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Option Pricing with Discrete Time Jump Processes

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Option Pricing with Discrete Time Jump Processes

Dominique Guégan* Florian Ielpo[†] Hanjarivo Lalaharison[‡]

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Abstract

In this paper we propose new option pricing models based on two classes of discrete Lévy-type processes. By combining these Lévy processes with several volatility dynamics of the GARCH type, we aim to take into account the dynamics of financial returns in a realistic way. The associated risk neutral dynamics of the time series models is obtained through two different specifications for the pricing kernel: we provide a characterization of the change in the probability measure using the Esscher transform and the Minimal Entropy Martingale Measure. We finally assess empirically the performance of this modelling approach, using a dataset of European options based on the CAC 40. Our empirical findings show that discrete time Lévy based models behave well regardless of the volatility dynamics and of the pricing kernel. A different specification is however required to obtain the best results for options with a shorter or longer maturity.

Keywords: Option pricing, Lévy processes, Incomplete market, exponential affine Stochastic Discount Factor, Minimal Entropy Martingale Measure, CAC 40.

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1 Introduction

When the concept of volatility aims at measuring the state of risks at work in financial markets, jumps offer a unique opportunity to represent the fundamental process through which market quotes integrate the relevant information flow over trading days. The inspection of both daily and intraday data sets reveals that the jump activity implied by the returns on various assets should not be ignored: the returns are characterized by tails whose thickness cannot be accounted for through any conditional Gaussian model, and whose fit is clearly improved through the use of jumps. Intraday data based tests improved our understanding over both the size and the intensity of the jump processes, unveiling how complex the blending of jumps and volatility can be. These considerations have led to the idea that a valuation model for financial assets should include both these components. This article proposes an empirical discrete time Lévy-process approach to the pricing of European options on equity indices. We discuss the key ingredients to be considered to achieve this objective and we empirically circumvent the usual pitfalls of such an approach, as the non-uniqueness of the pricing measure and the choice of a pricing kernel specification.

The introduction of jumps into financial models present various advantages, either in terms of time series analysis or in terms of option pricing. The deviation of historical returns from the Black Scholes (1973) hypothesis of Gaussianity is now well known and documented, see for example Eberlein and Prause (2002), Bouchaud and Potters (2003) or Embrecht *et al.* (2005). It led to the development of various time series models usually combining a GARCH component with a non-Gaussian distribution or a jump process. The use of Lévy processes is by no means common in time series analysis: various continuous time finance models are built on the combination of a process of the Heston (1993) type for the conditional variance with a jump component, either in the dynamics of the variance or of the log-returns. Examples of this sort are available in Merton (1973), Bates (1996) or Duffie *et al.* (2000). When the obvious interest of jumps in time series analysis lies within the fit of the tails of returns' distribution, the incorporation of jumps into a continuous time finance option pricing model is essential to the pricing of short term options: the convexity of the implied volatility smile is sharper for such options and stochastic volatility models are unable to replicate this stylized fact without a jump component. On this point, we refer to Duffie *et al.* (2000).

Even though continuous time finance provided market participants with a convenient and fast way to compute prices for various contingent claims, it offers a framework that can hardly handle the key issue of bringing together the historical and the risk neutral distributions. If the calibration of parameters using option prices is a relatively straightforward exercise, the estimation of the parameters solely using the historical time series of returns is often tricky: some references are for example the methodologies that are used in Eracker *et al.* (2003), Chacko and Viceira (2003) or Rockinger and Semenov (2005). Option pricing in a discrete time framework has over the years proved to be a fruitful and economically appealing means to understand how the time series of returns relate to the price of hedging vehicles such as European options, as presented for

example in Rosenberg and Engle (2002), Jackwerth (2001), Barone-Adesi *et al.* (2008), Chorro *et al.* (2010a, 2010b) among others.

In this article, we present and compare various option pricing models based on Lévy processes. Each of these models combine different volatility structures, conditional distributions and pricing kernels, the link between each of them being the use of an unconditional distribution which characterises a Lévy process. We define the change in the probability measure based on various assumptions regarding the pricing kernel. The volatility models used here are the EGARCH (Nelson, 1991), the GARCH-GJR (Glosten *et al.*, 1993) and the APARCH model (Ding *et al.*, 1993). Two Lévy process specifications are considered under the historical distribution. First, we introduce a structural Lévy model whose particular model is the Normal Inverse Gaussian model. Second, we consider the compound Poisson model proposed by Merton (1976). These processes are driven by a finite number of jumps within a finite time interval.

Appealing though the discrete time approach may be, it nonetheless requires coping with an additional issue: as explained in Chorro *et al.* (2010b), with such an approach, the risk neutral distribution is no longer unique and a choice must be made regarding the shape of the market's aggregated risk aversion implied by the selected pricing kernel. We focus on two representations for the pricing kernel: the Esscher transform (ESS) martingale measure which corresponds to an exponential affine stochastic discount factor and the Minimal Entropy Martingale Measure (MEMM) (or extended Girsanov principle) (Elliot and Madan, 1998). The equivalence between these two approaches has been proved inside a Gaussian framework (Badescu and Kulberger, 2007), but such equivalence is not available outside this framework. We prove the stability of the conditional distribution under the historical and risk neutral distributions and we exhibit the required change in the parameters of the historical distribution to obtain the risk neutral one. One novelty of our article is to compute the expression under the pricing measure of the pricing formula when we use GARCH-type processes with Lévy innovations, both using the ESS and MEMM approaches. We also provide the expression of the pricing distribution for a GARCH-type model with Poisson jumps using the Esscher transform. In the case of the MEMM approach, the result has already been established by Fujiwara and Miyahara (2003).

Finally, we assess the quality of the modelling approaches selected here using a dataset of European options on the CAC 40 index. Our empirical results point in several directions: first, this type of approach combining an empirical strategy with discrete time models provides minimum pricing errors that are broadly consistent with several articles based on a direct calibration of the models' parameters to option prices. Given that the processes used here are highly consistent with the empirical features of returns, the pricing errors are consistent across the models' specifications. The results show that the best model selected to price short term options is very different from the model that should be retained to price options with a longer time to maturity. To price short term options, a model mixing the APARCH volatility structure, the MEMM pricing kernel and a distribution disentangling jumps from volatility provide the lowest pricing errors.

In contrast, for options with a longer time to maturity, a model based on the EGARCH volatility structure and an Esscher transform ranks first among of the different competitors presented here. For this type of time horizon, we find that the model should be based on jumps affecting the volatility dynamics for in-the-money options. Instead, disentangling jumps from volatility seems to make sense empirically for out-of-the-money options.

This article is organized as follows. Section 2 presents the modelling approach retained here, along with the characterization of the risk neutral distribution associated with each model. Section 3 presents the empirical results and Section 4 concludes. All the proofs are given in the appendix.

2 A Lévy-Based Discrete Time Modelling Approach

This Section presents the Lévy-based time series models this article is based on. We discuss the model properties and then develop the change in probability measures required to obtain the risk neutral distribution essential to option pricing.

2.1 Lévy-based time series models for financial returns

Our approach combines different time varying volatility dynamics with Lévy processes. The models considered here are based on two different ways of dealing with jumps:

1. For the first specification, jumps affect both the returns $(Y_t)_t$ and the volatility $(h_t)_t$:

$$Y_t = \log \left(\frac{S_t}{S_{t-1}} \right) = \tilde{m}_t + \sqrt{h_t} z_t, \quad S_0 = s, \quad (1)$$

where $\tilde{m}_t = r + m_t$, $(h_t)_t$ is the conditional volatility obtained from any type of GARCH model. $(z_t)_t$ is a Lévy process with triplet (c, σ^2, U) , (Schoutens, 2003), $c \in \mathbb{R}$, $\sigma^2 \geq 0$ and U is the positive Lévy measure defined on $\mathbb{R} \setminus \{0\}$ satisfying

$$\int_{\mathbb{R}} \inf\{1, z^2\} U(dz) < \infty.$$

Inside these models we assume that the innovations $(z_t)_t$ follow a NIG-Lévy process which has no Brownian component and whose Lévy triplet is given by $(c, 0, U)$ with

$$c = \mu + \frac{2\delta\alpha}{\pi} \int_0^1 \sinh(\beta z) K_1(\alpha z) dz,$$

$(\alpha, \beta, \delta, \mu) \in \mathbb{R}^4$ with $\delta > 0$ and $0 < |\beta| < \alpha$, K_1 is the modified Bessel function of the third kind, and the Lévy measure which determines the jump behavior of discontinuous Lévy processes is:

$$U(dz) = u(z)dz = \frac{\delta\alpha}{\pi|z|} \exp(\beta z) K_1(\alpha|z|) dz.$$

2. In the second specification, jumps affect only the returns $(Y_t)_t$:

$$Y_t = \log\left(\frac{S_t}{S_{t-1}}\right) = \tilde{m}_t + \sqrt{h_t}z_t + \sum_{j=1}^{N_t} V_j, \quad S_0 = s, \quad (2)$$

where the innovations z_t are characterized by a standard normal distribution, N_t is a Poisson process with intensity λ , and V_1, V_2, \dots is a sequence of identically independent distributed random variables $N(\mu, \sigma^2)$, independent of the Poisson Process N_t .

In both models,

$$\begin{cases} m_t = \lambda_0 \sqrt{h_t} - \frac{1}{2}h_t; & \lambda_0 \in \mathbb{R}, \\ h_t = G(h_{t-1}, Y_{t-1}) & \text{is a related GARCH model.} \end{cases} \quad (3)$$

λ_0 is the constant unit risk premium. The conditional volatility is modelled by different GARCH models. Denoting $\varepsilon_t = \sqrt{h_t}z_t$, we use:

- the EGARCH(1,1) modelling (Nelson, 1991)

$$\log(h_{t+1}) = \alpha_0 + \beta_1 \log(h_t) + \alpha_1(|z_t| - E[|z_t|]) + \gamma_1 z_t, \quad (4)$$

- the GARCH-GJR(1,1) model introduced by Glosten, Jagannathan and Runkle (1993)

$$h_{t+1} = \omega + \beta h_t + \gamma \varepsilon_t^2 + \delta S_t^- \varepsilon_t^2, \quad (5)$$

where $S_t^- = 1$ if $\varepsilon_t < 0$, and $S_t^- = 0$ if $\varepsilon_t \geq 0$, and $\beta + \gamma + \frac{1}{2}\delta < 1$, $\omega > 0$, $\beta \geq 0$, $\gamma \geq 0$, and $\gamma + \delta \geq 0$.

- the APARCH(1,1) model, (Ding, Granger, and Engle, 1993)

$$h_{t+1}^{\frac{\delta}{2}} = \omega + a_1(|\varepsilon_t| - \gamma_1 \varepsilon_t)^\delta + b_1 h_t^{\frac{\delta}{2}}, \quad (6)$$

where $\delta > 0$, $|\gamma_1| < 1$.

2.2 The Stochastic Discount Factors and the corresponding risk neutral dynamics

A discrete time approach to option pricing implies the non-uniqueness of the pricing measure. This uniqueness can be obtained by specifying a stochastic discount factor. However, there has until now been no criterion to decide on a specific pricing kernel. In order to highlight the specific ability of Lévy processes to deliver a realistic pricing of options, we use two existing pricing kernels: first, the exponential affine stochastic discount factor, also known as the Esscher transform by Gerber and Shiu (1994b); second, the extended Girsanov principle developed by Elliott and Madan (1998) based on the existence of a Minimal Entropy Martingale Measure, as in Fujiwara and Miyahara (2003). By doing so, we will be able to interpret the interest of Lévy processes when it comes to option pricing, beyond the choice of a pricing kernel.

2.2.1 Pricing options with exponential affine stochastic discount factors

In this Section we consider that the stochastic discount factor M_t is characterized by the Esscher transform $\forall t \in \{0, \dots, T-1\}$:

$$M_{t,t+1} = \exp(\theta_{t+1}Y_{t+1} + \xi_{t+1}) \quad (7)$$

where Y_{t+1} is introduced in (1) or in (2), and where θ_{t+1} and ξ_{t+1} are \mathcal{F}_t -measurable random variables. We need to compute explicitly $(\theta_{t+1}, \xi_{t+1})$. The pricing formula for the risk free and the risky asset at different dates gives the following system:

$$\begin{cases} \varphi_t^{\mathbb{P}}(\theta_{t+1} + 1) = e^r \varphi_t^{\mathbb{P}}(\theta_{t+1}) \\ \varphi_t^{\mathbb{P}}(\theta_{t+1} + 1) = \exp(-\xi_{t+1}), \end{cases}$$

where $\varphi_t^{\mathbb{P}}(u) = E_{\mathbb{P}}[e^{uY_{t+1}}|\mathcal{F}_t]$ is the moment generating function under the historical distribution \mathbb{P} of Y_{t+1} , given \mathcal{F}_t . This system admits a unique solution $(\theta_{t+1}, \xi_{t+1})$. Thus, to characterize the process $(Y_t)_t$ under the risk neutral distribution that is denoted \mathbb{Q} , the following equation must be solved:

$$E^{\mathbb{P}}[M_{t,t+1}\Phi_{t+1}|\mathcal{F}_t] = e^{-r}E^{\mathbb{Q}}[\Phi_{t+1}|\mathcal{F}_t]$$

which is equivalent to:¹

$$E^{\mathbb{P}}\left[\frac{M_{t,t+1}}{E_{\mathbb{P}}[M_{t,t+1}|\mathcal{F}_t]}\Phi_{t+1}|\mathcal{F}_t\right] = E^{\mathbb{Q}}[\Phi_{t+1}|\mathcal{F}_t]. \quad (8)$$

Then the distribution of Y_t under the risk neutral distribution \mathbb{Q} is obtained through the moment generating function under \mathbb{Q} of Y_{t+1} , given \mathcal{F}_t .

Proposition 2.1 *The associated unique risk-neutral conditional distribution \mathbb{Q} of Y_{t+1} , given \mathcal{F}_t , has a probability density function (p.d.f.) with respect to the corresponding historical distribution given by $\frac{M_{t,t+1}}{E_{\mathbb{P}}[M_{t,t+1}|\mathcal{F}_t]}$ and a moment generating function given by:*

$$\varphi_t^{\mathbb{Q}}(v) = \frac{\varphi_t^{\mathbb{P}}(\theta_{t+1} + v)}{\varphi_t^{\mathbb{P}}(\theta_{t+1})}. \quad (9)$$

¹In the annex we recall why the relationship (8) ensures the uniqueness of the martingale measure \mathbb{Q} , as soon as we solve this last equation, recalling that

$$\begin{aligned} \frac{M_{t,t+1}}{E_{\mathbb{P}}[M_{t,t+1}|\mathcal{F}_t]} &= \frac{\exp(\theta_{t+1}Y_{t+1} + \xi_{t+1})}{E_{\mathbb{P}}[\exp(\theta_{t+1}Y_{t+1} + \xi_{t+1})|\mathcal{F}_t]} \\ &= \frac{\exp(\theta_{t+1}Y_{t+1}) \times e^{\xi_{t+1}}}{E_{\mathbb{P}}[\exp(\theta_{t+1}Y_{t+1})|\mathcal{F}_t] \times e^{\xi_{t+1}}} \\ &= \frac{\exp(\theta_{t+1}Y_{t+1})}{\varphi_t^{\mathbb{P}}(\theta_{t+1})} \end{aligned}$$

This proposition derives from Proposition 2 in Chorro *et al.* (2010a) as we introduce the stochastic process:

$$\left(L_t = \prod_{k=1}^t \frac{e^{\theta_k Y_k}}{\varphi_{k-1}^{\mathbb{P}}(\theta_k)} \right)_{t \in \{1, \dots, T\}} \quad (10)$$

that is a martingale under \mathbb{P} .

Note that the equation (10) corresponds to the risk neutral measure obtained through the conditional Esscher transformed measure with respect to the filtration \mathcal{F}_t . The relationship (9) is fundamental as it gives explicitly the conditional distributions of the log returns under the risk neutral distribution \mathbb{Q} . We now adapt Proposition 2.1 to the two types of Lévy process-based models presented earlier.

- Assuming that the returns $(Y_t)_t$ are governed by model (1) with innovations z_t characterized by a $NIG(\mu, \alpha, \beta, \delta)$ distribution with triplet $(c, 0, U)$,² then $(\sqrt{h_t} z_t)_t$ is a form of NIG process with triplet $(c', 0, \nu)$, with:

$$c' = c\sqrt{h_t} + \int_{\mathbb{R}} z\sqrt{h_t} \left(1_{\{|z| \leq \frac{1}{\sqrt{h_t}}\}}(z) - 1_{\{|z| \leq 1\}}(z) \right) u(z) dz,$$

then:

Proposition 2.2 *Given \mathcal{F}_{t-1} , assuming that the process $(Y_t)_t$ is defined in (1), under \mathbb{Q} , the process $(\sqrt{h_t} z_t)_t$ is again a Lévy process with triplet $(c^{\mathbb{Q}}, 0, \nu^{\mathbb{Q}})$ where:*

$$c^{\mathbb{Q}} = c\sqrt{h_t} - \int_{\{|z| \leq 1\}} z\sqrt{h_t} u(z) dz + \int_{\{|z| \leq \frac{1}{\sqrt{h_t}}\}} z\sqrt{h_t} e^{\theta_t \sqrt{h_t} z} u(z) dz$$

and

$$\nu^{\mathbb{Q}}(dx) = e^{\theta_t x} u\left(\frac{x}{\sqrt{h_t}}\right) \frac{dx}{\sqrt{h_t}}.$$

We can check that under \mathbb{Q} , the conditional mean return is not \tilde{m}_t but $\tilde{m}_t + \tilde{m}_t^{\text{shift}}$ with $\tilde{m}_t^{\text{shift}} = \int_{\mathbb{R}} z\sqrt{h_t}(e^{\theta_t \sqrt{h_t} z} - 1)U(dz)$. Moreover, the process $(Y_t)_t$ is no longer centered and its variance is not h_t but $\text{var}(Y_t) = h_t \left(\sigma^2 + \int_{\mathbb{R}} z^2 e^{\theta_t \sqrt{h_t} z} U(dz) \right)$.

- When the returns are governed by model (2), the moment generating functions under \mathbb{P} and \mathbb{Q} are given in the following proposition.

Proposition 2.3 *Given \mathcal{F}_{t-1} , the moment generating functions under \mathbb{P} and \mathbb{Q} of the process $(Y_t)_t$ introduced in (2) are respectively:*

$$\varphi_{t-1}^{\mathbb{P}}(u) = \exp \left[u\tilde{m}_t + \frac{1}{2} h_t u^2 + \lambda(e^{\frac{1}{2} \sigma^2 u^2} - 1) \right] \quad \text{and}$$

$$\varphi_{t-1}^{\mathbb{Q}}(u) = \exp \left[u\tilde{m}_t + h_t \left(\theta_t u + \frac{1}{2} u^2 \right) + \lambda e^{\mu \theta_t + \frac{1}{2} \sigma^2 \theta_t^2} \left(e^{\mu u + \sigma^2 (\theta_t u + \frac{1}{2} u^2)} - 1 \right) \right].$$

²The full characterization of the risk neutral dynamics is provided in the appendices.

Therefore, the risk neutral dynamics for the returns $(Y_t)_t$ belongs to the same family introduced under the historical measure \mathbb{P} , with $z_t \stackrel{\mathbb{Q}}{\sim} N(\sqrt{h_t}\theta_t, 1)$, $N_t \stackrel{\mathbb{Q}}{\sim} \mathcal{P}\left(\lambda e^{\mu\theta_t + \frac{1}{2}\sigma^2\theta_t^2}\right)$, $V_j \stackrel{\mathbb{Q}}{\sim} N(\mu + \sigma^2\theta_t, \sigma^2)$ ³. The characteristic triplets are $(\tilde{m}_t + h_t\theta_t, h_t, \nu^{\mathbb{Q}})$, where $\nu^{\mathbb{Q}}(x) = \lambda e^{\theta_t x} f(x) dx$ and f is the density of the normal distribution with mean μ and variance σ^2 . Consequently, under \mathbb{Q} , the returns distribution is obtained shifting the conditional mean and the conditional variance.

2.2.2 Pricing options with a Minimal Entropy Martingale Measure

In this subsection, we derive the risk neutral dynamics of the previous models using another pricing kernel, namely the Extended Girsanov Principle introduced by Elliot and Madan (1998). This approach is based on a multiplicative Doob decomposition of the discounted stock price into a predictable finite variation component and a Martingale one.

Under mild conditions Elliot and Madan (1998) built a risk neutral probability under which the conditional distribution of the discounted stock price is equal to the conditional distribution of its martingale component under the historical probability with changes in the parameters. The drift of the process is the only one being affected by this correction. The Esscher transform and the Minimal Entropy Martingale Measure are equivalent when the conditional distribution of the returns is Gaussian. This equivalence has not been proved for the models considered here.

- When the returns follow the dynamics specified by equation (1) with NIG-Lévy innovations $(z_t)_t$ and assuming that:⁴

$$\begin{aligned} \text{(i)} \quad & \int_{\{z>1\}} e^z e^{\beta(e^z-1)} U(dz) < \infty; \\ \text{(ii)} \quad & \tilde{m}_t + c' + \int_{\{|x|\leq 1\}} \left[(e^x - 1)e^{\beta(e^x-1)} - x \right] U\left(\frac{dx}{\sqrt{h_t}}\right) + \int_{\{|x|>1\}} (e^x - 1)e^{\beta(e^x-1)} U\left(\frac{dx}{\sqrt{h_t}}\right) = r \end{aligned}$$

where

$$c' = \left[c + \int_{\{|z|\leq \frac{1}{\sqrt{h_t}}\}} z U(dz) - \int_{\{|z|\leq 1\}} z U(dz) \right] \sqrt{h_t}.$$

Then, we obtain the following result:

Proposition 2.4 *Assuming that $(Y_t)_t$ is given by (1) with NIG-Lévy innovations $(z_t)_t$ under \mathbb{P} , then the process $(\sqrt{h_t}z_t)_t$ is a Lévy process under \mathbb{Q} conditionally to \mathcal{F}_{t-1} , with triplet $(c^{\mathbb{Q}}, 0, \nu^{\mathbb{Q}})$:*

$$c^{\mathbb{Q}} = c\sqrt{h_t} - \int_{\{|z|\leq 1\}} z\sqrt{h_t} u(z) dz + \int_{\{|z|\leq \frac{1}{\sqrt{h_t}}\}} z\sqrt{h_t} e^{\beta(e^z\sqrt{h_t}-1)} u(z) dz,$$

³Again, the proof of this proposition is included in the annex

⁴Again, a full mathematical developpement is provided in the appendices.

$$\nu^{\mathbb{Q}}(dx) = e^{\beta(e^x-1)} u \left(\frac{x}{\sqrt{h_t}} \right) \frac{dx}{\sqrt{h_t}}.$$

The characteristic functions of the process $(Y_t)_t$ given \mathcal{F}_{t-1} under \mathbb{P} and \mathbb{Q} are:

$$\begin{aligned} \log \phi_{t-1}^{\mathbb{P}}(u) &= iu\tilde{m}_t + iuc\sqrt{h_t} + \\ &\quad \int_{\mathbb{R}} (e^{iuz\sqrt{h_t}} - 1 - iuz\sqrt{h_t}1_{\{|z|\leq 1\}}) U(dz) \quad \text{and} \\ \log \phi_{t-1}^{\mathbb{Q}}(u) &= iu\tilde{m}_t + iuc\sqrt{h_t} + \\ &\quad \int_{\mathbb{R}} (e^{iuz\sqrt{h_t}} - 1 - iuz\sqrt{h_t}e^{-\beta(e^z\sqrt{h_t}-1)}1_{\{|z|\leq 1\}}) e^{\beta(e^z\sqrt{h_t}-1)} U(dz). \end{aligned}$$

Thus the characteristic function of $(Y_t)_t$ under the risk neutral distribution \mathbb{Q} has the same expression as the historical characteristic function with shift $\sigma^2 h_t \beta$ in the coefficient of iu and integrand $(e^{iuz\sqrt{h_t}} - 1)e^{\beta(e^z\sqrt{h_t}-1)}$ over \mathbb{R} .

In order to compute option prices, we simulate paths of the asset price under the historical measure \mathbb{P} . Then, we compute option prices as a weighted average of the payoff for each of the corresponding paths, where the weights are given by the Radon-Nikodym derivative that is defined by:

$$L_t = \frac{d\mathbb{Q}}{d\mathbb{P}} \Big|_{\mathcal{F}_t} = e^{\beta\hat{X}_t - bt}, \quad \text{with} \quad (11)$$

$$\begin{aligned} b &= \beta c' + \int_{\mathbb{R} \setminus \{0\}} \left[e^{\beta(e^x-1)} - 1 - \beta x 1_{\{|x|\leq 1\}} \right] U \left(\frac{dx}{\sqrt{h_t}} \right), \quad \text{and} \\ \hat{X}_t &= X_t + \frac{1}{2} \langle X^c \rangle_t + \sum_{u \in (0,t]} (e^{\Delta X_u} - 1 - \Delta X_u), \end{aligned} \quad (12)$$

where (X_t^c) is the continuous martingale part of (X_t) , and $\Delta X_u = X_u - X_{u-}$.

The same representation for the conditional mean as the one obtained using the exponential affine Stochastic Discount Factor can be derived as follows: $\tilde{m}_t^{\text{shift}} = \int_{\mathbb{R}} z\sqrt{h_t}(e^{\beta(e^z\sqrt{h_t}-1)} - 1)U(dz)$ and $\text{var}(Y_t) = h_t \left(\sigma^2 + \int_{\mathbb{R}} z^2 e^{\beta(e^z\sqrt{h_t}-1)} U(dz) \right)$.

- When the returns $(Y_t)_t$ are governed by model (2) under \mathbb{P} , then the risk neutral dynamics are characterized as follows:

Proposition 2.5 *Let the price process $(S_t)_t$ be defined by model (2), assuming that the condition (i) above holds, and there exists $\beta \in \mathbb{R}$ verifying:*

$$\begin{aligned} \tilde{m}_t + \left(\frac{1}{2} + \beta \right) h_t + \lambda \int_{\{|z|\leq 1\}} \left[(e^z - 1)e^{\beta(e^z-1)} - z \right] f(z) dz \\ + \lambda \int_{\{z>1\}} (e^z - 1)e^{\beta(e^z-1)} f(z) dz = r, \end{aligned}$$

then the following holds:

1. *There exists a unique constant b such that L_t is a \mathbb{P} -martingale, where $L_t = \frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}_t} = e^{\beta\hat{X}_t - bt}$, \hat{X}_t given by (12) and b verifies:*

$$b = \frac{\beta}{2}(1 + \beta)h_t + \beta\tilde{m}_t + \lambda \int_{\mathbb{R} \setminus \{0\}} \left[e^{\beta(e^z - 1)} - 1 - \beta z 1_{\{|z| \leq 1\}} \right] f(z) dz.$$

2. *Under \mathbb{Q} the stochastic process $(Y_t)_t$ is still a Lévy process, and the characteristics associated with the truncation function $\tau(z) := z 1_{\{|z| \leq 1\}}$ is given by*

$$\left(\beta h_t + \tilde{m}_t + \lambda \int_{\{|z| \leq 1\}} z(e^{\beta(e^z - 1)} - 1) f(z) dz, h_t, \nu^{\mathbb{Q}} \right) \text{ where,} \quad (13)$$

$$\nu^{\mathbb{Q}}(dz) = \lambda e^{\beta(e^z - 1)} f(z) dz. \quad (14)$$

Fujiwara and Miyahara (2003) proved this result in a particular case. The result provided here has been derived directly from theirs. When the characteristic triplets of the returns $(Y_t)_t$ under \mathbb{Q} are known, then we can define the prices under the risk neutral \mathbb{Q} .

3 Empirical Results

This Section is devoted to an empirical assessment of the option pricing models presented above.

3.1 The dataset

We use a dataset of closing option prices. These European options are options on the CAC 40 index. We use two types of data. First, we use daily returns on the CAC 40 index. This dataset starts on March 1, 1990 and ends on October 26, 2007. Descriptive statistics are presented in Table 1.

[Table 1 here]

The value of kurtosis leads us to seek the best distributions permitting the tails of these returns to be modelled. In Figure 1, we plot the empirical log-density (solid black line) vs. the estimated log density obtained with the NIG (green), Generalized Hyperbolic (red), Meixner (purple) and Normal (light blue) distributions. In Figure 2, we present the QQ-plots comparing the empirical quantiles of the CAC 40 log returns vs. the estimated quantiles obtained with the NIG (green), Generalized Hyperbolic (red), and Normal (light blue) distributions. If the data were from a normal distribution, we would expect most of the points to lie on the $y = x$ line. From these plots, we observe that the returns should be modelled by the NIG distribution. We test the goodness-of-fit using two goodness-of-fit tests, namely the Kolmogorov-Smirnov and the Anderson-Darling tests. The results are provided in Table 2. The Meixner distributions are rejected, whereas the other Lévy distributions are accepted.

[Figure 1 here]

[Figure 2 here]

[Table 2 here]

A second dataset is composed of option prices. This dataset starts on December 6, 2006 and ends on October 26, 2007. During this period, the CAC40 was sufficiently liquid to provide us interesting prices. It contains call option prices for a large range of moneyness and maturities. These options are marketed options for the following quarterly maturities: March, June, September and December and for various years. These contracts are the most liquid ones over an extended period of time such as ours.

3.2 Estimation

To estimate the parameters of these models, we first select a subsample containing 4,000 working days and ending on a date t between December 6, 2006 and October 26, 2007. Using a Quasi Maximum Likelihood estimation strategy, we estimate the parameters driving the volatility dynamics h_{t+1} . Tables 3 to 5 provide the values of the estimated parameters of the EGARCH, GARCH-GJR and APARCH models respectively with their standard deviations.

[Table 3 here]

[Table 4 here]

[Table 5 here]

Then, using the previous volatility dynamics, we estimate the unknown parameters $(\alpha, \beta, \delta, \mu)$ of the NIG distribution for model (1), and the parameters $\lambda, (\mu, \sigma^2)$ of the Poisson and the normal distributions for model (2), by maximum likelihood. Tables 6 and 7 provide the average estimated parameters with their standard deviation of the NIG distribution and the Poisson associated with normal distribution respectively, using the EGARCH, GARCH-GJR and APARCH's residuals obtained in the previous step.

[Table 6 here]

[Table 7 here]

3.3 Pricing results

We compute option prices by Monte Carlo simulation. In order to compute the European option prices for the models (1) and (2) for different strike prices and times to maturity, we denote \bar{P} the prices computed under \mathbb{Q} using Monte Carlo simulations. We now present the two ways to obtain these prices with respect to the pricing kernels that we use:

- The price of the European call option under \mathbb{Q} obtained using an exponential affine Stochastic Discount Factor is:

$$\bar{P}(t, T, K) = \frac{1}{N} \sum_{i=1}^N e^{-r(T-t)} f(Y_T^i) \quad (15)$$

where $f(Y_T^i) = (S_T^i - K)^+$ is the payoff, T is the maturity, K is the strike price, and (Y_T^i) for $i = 1, \dots, N$ are independent realizations of Y_T under \mathbb{Q} .

- The price of the European call option under \mathbb{Q} obtained using the Minimal Entropy Martingale Measure is given by:

$$\bar{P}(t, T, K) = \frac{1}{N} \sum_{i=1}^N e^{-r(T-t)} L_T^i f(Y_T^i) \quad (16)$$

where (Y_T^i) for $i = 1, \dots, N$ are independent realizations of Y_T under \mathbb{P} and L_T^i are realizations of L_T introduced in (11).

For each set of option prices we run $N = 10,000$ Monte Carlo simulations. In order to compare the pricing performances of the NIG-EGARCH, NIG-GJR, NIG-APARCH, Poisson-EGARCH, Poisson-GJR and Poisson-APARCH models using both the Esscher transform (ESS) and the Minimal Entropy Martingale Measure (MEMM), we use the average absolute relative pricing errors criterion given by:

$$AARPE = \frac{1}{M} \sum_{t=\tau_1}^{\tau_2} \sum_{j=1}^{J_t} \sum_{i=1}^{G_{t,j}} \left| \frac{P(t, T_j, K_i) - \bar{P}(t, T_j, K_i)}{P(t, T_j, K_i)} \right|,$$

where P is the corresponding quoted market option price, M is the total number of option prices involved in the computation of the criterion, J_t is the number of call option maturities T_j available at time t (t runs from December 6, 2006 (τ_1) to October 26, 2007 (τ_2)), and $G_{t,j}$ is the number of strikes K_i available at time t for this particular maturity T_j . In Table 8, we provide the pricing errors for different moneyness for the CAC 40 using the two previous models respectively with the Esscher transform and with the Minimal Entropy Martingale Measure. The computations have been done for various time to maturities 7 days $\leq T - t \leq 30$ days, 3 months $\leq T - t \leq 6$ months, and $T - t > 6$ months.

[Table 8 here]

Table 8 presents the AARPE criterion for the three maturity buckets considered here, and depending on the breakdown of moneyness retained in this article. Several empirical conclusions can be raised from the analysis of these results.

- (1) Regardless of the model selected, most of them provide fairly good results, especially given the fact that we do not estimate the parameters driving the various

models using the option prices. For a time to maturity below 30 calendar days, the average relative absolute errors range from 4.6% to 18.7% for moneynesses below or equal to 1. For the same maturity and for higher moneynesses, we obtain errors ranging from 189.8% and 403.7%. These results however remain interesting as the pricing of short term options is a challenging task. For options with a higher time-to-maturity, results are again encouraging and consistent with each other: even if – as discussed below – we do find best competitors, the different models and pricing kernel investigated here provide similar results: the AARPE for out-of-the-money options ranges from 5% to 38.1%. For in-the-money options, it ranges from 26.8% to 421.3%. The minimum errors obtained for each couple of time to maturity and moneyness is fairly comparable to what is presented in articles calibrating the parameters of the models to option prices, as in Christoffersen *et al.* (2006) or Barone Adesi *et al.* (2008).

- (2) For the options with the shortest time to maturities, the best performances are obtained for the model combining an APARCH volatility structure, with jumps affecting solely the returns and leaving the volatility unchanged and based on an MEMM pricing kernel. For moneynesses below 1.1, the selected error criterion ranges between 4.3% and 35.2%. For this type of maturity, the errors obtained are the most heterogenous ones, when comparing the various option pricing models specifications.
- (3) For options with a time to maturity higher than 30 days, the results are strongly consistent with each other. The volatility structure empirically favored is the EGARCH one. The best pricing kernel here is the Esscher transform one. In contrast, the conditional distribution should be selected differently, depending on the moneyness. For lower moneynesses, the model based on a Poisson distribution provides the best results. In contrast, for higher moneynesses, the NIG-based model delivers the lowest pricing errors. Hence, for this type of maturity, the question of mixing jumps and volatility is worth being raised: disentangling jumps from volatility seems an interesting choice when it comes to out-of-the-money options. On the other hand, it does not seem to matter for in-the-money options, as the NIG model uses the jumps in the dynamics of the volatility.

In the end, for a majority of results, out-of-the-money options are priced with lower errors when disentangling jumps from volatility. This seems to be the case for maturities which are both less and more than 30 days. In contrast, the volatility structure does not seem to matter, except for the APARCH model that generally provides the worst results, when considering the longest maturities.

4 Conclusion

This article presents different types of discrete time Lévy processes for option pricing purposes. These models are based on three different volatility dynamics and two different kinds of pricing kernels. We derive the characterization of the risk neutral distribution associated with each of these models. Combining jumps to volatility should provide

us with a realistic model of financial returns' dynamics, which is one of the major shortcomings of the Black and Scholes (1973) model. We provide an empirical assessment of these models using a dataset of option prices on the CAC 40 index. We find that most of these models provide interesting results. We also find that depending on the option's time to maturity, the type of Lévy process to be used should be different. So, it will be interesting to consider other data sets in order to understand the impact of the choice of the stochastic discount factor in a practical valuation under \mathbb{Q} .

5 Appendix:

5.1 The general results for the returns modelled by (1)

(a) - Model with an exponential affine SDF:

Using the relationship (9), we provide for model (1) the moment generating function of the log-returns under \mathbb{P} and \mathbb{Q} .

Proposition 5.1 *Given \mathcal{F}_{t-1} , the moment generating functions under \mathbb{P} and \mathbb{Q} of the process $(Y_t)_t$ introduced in (1) are respectively:*

$$\begin{aligned} \log \varphi_{t-1}^{\mathbb{P}}(u) &= u\tilde{m}_t + uc\sqrt{h_t} + \frac{u^2}{2}\sigma^2 h_t + \\ &\quad \int_{\mathbb{R}} (e^{uz\sqrt{h_t}} - 1 - uz\sqrt{h_t}1_{\{|z|\leq 1\}}) U(dz) \end{aligned}$$

and

$$\begin{aligned} \log \varphi_{t-1}^{\mathbb{Q}}(u) &= u\tilde{m}_t + uc\sqrt{h_t} + \frac{u^2}{2}\sigma^2 h_t \theta_t + \\ &\quad \int_{\mathbb{R}} (e^{uz\sqrt{h_t}} - 1 - uz\sqrt{h_t}e^{-\theta_t z\sqrt{h_t}}1_{\{|z|\leq 1\}}) e^{\theta_t z\sqrt{h_t}} U(dz), \end{aligned}$$

where \tilde{m}_t, c, θ_t have been defined previously.

The expression of the process under the risk neutral measure is now available:

Proposition 5.2 *If the returns $(Y_t)_t$ are modelled by (1) under \mathbb{P} , then under the risk neutral measure \mathbb{Q} , the process $(Y_t)_t$ follows the same process where $(X_t)_t$ is a Lévy process with triplet $(c^{\mathbb{Q}}, \sigma^2 h_t, \nu^{\mathbb{Q}})$ given by:*

$$c^{\mathbb{Q}} = c\sqrt{h_t} + \sigma^2 \theta_t h_t - \int_{\{|z|\leq 1\}} z\sqrt{h_t} U(dz) + \int_{\{|z|\leq \frac{1}{\sqrt{h_t}}\}} z\sqrt{h_t} e^{\theta_t \sqrt{h_t} z} U(dz)$$

and

$$\nu^{\mathbb{Q}}(dx) = e^{\theta_t x} U\left(\frac{dx}{\sqrt{h_t}}\right).$$

We observe that the conditional distribution of $(Y_t)_t$ under \mathbb{Q} belongs to the same family of distributions than under \mathbb{P} . Nevertheless, there are changes in the mean and in the variance.

(b) - Model with Minimal Entropy Martingale Measure:

To apply this methodology we recall briefly the framework in which we work, and specify some notations.

Let $G = (G_t)_{t \in [0, T]}$, $T > 0$, be a geometric Lévy process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, such that:

$$G_t = G_0 e^{z_t}, \quad (17)$$

where $G_0 > 0$ is a constant and $z = (z_t)_{t \in [0, T]}$ is a one-dimensional Lévy process. Let (c, σ^2, U) be the characteristics of $(z_t)_t$ associated with the truncation function $\tau(z) := z1_{\{|z| \leq 1\}}$. For a constant $r \in \mathbb{R}$, we set $B_t := e^{rt}$, and we also define $\tilde{G}_t := \frac{G_t}{B_t} = e^{-rt} G_t$: it is still a geometric Lévy process because $(z_t - rt)$ is a Lévy process, and is the discount price process of G_t . Now if $U = 0$, that is if $(G_t)_t$ does not have any jump, then this setup is nothing but the Black and Scholes model. Let $(\tilde{R}_t)_t$ be the return process for $(\tilde{G}_t)_t$. We introduce a condition on the driving Lévy process (z_t) .

(C): There exists a constant $\beta \in \mathbb{R}$ that satisfies the following conditions (i) and (ii):

(i)

$$\int_{\{z > 1\}} e^z e^{\beta(e^z - 1)} U(dz) < \infty; \quad (18)$$

(ii)

$$\begin{aligned} c + \left(\frac{1}{2} + \beta\right) \sigma^2 + \int_{\{|z| \leq 1\}} \left[(e^z - 1)e^{\beta(e^z - 1)} - z\right] U(dz) \\ + \int_{\{|z| > 1\}} (e^z - 1)e^{\beta(e^z - 1)} U(dz) = r. \end{aligned} \quad (19)$$

Then, we have the following result due to Fujiwara and Miyahara (2003).

Theorem 5.3 *Given the previous return process (\tilde{R}_t) , and assuming that the previous condition (C) holds, then*

(1) *We can define a probability measure \mathbb{Q} on \mathcal{F}_T by means of the Esscher transformation:*

$$\left. \frac{d\mathbb{Q}}{d\mathbb{P}} \right|_{\mathcal{F}_t} := \frac{e^{\beta \tilde{R}_t}}{E^{\mathbb{P}}[e^{\beta \tilde{R}_t}]} := \frac{e^{\beta \hat{z}_t}}{E^{\mathbb{P}}[e^{\beta \hat{z}_t}]} = e^{\beta \hat{z}_t - bt}, \quad (20)$$

for every $t \in [0, T]$, where β is the constant in **(C)**, and

$$\hat{z}_t = z_t + \frac{1}{2}\sigma^2 t + \int_{(0,t]} \int_{\mathbb{R} \setminus \{0\}} (e^z - 1 - z) N_p(du \, dz) \quad (21)$$

$$\begin{aligned} &= \sigma W_t + c_1 t + \int_{(0,t]} \int_{\{|z| \leq 1\}} (e^z - 1) \tilde{N}_p(du \, dz) \\ &\quad + \int_{(0,t]} \int_{\{|z| > 1\}} (e^z - 1) N_p(du \, dz) \end{aligned} \quad (22)$$

with

$$c_1 := \frac{1}{2}\sigma^2 + c + \int_{\{|z| \leq 1\}} (e^z - 1 - z) U(dz). \quad (23)$$

and

$$b = \frac{\beta}{2}(1 + \beta)\sigma^2 + \beta c + \int_{\mathbb{R} \setminus \{0\}} \left[e^{\beta(e^z - 1)} - 1 - \beta z 1_{\{|z| \leq 1\}} \right] U(dz). \quad (24)$$

(2) The stochastic process (z_t) is still a Lévy process under the probability measure \mathbb{Q} defined in (20), and the characteristics associated with the truncation function $\tau(z) := z 1_{\{|z| \leq 1\}}$ are:

$$\left(\beta\sigma^2 + c + \int_{\{|z| \leq 1\}} z(e^{\beta(e^z - 1)} - 1)U(dz), \sigma^2, U^{\mathbb{Q}} \right), \quad (25)$$

where:

$$U^{\mathbb{Q}}(dz) := e^{\beta(e^z - 1)} U(dz). \quad (26)$$

We now apply these theoretical results to model (1) we introduced previously for the stock price $(S_t)_t$ defined by:

$$S_t = S_{t-1} e^{\tilde{m}_t} \exp(X_t) \quad (27)$$

where $S_{t-1} e^{\tilde{m}_t}$ is \mathcal{F}_{t-1} -measurable, and $X_t = \sqrt{h_t} z_t$. Following Fujiwara and Miyahara (2003), we define a new probability measure \mathbb{Q} equivalent to \mathbb{P} , introducing the following technical condition on the driving Lévy process (z_t) .

(C₁): Constants c and $\beta \in \mathbb{R}$ exist that satisfy the following conditions (i) and (ii):

(i)

$$\int_{\{|z| > 1\}} e^z e^{\beta(e^z - 1)} U(dz) < \infty;$$

(ii)

$$\begin{aligned} \tilde{m}_t + c' + \left(\frac{1}{2} + \beta \right) \sigma^2 h_t + \int_{\{|x| \leq 1\}} \left[(e^x - 1) e^{\beta(e^x - 1)} - x \right] U \left(\frac{dx}{\sqrt{h_t}} \right) \\ + \int_{\{|x| > 1\}} (e^x - 1) e^{\beta(e^x - 1)} U \left(\frac{dx}{\sqrt{h_t}} \right) = r \end{aligned}$$

where:

$$c' = \left[c + \int_{\{|z| \leq \frac{1}{\sqrt{h_t}}\}} z U(dz) - \int_{\{|z| \leq 1\}} z U(dz) \right] \sqrt{h_t},$$

Proposition 5.4 *Let the price process $(S_t)_t$ be defined by (27), and the condition (C_1) , then:*

(i) *there exists a probability measure \mathbb{Q} defined on \mathcal{F}_T :*

$$L_t = \frac{d\mathbb{Q}}{d\mathbb{P}} \Big|_{\mathcal{F}_t} = e^{\beta \hat{X}_t - bt}, \quad \text{with} \quad (28)$$

$$b = \frac{\beta}{2}(1 + \beta)\sigma^2 h_t + \beta c' + \int_{\mathbb{R} \setminus \{0\}} \left[e^{\beta(e^x - 1)} - 1 - \beta x 1_{\{|x| \leq 1\}} \right] \nu(dx),$$

$$\hat{X}_t = X_t + \frac{1}{2} \langle X^c \rangle_t + \sum_{u \in (0, t]} (e^{\Delta X_u} - 1 - \Delta X_u), \quad (29)$$

where (X_t^c) is the continuous martingale part of (X_t) , and $\Delta X_u = X_u - X_{u-}$.

(ii) Under \mathbb{Q} , the price process $(S_t)_t$ is an exponential-Lévy process $S_t = S_{t-1} \exp(\tilde{m}_t + X_t)$ where \tilde{m}_t is introduced in (1), and $(X_t)_t$ is a Lévy process with Lévy triplet $(c^{\mathbb{Q}}, \sigma^2 h_t, \nu^{\mathbb{Q}})$ given by:

$$c^{\mathbb{Q}} = c\sqrt{h_t} + \sigma^2 \beta h_t - \int_{\{|z| \leq 1\}} z \sqrt{h_t} U(dz) + \int_{\{|z| \leq \frac{1}{\sqrt{h_t}}\}} z \sqrt{h_t} e^{\beta(e^z \sqrt{h_t} - 1)} U(dz),$$

$$\nu^{\mathbb{Q}}(dx) = e^{\beta(e^x - 1)} U\left(\frac{dx}{\sqrt{h_t}}\right).$$

Thus the dynamics of $(S_t)_t$ under \mathbb{Q} follows the same class of models than under \mathbb{P} , with changes inside the parameters of the Lévy process.

From the Theorem 5.3, we can derive the expression of the conditionnal moment generating functions for the process $(S_t)_t$ under \mathbb{P} and under \mathbb{Q} , and we will use them through Monte Carlo simulations to get option prices.

$$\begin{aligned} \log \varphi_{t-1}^{\mathbb{P}}(u) &= u\tilde{m}_t + uc\sqrt{h_t} + \frac{u^2}{2}\sigma^2 h_t + \\ &\quad \int_{\mathbb{R}} (e^{uz\sqrt{h_t}} - 1 - uz\sqrt{h_t} 1_{\{|z| \leq 1\}}) U(dz) \end{aligned}$$

and

$$\begin{aligned} \log \varphi_{t-1}^{\mathbb{Q}}(u) &= u\tilde{m}_t + uc\sqrt{h_t} + \frac{u^2}{2}\sigma^2 h_t \beta + \\ &\quad \int_{\mathbb{R}} (e^{uz\sqrt{h_t}} - 1 - uz\sqrt{h_t} e^{-\beta(e^z \sqrt{h_t} - 1)} 1_{\{|z| \leq 1\}}) e^{\beta(e^z \sqrt{h_t} - 1)} U(dz) \end{aligned}$$

5.2 The Relationship (8)

In this subsection, we prove Relationship (8).

The price C_t at time t of a European derivative asset with payoff $g(Y_{t+1}, \dots, Y_T)$ at T is

$$\begin{aligned} C_t(g, T) &= E_{\mathbb{P}}[M_{t,t+1} \times \dots \times M_{T-1,T} \times g(Y_{t+1}, \dots, Y_T) | \mathcal{F}_t] \\ &= E_{\mathbb{P}}[M_{t,T} \times g(Y_{t+1}, \dots, Y_T) | \mathcal{F}_t] \end{aligned}$$

which can be written as

$$\begin{aligned} C_t(g, T) &= E_{\mathbb{P}}[M_{t,T}|\mathcal{F}_t] \times E_{\mathbb{P}} \left[\frac{M_{t,T}}{E_{\mathbb{P}}[M_{t,T}|\mathcal{F}_t]} g(Y_{t+1}, \dots, Y_T) | \mathcal{F}_t \right] \\ &= e^{-r(T-t)} \times E_{\mathbb{P}} \left[\frac{M_{t,T}}{E_{\mathbb{P}}[M_{t,T}|\mathcal{F}_t]} g(Y_{t+1}, \dots, Y_T) | \mathcal{F}_t \right]. \end{aligned}$$

Defining $\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t,T}}{E_{\mathbb{P}}[M_{t,T}]}$, so $C_t(g, T)$ can be rewritten as

$$\begin{aligned} C_t(g, T) &= e^{-r(T-t)} E_{\mathbb{P}} \left[\frac{d\mathbb{Q}}{d\mathbb{P}} g((Y_{t+1}, \dots, Y_T) | \mathcal{F}_t) \right] \\ &= e^{-r(T-t)} E_{\mathbb{Q}} [g(Y_{t+1}, \dots, Y_T) | \mathcal{F}_t]. \end{aligned}$$

Consequently, the price C_t at time t of an asset with payoff $g(Y_{t+1})$ at time $t + 1$ is:

$$C_t(g) = e^{-r} E_{\mathbb{Q}} [g(Y_{t+1}) | \mathcal{F}_t]$$

and

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t,t+1}}{E_{\mathbb{P}}[M_{t,t+1}]}.$$

5.3 Proof of Proposition 2.3

Here we provide the proof of Proposition 2.3.

The moment generating function under \mathbb{P} of the process $(Y_t)_t$ introduced in (2) can be written as

$$\begin{aligned} \varphi_{t-1}^{\mathbb{P}}(u) &= E_{\mathbb{P}} \left[\exp \left(u\tilde{m}_t + uX_t + u \sum_{j=1}^{N_t} V_j \right) | \mathcal{F}_{t-1} \right] \\ &= \exp \left[u\tilde{m}_t + \frac{1}{2}u^2 h_t + \lambda \left(e^{\mu u + \frac{1}{2}\sigma^2 u^2} - 1 \right) \right], \end{aligned}$$

and the moment generating function under \mathbb{Q} is also derived from

$$\varphi_{t-1}^{\mathbb{Q}}(u) = \frac{\varphi_{t-1}^{\mathbb{P}}(\theta_t + u)}{\varphi_{t-1}^{\mathbb{P}}(\theta_t)}.$$

5.4 Proof of Proposition 5.1

Here we provide the proof of Proposition 5.1.

The moment generating function under \mathbb{P} of the process $(Y_t)_t$ introduced in (1) is

$$\begin{aligned}
 \varphi_{t-1}^{\mathbb{P}}(u) &= E_{\mathbb{P}} [\exp(uY_t) | \mathcal{F}_{t-1}] \\
 &= E_{\mathbb{P}} [\exp(u\tilde{m}_t) \exp(\sqrt{h_t}uz_t) | \mathcal{F}_{t-1}] \\
 &= \exp(u\tilde{m}_t) E_{\mathbb{P}} [\exp(u\sqrt{h_t}z_t) | \mathcal{F}_{t-1}] \quad \text{since } \tilde{m}_t \text{ is } \mathcal{F}_{t-1}\text{-measurable} \\
 &= \exp(u\tilde{m}_t) E_{\mathbb{P}} [\exp(u\sqrt{h_t}z_1)] \quad \text{since } z_t \text{ is i.i.d.} \\
 &= \exp(u\tilde{m}_t) \exp[\kappa(u\sqrt{h_t})] \quad \text{by the Lévy-Kintchine formula,}
 \end{aligned}$$

and the moment under \mathbb{Q} is derived from (9).

5.5 Proof of Proposition 5.2

In order to prove Proposition 5.2, we first introduce a lemma:

Lemma 5.5 *If z_t is a Lévy process with triplet (c, σ^2, U) , then $X_t = \sqrt{h_t}z_t$ is also a Lévy process with triplet $(c', \sigma^2 h_t, \nu)$ where:*

$$c' = c\sqrt{h_t} + \int_{\mathbb{R}} x (1_{\{|x| \leq 1\}}(x) - 1_S(x)) \nu(dx)$$

where S is the image by $\sqrt{h_t}$ of a unit ball in \mathbb{R} :

$$S = \{\sqrt{h_t}z : |z| \leq 1\}$$

and

$$\nu(dx) = U\left(\frac{dx}{\sqrt{h_t}}\right)$$

We now give the proof of Lemma 5.5.

Using Lévy-Itô decomposition, $X_t = \sqrt{h_t}z_t$ has the representation:

$$\begin{aligned}
 X_t &= \sigma\sqrt{h_t}W_t + c\sqrt{h_t}t + \int_{(0,t]} \int_{\{|z| \leq 1\}} z\sqrt{h_t}\tilde{N}_p(du \, dz) \\
 &\quad + \int_{(0,t]} \int_{\{|z| > 1\}} z\sqrt{h_t}N_p(du \, dz).
 \end{aligned} \tag{30}$$

In order to know the characteristics of $(X_t)_t$, we transform the point process $(p_t)_t$ into another one $(q_t)_t$ by

$$D_q := D_p \quad \text{and} \quad q_t := J(p_t),$$

where $J(z) := z\sqrt{h_t}$ for $z \in \mathbb{R}$, then:

$$\hat{N}_q(du \, dx) = du \, \nu(dx),$$

where $\nu(dx) := U \circ J^{-1}(dx)$ and

$$\begin{aligned}
X_t &= \sigma \sqrt{h_t} W_t + c \sqrt{h_t} t + \int_{(0,t]} \int_{\mathbb{R}} x 1_S(x) \tilde{N}_q(du \, dx) \\
&\quad + \int_{(0,t]} \int_{\mathbb{R}} x 1_{S^c} N_q(du \, dx) \\
&\quad \text{where } S = \{\sqrt{h_t} z : |z| \leq 1\} \text{ and } S^c = \{\sqrt{h_t} z : |z| > 1\} \\
&= \sigma \sqrt{h_t} W_t + c \sqrt{h_t} t + \int_{(0,t]} \int_{\{|x| \leq 1\}} x \tilde{N}_q(du \, dx) + \int_{(0,t]} \int_{\{|x| > 1\}} x N_q(du \, dx) \\
&\quad - \int_{(0,t]} \int_{\mathbb{R}} x 1_S(x) \, du \, \nu(dx) + \int_{(0,t]} \int_{\{|x| \leq 1\}} x \, du \, \nu(dx) \\
&= \sigma \sqrt{h_t} W_t + \left[c \sqrt{h_t} + \int_{\{|x| \leq 1\}} x \, \nu(dx) - \int_{\mathbb{R}} x 1_S(x) \, \nu(dx) \right] t \\
&\quad + \int_{(0,t]} \int_{\{|x| \leq 1\}} x \tilde{N}_q(du \, dx) + \int_{(0,t]} \int_{\{|x| > 1\}} x N_q(du \, dx) \\
&= \sigma \sqrt{h_t} W_t + \left[c \sqrt{h_t} + \int_{\mathbb{R}} x (1_{\{|x| \leq 1\}}(x) - 1_S(x)) \, \nu(dx) \right] t \\
&\quad + \int_{(0,t]} \int_{\{|x| \leq 1\}} x \tilde{N}_q(du \, dx) + \int_{(0,t]} \int_{\{|x| > 1\}} x N_q(du \, dx)
\end{aligned}$$

We obtain the Lévy-Itô decomposition of (X_t) associated with the truncation function $\tau(x) := x 1_{\{|x| \leq 1\}}$, and we derive the corresponding characteristics $(c', \sigma^2 h_t, \nu)$, where

$$c' = c \sqrt{h_t} + \int_{\mathbb{R}} x (1_{\{|x| \leq 1\}}(x) - 1_S(x)) \, \nu(dx).$$

We now come back to the proof of Proposition 5.2. From Proposition 2.1, we derive $\varphi^{\mathbb{Q}}(v)$

$$\begin{aligned}
\varphi^{\mathbb{Q}}(v) &= \frac{\varphi^{\mathbb{P}}(\theta_t + v)}{\varphi^{\mathbb{P}}(\theta_t)} \\
&= \exp[(\theta_t + v - \theta_t) \tilde{m}_t] \exp \left\{ \left[\kappa \left((\theta_t + v) \sqrt{h_t} \right) - \kappa \left(\theta_t \sqrt{h_t} \right) \right] t \right\} \\
&= \exp(v \tilde{m}_t) \exp \left\{ \left[\kappa \left((\theta_t + v) \sqrt{h_t} \right) - \kappa \left(\theta_t \sqrt{h_t} \right) \right] t \right\} \\
&= \exp(v \tilde{m}_t) \exp[\kappa^{\mathbb{Q}}(v \sqrt{h_t}) t]
\end{aligned}$$

where

$$\begin{aligned}
& \kappa^{\mathbb{Q}}(v\sqrt{h_t}) = \\
& = \kappa\left((\theta_t + v)\sqrt{h_t}\right) - \kappa\left(\theta_t\sqrt{h_t}\right) \\
& = c(\theta_t + v)\sqrt{h_t} + \frac{\sigma^2}{2}(\theta_t + v)^2 h_t + \int \left[e^{(\theta_t + v)\sqrt{h_t}x} - 1 - \tau(x)(\theta_t + v)\sqrt{h_t} \right] U(dx) \\
& \quad - c\theta_t\sqrt{h_t} - \frac{\sigma^2}{2}\theta_t^2 h_t - \int \left[e^{\theta_t\sqrt{h_t}x} - 1 - \tau(x)\theta_t\sqrt{h_t} \right] U(dx) \\
& = cv\sqrt{h_t} + \frac{\sigma^2}{2}v^2 h_t + \sigma^2\theta_t h_t v + \int \left[e^{v\sqrt{h_t}x} - 1 - \tau(x)v\sqrt{h_t} \right] e^{\theta_t\sqrt{h_t}x} U(dx) + \\
& \quad \int (e^{\theta_t\sqrt{h_t}x} - 1)\tau(x)\sqrt{h_t}vU(dx) \\
& = \left[c + \sigma^2\theta_t\sqrt{h_t} + \int (e^{\theta_t\sqrt{h_t}x} - 1)\tau(x)U(dx) \right] v\sqrt{h_t} + \frac{\sigma^2}{2}(v\sqrt{h_t})^2 + \\
& \quad \int \left(e^{v\sqrt{h_t}x} - 1 - \tau(x)v\sqrt{h_t} \right) e^{\theta_t\sqrt{h_t}x} U(dx).
\end{aligned}$$

Thus, under \mathbb{Q} , $(z_t)_t$ is a Lévy process with triplet $(b, \sigma^2, U^{\mathbb{Q}})$ where

$$b = c + \sigma^2\theta_t\sqrt{h_t} + \int (e^{\theta_t\sqrt{h_t}z} - 1)\tau(z)U(dz)$$

and

$$U^{\mathbb{Q}}(dx) = e^{\theta_t\sqrt{h_t}x}U(dx).$$

Using Lemma 4.1, $X_t = \sqrt{h_t}z_t$ is a Lévy process with triplet $(c', \sigma^2 h_t, \nu^{\mathbb{Q}})$ where

$$c' = c^{\mathbb{Q}} = b\sqrt{h_t} + \int_{\mathbb{R}} x \left(1_{\{|x| \leq 1\}}(x) - 1_S(x) \right) \nu^{\mathbb{Q}}(dx),$$

and

$$\nu^{\mathbb{Q}}(dx) = U^{\mathbb{Q}}\left(\frac{dx}{\sqrt{h_t}}\right) = e^{\theta_t x} U\left(\frac{dx}{\sqrt{h_t}}\right). \quad \square$$

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6 Tables and Figures

Number of observations	Mean	Minimum	Maximum
5052	1.4285e-004	-0.0947	0.1059
Median	Standard Deviation	Skewness	Kurtosis
3.3748e-004	0.0142	-0.0438	7.7029

Table 1: **Descriptive statistics**

This table presents the descriptive statistics for the CAC 40 log returns from March 1, 1990 to March 1, 2010.

	CAC 40
KS p-value for Normal	2.964e-14
KS p-value for NIG	0.9512
KS p-value for GH	0.964
AD p-value for Normal	$< 2.2e - 16$
AD p-value for NIG	0.12
AD p-value for GH	0.85
Chi-square p-value for Normal	$< 2.2e - 16$
Chi-square p-value for NIG	0.3656
Chi-square p-value for GH	0.5454
Chi-square p-value for Meixner	1.788e-10

Table 2: **Kolmogorov-Smirnov, Anderson-Darling and chi-square adequation tests**

This table presents the test statistics testing the adequation of the Normal, NIG and GH distributions to a data set of daily log returns of the French CAC 40 from March 1, 1990 to March 1, 2010.

EGARCH parameters	α_0	β_1	α_1	γ_1
Mean	-1.15E-01	0.986931623	0.100460765	-0.059513138
Stand.Dev	6.93E-03	0.0008161	0.008765466	0.003341666

Table 3: **Estimated parameters for the EGARCH model.**

This table presents the estimated parameters with their standard deviation for the EGARCH model by selecting a subsample containing 4000 working days and ending on the date t between December 6, 2006 and October 26, 2007, using a Quasi Maximum Likelihood Estimation.

GJR parameters	ω	β	γ	δ
Mean	2.07E-06	0.93402486	0.016211803	0.071794509
Stand.Dev	3.04E-07	0.003541235	0.001106381	0.005545625

Table 4: **Estimated parameters for the GARCH-GJR model.**

This table presents the estimated parameters with their standard deviation for the GARCH-GJR model by selecting a subsample containing 4000 working days and ending on the date t between December 6, 2006 and October 26, 2007, using a Quasi Maximum Likelihood Estimation.

APARCH parameters	ω	a_1	γ_1	b_1	δ
Mean	6.91E-05	0.0527	0.5707	0.9414	1.2204
Stand.Dev	2.07E-05	0.0031	0.0243	0.0036	0.0777

Table 5: **Estimated parameters for the APARCH model .**

This table presents the estimated parameters with their standard deviation for the APARCH model by selecting a subsample containing 4000 working days and ending on the date t between December 6, 2006 and October 26, 2007, using a Quasi Maximum Likelihood Estimation.

Models		α	β	δ	μ
NIG-EGARCH	Mean	2.484	-0.397	2.371	0.382
	Stand.Dev	0.229	0.065	0.215	0.064
NIG-GJR	Mean	2.487	-0.412	2.369	0.396
	Stand.Dev	0.212	0.059	0.198	0.056
NIG-APARCH	Mean	2.492	-0.396	2.366	0.381
	Stand.Dev	0.222	0.062	0.209	0.060

Table 6: **Estimated parameters for the NIG distribution.**

This table presents the estimated parameters for the NIG distribution using the 2 stages estimation procedure and 4000 log-returns. NIG-GARCH means the model (1) with NIG innovation associated with its GARCH model.

Models		λ	μ	σ^2
Poisson-EGARCH	Mean	0.141	3.72E-03	0.033
	Stand.Dev	4.92E-05	2.00E-02	0.016
Poisson-GJR	Mean	0.143	0.087	0.027
	Stand.Dev	0.009	0.026	0.045
Poisson-APARCH	Mean	0.139	0.059	0.038
	Stand.Dev	4.63E-05	0.022	0.015

Table 7: **Estimated parameters for the Poisson and normal distributions.**

This table presents the average estimated parameters for the Poisson and normal distributions using the 2 stages estimation procedure and 4000 log-returns. Poisson-GARCH means the model (2) associated with its GARCH model.

7 days \leq Maturity \leq 30 days

Moneyiness	<0.8	[.8-.9]	[.9-1]	[1-1.1]	>1.1
Nig-EGARCH Ess	0.051	0.079	0.137	2.105	1.971
Nig-GJR Ess	0.051	0.079	0.136	2.204	2.738
Nig-APARCH Ess	0.05	0.077	0.107	2.576	3.056
Nig-EGARCH MEMM	0.046	0.071	0.129	0.365	1.914
Nig-GJR MEMM	0.046	0.071	0.130	0.367	3.60
Nig-APARCH MEMM	0.044	0.067	0.099	2.235	2.835
Pois-EGARCH Ess	0.053	0.073	0.186	0.371	2.215
Pois-GJR Ess	0.054	0.073	0.187	0.373	2.842
Pois-APARCH Ess	0.051	0.071	0.095	1.568	5.021
Pois-EGARCH MEMM	0.049	0.068	0.154	0.486	3.482
Pois-GJR MEMM	0.049	0.068	0.154	0.482	4.037
Pois-APARCH MEMM	0.043	0.064	0.091	0.352	1.898

0.25 \leq Maturity \leq 0.5

Moneyiness	<0.8	[.8-.9]	[.9-1]	[1-1.1]	>1.1
Nig-EGARCH Ess	0.04	0.084	0.279	0.521	0.614
Nig-GJR Ess	0.040	0.084	0.283	0.526	0.619
Nig-APARCH Ess	0.061	0.102	0.402	0.868	4.125
Nig-EGARCH MEMM	0.057	0.115	0.752	3.084	3.389
Nig-GJR MEMM	0.056	0.114	0.755	3.052	3.342
Nig-APARCH MEMM	0.081	0.181	0.791	0.881	4.051
Pois-EGARCH Ess	0.032	0.071	0.362	0.765	2.562
Pois-GJR Ess	0.033	0.073	0.365	0.776	2.684
Pois-APARCH Ess	0.061	0.122	0.756	0.865	4.103
Pois-EGARCH MEMM	0.052	0.082	0.385	0.796	2.652
Pois-GJR MEMM	0.054	0.084	0.384	0.798	2.513
Pois-APARCH MEMM	0.07	0.192	0.825	1.004	4.301

	Maturity > 0.5				
Moneyiness	<0.8	[.8-.9]	[.9-1]	[1-1.1]	>1.1
Nig-EGARCH Ess	0.056	0.081	0.120	0.268	0.318
Nig-GJR Ess	0.057	0.082	0.121	0.269	0.321
Nig-APARCH Ess	0.058	0.074	0.142	0.302	0.351
Nig-EGARCH MEMM	0.051	0.068	0.249	0.436	0.454
Nig-GJR MEMM	0.051	0.067	0.245	0.445	0.466
Nig-APARCH MEMM	0.065	0.095	0.291	0.475	0.501
Pois-EGARCH Ess	0.05	0.066	0.111	0.302	0.321
Pois-GJR Ess	0.052	0.067	0.118	0.321	0.329
Pois-APARCH Ess	0.062	0.083	0.136	0.391	4.011
Pois-EGARCH MEMM	0.521	0.071	0.224	0.201	0.405
Pois-GJR MEMM	0.526	0.072	0.227	0.211	0.402
Pois-APARCH MEMM	0.068	0.076	0.381	0.445	4.213

Table 8: **Absolute Average Relative Pricing Errors (AARPE) for the CAC 40 French index.**

These tables present the AARPE using the NIG-EGARCH, NIG-GJR, NIG-APARCH, Poisson EGARCH, Poisson GJR and Poisson APARCH under an exponential affine stochastic discount factor (also known as Esscher transform ESS) or under a minimal entropy martingal measure (MEMM). For example, NIG-EGARCH ESS means the model (1) with NIG innovation and EGARCH volatility model associated with an exponential affine stochastic discount factor.

We put the minimal errors in bold face.

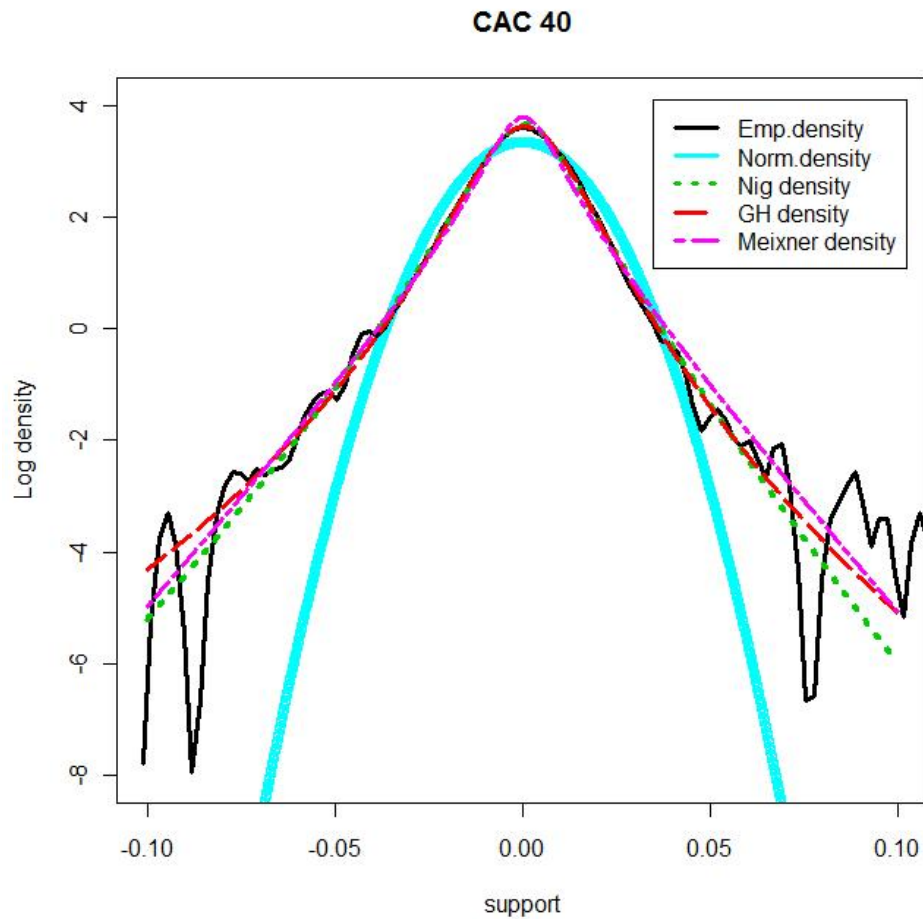


Figure 1: Empirical log-density (solid black line) vs. estimated log density obtained with the NIG (green), Generalized Hyperbolic (red), Meixner (purple) and Normal (light blue) distributions, using the CAC 40 returns data set.

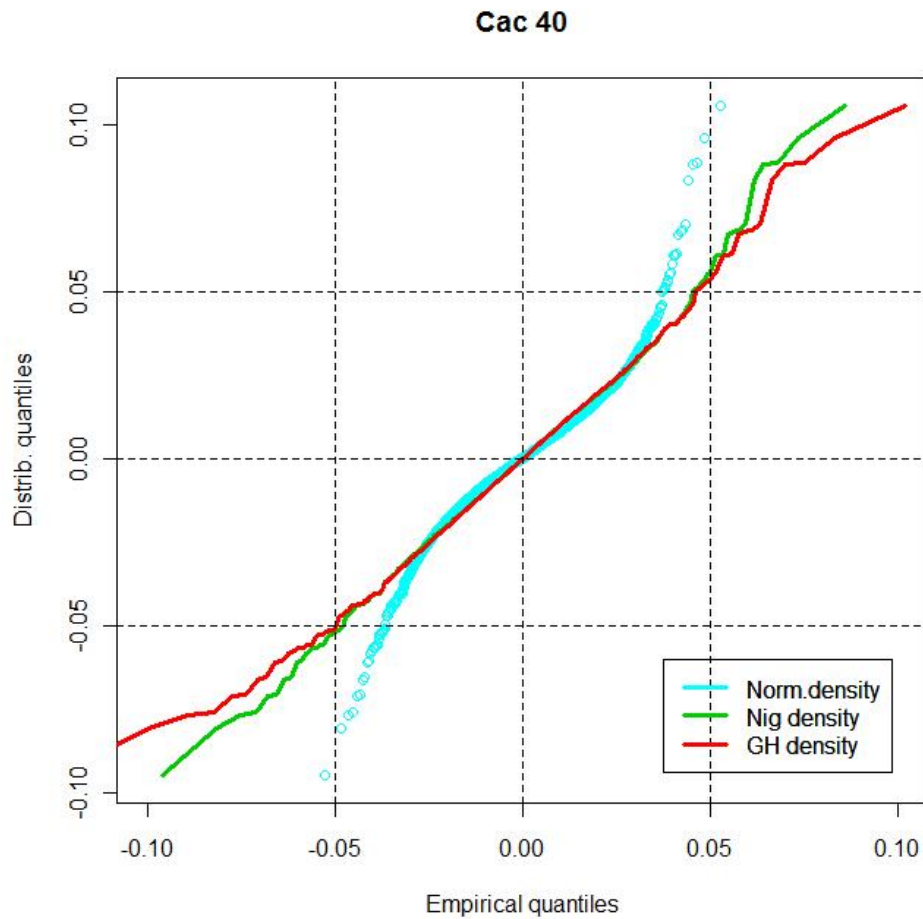


Figure 2: QQ-plots comparing the empirical quantiles of the CAC 40 log returns vs. the estimated quantiles obtained with the NIG (green), Generalized Hyperbolic (red), and Normal (light blue) distributions from March 1, 1990 to March 1, 2010.