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Interest Rate Rules and Global Determinacy: An Alternative to the Taylor Principle

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Abstract

A most wellknown determinacy condition on interest rate rules is the "Taylor principle", which says that nominal interest rates should respond more than hundred percent to inflation. Unfortunately, notably because interest rates must be positive, the Taylor principle cannot be satisfied for all interest rates, and as a consequence global determinacy may not prevail even though there exists a locally determinate equilibrium.

We propose here a simple alternative to the Taylor principle, which takes the form of a new condition on interest rate rules that ensures global determinacy. An important feature of the policy package is that it does not rely at all on any of the fiscal policies associated with the "fiscal theory of the price level", which was so far the main alternative for determinacy.

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1 Introduction

In the recent years an important concern of macroeconomists has been to find out which combinations of monetary and fiscal policies lead to determinacy of the dynamic equilibrium. Notably the early seminal work of Leeper (1991) shows how either monetary or fiscal policies can lead to local determinacy.

On the monetary side the focus has been recently on interest rate rules. A central result is the wellknown "Taylor principle", which says that the nominal interest rate should respond to inflation with an elasticity greater than one[1] (Taylor, 1993, 1998).

Unfortunately, as is wellknown, the Taylor principle is a condition for local, not global determinacy. Notably if one takes into account the fact that nominal interest rates must be positive, then the Taylor principle cannot be satisfied for all values of the interest rates, and as a consequence global determinacy may not prevail even though there exists a locally determinate equilibrium (Benhabib, Schmitt-Grohe and Uribe, 2001).

If the Taylor principle cannot deliver global determinacy, there is another wellknown determinacy condition (Sims, 1994, Woodford, 1994, 1995), which has been known under the name of "fiscal theory of the price level" (FTPL). The basic idea behind the FTPL is that the government pursues fiscal policies such that, in off-equilibrium paths, it will not satisfy its intertemporal budget constraint, and run an explosive debt policy. This leaves only one feasible equilibrium path[2]. Now although such off-equilibrium paths are not observed in the model's equilibrium, it would be extremely optimistic to assume that in real life situations the economy would follow at every instant the equilibrium path while the government pursues such policies. As a result many people would be reluctant to advise such policies to a real life government.

In view of the above qualifications the purpose of this article is to propose a simple condition on interest rate rules that will ensure global determinacy, even when no FTPL-type policies are implemented.

A key to the results is that we will make the realistic assumption that the economies we work in are *non-Ricardian*. By non-Ricardian economies we mean economies where, as in OLG models, new agents enter in time, so that in particular Ricardian equivalence fails (Barro, 1974). We shall see that in such a framework global determinacy can be obtained under reasonable fiscal and monetary policies.

[1] The original Taylor rule (1993) had actually both inflation and output as arguments, but the debate has mostly centered around inflation.

[2] Appraisals (with different degrees of criticism) of the FTPL appear in Buiter (2002), Christiano and Fitzgerald (2000), Cochrane (2005), Kocherlakota and Phelan (1999), McCallum (2001), Niepelt (2004) and Weil (2002).

2 The model

We want to have a non Ricardian monetary model that "nests" the traditional infinitely lived agent model, so we shall use a monetary model due to Weil (1987, 1991), and assume that new "generations" of households are born each period, but nobody dies. Denote as N_t the number of households alive at time t. So $N_t - N_{t-1} \geq 0$ households are born in period t. We will mainly work below with a constant rate of growth of the population $n \geq 0$, so that $N_t = (1+n)^t$. The Ricardian case is obtained by taking the limit case $n = 0$.

Consider a household born in period j. We denote by c_{jt}, y_{jt} and m_{jt} its consumption, endowment and money holdings at time $t \geq j$. This household maximizes the following utility function:

$$U_{jt} = \sum_{s=t}^{\infty} \beta^{s-t} Log\, c_{js} \tag{1}$$

and is submitted in period t to a "cash in advance" constraint:

$$P_t c_{jt} \leq m_{jt} \tag{2}$$

Household j begins period t with a financial wealth ω_{jt}. First the bond market opens, and the household lends an amount b_{jt} at the nominal interest rate i_t. The rest is kept under the form of money m_{jt}, so that:

$$\omega_{jt} = m_{jt} + b_{jt} \tag{3}$$

Then the goods market opens, and the household sells his endowment y_{jt}, pays taxes τ_{jt} in real terms and consumes c_{jt}, subject to the cash constraint (2). Consequently, the budget constraint for household j is:

$$\omega_{jt+1} = (1+i_t)\,\omega_{jt} - i_t m_{jt} + P_t y_{jt} - P_t \tau_{jt} - P_t c_{jt} \tag{4}$$

Aggregate quantities are obtained by summing the various individual variables. There are $N_j - N_{j-1}$ agents in generation j, so for example aggregate assets Ω_t and taxes T_t are equal to:

$$\Omega_t = \sum_{j \leq t} (N_j - N_{j-1})\,\omega_{jt} \qquad T_t = \sum_{j \leq t} (N_j - N_{j-1})\,\tau_{jt} \tag{5}$$

Similar formulas apply to output Y_t, consumption C_t, money M_t and bonds B_t. We now must describe how endowments and taxes are distributed among households. We assume that all households have the same income and taxes, so that:

$$y_{jt} = y_t = Y_t/N_t \qquad \tau_{jt} = \tau_t = T_t/N_t \tag{6}$$

and that real income per head grows at the rate ζ:

$$y_{t+1}/y_t = \zeta \qquad Y_{t+1}/Y_t = (1+n)\,\zeta \tag{7}$$

Let us now consider government. Households' aggregate financial wealth Ω_t has as a counterpart an identical amount Ω_t of financial liabilities of the government. These are decomposed into money and bonds:

$$\Omega_t = M_t + B_t \tag{8}$$

The evolution of these liabilities is described by the government's budget constraint:

$$\Omega_{t+1} = (1+i_t)\,\Omega_t - i_t M_t - P_t T_t \tag{9}$$

Note that, to simplify the exposition and to concentrate on tax policy, we assume that governement spending is zero. If not, the results would be essentially the same, but the formulas would be more clumsy.

2.1 Monetary policy

We shall consider "Taylor rules" where the nominal interest rate responds to inflation:

$$1 + i_t = \Phi(\Pi_t) \qquad \Phi(\Pi_t) \geq 1 \tag{10}$$

with $\Pi_t = P_t/P_{t-1}$. We shall also define the elasticity of this function:

$$\phi(\Pi_t) = \frac{\partial Log\Phi(\Pi_t)}{\partial Log\Pi_t} = \frac{\Pi_t \Phi'(\Pi_t)}{\Phi(\Pi_t)} \tag{11}$$

One says that the Taylor principle is satisfied at Π_t if $\phi(\Pi_t) > 1$.

2.2 Fiscal policy

If the budget was balanced, taxes would be equal to interest payments on bonds $i_t B_t$, so that one would have:

$$P_t T_t = i_t B_t \tag{12}$$

We shall actually assume that the government can run a deficit or a surplus, so taxes have the form:

$$P_t T_t = i_t B_t - D\left(\Omega_t, P_t Y_t\right) \tag{13}$$

where the function $D\left(\Omega_t, P_t Y_t\right)$, which represents the fiscal deficit in nominal terms, is homogeneous of degree 1 in its two arguments. The argument $P_t Y_t$ represents some type of income tax, while the argument Ω_t reflects the fact that the government may want to raise taxes in order to diminish its financial liabilities Ω_t.

Putting together equations (8), (9) and (13), we find:

$$\Omega_{t+1} = \Omega_t + D\left(\Omega_t, P_t Y_t\right) \tag{14}$$

3 Dynamics

Equation (14) is our first dynamic equation. Turning now to nominal income $P_t Y_t$, it is shown in the appendix that, assuming $N_{t+1}/N_t = 1 + n$, the dynamics of nominal income $P_t Y_t$ is given by:

$$P_{t+1} Y_{t+1} = \beta\left(1+n\right)\left(1+i_t\right) P_t Y_t - \left(1-\beta\right) n \Omega_{t+1} \tag{15}$$

Combining with equation (10) we obtain:

$$P_{t+1} Y_{t+1} = \beta\left(1+n\right)\Phi\left(\Pi_t\right) P_t Y_t - \left(1-\beta\right) n \Omega_{t+1} \tag{16}$$

We shall actually not use the two dynamic equations (14) and (16) as such, but rather use as working variables inflation Π_t and the predetermined variable X_t defined as[3]:

$$X_t = \frac{\Omega_t}{P_{t-1} Y_{t-1}} \tag{17}$$

Then dividing both sides of equation (16) by $\zeta\left(1+n\right) P_t Y_t$ it becomes:

$$\Pi_{t+1} = \frac{\Phi\left(\Pi_t\right)}{\xi} - \nu X_{t+1} \tag{18}$$

where $\xi = \zeta / \beta$ is the autarkic interest rate[4] and:

$$\nu = \frac{\left(1-\beta\right) n}{\left(1+n\right)\zeta} \tag{19}$$

[3] This representation is borrowed from Guillard (2004).

[4] The autarkic interest rate is the one which would prevail if each generation lived in complete autarky. Alternatively it is the real interest rate that would prevail in an economy with a single dynasty, and the utility function (1).

Let us now consider equation (14) and divide both sides by P_tY_t. We find, in view of the homogeneity properties of the function D:

$$X_{t+1} = \frac{\Omega_t + D\left(\Omega_t, P_tY_t\right)}{P_tY_t} = \frac{\Omega_t}{P_tY_t} + D\left(\frac{\Omega_t}{P_tY_t}, 1\right) \tag{20}$$

Let us define the "fiscal function" F:

$$F\left(Z_t\right) = Z_t + D\left(Z_t, 1\right) \tag{21}$$

Then equation (20) becomes, taking $Z_t = \Omega_t/P_tY_t$:

$$X_{t+1} = F\left(Z_t\right) = F\left(\frac{\Omega_t}{P_tY_t}\right) \tag{22}$$

Now we have:

$$Z_t = \frac{\Omega_t}{P_tY_t} = \frac{\Omega_t}{P_{t-1}Y_{t-1}}\frac{P_{t-1}Y_{t-1}}{P_tY_t} = \frac{X_t}{\zeta\left(1+n\right)\Pi_t} \tag{23}$$

so that finally:

$$X_{t+1} = F\left(Z_t\right) = F\left[\frac{X_t}{\zeta\left(1+n\right)\Pi_t}\right] \tag{24}$$

The fiscal function F will play an important role in what follows. We shall assume $F'\left(Z_t\right) > 0$ and denote as $f\left(Z_t\right)$ its elasticity:

$$f = f\left(Z_t\right) = \frac{\partial Log\left[F\left(Z_t\right)\right]}{\partial Log Z_t} \tag{25}$$

4 Ricardian economies and local determinacy

We begin our investigation with the traditional Ricardian version of the model. For that it is enough to take $n = 0$. The dynamic system (18), (24) simplifies as:

$$\Pi_{t+1} = \frac{\Phi\left(\Pi_t\right)}{\xi} \tag{26}$$

$$X_{t+1} = F\left(\frac{X_t}{\zeta\Pi_t}\right) \tag{27}$$

Steady states (Π, X) of this system (when they exist) are characterized by:

$$\Pi = \frac{\Phi(\Pi)}{\xi} \tag{28}$$

$$X = F\left(\frac{X}{\zeta\Pi}\right) \tag{29}$$

Linearizing (26), (27) around a steady state (Π, X) we find:

$$\Pi_{t+1} - \Pi = \phi(\Pi_t - \Pi) \tag{30}$$

$$X_{t+1} - X = f(X_t - X) - \frac{Xf}{\Pi}(\Pi_t - \Pi) \tag{31}$$

or in matrix form:

$$\begin{bmatrix} \Pi_{t+1} - \Pi \\ X_{t+1} - X \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ -Xf/\Pi & f \end{bmatrix} \begin{bmatrix} \Pi_t - \Pi \\ X_t - X \end{bmatrix} \tag{32}$$

with:

$$\phi = \phi(\Pi) = \frac{\Pi\Phi'(\Pi)}{\Phi(\Pi)} \qquad f = f(Z) = \frac{ZF'(Z)}{F(Z)} \tag{33}$$

The two roots are thus ϕ and f. Since there is one predetermined variable, X_t, and one non predetermined, Π_t, there will be local determinacy if one of the roots has modulus smaller than 1, the other bigger.

So we have two possibilities for local determinacy[5]. The first is:

$$\phi > 1 \qquad f < 1 \tag{34}$$

We recognize with $\phi > 1$ the Taylor principle. But we see that with the tax function appears a new possibility for local determinacy, i.e.:

$$\phi < 1 \qquad f > 1 \tag{35}$$

The condition $\phi < 1$ says that the Taylor principle is not satisfied. But $f > 1$ means that the ratio of government's financial liabilities to nominal income will have divergent dynamics along off-equilibrium paths (equation 31). As we indicated above, we want to avoid such policies, whose potentially destabilizing effects in real life situations certainly contributed making the FTPL controversial[6].

[5] This duality was uncovered in Leeper (1991).

[6] Note that the condition $f > 1$ should not be identified with the FTPL, since the FTPL is usually based on a global determinacy argument, whereas we give here a local determinacy condition. But the condition $f > 1$ is anyway a necessary one, and captures in a simple and elegant manner the controversial feature of the FTPL.

5 Global determinacy in the non-Ricardian case

We just saw that in the Ricardian framework there are two alternative conditions for local determinacy, corresponding to the Taylor principle and the FTPL, and expressed respectively as $\phi(\Pi_t) > 1$ and $f(Z_t) > 1$.

What we want to show is that in a non-Ricardian world it is possible to find some interest rate rules $\Phi(\Pi_t)$ such that (a) global determinacy obtains, and (b) no FTPL-type fiscal policy is used:

$$f(Z_t) \leq 1 \tag{36}$$

That such a combination is indeed possible is demonstrated in the following proposition:

Proposition 1: *Let us assume:*

$$\Phi'(\Pi_t) > \xi \qquad \forall \Pi_t \tag{37}$$

$$f(Z_t) \leq 1 \qquad \forall Z_t \tag{38}$$

Then there is a unique globally determinate equilibrium.

Proof: The proof will proceed in several steps. As a first step we shall show that the equilibrium is unique. From (18) and (24) the equations of the curves $X_{t+1} = X_t$ and $\Pi_{t+1} = \Pi_t$ are respectively:

$$X_t = F\left[\frac{X_t}{\zeta(1+n)\Pi_t}\right] \tag{39}$$

$$\frac{\Phi(\Pi_t)}{\xi} - \Pi_t = \nu F\left[\frac{X_t}{\zeta(1+n)\Pi_t}\right] \tag{40}$$

Let us differentiate logarithmically the two equations. We obtain for the locus $X_{t+1} = X_t$ (equation 39):

$$\frac{dX_t}{X_t} = f\left(\frac{dX_t}{X_t} - \frac{d\Pi_t}{\Pi_t}\right) \tag{41}$$

Assumption (38) implies that the curve $X_{t+1} = X_t$ is downward sloping.

Let us now differentiate the equation of the locus $\Pi_{t+1} = \Pi_t$ (equation 40). We find:

$$\frac{\Pi_t\left[\Phi'\left(\Pi_t\right)-\xi\right]}{\Phi\left(\Pi_t\right)-\xi\Pi_t}\frac{d\Pi_t}{\Pi_t}=f\left(\frac{dX_t}{X_t}-\frac{d\Pi_t}{\Pi_t}\right) \tag{42}$$

We may note that:

$$\Phi\left(\Pi_t\right)-\xi\Pi_t=\int_0^{\Pi_t}\left[\Phi'\left(\Pi_s\right)-\xi\right]d\Pi_s \tag{43}$$

so that assumption (37) implies that:

$$\Phi\left(\Pi_t\right)-\xi\Pi_t>0 \tag{44}$$

So using (42) and (44) we see that assumptions (37) and (38) imply that the curve $\Pi_{t+1}=\Pi_t$ is upward sloping.

Since the curve $X_{t+1}=X_t$ is downward sloping and the curve $\Pi_{t+1}=\Pi_t$ is upward sloping, their intersection, if it exists, is unique.

We can now move to the study of global determinacy of this equilibrium. The dynamics of the system is given by:

$$X_{t+1}>X_t \qquad \text{if} \qquad X_t<F\left[\frac{X_t}{\zeta\left(1+n\right)\Pi_t}\right] \tag{45}$$

$$\Pi_{t+1}>\Pi_t \qquad \text{if} \quad \nu F\left[\frac{X_t}{\zeta\left(1+n\right)\Pi_t}\right]<\frac{\Phi\left(\Pi_t\right)}{\xi}-\Pi_t \tag{46}$$

This is represented in figure 1 where it appears that the unique equilibrium has saddle path dynamics and global determinacy. Q.E.D.

Figure 1

There remains now to check that there exist functions $\Phi\left(\Pi_t\right)$ such that condition (37), which replaces the Taylor principle, is satisfied.

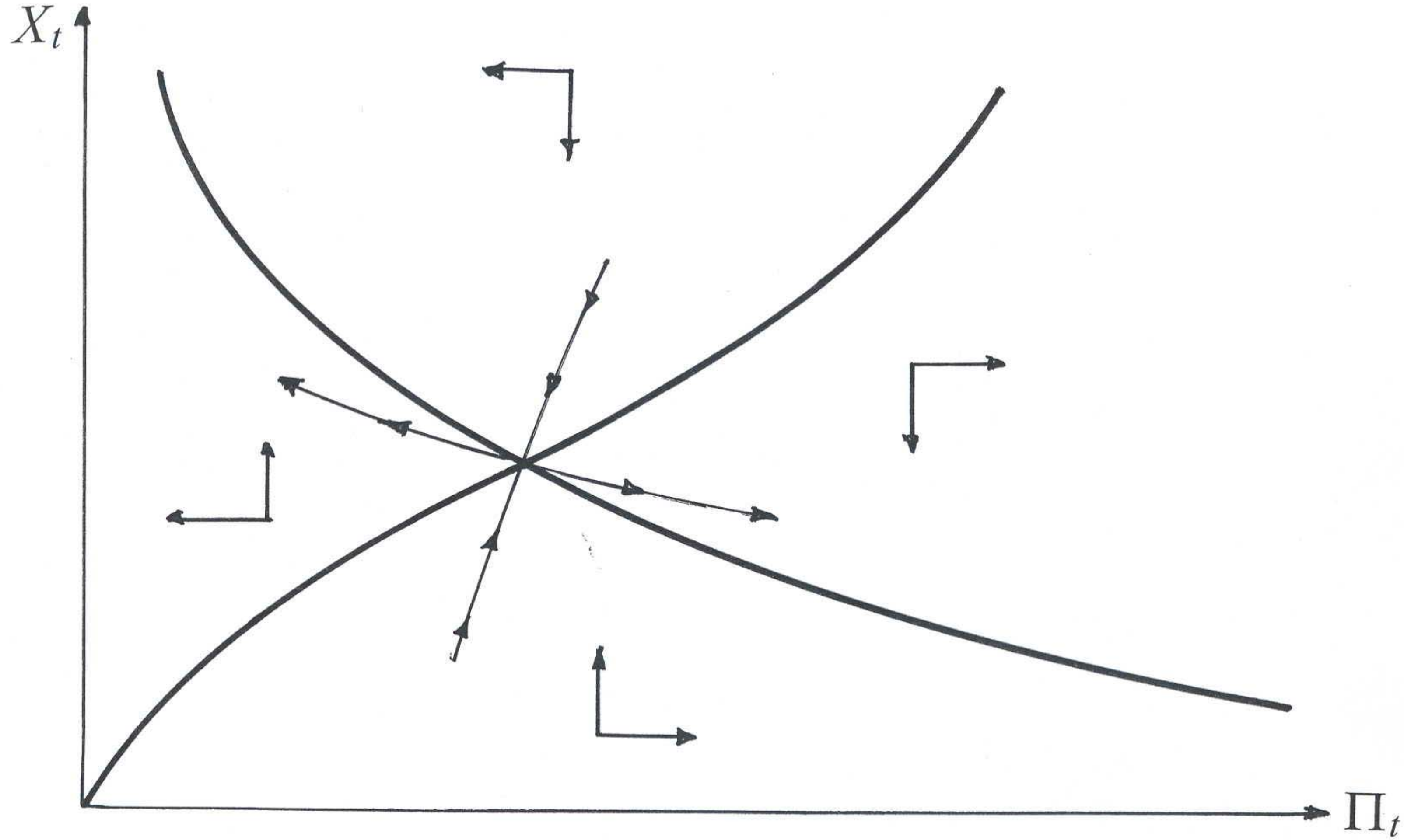
5.1 Linear interest rate rules

We shall now show that there are interest rate rules that satisfy condition (37). Let us consider indeed here simple linear interest rate rules:

$$\Phi\left(\Pi_t\right)=A\Pi_t+B \qquad A>0 \quad B>1 \tag{47}$$

Condition (37) will be satisfied if:

$$A>\xi \tag{48}$$


X_t
Π_t

We can further compute the elasticity of these policy rules:

$$\phi(\Pi_t) = \frac{\partial Log\Phi(\Pi_t)}{\partial Log\Pi_t} = \frac{\partial Log(A\Pi_t + B)}{\partial Log\Pi_t} = \frac{A\Pi_t}{A\Pi_t + B} < 1 \tag{49}$$

We note that this elasticity is always below 1. So, whatever the inflation rate, these interest rate rules do not satisfy the Taylor principle.

5.2 Fiscal policy

To make the example complete, we have to add to this monetary rule an explicit fiscal policy. We can take the following particularly simple policy:

$$F(Z_t) = \gamma Z_t \tag{50}$$

which corresponds to the following evolution of government liabilities:

$$\Omega_{t+1} = \gamma \Omega_t \tag{51}$$

For this policy $f = 1$, so condition (38) is satisfied. So the set of policies characterized by (47), (48) and (50) does satisfy the conditions of proposition 1, and therefore leads to global determinacy

6 Conclusions

A recent lively subject of research has been the search for conditions on monetary and fiscal policies that ensure global determinacy of the economic system. Two classic conditions, the Taylor principle and the fiscal theory of the price level (FTPL) turned out to be somewhat unsatisfactory: The Taylor principle delivers only local, but not global determinacy. The FTPL is implicitly based on fiscal policies that can make government's liabilities explosive in off-equilibrium paths, and this could be dangerous in real life situations.

So our purpose was to find a new interest rate rule that ensures global determinacy even when associated with "reasonable" fiscal policies (in the sense of condition 36). Proposition 1 shows that such a combination is actually feasible, and that condition (37) is an operational alternative to the Taylor principle.

A key to our results is the use of a "non Ricardian" framework where new agents enter the economy every period and Ricardian equivalence does not hold. This assumption seems more realistic than the traditional "Ricardian" framework with no birth. It has already proved quite useful in studying local determinacy (Bénassy, 2005), and appears here to be central for global determinacy as well.

It may finally be noted that the conditions given in proposition 1 can be extended to different frameworks. For example in appendix 2 the assumption of rational expectations is replaced by a very simple learning process, and it is shown that the very same policies assumed in proposition 1 lead to stable dynamics around the same unique equilibrium.

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Appendix 1

In this appendix we shall derive the fundamental dynamic equation (15). Consider the household's budget equation (4), and assume that i_t is strictly positive. The household will thus satisfy the "cash in advance" equation exactly, so that $m_{jt} = P_t c_{jt}$ and the budget constraint is written:

$$\omega_{jt+1} = (1 + i_t)\,\omega_{jt} + P_t y_t - P_t \tau_t - (1 + i_t)\,P_t c_{jt} \tag{52}$$

Let us define the following discount factors:

$$R_t = \frac{1}{(1 + i_0)\ldots(1 + i_{t-1})} \qquad R_0 = 1 \tag{53}$$

Maximizing the utility function (1) subject to the sequence of budget constraints (52) from time t to infinity yields household j's consumption function:

$$R_t P_t c_{jt} = (1 - \beta)\left[R_t \omega_{jt} + \sum_{s=t}^{\infty} R_{s+1} P_s\,(y_s - \tau_s)\right] \tag{54}$$

Summing this across the N_t agents alive in period t, and using the equilibrium condition $C_t = Y_t$ we obtain the equilibrium equation:

$$R_t P_t Y_t = R_t P_t C_t = (1 - \beta)\left[R_t \Omega_t + N_t \sum_{s=t}^{\infty} R_{s+1} P_s\,(y_s - \tau_s)\right] \tag{55}$$

Let us divide both sides by N_t, subtract from it the corresponding equation for $t + 1$, and then divide by R_{t+1}. We obtain:

$$(1 + i_t)\,P_t y_t - P_{t+1} y_{t+1} = (1 - \beta)\left[\frac{(1 + i_t)\,\Omega_t}{N_t} - \frac{\Omega_{t+1}}{N_{t+1}} + P_t\,(y_t - \tau_t)\right] \tag{56}$$

Divide the government's budget equation (9) by N_t and insert it into equation (56). This yields:

$$P_{t+1} y_{t+1} = \beta\,(1 + i_t)\,P_t y_t - (1 - \beta)\left(\frac{1}{N_t} - \frac{1}{N_{t+1}}\right)\Omega_{t+1} \tag{57}$$

Now multiply (57) by N_{t+1}, and assume $N_{t+1}/N_t = 1 + n$. We obtain equation (15).

Appendix 2

Learning

We shall now show that the sufficient conditions of proposition 1 are not limited to the rational expectations framework, but apply as well in a very simple learning process. Let us recall the two dynamic equations in the non-Ricardian system under rational expectations (equations 18 and 24):

$$\Pi_{t+1} = \frac{\Phi\left(\Pi_t\right)}{\xi} - \nu X_{t+1} \tag{58}$$

$$X_{t+1} = F\left[\frac{X_t}{\zeta\left(1+n\right)\Pi_t}\right] \tag{59}$$

Now in a framework with learning Π_{t+1} in the first equation has to be replaced by an inflationary expectation, denoted Π^e_{t+1}. We shall assume a particularly simple "learning scheme" by making the simple "static expectations" assumption:

$$\Pi^e_{t+1} = \Pi_{t-1} \tag{60}$$

so that the first equation becomes:

$$\frac{\Phi\left(\Pi_t\right)}{\xi} = \Pi_{t-1} + \nu X_{t+1} \tag{61}$$

The dynamic system now consists of the two equations (59) and (61). We want to show that the very same conditions that led to determinacy under rational expectations (proposition 1) will lead to a stable system under learning. This is done through the following proposition:

Proposition 2: *Let us assume:*

$$\Phi'\left(\Pi_t\right) > \xi \qquad \forall \Pi_t \tag{62}$$

$$f\left(Z_t\right) \leq 1 \qquad \forall Z_t \tag{63}$$

Then there is a unique dynamically stable equilibrium.

Proof: We shall first show that the equilibrium is unique. As in proposition 1 the equation of the curve $X_{t+1} = X_t$ is:

$$X_t = F\left[\frac{X_t}{\zeta(1+n)\Pi_t}\right] \tag{64}$$

and from (63) it is downward sloping. Now from (61) the equation of the locus $\Pi_t = \Pi_{t-1}$ is:

$$\frac{\Phi(\Pi_{t-1})}{\xi} - \Pi_{t-1} = \nu X_{t+1} = \nu F\left[\frac{X_t}{\zeta(1+n)\Pi_t}\right]$$

$$\nu F\left[\frac{X_t}{\zeta(1+n)\Pi_{t-1}}\right] = \nu F\left[\frac{F\left[X_{t-1}/\zeta(1+n)\Pi_{t-1}\right]}{\zeta(1+n)\Pi_{t-1}}\right] \tag{65}$$

or, forwarding one period, the equation of the locus $\Pi_{t+1} = \Pi_t$ is:

$$\frac{\Phi(\Pi_t)}{\xi} - \Pi_t = \nu F\left[\frac{F\left[X_t/\zeta(1+n)\Pi_t\right]}{\zeta(1+n)\Pi_t}\right] \tag{66}$$

which, combining (62) and (63) as in proposition 1, is upward sloping. Since the locus $X_{t+1} = X_t$ is downward sloping, and the locus $\Pi_{t+1} = \Pi_t$ upward sloping, the intersection of the two loci, when it exists, is unique.

We shall now determine the eigenvalues of the dynamic system. Let us call as before:

$$\chi = \chi(\Pi_t) = \frac{\Phi'(\Pi_t)}{\xi} \qquad \mu = \mu(\Pi_t) = \frac{\Phi(\Pi_t)}{\xi\Pi_t} \tag{67}$$

and linearize the two equations (59) and (61) around the steady state (Π, X):

$$X_{t+1} - X = f(X_t - X) - \frac{fX}{\Pi}(\Pi_t - \Pi) \tag{68}$$

$$\chi(\Pi_t - \Pi) = \Pi_{t-1} - \Pi + \nu(X_{t+1} - X) \tag{69}$$

From (61) we know that in steady state:

$$\frac{\nu X}{\Pi} = \frac{\Phi(\Pi)}{\xi\Pi} - 1 = \mu - 1 \tag{70}$$

So rearranging and combining (68) and (69) we obtain:

$$X_t - X = f(X_{t-1} - X) - \frac{f(\mu-1)}{\nu}(\Pi_{t-1} - \Pi) \tag{71}$$

$$[\chi + f(\mu-1)](\Pi_t - \Pi) = \Pi_{t-1} - \Pi + \nu f(X_t - X)$$

$$= \nu f^2 (X_{t-1} - X) + \left[1 - f^2 (\mu - 1)\right] (\Pi_{t-1} - \Pi) \tag{72}$$

or in matrix form:

$$\begin{bmatrix} \Pi_t - \Pi \\ X_t - X \end{bmatrix} = M \begin{bmatrix} \Pi_{t-1} - \Pi \\ X_{t-1} - X \end{bmatrix} \tag{73}$$

$$M = \begin{bmatrix} [1 - f^2 (\mu - 1)] / [\chi + f (\mu - 1)] & f^2 \nu / [\chi + f (\mu - 1)] \\ -f (\mu - 1) / \nu & f \end{bmatrix} \tag{74}$$

The characteristic polynomial is:

$$\Psi (\lambda) = \lambda^2 - \frac{1 + f\chi}{\chi + f (\mu - 1)} \lambda + \frac{f}{\chi + f (\mu - 1)} \tag{75}$$

We can compute:

$$\Psi (0) = \frac{f}{\chi + f (\mu - 1)} > 0 \tag{76}$$

$$\Psi (1) = \frac{(1 - f) (\chi - 1) + f (\mu - 1)}{\chi + f (\mu - 1)} > 0 \tag{77}$$

So both roots are on the same side of 1. The decisive number is the product of the roots which is $f / [\chi + f (\mu - 1)]$. The system will be locally stable if this product is smaller than one, i.e. if:

$$f < \chi + f (\mu - 1) \tag{78}$$

If $\chi (\Pi_t) > 1$, then $\mu (\Pi_t) > 1$, and condition (78) is satisfied for all fiscal rules such that $f (Z_t) < 1$. Note that the roots can be real or complex depending on the values of the parameters. Q.E.D.