's index: CAC, Dax, FTSE, Hang Seng and Nikkei.

For these indices, the non-rejection values of  $H_0 : \theta = 0$  occur always for values of  $d < 1$ . This implies that the estimated residuals from the cointegrating regression are of a lower order of integration than that of the individual series; this thus shows that there exists the possibility of a long run equilibrium relationship. When we compare these indices, the results vary slightly; the minimum of the absolute values of the Robinson (1994) test statistics corresponds to the values of the long memory parameter  $d$  equal to 0.725 for the CAC index, 0.775 for the Dax, and 0.800 for the Nikkei index, whatever the regressors. For the other two indices, the minimum of the absolute values of the statistics depend on the regressors: 0.700, 0.675 and 0.675 for the FTSE and 0.800, 0.750 and 0.775 for the Hang Seng index. On the whole, we can conclude that the fractional cointegration specification is accepted: a fractionally cointegrated relationship does exist between the Standard & Poor's index and the other indices. The corresponding equilibrium errors exhibit thus hyperbolic mean reversion. The half-life estimates of residuals from the cointegrating regression are given in Table 7 and the optimal ARFIMA models obtained by using the BIC criterion are shown in Table 8. Figures 3-7 depict the impulse response functions of the CAC, the Dax, the FTSE, the Hang Seng and the Nikkei indices; their estimated half-lives are given in Table 7.

[Insert Tables 7 and 8 here]  
 [Insert Figures 2 to 6 here]

The shorter half-lives correspond to the CAC (it is equal to 1.88) and the FTSE (about 3.7); they are longer for the HS (between 6.24 and 9.55), the DAX (8.74) and the NK (11.88).

## 4 Concluding remarks

Using linear cointegration tests, most of the empirical papers generally do not find pairwise cointegrating relationship between the US equity market and equity markets of the major industrial countries. However, adopting the two-step strategy of Gil-Alana (2003) and Caporale and Gil-Alana (2004a and b), based on the Robinson (1994) test, we show as a matter of fact that there exist pairwise fractional cointegration between the Standard & Poor's

index and the CAC, the Dax, the FTSE, the Hang Seng and the Nikkei indices; the corresponding equilibrium errors exhibit mean reversion, with half-life deviations lying between 2 and 12 days. It is worthwhile noting that emerging markets like Brazil and Argentina are not fractionally cointegrated with US stock market. These results suggest that there is some predictability of prices in the long-run (Winner-Loser effect) as well as in the short-run for the stock markets of industrialized countries, but also this evidence of fractional cointegration reduces, in the long-run, the potential benefits of international portfolio diversification. However, another important result is that these conclusions do not hold for some emerging countries.

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## Appendix

The Lagrange Multiplier (LM) statistic proposed by Robinson (1994), called  $\hat{r}$ , is given by:

$$\hat{r} = \left( \frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2} \quad (1)$$

where  $T$  is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j, \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{\sigma}^2 = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j, \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left[ \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right. \\ \left. \times \left( \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right],$$

$$\hat{\tau} = \arg \min_{\tau \in T^*} \hat{\sigma}^2, \quad \psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|, \quad \lambda_j = \frac{2\pi j}{T}$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j, \hat{\tau}), \quad g(\lambda, \tau) = \frac{2\pi}{\sigma^2} f(\lambda, \tau, \sigma^2);$$

$f$  is the spectral density of  $u_t$ ,  $T^*$  is a suitable set of  $\mathbb{R}^k$  and  $I(\lambda_j)$  is the periodogram of

$$\hat{u}_t = (1 - L)^d y_t - \hat{\beta}' W_t \quad (2)$$

evaluated at  $\lambda_j$  with

$$W_t = (1 - L)^d X_t$$

and

$$\hat{\beta} = \left( \sum_{t=1}^T W_t W_t' \right)^{-1} \sum_{t=1}^T W_t (1 - L)^d y_t.$$

Note that  $\sigma^2$  is generally no longer the variance of  $u_t$ , but rather the variance of the innovation sequence in a normalized Wold representation of  $u_t$ . Robinson (1994) shows that  $\hat{r}$  has a standard asymptotic distribution under some regularity conditions:

$$\hat{r} \xrightarrow{d} N(0, 1) \quad \text{as } T \rightarrow \infty.$$

**Table 1: Descriptive statistics on returns**  
( $100(\log(x_t) - \log(x_{t-1}))$ )

	Mean	Std Error
<i>BOV<sub>t</sub></i>	0.09187	1.9085
<i>CAC<sub>t</sub></i>	0.00770	1.3782
<i>DAX<sub>t</sub></i>	0.01157	1.5146
<i>FTSE<sub>t</sub></i>	0.00144	1.1347
<i>HS<sub>t</sub></i>	0.03487	1.4092
<i>NK<sub>t</sub></i>	0.00132	1.3462
<i>MERV<sub>t</sub></i>	0.06687	2.0891
<i>SP<sub>t</sub></i>	0.00499	1.1035

**Table 2: KPSS test on each individual series**

	$KPSS_0$	$KPSS_6$		$KPSS_0$	$KPSS_6$
$BOV_t$	39.947	5.746	$\Delta BOV_t$	0.101	0.105
$CAC_t$	41.821	5.998	$\Delta CAC_t$	0.125	0.150
$DAX_t$	46.395	6.650	$\Delta DAX_t$	0.112	0.120
$FTSE_t$	52.790	7.577	$\Delta FTSE_t$	0.068	0.089
$HS_t$	44.565	6.398	$\Delta HS_t$	0.110	0.108
$NK_t$	50.161	7.190	$\Delta NK_t$	0.144	0.149
$MERV_t$	31.346	4.506	$\Delta MERV_t$	0.094	0.085
$SP_t$	47.549	6.831	$\Delta SP_t$	0.068	0.080

Note: The period is January 4, 1999 - March 6, 2008 ( $T = 2389$ ).  $KPSS_i$  is the  $\tau$ -statistic of order  $i$  of the Kwiatowski, Phillips, Schmidt and Shin (1992) test; the critical values are: 0.119 (10%), 0.146 (5%) and 0.216 (1%).

**Table 3: Feasible local Whittle estimation of each variable**

	$\hat{d}$	$\hat{\sigma}$	$d_l$	$d_u$
$BOV_t$	1.027	0.040	0.949	1.106
$CAC_t$	0.995	0.040	0.916	1.073
$DAX_t$	1.015	0.040	0.937	1.093
$FTSE_t$	0.964	0.040	0.885	1.042
$HS_t$	1.018	0.040	0.939	1.096
$NK_t$	0.993	0.040	0.915	1.072
$MERV_t$	1.057	0.040	0.979	1.136
$SP_t$	0.994	0.040	0.915	1.072

Note: The period is January 4, 1999 - March 6, 2008 ( $T = 2389$ ).  $\hat{d}$  is FELW estimator developed by Shimotsu (2006) of the long memory parameter;  $\hat{\sigma}$  is the estimated standard-errors of the Shimotsu estimator;  $d_l$  and  $d_u$  are the lower and upper bounds of the 95% confidence intervals. For the feasible local Whittle estimation (see Shimotsu (2006)),  $m$  is chosen to be  $m = T^{0.65}$  with  $T$  is the sample size.

**Table 4: Order of integration of each variable  
with the test of Robinson (1994)**

<i>BOV<sub>t</sub></i>				<i>CAC<sub>t</sub></i>			
<i>d</i>	—	1	(1, <i>t</i> )	<i>d</i>	—	1	(1, <i>t</i> )
0.90	7.57	7.96	8.02	0.90	7.40	4.62	4.62
0.95	3.29	3.81	3.83	0.95	3.20	<b>0.81*</b>	<b>0.81*</b>
1.00	<b>-0.15*</b>	<b>0.48*</b>	<b>0.48*</b>	1.00	<b>-0.19*</b>	-2.17	-2.17
1.05	-2.96	-2.22	-2.23	1.05	-2.97	-4.56	-4.56
1.10	-5.27	-4.47	-4.48	1.10	-5.27	-6.52	-6.52

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<i>DAX<sub>t</sub></i>				<i>FTSE<sub>t</sub></i>			
<i>d</i>	—	1	(1, <i>t</i> )	<i>d</i>	—	1	(1, <i>t</i> )
0.90	7.33	7.15	7.14	0.90	7.51	1.93	1.93
0.95	3.15	2.66	2.65	0.95	3.29	<b>-1.26*</b>	<b>-1.26*</b>
1.00	<b>-0.23*</b>	<b>-0.85*</b>	<b>-0.85*</b>	1.00	<b>-0.13*</b>	-3.82	-3.82
1.05	-3.01	-3.66	-3.66	1.05	-2.92	-5.93	-5.93
1.10	-5.30	-5.94	-5.94	1.10	-5.24	-7.69	-7.69

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<i>HS<sub>t</sub></i>				<i>NK<sub>t</sub></i>			
<i>d</i>	—	1	(1, <i>t</i> )	<i>d</i>	—	1	(1, <i>t</i> )
0.90	7.50	8.71	8.71	0.90	7.40	7.18	7.18
0.95	3.26	4.21	4.21	0.95	3.16	2.83	2.83
1.00	<b>-0.17*</b>	<b>0.62*</b>	<b>0.62*</b>	1.00	<b>-0.26*</b>	<b>-0.58*</b>	<b>-0.58*</b>
1.05	-2.98	-2.28	-2.28	1.05	-3.06	-3.32	-3.32
1.10	-5.30	-4.66	-4.66	1.10	-5.38	-5.56	-5.56

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<i>MERV<sub>t</sub></i>				<i>SP<sub>t</sub></i>			
<i>d</i>	—	1	(1, <i>t</i> )	<i>d</i>	—	1	(1, <i>t</i> )
0.90	7.23	11.97	11.90	0.90	7.47	3.87	3.87
0.95	3.10	7.16	7.14	0.95	3.25	<b>0.45*</b>	<b>0.45*</b>
1.00	<b>-0.25*</b>	3.26	3.26	1.00	<b>-0.15*</b>	-2.32	-2.32
1.05	-3.00	<b>0.08*</b>	<b>0.09*</b>	1.05	-2.94	-4.60	-4.60
1.10	-5.28	-2.54	-2.53	1.10	-5.25	-6.51	-6.51

Note: The period is January 4, 1999 - March 6, 2008 ( $T = 2389$ ). We consider only the test where  $u_t$  is assumed to be white noise, with different specifications: with no regressors (—), with an intercept (1) and with a linear trend ((1,  $t$ )). In bold: the minimum (in absolute value) of the Robinson (1994) test statistic. \*: nonrejection values of the null hypothesis  $H_0 : \theta = 0$  at the 95% significance level (the critical value is 1.65 in absolute value).

**Table 5: Cointegration and Johansen's cointegration test**

	$\hat{\alpha}$	$\hat{\beta}$	$KPSS_0$	$KPSS_2$	$r \leq 1$	$r = 0$
$BOV_t - SP_t$	-1.846 (0.19)	1.995 (0.06)	18.838	6.405	0.0547	4.2781
$CAC_t - SP_t$	-0.537 (0.03)	1.358 (0.01)	19.377	6.523	3.5150	15.1261
$DAX_t - SP_t$	-1.711 (0.04)	1.756 (0.01)	21.138	7.121	3.1357	17.9492*
$FTSE_t - SP_t$	0.473 (0.02)	1.055 (0.00)	18.229	6.219	1.6811	5.9188
$HS_t - SP_t$	-0.399 (0.06)	1.474 (0.02)	9.099	3.102	0.1568	3.6410
$NK_t - SP_t$	-0.369 (0.04)	1.455 (0.01)	9.825	3.331	0.3735	10.2559
$MERV_t - SP_t$	-2.145 (0.25)	1.640 (0.08)	21.965	7.351	3.3482	10.0086

Note: The period is January 4, 1999 - March 6, 2008 ( $T = 2389$ ).  $\hat{\alpha}$  and  $\hat{\beta}$  are the estimated values of the coefficients in the cointegrating regression  $y_t = \alpha + \beta SP_t + noise$  where  $y_t$  is one of the seven stock indices under study (the series are expressed in logarithm); the estimation method is the ordinary least squares and standard errors are in parenthesis.  $KPSS_i$  is the statistic of order  $i$  of the Kwiatowski, Phillips, Schmidt and Shin (1992) test; the critical values are given in Shin (1994).

The last two columns presents the results of Johansen's cointegration test, using 15 lags and an unrestricted constant. Critical values for trace test are obtained from Johansen (1988) and given by

Significance level	$r \leq 1$	$r = 0$
5%	3.76	15.41
1%	6.65	20.04

where  $r$  denotes the number of cointegrating vectors.

**Table 6: Robinson (1994) test on estimated residuals from the cointegrating regression**

$BOV_t - SP_t$				$CAC_t - SP$			
$d$	—	1	(1, $t$ )	$d$	—	1	(1, $t$ )
0.925	3.35	1.53*	1.58*	0.675	6.47	5.15	5.26
0.950	1.77	<b>0.19*</b>	<b>0.22*</b>	0.700	3.37	2.15	2.22
0.975	<b>0.32*</b>	-1.04*	-1.03*	0.725	<b>0.72*</b>	<b>-0.38*</b>	<b>-0.34*</b>
1.000	-1.01*	-2.19	-2.19	0.750	-1.55*	-2.55	-2.52
1.025	-2.25	-3.26	-3.27	0.775	-3.50	-4.39	-4.37

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$DAX_t - SP_t$				$FTSE_t - SP_t$			
$d$	—	1	(1, $t$ )	$d$	—	1	(1, $t$ )
0.725	4.68	4.63	4.62	0.650	4.28	2.63	2.72
0.750	2.24	2.21	2.20	0.675	1.79	<b>0.35*</b>	<b>0.42*</b>
0.775	<b>0.10*</b>	<b>0.08*</b>	<b>0.07*</b>	0.700	<b>-0.37*</b>	-1.63*	-1.58*
0.800	-1.77	-1.78	-1.79	0.725	-2.27	-3.37	-3.33
0.825	-3.43	-3.43	-3.43	0.750	-3.94	-4.90	-4.87

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$HS_t - SP_t$				$NK_t - SP_t$			
$d$	—	1	(1, $t$ )	$d$	—	1	(1, $t$ )
0.725	6.95	3.04	3.28	0.750	3.92	3.92	3.90
0.750	4.43	<b>0.93*</b>	1.10*	0.775	1.73	1.73	1.72
0.775	2.21	-0.94*	<b>-0.83*</b>	0.800	<b>-0.20*</b>	<b>-0.20*</b>	<b>-0.21*</b>
0.800	<b>0.24*</b>	-2.61	-2.54	0.825	-1.93	-1.93	-1.94
0.825	-1.50*	-4.10	-4.05	0.850	-3.48	-3.48	-3.49

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$MERV_t - SP_t$			
$d$	—	1	(1, $t$ )
0.975	2.73	2.46	2.46
1.000	1.07*	0.88*	0.88*
1.025	<b>-0.42*</b>	<b>-0.54*</b>	<b>-0.54*</b>
1.050	-1.79	-1.85	-1.85
1.075	-3.03	-3.05	-3.04

Note: The period is January 4, 1999 - March 6, 2008 ( $T = 2389$ ). We consider only the test where  $u_t$  is assumed to be white noise, with different specifications: with no regressors (—), with an intercept (1) and with a linear trend ((1,  $t$ )). In bold: the minimum (in absolute value) of the Robinson (1994) test statistic. \*: rejections of the null hypothesis of "no cointegration" at the 95% significance level (the critical value is 1.65 in absolute value).

**Table 7: Half-life estimates of the residuals from the cointegrating regression**

	—	1	(1, $t$ )
<i>CAC</i>	1.88	1.88	1.88
<i>DAX</i>	8.74	8.74	8.74
<i>FTSE</i>	3.75	3.66	3.66
<i>HS</i>	6.24	7.55	9.55
<i>NK</i>	11.88	11.88	11.88

Note: The different specifications are: with no regressors (—), with an intercept (1) and with a linear trend ((1,  $t$ )). The half-life is estimated by using a linear interpolation as follows: if  $k$  is such that  $IRF[k] \geq 0.5 \geq IRF[k+1]$  then the linear approximation for the half-life estimate is given by

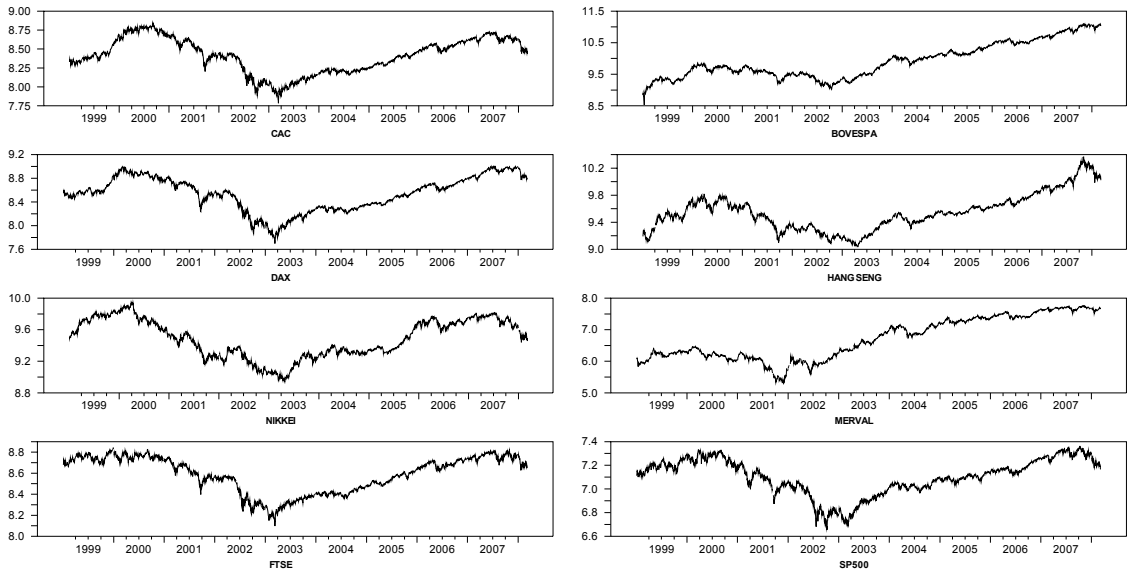
$$h = (0.5 - (k + 1)IRF[k] + kIRF[k + 1]) / (IRF[k + 1] - IRF[k]).$$



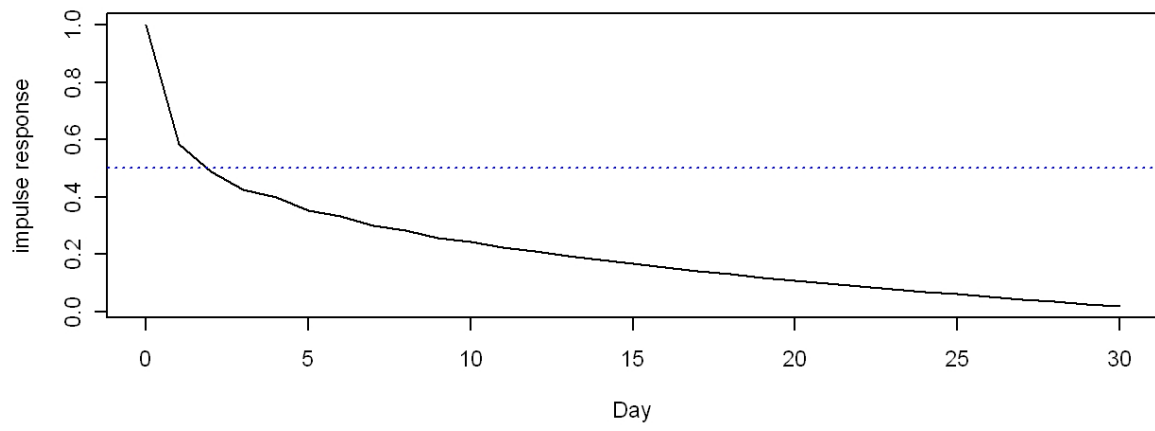
**Table 8: Optimal ARFIMA models  
obtained by using the BIC criterion**

<b>CAC</b>	$(1 - L)^{0.725} \left( 1 + \underset{(0.024)}{0.090L} + \underset{(0.033)}{0.782L^2} + \underset{(0.031)}{0.086L^3} + \underset{(0.021)}{0.033L^4} \right) e_t = (1 - \underset{(0.105)}{0.232L} - \underset{(0.476)}{0.803L^2}) \varepsilon_t$
<b>DAX</b>	$(1 - L)^{0.775} e_t = (1 - \underset{(0.020)}{0.048L} + \underset{(0.020)}{0.029L^2}) \varepsilon_t$
<b>FTSE</b>	with no regressors (—):
	$(1 - L)^{0.7} (1 - \underset{(0.008)}{0.054L} + \underset{(0.008)}{0.909L^2}) e_t =$
	$(1 - \underset{(0.001)}{0.045L} - \underset{(0.008)}{0.903L^2} + \underset{(0.001)}{0.139L^3}) \varepsilon_t$
	with an intercept (1) or a linear trend ((1, t)):
$(1 - L)^{0.675} (1 - \underset{(0.020)}{0.129L} + \underset{(0.007)}{0.944L^2} + \underset{(0.020)}{0.103L^3}) e_t =$	
$(1 - \underset{(0.0005)}{0.069L} - \underset{(0.006)}{0.928L^2}) \varepsilon_t$	
<b>HS</b>	with no regressors (—):
	$(1 - L)^{0.75} e_t = (1 - \underset{(0.020)}{0.017}) \varepsilon_t$
	with an intercept (1):
	$(1 - L)^{0.775} e_t = (1 - \underset{(0.020)}{0.048}) \varepsilon_t$
with a linear trend ((1, t)):	
$(1 - L)^{0.8} e_t = (1 - \underset{(0.021)}{0.077}) \varepsilon_t$	
<b>NK</b>	$(1 - L)^{0.8} e_t = (1 - \underset{(0.021)}{0.053L} + \underset{(0.020)}{0.015L^2}) \varepsilon_t$

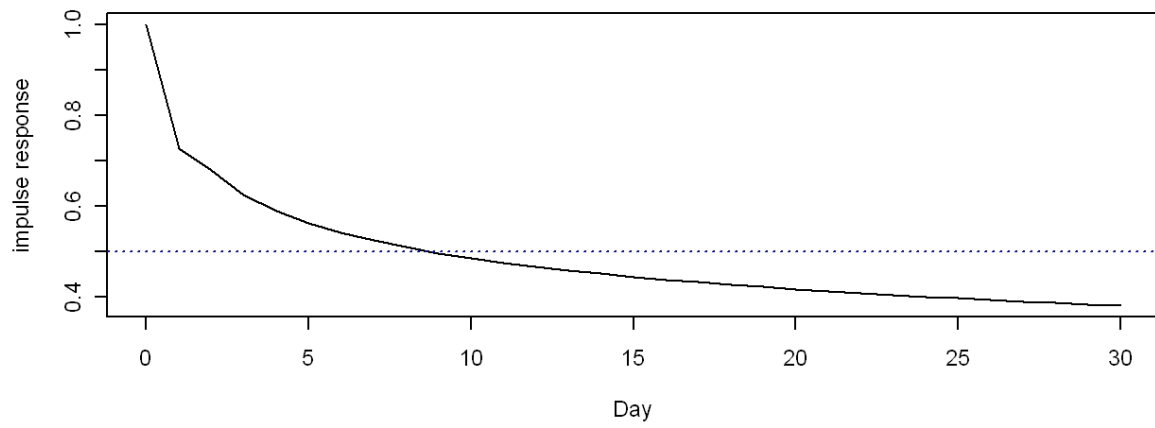
**Figure 1. The logarithmic stock markets indices**



**Figure 2. The impulse response function of the estimated residuals from the cointegrating regression ( $CAC_{rSP_t}$ )**

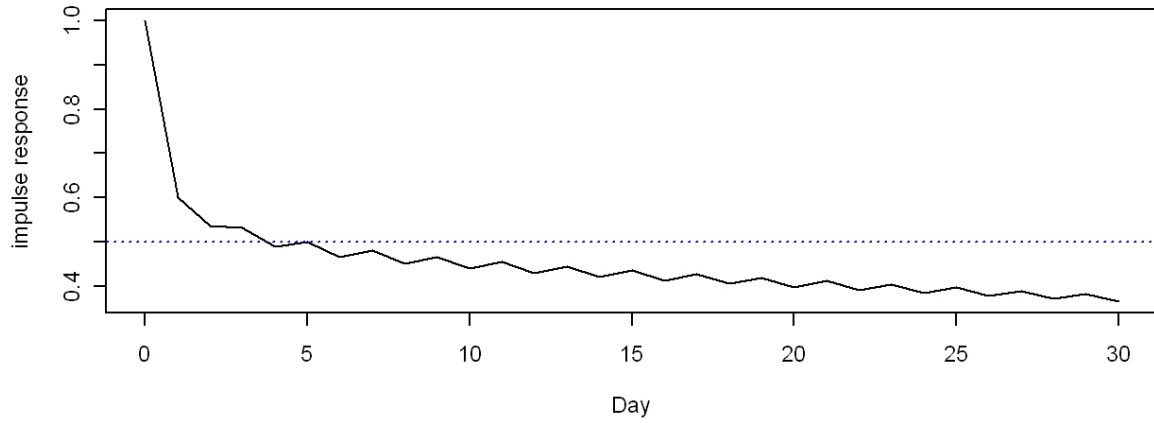


**Figure 3. The impulse response function of the estimated residuals from the cointegrating regression ( $DAX_t - SP_t$ )**

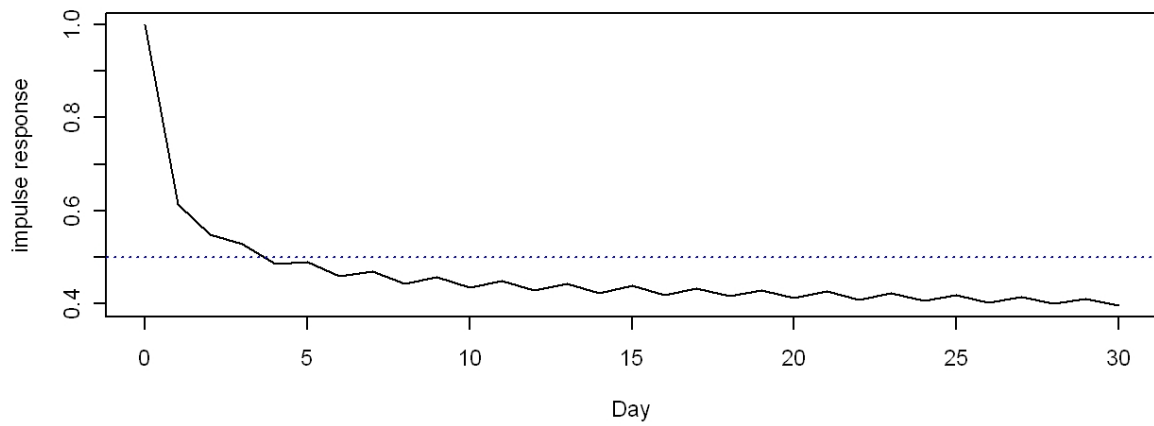


**Figure 4. The impulse response function of the estimated residuals from the cointegrating regression ( $FTSE_t - SP_t$ )**

**with no regressor (—)**

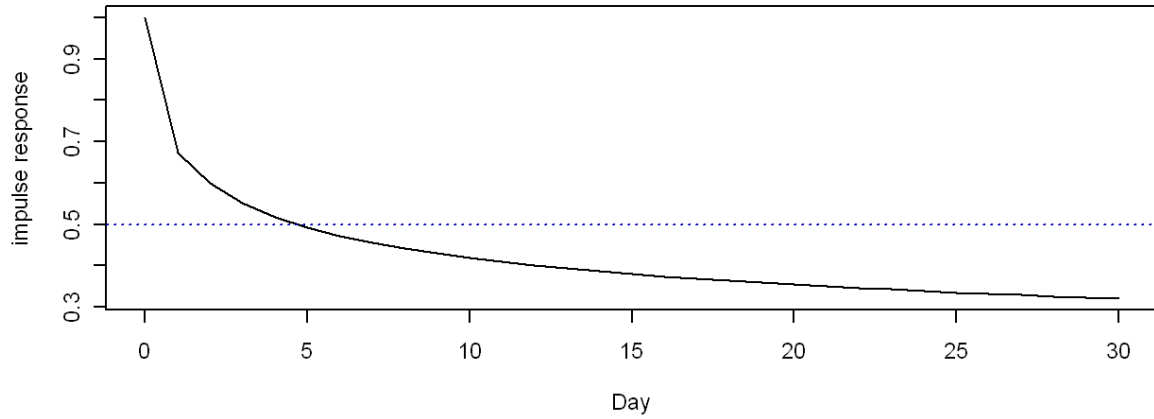


**with an intercept ( $I$ ) or a linear trend ( $(I,t)$ )**

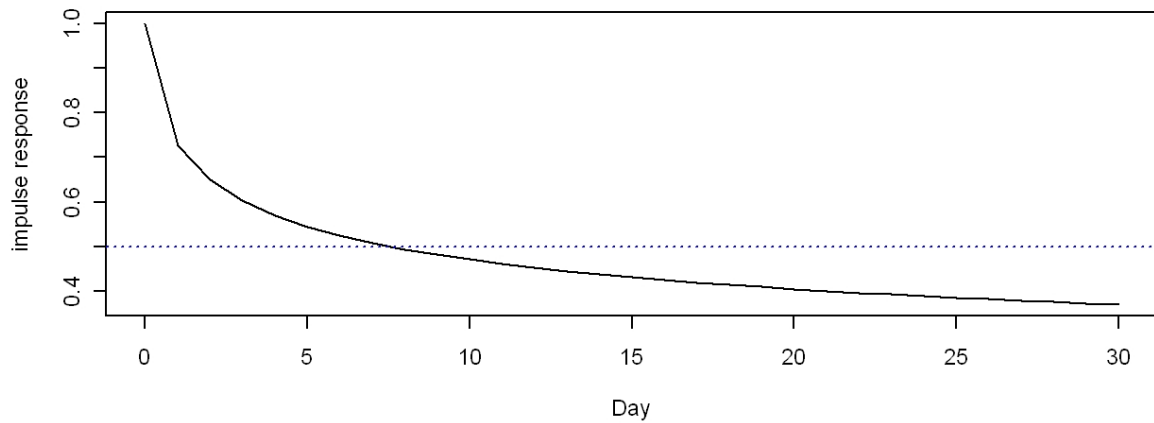


**Figure 5. The impulse response function of the estimated residuals from the cointegrating regression ( $HS_r-SP_t$ )**

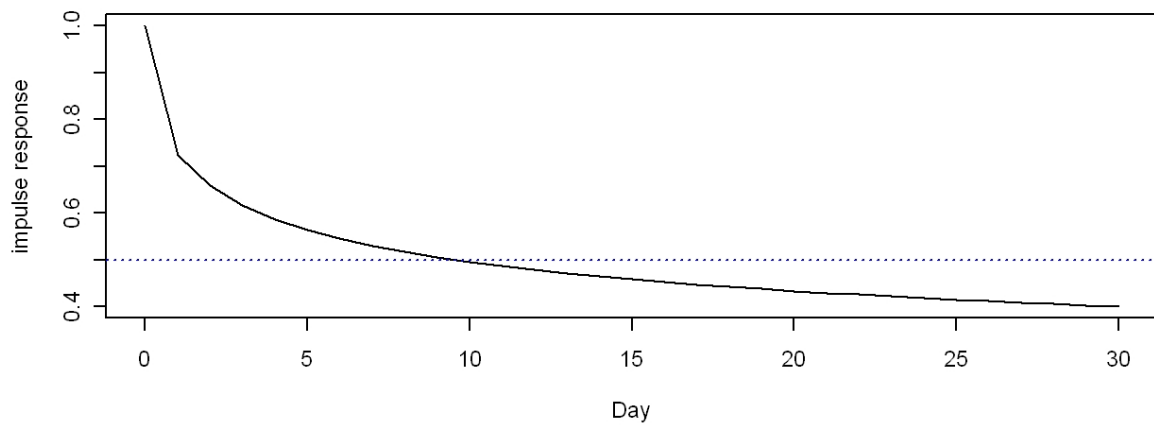
**with no regressor (—)**



**with an intercept ( $I$ )**



**with a linear trend ( $(I,t)$ )**



**Figure 6. The impulse response function of the estimated residuals from the cointegrating regression ( $NK_rSP_t$ )**

