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Antoine Billot. How to shake the Invisible Hand (when Robinson meets Friday). 2007. halshs-00588081

HAL Id: halshs-00588081

<https://shs.hal.science/halshs-00588081>

Preprint submitted on 22 Apr 2011

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PARIS SCHOOL OF ECONOMICS
ÉCOLE D'ÉCONOMIE DE PARIS

WORKING PAPER N° 2007 - 13

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HOW TO SHAKE THE INVISIBLE HAND

(When Robinson Meets Friday)

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May 14, 2007

Abstract

We propose to define the invisible hand by (i) modelling the mechanism itself (not to just assume its existence) and (ii) making explicit the limit conditions for its working. For that purpose, we simply assimilate the working of the invisible hand mechanism to the existence of a social preference such that individual and social optimalities are consistent. In introducing the possibility of interaction among individuals, we then suggest that the standard ‘Robinson case’ or social atomicity is just a degenerate feature of a more general requirement that we call the Global Network Agreement. Our main result is that the invisible hand mechanism does keep on working when there is an interaction between Robinson and Friday if the former (resp. the latter) is sensitive to the latter (resp. the former) in such a way that they exhibit some agreement in preferences. Hence, the ‘Robinson case’ naturally satisfies this property since nor Robinson neither Friday can disagree with himself. But more cooperative situations are also allowed in order to extent the invisible hand mechanism to cases with interactions.

Nous proposons de définir la Main Invisible sous la forme d’un mécanisme formel explicite. Pour cela, nous assimilons ce mécanisme à l’existence d’une relation de préférence sociale telle que les optimalités individuelle et collective soient compatibles. Puis en introduisant une possibilité d’interaction entre agents, nous suggérons que le cas standard (Adam Smith) d’‘atomicité sociale’ constitue le cas limite d’une classe plus générale de situations satisfaisant toutes une certaine contrainte appelée le “Global Network Agreement” (ou Accord Global de Réseau). Notre résultat principal est alors que le mécanisme de la main invisible s’applique même en présence d’interactions dès lors que les agents qui interagissent sont sensibles en préférence les uns aux autres d’une manière telle qu’ils manifestent entre eux un consensus minimal. Aussi, le cas standard (chaque agent est un Robinson) satisfait naturellement cette propriété,

*I thank Maurice Salles, Jacques-François Thisse, James Friedman, Claudio Mezzetti, the participants of the doctoral seminars at Venice International University and two referees for comments and suggestions. Furthermore, I specially thank Tzachi Gilboa whose remarks clearly improve the proof and even the meaning of the second theorem. Usual disclaimer applies. For correspondence: Université Panthéon-Assas, Paris 2, 92 rue d’Assas, 75006, Paris, France, e-mail address: billot@u-paris2.fr

cependant, d'autres cas, beaucoup plus coopératifs sont eux aussi aussi compatibles avec le mécanisme de la main invisible.

JEL Classification: A13-D71-J15

Keywords: Preferences, social interaction, invisible hand

1 Introduction

The idea that vice or selfishness leads to virtue or social good appears in 1723 with *La Fable des Abeilles* or *Les Vices Privés Font le Bien Public* proposed by Bernard Mandeville.¹ According to Mandeville, a hive is analogous to a human society. And what do we observe in a hive? Two things. First, the bees seem to behave in a coordinated and cooperative way while in reality they are just selfish maximizers of their own (private) welfare. Second, such decentralized actions of the individual bees promote the hive welfare better than could be achieved by a 'central planner bee'. How is it possible? In his famous book on *The Wealth of Nations* written in 1776, Adam Smith answered: because of the so-called 'invisible hand'. Two centuries later, the invisible hand metaphor still works: the liberal heirs of the classical economists can affirm that markets would spontaneously produce socially optimal results if only government refrains from interference. Yet, such a use of the Mandeville-Smith's intuition is purely ideological: it is a defence for laissez-faire capitalism. (The interpretation of the concept of the invisible hand is definitely subtle and consequently ambiguous: see Nozick, 1981, or Rothschild, 1994, for further discussions). The existence and the proper working of the invisible hand mechanism are taken for granted, without specifying the limit conditions for it to work. In addition, how should one precisely characterize an invisible hand mechanism? First, the outcome to be explained must be produced as a result of a decentralized process: there are no explicit agreements among the individuals. Second, the invisible hand mechanism must be non-intentional: the individuals do not intend to produce the outcome, that is social welfare; a by-product of their actions is the social desirability of the final outcome. Actually, since the process should work even if the individuals have no knowledge of the process, it is called invisible.²

In this paper, we propose to define the invisible hand mechanism in a more systematic *methodological* way. Specifically, we implicitly consider that it is contingent if the mechanism is actually working. What is interesting is rather (i) to model the mechanism itself, not to just assume its existence and (ii) to make explicit the limit conditions for it to work.

For that purpose, recall that an individual optimal decision is a maximal element of an individual preference order and a social optimal decision is a maximal element of a social preference order. Then, we suggest that an invisible hand mechanism naturally works when there is no conflict between each individual optimal decision and the social optimal one. Consequently, we consider

¹Actually, the first version of this text is a long poem published in 1705 under the title: *The Grumbling Hive* or *Knaves Turned Honest*.

²However, the invisibility is not an essential requirement for a mechanism to be considered as an invisible hand mechanism. The result in question needs not be unknown to the individuals participating in its production. Modern discussion about the invisible hand allows the individuals to know the result of their actions. It is nevertheless required that they do not intend to produce that result. The result should be an unintended consequence of the action, that is a by-product of intentions to do something else.

the working of an *invisible hand mechanism as the existence of a social preference for which individual and social optimalities are consistent*. The formal conditions of existence of such a social preference characterize the environment for the invisible hand mechanism to work.

Traditionally, one thinks that the invisible hand mechanism needs social atomicity, namely the fact that no individual can influence the social decision by means of his only preferences. However, in introducing interactions among individuals (*à la* Akerlof, 1997), we suggest that social atomicity, that is the ‘Robinson case’, is a particular feature of a more general requirement that we call *Global Network Agreement*: the invisible hand mechanism requires that individuals agree in some sense in preferences when they interact. In other words, each Robinson naturally satisfies this property since no one can disagree with himself. But other situations, definitely more cooperative, are allowed as well.

The organization of the paper is as follows: In Section 2, we present the model of interaction and the utility representation of individual preferences translating interactions (Theorem 1). In Section 3, we model the invisible hand mechanism and prove the limit condition of its working, namely the presence of a local network disagreement (Theorem 2). Finally, in Section 4, we illustrate our result with two examples where the invisible hand mechanism works: first, the Robinson case (no interaction) and second, an Interaction case with Global Network Agreement.

2 Individuals

2.1 Individual Preferences

Consider a given and finite set S of *individuals* i ($i : 1, \dots, n$). Call *coalition* any nonempty subset of S .

Consider an arbitrary given set $C \subset \mathbb{R}$ of (monetary) *consequences* expressed by real numbers and X a set of *acts*, that is functions x from the set S of individuals to that C of consequences. Any act x of X is such that $x(i)$ corresponds to the consequence of the act x for individual i .

For any coalition $A \subset S$, $x(A)$ denotes the tuple $[x(i)]_{i \in A}$. Furthermore, given a coalition A and two acts x and y , an act denoted (x_A, y_{-A}) is such that $(x_A, y_{-A})(i) = x(i)$ for all individuals $i \in A$ and $(x_A, y_{-A})(j) = y(j)$ for all other individuals $j \notin A$. By convention, we write (i) $x \geq y$ if $x(i) \geq y(i)$ for all individuals $i \in S$, (ii) $x > y$ if $x \geq y$ and $x(i) > y(i)$ for some individuals $i \in S$ and (iii) $x \gg y$ if $x(i) > y(i)$ for all individuals $i \in S$.

Each individual $i \in S$ is characterized by a *preference relation* \succsim_i , that is a binary relation defined over $X \times X$ such that $x \succsim_i y$ means that for individual i , x is at least as good as y . Two acts x, y are considered indifferent, which is denoted $x \sim_i y$, when we simultaneously have $x \succsim_i y$ and $y \succsim_i x$.

Besides, all preference relations are supposed to be (i) nontrivial: for all $i \in S$, $(x_i, z_{-i}) \succ_i (y_i, z_{-i})$ for some $x, y, z \in X$ and (ii) monotone: for all $i \in S$, if $x(i) > y(i)$, then $(x_i, z_{-i}) \succ_i (y_i, z_{-i})$ for all $z \in X$.

We also assume the following standard axioms.

Axiom 1 (Complete Weak Ordering) : $\forall i \in S$, the preference relation \succsim_i is a complete weak-order on X^2 .

Axiom 2 (Continuity) : $\forall i \in S, \forall x, y, z \in X$ such that $x \succ_i y \succ_i z$, there exists $\alpha \in [0, 1]$ such that $\alpha x + (1 - \alpha) z \sim_i y$.

Axiom 3 (Independence) : $\forall i \in S, \forall x, y, z \in X$ such that $x \succ_i y, \forall \alpha \in]0, 1[$: $\alpha x + (1 - \alpha) z \succ_i \alpha y + (1 - \alpha) z$.

2.2 Interaction and vNM Expected Utility Representation

Several economic situations involve basic interactions sometimes called ‘network externalities’ such as urban congestion, fashion effect, conformism or altruism and so on... The common idea of all these approaches is that individuals are assumed to be interdependent whenever their preferences may be affected by the social neighborhood (see e.g., Postlewaite, 1998, Billot, 2003). Here, if i interacts with j then i may be ‘sufficiently sensitive to j ’ to broaden his own set of preferences determinants in order to encapsulate the consequence for j of all acts i faces.³ For instance, Luttmer (2001) shows that numerous young black lawyers issued from the poor suburb of Chicago finally militate in favor of a fiscal policy that proposes to largely redistribute towards the poor social classes (even if, for that purpose, they will individually pay more taxes) rather than choosing to support a liberal program which maximizes their selfish interests.

2.2.1 Interaction

Consider that there exists a binary relation I defined on S^2 such that ‘ iIj ’ means that ‘ i interacts with j ’ and assume that this relation I is reflexive: $\forall i \in S, iIi$.

Consider now, for any $i \in S$, a probability distribution $p_i : 2^S \rightarrow [0, 1]$ with $\sum_{j \in S} p_i(j) = 1$ where $p_i(j)$ is the *probability for i to be sensitive to j* . Assume that if i has no interaction with j , then $p_i(j) = 0$, i.e. i ’s sensitivity to j is null. On contrary, each time iIj , $p_i(j) > 0$ and i is sensitive to what happens to j . (Intuitively, the more i interacts with j , the more he is sensitive to him.) Note that since we suppose I to be reflexive, then $p_i(i) > 0$. Finally, the support of the distribution p_i naturally corresponds to i ’s *individual network*, that is the set of all individuals with whom he has an interaction. We denote it S_i :

$$S_i = \{j \in S : iIj\} =_{def} \{j \in S : p_i(j) > 0\}. \quad (1)$$

Note that the reflexivity of I can also be written as $[\forall i \in S, i \in S_i]$.⁴

Two cases occur:

(1) (**Ic** as *Interaction case*). There exists at least one individual whose network is nonsingleton:

$$[\exists i, j \in S, i \neq j, \text{ s.t. } iIj] \iff [\exists i \in S \text{ s.t. } p_i(i) < 1] \iff [\exists i \in S \text{ s.t. } S_i \neq \{i\}].$$

For convenience, we say in that case that p_i is a nonsingleton distribution.

³This sort of interaction does not suppose any strategic behavior nor coordination: for instance, an individual i can be sensitive to what happens to another individual j without j being sensitive to i .

⁴In the language of epistemic logics *à la* Kripke (1963) (see for instance Hughes and Cresswell, 1989, or Billot and Walliser, 1999), the individual network S_i is called a *syntactical operator* and the distribution p_i is its valued *semantical* counterpart while the society S plays the role of the set of *possible worlds*.

(2) (**Rc** as *Robinson case*). All individual networks are singleton:

$$[\forall i \in S, \nexists j \in S, i \neq j, \text{ s.t. } iIj] \iff [\forall i \in S, p_i(i) = 1] \iff [\forall i \in S, S_i = \{i\}].$$

For convenience, we say in that case that any p_i is a singleton distribution.

The standard case, namely that which is commonly used in general microeconomics, is obviously given by **Rc**.

Remark 1 : *Transitivity of the interaction I — i.e. $\forall i, j, k \in S, [iIj \text{ and } jIk] \Rightarrow [iIk]$ —, can also be written as $[j \in S_i \Rightarrow S_j \subset S_i]$.*⁵

2.2.2 vNM Expected Utility Representation

Following Morris intuition in (1997, 2000), we then propose to translate the behavior of an individual who is sensitive to other individuals through the probability distribution which affects his utilities in a setting à la von Neumann-Morgenstern (vNM) where $u_i(x(i))$ translates i 's utility when facing the consequence of x for him, namely $x(i)$, while $u_i(x(j))$ corresponds to i 's utility when considering the consequence of x for j , namely $x(j)$.

Theorem 1 (vNM) : *Under **A1-A3**, for each individual $i \in S$, there exists a strictly increasing and continuous utility function $u_i : X \rightarrow \mathbb{R}$ such that:*

$$x \succsim_i y \iff \sum_{j \in S} p_i(j) u_i(x(j)) \geq \sum_{j \in S} p_i(j) u_i(y(j)), \quad (2)$$

where p_i is a given probability distribution, $p_i : 2^S \rightarrow [0, 1]$ with $\sum_{j \in S} p_i(j) = 1$, translating i 's interactions in S . Moreover, u_i is unique up to a linear transformation.

The proof is strictly equivalent to that of Proposition 5.4 in Kreps (1988, Ch. 5., 46-50).

Therefore each individual preference \succsim_i satisfying **A1-A3** has a *vNM representation* as defined in (2). This way, if iIj , i takes into account - by empathy - the consequence $x(j)$ of x weighted by the degree $p_i(j)$ of his sensitivity to j . On the contrary, if i does not interact with j , the influence of the consequence $x(j)$ in i 's evaluation of x is null.

In the Robinson case, (2) naturally becomes:

$$x \succsim_i y \iff u_i(x(i)) \geq u_i(y(i)) \quad (3)$$

since $S_i = \{i\}$ (see also Samet and Schmeidler, 2003, for the interpretation of such a constraint as the qualification rule of liberalism).

Throughout the sequel, we consider that each individual preferences has a vNM representation as defined by (2).

⁵See Billot and Walliser (1999, §2.4, 190-192) for details about the way semantical properties can be equivalently interpreted in syntax.

3 Invisible Hand

Consider a finite subset Π of acts, $\Pi \subset X$, and call it a *problem*. A problem corresponds to a particular decision which must be made by the society S .

For each problem $\Pi \subset X$ and each individual $i \in S$, it can be defined a set of i 's *optimal solutions*, that is the acts dominating in preferences all other acts that are candidates to solve the problem:

$$\Pi_i = \{x \in \Pi : x \succ_i y, \text{ for all } y \in \Pi\}. \quad (4)$$

Proposition 1 : Under **A1**, each individual optimal solution set Π_i associated to any problem Π is nonempty.

Proof: Straightforward by finiteness of Π and weak-ordering completeness of \succ_i . ■

Define now a *decision rule* as a function $\Phi : S \rightarrow \Pi$ and say Φ to be *optimal* when

$$\forall i \in S, \Phi(i) \in \Pi_i. \quad (5)$$

Remark 2 : A decision rule Φ does take into account the individuals' social network since each Π_i is based upon i 's preference and each i 's preference is related to S_i by its vNM representation as defined in (2).

When such an optimal decision rule is followed, $\Phi(i)$ determines the acts which dominate - for i - all other possible solutions of the problem. Such a rule generates an abstract new act $\overline{\varphi}$, not necessarily in Π , such that $\varphi(i) = \Phi_i(i)$, $\forall i \in S$ (where $\Phi_i(i)$ is the i 's consequence of the act $\Phi(i)$). The class of acts generated by optimal decision rules is denoted Π^* and defined as follows:

$$\Pi^* = \{x \in X : x(i) = \Phi_i(i), \forall i \in S, \text{ for some optimal } \Phi\}. \quad (6)$$

Proposition 2 : Under **A1-A3** the class Π^* of issues generated by optimal decision rules is nonempty.

Proof: This follows from Proposition 1 and (6). ■

Each element x of the nonempty class Π^* of acts generated by optimal decision rules is designed such that the individual consequence $x(i)$ is \succ_i -maximal by definition of Π_i and (5-6). Then, if there is no social preference such that each act of Π^* is weakly preferred to any other act of the problem Π , we know that a conflict arises between the society S as a whole and some of its members: hence the invisible hand does not seem to work. A contrario, if there is no contradiction between the society S as a whole and any of its members in the sense where the best decision for the former is consistent with each best decision of the others, the invisible hand seems to work.

Introduce one supplementary condition on the individual preferences.

Given individual preferences $(\succ_i)_{i \in S}$ and a coalition $A \subset S$, say that $x \succ_A y$ if $x \succ_i y$ for all $i \in A$, and $x \succ_A y$ if $x \succ_A y$ and $x \succ_i y$ for some $i \in A$.

Global Network Agreement : $\forall i \in S, x \succ_i y \Rightarrow x \succ_{s_i} y$ for all $x, y \in X$.

Global Network Agreement means that within each given individual network, no conflict arises between the individual and its own network. In return a local (network) disagreement corresponds to the fact that there exists an individual, say i , such that $x \succ_i y \not\Rightarrow x \succ_{S_i} y$ for all $x, y \in X$, i.e. $\exists x', y' \in X$ such that $x' \succ_i y'$ while $y' \succ_{S_i} x'$.

Lemma 1 : *In the Robinson case, there is a Global Network Agreement.*

Straightforward from **(Rc)**.

Note, indeed, that a Global Network Agreement does not imply that $S_i = \{i\}$, $\forall i \in S$. Thus, let us study other situations, especially situations where individual networks are nonsingleton.

Lemma 2 : *There is a Global Network Agreement iff the interaction I is transitive.*

Proof: (\Rightarrow) Transitivity of I can also be written as $[j \in S_i \Rightarrow S_j \subset S_i]$ (see Remark 1). It means that, $\forall i, j, k \in S$, if $i \in S_j$ and $j \in S_k$ then $i \in S_k$. Suppose now, on the contrary, that there exist three individuals $1, 2, 3 \in S$ such that $1 \in S_2$, $2 \in S_3$ but $1 \notin S_3$. Let $x(1) = 1$ and $x(i) = 0$ for all $i \neq 1$. By monotonicity of the preferences, $x \succ_1 0_n$ (since $1 \in S_1$ by reflexivity of I), $x \succ_2 0_n$ (since $1 \in S_2$), $x \sim_3 0_n$ (since $1 \notin S_3$) and finally $x \succ_i 0_n$ for all $i \in S$, where $0_n = (0, \dots, 0)$. Thus, $x \succ_{S_3} 0_n$ since $2 \in S_3$. But, $x \sim_3 0_n$ while $x \succ_{S_3} 0_n$ means that there is a local disagreement.

(\Leftarrow) Suppose that there is a local disagreement. It means that, for some $i \in S$, $x \succ_{S_i} 0_n$ while $x \sim_i 0_n$. Hence, there exists $j \in S_i$ such that $x \succ_j 0_n$. Let $x(k) = 1$ and $x(i) = 0$ for all $i \neq k$. If $x \succ_j 0_n$, this means that $k \in S_j$, $k \neq j$, i.e. $\{j, k\} \subset S_j$. Since $x \sim_i 0_n$, then $k \notin S_i$. Consequently $j \in S_i \not\Rightarrow S_j \subset S_i$ and I is not transitive. ■

Define a *social preference* \succ^* as a preference relation which satisfies **(A1-A3)** and a Strong Monotonicity condition:

$$\forall i \in S, \forall x, y \in X, (x_i, z_{-i}) \succ^* (y_i, z_{-i}) \Leftrightarrow x(i) \geq y(i). \quad (\text{SM})$$

This condition translates the fact that, ceteris paribus, the society is better off when allocating more than less to any given individual.

The modeling of an invisible hand mechanism that we propose is then that acts in Π^* (which may or may not be included in the problem Π) must be at least as good for the social preference \succ^* as acts in Π , that is $x \succ^* y$ for all acts $x \in \Pi^*$ and $y \in \Pi$.

Invisible Hand (IH) : There exists a social preference \succ^* such that for each finite problem $\Pi \subset X$ there exists an act $x \in \Pi^*$ such that $x \succ^* y$, for all $y \in \Pi$.

The Global Network Agreement condition implies that the sensitivity of an individual to another one, when it exists (i.e. in the Interaction case), can be translated through its individual network. Nevertheless, this result does not clarify the implication relation between **IH** and a Global Network Agreement.

Does **IH** implies Global Agreement or the other way round? If it is straightforward that **IH** is consistent with the Robinson case, that is when all the individual networks are singleton, we must know if it can be consistent as well with nonsingleton individual networks.

Theorem 2 : *If **IH** holds, then there is a Global Network Agreement.*

Lemma 1 shows that there is a Global Network Agreement iff the interaction I is transitive. Hence we have now to precise the relationship between **IH** and the properties of the interaction I :

Lemma 3 : *If **IH** holds, then the interaction I is transitive.*

Proof: Suppose that I does not satisfy transitivity, i.e. $[j \in S_i \not\Rightarrow S_j \subset S_i]$ for some $i, j, k \in S$. This means for instance that there exist three individuals $1, 2, 3 \in S$ such that $2 \in S_1$, $1 \in S_3$ but $2 \notin S_3$. Now, by **A2**, it is possible to define a problem $\Pi = \{x, 0\}$ where the act x is defined as $(\varepsilon_1, -1, -\varepsilon_3, 0, \dots, 0)$ (with $x(1) = \varepsilon_1$, $x(2) = -1$, $x(3) = -\varepsilon_3$ and $x(i) = 0$ for all $i \neq 1, 2, 3$ with $\varepsilon_1, \varepsilon_3$ positive and very small) such that the act $0_n \succ_1 x$ (since $\{1, 2\} \subset S_1$ and ε_1 very small), $0_n \succ_2 x$ (since $2 \in S_2$ while $1 \notin S_2$ and $3 \notin S_2$) but $x \succ_3 0_n$.⁶ Hence, if $y \in \Pi^*$, this implies $y(3) = -\varepsilon_3$ and $y(i) = 0$ for all $i \neq 3$, i.e. $\Pi^* = \{0, 0, y(3), 0, \dots, 0\}$. Thus, $0_n \succ^* y$ for all $y \in \Pi^*$. This is a contradiction with **IH**. ■

Proof of Theorem 2: It is straightforward from Lemmas 1 and 2.

If a conflict arises between the society as a whole and some of its members, the invisible hand does not work. This is clearly consistent with a society based on individuals who behave like Robinson. However, Theorem 2 shows that it is possible for the invisible hand mechanism to be relevant whenever ‘Robinson meets Friday who meets...’ if the interaction between the individuals is transitive such that there is no local disagreement. In that sense, if individuals through networks are sufficiently connected in preferences, the standard invisible hand mechanism can be extended to subsets of individuals.

4 Examples

As an illustration of situations where **IH** holds, consider the two following examples:

Example 1 (Robinson Case) : *Suppose three individuals $\{1, 2, 3\}$ whose individual distributions are singleton, i.e. $S_1 = \{1\}$, $S_2 = \{2\}$ and $S_3 = \{3\}$, and utilities are given by $u_i(x) = x$ for $i : 1, 2, 3$. Then, for two programs x, y :*

$$\begin{cases} x \succ_1 y & \Leftrightarrow & x(1) \geq y(1), \\ x \succ_2 y & \Leftrightarrow & x(2) \geq y(2), \\ x \succ_3 y & \Leftrightarrow & x(3) \geq y(3). \end{cases}$$

Consider now the following problem: $\Pi = \{x, 0\}$, where the act $x = (\varepsilon, -1, -\varepsilon^2)$, i.e. $x(1) = \varepsilon$, $x(2) = -1$ and $x(3) = -\varepsilon^2$. Now, it is obvious that the act

⁶Take for instance, $p_3(1) = p_3(3) = .5$ with $\varepsilon_1 > \varepsilon_3 > 0$.

$0_{n=3} = (0, 0, 0)$ is optimal for individuals 2 and 3 and the act x is optimal for 1. Thus, $\Pi^* = \{(x(1), 0, 0)\}$, that is $(\varepsilon, 0, 0)$. Now, by **SM**, the act $(\varepsilon, 0, 0) \succ^* (0, 0, 0)$ for each social preference and then **IH** holds.

Example 2 (Interaction Case with Global Agreement) : Suppose three individuals $\{1, 2, 3\}$ such that 1 and 3 have nonsingleton individual distributions, i.e.

$$\begin{cases} p_1(1) = \frac{1}{2} = p_1(2), \\ p_3(2) = \frac{1}{2} = p_3(3) \end{cases}$$

while 2's individual distribution is $p_2(2) = 1$. This corresponds to $S_1 = \{1, 2\}$, $S_2 = \{2\}$ and $S_3 = \{2, 3\}$. Suppose that all utilities are given by $u_i(x) = x$ for $i : 1, 2, 3$. Then, for two acts x, y , we have:

$$\begin{cases} x \succ_1 y & \Leftrightarrow \frac{1}{2}x(1) + \frac{1}{2}x(2) \geq \frac{1}{2}y(1) + \frac{1}{2}y(2), \\ x \succ_2 y & \Leftrightarrow x(2) \geq y(2), \\ x \succ_3 y & \Leftrightarrow \frac{1}{2}x(2) + \frac{1}{2}x(3) \geq \frac{1}{2}y(2) + \frac{1}{2}y(3). \end{cases}$$

Note that there is here a Global Network Agreement and define the social preference \succ^* as:

$$x \succ^* y \Leftrightarrow \frac{1}{6}x(1) + \frac{2}{3}x(2) + \frac{1}{6}x(3) \geq \frac{1}{6}y(1) + \frac{2}{3}y(2) + \frac{1}{6}y(3).$$

In order to illustrate that this social preference satisfies **IH**, consider any problem Π and optimal decision rule Φ . Then for any $y \in \Pi$, the optimality of Φ yields:

$$\begin{cases} \frac{1}{2}\Phi_1(1) + \frac{1}{2}\Phi_2(1) \geq \frac{1}{2}y(1) + \frac{1}{2}y(2), \\ \Phi_2(2) \geq y(2), \\ \Phi_2(2) \geq \Phi_2(1), \\ \Phi_2(2) \geq \Phi_2(3), \\ \frac{1}{2}\Phi_2(3) + \frac{1}{2}\Phi_3(3) \geq \frac{1}{2}y(2) + \frac{1}{2}y(3). \end{cases}$$

Multiplying the two first and the very last inequalities by $\frac{1}{3}$, the two others by $\frac{1}{6}$ and summing up gives rise to the following equation:

$$\frac{1}{6}\Phi_1(1) + \frac{2}{3}\Phi_2(2) + \frac{1}{6}\Phi_3(3) \geq \frac{1}{6}y(1) + \frac{2}{3}y(2) + \frac{1}{6}y(3).$$

Hence, we can conclude that $x \succ^* y$ for all $x \in \Pi^*$. Since it works for every $y \in \Pi$, then **IH** holds.

5 References

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