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Ricardian Equivalence and the Intertemporal Keynesian Multiplier

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Abstract

We show that Keynesian multiplier effects can be obtained in dynamic optimizing models if one combines both price rigidities and a "non Ricardian" framework where, due for example to the birth of new agents, Ricardian equivalence does not hold.

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1 Introduction

One of the most intriguing features of traditional Keynesian theory (Keynes, 1936, Hicks, 1937) is the so called "multiplier effect" by which an increase in governement spending can create an increase in consumption, whereas in Walrasian models it leads to a decrease, via the usual "crowding out" effect. This multiplier effect is customarily attributed to price or wage rigidities.

Now in the recent evolution of macroeconomic modelling most macroeconomic issues are reexamined within the rigorous framework of dynamic intertemporal maximizing models à la Ramsey (1928). The typical model depicts consumers as one single dynasty of infinitely lived agents. This model has notably the property of "Ricardian equivalence" (Barro, 1974), according to which, in a nutshell, the distribution of taxes across time is irrelevant as long as the government balances its budget intertemporally. The model has been extended to a monetary framework (Sidrauski, 1969, Brock, 1975), where Ricardian equivalence also holds.

In line with this recent evolution, a natural question to ask is whether a multiplier effect will arise in these dynamic models. The result has been actually disappointing: crowding out occurs in the usual DSGE model (see for example Fatas and Mihov, 2001), and even in models with price rigidities under standard parameterizations (see for example Collard and Dellas, 2005).

What we want to show in this paper is that, in order to obtain a strong enough multiplier effect, another ingredient, in addition to price rigidities, has to be introduced in dynamic models. Namely one should not only have price rigidities, but also model the economy as "non Ricardian". By non Ricardian economies we mean, as in Barro (1974), economies like overlapping generations (OLG) economies à la Samuelson (1958), where Ricardian equivalence does not hold. The non Ricardian economy we shall work with is due to Weil (1987, 1991). It is a monetary economy where, as in the Ricardian model, agents have an infinite life but, as in the OLG model, new agents arrive over time.

We shall call n the rate of growth of the population. In a nutshell the results are: If $n > 0$, there is a multiplier[1], and government spending leads to an increase in private consumption. This multiplier is larger, the higher n is, i.e. the more "non Ricardian" the economy is.

[1] By this we mean more precisely that the income multiplier is greater than one, so that there is no crowding out.

2 The model

We shall thus use the model of Weil (1987, 1991), which has the great advantage of having the Ricardian model as a particular case. Each household lives forever, but new "generations" are born every period. Denote as N_t the number of households alive at time t. We will work below with a constant rate of growth of the population $n \geq 0$, so that $N_t = (1+n)^t N_0$. The Ricardian model corresponds to the special limit case $n = 0$.

2.1 Households

Consider a household born in period j. We denote by c_{jt}, y_{jt} and m_{jt} his consumption, endowment and money holdings at time $t \geq j$. This household maximizes the following utility function:

$$U_{jt} = \sum_{s=t}^{\infty} \beta^{s-t} Log\, c_{js} \tag{1}$$

and is submitted in period t to a "cash in advance" constraint:

$$P_t c_{jt} \leq m_{jt} \tag{2}$$

Household j begins period t with a financial wealth ω_{jt}. First the bond market opens, and the household lends an amount b_{jt} at the nominal interest rate i_t. The rest is kept under the form of money m_{jt}, so that:

$$\omega_{jt} = m_{jt} + b_{jt} \tag{3}$$

Then the goods market opens, and the household sells his endowment y_{jt}, pays taxes τ_{jt} in real terms and consumes c_{jt}, subject to the cash constraint (2). Consequently, the budget constraint for household j is:

$$\omega_{jt+1} = (1+i_t)\,\omega_{jt} - i_t m_{jt} + P_t y_{jt} - P_t \tau_{jt} - P_t c_{jt} \tag{4}$$

2.2 Aggregation, endowments and taxes

Aggregate quantities are obtained by summing the various individual variables. There are $N_j - N_{j-1}$ agents in generation j, so for example aggregate taxes $\mathcal{T}_t$ are given by:

$$\mathcal{T}_t = \sum_{j \leq t} (N_j - N_{j-1})\, \tau_{jt} \tag{5}$$

The other aggregate quantities, Y_t, C_t, Ω_t, M_t and B_t, are deduced through similar formulas from the individual variables, $y_{jt}, c_{jt}, \omega_{jt}, m_{jt}$ and b_{jt}.

We now have to describe the distribution of endowments and taxes among households. We assume that all households have the same income and taxes:

$$y_{jt} = y_t = \frac{Y_t}{N_t} \qquad \tau_{jt} = \tau_t = \frac{\mathcal{T}_t}{N_t} \tag{6}$$

3 The dynamic equations

3.1 Taxes and government budget constraint

The dynamics of government liabilities Ω_t is:

$$\Omega_{t+1} = (1 + i_t)\,\Omega_t - i_t M_t + P_t G_t - P_t \mathcal{T}_t \tag{7}$$

Government budget balance corresponds to $\Omega_{t+1} = \Omega_t$, i.e. since $\Omega_t = M_t + B_t$:

$$P_t G_t = P_t \mathcal{T}_t - i_t B_t \tag{8}$$

We would like to have a tax index T_t such that budget balance is achieved under the traditional condition:

$$G_t = T_t \tag{9}$$

This will be the case if we define T_t through:

$$P_t T_t = P_t \mathcal{T}_t - i_t B_t \tag{10}$$

Budget balance now corresponds to (9), and the government budget constraint (7) is rewritten as:

$$\Omega_{t+1} = \Omega_t + P_t G_t - P_t T_t \tag{11}$$

3.2 Consumption dynamics

The dynamics of consumption is given by (see the appendix):

$$P_{t+1} C_{t+1} = \beta\,(1 + n)\,(1 + i_t)\,P_t C_t - (1 - \beta)\,n \Omega_{t+1} \tag{12}$$

The dynamic system consists of equations (11) and (12).

4 The multiplier

We shall assume that nominal prices are fully rigid, now and in the future. Besides simplifying the algebra this assumption, which is clearly the most favourable situation for a multiplier effect to arise, will allow to make crystalclear that a non Ricardian environment plays a central role in addition to price rigidities. We also assume that the nominal interest rate is pegged ($i_t = i$). Now equation (12) is rewritten:

$$P_t C_t = \gamma P_{t+1} C_{t+1} + \gamma (1-\beta) n \Omega_{t+1}$$

$$= \gamma P_{t+1} C_{t+1} + \gamma (1-\beta) n (\Omega_t + P_t G_t - P_t T_t) \tag{13}$$

with:

$$\gamma = \frac{1}{\beta (1+n)(1+i)} \tag{14}$$

We shall assume $\gamma < 1$[2]. Integrating forward equation (13) we obtain:

$$P_t C_t = \gamma (1-\beta) n \sum_{j=0}^{\infty} \gamma^j (\Omega_{t+j} + P_{t+j} G_{t+j} - P_{t+j} T_{t+j}) \tag{15}$$

Now from (11):

$$\Omega_{t+j} = \Omega_t + \sum_{k=0}^{j-1} P_{t+k} (G_{t+k} - T_{t+k}) \tag{16}$$

so that:

$$\sum_{j=0}^{\infty} \gamma^j \Omega_{t+j} = \sum_{j=0}^{\infty} \gamma^j \left[\Omega_t + \sum_{i=0}^{j-1} P_{t+k} (G_{t+k} - T_{t+k}) \right]$$

$$= \frac{\Omega_t}{1-\gamma} + \sum_{k=0}^{\infty} P_{t+k} (G_{t+k} - T_{t+k}) \sum_{j=k+1}^{\infty} \gamma^j$$

$$= \frac{\Omega_t}{1-\gamma} + \sum_{k=0}^{\infty} \frac{\gamma^{k+1}}{1-\gamma} P_{t+k} (G_{t+k} - T_{t+k}) \tag{17}$$

and finally, inserting (17) into (15):

[2] This condition also plays a role for price determinacy in models with market clearing. See Bénassy (2005) for a formal analysis and economic discussion.

$$P_t C_t = \frac{\gamma (1-\beta) n}{1-\gamma} \left[\Omega_t + \sum_{j=0}^{\infty} \gamma^j P_{t+j} \left(G_{t+j} - T_{t+j} \right) \right] \tag{18}$$

So with $n > 0$ consumption reacts positively to government spending, and from formula (18) the multiplier is greater, the higher n is.

5 Conclusion

We have thus seen that in order to obtain a "multiplier", price rigidities are not the only decisive ingredient, but it is also important to have a "non Ricardian" economy, corresponding here to $n > 0$. We shall now give a brief intuition for this.

Let us start with the limit case $n = 0$. As we shall now see the argument is quite reminiscent of the famous "balanced budget multiplier" of Haavelmo (1945). Consider indeed an increase in government spending, say G_t, all other spending variables being held constant. Because of the government's intertemporal budget constraint, G_t will have to be compensated by taxes, either now or in the future, and the total discounted value of these taxes must be exactly equal to G_t. As a result we have a "balanced budget" formula, as in Haavelmo.

Now if $n > 0$, new generations arrive over time, and the future, but yet unborn, generations will bear some of the future taxes. So there is part of government spending that will not be paid by currently alive generations, and as a result the traditional multiplier mechanism comes into play.

6 Appendix: derivation of equation (12)

We shall derive in this appendix the dynamic equation (12). Consider the household's budget equation (4). We assume that i_t is strictly positive, so households always want to satisfy the "cash in advance" equation (2) exactly. We thus have $m_{jt} = P_t c_{jt}$ and equation (4) is rewritten:

$$\omega_{jt+1} = (1 + i_t) \, \omega_{jt} + P_t y_t - P_t \tau_t - (1 + i_t) \, P_t c_{jt} \tag{19}$$

Define the intertemporal discount factors:

$$R_t = \prod_{s=0}^{t-1} \frac{1}{1 + i_s} \qquad R_0 = 1 \tag{20}$$

Applying the discount factors (20) to the budget constraint (19), it becomes:

$$R_{s+1}\omega_{js+1} = R_s\omega_{js} + R_{s+1}P_s\left(y_s - \tau_s\right) - R_sP_sc_{js} \tag{21}$$

If we aggregate all discounted budget constraints (21) from time t to infinity, and assume that $R_s\omega_{js}$ goes to zero as s goes to infinity (the usual transversality condition), we obtain the intertemporal budget constraint of the household:

$$\sum_{s=t}^{\infty} R_sP_sc_{js} = R_t\omega_{jt} + \sum_{s=t}^{\infty} R_{s+1}P_s\left(y_s - \tau_s\right) \tag{22}$$

Now maximizing utility function (1) subject to the intertemporal budget constraint (22) yields the following consumption function for a household j:

$$R_tP_tc_{jt} = (1-\beta)\left[R_t\omega_{jt} + \sum_{s=t}^{\infty} R_{s+1}P_s\left(y_s - \tau_s\right)\right] \tag{23}$$

Summing this across the N_t agents alive in period t, we obtain the aggregate consumption C_t:

$$R_tP_tC_t = (1-\beta)\left[R_t\Omega_t + N_t\sum_{s=t}^{\infty} R_{s+1}P_s\left(y_s - \tau_s\right)\right] \tag{24}$$

In equilibrium $Y_t = C_t + G_t$, so the equilibrium equation is:

$$R_tP_t\left(Y_t - G_t\right) = (1-\beta)\left[R_t\Omega_t + N_t\sum_{s=t}^{\infty} R_{s+1}P_s\left(y_s - \tau_s\right)\right] \tag{25}$$

Divide both sides by N_t and use $Y_t = N_ty_t$, $G_t = N_tg_t$:

$$R_tP_t\left(y_t - g_t\right) = (1-\beta)\left[\frac{R_t\Omega_t}{N_t} + \sum_{s=t}^{\infty} R_{s+1}P_s\left(y_s - \tau_s\right)\right] \tag{26}$$

Let us rewrite this equation for $t+1$ and subtract it from (26). We obtain:

$$R_tP_t\left(y_t - g_t\right) - R_{t+1}P_{t+1}\left(y_{t+1} - g_{t+1}\right) =$$

$$(1-\beta)\left[\frac{R_t\Omega_t}{N_t} - \frac{R_{t+1}\Omega_{t+1}}{N_{t+1}} + R_{t+1}P_t\left(y_t - \tau_t\right)\right] \tag{27}$$

Now multiply the government's budget equation (7) by R_{t+1}/N_t:

$$\frac{R_t\Omega_t}{N_t} = \frac{R_{t+1}\Omega_{t+1}}{N_t} + (R_t - R_{t+1}) P_t (y_t - g_t) - R_{t+1}P_t g_t + R_{t+1}P_t\tau_t \quad (28)$$

Insert this into equation (27):

$$R_{t+1}P_{t+1}(y_{t+1} - g_{t+1}) = \beta R_t P_t (y_t - g_t) - (1-\beta)\left(\frac{1}{N_t} - \frac{1}{N_{t+1}}\right) R_{t+1}\Omega_{t+1} \quad (29)$$

and multiply by N_{t+1}/R_{t+1}:

$$P_{t+1}(Y_{t+1} - G_{t+1}) = \beta\frac{N_{t+1}}{N_t}(1+i_t) P_t (Y_t - G_t) - (1-\beta)\left(\frac{N_{t+1}}{N_t} - 1\right)\Omega_{t+1} \quad (30)$$

Taking finally $N_{t+1}/N_t = 1+n$, and using $C_t = Y_t - G_t$ we obtain:

$$P_{t+1}C_{t+1} = \beta(1+n)(1+i_t) P_t C_t - (1-\beta) n\Omega_{t+1} \quad (31)$$

which is equation (12).

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