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| Arnold Chassagnon. Regular moral hazard economies. 2007. halshs-00588317

**HAL Id: halshs-00588317**

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Preprint submitted on 22 Apr 2011

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**PARIS SCHOOL OF ECONOMICS**  
ÉCOLE D'ÉCONOMIE DE PARIS

**WORKING PAPER N° 2007 - 03**

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**JEL Codes: D41, D82, D86**

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# Regular moral hazard economies\*

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February 2007

## Abstract

That paper formalizes the idea that when the magnitude of the moral hazard phenomenon is not important, the distortions like equilibria multiplicity or equilibrium discontinuity relative to the economic fundamentals disappear. We study a two states of nature insurance model, with a risk neutral principal, a risk averse agent and separable costs. Typically, in such economies, non convexities imply that the set of Pareto optimal allocations is not connected. Surprisingly, we prove that it is never the case under weak and realistic assumptions. That result is in particular valid under simple regularity assumptions on the cost function when the productivity of effort is always positive. We show that such regularity of the moral hazard economy is compatible with the remaining strong non convexities.

**Keywords:** Moral hazard, first order approach.

**JEL Classification:** D41, D82, D86

## 1 Introduction

Moral hazard economies usually display discontinuities. For instance, in a competitive insurance market devoted to provide insurance to only one unique type of agent, the equilibrium price may be a discontinuous function of the wealth of the agent. It has been recognized that non-convexities play a key role in such a phenomenon. Non-convexities come from the decentralized choice of the effort, a variable which benefits are ambiguous. Indeed, in the one hand, the benefits for the agent of increasing the effort are questionable, the utility gains being counterbalanced by the disutility costs and, while, in the other hand, the benefits reduction for the principal to increase the remuneration in the good state of the world are partially compensated by

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\*I'm grateful for comments from P. -Y. Geoffard in the genesis of this work and I also want to thank C. Bobtcheff, C. Collard, J. Fraysse and S. Piccolo for their efficient help. Remaining errors are mine.

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its consequence in terms of the chosen effort. Then, when the choice of effort is endogenously made by the agent, none of the preferences of the agents and of the profit of the principal are quasi-concave or quasi-convex functions. Three consequences: first the subset of maxima in the set of contracts in which indifference curves are tangent to an isoprofit curve is not necessarily connected. Say differently, the path of efficient risk sharing allocation are not continuous. Then, the equilibrium price could be a discontinuous function of the wealth of the agents, and/or there may be multiple equilibria.

That paper formalizes the idea that when the magnitude of the moral hazard phenomenon is not so important, the distortions mentioned above disappear. Our understanding of the moral hazard mechanism is originated in the first result that the non-convexities play a negative role only for high values of both incentives and effort (when they are possible). We then develop the idea that when the prevention technology is very costly for high levels of care, the set of contracts in which indifference curves are tangent to an isoprofit curve (let call it the contract curve) is regular. In such a context, the non-convexity remain, but they do not play a role. Our framework is the standard insurance model of Mossin (agents face a bernouilly risk distribution with two states of nature) and it entails separability of the VNM utility function and of the cost function. We show that the contract curve is increasing when the *cost function tends to infinity for larger values of effort, in a regular way*. The regularity assumption make more specific a necessary condition on the cost function second derivative, for obtaining infinite costs. It is then not very demanding.

Our analysis focus on the qualitative properties of the contract curve, which are a good indicator of the importance of distortions of the moral hazard model. By definition, each point of that set corresponds either to a maximum or to a minimum of the utility function under the profit function. However, when that curve is an increasing curve, it is immediate that any point corresponds to a maximum and that none of them is a local minimum. In the corresponding economies, there is neither multiplicity of equilibrium any more nor discontinuity of the equilibrium price.

Our paper is making a step forward relative to the state of the analysis of the moral hazard problem, particularly to the important literature concerned with the “first-order approach”. We can depict the difference and the affinities with that literature in a simple way. The “first-order approach” literature has been concerned about the monotonicity of the contingent contracts the agents obtain in standard agency problems. That requirement was not only conform to the intuition of the way the relationship between principals and agents should be designed, but also, it was simplifying the analysis. Our goal however, one step forward, is to understand how the relationship between the principal and the agents vary with the fundamentals of the economy. It has been recognized for long that moral hazard models have not a great power of prediction, even in its simple formulations. The result that we show by proposing an intuitive condition under which the remuneration increase in all the states of nature is then really innovative<sup>1</sup>. Moreover, it is in the flavor

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<sup>1</sup> « J'ai pénétré la forêt et je m'y suis perdu tant il faisait sombre, j'ai voulu l'éclairer mais ma petite flamme ne suffisait pas...alors la

of the “first-order approach” literature, as our assumptions only concern the prevention technology (through the cost of effort function). Indeed, our result do not depend on any specific assumption on the agents risk aversion.

Our assumptions encompasses cases like  $c(a) = 1/(\bar{a} - a)^\gamma$  with  $\gamma > 0$ . The cost should be strongly increasing, as we suppose that it is asymptotic in the neighborhood of  $\bar{a}$ . If we relax the assumption and consider convex marginal cost plus the condition  $c''' / c'' \geq 2$ , then the model is still regular while the profile of the contract curve could be more complex. Notice that the condition  $c''' / c'' \geq 2$  is compatible with cost still finite at the limit  $\bar{a}$ . For instance  $c(a) = a^4$  renders the model smooth.

Our result is a nice small piece of theory with huge potential applications. Let propose one of them. In a companion paper written with Estelle Malavolti, we investigate the effects of increasing the labor standards, in a context with some heterogeneity between employee, in which moral hazard constrain the relationship between employers and one of the employees, . We modeled the improvement of the labor standards as an alleviation of the cost function. Then, the analysis is in two steps. It should be first understood, in any case, the effect of increasing the labor standard is to improve the quality of the prevention technology. Then, for the employee which productivity is dependant on monetary incentives, that could make the story more complex, inducing discontinuity. [Improving the technology has an effect of the cost function, the employee being a priori induced to making more effort, and also an income effect. Both effect could lead to a discontinuity of the characteristics of the contract between that employee and the principal.] Second, if the prevention technology justify our regularity assumptions, then, we can show that increasing labor standards leads to increase the differences between the two types of employee. The coverage of the one which productivity is not linked to labor standard being increasing, while the one of the other being diminished.

The paper is organized as follows. Section 2 introduces the model Section 3 develops a representation of efficiency in the model through the set of contracts under which the derivative of the utility function and of the profit function are colinear. An example will be developped, showing how that set is a good indicator of the distortions of the moral hazard model. Section 4 develops the main result of the paper, concerning the unicity of the efficient allocation for a given transfer, and the continuity of the equilibrium path. Section 5 concludes the paper. Proofs given in the two personal states of nature framework are relegated to the appendix.

## 2 Model with additive desutility of effort and two states of nature

The endowment of each agent is a contingent good, the distribution support being included in  $\{W, W - L\}$ . The endowment distribution depends on the agent’s effort. That distribution satisfies the two conditions of

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lisière m’est apparue en pleine lumière.... et j’ai courru rejoindre la route...ma route...et lui la sienne...les deux légèrement infléchies. »  
P. Mérimée

Jewitt (1988) and Rogerson (1985), the monotonicity of the likelihood ratio (MLRC) and the convexity of the distribution function (CDFC). Without loss of generality, we can write that model in terms of the probability  $a \in [\underline{a}, \bar{a}]$  of the good state of nature —positively correlated to the effort— and also of the *additive* “cost” of obtaining that probability,  $c(a)$ . In that framework, the MLRC is equivalent to the condition that  $c(\cdot)$  is increasing and the CDFC, to the condition that  $c(\cdot)$  is convex. Throughout all the paper, we will suppose that condition satisfied. Moreover, we will concentrate on the case  $c''' \geq 0$ . For normalization, we suppose  $c'(\underline{a}) = 0$ .

**Assumption 1** (Main assumption on the cost function). *Functions  $a \mapsto c'(a)$  is increasing and convex on  $[\underline{a}, \bar{a}]$  and  $c'(\underline{a}) = 0$ .*

A standard insurance contract  $x = (x_1, x_2)$  is an exchange between the agent and the insurer of the support  $\{W, W - L\}$  of the initial distribution with the support  $\{x_1, x_2\}$ <sup>2</sup>. The utility of such contract for the risk-averse agent would depend on the level of its effort  $a$

$$U(x, a) = a u(x_1) + (1 - a) u(x_2) - c(a)$$

while the profit of the risk neutral insurer would depend on  $a$  and also on the parameters  $W$  and  $L$ :

$$\Pi(x, a) = a (W - x_1) + (1 - a) (W - L - x_2)$$

In a moral hazard context,  $a$  is chosen by the agent ; it depends on the value of the monetary incentives  $x_1$  and  $x_2$  ; due to the convexity of the cost function assumption, the optimal level is unique and it satisfies<sup>3</sup>:

$$u(x_1) - u(x_2) = c'(a) \quad \text{when } a \in (\underline{a}, \bar{a}).$$

The validity set of that condition,  $\mathcal{D} = \{x/0 \leq u(x_1) - u(x_2) \leq c'(\underline{a})\} \subset \{x/0 \leq x_2 \leq x_1\}$ , is called the domain. [ $\mathcal{D}$  depends only on  $c(\cdot)$ , and not on the dotation parameters  $W$  and  $L$ .] Effort  $a(\cdot)$  is thereafter endogeneized in the agent and principal objectives. Effort is differentiable, the derivatives being, in  $\mathcal{D}$ ,  $\frac{\partial a}{\partial x_1} = \frac{u'(x_1)}{c''(a)} > 0$  and  $\frac{\partial a}{\partial x_2} = \frac{-u'(x_2)}{c''(a)} < 0$ . By the envelop theorem, the gradient of the utility  $U(x_1, x_2)$  is

$$\nabla U = (a u'(x_1), (1 - a) u'(x_2))$$

which proves that indifference curves are decreasing in the space  $x_1, x_2$ . In the other hand, simple calculus shows that the gradient of the insurer payoff  $\Pi(x_1, x_2)$ ,

$$\nabla \Pi = \left( -a + \frac{\Pi_a}{c''} u'_1, -(1 - a) - \frac{\Pi_a}{c''(a)} u'_2 \right),$$

<sup>2</sup>Those notations are standard in the insurance literature but different from the ones of the First Order Approach literature.

<sup>3</sup>The uniqueness of the level of effort comes from conditions MLRC and CDFC, condition on which all the first order approach literature has been built. The fact that the optimal effort satisfies a first order condition will play a key role in our analysis.

with  $\Pi_a = L - (x_1 - x_2)$ , can be decomposed in two effects: a standard cost effect,  $(-a, -(1-a))$ , negative and proportional to the probabilities and an incentive one,  $(\frac{\Pi_a}{c''} u'_1, -\frac{\Pi_a}{c''} u'_2)$ , of ambiguous sign, implying that the isoprofit curve could be locally increasing.

### 3 Efficient allocation of effort and risk

The way efficiency is achieved in our model is the core of our study. We adopt a strategy of analysis similar to the one of the *first order approach theory* by focusing on the contracts satisfying a FOC condition, a condition which is usually not sufficient for efficiency. In that section, we will define the set of the points in which indifference curves and isoprofit curves are tangent and its properties will be linked with the way efficiency is achieved in the model. .

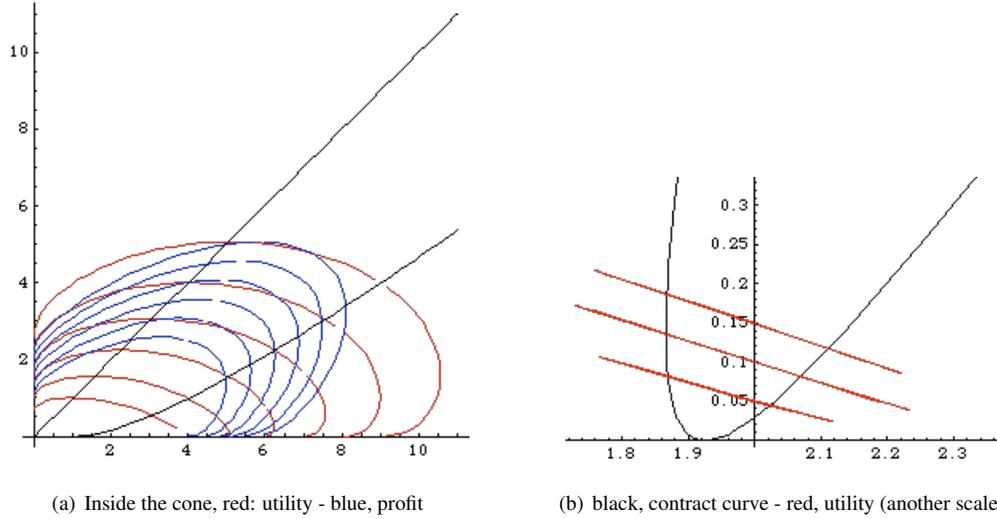
#### 3.1 Colinearity of the derivative of the utility and of the profit functions

We first consider necessary and sufficient conditions on the contracts for the colinearity of  $\nabla U$  and of  $\nabla \Pi$ . They are stated in lemma 1.

**Lemma 1.** *The derivatives of the utility function and of the profit function computed in a contract  $x$  are colinear if and only if conditions 1 and 2 below are satisfied. In that case conditions 3 and 4 are also true.*

1. *First order approach is valid:  $x \in \mathcal{D}$  ;*
2. *First order-equation is:  $\frac{1}{u'(x_1)} - \frac{1}{u'(x_2)} = \frac{\Pi_a}{a(1-a)c''(a)}$   $\left( \frac{1}{u'_i} = \frac{1}{\lambda} + \frac{\Pi_a}{c''(a)} \frac{\pi'_i}{\pi_i} \right)$  ;*
3. *Incentive effects are always dominated by “money” effects in  $x$  neighborhood, :  $\frac{\partial \Pi_i}{\partial x_i} \leq 0, i = 1, 2$  ;*
4. *Coverage is positive:  $\Pi_a \geq 0 \implies W - \pi - L \leq x_2 \leq x_1 \leq W - \pi$  (with  $\pi = \Pi(x_1, x_2)$ ).*

Conditions 1 to 4 of lemma 1 are fairly standard. Condition 1 states that points in which the derivative of the utility function and of the profit function should always belong to the domain, the subspace of  $\mathbb{R}_+^2$  in which monetary incentives matter. From now on, we will call such points *interior optima*. Condition 2 is the FOC condition. Condition 3 means that in the neighborhood of any interior optimum, increasing the remuneration in any state of nature is always costly. It follows that the isoprofit curve is locally decreasing in the neighborhood of an interior optimum. Condition 4 recall that any interior optimum is less risky than the contract  $(W - \pi, W - L - \pi)$  which exposure to risk is similar as the one of the initial endowment. Instead of full insurance, Moral Hazard constraints induces partial insurance.



Subfigure (a): consider only red and blue curves *only inside* the cone, which delimits the domain. Subfigure (b): black curves represents the contract curve, and red curves, some indifference curves. Subfigure (b) is entirely inside the domain.

Figure 1: Indifference curves, isoprofit curves and the FOC path

### 3.2 The set of contract curves

In a convex economy, Pareto optimality implies that utility and profit derivative functions are colinear. This is not necessarily like that in the insurance economy with moral hazard. Figure 1 depicts an example in which indifference and isoprofit curves are mainly concave and their tangency points do not correspond to Pareto optimal contracts<sup>4</sup>.

Interestingly, such an example is associated with a non increasing profile of the curve containing all the tangency points between indifference and isoprofit curves (drawn in subfigure (b)). In what follows, we study closely the properties of such FOC path, that we call the *contract curve*. In order to obtain general statements, we analyze all the contract curves when the dotation parameters  $W$  and  $W - L$  vary.

When  $W$  and  $W - L$  vary, the domain and the indifference curves remain similar, while isoprofit curves vary. Contract curve should then also vary. Lemma 2 study the set of all those curves. It shows that in any point of the domain there exists one and only one such curve passing through that point.

**Lemma 2.** *For any parameters  $W$  and  $L$ , consider the set of points such that  $\nabla U$  and  $\nabla \Pi$  are colinear:*

1. *it is invariant with the initial wealth  $W$  and it will be denoted  $\Phi_L$ ;*
2. *for different parameters  $L \neq L'$ , the intersection set  $\Phi_L \cap \Phi_{L'}$  is empty;*
3. *the family of curves  $(\Phi_L)_{L \geq 0}$  fill all the interior of the domain :  $\dot{\mathcal{D}} = \bigcup_{L \geq 0} \Phi_L$ .*

<sup>4</sup>The example is built with  $u(x) = \sqrt{x}$ ,  $c(a) = a^2$ , and  $L = 1.5$ . Curves are drawn in a  $x_1, x_2$  space.

The result comes easily from the fact that if  $\nabla U$  and  $\nabla \Pi$  are colinear in a particular contract  $x = (x_1, x_2) \in \dot{\mathcal{D}}$ , then, the first order condition of lemma 1 could be rewritten

$$L = (x_1 - x_2) + a(1 - a)c''(a) \left[ \frac{1}{u'(x_1)} - \frac{1}{u'(x_2)} \right] \quad (1)$$

with  $a$ , the optimal effort of the agent characterized by

$$a = (c')^{-1}(u(x_1) - u(x_2)) \quad (2)$$

making  $L$  a natural function of  $x_1, x_2$  and of  $\Phi_L$ , a set where  $L(\cdot)$  is constant. For the sake of simplicity, in what follows, we often consider that curve  $\Phi_L$  is the contract curve corresponding to the endowment parameters  $(L, 0)$ .

### 3.3 Efficient contract curves

The properties of  $\Phi_L$  are good indicators of the importance of distortions of the moral hazard model. In particular, the existence of singularity in the moral hazard model is linked with the relationship between  $\Phi_L$  and the set of Pareto optimal allocations. Singularity does not exist when the one-dimensional variety  $\Phi_L$  is included in the set of Pareto optimal allocations<sup>5</sup> and the reciprocal is true.

Whether the contract curve is included or not in the set of Pareto optimal allocations could be analyzed in terms of the geometric properties of  $\Phi_L$ . A particular case should be mentioned, when the one-dimensional variety  $\Phi_L$  is increasing: it is like in a standard Edgeworth box, the contract curve is included in the set of Pareto optimal allocations<sup>6</sup>. With more complex profiles, it is then quite natural to ask whether there are criteria for the inclusion of the contract curve inside the set of Pareto optimal allocation.

Cases in which contract curves are not increasing are also of interest. The economic literature does not say much on such cases, believing either that no conclusion could be drawn on efficiency or, worse, that non increasing profiles of the contract curves should immediately be the signal of inefficiency of some of its points. Our results investigate that kind of model. Figure 3-(b) suggest a situation in which a contract curve is not increasing (the fourth one) but all his elements are Pareto optimal allocations<sup>7</sup>. The picture depends in fact strongly on the variation of the utility along a contract curve.

We will analyze the situation for the whole class of model characterized by a VNM  $u(\cdot)$  and a cost function  $c(\cdot)$  and find sufficient conditions under which the whole collections of curves  $(\Phi_L)_L$  corresponds for any  $L$  to sets of Pareto optimal allocations.

<sup>5</sup>When  $\Phi_L$  is included in the set of Pareto optimal allocations, the path of efficient allocation is continuous. Then, neither multiple equilibria (in a pure competitive setting) nor other types of singularity could happen.

<sup>6</sup>The argument to prove that in such case, any element of this set is a Pareto optimal is standard.

<sup>7</sup>Remark that the figure 2 has the same s-shaped form as the graphical example from Mirrlees reported by Rogerson [1985] but is has not the same interpretation. In that figure, the optimal effort is endogenous. The two dimensions of the graph represent the contingent payments.

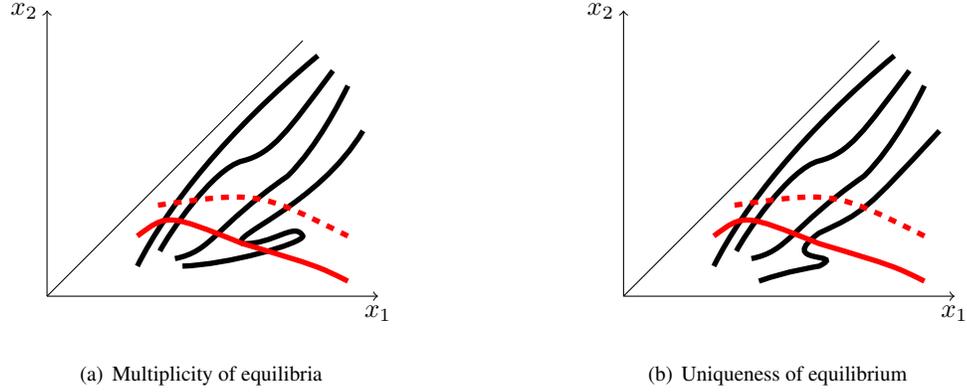


Figure 2: First order condition paths  $\Phi_L$  and intersections with iso-profit curves

**Definition 1.** *the whole class of model characterized by a VNM  $u(\cdot)$  and a cost function  $c(\cdot)$  is smooth if and only if, for any value of the parameter  $L$ , the contract curve  $\Phi_L$  is formed of Pareto optimal allocations of the model with the initial endowment  $(L, 0)$ .*

Following lemma proposes a full characterization of smooth economies.

**Lemma 3.** *An economy characterized by a VNM  $u(\cdot)$  and a cost function  $c(\cdot)$  is smooth if and only if, the deduced function  $L(\cdot)$  is increasing along any indifference curve.*

### 3.4 Robust properties of the contract curve

As a function of the two variables  $x_1$  and  $x_2$ , we can compute the partial derivatives of  $L$ . To simplify the calculs, denote by  $\varphi(a)$  the expression  $a(1-a)c''(a) \geq 0$  and its derivative  $\varphi' = (1-2a)c'' + a(1-a)c'''$ ,  $a$  being the optimal effort. Also, for notational convenience, we drop when there is no possible confusion both parenthesis and the names of intermediary variables: for instance  $u'_i$  denotes  $u'(x_i)$  and  $c''$ ,  $c''(a)$ . By composition, from equation 1, it comes:

$$\frac{\partial L}{\partial x_1} = 1 + \varphi' \frac{u'_1}{c''} \left( \frac{1}{u'_1} - \frac{1}{u'_2} \right) - \varphi \frac{u''_1}{u'^2_1} \quad (3)$$

$$\frac{\partial L}{\partial x_2} = -1 - \varphi' \frac{u'_2}{c''} \left( \frac{1}{u'_1} - \frac{1}{u'_2} \right) + \varphi \frac{u''_2}{u'^2_2} \quad (4)$$

A first consequence of the assumption  $c''' \geq 0$  is that the first derivative  $\frac{\partial L}{\partial x_1}$  is always positive, like the (lower) expression  $\frac{\partial L}{\partial x_1} + \varphi \frac{u''_1}{u'^2_1}$ ; indeed, the expression

$$\frac{\partial L}{\partial x_1} + \varphi \frac{u''_1}{u'^2_1} = 1 + (1-2a) \left( 1 - \frac{1}{u'_2} \right) + \frac{c'''}{c''} u'_1 \left( \frac{1}{u'_1} - \frac{1}{u'_2} \right) \quad (5)$$

is clearly positive in the case  $a \leq 1/2$ , as all of its terms are positive. In the case  $a \geq 1/2$ , equation 6 gives

also the result:

$$\frac{\partial L}{\partial x_1} + \varphi \frac{u''_1}{u_1^2} = 2(1-a) + (2a-1) \left( \frac{1}{u'_2} \right) + \frac{c'''}{c''} u'_1 \left( \frac{1}{u'_1} - \frac{1}{u'_2} \right) \quad (6)$$

Consider now the set of contract curves and the set of iso-effort curve. They satisfy the single crossing condition because a contract curve cannot be tangent to an iso-effort curve. Indeed, equation 7 shows that the difference  $\delta = \frac{\partial a}{\partial x_1} \frac{\partial L}{\partial x_2} - \frac{\partial a}{\partial x_2} \frac{\partial L}{\partial x_1}$  is positive, that is, the derivative vector cannot be colinear.

$$\delta c'' = (u'_2 - u'_1) + \varphi' (u'_1 u'_2 - u'_1 u'_2) \left( \frac{1}{u'_1} - \frac{1}{u'_2} \right) - \varphi \left( \frac{u''_1}{u_1^2} + \frac{u''_2}{u_2^2} \right) > 0 \quad (7)$$

A corollary is that given any  $L$ , the  $\Phi_L$  curve can be parametered by the effort.

## 4 Smooth moral hazard model

In that section, we study the profile of contract curve given two sets of natural assumptions and then, we illustrate our results by an example.

### 4.1 Asymptotic cost function case

We will restrict attention to cases in which the upper action *overline* cannot be attained, meaning that, whatever costly be the effort, the productivity of any extra unit of effort is positive. That seems to be particularly a natural assumption in situations where there remains some residual exposition to risk, independent of the human effort, i.e. when  $\bar{a} < 1$ . A natural consequence is that whatever the expenses in ameliorating the effort,  $\bar{a}$  is never obtained, which formally is translated by  $c(a) \rightarrow \infty$  when  $a \rightarrow \bar{a}$ .

Suppose that  $\lim_{a \rightarrow \bar{a}} c(a) \rightarrow \infty$ . Then, as the effort space is included in a compact set, such an assumption implies that all derivative of the effort cost function, must be unbounded over the effort space. It is also the case of the increasing function  $a \mapsto c(a) + (1-a)c'(a) = \int_0^a c'(x)dx + (1-a)c'(a)$  which can be interpreted when the effort  $a$  has been reach as a proxy of the total cost that would be needed to reduce uncertainty only to its residual part. It is also the case of the derivative of the preceding function,  $a \mapsto (1-a)c''(a)$ . In the following, we will show that if that (unbounded) function is also increasing, then the contract curve will have simple properties.

We will consider in fact a slightly less demanding assumption. Let define  $A^{Need}$  as the empty set when  $\bar{a} \leq \frac{1}{2}$  and  $A^{Need} = (\frac{1}{2}, \bar{a})$  otherwise.

**Assumption 2** (Cost regularity on the asymptotic case). *Functions  $a \mapsto (1-a)c''(a)$  is increasing on the set  $A^{Need}$ .*

That condition implies a strong convexity of the cost function which is needed to support any assumption such that  $\lim_{a \rightarrow \bar{a}} c(a) = +\infty$ . It encompasses cases like  $c(a) = (\bar{a} - a)^{-\gamma}$  with  $\gamma > 0$ . N Also remark that

the case  $c'''(a) = 0$  does not fit the regularity assumption, but remark that in that case, the limit of the cost function when  $a \rightarrow \bar{a}$  is a finite number.

**Theorem 1.** *Consider a moral hazard model economy characterized by a concave VNM  $u(\cdot)$  and a cost function  $c(\cdot)$  verifying assumption 1 and 2. Then, any contract curve is increasing*

1. *either under the regularity assumption ;*
2. *or under the assumption that there exists at least a parameter  $\gamma > 0$  such that function  $a \mapsto (\bar{a} - a)^\gamma c(a)$  is increasing and its derivative convex*

That result is in the spirit of the first order approach, as the regularity assumption does not depend on any particular specification of the VNM utility function. As already noticed, the regularity assumption is mainly an assumption about the regularity of the cost function, that is really natural in the case of possible infinite cost of effort (think about investment in order to reduce costs). Then, the scope of our result is very large.

**Proof of theorem 1 - point 1** The regularity assumption is equivalent to the condition that function  $a \mapsto (1 - a)c''(a)$  is increasing on the set  $A^{Need}$  which implies also that on that set function  $\varphi(\cdot)$  is increasing. On interval  $[0, \frac{1}{2}]$  function  $a \mapsto a(1 - a)$  is increasing and so  $\varphi(\cdot)$ . From equation 4 it is then immediate that the second derivative of  $L$  is negative. That's it!

**Proof of theorem 1 - point 2** That proof is relegated to the appendix ■

Another way to understand the scope of the result is to translate its assumptions with the notations of the Rogerson model. In the Rogerson model, the link between the probability and the investment to reduce risk is represented through a function  $1 - p = FAIL(c)$ ,  $p$  being the probability of failure<sup>8</sup>. If  $c(a) = 1/(\bar{a} - a)$  then,  $1 - p = FAIL(c) = \bar{a} + \frac{1}{c^\gamma}$ . Theorem 1 's second set of assumption cannot be translated like that. However, it is related a similar set of assumption saying that there exists a parameter  $\gamma$  such that  $c^\gamma FAIL(c)$  is a decreasing function and its derivative is a decreasing concave function.

Theorem 1 result is really strong as it do not depend on any assumption on the profile of the VNM function and it because implies to find complicated non regularity of the cost function in order that the contract curve be non increasing. It is a good tool in order to evaluate the choices of modelization in the moral hazard litterature. It is possible that some results in that litterature are directly related to the increasing contract curve.

## 4.2 Convex marginal cost function

Are there smooth models in which the profile of the contract curve is non trivial ? Such an issue is investigated in the following proposition.

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<sup>8</sup>I changed the meaning of letter  $a$ , in the Rogerson model,  $a$  is the unit to measure the cost and  $p(a)$  the probability of failure...

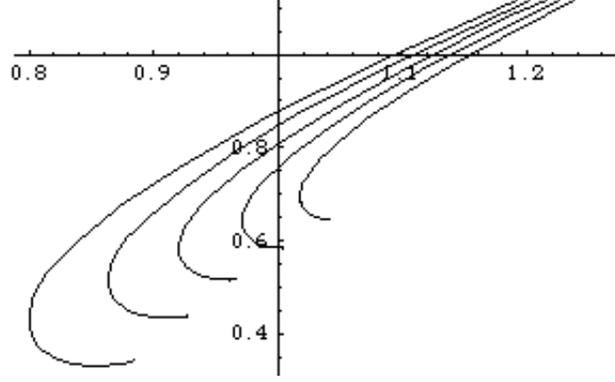


Figure 3: Representation of  $\Phi_L$  for  $L = .6, .7, .8, .9, 1$

Indeed, in some sense, assumption 2 is requiring a lot (and in particular, that the function  $a \mapsto (1 - a) c''(a)$  be bounded below by a positive number). And with such a strong assumption, not only the economy is smooth, but also the contract curve profile is trivial. It is then natural to ask if for less demanding assumption on the marginal cost function, one could find another result of smoothness of our economy. That will be done by sufficient condition 3.

**Assumption 3** (Marginal cost strong convexity). *Functions  $a \mapsto c'''(a)/c''(a) - 2$  is positive on the set  $A^{Need}$ .*

That condition is formally less demanding than condition 2. Indeed, condition 2 implies that on some interval  $A^{Need} \subset [\frac{1}{2}, 1]$  the cost function verifies

$$c'''(a)(1 - a) - c''(a) \geq 0 \iff c'''(a) \geq \frac{c''(a)}{1 - a}$$

which implies, as  $a \in A^{Need}$  that  $c'''(a) \geq 2c''(a), \forall a \in A^{Need}$ .

**Theorem 2.** *Consider a moral hazard model economy characterized by a concave VNM  $u(\cdot)$  and a cost function  $c(\cdot)$  verifying assumption 1 and 3. Then, the moral hazard model is smooth.*

**Proof.**

See appendix ■

What is really appealing in that kind of result is that one more time it does not depend on any assumption on the VNM morgerstern function.

### 4.3 An example of non increasing profiles of the contract curves family

Let study the variations of  $\Phi_L$  when  $u(x) = \sqrt{x}$  and  $c(a) = \frac{2}{3} \left( a + \frac{4}{5}(1 - a)^{\frac{5}{2}} \right)$ . The contract curve  $\Phi_L$  is characterized by two equations 1 and 2. However, as  $1/u'(x) = 2u(x) = 2\sqrt{x}$ , equation 1 can be simplified, when divided by equation 2, so that the system formed by those two equations is linear in  $\sqrt{x_1}$  and

$\sqrt{x_2}$ . The parameterization of  $\Phi_L$  for a particular  $L$  is then

$$\begin{cases} \sqrt{x_1} &= \frac{L}{2c'(a)} + a(1-a)c''(a) + \frac{c'(a)}{2} \\ \sqrt{x_2} &= \frac{L}{2c'(a)} + a(1-a)c''(a) - \frac{c'(a)}{2} \end{cases} \quad (8)$$

Joint function  $(x_1, x_2)$  is starting from  $+\infty$  (decreasing first) and is represented in figure 3.

## 5 Concluding remarks

Our analysis is beyond the first order approach problem, but it shares a similar approach, looking at first at the technology of effort. The fact that results on the distortion of the moral hazard depends only on the cost function and do not need more assumption on the Von Neuman Morgerstern utility function is noteworthy. It is a new piece on evidence on the fact the the effort separable model properties depends so much on the effort technology.

## References

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## Appendix

### Proof. of lemma 1

**Point 1** Consider  $x$  outside the domain. Then, the endogeneous effort is locally constant ( $a \in \{\underline{a}, \bar{a}\}$ ) and, at that point,  $\Pi_a = 0$ . Following,  $\nabla \Pi = (a, 1 - a)$ , and this vector is colinear to  $\nabla U$  only when  $u'(x_1) = u'(x_2)$ , i.e.,  $x_1 = x_2$ . However, the 45 degree line is inside the domain. So, outside the domain, the derivative of the utility function and of the profit function are never colinear.

**Point 2** It follows from the equality  $\nabla U = -\lambda \nabla \Pi$   $\left( \frac{1}{\lambda} = \frac{1}{u'(x_1)} - \frac{\Pi_a}{ac''(a)} \right)$  and  $\frac{1}{\lambda} = \frac{1}{u'(x_2)} + \frac{\Pi_a}{(1-a)c''(a)}$ .

**Point 3** It is a corollary of the fact that  $\nabla U = -\lambda \nabla \Pi$ . Indeed, all the coordinates of  $\nabla U$  are non negative.

**Point 4** Deduce from point 1 that  $x_1 \geq x_2$ . Then from point 2:  $\Pi_a = a(1-a)c''(a) \left[ \frac{1}{u'(x_1)} - \frac{1}{u'(x_2)} \right] \geq 0$ . The rest follows from the same profit as  $(W - \pi, W - \pi - L)$  condition:  $a \Pi_a + (W - \pi - L - x_2) = 0$  or  $-(1-a)\Pi_a - (W - \pi - x_1) = 0$ .

□

## Proof of lemma 2

**Point 1** Consider two sets of parameters,  $(W_1, W_1 - L)$  and  $(W_2, W_2 - L)$ . Consider two moral hazard model identical but those two sets of dotation. Then, the objective of the agent is the same, while the profit of the insurer differ. If we denote by  $\Pi_1$  and by  $\Pi_2$  the two respective profit functions, it is immediate to verify that  $\forall x : \Pi_1(x) - W_1 = \Pi_2(x) - W_2$ . Indeed, that equality follows from the fact that in any contract  $x$ , the action taken by the agent is the same in the two models, and that profit functions are linear in the dotation parameters. Then, a natural corollary is that

$$\forall x : \nabla \Pi_1(x) = \nabla \Pi_2(x)$$

from what follows directly statement 1 of the lemma.

**Point 2** Consider two sets of parameters,  $(W, W - L_1)$  and  $(W, W - L_2)$ . Consider two moral hazard model identical but those two sets of dotation. Then, the objective of the agent is the same, while the profit of the insurer differ. If we denote by  $\Pi_1$  and by  $\Pi_2$  the two respective profit functions, it is immediate to verify that  $\forall x : \Pi_1(x) + (1 - a(x))L_1 = \Pi_2(x) + (1 - a(x))L_2$ . Then, a natural corollary is that

$$\forall x : \nabla \Pi_1(x) - L_1 \nabla a(x) = \nabla \Pi_2(x) - L_2 \nabla a(x) \quad (9)$$

Consider then some  $x$  belonging to  $\Phi_{L_1} \cap \Phi_{L_2}$ . If  $L_1 - L_2 \neq 0$ , the fact that  $\nabla U$  is colinear to  $\nabla \Pi_1$  and to  $\nabla \Pi_2$  implies together with equation 9 that  $\nabla U$  is colinear to  $\nabla a(x)$ . However, that is a contradiction as we noticed that  $\frac{\partial a}{\partial x_1} * \frac{\partial a}{\partial x_2} < 0$  while  $\frac{\partial U}{\partial x_1} * \frac{\partial U}{\partial x_2} > 0$ .

**Point 3** For  $x$  in the interior of the domain, and  $a_x$ , the corresponding effort ( $a_x \in (0, 1)$ ), define  $K(x) = a_x(1 - a_x)c''(a_x) \left[ \frac{1}{u'(x_1)} - \frac{1}{u'(x_2)} \right] \geq 0$  and  $L(x) \in \mathbb{R}_+$  such that

$$L(x) = K(x) + (x_1 - x_2)$$

Then, from the definition of  $K(x)$  it is immediate that the point  $(x_1, x_2)$  verify conditions 1 and 2 of lemma 1, when the initial dotation is  $(L(x), 0)$ . That means that  $x \in \Phi_{L(x)}$ .

□

## Proof of theorem 1 - point 2

The derivative of functions  $x \mapsto \Psi = (1-x)^\gamma c(x)$  are

1.  $\Psi'(x) = c'(1-x)^\gamma - \gamma c(1-x)^{\gamma-1}$
2.  $\Psi''(x) = c''(1-x)^\gamma - 2\gamma c'(1-x)^{\gamma-1} + \gamma(\gamma-1)c(1-x)^{\gamma-2}$
3.  $\Psi'''(x) = c'''(1-x)^\gamma - 3\gamma c''(1-x)^{\gamma-1} + 3\gamma(\gamma-1)c'(1-x)^{\gamma-2} + \gamma(\gamma-1)(\gamma-2)c(1-x)^{\gamma-3}$

The first derivative being positive, it follows

$$\frac{(1-x)c'}{c} > \gamma \quad (10)$$

The second derivative being positive, plugging the inequalities of equation 10 implies

$$\frac{(1-x)c''}{c'} > \gamma + 1 \quad \text{and also} \quad \frac{\gamma(\gamma-1)c}{2(1-x)c''} > 2\gamma \frac{c'}{(1-x)c''} - 1 \quad (11)$$

The third derivative being positive, plugging the inequalities of equations 10 and 11 implies

$$\frac{(1-x)c'''}{c''} > 2(\gamma+1) - \frac{\gamma c'}{(1-x)c''} [\gamma+1] \geq \gamma+2 \geq 2 \quad (12)$$

from what is follows that  $(1-x)c''$  is an increasing function, that is, the cost function verifies the condition of point 1 of theorem 1.

□

## Proof of theorem 2

As we noticed, indifference curves are always decreasing, so, the value of the optimal effort strictly increases along an indifference curve when  $x_1$  increases and  $x_2$  decreases; it follows that an indifference curve can be parametered by the effort (denote by  $x_1(a)$  and  $x_2(a)$  the parametric equation of an indifference curve) and so, function  $L$  along that indifference curve. It is well know that  $x'_1(a) = \lambda(a)(1-a)u'(x_2) > 0$  and that  $x'_2(a) = -\lambda(a)au'(x_2)$ . Then, the variations of  $L$  along the indifference curve can be represented by

$$\frac{L'(a)}{\lambda(a)} = ((1-a)u'_2 + a u'_1) + \frac{\varphi'}{c''} u'_1 u'_2 \left( \frac{1}{u'_1} - \frac{1}{u'_2} \right) \quad (13)$$

$$- \varphi \left( (1-a)u'_2 \frac{u''_1}{u'^2_1} + a u'_1 \frac{u''_2}{u'^2_2} \right) \quad (14)$$

$$= [(3a-1)u'_1 + (2-3a)u'_2] + a(1-a)(u'_2 - u'_1) \frac{c'''}{c''} \quad (15)$$

$$- \varphi u'_1 u'_2 \left( (1-a) \frac{u''_1}{u'^3_1} + a \frac{u''_2}{u'^3_2} \right)$$

In equation 15, the two last terms of the sum are always positive. A finer analysis is needed for the first term (in bracket). In case  $a \in [\frac{1}{3}, \frac{2}{3}]$ , the two coefficients  $3a - 1$  and  $2 - 3a$  are positive, and so the first term. In case  $a \in [0, \frac{1}{3}]$ , the first term of the sum can be written  $u'_1 + (2 - 3a)(u'_2 - u'_1)$  which is positive (remember  $x_1 \geq x_2$ ). More travail is needed when  $a \geq 2/3$ . The last term of the sum is positive: it can be lowered by 0. The second term of the sum can be lowered by  $2a(1 - a)(u'_2 - u'_1)$  under the assumption of proposition 2. Then,

$$\frac{L'(a)}{\lambda(a)} \geq u'_1 [3a - 1 - 2a(1 - a)] + u'_2 [2 - 3a + 2a(1 - a)] \quad (16)$$

$$= (1 + a)(2a - 1) u'_1 + (1 - a)(2 + a)u'_2 > 0 \quad \forall a \geq \frac{2}{3} \quad (17)$$

the economy is then smooth.  $\square$