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Abstract

This paper explores the relationship between the severity of credit constraints and long run inflation in a simple non Ricardian setting. It is shown that a low positive inflation can loosen credit constraints and that this effect yields a theory of the optimal long run inflation target with no assumption concerning nominal rigidities or expectation errors. Credit constraints introduce an un-priced negative effect of the real interest rate on investment. Because of this effect, the standard characterization of economic efficiency with the Golden Rule fails to apply. When fiscal policy is optimally designed, the first best allocation can be achieved thanks to a positive inflation rate and a proportional tax on consumption.

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1 Introduction

The question of the optimal long run inflation rate has been rejuvenated by the discussion of the objective function of central banks (Bernanke *and al.,* 1999). This hot debate has of course produced many empirical and theoretical contributions (e.g., Walsh, 1998 and Lucas 2000). Many of these outlined the cost of inflation either theoretically (Sargent and Lungqvist, 2000, chap. 17) or empirically (Fisher, 1993). Most of the theoretical models yield the Friedman Rule as a long run optimal evolution of inflation (Abel, 1987). This rule stipulates that monetary authorities must induce a long run deflation such that the nominal interest rate is driven to 0. This kind of policy recommendation is at odds with the actual practice of central banks, which is always to target small positive value of long run inflation, usually between 1 and 3 % (Bernanke and Mishkin, 1997). In the literature, the rationale for a positive value of inflation invokes many arguments, including among which nominal downward rigidities (Akerlof, Dickens and Perry, 1996), the zero bound on the nominal interest rate, measurement problems and the fear of deflation (Delong 1999), which could lead to credit problems. This paper explores precisely the relationship between credit problems and inflation. The goal is not only to assess whether credit problems are worsened because of deflation but also, symmetrically, if a small positive inflation can loosen credit constraints and hence increase welfare in the long run, which could justify the positive inflation target of the central banks.

The basic intuition can be written simply. First, because of credit constraints the decentralized level of investment can be too low and the long run capital stock may be to small. Second, inflation decreases the financial return on money and thus induces a shift away from money toward financial savings. This is the so-called "Tobin effect" (Tobin, 1965). The resources available to finance investment increase, which can increase the welfare of the economy. As noted by Walsh (1998), this effect has not been modeled with explicit micro-foundations yet, although they are necessary for at least two reasons. First, one must have an explicit social welfare function to derive the effect of inflation on welfare and not only on real variables. Indeed, inflation decreases the return on money balances, which reduces the ability of private agents to smooth their consumption. Second, one has to prove that the effect of inflation on capital accumulation holds in the long run and not only during the adjustment process.

This paper introduces explicit microfoundations based on rational expectations and on flexible prices and shows that there is a positive *long run* inflation rate, around 1.5%, which maximizes the social welfare. This result is the effect of three standard imperfections.

The first one is a standard shopping time constraint used in a growth model. This constraint yields a money demand, even when money is dominated by other assets. More importantly, the second imperfection is the introduction of credit constraints in monetary growth models. The empirical relevance of credit constraints has been highlighted by many papers (see Chirinko and Schaller, 1995; Hubbard, 1998). The theoretical researches on microfounded credit constraints, such as Holmstrom and Tirole (1997, 1998a,b) based on moral hazard or Kiyotaki and Moore (1997) based on lack of commitment, imply that the amount of borrowing available to private agents is below its optimal value and that it is limited by the amount of pledgeable income which can be used as collateral.

The third one is a non Ricardian economy based on an overlapping generation structure. The choice of a non Ricardian structure stems first from empirical studies which reject the altruistic model of the family (Hurd, 1990; Altonji, *et al.,* 1992, 1997). If the bequest motive is not crucial to the saving behavior, then the infinitely living representative agent may not be a satisfactory assumption for studying the long run effect of monetary policy. Indeed, from the work of Weiss (1980), Abel (1987), Buiter (1988) and Weil (1991), it is known that in these non Ricardian frameworks money is not super-neutral : changes in the inflation rate have long run real effects on capital, even with the assumption of rational expectations and flexible prices. Indeed, to put it like Weil (1991), inflation is a transfer of resources between generations, hence it affects long run real allocations. As was shown by Buiter (1988), as soon as living households expect agents to enter the economy without any bequest, the economy becomes non Ricardian. Various formalizations of non Ricardian environments such as the two-period overlapping generations framework (Weiss, 1980), or the overlapping generations structure with infinitely living agents (Weil, 1991) have been studied in the literature. To allow for a simple analysis of credit constraints with analytical results, I use the simplest non Ricardian monetary setting, which is a two-period OLG structure with a shopping time constraint.

It is shown that the standard rules for characterizing economic efficiency, such as the Golden Rule (Abel, 1987), do not apply when credit constraints are binding. It induces under-investment compared to the optimal level. Because of the non Ricardian structure, a positive inflation decreases the long run real interest for the new money to be accepted and increases the long run capital stock. This effect can alleviate the initial negative effect of credit constraints. But, inflation has also the standard distorting effect on consumption because it prevents households to smooth optimally their consumption (Abel, 1987; Lucas, 2000). Hence, inflation is related to a trade-off between investment and the allocation of consumption.

The model yields two results. First, the optimal inflation rate is determined for the case of an un-optimized fiscal policy. With realistic parameter values, it is shown that the optimal annual inflation rate is around 1.5%. Moreover, this value increases as credit constraints become more severe. Second, I show that the first best can be achieved if both fiscal and monetary policy are optimally designed. A simple fiscal policy can alleviate the distorting effect of inflation by introducing a proportional tax on consumption. The value of inflation which allows the first best to be reached is again positive.

This paper is related to two strands of literature. The first one concerns the economy of credit constraints. More precisely this paper is related to the recent models which give explicit microfoundations based on asymmetries of information to credit constraints and to the role of collateral, such as Holmstrom and Tirole (1998) or Kiyotaki and Moore (1997). Kiyotaki and Moore introduce credit constraints to model credit cycles and do not focus on long run inflation. The question of my model is close to the one addressed by Holmstrom and Tirole (1998a), which concerns the supply of public liquidity in the economy. Whereas their recommendation is that the State should provide pledgeable income to private agents to reach the optimal level of production, I show that monetary policy is a simple tool with which to alleviate the effects of credit constraints thanks to a decline in the real interest rate.

Second, this paper is related to the literature on optimal monetary policy in non Ricardian environments (Weiss, 1980; Abel, 1987; Weil, 1995; Benassy, 2003a,b). It builds on Weiss (1980) and Abel (1987). The basic framework is an OLG model, with a shopping time constraint. Such a framework yields a cost of inflation and the Golden Rule and the Friedman Rule are optimal when there are no credit constraints. Credit constraints are introduced as a simple deviation from this benchmark case. The introduction of a non Ricardian structure with credit constraints distinguishes this model from other study of optimal monetary policy (Chari *et al.*, 1996; Sargent and Lunqvist, 2000 among others).

The paper is presented in eight sections. Section 2 introduces the OLG structure. Section 3 presents the production sector and the formalization of credit constraints. Section 4 determines the first best level of production and exhibits the effects of market imperfections. Section 5 presents market equilibria and determines the link between inflation and the real interest rate. The effect of inflation on welfare is analyzed in section 6, which yields an optimal inflation rate when fiscal policy is not optimized. Section 7 exhibits the optimal fiscal and monetary policy and shows that the first best can be achieved. Section 8 is the conclusion.

2 Population

The model has an overlapping generations structure, where each agent lives for two periods. There are two types of agents at each period, workers and entrepreneurs. To simplify the algebra, I assume that the populations of entrepreneurs and workers grow at the same rate n, and that their size is equal to L_t at each period.

$$L_t = (1+n)^t \tag{1}$$

The economic role of entrepreneurs is presented in section 3.2 and concerns the intermediate good sector.

Each worker can sell inelastically one unit of labor when young and consumes both when young and old. The net nominal revenue of a young worker is composed of two elements. The first one is the wage income w_t earned in the first period of life. The second element is the amount of money given by the state, $P_t \mu_t$, where μ_t is the quantity of money given in real terms, and where P_t is the price level of the final good.

The net nominal revenue of old workers is the sum of the gross revenue of savings s_t held in financial assets, that is $s_t (1 + r_{t+1})$ where r_{t+1} is the riskless nominal interest rate between period t and period $t+1$, and the amount of money held M_t. Moreover, I assume that the workers can fully diversify their risks on the financial markets. As there is no aggregate risk, the revenue from financial savings is thus deterministic. To simplify the algebra, I assume that the instantaneous utility function is $u(.) = \ln(.)$. This specification allows for a simple derivation of analytical results for all specifications of the model.

The money demand is modeled by a shopping time model[1]. Each young household spends a

[1] The same money demand can be obtained with the cash-in-advance constraint introduced by Alvarez *et al.*

given amount of time $\bar{l}$ to acquire consumption good. As is standard in shopping time model, the time spent is l_t negatively related to the level of consumption, and negatively related to the holdings of real money balances $\frac{M_t}{P_t}$. I use the following standard transaction technology

$$l_t = \frac{c_t}{M_t/P_t}\tilde{\theta}$$

where $\tilde{\theta}$ is a parameter corresponding to the time cost per trip to the bank as in Baumol (1952). As a consequence, the demand of real money balances is

$$M_t/P_t = \theta c_t$$

where $\theta = \frac{\tilde{\theta}}{\bar{l}} < 1$ (at least one trip to the bank). The rationale for money demand is crucial in non Ricardian framework to assess the long run effect of inflation. Hence, this money demand function is discussed below in relation to basic empirical facts.

A worker born at date t chooses his consumption at date t, $t+1$, namely c_t^y, c_{t+1}^o to solve the standard following program

$$\max_{c_t^y, c_{t+1}^o, s_t, M_t} \ln\left(c_t^y\right) + \gamma \ln\left(c_{t+1}^o\right)$$

$$s.t. \qquad P_t c_t^y + s_t + M_t = w_t + P_t \mu_t$$

$$P_{t+1} c_{t+1}^o = s_t\left(1 + r_{t+1}\right) + M_t$$

$$M_t = \theta P_t c_t^y \quad \text{with } \theta \leq 1$$

The first two constraints are the budget constraints at period t and $t+1$, the third constraint is the money demand for liquidity services. The solution of this program is simply

$$c_t^y = \frac{1}{1+\gamma}\frac{1+r_{t+1}}{1+(1+\theta)\,r_{t+1}}\left(\frac{w_t}{P_t}+\mu_t\right) \tag{2}$$

$$c_{t+1}^o = \frac{\gamma}{1+\gamma}R_{t+1}\left(\frac{w_t}{P_t}+\mu_t\right) \tag{3}$$

(2001), which is that *current* consumption is limited by current money holdings. Hence $c_t \leq \nu M_t/P_t$

Where, with standard notations, R_{t+1} denotes the real *gross* interest rate between period t and $t+1$:

$$R_{t+1} = \frac{(1+r_{t+1})P_t}{P_{t+1}} \tag{4}$$

Consumption of workers satisfy the following relationship

$$\frac{c^o_{t+1}}{c^y_t} = \gamma \frac{1+(1+\theta)\,r_{t+1}}{1+r_{t+1}} R_{t+1} \tag{5}$$

The demand for money is simply given by

$$\frac{M_t}{P_t} = \theta \frac{1}{1+\gamma} \frac{1+r_{t+1}}{1+(1+\theta)\,r_{t+1}} \left(\frac{w_t}{P_t} + \mu_t \right) \tag{6}$$

This relationship proves that the demand for money is a decreasing function of the nominal interest rate, everything else being equal. Indeed, the nominal interest rate should be seen here as the difference between the real return on financial markets, $r_{t+1} - \pi_{t+1}$, if π_{t+1} is the annual net inflation rate, minus the real return on money, which is $-\pi_{t+1}$. As a consequence, controlling for the real interest rate, the real money demand is negatively related to the inflation rate.

This relationship is a first justification for the choice of shopping time model. Indeed, the negative correlation between inflation and real money demand is a well established empirical fact (Goldfeld and Sichel, 1990; Attanasio *et al.*, 2002). In this framework, the formalization of money demand by the standard cash-in-advance assumption yields a money demand which is an increasing function of the nominal interest rate, what is inconsistent[2]. This relation between the demand for money and the nominal interest rate can also be obtained in models where the money enters the utility function (Weiss 1980; Abel 1987). The choice of a shopping time

[2] The standard cash-in-advance model is $c_{t+1} \leq \nu M_t / P_{t+1}$ with $\nu \leq 1$. In this model, it would yield the money demand

$$\frac{M_t}{P_t} = \nu \frac{\gamma}{1+\gamma} \frac{1+r_{t+1}}{1+\nu r_{t+1}} \left(\frac{w_t}{P_t} + \mu_t \right)$$

which is increasing in r_{t+1}.

model instead of a money in the utility function model stems from the fact that it simplifies the algebra and that it allows the optimality of the Friedman Rule[3] to be studied. Finally, the assumption that households spend a given amount of time shopping is introduced to make the algebra tractable. This assumption does not reduce the generality of the model because for the low inflation equilibria below, it does not seem realistic to assume that there is a trade-off between time spent shopping and time spent in production. Moreover, this formalization of money demand preserves the cost of inflation extensively studied by Lucas (2000). Indeed, because if this constraint household must keep some resources remunerated at a rate $-\pi$, whereas the financial titles are remunerated at a rate $r_{t+1} - \pi_{t+1} \geq -\pi_{t+1}$. Note that the opportunity cost to hold money disappears if the Friedman rule is applied. In this case, $r_{t+1} = 0$, and the return on money is the equal to the return on financial markets.

The total financial savings of the workers at period t are

$$L_t s_t = \frac{1}{1+\gamma}\left(\gamma - \frac{\theta}{1+(1+\theta)\,r_{t+1}}\right)(w_t + P_t\mu_t)\,L_t \tag{7}$$

If there is no transaction constraint ($\theta = 0$), the real value of savings $\frac{L_t s_t}{P_t}$ is simply proportional to the real total revenue $\left(\frac{w_t}{P_t} + \mu_t\right) L_t$. Because of transaction constraint, the total amount of savings is an increasing function of the nominal interest rate. As a consequence, an increase in inflation for a given real interest rate increase the amount of financial savings. Indeed, it decreases the demand for money and induces a shift toward remunerated financial instruments. This the so-called Tobin effect described in Tobin (1965), and which can be found in empirical studies (Loayza *et al.*, 2000). Of course, this partial equilibrium result has to be studied in general equilibrium and in interaction with credit constraints.

[3] In money-in-the-utility function models, normative conclusions on the Friedman Rule can be obtained only if a satiation point is introduced in the utility function (Abel, 1987), which is not the case in Weiss (1980) or Weil (1991). The introduction of such a satiation point makes it impossible to obtain analytical results with credit constraints.

Finally, total consumption from workers can be written in real terms as

$$C_t^{work} = L_t c_t^y + L_{t-1} c_t^o \tag{8}$$

3 Production

3.1 Final Goods Sector

The representative firm in the final goods sector produces at period t with constant returns, using labor and a quantity K_t of intermediate goods.

$$Y_t = L_t^{1-\beta} K_t^{\beta} \tag{9}$$

The total amount of intermediate goods K_t is bought from N_t intermediate goods producers which are assumed to be perfect substitutes :

$$K_t = \int_0^{N_t} y_i di$$

The price of one unit of final goods at period t is denoted P_t. The price of the intermediate goods is the same for all producers because of perfect substitutability and is denoted p_t^{int} at period t. The wage rate is denoted w_t. The program of the firm is thus

$$\max_{L_t, y_i} \;\; P_t Y_t - \int_0^{N_t} p^{int} y_i di - w_t L_t$$

The first order condition yields the real wage

$$\frac{w_t}{P_t} = (1-\beta)\frac{Y_t}{L_t} \tag{10}$$

If each intermediate goods producer sells only an amount of goods $y = 1$, which will be the case, the above program gives the price of intermediate goods as a function of N_t :

$$p_t^{int} = \beta P_t \left(\frac{L_t}{N_t}\right)^{1-\beta} \tag{11}$$

3.2 Intermediate Sector

There are L_t entrepreneurs who live for two periods. They are risk-neutral and their utility depends on their consumption of final good in their second period of life. At period t, each newborn entrepreneur faces a shock ε, which determines the amount of final goods he must buy to be able to produce one unit of intermediate good at period $t+1$, hence $y = 1$. The value ε is a productivity shock: the higher is ε, the lower the productivity. The shock is idiosyncratic and is drawn out of a density of distribution f which is common knowledge. If an entrepreneur does not invest the amount ε then production does not take place. The financing decision will be made in a situation of asymmetries of information which will yield credit constraints. I use in this section the formalization of credit constraints used by Holmstrom and Tirole (1997), which is based on moral hazard. The results would be the same if credit rationing were based on lack of commitment as introduced by Kiyotaki and Moore (1997). I have chosen Holmstrom and Tirole's formulation only because it is easier to compare the results with the first best outcome of the model.

After the shock at period t, each entrepreneur can make two types of unverifiable effort. Either he makes a high effort, in which case the probability of success is q_H, or he cheats. In this case, he can sell an amount $B < 1$ of intermediate goods for private use at period $t+1$ and the probability of success in the production of the unit of final good is $q_L < q_H$. Because of the probability of failure, the nominal interest rate paid by entrepreneurs includes a risk premium. This interest rate is denoted $\tilde{r}_{t+1}$ and is determined below.

As $y = 1$, the profits derived from production are $p_{t+1}^{int} - P_t\left(1 + \tilde{r}_{t+1}\right)\varepsilon$. Hence, in case of high effort, the expected nominal remuneration of the entrepreneur is $q_H\left(p_{t+1}^{int} - P_t\left(1 + \tilde{r}_{t+1}\right)\varepsilon\right)$. If he cheats, his expected nominal remuneration becomes $p_{t+1}^{int}B + q_L\left(p_{t+1}^{int} - P_t\left(1 + \tilde{r}_{t+1}\right)\varepsilon\right)$. The condition for the entrepreneur not to cheat is

$$q_H\left(p_{t+1}^{int}-P_t\left(1+\tilde{r}_{t+1}\right)\varepsilon\right)\geq p_{t+1}^{int}B+q_L\left(p_{t+1}^{int}-P_t\left(1+\tilde{r}_{t+1}\right)\varepsilon\right)$$

As lenders anticipate this constraint, they finance entrepreneurs only if the previous relationship is fulfilled, giving the incentive not to cheat and thus to get the highest return[4]. After few calculations, and using the value of p_t^{int} given by equation (11), and with $\Delta q=q_H-q_L$ one finds that the condition to be financed is $\varepsilon\leq\varepsilon_t^*$, where ε_t^* is the threshold defined by

$$\varepsilon_t^*=\beta\left(1-\frac{B}{\Delta q}\right)\left(\frac{L_{t+1}}{N_{t+1}}\right)^{1-\beta}\frac{P_{t+1}}{\left(1+\tilde{r}_{t+1}\right)P_t} \tag{12}$$

The total number of firms which produce at period $t+1$ is the number of newborn entrepreneurs at period t whose shock satisfies $\varepsilon\leq\varepsilon_t^*$, and who produce with a probability q_H. Hence, if F denotes the cumulative distribution of shocks, $F\left(\varepsilon\right)\equiv\int_0^{\varepsilon}f\left(x\right)dx$, the total number of intermediate good producers at period $t+1, N_{t+1}$ is

$$N_{t+1}=q_HL_tF\left(\varepsilon_t^*\right) \tag{13}$$

As all the financed entrepreneurs have incentives to make the high effort, the probability of success is q_H. The return of 1 unit of money invested in production is $q_H\left(1+\tilde{r}_{t+1}\right)$ and as the riskless interest rate is r_{t+1}, the competition among financiers yields

$$q_H\left(1+\tilde{r}_{t+1}\right)=1+r_{t+1}$$

The risk premium[5] is thus $\frac{1}{q_H}$. Using the previous equality and the value of N_{t+1} given by (13) in (12) yields

$$\varepsilon_t^*F^{1-\beta}\left(\varepsilon_t^*\right)=\beta\left(\frac{q_H}{1+n}\right)^{\beta}\frac{1+n}{R_{t+1}}\left(1-\frac{B}{\Delta q}\right) \tag{14}$$

[4] More formally, I assume that q_L is low enough such that the lenders can be repaid only in the high effort case, as in Holmstrom and Tirole (1998a). A benchmark case is $q_L=0$.

[5] As households can fully diversify their risks, the net nominal revenue from savings is deterministic and equal to r_{t+1}. Moreover, aggregation in the final good sector makes the production of final goods and the real wage deterministic.

where the real interest rate R_{t+1} was defined in equation (4). This equality defines ε_t^* as a function of R_{t+1} and of the parameters of the model.

Because of moral hazard, some entrepreneurs with a positive net present value (NPV) are not financed : entrepreneurs with a positive NPV have a shock such that the selling price is above the financing cost $p_{t+1}^{int} \geq P_t (1 + \tilde{r}_{t+1}) \varepsilon$, with as before $q_H (1 + \tilde{r}_{t+1}) = 1 + r_{t+1}$. This yields the threshold ε^{**} below which entrepreneurs have a positive NPV. Substituting p_{t+1}^{int} by its expression given by (11), and as the number of intermediate good producers is $N_{t+1}^{NPV} = q_H L_t F (\varepsilon_t^{**})$, one finds that the threshold ε^{**} satisfies the equality

$$\varepsilon_t^{**} F^{1-\beta} (\varepsilon_t^{**}) = \beta \left(\frac{q_H}{1+n} \right)^{\beta} \frac{1+n}{R_{t+1}} \tag{15}$$

The threshold ε^* is always below the threshold ε^{**} because $F^{1-\beta}$ is increasing. Strictly speaking, the credit constrained entrepreneurs are those who have a project with a positive NPV, but who are not financed. Hence those for whom $\varepsilon^* \leq \varepsilon \leq \varepsilon^{**}$. The fraction of credit constrained firms is thus $\frac{F(\varepsilon^{**}) - F(\varepsilon^*)}{F(\varepsilon^{**})}$. Note that the two thresholds are equal when the private gains B are equal to 0. Indeed, in this case, there is no incentive problem and the complete contract result can be obtained. The two thresholds increase when the real interest rate decreases. In particular, the credit constraint threshold decreases, that is, more entrepreneurs are financed when the real interest rate R_{t+1} decreases. Indeed, the basic idea of this formulation of credit constraint is that the financing contract must leave enough surplus for the entrepreneur in order to create the incentives to make a high effort. When the real interest rate increases, the firms have to devote more resources to pay back their debt. As a consequence the surplus left for incentives decreases, and more firms become credit constrained. A decrease in real interest rate is a means to decrease the severity of credit constraint, for the very same mechanism[6].

[6] This is a standard result in the credit constraints literature (Kiyotaki and Moore,1997; Holmstrom and Tirole, 1997)

To give a simple specification for calibration purpose, I assume that the shock is uniformly distributed on the segment [0..1]. As a consequence, $f = 1$, and if $x \leq 1$, $F(x) = x$ and $G(x) = \frac{x^2}{2}$. In this case, the threshold ε^* can be written, with equation (14) :

$$\varepsilon^*(R) = \left[\beta\left(\frac{q_H}{1+n}\right)^{\beta}\frac{1+n}{R}\left(1-\frac{B}{\Delta q}\right)\right]^{\frac{1}{2-\beta}} \tag{16}$$

$\varepsilon^*(R)$ is a decreasing function of R. The previous equality is true only when $\varepsilon^*(R) < 1$. When $\varepsilon^*(R) \geq 1$, the real interest rate is small enough such that all entrepreneurs are financed. I will assume for now that $\varepsilon^*(R) < 1$, which will be checked in the calibration exercise. With this specification, the fraction of credit constrained firms is

$$\frac{\varepsilon^{**}-\varepsilon^*}{\varepsilon^{**}} = 1-\left(1-\frac{B}{\Delta q}\right)^{\frac{1}{2-\beta}} \tag{17}$$

An entrepreneur with a shock ε_t demands a quantity ε_t of final goods at period t, conditional on the fact that he is financed : $\varepsilon_t \leq \varepsilon_t^*$. As a consequence, the total amount of final goods bought by entrepreneurs at period t, I_t, is

$$I_t = L_t G(\varepsilon_t^*) \tag{18}$$

where the function G is defined by $G(\varepsilon) \equiv \int_0^{\varepsilon} x f(x)\, dx$.

The total consumption of final goods by the financed entrepreneurs at period $t+1$ is the value of the production in final good $\frac{p_{t+1}^{int}}{P_{t+1}}$, minus the financing cost expressed in final goods $\frac{P_t(1+\tilde{r}_{t+1})}{P_{t+1}}\varepsilon$, times the probability of production q_H. As a consequence, the total consumption of entrepreneurs at period $t+1$, C_{t+1}^{entr} is

$$C_{t+1}^{entr} = \int_0^{\varepsilon_t^*} q_H\left(\frac{p_{t+1}^{int}}{P_{t+1}} - \frac{P_t(1+\tilde{r}_{t+1})}{P_{t+1}}\varepsilon\right) L_t f(\varepsilon)\, d\varepsilon$$

Using the uniform distribution hypothesis, such that $\varepsilon_t^* F^{\beta}(\varepsilon_t^*) = 2G(\varepsilon_t^*) = (\varepsilon_t^*)^2$, the definition of R_{t+1} and the expression p_{t+1}^{int} given by equation (11), equations (13) and (16), one finds

$$C_{t+1}^{entr} = L_t \frac{1+\frac{B}{\Delta q}}{1-\frac{B}{\Delta q}} R_{t+1} G(\varepsilon_t^*) \tag{19}$$

Using the expression of the number of intermediate goods producers (13) with equation (9), one can determine the production of final goods at period t :

$$Y_t = L_t^{1-\beta} N_t^{\beta} = \left(\frac{q_H}{1+n}\right)^{\beta} L_t F^{\beta}\left(\varepsilon_{t-1}^{*}\right) \tag{20}$$

Finally, using equation (10), one finds the expression of the real wage :

$$\frac{w_t}{P_t} = (1-\beta)\left(\frac{q_H}{1+n}\right)^{\beta} F^{\beta}\left(\varepsilon_{t-1}^{*}\right) \tag{21}$$

This relationship proves that the workers also suffer from credit constraints, which decrease the threshold ε_{t-1}^{*}. Indeed, it reduces capital accumulation and hence labor productivity and the real wage.

4 First Best Allocation

The inefficiencies created by the shopping time and the credit constraints can be exhibited by comparing the previous results and the first best outcome of the model. As the focus of this paper is on long run inflation, the following analysis studies the balanced growth path where the real gross interest rate R and the credit threshold ε^{*} are constant. As a consequence, consumption per capita will be constant, and the growth rate of the economy will be simply given by the growth rate of the population n. To determine the first best allocation, I assume that the central planner maximizes the utility of a representative generation as in Weiss (1980). The central planner has access to all resources in the economy and it faces the technological shock ε on all types of intermediate goods, but it can observe the effort of entrepreneurs and hence it can force them to make the high effort.

For the sake of generality, I assume that the central planner gives a weight η to workers and a weight $1-\eta$ to entrepreneurs in the social welfare function. As entrepreneurs are risk neutral, I simply assume that their utility function is $u(c) = c$.

As the utility of private agents only depends on the consumption of final goods, the program of the central planner is very simple[7]. It first maximizes the amount of final goods available for consumption, that I denote as Y^f, and then redistributes it optimally. Y^f is equal to total production minus the resources given to newborn entrepreneurs to be able to produce. Using directly $y = 1$, Y^f can be written as

$$Y^f = L_t^{1-\beta} \left(q_H L_{t-1} \int_0^{\varepsilon^{opt}} f(\varepsilon)\, d\varepsilon \right)^{\beta} - L_t \int_0^{\varepsilon^{opt}} \varepsilon f(\varepsilon)\, d\varepsilon$$

where ε^{opt} is the threshold below which the entrepreneurs are not financed. The central planner solves simply the following program

$$\max_{\varepsilon^{opt}} Y^f$$

It yields the threshold ε^{opt} defined by the first order condition

$$\varepsilon^{opt} F\left(\varepsilon^{opt}\right)^{1-\beta} = \beta \left(\frac{q_H}{1+n} \right)^{\beta} \tag{22}$$

To redistribute the final goods optimally the central planner maximizes the social welfare function, U^s

$$\max_{c^y, c^o, c^{entr}} \eta \left(\ln c^y + \gamma \ln c^o \right) + (1-\eta)\, c^e \tag{23}$$

$$s.t. \quad L_t c^y + L_{t-1} c^o + L_{t-1} c^e = Y^f$$

Note that the amount c^e is the average utility of all entrepreneurs, financed or not. It yields the allocation for workers

$$\frac{c^o}{c^y} = \gamma (1+n) \tag{24}$$

[7] It is thus assumed that the shopping time does not provide additional utility. If it is not the case, the results in the following sections would be the same for the optimal value of inflation. But, only a second best could be reached because workers would take time shopping in any case.

Comparing the optimal values given by (22) and (24) with the decentralized values given by (5) and (14), one can exhibit the effects on welfare of the two market imperfections, the shopping time and the credit constraints

First, comparing the equality (22) with equations (14), it is easy to show that in absence of credit constraints ($B = 0$) the optimal level of production is obtained when the equality $R = 1 + n$ is satisfied. This expression is simply the Golden Rule which stipulates that the real net interest rate must be equal to the growth rate of the economy. This result is standard in OLG models where the central planner does not discount the utility of future generation (Abel, 1987). When $B = 0$, the shopping time constraint can be cancelled by setting a nominal interest rate equal to 0. Comparing equation (5) and equation (24), one sees that when $r = 0$ and $R = 1 + n$ the workers, who face the shopping time constraint, can smooth optimally their consumption, which is the standard result. Hence, when $B = 0$ the first best can be achieved if $R = 1 + n$ and $r = 0$, with lump sum transfers between entrepreneurs and workers.

Second, when credit constraints are binding, that is when $B > 0$, there is a new trade-off between production and intertemporal allocation of consumption. Indeed, in this case, equations (14) and (22) show that the financing threshold is equal to the optimal threshold, $\varepsilon^* = \varepsilon^{opt}$ when $R = R^{opt}$, with

$$R^{opt} = (1+n)\left(1 - \frac{B}{\Delta q}\right) < 1 + n \tag{25}$$

But then, intertemporal consumption is not optimal in the general case :

$$\frac{c^o}{c^y} = \gamma\,(1+n)\left(1 - \frac{B}{\Delta q}\right)\frac{1 + (1+\theta)\,r}{1+r} \neq \gamma\,(1+n)$$

Note that even if the Friedman Rule prevails, that is if $r = 0$, the allocation is not optimal: $\frac{c^o}{c^y} = \gamma\,(1+n)\left(1 - \frac{B}{\Delta q}\right)$. This trade-off between production and allocation will be at the core of the model. Indeed, in the following section, I prove that monetary policy can decrease the long run real interest rate below $1 + n$ by setting a positive nominal interest rate, which creates an additional distorting effect on the consumption of workers compared to the first best. As a consequence, the trade-off, which yields the optimal nominal interest rate, will be between increasing production to come closer to the first best level, and increasing the distorting effect of a positive nominal interest rate.

When credit constraints are binding, the optimal level of production can be achieved only if the economy is dynamically inefficient in the sense of the seminal paper of Diamond (1965) and of Abel *et al.* (1989). If the Golden Rule is verified in this credit constrained economy, then there is under-investment and under-production in the long run. This result is not surprising. Indeed, because of the asymmetries of information, the real interest rate does not convey the right information about the social return on investment. The productivity of capital is higher than the real interest rate if the Golden Rule prevails. A decrease in the real interest rate (compared to the situation without credit constraint) is a means to stimulate investment. As a direct consequence, the optimal real interest rate defined below will be a decreasing function of severity of the incentive problems, measured by $\frac{B}{\Delta q}$.

5 Market Equilibria and Solution of the Model

The following analysis concentrates on the balanced growth path where the exogenous money supply grows at a constant rate, where the nominal interest rate r, the real gross interest rate R, the credit threshold ε^* and the gross inflation rate $\Pi = \frac{P_{t+1}}{P_t}$ are constant. As a consequence, consumption of households and entrepreneurs are constant. I assume that the monetary authorities choose the inflation rate and determine as a result the amount of money given by helicopter drops to the young workers[8]. Hence, this section exhibits the solution of the

[8] I have also solved the model with a more realistic process of money creation, which is open-market operations. The results did not differ qualitatively. Hence, the simple and well known framework is used.

model as a function of the exogenous inflation rate.

There are four market equilibria which must be studied: the labor market, the final goods market, the financial market and the money market. Because of the Walras Law, only three of these equilibria need to be exhibited.

The labor market equilibrium simply states that all workers are employed. Hence, the size of the labor working in the final goods sector at period t is L_t.

The good market equilibrium is the equality between total production in the final goods sector, Y_t and the sum of three terms: the consumption of workers C_t^{work}, the consumption of entrepreneurs C_t^{entr} and the investment of newborn entrepreneurs, I_t:

$$C_t^{work} + C_t^{entr} + I_t = Y_t \tag{26}$$

The equilibrium on the financial market simply states that total investment is financed by the financial savings of the workers.

$$L_t s_t = P_t I_t \tag{27}$$

I denote Λ_t the stock of money in circulation at each end of period. As money is only held at each period by the young workers, who hold M_t, the period t money market equilibrium is

$$L_t M_t = \Lambda_t \tag{28}$$

Money growth comes from the helicopter drops to all young workers, which were denoted by μ_t in real terms. Hence,

$$\Lambda_t - \Lambda_{t-1} = L_t P_t \mu_t \tag{29}$$

Using the market equilibria, one can exhibit a relationship between inflation and the real interest rate along a balanced growth path. This relationship is the result of the financial market equilibrium which determines the real interest rate R, when it is taken into account the fact that savings depends on inflation and the money transfer μ necessary to induce the level of inflation Π. The calculations to reach this expression are not insightful and are thus presented in appendix. One finds

$$\Pi = \frac{\frac{1}{1+\gamma} + \frac{1-\beta}{\beta}\frac{2}{1-\frac{B}{\Delta q}} - \frac{1+n}{R}}{\frac{1}{1+\gamma} - \frac{1+\theta}{\theta} + \frac{1+\theta}{\theta}\frac{\gamma}{1+\gamma}\frac{1-\beta}{\beta}\frac{2}{1-\frac{B}{\Delta q}}\frac{R}{1+n}}\frac{1}{1+n} \tag{30}$$

The previous equality is the central relationship of the model. It exhibits the long run relationship between the inflation rate and the real interest rate. First, even when there is no credit constraints, that is when $B = 0$, a change in the inflation rate affects the real interest rate and the long run stock of capital. As a consequence, money is not super-neutral even when there are no credit constraints. The long run effects of inflation come from the non Ricardian structure, as has been shown in previous works, notably Weiss (1980), Buitter (1988) and Weil (1991). New agents will enter the economy in the future and receive new money. As a consequence, agents not yet born will benefit from money creation and agents living today face the cost of a high nominal interest rate and a high inflation rate, which increases the opportunity cost to hold money. This taxation of their second period wealth tends to increase real savings and hence to decrease the real interest rate. Indeed, the calibration below shows that for realistic values of the parameters, the real interest rate is a decreasing function of the inflation rate. This result is found in various settings such as the two periods OLG (Weiss, 1980) or the infinitely living agents OLG model (Weil, 1991).

However, in the general case, two effects are at stake depending on the gains for the household of a decrease in the real interest rate. Indeed, assume that the workers gain all the revenue from production such that β tends toward 0. In this case, the inflation rate can be written as

$$\Pi = \frac{1+\gamma}{\gamma}\frac{\theta}{1+\theta}\frac{1}{R}$$

which defines a decreasing relationship between inflation and the real interest rate. If β tends toward 1, the effect on the denominator in (30) vanishes and the inflation rate is an increasing function of the real interest rate. The reason for this is that the total effect on savings of a decrease in the nominal interest depends on the effect on the revenue of workers given by equation (21). If the real wage does not depend on R, then money creation has a negative effect on savings. Indeed, an increase in money creation μ decreases the nominal interest rate. Indeed, as $\frac{\partial \frac{m_t}{P_t}}{\partial \mu_t} = \theta \frac{1}{1+\gamma} \frac{1+r_{t+1}}{1+(1+\theta) r_{t+1}} < 1$, the nominal interest rate r_{t+1} has to decrease for the extra money to be accepted. This decrease in the nominal interest rate contributes to decrease the savings, which contributes to an increase in the real interest rate.

Before turning to the effect of inflation on welfare, one can exhibit the inflation rate which yields the optimal level of production. Indeed, using (30), the inflation rate Π^{prod} which allows to reach $R^{opt} = \left(1 - \frac{B}{\Delta q}\right)(1+n)$ is

$$\Pi^{prod} = \frac{\frac{1}{1+\gamma} + \left(2\frac{1-\beta}{\beta} - 1\right)\frac{1}{1-\frac{B}{\Delta q}}}{\frac{1}{1+\gamma} + \left(2\frac{\gamma}{1+\gamma}\frac{1-\beta}{\beta} - 1\right)\frac{1+\theta}{\theta}} \frac{1}{1+n}$$

When $\beta < 2/3$, this inflation rate Π^{prod} is an increasing function of the severity of the credit constraints $\frac{B}{\Delta q}$. The value β is capital share in total revenue, which is around $\frac{1}{3}$. The assumption $\beta < \frac{2}{3}$ is thus quite realistic. The more severe the credit constraints, the higher the inflation rate which allows the first best level of production to be reached. But, although it reaches the first best value of production, this inflation rate is not optimal because it does not consider the distorting effect of inflation on the consumption of households. The following section studies the effect of inflation on welfare.

6 Inflation and Welfare

This section studies the effect of inflation on welfare to determine the optimal inflation rate. The social welfare function is the one given above:

$$U^s = \eta U^{work} + (1-\eta) c^{entr} \tag{31}$$

where U^{work} is the utility of workers and c^{entr} is the utility of entrepreneurs which is linear in consumption. Some algebra is necessary to express the consumption of each agent as a function of the real interest rate R and the inflation rate. Then using the relationship (30), one can compute social welfare as a function of the inflation rate Π. These calculations are performed in appendix B. Because the results are too involved, it is not insightful to deduce explicit values of the optimal inflation rate. Instead, a simple calibration of the model yields interesting results.

Six parameters have to be determined, γ, β, n, q_H and $\frac{B}{\Delta q}$ and θ. Different values for these parameters are used in the literature (Rios-Rull, 1996; De La Croix and Michel, 2002). As the main focus of this paper is on the production process, it is natural to assume that a generation corresponds to the average utilization period of an investment good in national account, which is 12 years. The value of the household annual discount rate is set to the standard value of 0.97. The annual real growth rate of the economy is assumed to be 2%. The capital share in GDP is set to $\beta = 0.33$. I assume that $q_H = .99$, such that there is 1% of bankruptcies at each period. This number is the average rate of business failures in the US[9].

To determine a realistic value of $\frac{B}{\Delta q}$, one can use the empirical literature on financing constraint and corporate investment such as Fazzari, Hubbard and Petersen (1988) or Chirinko and Schaller (1995). The goal of these papers is to show that firms are credit constrained by proving that additional cash flow increases investment after controlling for the opportunities of investment. Without credit constraints, the cash flow would not influence investment. Using different samples, these papers usually find that one additional dollar increases investment by an amount from 0.1 to 0.4 dollars. I take the conservative value of 0.1. As only the credit constrained firms would invest an additional dollar, this number implies that 10% of the firms are credit constrained. Using equation (17) to set $\frac{\varepsilon^{**}-\varepsilon^{*}}{\varepsilon^{**}} = 10\%$, I get the value $\frac{B}{\Delta q} = 0.16$

[9] This number is taken from the *Statistical Abstract of the US*, section 17 for the period from 1980 to 1998.

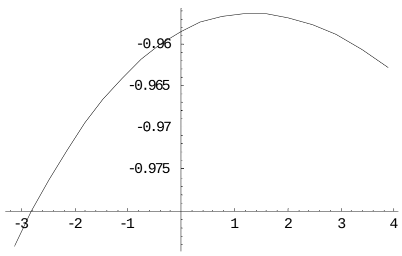

Figure 1: Welfare as a function of the Annual Net Inflation Rate in Percent

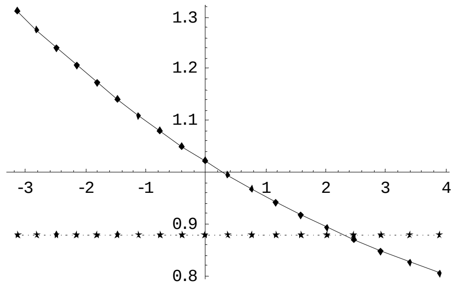

Figure 2: $\frac{c^o}{c^y}$ as a function of π

The value of θ is determined to reach a realistic value of the real money balances in the economy, $M1/GDP$. This ratio varies from 14% in the US to 30% in the Euro area in 2003. I take the average value of 20%. The expression of money demand is provided in appendix B and yields a value $\theta = 0.55$ for realistic values of the real interest rate (less than 5%). Finally, I assume that $\eta = 0.5$ such that the entrepreneurs and the workers enter with the same weight in the social welfare function.

Figure 1 plots the value of the social welfare as a function of the annual net inflation rate. The welfare reaches its maximum at an annual inflation rate equal to 1.4%. Using the relationship (30), one finds that the annual nominal interest rate is equal to 2.7%. The value of ε^* stays below 1 for the whole range of values of inflation. To understand the trade-off behind this graph, figures 2 and 3 plots the surplus left for consumption and the ratio $\frac{c^o}{c^y}$. Figure 2 plots the ratio of consumption of old workers on the consumption of young workers. The horizontal line is

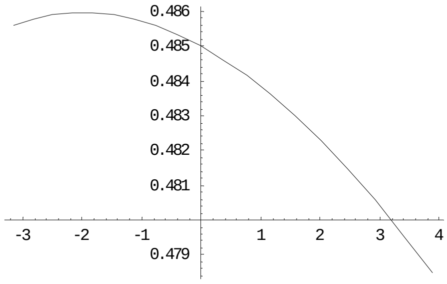

Figure 3: Total Consumption as a function π

the first best value of this ratio, given by $\gamma(1+n)$. The ratio is downward slopping because inflation decreases the return on savings, and thus creates incentives to consume more when young. Hence, young workers save more on financial markets to compensate only partially for the decrease on the return on money holdings. The optimal smoothing is obtained for a value of inflation equal to 2.3%.

Figure 3 plots the surplus per capita left for the consumption of private agents (that is $(Y-I)/L$). The surplus is maximized for $\pi = -1.8\%$. If workers could perfectly smooth their consumption, the optimal inflation rate would be the value that maximizes this surplus. But at this maximum, one can see in figure 2, that consumption of old workers is too high. Hence, a higher inflation than the one which maximizes surplus permits a better consumption smoothing and is optimal although total consumption is smaller. For this reason, the optimal inflation rate is between the inflation rate which optimizes smoothing and the one which maximizes surplus. The result that the value of inflation which maximizes the surplus is lower than the value which allows optimal smoothing is not robust. It is actually reversed when 50% of the firms are credit constraint. What is robust is that these two values differ and hence that the optimal value of inflation yields a second best.

To exhibit the effect of credit constraints on the optimal inflation rate, figure 3 plots the optimal annual net inflation rate as a function of the percentage of credit constrained firms,

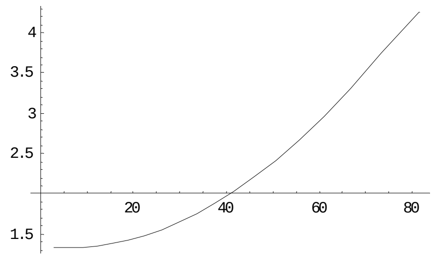

Figure 4: Optimal Inflation Rate as a function of the Percentage of Credit Constraint Firms

which was previously set to 10%. It is checked that credit constraints are binding for every value of the parameter, that is $\varepsilon^* < 1$. As previously explained, the optimal inflation rate is an increasing function of the severity of credit constraints. Indeed, when the number of credit constraint firms increases the production inefficiency worsened compared to the allocating inefficiency. In this case, a higher inflation permits to increase production, at the cost of a greater allocating inefficiency. The convex shape of this function comes from decreasing returns in production. When more firms are credit constrained, the marginal increase in the number of credit constrained firms becomes more costly and it is thus optimal to raise the marginal increase in inflation.

Note that the optimal inflation rate does not converge toward the value implied by the Friedman Rule, $\Pi = \frac{1}{1+n}$, as $\frac{B}{\Delta q}$ tends toward 0, that is when credit constraints disappear. Indeed, since Abel (1987), it is known that the Friedman Rule is optimal in non Ricardian Frameworks only if fiscal policy is designed to reach the optimal real interest rate : fiscal policy cancels the imperfections created by the non Ricardian Structure, and monetary policy with the Friedman Rule cancels the effect of the monetary constraints. As there is no fiscal policy here, there is no reason why the Friedman Rule should be optimal.

The goal of this simple calibration was to show that the trade-off between production and allocation yields realistic values for the long run inflation rate, which is close to the actual target of monetary authorities in developed countries. This simple model also gives a simple reason for which the inflation rate is higher in less developed countries. Indeed, as financial markets are less efficient, credit constraints are more severe, and the increase in production may be more important than the allocation efficiency. Of course, many others factors are at stake and a detailed empirical study is left for future works.

Until now it has been assumed that only monetary policy was used to increase welfare. The next section introduces a simple fiscal policy to reach the first best.

7 A Simple Fiscal Policy to Reach the First Best

The first best outcome could not be reached in the previous section because only monetary policy was available. In this section, a simple fiscal policy is introduced to counter this distorting effect and to reach the first best allocation.

There are three imperfections in this model. The first one is the non Ricardian structure which entails that the equilibrium long run real interest rate may not be optimal. The second is the monetary constraint modeled by a shopping time model. The third is the credit constraint. As was explained above, it is known that in the presence of the first two constraints the Friedman Rule and the Golden Rule are optimal. Here, the third constraint entails that the Golden Rule is no longer optimal on the production side. As a consequence, the Friedman Rule is no longer a natural benchmark, because of the new trade-off between production and allocation. The goal of monetary and fiscal policy is now to allow jointly for both optimal production and optimal consumption.

The basic idea to reach the first best is to introduce a distorting taxation scheme, which cancels the distorting effect of inflation. The main constraint on this fiscal policy is that the information set available to the State to define its transfers must be realistic. For this reason, I assume that the State does not observe the shock faced by entrepreneurs. Hence, it has less information than financial markets on the ability of entrepreneurs to produce and no transfer of capital is possible. Moreover, I assume that the budget of the State is balanced at each period. Even with these constraints, a simple fiscal policy which affects only the workers can reach the first best allocation, when it is used with an optimal monetary policy. The only information that the State must know is the consumption of young workers, or equivalently the money they hold. Roughly speaking, as there are three imperfections, one needs three tools. These are a lump-sum transfer, a distorting transfer and the inflation rate.

Assume that the State introduces a lump sum real tax τ^y taken from young workers, and gives a net nominal amount $\lambda P_t c_t^y$ to young workers at period $t+1$ if they have consumed c_t^y at period t. λ can be either positive or negative. The transfer is proportional to consumption, what is known by young workers. The program of the workers is now

$$\max_{c_t^y, c_{t+1}^o, s_t, M_t} \ln\left(c_t^y\right) + \gamma \ln\left(c_{t+1}^o\right)$$

$$s.t. \qquad P_t c_t^y + s_t + M_t = w_t + P_t \mu_t - P_t \tau^y$$

$$P_{t+1} c_{t+1}^o = s_t \left(1 + r_{t+1}\right) + M_t + \lambda_t P_t c_t^y$$

$$M_t = \theta P_t c_t^y \text{ with } \theta \leq 1$$

It yields

$$\frac{c_{t+1}^o}{c_t^y} = \gamma \frac{P_t}{P_{t+1}} \left(\left(1+\theta\right)\left(1+r_{t+1}\right) - \left(\lambda_t + \theta\right)\right) \tag{32}$$

Now assume that at each period the coefficient λ_t is set by the fiscal authorities to the value

$$\lambda_t = \left(\theta + 1 - \frac{1}{1 - \frac{B}{\Delta q}}\right)\left(1 + r_{t+1}\right) - \theta \tag{33}$$

which depends only on the parameters of the model and on r_{t+1}. With this value, the ratio (32) becomes

$$\frac{c_{t+1}^{o}}{c_{t}^{y}} = \gamma \frac{R_{t+1}}{1 - \frac{B}{\Delta q}} \tag{34}$$

As a consequence, for the value of the gross real interest rate which yields the first best level of production, given by (25), $R^{opt} = (1+n)\left(1 - \frac{B}{\Delta q}\right)$, the allocation of consumption between the two life periods of workers is optimal, as can be seen from equation (24). The inflation rate and the lump-sum transfer have now to be determined jointly to balance the budget of the State and to yield, as a decentralized outcome, $R = R^{opt}$.

For the sake of generality, I introduce a tax on the consumption of entrepreneurs. Each entrepreneur pays a fraction $\zeta < 1$ of its revenue to the State. Entrepreneurs who do not produce pay no tax. The total consumption of entrepreneurs is

$$\tilde{C}_{t+1}^{entr} = (1 - \zeta)\, C_{t+1}^{entr}$$

with the value C_{t+1}^{entr} given as before by equation (19). This tax does not affect the moral hazard problem of entrepreneurs because, firstly, only entrepreneurs for which the incentive constraint is binding are financed in any case. Secondly, entrepreneurs derive utility only from the consumption of final good. Hence, as soon as they can consume something, that is $\zeta < 1$, they have incentive to try to be financed.

The budget constraint of the state at period $t+1$ is

$$\zeta C_{t+1}^{entr} + L_{t+1}\tau^{y} = L_t \lambda_t c_t^{y} \tag{35}$$

The market equilibria are the same as the ones given in section 5. Using these market equilibria, one can find the value of inflation for which the interest rate is equal to its optimal value $R^{opt} = (1+n)\left(1 - \frac{B}{\Delta q}\right)$. The detailed calculations are performed in the appendix C. With the parameters given above, figure 5 plots the optimal inflation rate[10] for two different values of the

[10] There are other solutions for the optimal inflation rate, but below -2%. These ones do not define equilibria

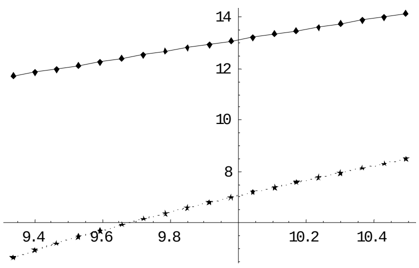

Figure 5: Annual Optimal Inflation for $\zeta = 0$ and $\zeta \simeq 1$.

tax ζ on the consumption of entrepreneurs, as a function of the percentage of credit constrained firms (in the neighborhood of 10%). Different values of ζ corresponds to different value of η is the social welfare function. The limit case where $\zeta \simeq 1$ corresponds obviously to the case where the only the utility of workers enter in the social welfare function, $\eta = 1$.The solid line corresponds to the case $\zeta \simeq 1$, and the dash line which is below corresponds to $\zeta = 0$. The optimal inflation rate is an increasing function of the fraction of constrained firms, for the same reason as the ones of the previous section. The first best inflation rate is above the second best one because of the effect of fiscal policy on consumption: first, inflation has to be high enough to reach the optimal saving rate. But, consumption of old workers is decreasing with inflation because, although workers save more because of inflation, they do not fully compensate the decrease in the return on money. Hence, consumption of old workers has to subsidize to reach the first best. Indeed, with the given parameters one finds an average value of $\lambda = 0.3$. But then, young workers have less incentives to save because their money holdings (or their consumption when young) are remunerated. Hence, inflation has to increase more to provide the correct incentives to save. Finally, when workers are favored in the social welfare function, inflation is higher because higher inflation decreases the real interest rate what favors investment

because the return on money would be greater than the real interest rate R^{opt}, hence no resources would be lent on the financial markets and production would collapse.

and the real wage.

The result of this section is that the first best outcome of the model can be reached if fiscal policy is designed jointly with monetary policy, although credit constraints are binding and inflation is positive. But, fiscal policy is often used for redistributive purposes independently of the monetary policy. Hence, it is difficult to argue that the actual distorting transfers correspond to the optimal fiscal policy presented here. As a consequence, the result of the previous section without any optimization on fiscal policy yields more interesting results both on the positive side and on the normative side. For this reason, the optimal fiscal policy has been introduced as an extension and the inflation target determined in the previous section may be more realistic. Nevertheless, the result that the optimal inflation rate depends on the nature of the fiscal policy is of independent interest.

8 Conclusion

This paper studies the optimal inflation rate in a simple monetary non Ricardian setting with credit rationing. Because of credit constraints, the standard Golden Rule, which stipulates that the real interest rate must be equal to the growth rate of the economy, yields under-investment. A decrease in the real interest rate can increase investment and bring it closer to its first best value. Second, because of the non Ricardian framework the inflation rate has a long run effect on the real interest rate. More precisely, it has been shown that an increase in the inflation rate decreases the real interest rate and increases capital accumulation. This "Tobin effect" of inflation on investment (Weil, 1991) is a first effect of inflation in the long run. The second effect of inflation is the standard distorting effect in monetary economies: inflation affects the opportunity cost of holding money and prevents workers from smoothing optimally their consumption. It has been shown that the trade-off between these two effects yield a theory of the optimal long run inflation rate, which is consistent with the actual practice of central banks. Moreover, when the level of credit rationing increases, the optimal real interest rate decreases and the optimal inflation rate increases. As a consequence, the more efficient the financial market, the lower the optimal long run inflation rate. The previous results provide a second best theory of inflation because the first best can not be reached with only monetary policy. As a simple extension, a simple fiscal policy which is basically a proportional tax (or subsidy) on consumption is introduced. The first best allocation can be reached if fiscal and monetary policy are jointly optimally defined. In this case, the first best value of inflation is higher than the second best value. As a consequence, the optimal inflation rate depends on the nature of the fiscal policy. Finally, as fiscal policy is often used for redistributive purposes independent of inflation, it may not be realistic to assume that this fiscal policy can not be implemented. For this reason, the optimal inflation rate based only on an optimal monetary policy seems a more realistic target.

A Proof of equality 30

Using the financial market equilibrium (27), together with the amount of savings (7), and total investment (18), one finds

$$\frac{1}{1+\gamma}\left(\gamma-\frac{\theta}{1+(1+\theta)(R\Pi-1)}\right)\left(\frac{w_t}{P_t}+\mu\right)=G\left(\varepsilon^*\right) \tag{36}$$

Using the demand for money (6), together with the money market equilibrium (28), one finds

$$\frac{\Lambda_t}{L_t}\frac{1}{P_t}=\theta\frac{1}{1+\gamma}\frac{R\Pi}{1+(1+\theta)(R\Pi-1)}\left(\frac{w_t}{P_t}+\mu\right)$$

where I have substituted the nominal interest rate by its value $r=R\Pi-1$. The right hand side is constant along a steady state, because R, Π and μ are constant by assumption. Hence, ε^* and $\frac{w_t}{P_t}$ are constant because of equations (16) and (21). The previous equation yields that $\frac{\Lambda_t}{L_t}\frac{1}{P_t}$ is constant and hence

$$\Pi=\frac{\Lambda_t}{\Lambda_{t-1}}\frac{1}{1+n}$$

This is obviously the determination of inflation by the quantity of money.

Money growth comes from the helicopter drops to all young workers, which were denoted by μ_t in real terms. Hence,

$$\Lambda_t-\Lambda_{t-1}=L_tP_t\mu_t$$

It yields

$$\mu_t=\frac{\Lambda_t-\Lambda_{t-1}}{L_tP_t}=\frac{\Lambda_t}{P_tL_t}\left(1-\frac{\Lambda_{t-1}}{\Lambda_t}\right)$$

Using the four previous equations with (28), one finds

$$\mu=\left(1-\frac{1}{\Pi(1+n)}\right)\theta\frac{1}{1+\gamma}\frac{R\Pi}{1+(1+\theta)(R\Pi-1)}\left(\frac{w_t}{P_t}+\mu\right) \tag{37}$$

This equality relates the amount of money given to each young workers to the inflation rate, the real interest rate and the total revenue of young workers.

The equation (37) can be used to find the relationship between inflation and the real interest rate. Indeed, the real wage is given by equation (21). As $F(\varepsilon^*) = \varepsilon^*$ and $G(\varepsilon^*) = \frac{(\varepsilon^*)^2}{2}$, and because of the definition of ε^* given by (16), one gets

$$\frac{F^{\beta}(\varepsilon^*)}{G(\varepsilon^*)} = \frac{2\left(\frac{q_H}{1+n}\right)^{-\beta}}{\beta\left(1-\frac{B}{\Delta q}\right)}\frac{R}{1+n} \tag{38}$$

Using equation (21), (37) and (38) to substitute for $\frac{w}{P}$ and μ in equation (36), one finally finds the equation (30).

B Welfare as a function of Inflation

The average consumption of an entrepreneur, $c^{entr} = C^{entr}_{t+1}/L_t$, can be easily calculated with equations (16) and (19). It yields

$$c^{entr} = A\left(1-\frac{B}{\Delta q}\right)^{\frac{\beta}{2-\beta}}\left(1+\frac{B}{\Delta q}\right)R^{-\frac{\beta}{2-\beta}} \tag{39}$$

where the constant $A = \frac{1}{2}\left[\beta\left(\frac{q_H}{1+n}\right)^{\beta}(1+n)\right]^{\frac{2}{2-\beta}}$ does not depend on $\frac{B}{\Delta q}$. The average consumption of an entrepreneur is a decreasing function of the real interest rate. This result is obvious because first the real interest is only a cost for the entrepreneurs. Second, an increase in the real interest rate increases credit rationing and decreases production. As entrepreneurs are risk neutral, and as the shocks are i.i.d, this value is also the expected utility before the shock.

The utility of a worker is $U^{work} = \ln\left(c^j\right) + \gamma\ln(c^v)$. Using equations (2), (3), together with equations (37) and (36), one finds

$$\begin{aligned} c^y &= \Pi\frac{RG(\varepsilon^*)}{\gamma(1+(1+\theta)(R\Pi-1))-\theta} \\ c^o &= \frac{\gamma RG(\varepsilon^*)}{\gamma-\frac{\theta}{1+(1+\theta)(R\Pi-1)}} \end{aligned}$$

Using these expressions in the utility function $U^{work} = \ln(c^y) + \gamma\ln(c^o)$ yields the following equation:

$$U^{work} = (1+\gamma)\ln\frac{RG(\varepsilon^*)}{\gamma - \frac{\theta}{1+(1+\theta)(R\Pi-1)}} - \ln\left((\theta+1)R - \frac{\theta}{\Pi}\right) + \gamma\ln\gamma \tag{40}$$

This expression yields a relationship $U^{work}(R)$, with the expression of ε^* given by equation (16) and the expression of the inflation rate as a function of R given by the relationship (30).

The optimal inflation rate is now simple to determine. With relationships (39) and (40), one can express the social welfare U^s given by (31) as a function of the real interest rate R. With the equality (30), the relationship between Π and U^s can be deduced.

The demand for money (6), with equation (10) gives

$$\frac{M_t L_t}{P_t Y} = \theta\frac{1-\beta}{1+\gamma}\frac{1+r_{t+1}}{1+(1+\theta)r_{t+1}}\frac{\frac{w_t}{P_t}+\mu_t}{\frac{w_t}{P_t}}$$

Using (21), (37) and (38), one finds

$$\frac{M_t L_t}{P_t Y} = \frac{1}{2}\theta\beta(1+n)\Pi\frac{1-\frac{B}{\Delta q}}{\gamma(1+(1+\theta)r)-\theta} \tag{41}$$

C Optimal Inflation Rate with fiscal Policy

The goal of this appendix is to construct an equilibrium such that $R = R^{opt}$, with the fiscal policy given in the text.

The solution of the program of workers yields the consumption, the real savings and the real demand for money

$$\begin{aligned}
c_t^y &= \frac{1}{1+\gamma}\left(1-\frac{B}{\Delta q}\right)\left(\frac{w_t}{P_t}+\mu_t-\tau^y\right) && (42)\\
c_{t+1}^o &= \gamma\frac{R_{t+1}}{1+\gamma}\left(\frac{w_t}{P_t}+\mu_t-\tau^y\right)\\
\frac{s_t}{P_t} &= \left(1-\frac{1+\theta}{1+\gamma}\left(1-\frac{B}{\Delta q}\right)\right)\left(\frac{w_t}{P_t}+\mu_t-\tau^y\right)\\
\frac{M_t}{P_t} &= \theta\frac{1}{1+\gamma}\left(1-\frac{B}{\Delta q}\right)\left(\frac{w_t}{P_t}+\mu_t-\tau^y\right) && (43)
\end{aligned}$$

Using the money demand (43) instead of equation (6), the money market equilibrium yields

$$\mu_t = \left(1 - \frac{1}{\Pi\left(1+n\right)}\right)\theta\frac{1}{1+\gamma}\left(1 - \frac{B}{\Delta q}\right)\left(\frac{w_t}{P_t} + \mu_t - \tau^y\right)$$

instead of equation (37).

The financial market equilibrium yields

$$\frac{w_t}{P_t} + \mu_t - \tau^y = \frac{1}{1 - \frac{1+\theta}{1+\gamma}\left(1 - \frac{B}{\Delta q}\right)}G\left(\varepsilon^*\right) \tag{44}$$

instead of equation (36).

Substituting for μ_t and using the equality (21) and the relationship (38), one finds

$$\begin{aligned} &\frac{1}{1 - \frac{1+\theta}{1+\gamma}\left(1 - \frac{B}{\Delta q}\right)} \\ = \ &\frac{1-\beta}{\beta}\frac{2}{1 - \frac{B}{\Delta q}}\frac{R}{1+n} + \left(1 - \frac{1}{\Pi\left(1+n\right)}\right)\frac{\theta}{1+\gamma}\frac{1 - \frac{B}{\Delta q}}{1 - \frac{1+\theta}{1+\gamma}\left(1 - \frac{B}{\Delta q}\right)} - \frac{\tau^y}{G\left(\varepsilon^*\right)} \end{aligned}$$

The value of the real interest rate which yields the optimal value of production is $R^{opt} = (1+n)\left(1 - \frac{B}{\Delta q}\right)$. Substituting R by this value in the previous equation, one finds the optimal value of inflation, after few calculations:

$$\Pi^{opt} = \frac{1}{1+n}\frac{1}{1 - \frac{1+\gamma}{\theta}\frac{1}{1-\frac{B}{\Delta q}}\left(1 + \left(1 - \frac{1+\theta}{1+\gamma}\left(1 - \frac{B}{\Delta q}\right)\right)\left(\frac{\tau^y}{G(\varepsilon^{opt})} - 2\frac{1-\beta}{\beta}\right)\right)}$$

This expression is the value of inflation which yields as an outcome of the financial market and the money market equilibria $R = R^{opt}$. It still depends on the the tax paid by young workers, which is endogenous. This one is given by the budget constraint of the State. It yields (35)

$$\tau^y = \frac{1}{1+n}\left(\lambda c^y - \zeta\frac{1 + \frac{B}{\Delta q}}{1 - \frac{B}{\Delta q}}RG\left(\varepsilon^{opt}\right)\right) \tag{45}$$

One can use equations (42), (44), (19) and the equality $R = (1+n)\left(1 - \frac{B}{\Delta q}\right)$ to substitute for c^y_t. It gives the following equality (46).

$$\tau^y = \left(\frac{\lambda}{1+n}\frac{1}{1+\gamma}\frac{1}{1 - \frac{1+\theta}{1+\gamma}\left(1 - \frac{B}{\Delta q}\right)} - \zeta\frac{1 + \frac{B}{\Delta q}}{1 - \frac{B}{\Delta q}}\right)\left(1 - \frac{B}{\Delta q}\right)G\left(\varepsilon^*\right) \tag{46}$$

Finally, the value of λ is obtained with the equality $1 + r = R\Pi = (1+n)\left(1 - \frac{B}{\Delta q}\right)\Pi$ used in equation (33). It yields :

$$\lambda = \left(\theta - (\theta + 1)\frac{B}{\Delta q}\right)(1+n)\,\Pi - \theta \tag{47}$$

Using this value of λ in equation (46), to substitute for the value of τ^y in equation (45), one finds an implicit function in Π. The solution Π^{opt} of this equation is the value of the inflation rate which yields the equilibrium real interest rate R^{opt}. Indeed, if monetary authorities set $\Pi = \Pi^{opt}$, then there exists a decentralized equilibrium such as $R = R^{opt}$. By construction, the budget of the State and of private agents are balanced, first order conditions of workers and entrepreneurs are fulfilled, the money market, the financial market (and hence the good market because of the Walras Law) are at equilibrium. In the general case, there may be multiple equilibria. This issue is discussed in the calibrated example given in the text.

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