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Selective Immigration Policies, Human Capital Accumulation and Migration Duration in Infinite Horizon*

by

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Abstract

An increasing literature encourages the use of selective immigration policies as a tool to promote incentives to education. It is argued that, since not everybody is allowed to migrate, under these policies a poor country may well turn out with more human capital than in autarchy. The implicit assumption is that migrations are permanent. However, this assumption has recently been dropped: a large literature studies the optimal migration duration in an intertemporal framework. In our work we study how selective immigration policies affect the human capital accumulation and the migration duration. Unlike most of the existing literature, the probability of entering abroad is endogenous and our analisys is not limited to two periods: there is no reason to consider a single migration spell, and our infinite-horizon model includes an aggregate shock as a source of constrained migration. Contrary to the "brain gain with a brain drain" reasoning, we show that selective policies may be harmful for human capital accumulation. As a consequence, their effectiveness is questionable, and they may produce a "brain loss" rather than a brain gain. Besides, borders closure backfires on migration duration especially for unskilled workers.

Keywords: return migration, human capital, brain drain.

JEL Classification: F 200, F220.

1 Introduction

The role of international migrations in international factor movements has always been peculiar: unlike flows of capital or goods, inflows of immigrants can generate frictions with natives and xenophobia, particularly when combined to high unemployment.

The eastward enlargement of the European Union is going to add approximately 50 millions of people to the existing labor force. Large and persistent wage diffentials support the incentives for extensive mass migration from low-wage, densely populated countries, to the developed world. This is the case of the Latin America with respect to the U.S., and of North Africa with respect to the E.U. (Lundborg and Segerstrom 2002). Concerns about the effects of mass immigration push the governments of destination countries to raise entry barriers, and the governments of source countries to be concerned about the risk of a brain drain. OECD (1999) reports that in the years 1997-98 many countries have modified their entry regulations, essentially "to reinforce borders control and restrict the requirements of entry, residence and work". OECD (2001) confirms this legislative trend.

Entry barriers based on human capital requirements have an important effect on the migrants' behaviour, and they are renewing the brain drain concerns[1]. Some recent contributions, however, (Mountford, 1997; Stark and Wang, 2002) argue that, as long as there exist a severe immigration restriction, source countries may benefit as well from the incentive to migrate, because eventually most human capital is retained within the homeland. However, as it is well-known in the literature, what really matters for a brain drain to exist is that the emigration decision is permanent.

Most of the early economic analyses of migrations rely on wage differentials in a static context (Sjaastadt 1962; Harris and Todaro, 1970). In these models, migration towards the rich countries increases with wage differentials. Within such a framework, the decision to migrate can only be permanent and voluntary. Both these assumptions are at odds with reality, the first because there are both inflows and outflows of migrants, and the second because migration waves are also driven by aggregate shocks, like wars, macroeconomic crises, climate changes.

Temporary migration is receiving an ever-increasing attention in the literature.

[1] For a recent survey about the brain drain literature we refer to Commander et al. (2003).

Dustmann (2001) points out that "temporary migrations are not uncommon, and often they are the rule rather than the exception", even in spite of persistent higher wages in the destination country[2]. OECD (1999, 2001) dedicates as well particular attention to this subject. As for the importance of aggregate shocks, OECD (2001) emphasizes the role of the recent regional conflicts multiplication in increasing the migratory flows. It is well-known that the emigration wave from Europe in the 1840s was associated with famine and revolutions. Bonifazi and Strozza (2001) describe the huge population relocation occurred in the decade following World War II. More recently, Africa civil wars typically displaced 64 per thousand of the population per year. In 1975, the fall of Saigon produced a large scale exodus from Vietnam, Laos and Cambodia, and, twenty years later, the disintegration of Yugoslavia generated large flows to the E.U. (Chiswick and Hatton 2002).

A satisfactory model should deal with the possibility of forced emigration. Moreover, once selective policies are established, the migration duration and human capital accumulation decisions can't be considered separately. Unlike most of the existing literature, we are able to account for both voluntary and constrained migration: wage differentials cause the former and aggregate shocks cause the latter. The importance of these factors for studying migration from poor to rich countries is self-evident. In our model a country-specific shock affecting the source country can generate a migration wave.

To give an idea of the macroeconomic risk associated to several developing countries, in Table 1 we reproduce a table from "The Economist" (may 27th 2004). "The rankings combine measures of political risk (such as the threat of war) and economic risk (such as the size of fiscal deficits). They also include measures that affect a country's liquidity and solvency (eg, its debt structure and foreign-exchange reserves)". Similar indicators are widely used in the business community and in the academic literature (Beine et al., 2003; Easterly and Levine, 1997).

Another common assumption in the literature on return migration is that the return decision is permanent[3]. From a theoretical perspective, it is not evident why migration should be limited to a single spell. There exists, indeed, clear evidence that

[2] We refer to Dustmann (2001) for further references about temporary migrations.

[3] See for example Galor and Stark (1990), and Dustmann (1997). See Hill (1987) for a model of multiple migrations.

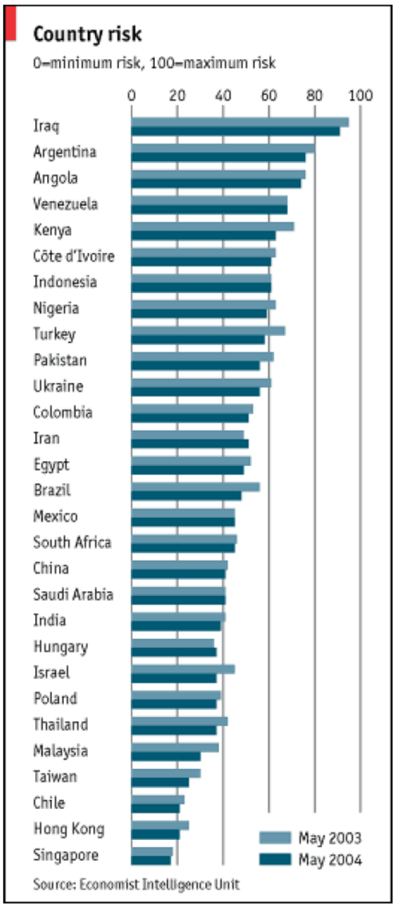

Figure 1: Table 1

even at the end of the XIX century repeated migration spells of 3-4 years were not uncommon (Baines 1991)[4]. According to Chiswick and Hatton (2002), over the same period the outflow of returning migrants from the U.S. grew from less than 10% up to 30% of the inflow. In more recent times, similar results are reported by Byerlee (1974) for African migrants, and by Cornelius (1978) and North and Houstoun (1976) for Mexican ones.

In this paper we try to overcome this restriction, and we are able to study the migratory behaviour and the human capital accumulation when migration can be both voluntary and constrained, in presence of selective policies for the access to the destination country. To this aim, we develop an infinite horizon model where an individual determines jointly her migration duration and her human capital. Our findings confirm the importance of migration prospects as an incentive to human capital accumulation (Mountford, 1997; Stark et al., 1997; Stark and Wang 2002). Under more general assumptions, however, it turns out that restrictive immigration policies may hinder human capital accumulation. This happens because entry restrictions reduce expected returns to human capital, and, under certain circumstances, they cause a "brain loss". The reasoning behind the "brain gain with a brain drain" is therefore not always valid.

We stress that a brain drain arises when migration is permanent. Talented individuals can often choose their location according to their preferences: as long as the preference for home consumption is not outweighted by macroeconomic risk, migrations are temporary. As a consequence, promoting political and economic stability of the sending countries may be more effective than entry barriers.

Besides, our results question the consistency of restrictive immigration policies with the objective of reducing the immigrants' stock: especially for low-skilled workers, entry closure biases the incentive structure towards longer migration spells and, eventually, permanent migration. Faini (1996) points it out clearly: "If a migrant is not certain that he would be allowed back in the host country if he ever returned

[4] "One reason for thinking that the emigrants *intended* to remain abroad for only a relatively short period is that many made a second emigration just after returning. For example, ten per cent of the Italian immigrants into the U. S. in 1904 were entering for the second time" (Baines, 1991, p. 36); and "As transport improved, emigration became less final. [...] The changes also favoured a relatively new kind of emigrant -one who expected to return within a relatively short period" (Baines, 1991, p. 41)

home, his propensity to return will obviously decline".

The paper is organised as follows: next Subsection reviews some main findings in the literature. Our model is developed in Sections 2 and 3. In Section 4 we discuss our results, and a sensitivity analysis is used to illustrate our findings in Section 5. Section 6 contains a comparison of our results to those present in the literature. Conclusions are reported in Section 7. The proofs are gathered in the Appendix.

1.1 Related Literature

The possible benefits of a brain drain and the focus on return migration are recent topics in the literature. In an OLG framework there are several mechanisms able to generate some benefits from the brain drain, and they rely basically on the existence of externalities on the human capital: Vidal (1998) points to enhanced intergenerational transmission of skills and education; Mountford (1997) and Beine et al. (2001) stress the possibility of intergenerational spillovers between skilled workers.

The possibility of migration increases the expected returns to human capital, and thus the incentive to education. Stark et al. (1997) distinguish between education and ability. When there exists asymmetric information about the worker's ability, the incentive to invest in education and migrating is even stronger for low-ability individuals; however, after their real productivity is observed, they will find convenient to return. Stark and Wang (2002) use a static model to state some conditions under which a restrictive immigration policy in the *destination* country increases the welfare of the *sending* country: the idea is that entry rationing in the developed countries can keep most of the human capital at home.

The literature about return migration adopts mainly life-cycle models.

An early contribution to the study of migration duration it given by Djajic and Milbourne (1988). They develop a two-period model to study the effect of wage differentials in determining migration flows and their final effect on the equilibrium wages, but they are aware that more research is needed to understand why "some migrants make several trips, some stay longer than others, and some never return".

Hill (1987) stresses, interestingly, the importance of "the repetitive character of contemporary labor migration"; in spite of that, his assumption of an identical duration for each migration spell can be deceptive.

Dustmann (2001) shows that an increase in the host country wage may lead both to a decrease and an increase in migration duration.

Dustmann (1997) develops a life-cycle model to study the effect of the correlation between the shocks in the host and the origin labour markets. Both the optimal migration duration and migrants' saving behaviour depend heavily on the sign of this correlation; nonetheless, the final result is undetermined because it depends on both the wage differential and on the relative risk in the two labour markets.

Mesnard (2004) considers temporary migration as a possible way to overcome rationing in the credit market.

Galor and Stark (1990) suggest that an exogenous probability of returning may induce migrants to work harder and save more than natives to smooth their consumption path.

2 Migration in infinite horizon

The life-cycle model is the basis of the literature surveyed in the previous Section. However, the use of only two periods tends to hide "the repetitive character of contemporary labor migration" (Hill, 1987), and this is why the return decision is generally considered as permanent. In this Section, we shall try to overcome this restriction. When we shift to infinite horizon, it appears immediately that considering only one stay abroad is somewhat arbitrary. In our model migrations are a recurrent phenomenon, driven by wage differentials and by aggregate shocks.

For simplicity, we consider a risk-neutral potential migrant with an infinite life horizon. She must first choose whether or not to migrate to a destination country D, and then after how much time to return to her origin country O.

In both countries there exists a unique consumption good produced using only capital by means of a linear technology:

$$c_{ti} = k_{ti} \qquad i = O, D \quad t = 0, 1, 2....$$

For simplicity, country O is not endowed with capital, that is accumulated in D by means of inelastically supplied labor.

2.1 Destination country

The law regulating capital accumulation in D is

$$k_{t+1} = (1 - \delta) k_t + (1 + h) \tag{1}$$

where $0 < \delta < 1$ is the depreciation rate and $(1 + h) > 0$ and finite is the per-period inelastic supply of "effort": it is given by the sum of a basic effort (1) delivered by any individual irrespective of her skill plus her human capital endowment ($h \geq 0$). In other words, capital accumulation is faster for skilled individuals. This reflects differences in productivity among agents with different human capital endowment (h). The per-period utility in D is

$$u^D \left(c_t^D \right) = c_t^D$$

2.2 Origin country

To keep our model as simple as possible, in O there is neither capital endowment ($k_0 = 0$) nor capital production, thus migration is needed in order to accumulate (in principle, we could allow for a more realistic slower capital accumulation without changing our results).

To stress the importance of macroeconomic risk in developing countries we assume that in each period the accumulated capital is confiscated with a probability $0 < p < 1$, whereas with a probability $(1 - p)$ it is perfectly conserved. Such an assumption requires some words of explanation: our aim is to account for *both* economic migration *and* constrained migration. While most of the literature is focused on the latter, we are including in our model a push factor able to generate a constrained migration wave. Such shocks can be currency crisis, hyperinflations, wars, *coups d'état,* and they are typical of developing countries. In each of these cases, most of the accumulated wealth of the population can be destroyed [5]. After a capital confiscation, an individual is forced to re-migrate.

[5] Consider that temporary migration is often used to build homes or to start a business (See Dustmann and Kirchkamp 2002, Mesnard 2004 and the references quoted therein). Wealth confiscation may occur in case of dictatorship or under a bank rush: more generally, it can be associated with political regime changes.

Therefore, the capital stock in O is

$$k_{t+1} = (1-p)\, k_t \tag{2}$$

In spite of poor economic conditions, in O it is possible to accumulate human capital, as it is often the case for developing countries. An individual can accumulate her human capital by means of her effort e:

$$\begin{aligned} e &= e(h) \\ e'(h) &> 0 \\ e''(h) &> 0 \\ e(0) &= 0 \end{aligned} \tag{3}$$

Individuals differ with respect to their abilities:

$$\begin{aligned} e_i &= \theta_i e(h) \\ \theta_i &> 0 \\ \theta_i &\sim f(\theta) \end{aligned}$$

A key assumption of the literature about return migration is that the marginal utility of home consumption is higher:

$$u^O\left(c_t^O\right) = \alpha c_t^O, \qquad \alpha > 1.$$

where c_t^O is consumption in O and α is a coefficient used to depict the preference for home consumption. As a consequence, consumption in O dominates consumption in D, but in D it is possible to accumulate capital (k).

Confiscation of the capital stock is ruled out in D, where the economic and political environment is comparatively highly stable.

Given the linear technology used to produce the consumption good, consumption in O is equal to the endowment of capital, if the shock is not realized, or to zero, in the opposite case.

3 Migration duration and human capital accumulation

Now we turn to the individual optimization problem: we assume that the physical capital endowment in $t = 0$ is zero, thus consumption, and therefore the utility is nil. To make things clearer, it is useful to begin assuming no entry rationing in D. The migrant has to choose the optimal time to spend in D, and her optimal human capital level. We denote with $U_0^M(h, T)$ the utility corresponding to T periods of migration with zero initial physical capital endowment:

$$U_0^M(h, T) = V^D(h, T) + \beta^T V^O(h, k_T) \tag{4}$$

where $0 < \beta < 1$ is the discount factor, $V^D(T)$ is the utility of staying T periods abroad and $V^O(k_T)$ is the utility of returning to O with the accumulated capital k_T. The effort required to acquire human capital $(\theta_i e(h))$ enters negatively the utility.

However, as we have stressed in the Introduction, migrating is not the result of a purely individual choice: there exist several institutional barriers rationing entry into foreign countries. Human capital endowment is a crucial variable enabling a migrant to enter D[6]. Usually highly skilled individuals have better opportunities of mobility, whereas restrictive immigration policies are targeted to less qualified or unskilled individuals. As a result of these restrictions, only a fraction of migrants are allowed to cross the border.

In our model the probability q of entering D is a function of individual and institutional characteristics. Human capital-based screening gives everybody a probability of entering $\pi(h)$, while the parameter $\Psi \in [0, 1]$ depicts the weight that immigration policy places on human capital: with Ψ close to 1, immigrants are screened according to the human capital they bring into the country[7]. When Ψ is close to 0, entry is free for anybody. This is a convenient way to represent the complex combination of individual and institutional characteristics enabling an individual to cross the border. The overall probability $q(\pi(h), \Psi)$ of entering D is therefore given by

[6] Regulations differ among different countries, but, generally, entry requirements are homogeneous and stable over time for large areas. For example, we may think of the EU as a single macro-region, as well as the US.

[7] For example, Canada uses a point scheme since 1965; Australia has adopted a similar policy -recently refined- since 1984, and so does the UK.

$$q(\pi(h),\Psi) = \Psi\pi\left(h\right) + (1-\Psi) \tag{5}$$

$\pi(h)$ has the following properties:

$$\begin{aligned}
\pi'\left(h\right) &> 0 \quad \textit{and bounded} \\
\pi\left(H\right) &= 1 \qquad (6) \\
\pi'\left(H\right) &= 0 \qquad (7) \\
\pi\left(0\right) &= \pi_0 > 0
\end{aligned}$$

Quite intuitively, $\pi\left(h\right)$ is increasing in h. $\pi\left(H\right) = 1$ means that the government establishes a threshold level of human capital (H) which entitles to free mobility[8]. $\pi(0) = \pi_0 > 0$ is the probability of entering as a totally unskilled worker (it may well be the probability of entering illegally).

When entry is made uncertain, the utility of migrating is

$$U_0^M\left(h,T\right) = V^D\left(T\right) + \beta^T\left\{\left(1-p\right)V^O\left(k_T\right) + pqU_0^M\left(h,T\right) + p\left(1-q\right)V^O\left(0\right)\right\} \tag{8}$$

Which is to be interpreted as follows: The agent returns home after T periods abroad, which give her an utility $V^D\left(T\right)$. Once returned, her capital is conserved with a probability $(1-p)$, yielding a utility $V^O\left(k_T\right)$ .Conversely, her capital is confiscated with a probability p, and she will immediately try to re-migrate. With a probability q she will succeed and re-build her capital stock getting a utility $U_0^M(h,T)$. With a probability $(1-q)$ she won't succeed, and she will simply get the utility of living in O without capital $(V^O\left(0\right))$[9].

The lifetime utility $U(h,T)$ is given by the utility of migrating minus the disutility of the effort:

$$U(h,T) = U_0^M\left(h,T\right) - \theta_i e(h) \tag{9}$$

[8] Obviously, when $h \geq H$, $\pi'\left(H\right) = 0$ because any unit of human capital beyond the threshold cannot increase further the probability of entering D. Notice, however, that there may exist an incentive to accumulate more human than H because it still affects the accumulation of physical capital k. Finally, a government may desire not to give free entry to anybody. In such a case, $H- > \infty$.

[9] The procedure to compute $U_0^M(T)$ is shown in the Appendix.

substituting (8) into (9) we finally get

$$U(h,T) = V^D(T) + \beta^T \left\{(1-p)\,V^O(k_T) + pqU_0^M(h,T) + p(1-q)\,V^O(0)\right\} - \theta_i e(h) \tag{10}$$

Given the simple structure of the model, $V^D(T)$ is the indirect utility of the following maximization problem:

$$V^D(T) = \max \sum_{t=0}^{T-1} \beta^T c_t \tag{11}$$

subject to the constraints

$$\begin{aligned} c_t &= k_t \\ k_{t+1} &= (1-\delta)\,k_t + (1+h) \\ k_0 &= 0. \end{aligned}$$

$V^O(k_T)$ is simply the utility of living in O with an initial capital endowment k_T.

Constructing the expressions for $V^D(T)$ and $V^O(k_T)$ (see the Appendix) gives the expected lifetime utility associated to T periods of migration:

$$U(h,T) = \frac{\frac{(h+1)\beta}{\delta}\left[\frac{1-\beta^{T-1}}{1-\beta} - (1-\delta)\left(\frac{1-(\beta(1-\delta))^{T-1}}{1-(\beta(1-\delta))}\right)\right] + \frac{(h+1)\beta}{\delta}\left[\frac{\beta^{T-1}\alpha(1-p)\left(1-(1-\delta)^T\right)}{1-\beta(1-p)}\right]}{1 - \frac{p\beta^T q(\pi(h),\Psi)}{[1-\beta(1-p)][1-\beta(1-q(\pi(h),\Psi))]}} - \theta_i e(h) \tag{12}$$

The emigrant has to maximize (12) with respect to T and h. This two-variable optimization program does not admit a closed-form solution. However, it is possible to show its characteristics, and derive the sufficient conditions for the existence of a solution (T^*, h^*) where T^* is finite and $h^* \geq 0$.

Proposition 1 *(Optimal migration duration under uncertainty) A sufficient condition to have an interior solution to the maximization program is that $\alpha > \alpha_1$. The value of α_1 is shown in the Appendix. When this condition hold, the optimal migration duration is positive and finite for any $h^* \geq 0$.*

Proof. See the Appendix. ■

Intuitively, the above Proposition states that for having an interior solution we need a sufficiently high preference for home consumption. Notice that we are reporting a *sufficient* condition. This means that a finite migration exists under quite general conditions and that the incentive to accumulate abroad and return exists for unskilled workers as well ($h^* \geq 0$).

It is interesting to remark that this result reverses the "common wisdom": as long as α is sufficiently high, permanent migration is somewhat more difficult to explain than temporary migration.

In Figure 3.1, we show a plot of (12)[10].

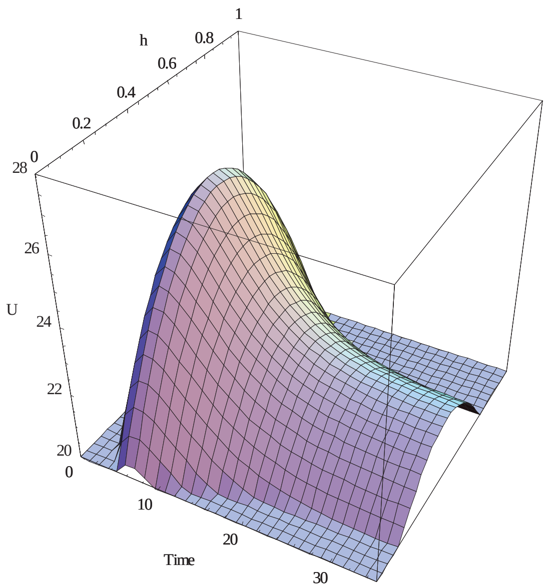

Figure 3.1

4 Brain gain or brain loss?

A new immigration policy is a change in Ψ. Generally, a government may affect π_0 and the shape of $\pi(h)$ as well; however, varying Ψ is the most direct intervention to make entry rationing more -or less- severe[11]. We are now going to examine the impact of Ψ on T^* and h^*. By means of the implicit function theorem it is easy to

[10] The plot is obtained with the following functional forms: $e(h) = h^2$; $q(h, \Psi) = 1 - \Psi + \left(1 - e^{-h}\right) \Psi$; the parameters are: $\alpha = 4$; $\beta = .8$; $\delta = .03$; $\Psi = .5$; $p = .25$; $\theta = 20$.

[11] It is useful to recall that $\Psi = 1$ indicates that entry is totally screening-based.

show (see the Appendix) that the sign of $\frac{\partial h^*}{\partial \Psi}$ is equal to the sign of

$$\frac{\partial}{\partial \Psi}\left(\frac{\partial U(T,h)}{\partial h}\right) \tag{13}$$

Studying the properties of (13) enables us to write the following Proposition:

Proposition 2 *(Brain Gain and Brain Loss) When θ_i is sufficiently large and $\frac{\partial q(\pi(h),\Psi)}{\partial h}$ is sufficiently small, selective immigration policies are harmful to human capital accumulation.*

Proof. See the Appendix. ■

Proposition 2 states that the argument for "a brain gain with a brain drain" can easily be reversed, because it depends on the sign of $\frac{\partial h^*}{\partial \Psi}$.

Notice that the "brain loss" of our model differs from the "brain drain", which stems from permanent migration. The brain loss occurs because expected returns to human capital are reduced when entering D is made difficult.

In the Appendix, we show that, when the probability of admission is little sensitive to an increase in h, agents with θ_i large enough *reduce* their equilibrium human capital. Moreover, the reduction is most important for the most disadvantaged individuals.

Since agents with the "wrong" sign of $\frac{\partial h^*}{\partial \Psi}$ reduce their education, the case for "a brain gain with a brain drain" argued, for example, by Stark and Wang (2002) is not general: selective immigration policies may be an incentive to human capital accumulation as well as a disincentive, and their use should be carefully evaluated[12].

Generalizing the reasoning behind the "brain gain with a brain drain" implies that a migrant has no chance whatsoever to enter D without human capital or that it is only useful to cross the border. Otherwise, wage differentials supply the "right" incentives by themselves.

It is important to stress that our agents are risk-neutral: risk aversion would produce even stronger results; moreover, the denominator of (12) is minimum for $\Psi = 0$, thus, in terms of individual utility, free entry always dominates entry rationing.

[12] It is important to mention the result in Mountford (1997): he clearly shows that the equilibrium human capital is increasing with the probability of migrating. Nonetheless, in his model, he can't provide for free migration without causing a complete human capital depletion in the source country.

Finally, we can get an important insight from our model: if we interpret θ_i as a country-specific parameter rather than an individual charachteristic, we can apply the above reasoning on a country scale: a brain loss may damage the economies turned up with $\frac{\partial h^*}{\partial \Psi} < 0$.

Remark 3 *Since the sign of* $\frac{\partial h^*}{\partial \Psi}$ *may be different for different agents, applying a uniform policy towards different individuals or countries can increase the world human capital dispersion.*

This finding casts some doubts on the effects the widespread adoption of point schemes may have in the long run: such policies may benefit the receiving countries, but they are not a panacea and there exists a possibility that they exacerbate inequality. Beine et al. (2003) report evidence consistent with the hypothesis that the possibility of migrating is a powerful incentive to acquire human capital. Indeed, in their estimates the probability of migrating is substituted to wage differentials. This incentive effect, evidently, *does not depend on the selective policies, but simply on wage differentials.*

5 Numerical examples

5.1 Optimal migration duration

Since it is not possible to use the implicit function theorem to evaluate the effect of Ψ and p on T^*, we have to use a series of simulations.

The model has been simulated for different parameter values and different functional forms of $e(h)$ and $\pi(h)$[13]. When searching the effect of Ψ over T^*, we have computed T^* when Ψ ranges from 0 to 1, in steps of 0.01. We found that T^* tends to increase with Ψ especially for individuals with low human capital. This is quite intuitive: skilled workers can easily recover abroad when the shock occurs, whereas unskilled ones cannot, thus they stay abroad longer. Figure 5.1 below gives an example with $\theta_i = 5$ (lower curve), $\theta_i = 10$ (middle curve) $\theta_i = 20$ (upper curve)[14]:

[13] Our simulations are available upon request.

[14] The parameters used are:
$\alpha = 4$, $\beta = .8$; $\delta = .03$; $p = .25$, $\Psi = .5$. $e(h) = h^2$, $q = \Psi(1 - e^{-h}) + (1 - \Psi)$

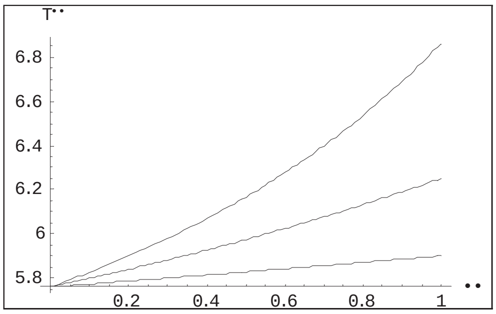

Figure 5.1

Not surprisingly, the effect of p on the optimal migration duration is even more important. In Figure 5.2 we have plotted T^* against p ranging from 0 to .4[15] for two individuals ($\theta = 10$, $\theta = 20$). Notice that in the upper $\Psi = 1$, and in the lower curve $\Psi = 0$. Again, the selective policy biases the result towards a longer migration spell especially for the less skilled worker.

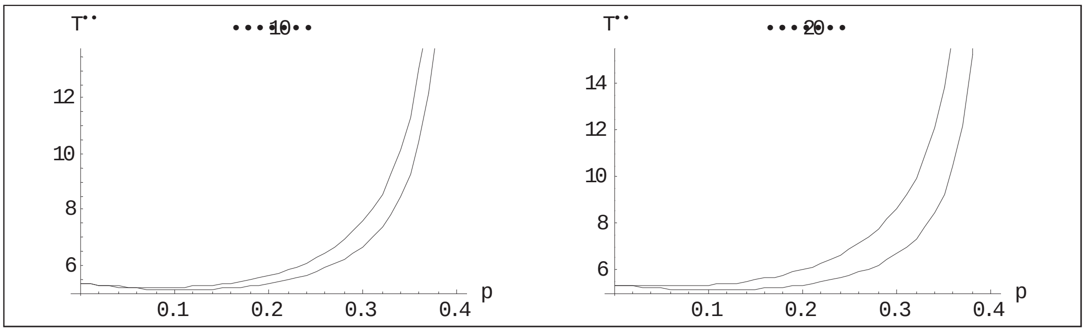

Figure 5.2: risk and migration duration.

In Fig. 5.3 we show an example of permanent migration. Such a result can be obtained easily setting α close to one[16].

[15] In these simulations, when $p > .4$ T^* tends to infinity. The values used are $\alpha = 4$; $\beta = .8$; $\delta = .03$; $\theta = 20$. $q = \Psi(1 - e^{-h}) + (1 - \Psi)$; $e(h) = h^2$.

[16] We have set $\alpha = 1.2$ in the previous simulations.

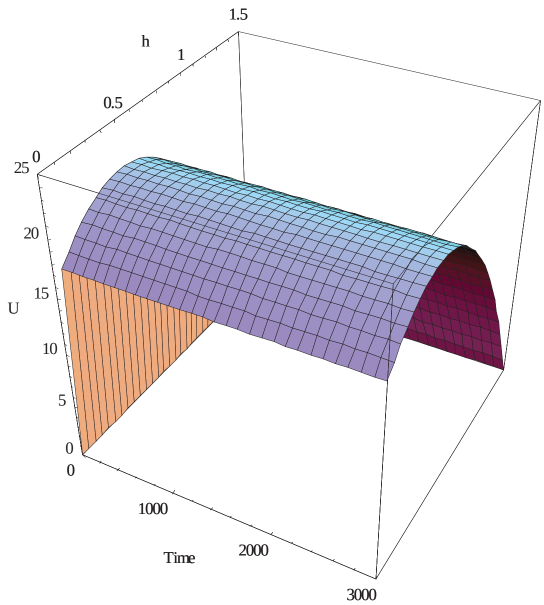

Figure 5.3: permanent migration

As α reflects the importance of cultural and ethnic factors, one may think that it is close to one when O and D are homogeneous in language, culture and traditions. This implies that the incentive to return is more important the more different are O and D. In other words, migrants who experience a more difficult assimilation are the most likely to return[17].

Finally, in Figure 5.4 we show a plot of T^* against θ[18] The upper curve is obtained with $\Psi = 1$, and the lower curve with $\Psi = .5$. We have used $\Psi = .5$ instead of $\Psi = .0$ because in the latter case the effect is too small to be plotted on the same diagram. Again, the line corresponding to the higher $\Psi = 1$ lies above the one corresponding to the lower one.

Summarizing, our simulations show a trade-off between entry rationing and migration duration: frontier closure tends to increase time spent abroad and reduces total utility. This is particularly true for unskilled migrants, i.e. those with low values of h^* : since they can't react to a restrictive policy by increasing their human capital, they simply stay longer. This can be seen as a simple result of the Lucas critique. Nonetheless, closed-door policies are especially intended to reduce the number

[17] For an enlightening analysis of the assimilation problem, see the seminal Lazear's (1999) article.

[18] The values used are: $\alpha = 2$; $\beta = .7$; $\delta = .1$; $p = .1$; $e(h) = h^2$; $q = \Psi(1 - e^{-h}) + (1 - \Psi)$

of unskilled immigrants, who are considered a burden for the welfare system.

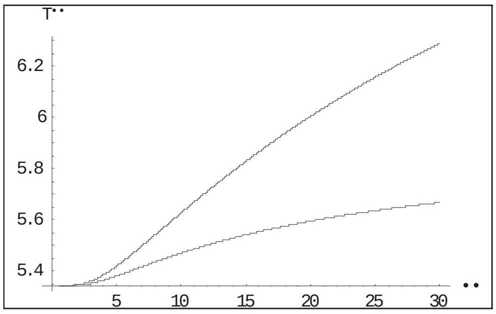

Figure 5.4: abilities and migration duration

5.2 Risk and optimal human capital accumulation

Using the Implicit Function Theorem, it is easy to see that the sign of the derivative $\frac{\partial h^*}{\partial p}$ is the sign of

$$\frac{\partial}{\partial p}\left(\frac{\partial U(T,h)}{\partial h}\right).$$

In principle, this sign is ambigous: while risk makes less attractive to invest in human capital, at the same time a skilled migrant can recover abroad easier. From this point of view, human capital has an insurance effect. Which effect prevails is in principle undetermined. Nonetheless, our simulations suggest that the final impact on h^* is negative for realistic parameter configurations. This finding, however, should not be generalized. We give an example in figure 5.5.[19], where, in both diagrams, the upper curve is obtained with $\Psi = 0$, and the lower one with $\Psi = 1$.

[19]The parameter values are $\alpha = 4$, $\beta = .85$, $\delta = .05$, $e(h) = h^2$, $q = \Psi(1 - \exp(-h)) + 1 - \Psi$. Other simulations used, for example, $e(h) = E^h$, $e(h) = h^2 + h$.

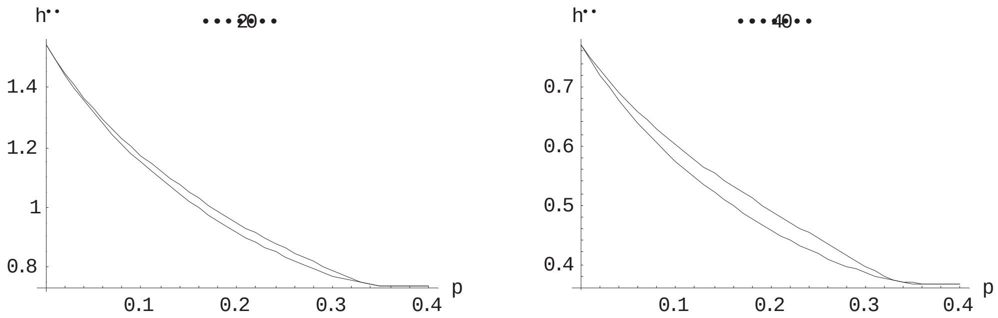

Figure 5.5: macroeconomic risk and equilibrium human capital.

Again, the comparison between closed-door and open-door policies shows that a pure selective policy can be a disincentive to human capital accumulation. This is not surprising, since the denominator of (12) is minimum for $\Psi = 0$, thus utility is higher for any (h, T) when $\Psi = 0$.

Diagrams in Figure 5.5 have been plotted for two individuals, with $\theta_i = 20$ and $\theta_i = 40$ respectively. Even though both agents reduce h^* when $\Psi = 1$, the effect is more important when $\theta_i = 40$, i.e. for the most disadvantaged individual.

In the following Sections, we are going to discuss the consequences of a "brain loss".

6 Brain drain Vs. brain gain

The contributions mentioned in Section 1.1 stress that migration is not necessarily a cause of brain drain because the possibility of migrating establishes an incentive for human capital accumulation and, as long as not everybody is allowed to migrate, the origin country may end up with more human capital than in autarchy. Mountford (1997), and Stark and Wang (2002) state some conditions under which a restrictive immigration policy in the *destination* country enhances the *origin* country's welfare. Contrary to our work, in these articles migration is permanent, and the probability of entering D is exogenous. Our work does not deal with the growth problem associated to the brain drain; however, it is possible to draw some conclusions about the stock of human capital in O.

First of all, we reproduce the well-known result that, as long as migration duration is finite, there is no brain drain. Moreover, in our setting total freedom of immigration would not generate a brain drain *per se*.

From this point of view, the true causes of brain drain are the individual preferences and the country-specific risk, rather than wage differentials. The most gifted individuals are contended on the international markets, and often they can decide their location according to their preferences: if $\alpha \leq 1$ there is no reason to stay in O. On the contrary, too high a risk of a negative shock may push an agent with $\alpha > 1$ out of her country.

Unlike most of the current literature, in our paper the probability of entering abroad is endogenous, and it depends on the amount of human capital. Moreover, wage differentials are proportional to the accumulated human capital, thus incentives to education exist *independently of selective immigration policies*. In this case, rationing at the borders may reinforce the incentive to acquire human capital for some individuals, but may destroy it for others. More precisely, immigration policy determines endogenously the share of individuals with $h^* = 0$, whereas in Stark and Wang (2002) it is simply assumed that not all workers can build sufficient human capital to have a positive probability of entry. While supporters of selective policies move from the unlikely assumption that there is no chance to cross the border without human capital, we argue that the correct benchmark should be free immigration.

Since restrictive immigration policies can deter some individuals to take any education, their use should be careful: under some not unlikely circustances, they can generate more inequality in the human capital distribution.

Finally, let us recall that, if we consider θ as a country-specific coefficient, adopting the *same* policy towards *different* countries may have undesired effects.

7 Conclusions

Ten years ago, Crettez, Michel and Vidal (1996) claimed that "policies aimed at promoting or regulating inter-country migration flows are often made without any underlying conceptual framework". Nowadays, the study of migration duration is receiving increasing attention, as well as the effect of migration prospects on human capital accumulation. In our work, we have attempted to connect these streams of literature.

We think that we have carefully modelled this major point of our paper: we have generalised the choice about migration duration to an infinite-horizon framework, at the cost of using a rough carachterization of the consumption behaviour. This has proved necessary to preserve simplicity and analytic tractability. Our findings question the effectiveness of human-capital based immigration policies. First, we confirm that migration duration is not independent of the immigration policy. Second, macroeconomic risk may be even more important than wage differentials when deciding whether or not to return. Third, selective policies are a double-edged weapon: they can both foster and harm the equilibrium level of human capital.

With respect to a *laissez-faire* policy they decrease returns to human capital, and they can have ambigous effects on incentives to education. This might cause a "brain loss", rather than a brain drain. At least, this result suggest that selective immigration policies should not be used unconditionally, and that the logic behind the "brain gain with a brain drain" is not always correct.

The empirical implications of this finding are quite important. Depending on the interpretation of θ, when uniform selective policies are applied we should observe more or less dispersion in human capital distribution within or between the origin countries, according to the sign of $\frac{\partial h^*}{\partial \Psi}$. We hope to develop our research in this way in the future.

With respect to the optimal migration spell, we find that closed-door policies backfire on migration duration especially for unskilled immigrants ($h^* = 0$). Such policies underestimate the importance of this effect. Kossoudji (1992), indeed, finds that attempts to enforce the U.S.-Mexican border eventually "alter lengths of spells of future trips to the U.S.". This outcome is well-known among demographers: Bonifazi and Strozza (2001) consider the introduction of entry barriers in Germany after the oil shocks. After 1975, inflow was reduced, but new entries occurred mainly through family reunification[20]. Family reunification indicates that expectations about migration duration have changed: the costs of returning home may be too high to permit an easy reversal[21]. Currently, family reunifications account for at least one half of the

[20]See King (1993) for similar results. For further references on the effect of the post-oil shocks frontier closure on family reunification we refer to Venturini (2001, p. 217-221 and the authors quoted therein).

[21]Think, for example, to the children's education.

legal inflow into the E.U. (OECD; 2001, 2004). Only the migrants from the riskiest countries are likely to stay forever anyway, and in this case entry restrictions are effective. Undoubtely, more empirical evidence is necessary on this topic; unfortunately, a serious lack of data makes difficult the research on migration duration: the available databases track quite rarely the different trips of the same individuals.

Finally, in this paper, α (the preference for home consumption) is constant. Indeed, it may well be time-dependent: we can imagine that, in the long run, an assimilation effect may drive this paramenter towards 1, making consumption in either country indifferent. This would reinforce our concerns about the effectiveness of entry restrictions.

On the other hand, economic policies can affect p : policies aimed to reduce the risk in the developing countries reinforce by themselves the incentive to return. Though it may be difficult to influence these processes, there are no theoretical reasons why international, co-ordinated development policies should be less effective or more costly than enforcing strict frontiers closure. It is also important to mention the literature showing that trade liberalization can be the best option for an incentive-compatible immigration reduction (Trefler, 1997).

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Appendix

Derivation of $V^D(T)$**:** the optimization problem is

$$V^D(T) = \max \sum_{t=0}^{T-1} \beta^T c_t \tag{A.1}$$

subject to the constraints

$$\begin{aligned} c_t &= k_t \\ k_{t+1} &= (1-\delta) k_t + (1+h) \\ k_0 &= 0. \end{aligned}$$

Integrating the law of motion of the capital, we get

$$V^D(T) = \frac{(1+h)}{\delta} \left[\frac{1-\beta^T}{1-\beta} - \frac{1-(\beta(1-\delta))^T}{1-(\beta(1-\delta))} \right]. \tag{A.2}$$

Derivation of $V^O(0)$**:**

Let us now first compute $V^O(0)$. In the current period consumption is zero, and the agent is going to re-migrate in the following period with a probability q. If she succeeds, her utility will be $U_0^M(h,T)$, otherwise she will get again $V^O(0)$. Therefore, we have

$$V^O(0) = 0 + \beta \left\{ q U_0^M(h,T) + (1-q) V^O(0) \right\}$$

from which it is easy to get the expression for $V^O(0)$:

$$V^O(0) = \frac{\beta q U_0^M(h,T)}{1-\beta(1-q)}. \tag{A.10}$$

The computation of $V^O(k_T)$ is less straightforward: the capital k_T yields a utility αk_T in the first period. It is easy to compute

$$k_T = \frac{(1+h)}{\delta} \left(1 - (1-\delta)^T \right) \tag{A.11}$$

In the following period, with a probability $(1-p)$ the adverse shock does not occur and therefore the utility is still $V^O(k_T)$. Conversely, the individual will re-migrate with a probability q or will get an utility equal to $V^O(0)$ with a probability $(1-q)$. We have then the following expression for $V^O(k_T)$:

$$V^O(k_T) = \alpha k_T + \beta\left\{(1-p)V^O(k_T) + pqU_0^M(h,T) + p(1-q)V^O(0)\right\} \qquad \text{(A.12)}$$

Solving (A.12) with respect to $V^O(k_T)$, and using (A.10) and (A.11), we immediately get:

$$V^O(k_T) = \alpha\frac{(1+h)}{\delta}\left[\frac{\left(1-(1-\delta)^T\right)}{1-\beta(1-p)}\right] + \frac{\beta pqU_0^M(h,T)}{1-\beta(1-p)} + $$
$$+\left[\frac{\beta p(1-q)}{1-\beta(1-p)}\right]\left[\frac{\beta qU_0^M(h,T)}{1-\beta(1-q)}\right]. \qquad \text{(A.13)}$$

Now, substituting (A.10) and (A.12) in (9):

$$U_0^M(h,T) = \frac{(1+h)}{\delta}\left[\frac{1-\beta^T}{1-\beta} - \frac{1-(\beta(1-\delta))^T}{1-(\beta(1-\delta))}\right] + \beta^T\left[pqU_0^M(h,T) + \frac{p(1-q)\beta qU_0^M(h,T)}{1-\beta(1-q)}\right] +$$
$$+\beta^T\left[\frac{\alpha(1-p)\left(1-(1-\delta)^T\right)}{1-\beta} + \frac{\beta pqU_0^M(h,T)}{1-\beta(1-p)}\right] +$$
$$+\beta^T\left[\left(\frac{\beta p(1-q)}{1-\beta(1-p)}\right)\left(\frac{\beta qU_0^M(h,T)}{1-\beta(1-q)}\right)\right] \qquad \text{(A.14)}$$

rearranging the above expression we obtain the expression for (10).

It is useful to remark that $U(0,0)=0$ and that

$$\lim_{T->\infty} U(h,T) = \frac{(1+h)}{\delta}\left[\frac{1}{(1-\beta)} - \frac{1}{1-\beta(1-\delta)}\right] - \theta_i e(h) \qquad \text{(A.15)}$$

Proof of Proposition 1)

Notice that the equilibrium human capital h^* must always be finite: indeed, for any $h^* > H$ the utility (12) is linear in h, while the effort $\theta_i e(h)$ is convex. Thus, beyond H, the marginal cost of accumulating human capital grows faster than the marginal benefit and the optimal h must be finite.

Now we have to prove that T^* can be finite as well. Since (A.15) exists for any finite value of h, we can write the difference function (12) - (A.15):

$$DIFF = \frac{\frac{(h+1)\beta}{\delta}\left[\frac{1-\beta^{T-1}}{1-\beta} - (1-\delta)\left(\frac{1-(\beta(1-\delta))^{T-1}}{1-(\beta(1-\delta))}\right)\right] + \frac{(h+1)\beta}{\delta}\left[\frac{\beta^{T-1}\alpha(1-p)\left(1-(1-\delta)^T\right)}{1-\beta(1-p)}\right]}{1-\frac{p\beta^T q(\pi(h),\Psi)}{[1-\beta(1-p)][1-\beta(1-q(\pi(h),\Psi))]}} - \theta_i e(h) -$$

$$-\left\{\frac{(1+h)}{\delta}\left[\frac{1}{(1-\beta)}-\frac{1}{1-\beta\,(1-\delta)}\right]-\theta_i e(h)\right\} \tag{A.16}$$

a *sufficient* condition for a max of (12) with respect to (T,h) to exist is that (12) approaches (A.15) from above *for any* non-negative h. This happens when (A.16) is positive *for any* non-negative h in a neighborhood of $T\to\infty$.

To study the sign of (A.16) around $T\to\infty$, remark first that the terms depending on T are β^T and $(1-\delta)^T$, which tend to zero as T goes to infinity.

Thus, we can perform the following variable substitution in (A.16): $\beta^T\equiv b$, and $(1-\delta)^T\equiv d$. We call this new function $\Phi(b,d)$.

Let Ω be the coefficient of the first term of the Maclaurin expansion of $\Phi(b,d)$. The sign of $\Phi(b,d)$ for $b\to 0$ and $d\to 0$ is of course the same of (A.16) for $T\to\infty$, and it is given by the sign of Ω.

Finally, we can construct the following system of inequalities:

$$\left\{\begin{array}{c}\Omega>0\\ 0<\beta<1\\ 0<\pi\leq 1\\ 0\leq\Psi\leq 1\\ 0<\delta<1\\ \alpha>1\\ 0<p<1\\ e\geq 0\\ h\geq 0\\ \theta>0\\ 0<d<(1-\delta)\\ 0<b\leq\beta\end{array}\right.$$

this system admits solutions for[22]

$$\begin{aligned} 0 &< \beta < 1 \\ 0 &< \pi \leq 1 \\ 0 &\leq \Psi \leq 1 \\ 0 &< \delta < 1 \\ \alpha &> \alpha_1 \\ 0 &< p < 1 \\ e &> 0 \\ h &\geq 0 \\ \theta &> 0 \\ 0 &< d < 1 \\ 0 &< b < \beta \end{aligned}$$

where

$$\alpha_1 = \frac{1 - \beta(-1+p)\ (-1+\delta) - \beta\Psi(-1+\pi(h))\ (-1+(-1+p)\ \beta\ (-1+\delta)+p\,\delta)}{(-1+d)\ (-1+p)\ (1+\beta\ (-1+\delta))\ (1+(-1+\pi(h))\ \beta\,\Psi)} +$$
$$+ \frac{d\ (1+(-1+p)\ \beta)\ (-1+\beta\ (\Psi - \pi(h)\ \Psi))}{(-1+d)\ (-1+p)\ (1+\beta\ (-1+\delta))\ (1+(-1+\pi(h))\ \beta\,\Psi)}$$

α_1 is a function of h and T, because it contains $\pi(h)$ and d.

To prove that there exist $\alpha > \alpha_1$, we have to prove that α_1 is bounded for any h and any $T \geq 1$. If α_1 is continous and it is defined over a compact domain, we know that it is bounded. Since $d \in [0, (1-\delta)]$ for any $T \geq 1$ and $\pi(h) \in [\pi_0, 1]$ for any h, the first requirement is satisfied[23]. Then we have to prove that α_1 is continous.

To do so, we are going to study α_1 along its boundaries: consider first the case when $\pi(h) \to \pi_0$:

$$\lim_{\pi(h)\to\pi_0} \alpha_1 = \frac{1 - \beta(-1+p)\ (-1+\delta) - \beta\Psi(-1+\pi_0)\ (-1+(-1+p)\ \beta\ (-1+\delta)+p\,\delta)}{(-1+d)\ (-1+p)\ (1+\beta\ (-1+\delta))\ (1+(-1+\pi_0)\ \beta\,\Psi)} +$$

[22] The routines written to solve the system are available upon request.

[23] Notice that we can set $\pi_0 = 0$ without altering our results. Obviously when $T = 0$, α_1 tends to infinity, but this case is not important since it yields zero utility.

$$+\frac{d\ (1+(-1+p)\ \beta)\ (-1+\beta\ (\Psi-\pi_0\,\Psi))}{(-1+d)\ (-1+p)\ (1+\beta\ (-1+\delta))\ (1+(-1+\pi_0)\ \beta\,\Psi)}$$

this limit is bounded for any $d \leq 1$, i.e. for any $T \in [1, \infty)$.

$$\lim_{\pi(h)\to 1} \alpha_1 = \frac{1+d(-1+\beta-p\beta)+\beta\ (-1+p+\delta-p\delta)}{(-1+d)\,(-1+p)\,(1+\beta\ (-1+\delta))}$$

this limit is finite for any $d \leq 1$ as well. Now we have to study the boundaries for $d \to 0$ and $d \to (1-\delta)$.

$$\lim_{d\to 0} \alpha_1 = \frac{-1+\ \beta\,(-1+p)\ (-1+\delta)+\beta\Psi\,(-1+\pi\,(h))\ \ (-1+(-1+p)\ \beta\ (-1+\delta)+p\,\delta)}{(-1+p)\ (1+\beta\ (-1+\delta))\ (1+(-1+\pi\,(h))\ \beta\,\Psi)}$$

On this boundary, α_1 is bounded for any $\pi\,(h) \in [\pi_0, 1]$. Finally, we have

$$\lim_{d\to(1-\delta)} \alpha_1 = \frac{-1+(-1+p)\ (-1+\pi\,(h))\ \beta\,\Psi}{(-1+p)\ (1+\beta\ (-1+\delta))\,(1+(-1+\pi\,(h))\ \beta\,\Psi)}$$

which is defined for any $\pi\,(h) \in [\pi_0, 1]$. Thus, α_1 is continous on its boundaries. In its interior the denominator is never zero, Since numerator and denominator of α_1 cannot be contemporaneously 0, we conclude that α_1 is continous. Thus there exists a value M such that $M > \alpha_1$ for any pair $\pi\,(h) \in [\pi_0, 1]\,,\ \ d \in [0, (1-\delta)]$.

Moreover

$$\frac{\partial\alpha_1}{\partial\Psi} = \frac{p(1-\pi(h))\beta\delta}{(-1+d)\,(-1+p)\,(1+\beta(-1+\delta))\,(1+(\pi\,(h)-1)\,\beta\Psi)^2} > 0.$$

Proof of Proposition 2)

To prove the Proposition, we only need to use the Implicit Function Theorem. To simplify the notation, we indicate the utility in its maximum $U(T^*, h^*)$ as $U(T^*(\Psi), h^*(\Psi), \Psi)$, and its partial derivatives with$U_{ij}\quad i,j = h, \Psi$. Remark that, since T is discrete, in a neighborhood of (h^*), T^* does not change. The derivative $\frac{\partial h^*}{\partial\Psi}$ is thus given by$\frac{\partial h^*}{\partial\Psi} = -\left[\frac{U_{h\Psi}}{U_{hh}}\right]$

notice that the denominator of $(\frac{U_{h\Psi}}{U_{hh}})$ is negative because U_{hh} is the second derivative of the utility in its max. Therefore, the sign of $(\frac{U_{h\Psi}}{U_{hh}})$ in h^* is the sign of $U_{h\Psi}$.

The derivative $U_{h\Psi}$ is

$$U_{h\Psi} = -\left(\frac{(y+m)\left(-\left(\frac{p\,\beta^{1+T}\,(1-\pi(h))\,(1-\Psi+\Psi\,\pi(h))}{(1-(1-p)\,\beta)\,(1-\beta\,(\Psi-\Psi\,\pi(h)))^2}\right)-\frac{p\,\beta^T\,(-1+\pi(h))}{(1-(1-p)\,\beta)\,(1-\beta\,(\Psi-\Psi\,\pi(h)))}\right)}{x^2}\right)-$$
$$-\frac{((1+h)\,(y+m))\left(\frac{p\,(-1+\beta)\,\beta^T\,(-1-\beta\,\Psi+\beta\,\Psi\,\pi(h))\,\pi'(h)}{(1+(-1+p)\,\beta)\,(1-\beta\,\Psi+\beta\,\Psi\,\pi(h))^3}\right)}{x^2}+$$
$$+\frac{2\,((1+h)\,(y+m))\left(\frac{p^2\,(-1+\beta)^2\,\beta^{2\,T}\,\Psi\,(-1+\pi(h))\,\pi'(h)}{(1+(-1+p)\,\beta)^2\,(1-\beta\,\Psi+\beta\,\Psi\,\pi(h))^4}\right)}{x^3}$$

where

$$x \equiv 1-\frac{p\,\beta^T\,(1-\Psi+\Psi\,\pi(h))}{(1-(1-p)\,\beta)\,(1-\beta\,(\Psi-\Psi\,\pi(h)))}.,$$

$$y \equiv \frac{\beta\left(\frac{1-\beta^{-1+T}}{1-\beta}-\frac{\left(1-(\beta\,(1-\delta))^{-1+T}\right)(1-\delta)}{1-\beta\,(1-\delta)}\right)}{\delta}$$

and

$$m \equiv \frac{(1-p)\,\alpha\,\beta^T\,\left(1-(1-\delta)^T\right)}{(1-(1-p)\,\beta)\,\delta}$$

We want to prove that this derivative can be negative in (T^*, h^*), where T^* is finite. To do so, it is sufficient to find an example of a parameters set for which $\frac{\partial h^*}{\partial \Psi}$ is always negative. We have performed several numerical simulations, and we are going to report an example.

Set for instance $\beta = .8$; $\delta = .05$; $\Psi = .7$; $p = .2$. We substitute these values into α_1 and multiply it by 5 to get $\alpha > \alpha_1$.

After these substitutions, $U_{h\Psi}$ is still a function of $\pi(h)$, $\pi'(h)$, T, h.

Now, let

$$\begin{aligned}
\beta^T &\equiv b \quad (0 < b \leq \beta) \\
(1-\delta)^T &\equiv d \quad (0 < d \leq (1-\delta)) \\
\pi(h) &\equiv \pi \quad (\pi_0 < \pi \leq 1) \\
\pi'(h) &\equiv \pi p h > 0.
\end{aligned}$$

We can now solve the following inequality system:

$$\begin{aligned} U_{h\Psi} &< 0 \\ 0 &< b \leq .8 \\ 0 &< d \leq .95 \\ \pi_0 &< \pi \leq 1 \\ \pi p h &> 0 \\ h &\geq 0 \end{aligned}$$

it is important to remark that the dependence on time is given by b and d. The system has two sets of solutions, and we report for brevity the most interesting one:

$$\begin{aligned} 0 &< b \leq .8 \\ 0 &< d \leq .95 \\ 0 &< \pi \leq 1 \\ 0 &< \pi p h < \frac{-11 - 3\pi + 14\pi^2}{-39 + 14\pi} \\ 0 &< h < \frac{(\pi - 1)\left[25b\left(3 + 7\pi\right) - 18\left(11 + 14\pi\right)\right]}{\pi p h\left[702 - 252\pi + 25b\left(-17 + 7\pi\right)\right]} - 1 \end{aligned}$$

Thus, in order, to have $\frac{\partial h^*}{\partial \Psi} < 0$ we need at the same time that both h^* and $\pi'(h^*)$ be sufficiently small. In other words, the optimal human capital has to be sufficiently small, and the selective policy has to be "severe", in the sense that the marginal increase in the probability of admission is small as h grows.

It is important to remark that:

1) this result is independent of T: it holds for any b and d, and thus for any T^*;

2) it is always possible to find $0 < h^* < \frac{(\pi(h^*)-1)[25b(3+7\pi(h^*))-18(11+14\pi(h^*))]}{\pi'(h^*)[702-252\pi(h^*)+25b(-17+7\pi(h^*))]} - 1$ because the first-order condition is linear in θ_i, and we can solve it with respect to θ_i.Then, it is always possible to choose θ_i such that as $0 < h^* < \frac{(\pi(h^*)-1)[25b(3+7\pi(h^*))-18(11+14\pi(h^*))]}{\pi'(h^*)[702-252\pi(h^*)+25b(-17+7\pi(h^*))]} - 1$. As a consequence, the condition on h^* is indeed a condition on θ.

[[[$\theta_i- > \infty$, $h^* \rightarrow 0$, **and we can appropriately choice θ_i to find an individual who meets the condition.**]]]

The other set fo solutions allows for a less severe policy, but is not valid for any T.

In general, repeating the procedure with different values for β, δ, Ψ, p, gives two sets of solutions with the properties discussed above: when the selective policy is "severe", and that h^* is low enough, the effect is negative for any T; when the policy is less severe we need that T^* be sufficiently high.