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To cite this version:

Tai-Yu Ma, Jean-Patrick Lebacque. A Lagrangian discretization multiagent approach for large-scale multimodal dynamic assignment. 2011. halshs-00592273

HAL Id: halshs-00592273

https://shs.hal.science/halshs-00592273v1

Preprint submitted on 11 May 2011

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A Lagrangian discretization multiagent approach for large-scale multimodal dynamic assignment

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Abstract

This paper develops a Lagrangian discretization multiagent model for large-scale multimodal simulation and assignment. For road traffic flow modeling, we describe the dynamics of vehicle packets based on a macroscopic model on the basis of a Lagrangian discretization. The metro/tram/train systems are modeled on constant speed on scheduled timetable/frequency over lines of operations. Congestion is modeled as waiting time at stations plus induced discomfort when the capacity of vehicle is achieved. For the bus system, it is modeled similar to cars with different speed settings, either competing for road capacity resources with other vehicles or moving on separated bus lines on the road network. For solving the large-scale multimodal dynamic traffic assignment problem, an effective-path-based cross entropy is proposed to approximate the dynamic user equilibrium. Some numerical simulations have been conducted to demonstrate its ability to describe traffic dynamics on road network.

Keywords: multimodal transportation systems; Lagrangian discretization; traffic assignment; multiagent systems

1. Introduction

Traffic assignment problem has been recognized as an important research topic in transportation science. It provides a useful tool for decision making on traffic management, planning and control strategy implementations. In recent years, increasing applications of multiagent framework on travel demand or supply modeling have been proposed (Cetin et al., 2002; Raney et al., 2003; Rieser et al., 2007). The multiagent framework is very convenient to represent the dynamics of the transportation system by specifying the behavior of homogeneous/heterogeneous agents (travelers/vehicles). Moreover, it can combine the desired details on either travel behavior modeling (destination/departure time/route/activity chain choice) or multimodal traffic flow modeling. In general, agent's learning process is modeled in an iterative day-to-day adjustment process based on experienced performance of travel choice. However, there are still few studies on efficient algorithms for computing approximate solutions of dynamic user equilibrium on such a non-cooperative multiagent system.

The calculation of mode, departure time and route choice in a multimodal simulation model is not a trivial work. The problem is to determine the probabilities of users' travel choices in such a way that Wardrop's user equilibrium is achieved. Several solution algorithms have been proposed in the past, including projected dynamical system approaches (Bertsekas and Gafni, 1982; Nagurney, 1993), dynamical system approaches (Smith, 1993; Jin, 2007), and the method of successive averages (Tong and Wong, 2000). However, when multiple modes/classes are presented in a link, the monotonicity of link travel cost cannot be held, which makes it difficult to obtain equilibrium

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solutions (Wynter 2001). Moreover, iterative path-based network loading algorithms become impractical when the size of network becomes large.

Another difficulty is how to efficiently represent the traffic dynamics on large-scale multimodal network by guaranteeing its numerical correctness and accommodating the flexibility of agent's behavior adaptation. One promising way is based on Lagrangian discretization of a macroscopic model, such as GSOM (generic second order modeling (Lebacque et al, 2007, 2008). In this method, homogeneous travelers are grouped in packets, and their movements on the network are described. Note that in the literature several packet-based mesoscopic models have been proposed with different concepts.

This paper aims to develop a Lagrangian discretization multiagent model for large-scale multimodal simulation and assignment. We propose a multilayer multimodal network permitting to present heterogeneous packet flows on different transport subnetworks (Ma and Lebacque, 2008). The dynamics of vehicle packets on road network is described on a LWR model (Lighthill and Whitham, 1955; Richards, 1956) on the basis of a Lagrangian discretization. For the intersection modeling, it is based on the traffic supply-demand approach (Lebacque, 1996; Lebacque and Khoshyaran, 2005; Lebacque et al., 2008; Khoshyaran and Lebacque, 2008). The flow dynamics in intersection is characterized by the demands on incoming links and the supply at interaction node, constrained by the number of vehicles stocked in a node. For public transport system modeling, we distinguish metro/tram/train (non-road mode) systems and bus (road-mode) system. The former is modeled on constant speed on scheduled timetable/frequency over transit lines. Congestion is modeled as waiting time at stations plus induced discomfort when the capacity of vehicle is achieved. For the bus system, it is modeled similar to cars with different speed settings, either competing for road capacity resources with other vehicles or moving on separated bus lines on the road network.

For solving large-scale multimodal dynamic user equilibrium problem, an effective-path-based cross entropy (CE) method based algorithm (Ma and Lebacque, 2007) is proposed to obtain the approximate equilibrium solution. The proposed CE approach is the generalization of Jin's dynamical system approaches, but differently; it is based on a subset of routes, avoiding the difficulty of path enumeration for all origin-destination pairs.

The remained part of this paper is organized as follows. Section 2 presents multimodal transportation network structure, its modeling and k-shortest hyperpath algorithm. Section 3 describes traffic flow dynamics based on Lagrangian discretization with the presence of different classes of vehicles. Section 4 describes the proposed solution algorithm based on cross entropy method. Section 5 is devoted to numerical studies for intersection simulation. Finally, we conclude the study in Section 6.

2. Multimodal network structure and demand modeling

2.1. Multimodal network structure

The multimodal transportation network is structured as multi-layers in which each layer represents one subnetwork. The multilayer structure is an efficient way to represent the multimodal transportation network and has been widely applied in the past (Bielli et al., 2006; Meschini et al., 2007). Following the previous work (Ma and Lebacque, 2008), the multimodal transportation system is composed of a set of private modes and a set of transit modes for which the network is represented by a directed graph composed of a set of subgraphs. Fig. 1 shows the network structure. For road network, with the presence of multiclasses of vehicles (cars and buses), traffic flow dynamics is modeled based on Lagarangian discretization of macroscopic traffic flow model (described in Section 3). Note that we simulate bus flow on road network, but the corresponding passenger flow is simulated on bus hypernetwork; in such a way the hyperpaths used can be explicitly represented. The moving speed of users on the bus lines is duplicated by bus vehicle(s) on road network. For non-road mode transit system, its components are described as follows:

- Transit service operations: transit modes such as metro/tram/train operate on respective transit lines with fixed frequency and capacity constraints. For simplicity, transit-vehicle agents move in the opposite directions with constant speed and a short stop times at line nodes, assuming all users can board the vehicle within the stop time interval until the capacity of the vehicle is achieved.

- Travel time on transit system: the travel time of hyperpath on transit system is composed of walking time on boarding/alighting arcs plus waiting time at line nodes. The boarding/alighting time represents average walking time from the station to the access point of the vehicle. The first-in-first-out (FIFO) principle is applied for users when they are boarding the arriving vehicles. If the capacity of vehicle is reached, the users cannot board their vehicles and need to wait for the next vehicle.
- Modal transfer within transit system: as shown in Fig. 1, the transfer time is determined by the minimum time for joining two different metro/bus line nodes.

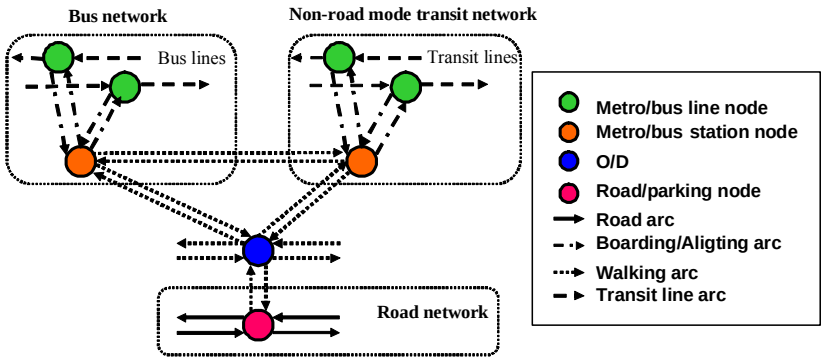

Fig. 2 Multimodal transportation network structure

The transit network includes bus network and non-road mode transit network as shown in Fig. 2. The multimodal route choice and transit assignment will be described in the next section.

2.2. Multimodal route choice and transit assignment

The traffic assignment problem in transit network has been widely studied in the past (Nguyen and Pallottino, 1988; De Cea and Fernandez, 1993; Spiess and Florian, 1989; Schmöcker et al., 2008). The methods used can be classified into two categories: frequency-based approach and schedule-based approach. The frequency-based approach doesn't simulate explicitly the operations of transit system. The congestion of users is estimated by delay function to estimate the waiting time at the boarding arcs or stops. The scheduled-based approach simulates the operations of transit system. The waiting time at stop is explicitly calculated as the difference between the arrival time and the time of boarding on transit vehicle following FIFO principle. The transit assignment consists of two steps. First, we need to generate the set of the shortest hyperpaths for users' route choice and then to assign them on the network to iteratively achieve user equilibrium (Poon et al., 2004). The generalized cost of path in the transit system includes an in-vehicle time, waiting time, walking time, and a line change penalty. Hamdouch and Lawphongpanich (2008) proposed a scheduled-based transit assignment model, in which the transit vehicle capacity is explicitly considered. Such users' route choice behavior is assumed that they can switch to different preferred route list (called travel strategies) facing transit line congestion. The method of successive averages was used for computing user equilibrium assignment on a time-expended network. Although these studies calculated explicitly users' waiting time at station/boarding arc, the hyperpaths are not explicitly considered in these studies.

The algorithms of finding the viable shortest hyperpaths in a multimodal network have been proposed in the literature (Lozano and Storchi, 2001, 2002). These algorithms aim to find the shortest cost paths under the upper limit of modal transfers. However, as mentioned above, users may use different hyperpaths to adapt the congestion situations occurred on some transit lines during some period periods of time. Here, we propose a different dynamic transit assignment scheme as follows. We assume that users utilize a set of *attractive hyperpaths* to arrive to their destinations. The set of attractive hyperpaths are time-dependent based on the scheduled services. Users learn iteratively the optimal departure time and hyperpath choice based on CE approach (described later) such as their total generalized travel costs are minimized. To generate attractive hyperpaths, we propose a k-shortest viable hyperpath algorithm based on Yen's ranking loopless paths algorithm (Martins and Pascoal, 2003). The idea is that we associate each mode-attribute with a node of transit network. The k-shortest paths found by Yen's algorithm are

re-evaluated by checking the constraint of the upper modal transfer and adding related transfer penalty cost. The reader is referred to Martins and Pascoal (2003) for the implementation of basic k-shortest paths algorithm.

3. Lagrangian discretization scheme for road traffic flow modeling

For large scale road network simulation, one important issue is how to efficiently represent the heterogonous traffic flow in conforming to macroscopic or microscopic traffic models. One flexible way is encompassing vehicles in parcel of vehicles (called packets hereafter) based on Lagrangian discretization and describing their movement on the network. Based on the work of Courant and Friedrichs (1948) and Aw et al. (2000), the LWR model can be transformed from Eulerian coordinates to Lagrangian coordinates. Such transformation allows us building an agent-based traffic model conforming to macroscopic traffic fluid models.

We recall here the formulation of the LWR model in Lagrangian coordinates. Consider N being the cumulative number of vehicles that have passed location x by time t on a road section. The LWR model based on Eulerian coordinates (x, t) can be transformed into Lagrangian coordinates (N, t):

$$\begin{cases} \partial_t r + \partial_N v = 0 \\ v = V_e(r) \end{cases} \tag{1}$$

where r is the distance between two consecutive vehicles, or the inverse of density; $V_e(r)$ represents the equilibrium speed based on the fundamental diagram.

The resolution of the above equation based on Godunov method can obtain the following particle discretization of traffic flow dynamics:

$$x_a^{t+1} = x_a^t + v_a^t \Delta t \text{, with } v_a^t = V_e\left(\frac{N_a}{x_{a-1}^t - x_a^t}\right) \tag{2}$$

where x_a^t is the position of the packet a at time t; v_a^t is the average speed of the packet a at time t; and N_a is the size of the packet, defined as the number of vehicles between locations x_a^t and x_{a-1}^t at time $t\Delta t$ (see Fig. 2). The discretization of time Δt should satisfy the Courant-Friedrichs-Lewy's (CFL) stability condition.

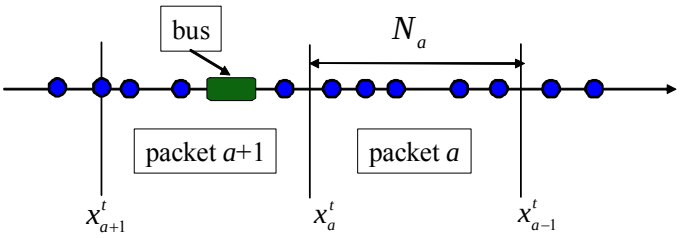

Fig. 2 Lagrangian discretization of the packet of vehicles and bus

When two types of vehicles (car and bus) are presented in a road section, we describe here their interaction. In the case when a bus is located at the downstream of a packet, if the position of the packet calculated by (2) overtakes the bus by one time step, its speed should then be corrected as:

$$v_a^t = \begin{cases} \min[v_{bus}, V_e(\frac{N_a}{x_{bus}^t - x_a^t})], & \text{if the number of lane in the raod section is } 1 \\ V_e(\frac{\lambda N_a}{x_{a-1}^t - x_a^t}), & \text{otherwise} \end{cases} \tag{3}$$

where λ ($\lambda > 1$) is the passenger car equivalent (PCE) of bus. The above equation describes that if the number of lanes in a link is greater than 1, then the packet can overtake the bus in front of it with reduced speed.

The speed of a bus is assumed constant in fluid regime. However, in congestion regime, its speed is bounded at the downstream traffic state. The speed of a bus is given by

$$v_a^t = \min[v_{bus}, V_e(\frac{\lambda}{x_a^t - x_{bus}^t})] \tag{4}$$

Note that in (4) we use the fundamental diagram of bus for calculating V_e.

3.1. Intersection modeling

The intersection is modeled based on internal state model (Lebacque et al., 2008; Khoshyaran and Lebacque, 2008). This model considers the intersection as a point with limited stock capacity. In this section, we describe the boundary conditions when the packets pass through an intersection.

The notation is defined as follows.

z: node representing the intersection
i: upstream link
j: downstream link
N_z: number of vehicles, which can be stocked in the intersection *z*
$\delta_i(t)$: upstream link demand of link *i* at time *t*
$\Sigma_i(t)$: partial supply for upstream link *i* at time *t*
$q_i(t)$: upstream link flow of link *i* at time *t*
$\Delta_j(t)$: partial demand of downstream link *j* at time *t*
$\sigma_j(t)$: downstream supply of link j at time *t*
$r_j(t)$: downstream link inflow at time *t*
$N_z(t)$: number of vehicles entering at *z* at time *t*
$n_a(t)$: number of vehicles of packet *a* not yet entering the intersection at time *t*
$n_b(t)$: number of vehicles of packet *b* not yet leaving (still stocked in) the intersection at time *t*

The partial supply $\Sigma_i(t)$ is calculated as the fraction of the number of lanes of link *i* with respect to the summation of the lanes of upstream links. We apply FIFO principle to the packets leaving the downstream link *j*. The inflow $q_i(t)$ and outflow $r_j(t)$ are determined based on supply-demand approach (Lebacque, 1996) as

$$q_i(t) = \min[\delta_i(t), \Sigma_i(t)] \tag{5}$$

$$r_j(t) = \min[\Delta_j(t), \sigma_j(t)] \tag{6}$$

The variation of the number of vehicles stocked at the intersection *z* at one time step is obtained based on the conservation of vehicles at the intersection:

$$N_z(t+1) - N_z(t) = \sum_i q_i(t)\Delta t - \sum_j r_j(t)\Delta t \tag{7}$$

The position of packet/bus when traversing the intersection *z* is determined by the steps described in Table 1.

Table 1 Computation of inflows and outflows at intersection

1. Determine $q_i(t)$ and $r_j(t)$ based on (5) and (6).
2. **Inflow computation**:

   IF $q_i(t)\Delta t \geq n_a(t)$ **THEN** the packet *a* enters the intersection *z*

   ELSE $n_a(t+\Delta t) = n_a(t) - q_i(t)\Delta t$, $v_a^t = V_e(\frac{n_a(t)}{x_z - x_a^t})$ and $x_a^{t+1} = x_a^t + v_a^t \Delta t$
3. **Outflow computation**:

   IF $r_j(t)\Delta t \geq n_b(t)$ **THEN** the packet *b* enters downstream link *j*, $v_b^t = V_e(\frac{N_b}{x_{b-1}^t - x_z})$, and $x_b^{t+1} = x_b^t + v_b^t \Delta t$.

   Update the position of the second packets as the first in the queue bound for link j, the queue size reduces by 1.

   ELSE $n_b(t+\Delta t) = n_b(t) - r_j(t)\Delta t$

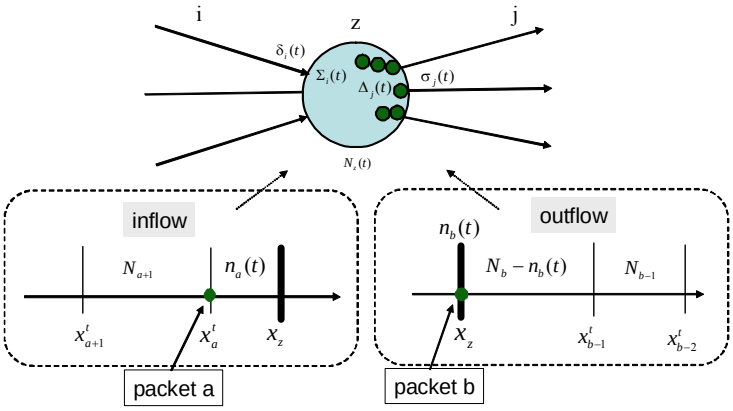

Fig. 3 Intersection modeling based on internal state model

4. Solution algorithm

In this section, following the previous studies (Ma and Lebacque, 2007; Ma, 2007; Lebacque et al., 2009) the CE approach is proposed for solving multimodal dynamic traffic assignment problem. The CE method derives iteratively a mode, a departure time and the route choice probabilities towards user equilibrium based on minimizing the expectation of travel cost relative to choice alternatives. Let recall basic concept of the CE method as follows.

Consider a traffic assignment problem with fixed demand for a set of paths R_k connecting a pair of origin and destination *k*. Travelers' path choice is based on the probability distribution P_k, iteratively updated based on travelers' experienced travel costs. The path performance function is defined by Boltzmann distribution with the control parameter γ as

$$H_r(\gamma) = e^{-C_r(d_r)/\gamma}, \quad \forall r \in R_k \tag{8}$$

where $-C_r(d_r)$ is the travel cost of path r and d_r is its travel demand.

Based on maximizing the expectation of performance function, the optimal probability distribution can be derived (Rubinstein, 1999; Helvik and Wittner, 2001; Ma and Lebacque, 2007):

$$p_r^{w+1} = p_r^w \frac{e^{-C_r^w/\gamma^w}}{\sum_{s\in R_k} p_s^w e^{-C_s^w/\gamma^w}}, \quad \forall r \in R_k \tag{9}$$

where p_r^w is the choice probability of path r at iteration w.

The control parameter γ in (9) is determined by solving:

$$\text{Min } \gamma^w \quad \text{subject to} \quad \sum_{r\in R_k} | p_r^{w+1} - p_r^w | \le \alpha^w \tag{10}$$

where α^w is a numerical divergent series such that the flow adjustment converges to fixed points.

As in large scale transportation network, the enumeration of all possible paths for assignment is inapplicable. Two possible algorithms are proposed for this issue:

a. Node-based assignment scheme (Ma and Lebacque, 2009): we associate the outgoing node choice probabilities with each node for each class of users. The class may be the users of the same origin and destination and with the same mode choice. The outgoing node choice probability means the probability of using arc (x, y) when arriving at node x. The probability is iteratively updated based on (9)-(10). The path choice probability is obtained by the multiplication of conditional choice probability determined at node level when the users move in the network.

b. Effective paths assignment: we find k-shortest multimodal viable hyperpaths and then assign users based on the choice probabilities of the effective paths determined by (9)-(10).

Previous work (Ma and Lebacque, 2009) has showed that the node-based CE approach can find the approximation of user equilibrium in multiclasses road network. However, when the size of network becomes large, the computation for optimal outgoing node choice probability could be very time consuming. Hence, the solution algorithm (b) may be preferred. A comparative study is currently being performed to confirm this conjecture.

5. Numerical study

In this section, we present some simulation results on intersection modeling with two classes of vehicles. These primary studies are essential for further applications on the proposed large scale multimodal transportation network.

5.1. Numerical simulation of traffic flow on road network

A simple network presented in Fig. 4 contains an intersection at node 2. The study presents how the car packets overtake the bus if there is more than one lane in a link and also the flow evolution at the intersection by tracing the trajectories of car packets and buses.

We use a distinguished triangular fundamental diagram for cars and buses, stating the density-speed relationship as follows:

$$V_{e,h}(\rho) = \begin{cases} V_{\max,h} & \text{if } 0 \le \rho < \rho_{\text{cr},h} \\ V_{\max,h} \dfrac{\rho_{\max,h} - \rho}{\rho_{\max,h} - \rho_{\text{cr},h}} & \text{if } \rho_{\text{cr},h} \le \rho < \rho_{\max,h} \end{cases}, \; h = \text{car, bus} \tag{11}$$

The simulation time step is determined by verifying the Courant-Friedrich-Lewy's (CFL) condition as

$\Delta t \le \min_l \frac{1}{w_{car}\rho_{car}^{\max} n_l} = \frac{1}{5*0.6} = 0.33$, hence we use Δt =0.3 sec. The simulation settings are the following:

for passenger cars : $v_{car}^{\max}$ =20 m/s, $w_{car} = v_{car}^{\max}/4$=5 m/s, $\rho_{car}^{\max}$ =0.2 veh/m/lane, $q_{car}^{\max} = \rho_{cr}^{\max} * v_{car}^{\max}$;

for buses: $v_{car}^{\max}$ =10 m/s; $w_{bus} = v_{bus}^{\max}/4$=2.5 m/s, $\rho_{bus}^{\max}$ =0.1 bus/m/lane, $q_{bus}^{\max} = \rho_{bus}^{\max} * v_{bus}^{\max}$.

The size of packets is set homogeneously as 10 vehicles/packet. The results show that the proposed intersection model produces the expected shockwave propagation at intersection when downstream link capacity is inferior to the demand. The simulation results of behavior of different classes of vehicles (car and bus) in a link with two lanes (the left part of Fig. 5) and one lane (the right part of Fig. 5) are expected. It is shown that when some packets overtake a bus, its speed is reduced. However, when there is only one lane in a link, the packets follow the speed of the bus at the downstream.

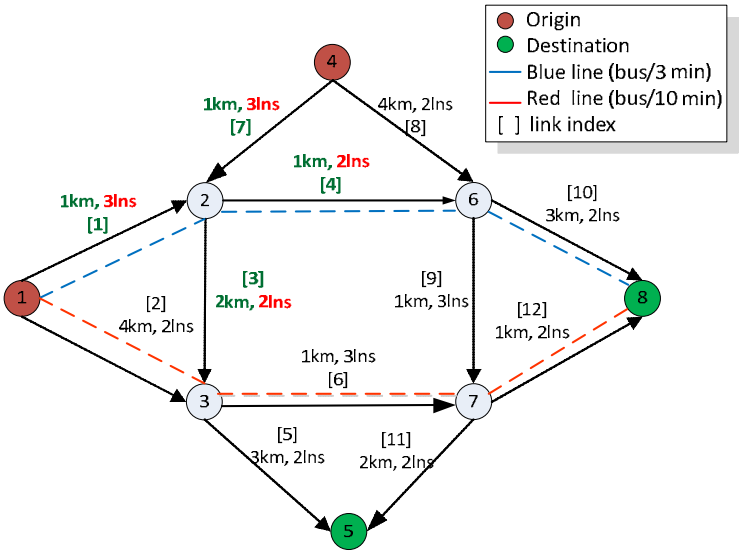

Fig. 4 A simple road network

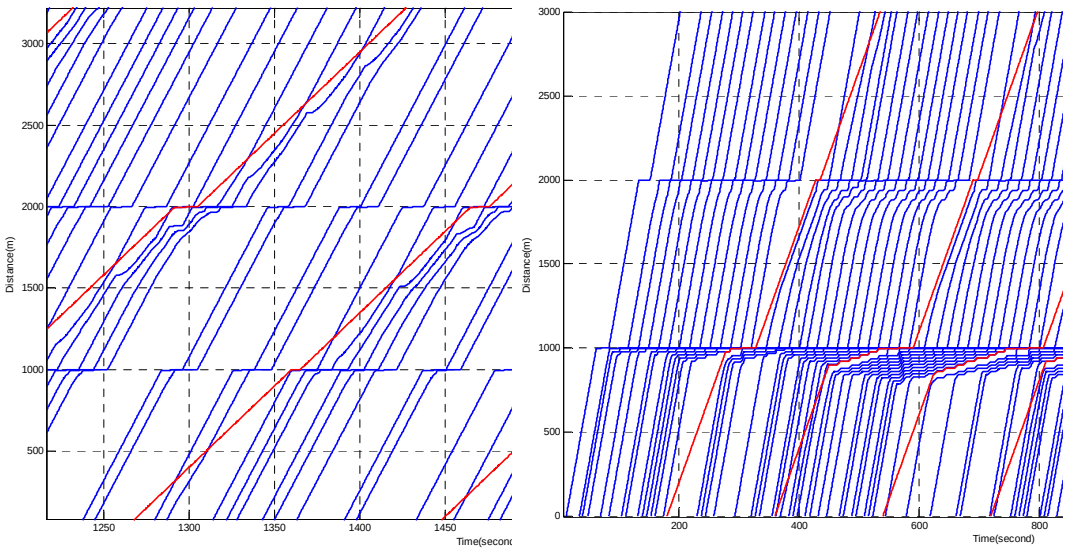

Fig. 5 The trajectories of car packets and buses on path 1-2-6-8 with two lanes in link (2, 6) (Left) and with one lane in link (2, 6) (Right)

6. Conclusion

This paper proposes a Lagrangian discretization of multiaget approach for multimodal transportation system modeling. The multimodal network is represented by a multilayer hypergraph in order to explicitly model vehicles'/passangers' flow on it. The simulation study of internal-state-based intersection model has been conducted to describe efficiently traffic dynamics. We propose the cross entropy approach for solving multimodal dynamic traffic assignment problem. The numerical test is currently under study.

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