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Bubbles and self-fulfilling crises

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JEL Codes : G12, G33

Keywords : Credit market imperfections, self-fulfilling expectations, financial crises.

Bubbles and Self-fulfilling Crises*

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Abstract: Financial crises are often associated with an endogenous credit reversal, followed by a fall in asset prices and serious disruptions in the financial sector. To account for this sequence of events, this paper constructs a model where excessive risk-taking by investors leads to a bubble in asset prices, and where the supply of credit to these investors is endogenous. We show that the interplay between excessive risk-taking and the endogeneity of credit may give rise to multiple equilibria associated with different levels of lending, asset prices, and output. Stochastic equilibria lead, with positive probability, to an inefficient liquidity dry-up, a market crash, and widespread failures by borrowers. The possibility of multiple equilibria and self-fulfilling crises is shown to be related to the severity of the risk-shifting problem in the economy.

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1 Introduction

The resurgence of financial crises over the past twenty years, both in OECD and developing countries, has sparked renewed interest in the potential sources of financial fragility and market imperfections from which they originate. Although each crisis had, of course, its own particular features, it is now widely agreed that many of them were characterised by a common underlying pattern of destabilising developments in credit and asset markets. Amongst OECD countries in the 1980s and early 1990s, such as Japan or the Scandinavian countries, financial crises were an integral part of a broader ‘credit cycle’ whereby financial deregulation led to an increase in available credit, fuelled a period of overinvestment in real estate and stock markets, and led to high asset-price inflation. These events were then followed by a credit contraction (or ‘crunch’) and the bursting of the asset bubble, causing the actual or near bankruptcy of the financial institutions which had initially levered the asset investment¹. A similar sequence of events has been observed in a number of Asian and Latin American countries, where capital account liberalisation allowed large amounts of capital to flow in during the 1990s, with a similar effect of raising asset prices to unsustainable levels. This phase of overlending often ended in a brutal capital account reversal followed by a market crash and a banking crisis.²

An important theoretical issue, as yet largely unanswered, is whether the credit turnaround that typically accompanies financial crises is the outcome of an autonomous, ‘extrinsic’, reversal of expectations on the part of economic agents, or simply the natural outcome of accumulated macroeconomic imbalances or policy mistakes, i.e., the intrinsic fundamentals of the economy. For a time, the consensus was to interpret crises simply as the outcome of extraneous ‘sunspots’ hitting the beliefs of investors, regardless of the underlying fundamental soundness of the economy. For example, early models of crises would emphasise the inherent instability of the banking system, whose provision of liquidity insurance made banks sensitive to self-fulfilling runs, as the ultimate source of vulnerability to crises³. In a similar vein, ‘second-generation’ models of currency crises would insist on the potential

¹See Borio, Kennedy and Prowse (1994) and Allen and Gale (1999, 2000), as well as the references therein, for a more detailed account of these events.

²See Calvo (1998), Kaminsky (1999) and Kaminsky and Rheinart (1998, 1999) for the evidence on this sequence of events, often referred to as ‘sudden stop’.

³See Diamond and Dybvig (1983), as well as Chang and Velasco (2002) for an open-economy model.

existence of multiple equilibria in models of exchange rate determination, where the defense of a pre-announced peg by the central bank is too costly to be fully credible⁴.

Although such expectational factors certainly play a rôle in triggering financial crises, theories based purely on self-fulfilling expectations clearly do not tell the full story. In virtually all the recent episodes briefly mentioned above, specific macroeconomic or structural sources of fragility preceded the actual occurrence of the crisis. In OECD countries, for example, financial crises usually followed periods of loose monetary policy or poor exchange-rate management (e.g., Borio *et al.*, 1994). In emerging countries, the culprit was often to be found in the weakness of the banking sector, due to poor financial regulation, as well as other factors such as unsustainable fiscal or exchange rate policies (Summers, 2000). Overall, the evidence from this latter group of countries indicates that factors of fundamental weakness explain only some of the probability of a crisis, suggesting that both fundamental and non-fundamental elements are at work in triggering financial crises (see Kaminsky, 1999, and the discussion in Chari and Kehoe, 2003).

The model of financial crises that we develop below aims to account for both the credit-asset price cycle typical of recent crises and the joint role of fundamental and nonfundamental factors in making crises possible. In so doing, we draw on Allen and Gale (2000), for whom financial crises are the natural outcome of credit relations where portfolio investors borrow to buy risky assets, and are protected against bad payoff outcomes by the use of debt contracts with limited liability. Investors' distorted incentives then lead them to overinvest in risky assets (i.e., a *risk-shifting* problem arises), whose price consequently rises to high levels (leading to an *asset bubble*), with the possibility that investors go bankrupt if asset payoffs turn out badly (a *financial crisis* occurs). Unlike Allen and Gale, however, who study the risk-shifting problem in isolation and make the partial-equilibrium assumption that the amount of funds available to investors is exogenous, we allow for endogenous variations in the supply of credit resulting from lenders' utility-maximising behaviour. We regard this alternative specification as not only more realistic, but also particularly relevant to our understanding of recent crises episodes, where the endogeneity of aggregate credit was frequently identified as being an important source of financial instability⁵.

Our results indicate that the interdependence between excessive risk-taking by investors

⁴E.g., Obstfeld (1996) and Velasco (1996).

⁵See, for example, Edison, Luangaram and Miller (2000) for a contribution representative of this view.

and the elasticity of aggregate credit is indeed a serious factor of endogenous instability. First, we show that, under risk-shifting, the equilibrium return that lenders expect from lending to investors may be non-monotonic and *increase* with the aggregate quantity of loans, rather than *decrease*, as standard marginal productivity arguments would suggest. The explanation is that investors' optimal portfolio composition typically changes as the amount of funds that is lent to them varies, i.e., the 'assets' and 'liabilities' sides of investors' balance-sheets are not independent. In certain circumstances, which we derive and explain in the paper, an increase in investors' liabilities may increase the share of safe assets in their portfolios, which tends to raise the *ex ante* return on loans. When strong enough, this 'portfolio composition' effect may dominate the usual 'marginal productivity' effect, so that the expected return on loans increases with aggregate loans (for some range of total loans at least). This strategic complementarity naturally leads to the existence of multiple equilibria associated with different levels of aggregate lending, asset prices, and output. We relate the intensity of these strategic complementarities, and the resulting possibility of multiple equilibria, to the severity of the risk-shifting problem in the economy.

We then consider the case where multiple equilibria do exist, and where the selection of an equilibrium with low lending follows a 'sunspot', i.e., an extraneous signal of any *ex ante* probability on which agents coordinate their expectations. We show that such stochastic equilibria generate *self-fulfilling crises* with the following characteristics; i) lending to portfolio investors drops off as lenders choose to consume or store, rather than lend, a large share of their endowment (credit contraction), ii) this causes a fall in investors' resources and a drop in their demand for fixed-supply assets, whose price consequently falls to low levels (market crash), and iii) this fall in prices forces into bankruptcy investors who had previously borrowed to buy assets, as the new value of their assets falls short of their liabilities (financial sector disruptions). In short, *weak fundamentals* make multiple equilibria possible, while *self-fulfilling expectations* trigger the actual occurrence of the crisis. We also provide a full welfare analysis of the model. Crises are shown to unambiguously decrease *ex ante* welfare, with a principal source of this welfare loss being the negative *wealth effects* of the crash on lenders' consumption levels.

Although our theory of financial crises draws on recent related contributions, it also differs from them in several dimensions. While Allen and Gale (2000) and Edison *et al.* (2000)

both emphasise the interdependency between asset price movements and aggregate credit during crises, they do so in the framework of single-equilibrium models where crises are entirely explained by exogenous fundamentals. Building on the empirical results of Kaminsky (1999) discussed above, Chari and Kehoe (2003) account for the probability of crises unexplained by fundamental factors by relying on investors' 'herd behaviour' in an environment with heterogeneous information; in contrast, our results are derived within a rational expectations framework where all investors share the same information about asset payoffs. Finally, within the class of multiple-equilibrium based theories, our framework differs from 'third generation' models of currency crises (e.g., Aghion, Bacchetta and Banerjee, 2001 and 2004) by focusing on the instability of aggregate credit, rather than the volatility of nominal exchange rates; it also differs from infinite-horizon models where self-fulfilling asset-price movements are the outcome of 'steady state indeterminacy', i.e., the multiplicity of converging perfect-foresight equilibrium paths (as in Challe, 2004, for example).⁶

The remainder of the paper is organised as follows. Section 2 introduces the model and derives its unique fundamental (i.e., first-best efficient) equilibrium. Section 3 shows how the interdependency between endogenous lending and the excessive risk-taking of portfolio investors may give rise to multiple equilibria associated with different levels of lending, asset prices, and output. Section 4 derives the stochastic equilibria of this economy (i.e., equilibria featuring self-fulfilling crises), and analyses their welfare properties. Section 5 offers two extensions to the basic model, while Section 6 concludes.

2 The model

2.1 Timing and assets

There are three dates, 0, 1 and 2, and two real assets. One asset, safe and in variable supply, is two-period lived and yields $f(x)$ units of the (all-purpose) good at date $t+1$ for $x \geq 0$ units invested at date t , $t = 0, 1$. It is assumed that $f(\cdot)$ is a twice continuously differentiable

⁶Caballero and Krishnamurthy (2006) offer a model of emerging country bubbles where the bursting of the bubble is associated with a capital flow reversal. In their model, the existence of bubbles is related to the relative scarcity of available stores of value (as in Tirole (1985)), while our bubbles owe their existence to agency problems in the financial sector leading to excessive risk-taking by investors.

function satisfying $f'(x) > 0$, $f''(x) < 0$, $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$. Moreover, the following standard assumption is made to limit the curvature of $f(\cdot)$, for all $x > 0$:

$$\eta(x) \equiv -xf''(x)/f'(x) < 1. \quad (1)$$

The other asset is risky, in fixed supply (normalised to 1), and three-period lived –it is available for buying at date 0 and delivers a terminal payoff R at date 2, where R is a random variable at dates 0 and 1 that takes on the value R^h with probability $\pi \in (0, 1]$, and 0 otherwise, at date 2. Although more general distributions for the fundamental uncertainty affecting the asset payoff can be considered, we choose this simple specification in order to focus on the extrinsic uncertainty generated by the presence of multiple equilibria.

The interpretation of this menu of available assets is that the supply of the risky asset responds slowly to changes in its demand (think of real estate, for example), while that of the safe asset adjusts quickly, and we consider the way markets clear in the short run. The market price of the risky asset at date t , in terms of the good (which is taken as the numeraire), is denoted P_t , $t = 0, 1$.

2.2 Agents and market structure

The economy consists of four types of risk-neutral agents in large numbers⁷. There is a continuum of three-period lived *lenders* of mass 1, who enter the market at date 0 and leave it at date 2. Their intertemporal utility is $u(c_1, c_2) = c_1 + \beta c_2$, where c_t , $t = 1, 2$, is date t consumption and $\beta > 0$ is the discount factor (lenders do not enjoy date 0 consumption). Lenders receive an endowment $e_0 > 0$ at date 0 and e_1 at date 1, regarding which the following technical assumption is made:

$$e_1 > f'^{-1}(1/\beta) + \beta\pi R^h. \quad (2)$$

As will become clear below, condition (2) is necessary and sufficient for all the equilibria that we analyse in the paper to correspond to interior solutions (i.e., where both c_1 and c_2 are positive). Given the lenders' assumed utility function, the entire endowment e_0 is saved at date 0 (provided that the *ex ante* return on saving at date 0 is non negative, as will

⁷The paper focuses on the risk-neutral case, in which all results can be derived analytically. The risk-averse case is explored numerically as an extension to the baseline model in Section 5.2.

always be the case), while savings decisions at date 1 depend on the comparison between the expected return on savings then and the gross rate of time preference, $1/\beta$. This possibility that lenders consume, rather than lend, part of their wealth at date 1 renders aggregate lending endogenous at that date, and is the novel and crucial feature of our model.

Lenders face overlapping generations of two-period lived *investors* and *entrepreneurs* with positive mass, entering the economy at dates 0 and 1 and maximising end-of-life consumption. In the remainder of the paper, we shall refer to ‘date t investors (entrepreneurs)’ as the investors (entrepreneurs) who enter the economy at date t , $t = 0, 1$, and leave it at date $t + 1$. Neither investors nor entrepreneurs receive any endowment. Finally, the stock of risky assets is initially held by a class of one-period lived *initial asset holders*, who sell them to investors at date 0 and then leave the market.

There is *market segmentation* (i.e., restrictions on agents’ asset holdings) in the two following senses. First, only entrepreneurs have access to the production technology $f(\cdot)$. Since they have no wealth of their own, they borrow funds by issuing X_{St} corporate bonds (at the normalised price 1) at date t ($= 0, 1$). Entrepreneurs’ utility maximisation under perfect competition then ensures that the gross interest rate on corporate bonds at date t ($= 0, 1$), called r_t , is equal to the marginal product of capital at the same date, $f'(X_{St})$.

Second, lenders cannot directly buy risky assets or corporate bonds, and must thus lend to investors to finance future consumption. This restriction implies that market equilibria at dates 0 and 1 are *intermediated*, with lenders first entrusting investors with their savings, and investors then lending to entrepreneurs (i.e., buying X_{St} corporate bonds at price 1) and investing in risky assets (i.e., buying X_{Rt} assets at price P_t)⁸. We denote B_t , $t = 0, 1$, the demand for loans by date t investors (which, in equilibrium, equals lenders’ savings at the same date). Finally, we follow Allen and Gale (2000) in assuming that lenders and investors are restricted to simple debt contracts, where the contracted rate on these loans, denoted r_t^l , $t = 0, 1$, cannot be conditional on the loan size or, due to asymmetric information, the investor’s portfolio. As will be shown below, the use of debt contracts with limited liability causes lenders’ and investors’ incentives to be misaligned, and is the basic market imperfection in the model.

⁸This structure implies that lenders have no choice but to lend to investors to finance future consumption. Section 5.1 shows that all our results carry over when lenders have access to a storage technology whose positive gross return competes with risky lending.

2.3 Fundamental equilibrium

In the intermediated economy described above, investors are granted exclusive access to the markets for risky assets and corporate bonds. Before analysing the resulting market outcome in more detail, it is useful to first derive the equilibrium that would prevail without these restrictions, i.e., if lenders could directly buy both real assets. The corresponding ‘fundamental’ equilibrium, in which prices and quantities are first-best efficient, will provide a natural benchmark against which the intermediated equilibrium can be compared. As is usual with finite horizon economies, we work out equilibrium prices and quantities backwards, using date 1 outcomes to solve for date 0 equilibrium conditions.

Subgame equilibrium at date 1. Given their date 1 wealth, denoted W_1 , lenders maximise $E_1 u(c_1, c_2) = c_1 + \beta E_1 c_2$. Since lenders’ date 1 savings, B_1 , equal safe asset investment, X_{S1} , plus risky asset investment, $X_{R1}P_1$, lenders’ expected utility from saving B_1 and choosing a portfolio (X_{S1}, X_{R1}) at date 1 is

$$W_1 - B_1 + \beta E_1 (r_1^F X_{S1} + R X_{R1}) = W_1 - B_1 + \beta (r_1^F B_1 + X_{R1} (\pi R^h - r_1^F P_1^F)), \quad (3)$$

where P_1^F and r_1^F denote the date 1 fundamental values of the risky asset and the interest rate, respectively. Given B_1 , the price of the asset in the fundamental equilibrium must be:

$$P_1^F = \pi R^h / r_1^F. \quad (4)$$

If the fundamental value of the risky assets were lower than $\pi R^h / r_1^F$, then the net return on trading them, $\pi R^h - r_1^F P_1^F$, would be positive for all positive values of X_{R1} and lenders would want to buy an infinite quantity of risky assets; if it were higher than $\pi R^h / r_1^F$, then this net return would be negative and the demand for risky assets would be zero. Since the risky asset is in positive and finite supply, neither $P_1^F < \pi R^h / r_1^F$ nor $P_1^F > \pi R^h / r_1^F$ can be equilibrium situations.

Using equation (4) and the fact that in equilibrium $X_{R1} = 1$ and thus $r_1^F = f'(X_{S1}) = f'(B_1 - P_1^F)$, market-clearing for corporate bonds implies:

$$f'^{-1}(r_1^F) + \pi R^h / r_1^F = B_1. \quad (5)$$

Given the properties of $f(\cdot)$, equation (5) defines r_1^F uniquely for all positive values of B_1 . It can thus be inverted to yield the interest rate function $r_1^F(B_1)$, where $r_1^F(B_1)$ is continuous, strictly decreasing, and such that $r_1^F(0) = \infty$ and $r_1^F(\infty) = 0$.

Substituting (4) into (3), we can see that lenders' expected utility is $W_1 - B_1 + \beta B_1 r_1^F(B_1)$. Given lenders' utility functions and our assumption of a high enough date 1 endowment (see (2)), lenders increase savings up to the point where the rate of return on savings, $r_1^F(B_1)$, is equal to the gross rate of time preference, $1/\beta$ (see figure 1 below). Substituting $r_1^F = 1/\beta$ into equations (4) and (5), we find that asset prices and aggregate savings in the fundamental equilibrium are uniquely determined and given by:

$$P_1^F = \beta\pi R^h, \quad (6)$$

$$B_1^F = f'^{-1}(1/\beta) + \beta\pi R^h, \quad (7)$$

where inequality (2) ensures that $B_1^F < e_1$, i.e., that the fundamental equilibrium is interior. In short, lenders' risk neutrality implies that the fundamental value of the asset, P_1^F , is equal to the discounted expected dividend stream, $\beta\pi R^h$, while capital investment, X_{S1}^F , is at the point where its rate of return equals lenders' rate of time preference, $f'^{-1}(1/\beta)$.

Equilibrium at date 0. The fundamental price vector at date 1, (P_1^F, r_1^F) , can now be used to derive that at date 0, (P_0^F, r_0^F) , by simply noting that the equilibrium price of risky assets at date 1, P_1^F , is also the payoff from holding them from date 0 to date 1. Lenders' total (deterministic) payoff at date 1 from choosing a portfolio (X_{S0}, X_{R0}) at date 0 is then $r_0^F X_{S0} + P_1^F X_{R0}$, which they maximise subject to the portfolio choice constraint $X_{S0} + P_0^F X_{R0} = e_0$, while taking r_0 and P_0 as given. They thus maximise:

$$r_0^F X_{S0} + P_1^F X_{R0} = e_0 r_0^F + X_{R0} (P_1^F - r_0^F P_0^F).$$

Given $e_0 r_0^F$, the fundamental value of the risky asset at date 0 cannot be higher (lower) than P_1^F/r_0^F , since asset demand would then be equal to zero (infinity). It must thus be:

$$P_0^F = P_1^F/r_0^F = \beta\pi R^h/r_0^F. \quad (8)$$

Using (8), the properties of $f(\cdot)$, and the fact that $X_{R0} = 1$ and thus $r_0^F = f'(X_{S0}) = f'(e_0 - P_0^F)$ in equilibrium, r_0^F is uniquely determined by the following equation:

$$f'^{-1}(r_0^F) + \beta\pi R^h/r_0^F = e_0. \quad (9)$$

Equations (8)–(9) fully characterise equilibrium prices and quantities at date 0 and complete our derivation of the fundamental equilibrium of this economy. The remainder of the paper then works out equilibrium prices and quantities for the intermediated case, i.e., where lenders no longer have direct access to the markets for risky assets and corporate bonds.

3 Endogenous lending and multiple equilibria

This Section and the following one derive the intermediated equilibrium (equilibria) of the economy, using a method similar to that used for the fundamental case above. The present Section solves for the equilibrium at date 1, and shows how the interplay between endogenous lending and the risk-shifting problem may lead to multiple equilibria. Section 4 then uses date 1 outcomes to derive the stochastic equilibria of the full model.

3.1 Market clearing at date 1

Contracted loan rate. Date 1 investors borrow B_1 (≥ 0) from lenders, which they use to buy X_{S1} corporate bonds at price 1 and X_{R1} risky assets at price P_1 (so that $B_1 = X_{S1} + X_{R1}P_1$). The use of debt contracts with limited liability allows investors to default, and earn 0, when their total payoff at date 2, $r_1X_{S1} + RX_{R1}$, is less than the amount owed to lenders, $r_1^l B_1$. Thus, the terminal consumption of date 1 investors is:

$$\max [r_1X_{S1} + RX_{R1} - r_1^l B_1, 0] = \max [X_{R1} (R - r_1P_1) + B_1 (r_1 - r_1^l), 0].$$

Note from the latter equation that the contracted rate on loans between lenders and investors, r_1^l , must be equal to the interest rate on corporate bonds, r_1 . If $r_1 > r_1^l$, then investors would want to borrow an unlimited amount of funds from lenders and use them to buy corporate bonds; they would then reach the finite limit of available funds, and from then compete for loans until $r_1 = r_1^l$. If $r_1 < r_1^l$ then investors' loan demand would be nil, implying that the return on corporate bonds would be $r_1 = f'(0) = \infty$, a contradiction. Thus, any equilibrium in the markets for loans and corporate bonds must satisfy $r_1^l = r_1 = f'(X_{S1})$. At this loan rate, perfect competition amongst investors drives down the net return on trading corporate bonds to zero.

Asset prices and interest rate. Since $B_1 (r_1 - r_1^l) = 0$, investors' terminal consumption is simply $\max [X_{R1} (R - r_1P_1), 0]$. Because $X_{R1} (0 - r_1P_1) < 0$ for all $P_1 > 0$, investors default on loans when the asset payoff is 0, and this occurs with probability $1 - \pi$. Their expected date 2 consumption is thus $\pi X_{R1} (R^h - r_1P_1)$, provided they do not default when the asset payoff is R^h (i.e., provided $X_{R1} (R^h - r_1P_1)$ is non-negative, as is always the case in equilibrium). Given their objective of maximising expected terminal consumption, market clearing for the

risky asset implies that its equilibrium price must be:

$$P_1 = R^h/r_1. \quad (10)$$

If the price of the asset were lower (higher) than R^h/r_1 , then $R^h - r_1P_1$ would be positive (negative) for all positive values of X_{R1} and date 1 investors would want to buy infinitely many (zero) risky assets. Given (10), investors' consumption when $R = R^h$ is $X_{R1}(R^h - r_1P_1) = 0$. The reason for this is intuitive: because markets are competitive, investors must make zero expected profits on trading risky assets. Since they earn zero when $R = 0$ and they default, they must also earn zero when $R = R^h$, which is ensured by the equilibrium price (10). Thus, in equilibrium the terminal consumption of date 1 investors is zero under both possible values of R at date 2.

Using equation (10) and the fact that in equilibrium $X_{R1} = 1$ and $r_1 = f'(X_{S1})$, we have $r_1 = f'(B_1 - P_1)$. Market clearing for corporate bonds at date 1 then implies:

$$f'^{-1}(r_1) + R^h/r_1 = B_1. \quad (11)$$

From the hypothesised properties of $f(\cdot)$, equation (11) uniquely defines the equilibrium interest rate for all positive values of B_1 . The implied interest rate function, $r_1(B_1)$, is continuous and such that $r'_1(B_1) < 0$, $r_1(0) = \infty$ and $r_1(\infty) = 0$. Equations (10)–(11) then fully characterise the intermediated equilibrium price vector at date 1, (P_1, r_1) , *conditional* on the amount of aggregate lending, B_1 .

Note from (5) and (11) that, for a given quantity of savings B_1 , the intermediated interest rate, r_1 , is higher than its fundamental analogue, r_1^F . This can be explained as follows. For a given value of B_1 , the expected asset payoff that accrues to investors in the intermediated equilibrium, R^h , is higher than the expected payoff to lenders in the fundamental equilibrium, πR^h . In consequence, risky assets are bid up in the intermediated equilibrium and safe asset investment, X_{S1} , is crowded out, which in turn raises the equilibrium interest rate, r_1 (relative to the fundamental rate, r_1^F). The intermediated equilibrium is thus characterised by *risk shifting*, in the sense that portfolio delegation to debt-financed investors leads to an excessive share of risky asset investment, and too little safe asset investment, relative to the efficient portfolio (i.e., the fundamental equilibrium). The implications of this distortion for equilibrium asset prices and savings are analysed further in Section 3.4.

3.2 Expected return on loans

Given lenders' utility functions, individual lending decisions at date 1 depend on the expected return on the loans they make to investors, denoted ρ_1 , as compared to the gross rate of time preference, $1/\beta$. Note that ρ_1 in general differs from the contracted loan rate, r_1 , because of the possibility that date 1 investors default on loans at date 2.

When date 1 investors do not default on loans (i.e., when $R = R^h$), the contracted loan rate applies and they repay lenders $r_1 B_1$. When they do default, lenders gather the residual value of investors' portfolio, i.e., the capitalised value of corporate bonds, $r_1 X_{S1} = r_1 (B_1 - P_1)$. The *ex ante* unit loan return is thus $\pi r_1 + (1 - \pi) r_1 (1 - P_1/B_1)$ or, using (10) and the interest rate function $r_1 = r_1(B_1)$,

$$\rho_1(B_1) = r_1(B_1) - \frac{(1 - \pi) R^h}{B_1} (> 0). \quad (12)$$

Note from equations (5), (11) and (12) that the probability that investors go bust at date 2, $1 - \pi$, indexes the distance between the contracted and actual *ex ante* returns on savings, r_1 and ρ_1 . When $\pi = 1$ the risk-shifting problem disappears since portfolio investors never default; the intermediated loan return, $\rho_1(B_1)$, is then identical to the contracted loan rate, $r_1(B_1)$, which in turn equals the fundamental return, $r_1^F(B_1)$; in this case, the date 1 intermediated equilibrium is uniquely determined by equations (6)–(7). When $\pi < 1$, investors' and lenders' incentives become misaligned, and a gap $(1 - \pi) R^h/B_1 > 0$ appears between r_1 and ρ_1 . Thus, $1 - \pi$ measures both the severity of the risk-shifting problem in the economy (i.e., the extent to which investors take more risk than if they were playing with their own funds) and the implied distortion in the intermediated return on loans (i.e., $r_1 - \rho_1$).

To analyse the existence and properties of the intermediated equilibrium when $\pi < 1$, we have to characterise the behaviour of $\rho_1(B_1)$ as total loans, B_1 , vary over $(0, \infty)$. First, note that $\rho_1(B_1)$ is continuous and such that $\rho_1(\infty) = 0$ and $\rho_1(0) = \infty$.⁹ Although this implies that $\partial\rho_1(B_1)/\partial B_1$ must be negative somewhere, the two terms on the right-hand side of (12) indicate that, over a given interval $[B_a, B_b] \subset (0, \infty)$, the change in $\rho_1(B_1)$ as a function of B_1 is of ambiguous sign.

The first term of the right-hand side of (12), $r_1(B_1)$, is the (decreasing) interest rate

⁹That $\rho_1(0) = \infty$ can be seen from the facts that $r_1(0) = \infty$ (see (11)) and $X_{S1}/B_1 \geq 0$ in (14).

function defined by equation (11): an increase in B_1 raises the amount invested in the safe asset, X_{S1} , which reduces the equilibrium interest rate, $r_1 = f'(X_{S1})$, and thus the average return on loans; this is the usual ‘marginal productivity effect’ of aggregate savings on the loan return. In contrast, the second term, $-(1 - \pi) R^h/B_1$, *increases* with B_1 ; this latter effect reflects the impact of the total loan amount on the average riskiness of loans as the *composition* of the optimal portfolio varies with B_1 . To analyse this second effect in more detail, first use (11) to write the relationship between safe asset investment, X_{S1} , and aggregate lending, B_1 , as follows:

$$B_1 = X_{S1} + R^h/f'(X_{S1}). \quad (13)$$

From (13) and assumption (1) regarding the concavity of $f(\cdot)$, it is easy to check that an increase in B_1 raises *both* the quantity of safe assets, X_{S1} , and the share of safe asset investment in investors’ portfolio, X_{S1}/B_1 (i.e., it lowers $B_1/X_{S1} = 1 + R^h/X_{S1}f'(X_{S1})$). In other words, even though an increase in B_1 lowers r_1 and thus raises asset prices, R^h/r_1 , the relative size of risky asset investment, $P_1/B_1 = 1 - X_{S1}/B_1$, tends to *decrease* as B_1 increases. This ‘portfolio composition effect’ in turn limits the loss to lenders in case of investors’ default and tends to raise the *ex ante* return on loans.

Given these two effects, the crucial question is: Are there intervals of B_1 over which $\rho_1(B_1)$ may be increasing, i.e., where the portfolio composition effect dominates the marginal productivity effect? To get an insight into the conditions under which this is the case, solve (11) for R^h and substitute the resulting expression into (12) to obtain:

$$\rho_1(B_1) = r_1(B_1) (\pi + (1 - \pi) (X_{S1}/B_1)). \quad (14)$$

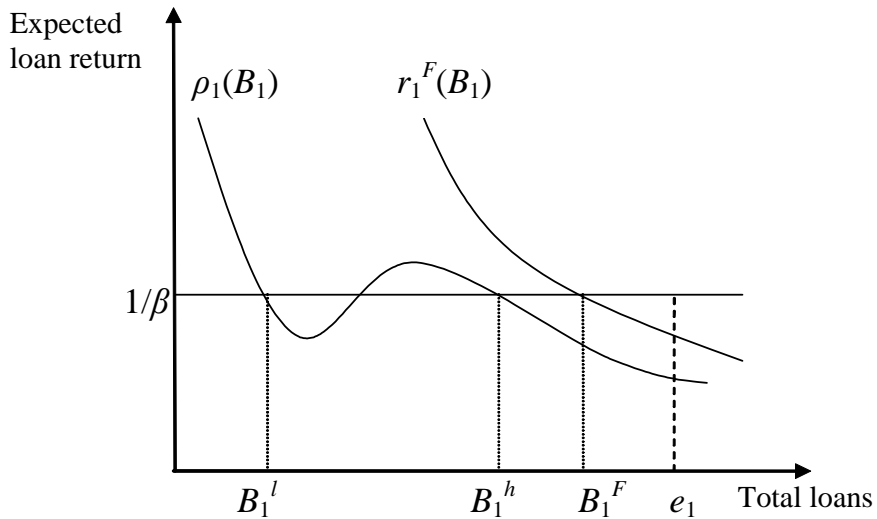
Both effects are made explicit in (14). Intuitively, for the increase in X_{S1}/B_1 to dominate the decrease in $r_1(B_1)$ induced by a marginal increase in B_1 , $1 - \pi$ must be sufficiently large (i.e., the risk-shifting problem must be large enough), and $-r'_1(B_1) (> 0)$ must be not too large (i.e., the marginal productivity effect must be weak enough). When this is the case, ‘strategic complementarities’ (in the sense of Cooper and John, 1988) in lending decisions appear, as a symmetric decision by other lenders to increase their loans may then lead any individual lender to do the same. Proposition 1 formally establishes the conditions for such complementarities to occur in the general case, as well as for a more specific class of production functions.

Proposition 1 (Strategic complementarities). The loan return curve, $\rho_1(B_1)$, is increasing in total loans, B_1 , provided π and $-f''(x)$ are not too large. In the isoelastic case where $f(x) = x^{1-\eta}/(1-\eta)$, $\eta \in (0, 1)$, $\rho_1(B_1)$ has exactly one (zero) increasing interval if

$$2\eta + \sqrt{\pi} < (\geq) 1.$$

The proof is in the Appendix. For a general function $f(\cdot)$, there may be several intervals of B_1 over which $\rho_1(B_1)$ is increasing, i.e., over which the implied $-f''(X_{S1})$ is sufficiently small (provided π is not too large). In the isoelastic case, a high value of η increases the curvature of $f(\cdot)$ and strengthens the marginal productivity effect; thus, neither π nor η must be too large for the portfolio composition effect to dominate the marginal productivity effect. In the remainder of the paper, we shall focus on a particularly simple case of non-monotonicity by assuming that $\rho_1(B_1)$ has one single increasing interval, as depicted in Figure 1, and as implied by the isoelastic case when $2\eta + \sqrt{\pi} < 1$ (all of our results generalise straightforwardly to the case of multiple increasing intervals).

Figure 1: Loan market equilibrium at date 1



3.3 Loan market equilibrium

Having characterised the *ex ante* loan return, ρ_1 , as a function of aggregate loans, B_1 , we may now analyse the way the latter is determined in equilibrium. At date 1, lenders choose the

individual level of loans, \hat{B}_1 , that maximises expected utility, $c_1 + \beta E_1 c_2$, taking $\rho_1 = \rho_1(B_1)$ as given. Given the lenders' utility function, they find it worthwhile to increase (decrease) savings whenever $\rho_1 > (<) 1/\beta$. Any interior equilibrium must thus satisfy $\rho_1 = 1/\beta$. We focus on symmetric Nash equilibria, where consumption/savings plans are identical across lenders (i.e., $\hat{B}_1 = B_1$) and no lender finds it worthwhile to individually alter his own plan.

Figure 1 shows how multiple intersections between the $\rho_1(B)$ -curve and the $1/\beta$ -line, when they occur, give rise to multiple equilibria.¹⁰ B_1^l and B_1^h represent two stable levels of aggregate lending, i.e., where a symmetric marginal move away from equilibrium by all lenders alters the loan return in such a way as to move the economy back to equilibrium. The value of B_1 where the $\rho_1(B_1)$ -curve crosses the $1/\beta$ -line from below is not stable and will not be discussed any further (starting from this point, an arbitrarily small increase (decrease) in B_1 tends to increase (decrease) $\rho_1(B_1)$, triggering a further move away from equilibrium). In both stable equilibria the *ex ante* return on loans is $1/\beta$, and lenders (expected) date 2 consumption, conditional on the selection of equilibrium j , $j = l, h$, is $\rho_1(B_1)B_1^j = B_1^j/\beta$.

Recall from equation (13) that an increase in B_1 lowers marginal productivity but also reduces the share of risky assets in investors' portfolios. The low-lending equilibrium is thus characterised by a high safe return but a high share of risky assets in the portfolio, while the high-lending equilibrium exhibits a low safe return but a safer average portfolio. Finally, notice that even though both equilibria yield the same *ex ante* return on loans, $1/\beta$, they are always associated with different levels of interest rates, asset prices, productive investment, and (expected) date 2 output: equation (11) and the fact that $B_1^h > B_1^l$ implies that $r_1(B^h) < r_1(B^l)$. Then, denoting P_1^j the asset's price, X_{S1}^j productive investment, and $E_1(Y|j)$ expected date 2 output (in the sense of the total quantity of goods available for agents' consumption) when total lending is B_1^j , we have:

$$\begin{aligned} P_1^h &= R^h/r_1(B^h) > P_1^l = R^h/r_1(B^l), \\ X_{S1}^h &= f'^{-1}(r_1(B^h)) > X_{S1}^l = f'^{-1}(r_1(B^l)), \\ E_1(Y|h) &= f(X_{S1}^h) + \pi R^h > E_1(Y|l) = f(X_{S1}^l) + \pi R^h. \end{aligned}$$

¹⁰Assumption (2), together with the fact (as proved and analysed further in Section 3.4) that $B_1^h < B_1^f$, ensures that both B_1^l and B_1^h are interior solutions which are independent of the amount of goods that lenders receive from the loans they made at date 0. Any income from these loans is thus consumed at date 1 (the effects of date 0 loans on lenders' date 1 wealth and consumption are analysed in Section 4.2).

In short, the selection of the low-lending equilibrium raises the interest rate and depresses asset prices, productive investment, and future output, relative to the equilibrium with high lending. (More generally, there may be more than two stable equilibria if $\rho_1(B_1)$ has more than one increasing interval, but their properties are similar to the 2-equilibrium case, i.e., the higher is B_1 , the lower is $r_1(B_1)$, and the higher are P_1 , X_{S1} and $E_1(Y)$).

3.4 Comparison with the fundamental equilibrium

We emphasised above that the risk-shifting problem arising under market segmentation leads investors to overinvest in risky assets, relative to the fundamental equilibrium. Proposition 2 summarises the implications of this distortion for the price of the risky asset and the amount of aggregate saving in equilibrium.

Proposition 2 (Asset bubbles and crowding out). In both intermediated equilibria, asset prices are higher than in the fundamental equilibrium (i.e., $P_1^j > P_1^F$, $j = l, h$), while aggregate savings are lower than in the fundamental equilibrium (i.e., $B_1^j < B_1^F$, $j = l, h$).

The proof is in the Appendix. That $P_1^j > P_1^F$, $j = l, h$, indicates that assets are overpriced at date 1 in both intermediated equilibria, i.e., both equilibria are associated with a positive *bubble* in asset prices (the bubble being larger, the larger is aggregate credit). Because investors are protected against a bad value of the asset payoff by the use of simple debt contracts, they bid up the asset, with the consequence of raising its price and its share in equilibrium portfolios (relative to the fundamental equilibrium).

The reason why savings are lower in both intermediated equilibria than in the fundamental equilibrium (i.e., $B_1^l < B_1^h < B_1^F$) follows naturally: excessive risky asset investment by portfolio investors implies that, for any given level of savings B_1 , the intermediated *ex ante* loan return, $\rho_1(B_1)$, is *lower* than the fundamental return, $r_1^F(B_1) = 1/\beta$ (see our analysis in Section 3.1). Lenders thus optimally reduce lending in the intermediated equilibrium (relative to the fundamental one) up to the point where this intermediated return equals the fundamental return, i.e., the gross rate of time preference $1/\beta$. Note, as a consequence, that a double crowding out effect is in fact at work on X_{S1} in the intermediated equilibrium. First, for a given level of aggregate savings B_1 , bubbly asset prices crowd out safe asset investment, X_{S1} , which raises the equilibrium interest rate, $r_1 = f'(X_{S1})$. Second, lenders' optimal re-

action to the resulting price distortion is to reduce savings, B_1 , which lowers X_{S1} (and raises r_1) even further. The crowding out of productive investment by the asset bubble is the basic source of output loss in the intermediated economy, relative to the fundamental equilibrium. The implications of this loss as to the welfare ranking of the (many) intermediated equilibria are analysed in the context of the full stochastic model below.

4 Self-fulfilling financial crises

The previous Section showed that the excessive risk taking of portfolio investors may lead, under endogenous credit, to the existence of multiple equilibria at date 1 associated with different levels of aggregate lending, interest rates, and asset prices. We now analyse the full time span of the model to demonstrate the possibility of a *self-fulfilling financial crisis* associated with the risk that the low-lending equilibrium is selected.

4.1 Market clearing at date 0

Crisis equilibria are constructed by randomising over the two possible lending equilibria that may prevail at date 1. More specifically, assume that, from the point of view of date 0, high lending is selected with probability $p \in (0, 1)$ at date 1, so that the ‘sunspot’ on which agents coordinate their expectations causes lending and asset prices to drop down to low levels with probability $1 - p$. With this specification for extraneous uncertainty at the intermediate date, the model potentially has a *continuum* of stochastic equilibria indexed by the *ex ante* probability of a market crash, $1 - p$. Since the asset’s price at date 1 is the asset payoff accruing to date 0 investors, this uncertainty about asset prices creates a risk-shifting problem at date 0 similar to that created at date 1 by intrinsic uncertainty about the terminal payoff of the asset. This causes the asset to be bid up at date 0, with the possibility that a self-fulfilling crisis (i.e., a drop in asset prices forcing date 0 investors into bankruptcy) occurs at date 1 if the low lending/low asset prices equilibrium is selected at that date.¹¹

¹¹For the sake of conciseness, we focus on equilibria where financial crises may actually occur at date 1 (i.e., where date 0 investors may go bankrupt), and thus leave out of the analysis equilibria with deterministic date 1 outcomes, i.e., $p = 1$ (high lending is selected for sure) and $p = 0$ (low lending for sure).

Contracted loan rate. Denote (P_0, r_0) the equilibrium price vector, r_0^l the contracted loan rate, and (X_{S0}, X_{R0}) the portfolio of date 0 investors, all at date 0. Limited liability and the portfolio constraint $B_0 = X_{S0} + P_0 X_{R0}$ imply that investors' terminal consumption is:

$$\max [r_0 X_{S0} + P_1 X_{R0} - r_0^l B_0, 0] = \max [X_{R0} (P_1 - r_0 P_0) + B_0 (r_0 - r_0^l), 0],$$

where, given our specification for extraneous uncertainty about aggregate lending and asset prices, P_1 is a random variable at date 0, taking on the value P_1^h with probability p (i.e., B_1^h is selected), and P_1^l otherwise (B_1^l is selected), at date 1. The contracted rate on loans at date 0, r_0^l , must necessarily be equal to the rate on corporate bonds at the same date, r_0 . If the former were lower (higher) than r_0 , then date 0 investors would want to borrow infinitely many (zero) units of goods and use them to buy corporate bonds, while the loan supply at date 0 is exactly e_0 (the gross expected return on loans at date 0 is always non-negative, because the liquidation value of date-0 portfolios cannot be negative). Thus, any equilibrium must satisfy $r_0 = r_0^l$ and $B_0 = e_0$.

Asset prices and interest rate. In the equilibria that we are considering, date 0 investors default on loans when the asset price at date 1 is P_1^l , but not when it is P_1^h . Since $B_0 (r_0 - r_0^l) = 0$, their terminal consumption is $X_{R0} (P_1^h - r_0 P_0) \geq 0$ with probability p , and 0 otherwise. Date 0 investors choose the level of X_{R0} that maximises expected consumption, $p X_{R0} (P_1^h - r_0 P_0)$, while any potential solution to their decision problem must be such that they do not default on loans if the asset price at date 1 is P_1^h , but do default if it is P_1^l , i.e.,

$$P_1^h - r_0 P_0 \geq 0, P_1^l - r_0 P_0 < 0. \quad (15)$$

The demand for risky assets by date 0 investors, X_{R0} , is infinite (zero) if $P_1^h - r_0 P_0 > 0 (< 0)$. Market clearing thus requires that the equilibrium price of the risky asset be:

$$P_0 = P_1^h / r_0, \quad (16)$$

which satisfies both inequalities in (15). Again, the interpretation of this equilibrium price is straightforward. Perfect competition for the risky asset by investors implies an asset price such that they make zero expected profit. Because they make zero profit from holding risky assets when the asset payoff is P_1^l (i.e., when they default), they must also earn zero when it is P_1^h ; this is exactly what the equilibrium price P_1^h / r_0 ensures.

Aggregate lending from date 0 to date 1 is e_0 . In equilibrium we have $X_{R0} = 1$ and $r_0 = f'(X_{S0}) = f'(e_0 - P_0)$. Thus, r_0 is uniquely determined by the following equation:

$$f'^{-1}(r_0) + P_1^h/r_0 = e_0, \quad (17)$$

where $P_1^h = R^h/r_1(B_1^h)$ is independent of e_0 , due to the interiority of B_1^h allowed by assumption (2). Note from (16)-(17) that the equilibrium price vector at date 0, (P_0, r_0) , is uniquely determined and does not depend on the probability of a crisis, $1 - p$: as date 0 investors are protected against a bad shock to the value of their portfolio by the use of simple debt contracts, they simply disregard the lower end of the payoff distribution (i.e., the payoff P_1^l with probability $1 - p$) when selecting their optimal portfolio.

Asset bubbles and crowding out. We complete this Section by showing that the risk-shifting problem due to date 1 extraneous uncertainty and the limited liability of date 0 investors causes asset prices to be overvalued at date 0, and to crowd out real investment at that date, X_{S0} . From (8)–(9) and (16)–(17), the mispricing of risky assets at date 0 is given by:

$$P_0 - P_0^F = f'^{-1}(r_0^F) - f'^{-1}(r_0).$$

Using (9) and (17), together with the fact that $P_1^h > P_1^F$ (which was established in Proposition 2), it is easily seen that $r_0 > r_0^F$. Since $f'^{-1}(\cdot)$ is decreasing, $P_0 - P_0^F > 0$ and there is a positive asset price bubble at date 0. Note that e_0 being exogenously given, the amount of crowding out caused by this bubble is simply $X_{S0}^F - X_{S0} = P_0 - P_0^F$. The implied lower level of capital investment at date 0 in turn lowers date 1 output, $f(X_{S0})$, in the same way as date 2 (expected) output, $f(X_{S1}) + \pi R^h$, was lowered by the asset bubble at date 1.

4.2 The wealth effect of crises

Having shown the existence of a continuum of stochastic equilibria indexed by the probability of a self-fulfilling crisis, we are now in a position to study the welfare properties of these equilibria in more details. We first analyse the way crises affect lenders' wealth and intertemporal consumption plans, and then turn to the effect of crises on other agents' utility.

To see why lenders' wealth at date 1 is contingent on whether a crisis occurs at date 1 or not, we calculate how it is affected by the possible default of date-0 investors. When these investors do not default, they owe lenders the capitalised value of outstanding debt at date

1, r_0e_0 . As lenders receive an endowment e_1 at date 1, their date 1 wealth if no crisis occurs is simply $W_1^h = e_1 + r_0e_0$. When investors do default, on the contrary, lenders' wealth at date 1 is their date 1 endowment, e_1 , *plus* the residual value of the date 0 investors' portfolio, $r_0X_{0S} + P_1^l$. Using (17), lenders' date 1 wealth, W_1^j , conditional on whether a crisis occurs ($j = l$) or not ($j = h$), is thus given by:

$$W_1^j = e_1 + r_0X_{S0} + P_1^j, \quad j = l, h. \quad (18)$$

Obviously, the total quantity of goods available at date 1 is the same across equilibria, because initial capital investment, X_{S0} , is uniquely determined (i.e., it does not depend on p). This quantity amounts to lenders' date 1 endowment, e_1 , *plus* entrepreneurs' production, $f(X_{S0})$, the latter being shared between date 0 entrepreneurs, who gather the surplus $f(X_{S0}) - r_0X_{S0}$ in competitive equilibrium, and lenders, who receive r_0X_{S0} (recall that P_0 is such that date 0 investors consume zero whether $P_1 = P_1^l$ or P_1^h).¹²

From condition (2) and the second inequality stated in Proposition 2, we have $B_1^j < B_1^F < W_1^j$, $j = l, h$, implying that both possible levels of wealth give rise to interior solutions for consumption-savings plans at date 1 where $\rho_1(B_1^j) = 1/\beta$. If a crisis occurs at date 1, then lenders' wealth and savings at that date are W_1^l and B_1^l , respectively, while their date 1 and (expected) date 2 consumption levels are $W_1^l - B_1^l$ and B_1^l/β , respectively; it follows that their discounted utility flow from date 1 on is simply $W_1^l - B_1^l + \beta \times B_1^l/\beta = W_1^l$. Similarly, if a crisis does *not* occur at date 1, then lenders' date 1 and date 2 consumption levels are $W_1^h - B_1^h$ and B_1^h/β , respectively, yielding a discounted utility from date 1 on of W_1^h . Weighing these possible outcomes with the probabilities that they actually occur, and then using (18), we find that lenders' *ex ante* utility (i.e., from the point of view of date 0) depends on the crisis probability, $1 - p$, as follows:

$$\begin{aligned} E_0W_1 &= pW_1^h + (1 - p)W_1^l \\ &= e_1 + r_0X_{S0} + pP_1^h + (1 - p)P_1^l. \end{aligned}$$

¹²There are two equivalent ways of characterising lenders' budget sets at date 1: looking at their *wealth*, W_1^j is assigned to date 1 consumption and date 1 lending, so that, using (18), $W_1^j = e_1 + r_0X_{0S} + P_1^j = c_1^j + B_1^j$, $j = l, h$; the total quantity of *goods* accruing to lenders at date 1 is ultimately shared between date 1 consumption, c_1^j , and date 1 capital investment, X_{S1}^j , so that $e_1 + r_0X_{0S} = c_1^j + X_{S1}^j$, $j = l, h$. Since $B_1^j = X_{S1}^j + P_1^j$, these two formulations are, obviously, mutually consistent.

E_0W_1 is decreasing in $1-p$, since $P_1^h > P_1^l$ and $e_1+r_0X_{S0}$, P_1^l and P_1^h do not depend on p . Note that it is the selection of the low-lending equilibrium itself that triggers the crisis which lowers lenders' wealth and discounted consumption flow. Thus, the utility loss incurred by lenders when a crisis occurs is akin to a pure coordination failure in consumption/savings decisions –rather than an exogenously assumed destruction of value associated with the early liquidation of the long asset, as is often assumed in liquidity-based theories of financial crises (e.g., Diamond and Dybvig, 1983, Allen and Gale, 2000, and Chang and Velasco, 2002).

4.3 Aggregate welfare

We can now complete the welfare analysis of the model by studying the effect of the *ex ante* crisis probability on other agents' consumption. With respect to *investors*, Sections 3.1 and 4.1 have established that both date 0 and date 1 investors consume zero in equilibrium, whatever the realisation of extrinsic (date 1) and fundamental (date 2) uncertainty. Investors' *ex ante* welfare is thus zero in all equilibria. With respect to *entrepreneurs*, the terminal consumption of date-1 entrepreneurs is $f(X_{S1}) - X_{S1}f'(X_{S1})$, which is increasing in X_{S1} . Since $X_{S1}^h > X_{S1}^l$ (see Section 3.3), their *ex ante* welfare, from the point of view of date 0, is $p(f(X_{S1}^h) - X_{S1}^h f'(X_{S1}^h)) + (1-p)(f(X_{S1}^l) - X_{S1}^l f'(X_{S1}^l))$, which decreases with $1-p$. Date 0 entrepreneurs consume $f(X_{S0}) - f'(X_{S0})X_{S0}$, where $X_{S0} = f'^{-1}(r_0)$ does not depend on p . Finally, *initial asset holders'* consumption is just the selling price of the asset at date 0, P_0 , which is independent of p . In short, neither investors nor initial asset holders or date 0 entrepreneurs are affected by the crisis probability. Lenders are, because the crisis reduces their asset wealth and discounted consumption flow. Date 1 entrepreneurs are, because low lending reduces their surplus and terminal consumption. We summarise the main results derived from Section 4 in the following proposition.

Proposition 3 (Crises and welfare). When multiple levels of lending may prevail at date 1, the model has a continuum of stochastic equilibria indexed by the probability of a self-fulfilling crisis, $1-p \in (0, 1)$. The higher this probability, the lower is *ex ante* welfare.

5 Extensions

Propositions 1-3 were derived under stark simplifying assumptions about agents' preferences and the technologies that are available to them. We now test the robustness of our results by relaxing two significant hypotheses, namely, i) the absence of investment opportunities other than risky lending available to lenders, and ii) the risk-neutrality of all agents.

5.1 Storage and 'flight to quality'

Our baseline model specification implied that lenders' choices at date 1 were limited to either consumption or risky lending to investors. Assume instead that lenders may protect themselves from excessive risk-taking by investors by investing part of their wealth at the safe return $\tau > 0$. The latter may reflect the possibility for lenders to store wealth in the form of cash balances or government bonds in a closed economy; alternatively, it may be interpreted as the world interest rate faced by agents in a small, open economy. We show that the model with storage generates self-fulfilling crisis equilibria similar to those analysed in Sections 3-4, where all lenders symmetrically turn away from risky lending at date 1 to seek safer investment opportunities. In other words, a *flight to quality* (rather than a contraction in aggregate savings) triggers the market crash and financial crisis.

To keep this alternative formulation of the model concise, assume that the storage technology is available between dates 1 and 2 only.¹³ At date 0, lenders lend their entire date 0 wealth, e_0 , to investors as before. At date 1, they may now spread their current wealth, W_1 , between loans to investors, \hat{B}_1 , storage, \hat{S}_1 , and current consumption, $c_1 = W_1 - \hat{B}_1 - \hat{S}_1$.

Subgame equilibrium at date 1. The problems faced by entrepreneurs and investors at date 1 are exactly the same as those in the baseline model. Consequently, they yield the same equilibrium pricing equations (10)–(11) and implied *ex ante* loan return function (12). Given their current wealth level W_1 , lenders now choose the quantities (\hat{B}_1, \hat{S}_1) that maximise:

$$E_1(c_1 + \beta c_2) = W_1 - B_1 - \hat{S}_1 + \beta \left(\rho_1 \hat{B}_1 + \tau \hat{S}_1 \right),$$

subject to $\hat{B}_1 + \hat{S}_1 \leq W_1$, and taking τ and ρ_1 as given. If $\tau < 1/\beta$, then lenders will never find it worthwhile to store and thus choose $\hat{S}_1 = 0$, in which case all potential equilibria are

¹³Our results can be generalised to the situation where it is also available from date 0 to date 1, but the full analysis of this case requires substantial algebra without yielding any extra insights.

identical those analysed in Sections 2-4. We focus on the only robust interesting case by assuming:¹⁴

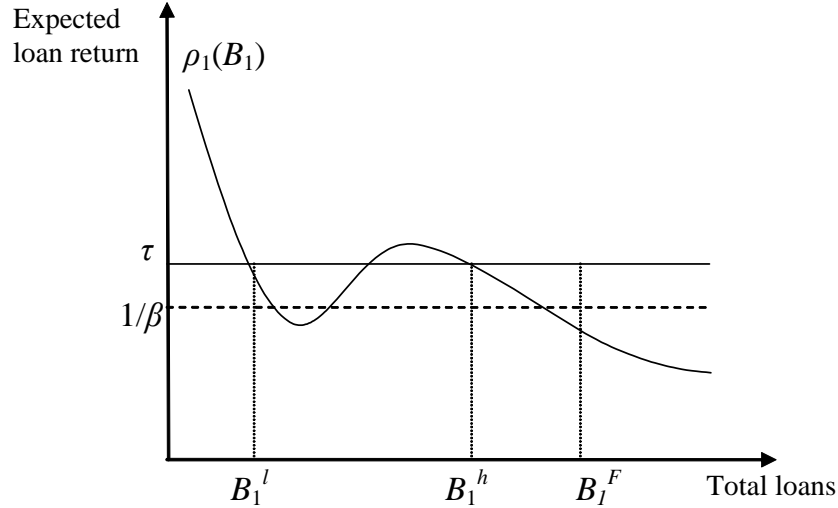
$$\tau > 1/\beta. \quad (19)$$

When (19) holds, lenders will never find it worthwhile to consume at date 1 and choose $c_1 = 0$. They thus maximise $E_1(c_2) = \rho_1 \hat{B}_1 + \tau \hat{S}_1$, subject to $W_1 = \hat{B}_1 + \hat{S}_1$. All symmetric, interior solutions to this problem are such that $\hat{B}_1 = B_1 > 0$, $\hat{S}_1 = S_1 > 0$, and

$$\rho_1(B_1) = r_1(B_1) - (1 - \pi) R^h / B_1 = \tau,$$

where $r_1(B_1)$ is defined by equation (11). It is easy to check that assumptions (2) and (19) imply that $e_1 > f'^{-1}(\tau) + \pi R^h / \tau$, which in turn ensures that every potential equilibrium is interior. Figure 2 shows how multiple intersections between the ρ_1 -curve and the τ -line, when they occur, give rise to multiple, symmetric Nash equilibria at date 1, associated with different levels of lending and storage. The only difference with the baseline model here is that the appropriate required rate of return, against which ρ_1 is compared by lenders, is now τ rather than $1/\beta$.

Figure 2: Loan market equilibrium with storage



Equilibrium at date 0. Just as in the baseline model, assume that the model has two stable subgame equilibria at date 1, B_1^l and B_1^h , and that, from the point of view of date 0, B_1^h is

¹⁴In the knife-edge situation where $\tau = 1/\beta$, B_1 and $S_1 + C_1$ depend on which equilibrium is selected, but neither S_1 nor C_1 are determined (i.e., lenders are indifferent between storing and consuming at date 1).

selected with probability $p \in (0, 1)$. It is easy to check that investors', entrepreneurs' and lenders' problems are exactly the same as those described in Section 4.1. Consequently, they yield the same price vector (P_0, r_0) as that implied by (16)-(17). Then, it is straightforward to show the the two statements contained in proposition 3 apply: the model with storage generates a continuum of stochastic of crisis equilibria, while aggregate welfare unambiguously decreases with the probability of crisis $1 - p$.

5.2 Risk-averse agents

The assumption of limited investor's liability, coupled with the hypothesis of all agents' risk neutrality, introduces a great deal of 'risk-loving' behaviour in the economy. This naturally raises the question whether our results are still valid when agents, especially lenders, are risk-averse. To investigate this case, assume that investors and entrepreneurs maximise a utility function $v(\cdot)$ of terminal consumption, defined over $(0, \infty)$ and such that $v'(\cdot) > 0$, $v''(\cdot) \leq 0$, while lenders' intertemporal utility is now $u(c_1, c_2) = c_1 + \beta v(c_2)$. Entrepreneurs' choices at dates 0 and 1 are not altered by this generalisation, since their terminal consumption is positive and deterministic. It is easy to check that investors' decisions are not modified either, relative to the risk-neutral case, provided they receive an (arbitrarily small) extra terminal endowment $\tilde{e} > 0$.¹⁵ At date 1, lenders now choose individual lending, \hat{B}_1 , which maximises $c_1 + \beta E_1 v(c_2)$, taking aggregate lending, B_1 , asset prices, P_1 , and the interest rate, r_1 , as given. If date 1 investors do not default, any individual lender having lent \hat{B}_1 receives the contractual repayment $r_1 \hat{B}_1$ at date 2. If investors do default, this lender is entitled to a share of the residual portfolio, $r_1 (B_1 - P_1)$, proportional to his share in investors' liabilities, \hat{B}_1/B_1 . Lenders thus solve:

$$\max_{\hat{B}_1} W_1 - \hat{B}_1 + \beta \left(\pi v(r_1 \hat{B}_1) + (1 - \pi) v \left(\hat{B}_1 \times \frac{r_1 (B_1 - P_1)}{B_1} \right) \right). \quad (20)$$

Recall that lenders' date 1 wealth, W_1 , is state contingent, as it depends on the capitalised value of lenders' date 0 loans (see Section 4.3). However any possible equilibrium value of

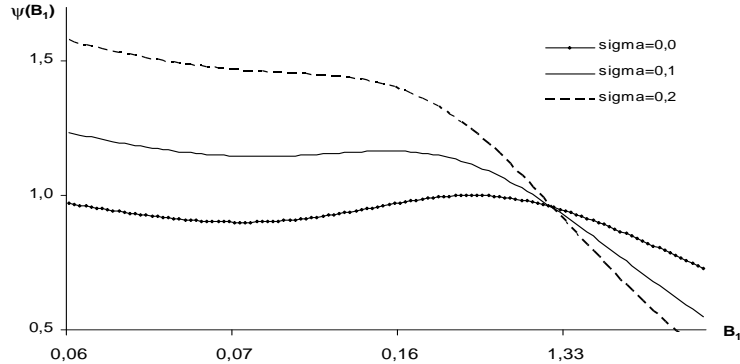
¹⁵The expected utility of date 1 investors is then $(1 - \pi) v(\tilde{e}) + \pi v(X_{R1}(R^h - r_1 P_1) + \tilde{e})$, yielding the asset demand $(R^h - r_1 P_1) v'(X_{R1}(R^h - r_1 P_1) + \tilde{e}) = 0$; in equilibrium $X_{R1} = 1$ and $R^h - r_1 P_1 = 0$ since $v'(\tilde{e})$ is positive and finite. Similarly, the date 0 investors' asset demand is such that $(P_1^h - r_0 P_0) v'(X_{R0}(P_1^h - r_0 P_0) + \tilde{e}) = 0$, yielding (16) in equilibrium. An alternative assumption is that $\tilde{e} = 0$ but $\lim_{x \rightarrow 0} x v'(x) = 0$.

W_1 can be made large enough, by increasing e_1 sufficiently, for all corresponding values of B_1 to be interior. Assuming interiority, solving (20) for \hat{B}_1 , and then using $P_1 = R^h/r_1$ and imposing symmetry across lenders ($\hat{B}_1 = B_1$), we find that any equilibrium lending level of the intermediated economy must satisfy:

$$\psi(B_1) \equiv \pi r_1 v'(r_1 B_1) + (1 - \pi) \left(r_1 - \frac{R^h}{B_1} \right) v'(r_1 B_1 - R^h) = \frac{1}{\beta}, \quad (21)$$

where, from investors' optimal portfolio choice, $r_1 = r_1(B_1)$ is defined by equation (11) above. Note that when $v(x) = x$ then $\psi(B_1) = \rho_1(B_1)$ and (21) is reduced to $\rho_1(B_1) = 1/\beta$, our equilibrium condition under risk neutrality. Thus, $\psi(\cdot)$ generalises the $\rho_1(\cdot)$ function for the risk-averse case, and can consequently be interpreted as the 'risk-corrected' *ex ante* return that lenders expect from their loans to investors (which is $1/\beta$ in equilibrium).

Figure 3: The risk-corrected expected loan return



The existence of multiple equilibria requires that $\psi(\cdot)$ be increasing over at least one interval of B_1 . Since we could derive no simple analytical condition ensuring that this is the case, we computed the $\psi(B_1)$ function numerically for the isoelastic case, where $f(x) = x^{1-\eta}/(1-\eta)$, $\eta \in (0, 1)$, and $v(x) = x^{1-\sigma}/(1-\sigma)$, $\sigma \geq 0$, for a variety of parameter values. We found that $\psi(B_1)$ may have an increasing interval if the risk-shifting problem is large enough (i.e., $1 - \pi$ is not too small), and if neither $f(\cdot)$ nor $u(\cdot)$ are too concave (i.e., neither η nor σ are too large). We know from proposition 2 and the discussion in Section 3.2 that high values of π or η are detrimental to multiple equilibria because they make it less likely that the portfolio composition effect dominate the marginal productivity effect; a positive value of σ strengthens the marginal productivity effect further by making lenders less willing to substitute current consumption, c_1 , for future risky consumption, c_2 .

For sake of illustration, Figure 3 represents the risk-corrected loan return curve when $\eta = R^h = 0.1$ and $\pi = 0.5$, for different values of σ ; As σ gradually increases, the increasingness of $\psi(\cdot)$ becomes less and less pronounced over the relevant range of B_1 , until $\psi(\cdot)$ decreases over the entire $(0, \infty)$ interval. Since date 0 equilibrium values are not affected by this generalisation, we conclude that multiple equilibria and self-fulfilling crises may still exist in the risk-averse economy, provided that lenders are not ‘too’ risk averse.

6 Concluding remarks

This paper offers a simple theory of self-fulfilling financial crises based on the excessive risk taking of debt-financed portfolio investors. In our model, the interplay between the amount of funds available to investors, the composition of their portfolio, and the return that they are able to offer in competitive equilibrium, creates a strategic complementarity between lenders’ savings decisions, which naturally give rise to multiple equilibria associated with different levels of lending, interest rates, asset prices and future output. Expectations-driven financial crises may then occur with positive probability as soon as the economy exhibits (at least) two possible equilibrium levels of lending, and the coordination of lenders on a particular equilibrium is determined by an extraneous ‘sunspot’. We showed that such crises are characterised by a self-fulfilling credit contraction, followed by a market crash, widespread failures of investors, and a fall in productive investment.

Apart from demonstrating that credit intermediation based on debt contracts is a potential source of endogenous financial instability, the model also provides new insights into the potential welfare costs of financial crises. In our model, the dramatic reduction in lending and asset prices associated with the crisis equilibrium has two implications. First, it brings about a reduction in lenders’ wealth due to a fall in the total value of their capitalised investment, which reduces their discounted consumption flow from the time of the crisis onwards. Second, the credit contraction associated with the crisis causes a fall in productive investment and output, and consequently reduces entrepreneurs’ profits and consumption. Thus, both savers and final producers are hurt by the financial crisis, while intermediate investors, whose risk is hedged by the use of debt contracts, are ultimately left unharmed.

Appendix

Proof of proposition 1

From equation (12), we have that $\partial\rho(B_1)/\partial B_1 > 0$ if and only if

$$-r'_1(B_1) B_1^2 < (1 - \pi) R^h. \quad (\text{A1})$$

Given π and R^h , (A1) may hold if $-r'_1(B_1)$ is small enough over some interval of B_1 , that is if the interest rate, r_1 , is not very responsive to changes in the implied level of safe asset investment, X_{S1} . This in turn holds if $f(X_{S1})$ is ‘flat enough’ over the relevant range of X_{S1} , so that $r_1 = f'(X_{S1})$ responds only little to changes in X_{S1} . Using (11), together with the fact that $\partial f'^{-1}(r_1)/\partial r_1 = 1/f''(X_{S1})$, the left-hand side of (A1) yields:

$$-r'_1(B_1) B_1^2 = \frac{(R^h + X_{S1} f'(X_{S1}))^2}{R^h + f'(X_{S1})^2 / (-f''(X_{S1}))} (> 0).$$

For $X_{S1} \in [\underline{X}, \bar{X}]$, i.e. when $B_1 \in [\underline{X} + R^h/f'(\underline{X}), \bar{X} + R^h/f'(\bar{X})]$, $-r'_1(B_1) B_1^2$ can be made gradually smaller by decreasing the curvature of $f(\cdot)$ over $[\underline{X}, \bar{X}]$; in this case $f'(X_{S1})$ is bounded both above and below, and $-f''(X_{S1})$ can be made arbitrarily small, producing a value of $-r'_1(B_1) B_1^2$ small enough for (A1) to hold (provided $\pi \neq 1$). The larger is $1 - \pi$, the more likely it is that inequality (A1) is satisfied, for a given $r_1(B_1)$ function.

Consider now the isoelastic case. When $f(X_{S1}) = X_{S1}^{1-\eta}/(1-\eta)$, equation (11) becomes $B_1(r_1) = r_1^{-1/\eta} + R^h r_1^{-1}$, which in turn implies:

$$r'_1(B_1) = \frac{1}{B'_1(r_1)} = \frac{1}{(-1/\eta) r_1^{-1-1/\eta} - R^h r_1^{-2}},$$

where $r_1 = r_1(B_1)$. From equation (12), $\partial\rho_1(B_1)/\partial B_1 > 0 (< 0)$ when $r'_1(B_1) + (1 - \pi) R^h/B_1^2 > 0 (< 0)$, that is, when

$$\frac{1}{(-1/\eta) r_1^{-1-1/\eta} - R^h r_1^{-2}} + \frac{(1 - \pi)/R^h}{(r_1^{-1/\eta} + R^h r_1^{-1})^2} > 0 (< 0).$$

Defining $Y \equiv r_1^{1-1/\eta}$ and rearranging, we find that $\rho_1(B_1)$ increases (decreases) when

$$\Psi(Y) = Y^2 + R^h \left(2 - \frac{1 - \pi}{\eta}\right) Y + \pi (R^h)^2 < 0 (> 0).$$

The expression $\Psi(Y)$ changes sign over $(0, \infty)$ if $\Psi(Y) = 0$ has two real roots, including at least one positive root. A necessary condition for this to hold is that the discriminant of

$\Psi(Y) = 0$ be positive, i.e., the following inequality must hold:

$$1 + 4\eta(\eta - 1) > \pi. \quad (\text{A2})$$

When (A2) holds, the roots Y_a, Y_b of $\Psi(Y) = 0$ are:

$$Y_{a,b} = \frac{R^h}{2} \left(\left(\frac{1-\pi}{\eta} - 2 \right) \mp \sqrt{\left(\frac{1-\pi}{\eta} - 2 \right)^2 - 4\pi} \right).$$

Both roots are positive (negative) if $1 - 2\eta > (<) \pi$. Combined with inequality (A2), this means that $\Psi(Y)$ changes signs over $(0, \infty)$ if and only if

$$2\eta + \sqrt{\pi} < 1. \quad (\text{A3})$$

$\Psi(Y)$ is negative for $Y \in (Y_a, Y_b)$, and positive for $Y \in (0, Y_a) \cup (Y_b, \infty)$. Since $Y = r_1^{1-1/\eta}$, this means that $\Psi(Y)$ is negative for intermediate values of r_1 and positive otherwise. Using (11) again, this in turn implies that, provided (A3) holds, $\rho_1(B_1)$ is strictly increasing for intermediate values of B_1 and strictly decreasing otherwise. When (A3) does not hold, then $\Psi(Y)$ is non-negative and $\rho_1(B_1)$ is decreasing or flat over $(0, \infty)$.

Proof of proposition 2

Comparing equations (6) and (10), we have that $P_1^j > P_1^F$, $j = l, h$, if and only if

$$\pi r_1(B_1^j) < 1/\beta, \quad j = l, h.$$

In equilibrium, $\rho_1(B_1^j) = 1/\beta$. Then, substituting (14) into the above inequality, we find that $P_1^j > P_1^F$ if and only if $X_{S1}^j/B_1^j > 0$, which is always true whether $j = l$ or h .

Turning to the second inequality in this proposition, first use $\rho_1(B_1^j) = 1/\beta$, together with equations (11) and (12), to rewrite B_1^j as follows:

$$B_1^j = r_1(B_1^j) f'^{-1}(r_1(B_1^j)) + \beta \pi R^h, \quad j = l, h.$$

Comparing the latter equation with (7), we find that $B_1^j < B_1^F$ if, and only if,

$$r_1(B_1^j) f'^{-1}(r_1(B_1^j)) < (1/\beta) f'^{-1}(1/\beta), \quad j = l, h.$$

$r_1 f'^{-1}(r_1)$ falls with r_1 since $f'^{-1}(r_1) + r_1 f'^{-1\prime}(r_1) = X_{S1} + f'(X_{S1})/f''(X_{S1})$ is negative by assumption (1). Thus, $r_1 f'^{-1}(r_1) < (1/\beta) f'^{-1}(1/\beta)$ if and only if $r_1(B_1^j) > 1/\beta$, $j = l, h$, which is necessarily true from (12) and the fact that $\rho_1(B_1^j) = 1/\beta$.

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