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The Case for a Financial Approach to Money Demand*

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Abstract

The distribution of money across households is much more similar to the distribution of financial assets than to that of consumption levels, even controlling for life-cycle effects. This is a puzzle for theories which directly link money demand to consumption, such as cash-in-advance (CIA), money-in-the-utility function (MIUF) or shopping-time models. This paper shows that the joint distribution of money and financial assets can be explained by an incomplete-market model when frictions are introduced into financial markets. Money demand is modeled as a portfolio choice with a fixed transaction cost in financial markets.

JEL codes: E40, E50.

Keywords: Money Demand, Money Distribution, Heterogenous Agents.

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1 Introduction

Why do households hold money? Various theories of money demand have proposed answers to this question by focusing on the transaction role money plays in goods markets (e.g., shopping-time and cash-in-advance (CIA) models), transaction costs in financial markets (Allais, 1947; Baumol, 1952; Tobin, 1956) or simply assuming a liquidity role of money, as the money-in-the-utility function (MIUF) literature. These theories are observationally equivalent using aggregate data: they can be realistically calibrated to match aggregate estimates, such as the interest elasticity of money demand. In this paper, I argue that household data can be used to assess the relevance of these different theories: Household data strongly reject standard models of money demand, such as CIA, baseline MIUF or shopping-time models, while theories based on financial frictions are able to reproduce realistic distributions of money, consumption and wealth.

In both Italian and US data, the distribution of money (M1) is similar to that of financial wealth, and much more unequally distributed than is that of consumption (as measured by the Gini index, for example). The ranking in the US in 2004 is as follows: the Gini indices are around 0.3 for consumption, 0.5 for income, 0.8 for net wealth and 0.8 for money. This result, further detailed below, continues to hold with different definitions of money, for various time periods, and after controlling for life-cycle effects. This distribution of money cannot be understood in standard macroeconomic models where money demand is introduced via CIA, MIUF or shopping-time assumptions. In these models, real money balances are proportional to consumption and the distributions of both money holdings and consumption should be equally unequal (i.e. have the same Gini coefficient). As is shown below, this difficulty remains even when we consider more general transaction technologies on the goods market, which may produce scale economies. In addition to its theoretical interest, the ability to reproduce the distribution of money is crucial for the assessment of the real and welfare effects of inflation. When money holdings are

highly dispersed, inflation can be expected to have significant distributional effects.

We here show that a realistic joint distribution of consumption, money and financial assets can be reproduced via a transaction cost in financial markets only, without any assumptiona regarding frictions in the goods market. Money demand is modeled as a portfolio choice between money and a riskless interest-bearing asset. Money holdings can be freely adjusted, but there is a transaction cost of adjusting the quantity of financial assets. This foundation of money demand was introduced by Allais, Baumol and Tobin (1956), in their inventory approach to monetary theory. Jovanovic (1982) and Romer (1986) provide general-equilibrium extensions, but without focusing on the properties of the equilibrium distributions. Contrary to these papers, I assume, as in Heller (1974) and Chatterjee and Corbae (1994), that households do not need money to make purchases, and do not face a cash-in-advance constraint. I thus assume that the payment system is well organized, so that the transaction demand for money is close to zero. This financial approach to money demand makes it possible to consider money as a special asset in the theory of asset prices with transaction costs in financial markets (see Lo et al. 2004 for references). In this literature, transaction costs capture the differences in liquidity between assets (Huang, 2003, for example).

This portfolio choice is introduced into a production economy where infinitely-lived agents face uninsurable income fluctuations and borrowing constraints, a framework often described as the "Bewley-Huggett-Aiyagari" environment. In this type of economy, households can choose between two assets with different returns, but with different transaction costs, in order to smooth idiosyncratic income fluctuations. Due to the transaction cost, households participate only infrequently in financial markets to rebalance their portfolio, as documented by Vissing-Jorgensen (2002) among others. This type of economy does not introduce life-cycle considerations and is thus well suited for the analysis of heterogeneity within generations. The model is calibrated to reproduce the idiosyncratic income fluctuations faced by US households. The transaction cost is chosen to match the average quantity of money

held by households in the US economy.

This model generates a realistic joint distribution of money and financial assets, with the transaction cost being the only deviation from the baseline heterogenous-agents model. This result is robust to various changes in the model parameters, and to modeling choices. In particular, although the participation cost in the financial market affects the average amount of money held, it does not significantly change the dispersion of the distribution of money holdings. The main reason for this is that households hold money to smooth consumption without paying transaction costs in financial markets. They only participate in financial markets to increase their financial savings when their money holdings are high, and participate in financial markets to dis-save when their money holdings are low. Between these two boundaries, which depend on household wealth, money is used as an asset to smooth consumption. In consequence, although the amount saved in money is on average much less than that invested in financial markets, the dispersion of the distributions of money and assets remain close to each other.

Other Related Literature

Although there is a vast literature on money demand, to my knowledge this paper is the first to focus on the properties of the distribution of money across households in order to assess the relevance of theories of money demand. The paper belongs first to the literature on money demand, and more specifically to the Allais-Baumol-Tobin model in general equilibrium. In a recent paper, Alvarez et al. (2002) introduce both a fixed transaction cost and a cash-in-advance constraint in a general-equilibrium setting. To simplify their analysis of the short-run effect of money injections, they assume that markets are complete and, in consequence, that all agents have the same financial wealth. As my goal is to introduce only essential departures from the benchmark settings to reproduce the joint distribution of money and wealth, I only assume a transaction cost in financial markets and do not introduce a cash-in-advance constraint. Heller (1974) has proved that the fixed

transaction cost in financial markets suffices for money to have a positive value in equilibrium.

Second, this paper belongs to the literature on money demand in economies with idiosyncratic shocks and incomplete markets. The initial papers in this literature considered money as the only available asset for self-insurance against idiosyncratic shocks (Bewley, 1980 and 1983; Scheinkman and Weiss 1986; Imohoroglu, 1992). More recent papers have introduced another financial asset with some additional frictions to justify positive money demand. Imrohoroglu and Prescott (1991) use a per-period cost, so that households hold either money or financial assets, but never both, and consider the real effects of various monetary arrangements. Erosa and Ventura (2002) introduce a cash-in-advance constraint and a fixed cost of withdrawing money from financial markets to study the inflation tax. Akyol (2004) analyses an endowment economy where the timing of market openings implies that only high-income agents hold money. Bai (2005) also introduces transaction costs in financial markets, but in the context of an endowment economy to study the real effect of inflation on the real interest rate (the so-called Mundell-Tobin effect) and on welfare. He does not consider the cross-sectional distributions of money and assets. Algan and Ragot (2008) introduce money in the utility function to study non-neutralities induced by binding credit constraints. None of these papers describes or reproduces the empirical distribution of money. This has, however, been analysed in some of the more recent papers in the search-theoretic literature (Molico, 2006), which has also explained the coexistence of money and financial assets by introducing centralized financial markets and decentralized goods markets (Chiu and Molico, 2007). However, the distribution of money herfe is similar to that of consumption. Finally, Heer et al. (2007) consider the money-age distribution and conclude that standard monetary models fail to reproduce this distribution, but do not provide an alternative model. This paper proves that the same puzzle pertains within a given age group, and that a model with fixed participation costs can explain these distributions. To my knowledge, this paper is the first to reproduce a realistic joint distribution of

money and wealth.

This paper is also related to the empirical work which has estimated money demand using household data. Mulligan and Sala-i-Martin (2000) introduce a fixed adoption cost of the technology to participate in financial markets, in addition to a shopping-time constraint. They estimate the adoption cost via various economic and econometric models using US household data. Attanasio et al. (2002) estimate a shopping-time model *à la* McCallum and Goodfriend (1987), using Italian household data. Finally, Alvarez and Lippi (2007) use Italian household data to estimate a model where households face a cash-in-advance constraint, a fixed transaction cost and a stochastic cost of withdrawing money. They show that this stochastic component improves the outcome of the model as compared to a deterministic Baumol-Tobin framework. Although I also use household data, my goal is different: I reproduce a realistic joint distribution of money, wealth, and consumption as a general equilibrium outcome, and show that a simple friction in financial markets suffices to generate these results.

The paper is organized as follows. Section 2 presents empirical facts about the distribution of money in Italy and the US. Section 3 shows that the usual assumptiona regarding money demand fails to reproduce these facts. Section 4 describes the fixed transaction-cost model, and the parameterization is presented in section 5. Section 6 presents the results and the distribution of money and assets, and Section 7 discusses some robustness tests. Finally, Section 8 concludes.

2 The Distribution of Money

This section presents some empirical facts about the distribution of money and assets in the Italian and US economies. Although the model below will be calibrated using US data, I use Italian data to verify that the properties of the distribution of money are similar across countries. In the following, I use a narrow definition of money, M1, to emphasisse the distinction between money and other financial assets. The main

and robust result of the analysis is that, even with this definition, the distribution of money appears to be similar to the distribution of assets. The same analysis has been carried out for various monetary aggregates and the results are quantitatively similar. As a summary of the following analysis, Fig. 1 depicts the Lorenz curves of the money, income and net worth[1] distributions using the 2004 Survey of Consumer Finance, and the Lorenz curves of the consumption, income, net worth, and money distributions using Italian data from the 2004 Survey of Households' Income and Wealth. In both cases, I only consider households whose head is aged between 35 and 44 to avoid life-cycle effects. Money is more unequally distributed than income and net wealth in both countries.

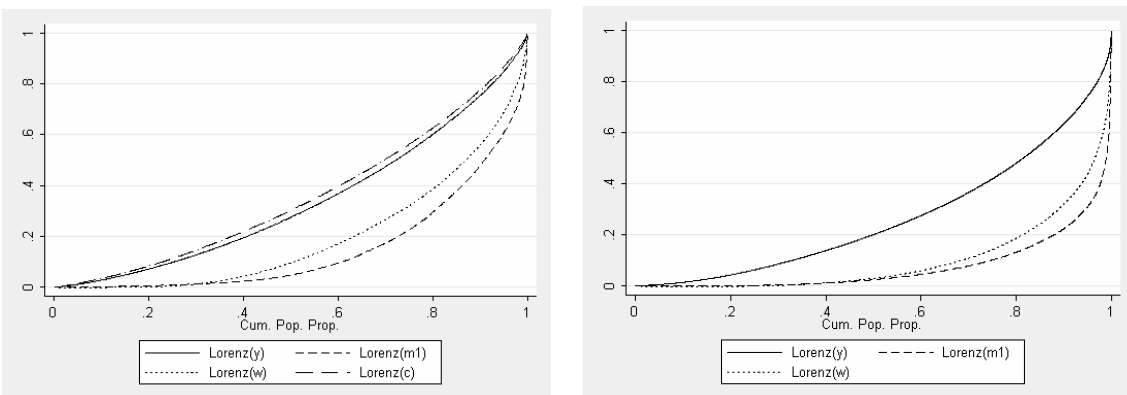


Figure 1: Lorenz Curves of Income (y), Money (m1), Wealth (w) and Consumption (c), in Italy (left) and the US (right), for households whose head is aged between 35 and 44.

2.1 2004 Italian Data

This section uses the 2004 Italian Survey of Households Income and Wealth to examine the distribution of money. This periodic survey provides data for various deposit accounts, currency, income and wealth in the Italian population. Each survey is conducted on a sample of about 8,000 households, and provides representative weights.

[1] As is fairly usual, I use net worth as a summary statistic for all types of assets. The Lorenz curve of financial assets is very similar to the Lorenz curve of net wealth.

A number of recent papers have used this data set to analyse money demand at the household level (Attanasio, et al. 2002; Alvarez and Lippi 2007, amongst others).

Table 1: Distribution of Money and Wealth, Italy 2004

Gini Index of	Cons.	Income	Net W.	Money
Total Population	.30	.35	.59	.68
Pop., 35≤age≤44	.29	.32	.61	.70
Pop., 35≤age≤44, 99%.	.27	.31	.57	.63

Table 1 shows the Gini index of the distributions of consumption, income, net worth and money (in the columns) for three different types of households (in the rows). The first column presents the Gini coefficient for total consumption, and the first row shows the results for the whole population. This is fairly low, at .30. To avoid life-cycle effects the second line focuses on households whose head is aed between 35 and 44. The Gini coefficient is almost unchanged at .29. The second column shows the results for the distribution of income. The Gini coefficeinjt is a little higher than that of consumption at .35, decreasing to .32 for the 35-44 age group. The third column performs the same exercise for the distribution of net wealth. This is more dispersed than consumption or income: the Gini coefficient for net worth is .59, increasing slightly to .61 for the 35-44 age group.

I use Italian data to construct the quantity of money (M1) held by each households, as the sum of the amount held in currency and in checking accounts. Although checking accounts are interest-bearing in Italy, the interest rate is low enough for this aggregation to be relevant: the average interest rate on checking accounts is below 1%, whereas the average yearly yield of 10 year securities was over 4% in Italy in 2004. The last column of Table 1 shows the distribution of money. The Gini coefficient is very high here, at .68, and increases to .70 for the 35-44 age group. As a robustness check, I consider the distribution of money without including the 1% of the households who hold the nost money. Some households may hold money to buy very expensive durable goods (houses) in their checking accounts for a few

days prior to the transaction. If the survey interview occurs during this period, we observe high levels of money balances that are not relevant[2]. The Gini coefficient on money holdings falls from .70 to .63 after this exclusion, but remains high.

The distribution of money is thus similar to that of net wealth, and is very different from that of consumption. For space reasons, this section has characterized the distribution by the Gini coefficient. Howeve, other measures of inequality yield the same results. This can be seen graphically in Figure 1, which shows the four Lorenz curves for the population aged between 35 and 44.

Table 2 presents the empirical correlations between money holdings, consumption levels, income and wealth. Money is positively correlated with consumption, income and wealth, with a coefficient of between .2 and .3. The correlation between the ratio of money over total financial assets and wealth is negative. That is, the share of money in the financial portfolio falls with wealth. This property of the money/wealth distribution had already been noted by Erosa and Ventura (2002) in the US economy.

Table 2: Empirical Correlations, Italy 2004, 35≤age≤44

Money & Income	.21
Money & Consumption	.27
Money & Net Wealth	.30
(Money/Fin. W.) & Net .W.	-0.13

2.2 US Data

US data do not allow us to carry out the same detailed analysis: Income, money and financial wealth come from by the Survey of Consumer Finance (SCF), and the distribution of consumption can be found in the survey of Consumer Expenditures

[2] I carry out this exercise even though it is problematic to justify the exclusion of this 1% of households. If households keep money to buy a house over a period of one week, and buy a new house as often as every five years, the probability that they will be observed with this money the day of the interview is only $(1/52) * (1/5) = 0.4\%$.

(CE). Hence, we cannot calculate the correlation between consumption and money. I use a conservative definition of money, which is the amount held in checking accounts. This is the only fraction of M1 which is available in the data. I also provide statistics for the amount held in all transaction accounts, which correspond to the M2 aggregate.[3]

The distribution of money in the SCF[4] 2004 is investigated in Table 3. The Gini index of the distribution of money held in checking accounts is given in the first row. This is very high at .81. As before, to exclude life cycle effects, the second row focuses on households whose head is aged between 35 and 45. The Gini coefficient increases to .83 here. Finally the third row excludes the 1% money-richest households: the Gini coefficient falls, but is still high at .75. The second column performs the same analysis for money held in all transaction accounts, such as checking, savings and money market accounts. The Gini coefficient here is of the same order of magnitude, and decreases from .85 to .79. excluding the excludes the 1% money-richest households.

The results for the distribution of net wealth are given in column 3. The values of the Gini index are very similar between specifications. Last, column 4 shows the results for the distribution of income. The Gini index is lower than that for the distribution of money for all definitions of money and for all sets of households. As a result, the distribution of money is much closer to the distribution of net wealth than to the distribution of income.

The correlation between money (checking account), income and other assets is presented in Table 4. Money is positively correlated with both income and net wealth: richer households hold more money on average. The last line of Table 4

[3]Note that this measure of money does not include currency, which is not available for US households.

[4]The same exercise can be carried out for a number of years of the SCF. The results are quantitatively similar.

Table 3: Distribution of Money and Wealth

Gini Index of	Check. Acc	Trans. Acc	Net W.	Income
Total Population	.81	.85	.81	.54
Pop., 35≤age≤44	.83	.85	.80	.47
Pop., 35≤age≤44, 99%.	.75	.79	.73	.41

shows the correlation between the ratio of money in financial wealth and total net wealth. This correlation is negative. As in the Italian data, richer households hold more money but as a smaller fraction of their financial wealth.

Table 4: Empirical Correlations

US, 2004, 35≤age≤44

Money & Income	.12
Money & Net Wealth	.17
(Money/Fin. W.) & Net Wealth	-0.08

Table 5 below presents some additional properties of the joint distribution of money and assets in the US economy, which will be used to illustrate the model's outcome. The table represents the fraction of total wealth and total money held by the richest 1% of the population (line 1), the richest 10% (line 2), the richest 20% (line 3) and the poorest 40% (line 4). First, the richest households hold a significant fraction of money, whereas te 40% poorest households hold a much lower fraction. Second, we can check that the proportion of money in total wealth is higher for the poorest households than for the richest households. Poor households hold relatively more money than financial assets, but they hold a smaller fraction of the total quantity of money.

Finally, the distribution of consumption can be obtained from the survey of Consumer Expenditures (CE). Krueger and Perri (2002) note that the distribution of consumption is much less unequally distributed than the distribution of income.

Table 5: Asset Holding Distribution

US Data, 35≤age≤44

	Fract. of Wealth	Fract. of Checking
Wealth 99-100	32.7%	10.9%
Wealth 90-100	70.0%	60.8%
Wealth 80-100	82.4%	72.6%
Wealth 0-40	1.03%	5.4%

The consumption Gini coefficient is around 0.27 and changes only little over time. I calculate the same Gini coefficient for total consumption using the NBER extract of the Consumer Expenditures in 2002, which is the latest year available. I find a Gini coefficient of .28. There is substantial empirical debate about the quality of the data and the estimated changes in consumption inequality (Attanasio et al. 2004, for example). The consensus view is that consumption levels are less unequally distributed than income. As a result, the distribution of money is much closer to the distribution of total wealth than to the distribution of consumption.

To summarize these US and Italian findings: 1) inequality in money holdings is more similar to inequality in net wealth and very different from inequality in consumption; 2) money is positively correlated with wealth, income and consumption levels; and 3) the ratio of money over financial assets falls with wealth.

3 Some Difficulties in Linking Money and Consumption

Simple models of money demand. Simple models of money demand cannot reproduce the shape of the distribution of money, when they link money demand to consumption. They assume that the real money holdings of a household i, m^i, are

simply proportional to consumption, c^i

$$m^i = Ac^i$$

where A is a constant, the same for all households, which may depend on the nominal interest rate, real wages and preference parameters. This form is used for instance in Cooley and Hansen (1989) to assess the welfare cost of inflation. It also results in all models with money-in-the utility function (MIUF) where the utility function is homothetic in money and consumption in the sense of Chari et al. (1996), which is the benchmark case in this literature. It is also obtained in a simple specification of the shopping-time model (McCallum and Goodfriend 1987).

In this case, the distributions of money and consumption are homothetic, and their Gini indexes are equal. The difference in the data cannot therefore be explained.

Economies of scale in the transaction technology. Some authors have noted that the share of money holdings in total wealth falls with total wealth and have concluded that the transaction technology exhibits scale economies: Richer households, even if they consume more, need less money because they buy more goods via credit. Dotsey and Ireland (1996) provide a microfoundation of this transaction technology, which uses the flexibility provided by the definition of cash and credit goods of Stokey and Lucas (1987). Erosa and Ventura (2002) use this formulation of a heterogenous-agents setting. This implies that the quantity of money and the consumption level of household i satisfy the following relationship:

$$\frac{m^i}{c^i} = A\left(c^i\right)^{-\theta} \text{ with } \theta > 0 \tag{1}$$

However, this specification is not able to reproduce a realistic distribution of money. With moderate returns (a low value of θ), the distribution of money is more equally distributed than the distribution of consumption, because the households with higher consumption levels hold fewer real balances. A more dispersed distribution of money can only be obtained with a very high increasing return in the

transaction technology. In this case, households who consume the most hold almost no money, whereas households who consume little hold higher levels of money balances. However, one counterfactual implication of this assumption is that consumption and money will be *negatively* correlated, as higher consumption implies lower money holdings and vice versa.

To illustrate, I consider the equilibrium distribution of consumption of Italian households aged between 35 and 44. I generate fictitious money distributions with various transaction technologies, using the general form of the transaction technology (1) for various values of θ. I finally analyze the distributional properties of the joint distribution of money and consumption. The results are summarized in Table 6.

Table 6: Properties of the Distribution of Money for Different Transaction Technologies

Values of θ	Data	0	0.5	1	2	3.7	-1.7
Gini of consumption	.29	.29	.29	.29	.29	.29	.29
Gini of Money	.70	.29	14	0	0.30	.70	.70
Corr. Money Consumpt.	.27	1	.97	0	-0.63	-0.36	0.69

Table 6 presents the value of the Gini coefficient and the correlation between money and consumption for various values of θ. For θ less than 1, the distribution of money is more concentrated than the distribution of consumption. To obtain a wider dispersion of money, the returns on the transaction technology must be higher than 1, but the correlation between money and consumption then becomes negative, which is counterfactual.

The same type of experiment can be carriet out with the US Data. Using the distribution of money, I generate a fictitious distribution of consumption using (1). I determine the value of θ for which the distribution of consumption is realistic in terms of the Gini coefficient. One again, we need a value of θ of over 3 to obtain a Gini coefficient under .47, which is the Gini coefficient on income.

Finally, note that the microfoundation of money demand with scale economies in Dotsey and Ireland (1996) requires increasing returns to scale to obtain the correct sign on the interest elasticity of money demand. Erosa and Ventura (2002), who do not explicitly reproduce the distribution of money, use a value of θ of over 3. Algan and Ragot (2008), who use a MIUF framework, assume that $\theta = 0$.

The following model proves than we can obtain a realistic distribution of money by focusing on transaction technologies in the *financial* market and not in the *goods* markets. The correlation between money and consumption will appear as an outcome, rather than as a specific utility function imposed on the households.

4 The Model

The economy is populated by a unit mass of households, a representative firm and the Government. There is a consumption-investment good and there are two assets: money and a riskless asset issued by firms. The Government finances a public good vai the inflation tax and distortionary taxes on labor and capital.

Time is discrete and $t = 0, 1, ..$ denotes the period. There is no aggregate uncertainty, but households face idiosyncratic productivity shocks. These shocks are not insurable, and households can partially self-insure by holding money or riskless assets. The crucial assumption is that households must pay a fixed cost λ in terms of the final good[5] to enter the financial market in order to adjust their financial position, and pay no cost to adjust their monetary holdings.

4.1 Households

There is a continuum of length 1 of infinitely-lived households who enjoy utility from consumption c and disutility from hours worked n. For simplicity only, I follow Greenwood, Hercowitz and Huffman (1988) and Domeij and Heathcote (2004)

[5] The results do not significantly change if we assume that this cost is paid in labor, and thus affects labor supply.

in assuming the following functional form for the period utility function (see also Heathcote 2005 for a discussion of the properties of this functional form):

$$u(c,n) = \frac{1}{1-\gamma}\left[\left(c - \psi\frac{n^{1+\frac{1}{\varepsilon}}}{1+1/\varepsilon}\right)^{1-\gamma} - 1\right]$$

In this specification, ε is the Frisch elasticity of labor supply, ψ scales average labor supply, and γ is the risk-aversion coefficient. In each period, a household i can be in one of three states according its labor market status. Its productivity e_t^i is then either e^1, e^2 or e^3. For instance, a household whose productivity is e^1 and which works n_t hours earns labor income of $e^1 n_t w_t$, where w_t is the after-tax wage by efficiency unit. Labor productivity e_t^i follows a three-state first order Markov chain with transition matrix T. $N_t = [N_t^1, N_t^2, N_t^3]'$ is the distribution vector of households according to their state on the labor market in period $t = 0, 1....$ The distribution in period t is $N_0 T^t$. Given standard conditions, which will be fulfilled here, the transition Matrix T has an unique ergodic set $N^* = \{N_1^*, N_2^*, N_3^*\}$ such that $N^* T = N^*$. To simplify the dynamics, I assume that the economy starts with the distribution N^* of households.

The variables a_t^i and m_t^i denote respectively the *real* quantity of financial assets and money held at the end of period $t-1$, and r_t is the after-tax real interest rate on the riskless asset. P_t denotes the money price of one unit of the investment-consumption good, and $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate between periods $t-1$ and t. The real revenue at the beginning of period t of a household holding a_t^i and m_t^i is thus $\frac{m_t^i}{\Pi_t} + (1+r_t)\, a_t$.

Households pay proportional taxes on capital and labor income: τ^{cap} is the tax rate on capital and τ^{lab} is the tax rate on labor. The variables $\tilde{w}_t$ and $\tilde{r}_t$ are respectively the real wage and the real interest rate before taxes:

$$w_t = \left(1 - \tau_t^{lab}\right)\tilde{w}_t$$
$$r_t = \left(1 - \tau_t^{cap}\right)\tilde{r}_t$$

In period t, each household can choose to participate or not in the financial market.

If it participates, it pays a cost λ and can freely use its total monetary and financial resources $\frac{m_t^i}{\Pi_t} + (1 + r_t) a_t$ to consume the amount c_t^i, and to save a quantity a_{t+1}^i in financial assets and a quantity m_{t+1}^i in money. If the household does not participate, it can only use its monetary revenue m_t^i/Π_t to consume c_t^i and to keep a fraction m_{t+1}^i in money. It is assumed that its financial wealth is reinvested in financial assets[6]: $a_{t+1}^i = (1 + r_t) a_t^i$. This transaction choice is summarized by the dummy variable I_t^i, which equals 1 when the household participates and 0 otherwise.

No private households can issue money $m_t^i \geq 0$, and households face a simple borrowing limit when participating in financial markets: $a_t^i \geq 0$, for $t = 0, 1..$ and $i \in [0, 1]$.

The program of households i can be summarized as follows:

$$\max_{\{m_{t+1}^i, a_{t+1}^i, c_t^i, n_t^i, I_t^i\}_{t=0,1..}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^i, n_t^i\right)$$

subject to

$$\begin{aligned} c_t^i + m_{t+1}^i + I_t^i \left(a_{t+1}^i - (1 + r_t) a_t^i + \lambda\right) &= e_t^i w_t n_t^i + \frac{m_t^i}{\Pi_t} \\ \left(1 - I_t^i\right) \left(a_{t+1}^i - (1 + r_t) a_t^i\right) &= 0 \\ c_t^i, n_t^i, m_{t+1}^i, a_{t+1}^i &\geq 0, I_t^i \in \{0, 1\} \\ a_0^i, m_i^0 &\text{ given} \end{aligned}$$

Recursive Formulation

The program of the households can be written in a recursive way as follows (See Baim 2005, for a proof of the existence of Bellman equations in a similar economy). Define $V_t^{par}\left(a_t^i, m_t^i, e_t^i\right)$ as the maximum utility that a household with productivity e_t^i can reach at period t if it participates in financial markets at period t and if it holds an amount m_t^i and a_t^i of monetary and financial wealth respectively; $V_t^{ex}\left(a_t^i, m_t^i, e_t^i\right)$ is the analogous utility if the household does not participate.

[6] This is the standard assumption made by Romer (1986) for instance. The quantitative results do not change if interest is paid in money.

The Bellman value $V_t^{par}\left(a_t^i, m_t^i, e_t^i\right)$ satisfies

$$V_t^{par}\left(m_t^i, a_t^i, e_t^i\right) = \max_{a_{t+1}^i, m_{t+1}^i, n_t^i, c_t^i} u\left(c_t^i, n_t^i\right) + \beta E_t \max\{V_{t+1}^{par}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right), V_{t+1}^{ex}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right)\}$$

$$m_{t+1}^i + a_{t+1}^i + c_t^i = w_t e_t^i n_t^i + \frac{m_t^i}{\Pi_t} - \lambda + (1 + r_t)\, a_t^i$$

$$c_t^i, n_t^i, a_{t+1}^i, m_{t+1}^i \geq 0$$

The value $V_t^{ex}\left(a_t^i, m_t^i, e_t^i\right)$ satisfies

$$V_t^{ex}\left(m_t^i, a_t^i, e_t^i\right) = \max_{m_{t+1}^i, n_t^i, c_t^i} \left\{u\left(c_t^i, n_t^i\right) + \beta E \max\{V_{t+1}^{par}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right), V_{t+1}^{ex}\left(m_{t+1}^i, a_{t+1}^i, e_{t+1}^i\right)\}\right\}$$

$$m_{t+1}^i + c_t^i = w_t e_i^t n_t^i + \frac{m_t^i}{\Pi_t}$$

$$a_{t+1}^i = a_t^i\,(1 + r_t)$$

$$c_t^i, n_t^i, m_{t+1}^i \geq 0$$

Note first that the definition of the Bellman equations differs according to the budget constraints of the household: either the household faces one budget constraint, or the consumption choice is made facing both monetary and financial budget constraints. Second the household anticipates that its current portfolio choice may affect it participation choice tomorrow.

Finally, the maximum utility that a household with productivity e_t^i and assets m_t^i and a_t^i can reach is

$$V_t\left(m_t^i, a_t^i, e_t^i\right) = \max\{V_t^{par}\left(m_t^i, a_t^i, e_t^i\right), V_t^{ex}\left(m_t^i, a_t^i, e_t^i\right)\}$$

According to this last maximization, the household either chooses to participate, in which case $I_t^i = 1$, or not, $I_t^i = 0$.

The solution of the household's problem produces a set of optimal decision rules defined over the productivity set $E = \{e^1, e^2, e^3\}$ and the set of assets:

$$\left.\begin{array}{c} c_t(.,.,.) : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\ a_{t+1}(.,.,.) : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\ m_{t+1}(.,.,.) : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \mathbb{R}^+ \\ n_t(.,.,.) : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow [0,1] \\ I_t(.,.,.) : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow \{0,1\} \end{array}\right\} t = 0, 1, ...$$

4.2 Firms

The consumption-investment good is produced by a representative firm in a competitive market. Capital depreciates at a rate of δ and is installed one period before production. We denote by K_t and L_t aggregate capital and aggregate effective labor used in production in period t . Output Y_t is given by

$$Y = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha},\ 0 < \alpha < 1$$

Effective labor supply is:

$$L_t = e^1 L_t^1 + e^2 L_t^2 + e^3 L_t^3$$

where L_t^1, L_t^2 and L_t^3 is the aggregate labor supply of workers of productivity $1, 2$ and 3 respectively. Profit maximization yields the following relationships

$$\tilde{w}_t = F_L'(K_t, L_t) \tag{2}$$

$$\tilde{r}_t + \delta = F_K'(K_t, L_t) \tag{3}$$

where $\tilde{w}_t$ and $\tilde{r}_t$ are before-tax real wages per efficient unit and the real interest rate.

4.3 Monetary Policy

At each period t, monetary authorities create an amount of new money Δ_t. Let M_t be the total amount of nominal money in circulation at the *end* of period t. The law of motion of the nominal quantity of money is thus

$$M_t = M_{t-1} + \Delta_t \tag{4}$$

The real value of the inflation tax is thus Δ_t / P_t.

I focus below on stationary equilibria where monetary authorities create a quantity of money proportional to the total nominal quantity of money of the previous period, with a coefficient of π. In this case $\Delta_t = \pi M_{t-1}$ and the revenue from the inflation tax is $\pi M_{t-1}/P_t$.

4.4 Government

The Government finances a public good, which costs G_t units of goods in period t. It receives the inflation tax Δ_t/P_t and the proportional taxes on capital and labor income, with coefficients τ_t^{cap} and τ_t^{lab} respectively. It is assumed that the Government does not issue any debt. Its budget constraint is

$$G_t = \tau_t^{cap}\tilde{r}_t K_t + \tau_t^{lab}\left(N_t^1 L_t^1 e_t^1 + N_t^2 L_t^2 e_t^2 + N_t^3 L_t^3 e_t^3\right)\tilde{w}_t + \frac{\Delta_t}{P_t} \tag{5}$$

where N_t^1, N_t^2 and N_t^3 are the number of type $1, 2$ and 3 households respectively.

4.5 Market Clearing

Denote $\Phi_t : \mathbb{R}^+ \times \mathbb{R}^+ \times E \longrightarrow [0,1]$ as the joint distribution of households over financial assets, money holdings and productivity in period t. Money and capital market equilibria state that money is held by households at the end of each period, and that financial savings are lent to the representative firm. These can be written as, for $t \geq 0$:

$$M_t = \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} P_t m_{t+1}(a,m,e)\, d\Phi_t(a,m,e) \tag{6}$$

$$K_{t+1} = \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} a_{t+1}(a,m,e)\, d\Phi_t(a,m,e) \tag{7}$$

The goods-market equilibrium requires that the amount produced is consumed by the State, invested in the firm, consumed by the households, but also destroyed in the transaction cost. This can be written as

$$\begin{aligned} G_t + K_{t+1} + \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} & c_t(a,m,e)\, d\Phi_t(a,m,e) \\ & + \lambda \int_{\mathbb{R}^+\times\mathbb{R}^+\times E} I_t(a,m,e)\, d\Phi_t(a,m,e) = F(K_t, L_t) + (1-\delta) K_t \end{aligned} \tag{8}$$

4.6 Equilibrium

For a given path of Government spending $\{G_t^{cap}\}_{t=0..\infty}$ and money creation $\{\Delta_t\}_{t=0..\infty}$, an equilibrium in this economy is a sequence of decision rules $c_t(.,.,.)$, $a_t(.,.,.)$, $m_t(.,.,.)$, $n_t(.,.,.)$

$I_t(.,.,.)$ defined over $\mathbb{R}^+ \times \mathbb{R}^+ \times \{e^1, e^2, e^3\}$ for $t = 0..\infty$, sequences of prices $\{P_t\}_{t=0..\infty}$, $\{\tilde{\omega}\}_{t=0..\infty}$ and $\{\tilde{r}\}_{t=0..\infty}$, and sequences of taxes $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau^{cap}\}_{t=0..\infty}$ such that:

1. The functions $c_t(.,.,.)$, $a_t(.,.,.), m_t(.,.,.), n_t(.,.,.)$ $I_t(.,.,.)$ solve the household's problem for a sequence of prices $\{P_t\}_{t=0..\infty}$, $\{\tilde{\omega}\}_{t=0..\infty}$ and $\{\tilde{r}\}_{t=0..\infty}$, and taxes $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau^{cap}\}_{t=0..\infty}$.

2. The joint distribution Φ_t over productivity and wealth evolves according to the decision rules and the transition matrix T.

3. Factor prices are competitively determined by firm optimal behavior (2)-(3).

4. The quantity of money in circulation follows the law of motion (4).

5. Markets clear: equations (6)-(8).

6. Tax rates $\{\tau_t^{lab}\}_{t=0..\infty}$ and $\{\tau_t^{cap}\}_{t=0..\infty}$ are such that the government budget (5) is balanced.

A *stationary* equilibrium is an equilibrium where the nominal money growth rate, the values $G, \tau^{lab}, \tau^{cap}$, r, w, the gross inflation rate $\Pi = \frac{P_t}{P_{t-1}}$, the joint distribution Φ and the decision functions $c(.,.,.)$, $a(.,.,.), m(.,.,.), n(.,.,.)$ $I(.,.,.)$ are time-invariant. In such an equilibrium, the aggregate real variables are constant whereas the nominal variables all grow at the same rate.

5 Parameterization

The model period is one quarter. Table 7 summarizes the parameter values at a quarterly frequency in the stationary benchmark equilibirum

Table 7: Parameter Values

α	β	ψ	σ	ε	τ^{cap}	τ^{lab}	δ	π	λ
0.36	0.99	117	1	0.3	0.397	0.296	0.015	0.007	0.13

Preference and fiscal parameters

The preference and technology parameters have been set to standard values. The capital share is fixed at $\alpha = 0.36$ (Cooley and Prescott, 1995) and the depreciation rate is $\delta = 0.015$, such that the annual depreciation rate is 6% (Stokey and Rebelo, 1995). The discount factor β is set to 0.99, to obtain a realistic capital-output ratio of around 3. The risk-aversion parameter, σ, is set to 1. The Frisch Elasticity of labor supply ε is estimated to be between 0.1 and 1. I follow Heathcote (2005) and use a conservative value of 0.3. Given this value, ψ is set such that aggregate effective labor supply is close to 0.33. The fiscal parameters are calibrated to match the actual tax distortions in the US economy. Following Domeij and Heathcote (2004), the average tax rate on capital income τ^{cap} is 39.7 percent, whereas the average tax rate on labor income τ^{lab} is 26.9 percent. The implied government consumption to annual output ratio is 0.24, which is a little higher than, but not too dissimilar to, the U.S. average of 0.19 over the 1990-1996 period.

The household productivity process

Different models of the income process are now used in the literature. Our modeling strategy is to use a simple process which yields realistic distributions for consumption, income and wealth. I consequentlyh use that in Domeij and Heathcote (2004), with endogenous labor used at a quarterly frequency. They estimate a three-state Markov process, which reproduces the process for logged labor earnings using PSID data. The Markov chain is estimated under two constraints: (i) The first-order autocorrelation in annual labor income is 0.9; and (ii) The standard deviation of the residual in the wage equation is 0.224. These values are consistent with estimations found in the literature (Storesletten, Telmer and Yaron, 2007; Floden and Lindé 2001, amongst others).

The transition matrix is

$$T = \begin{bmatrix} 0.974 & 0.026 & 0 \\ 0.0013 & 0.9974 & 0.0013 \\ 0 & 0.026 & 0.974 \end{bmatrix}$$

The three productivity levels are $e^1 = 4.74$, $e^2 = 0.848$, $e^3 = 0.17$. The long-run

distribution of productivity across the three states is $N^* = [0.045 \quad 0.91 \quad 0.045]'$. This parametrization yields a realistic distribution of both wealth and consumption, which is very useful for the issue that we address.

Monetary Parameters

The remaining parameters concern monetary policy and the transaction cost. First, I consider the average US annual inflation rate in 2004, 2.8 percent. Consequently, the quarterly inflation rate is $\pi = 0.007$. I calibrate the transaction cost $\lambda = 0.13$ to reproduce the ratio of money over income of households between 35 and 44 years old. Money is defined as above as the amount in checking accounts in SCF 2004. The ratio of money over income is 8%. I find a value of 0.19 for λ, which corresponds to 3% of households' average annual income. Scaling by average income per capita in the US of \$43000, we find an annual transaction cost to financial markets for the riskless asset of around \$1450. To my knowledge, there is no consensus in the empirical literature regarding the level of such costs. The empirical strategy of Mulligan and Sala-i-Martin (2000) and Paiella (2001) only provides the median cost or the lower bound of the participation cost. Some insights can be obtained from the literature which estimates the cost of participating in the risky-asset market. Vissing-Jorgensen (2002) estimates this participation cost to be as high as 1100 dollars in order to understand the transaction decisions of 95% of non-participants, whereas a cost of 260 dollars suffices to explain the choices of 75% of non-participants. In consequence, although our cost is towards the top-end of these estimates, it is not inconsistent with current empirical results. I show below the results of the model at a monthly frequency. This estimation of the transaction cost is robust to changes in the definition of the period.

6 Results

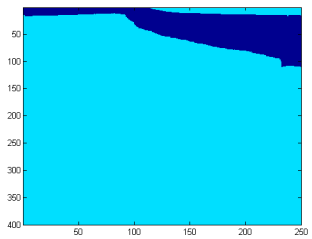
This section first presents the household policy rules and then the properties of the distribution of consumption, income, money and total wealth.

6.1 Participation Decisions

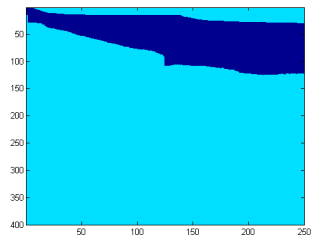
The economic behavior of households is best described by the participation decision in financial markets. Fig. 8 presents the household decision rule of according to their productivity. The x-axis measures the quantity of financial assets held at the

Table 8: Participation Decisions

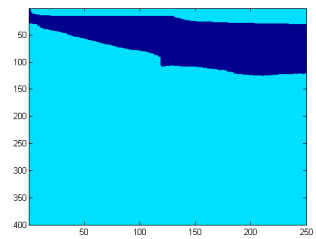
Type 1 Households



Type 2 Households



Type 3 Households



beginning of the period, and the y-axis shows the quantity of money held at the beginning of the period. Each point is a begining-of-period portfolio. For each household type, the dark area represents the portfolio for which households choose not to participate in financial markets. The lighter area represents portfolios for which households choose to participate in financial markets. Households holding a high quantity of money and a small quantity of financial assets (in the South-West corner) and households holding a small quantity of money and high quantity of assets (in the North-East corner) both participate in financial markets; the households who are inbetween do not participate. Households with a large amount of money and few assets (NE) participate to save in financial assets and dis-save money. Households with little money and many assets participate to dis-save in financial assets and save in money. It can be seen that the lower is productivity the bigger ius the area where households dis-save, and the smaller is the area where households save. Households thus hold both money and financial assets in equilibrium, although the (marginal) return on money is lower than that on financial assets.

6.2 The Distribution of Money and Financial Assets

The distribution of consumption, income, financial wealth and money in the benchmark economy is summarized in Table 9, which presents the associated Gini coefficients.

Table 9: Gini Indexes

	Consumption	Income	Money	Wealth
US Data (age 35-44)	.28	.47	.83	.80
Model	.32	.37	.85	.81

First note that the ranking of the Gini coefficients is the same as that in the data, and that the model performs quantitatively well in reproducing the inequality in the distribution of consumption, income, money and wealth. The Gini of the total wealth distribution is 0.81. Domeij and Heathcote (2004) find a Gini coefficient of 0.78 in a similar economy with only financial assets. The introduction of money does not thus significantly modify the shape of the distribution of assets. The Gini coefficient for money is 0.85, which is very similar to that actually observed in the US economy. The Gini coefficient for consumption is a little higher than its empirical counterpart, whereas the Gini coefficient for income is greater than that found in the data.

Table 10 below investigates the distributional properties of the model. As in Table 5 above, the table shows the fraction of wealth and money held by various subpopulations, ranked by their wealth. The right-hand side of the table presents the values produced by the model. For ease of comparison, the left-hand side reproduces the empirical counterparts in the US in 2004. The model performs relatively well in reproducing the wealth and money holdings of the poorest households. However, it

does not reproduce a realistic distribution of wealth for the richest 1% of households. This difficulty is known in this class of models. One solution is to introduce life-cyle effects and bequest motives, as in Di Nardi (2005) for instance.

Table 10: The Distribution of Asset Holdings

	US Data, 35≤age≤44		Model	
	Fract. of Wealth	Fract. of Checking	Fract. of Wealth	Fract. of Money
Wealth 99-100	32.7%	10.9%	14.0%	0.3%
Wealth 90-100	70.0%	60.8%	68.7%	43.2%
Wealth 80-100	82.4%	72.6%	84.7%	79.4%
Wealth 0-40	1.0%	5.4%	1.5%	2.1%

Table 11 presents the correlation between money, income and financial wealth generated by the model. The left-hand side shows values in US data for the relevant age group, and the right-hand side shows the model results. All of the model correlations have the right signs. Further, the correlations between money & consumption and money & wealth are higher than those in the data. There are therefore other sources of heterogeneity in money holdings which are not captured by the model. Finally, the model is able to reproduce the average negative correlation between wealth and the ratio of money to total wealth.

7 Robustness Checks

The model thus yields a realistic joint distribution of money and financial assets. Three robustness checks are considered in this section. First, the model is parameterized with different values for transaction costs, to produce changes in the distribution

Table 11: Empirical Correlations,

	US Data	Model
Money & Income	.12	.40
Money & Consumption	-	.45
Money & Wealth	.17	.39
(Money/ Wealth) & Wealth	-0.08	-0.07

of money and wealth. Second, the model is parameterized at a monthly frequency. Third, we see whether the results are robuse to an alternative labor-income process.

7.1 The Effect of Transaction Costs

To assess the effect of transaction costs on the inequality in the money distribution, Table 12 presents the change in relative money holdings as the cost λ varies. To simplify the exposition, this cost is presented as a percentage of the benchmark value.

The first line shows the change in financial wealth, and the second represents the change in money holdings. All figures are expressed as a percentage of the value of the financial wealth held in the benchmark case. For example, in the benchmark case, households hold money equal to 2% of their amount held in financial wealth. The third and fourth lines present the Gini coefficients for money and financial wealth respectively.

Table 12: Share of Money Holdings

λ	0	50	100	200	∞
Fin. Wealth	278	117	100	80	0
Money	0	1.26	2.02	2.67	12.5
Gini Money	–	0.81	0.82	0.84	0.94
Gini Fin. W.	0.63	0.79	0.77	0.79	–

When $\lambda = 0$, there is no transaction cost in entering financial markets, and as a result money is a dominated asset which is not held in equilibrium. This is the benchmark Hugget (1993) and Aiyagari (1994) models. When $\lambda = \infty$, the transaction cost of entering the financial market is too high and households only hold money. This corresponds to the Bewley (1983), Scheinkman and Weiss (1986) and Imrohoroglu (1992) models. For values of λ between these two extremes, households hold both money and financial assets in equilibrium. The quantity of money (financial assets) held decreases (increases) with the transaction cost. In all cases, the Gini coefficient for money remains high, and actually rises with the transaction cost. The greater the quantity of money in the economy, the more unequally it is distributed.

7.2 A Monthly Specification

As a robustness check, the model is re-calibrated at a monthly frequency. The discount factor, the financial market transaction cost, and the preference for leisure have been re-calibrated to match roughly the same capital-output ratio, the same money-output ratio and the same labor supply. The modified parameters are listed below.

ψ	β	λ
210	0.9968	0.75

The labor process has been modified accordingly. The new transition matrix T^* is such that $T = (T^*)^3$.

The results are very similar to those from the quarterly specification. For instance the Gini coefficient for money is 0.82 and that for wealth is 0.82. The transaction cost λ is 2.3% of annual average income, whereas it was 3% in the quarterly calibration. The results of the model are therefore roughly invariant to the definition of the period. I use the quarterly specification to increase the converge speed of the algorithm.

7.3 An Alternative Income Process

Before using this model to assess the effect of inflation, I check that its ability to reproduce the distribution of money is not due to the particular values of the parameters retained in the calibration exercise. I here follow the calibration described in Diaz, Pijoan-Mas and Rios-Rull (2003) which is a simplified version of Castaneda, Diaz-Gimenez and Rios-Rull (2003). I use this calibration at a quarterly frequency. Here, labor supply is exogenous and households face the earning process

$$\{e_1, e_2, e_3\} = \left\{ \begin{array}{ccc} 1.000, & 5.290, & 46.550 \end{array} \right\}$$

$$T = \left[\begin{array}{ccc} 0.998 & 0.002 & 0 \\ 0.0022 & 0.9949 & 0.0029 \\ 0 & 0.0215 & 0.9785 \end{array} \right]$$

$$N^* = \left[\begin{array}{ccc} 0.4922 & 0.4475 & 0.064 \end{array} \right]$$

I calibrate the discount factor β and the transaction cost λ to obtain the same capital-output ratio and the same money-output ratio as in the benchmark economy. This produces

σ	β	λ
2	0.981	0.2

The Gini coefficients are as follows.

Gini Index of		
Income	Money	Wealth
0.61	0.65	0.7

As before, we have the correct ranking of the Gini coefficients, and a high Gini coefficient for money. In this specification, however, consumption appears to be too dispersed, with a Gini coefficient of 0.52, which is greater than the value found in US data (Krueger and Perri, 2005). Nevertheless, the fact that the money distribution is much more dispersed than consumption and income is a robust outcome of the formalization of money demand in this paper.

8 Conclusion

This paper uses a remarkable fact about household money holdings to discuss the relevance of various theories of money demand: the distribution of money across households is similar to the distribution of financial assets, and very different from the distribution of consumption. The theories which provide a foundation of money demand with frictions in the goods market cannot reproduce this fact with realistic assumptions, and yield a distribution of money similar to that of consumption. This paper provides a model where a simple transaction cost in financial markets generates a realistic joint distribution of money and financial assets. Due to this cost, the average expected return to money can be higher than that on financial assets without any other frictions. Of course, this paper does not claim that the existence of a cash-in-advance constraint for a range of goods is unimportant for some aspects of money demand, for example for currency holdings. But it does seem that a financial approach to money demand is more relevant when we consider broader money aggregates. The analysis of the real and welfare effects of various types of shocks in this framework is a useful avenue for future research.

A Appendix

A.1 Equilibrium Values

The following table provides the equilibrium values of the model.

K	M	Y	C	L	r	w
15.7992	0.4231	1.2944	0.6694	0.3169	0.0087	1.9108

A.2 Computational Strategy

The computational strategy for the stationary equilibrium of the type of model used in this paper is now well-defined. The stationary value functions are solved by iteration over the value function. For a given real interest rate r and a fixed inflation rate,

the other six value functions $\{V_j^{par}(.,.,e^1), V_j^{par}(.,.,e^2), V_j^{par}(.,.,e^3), V_j^{ex}(.,.,e^1), V_t^{ex}(.,.,e^2), V_t^{ex}(.,.,e^3)\}$ are iterated using standard techniques. The value function iteration avoids any assumptions regarding the differentiability of the value function, which is not guaranteed here. After convergence, I determine the stationary distribution to compute aggregate financial savings and the total real quantity of money. I then adjust the real interest rate until the market for the riskless financial assets clears. Note that no equilibrium condition is necessary for the real quantity of money, as the ratio of the nominal to the real quantity of money endogenously determines the equilibrium money price of the final good.

To speed up convergence, I compute the level of the value function on a first grid where maxima are found by a hill-climbing algorithm. To obtain precise value functions, I extrapolate this first grid via a second grid. One important point is that this must be a *square* grid (and not rectangular), so as not to favour an asset in term of portfolio adjustment. The first grid has 30 values for money and 700 values for financial assets. The second grid decomposes each increment into 50 separate points for both money and financial assets. Value maximization is thus carried out for 35000 different values of the financial asset and 1500 values for money.

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