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Hubert de La Bruslerie

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The Equal Opportunity Rule in Transfer of Control: A Contractual

Model

Hubert de La Bruslerie
Professor of Finance
University Paris-Dauphine *

Abstract:

The equal opportunity rule is seen as protecting investors in the event of a transfer of control. In order to better analyze the consequence of such a rule, we need to account for the information asymmetry that exists between new controlling shareholders and outside investors with private benefits. Both parties need to design a new implicit contract to share the firm's ownership. The first one seeks to control the firm; the other agrees to stand as minor investors. Using a signaling model, we show that the new controlling shareholder issues signals to outside shareholders to deliver private information on the firm's future economic return and his private rate of appropriation. We highlight the fact that ownership is a good signal: the higher the share of capital of the controlling shareholder, the better the prospects for future economic return, as perceived by outsiders. Another signal results from the premium embedded in the acquisition price, which also gives information on the future economic prospects of the firm. In a controlling ownership system, the equal opportunity rule modifies the relative behaviors of controlling and outside shareholders. The quality of information deteriorates despite the fact that the discipline may be stronger.

Keyword: Equal opportunity rule, transfer of control, takeover, controlling shareholder, investor protection, signaling equilibrium, private benefits

JEL: G3 / G34 / G38

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* DRM-Finance, Place du Mal de Lattre 75116 Paris, mail : hlb@dauphine.fr
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The Equal Opportunity Rule in Transfer of Control: A Contractual Model

Introduction

Transfers of control are aimed at disciplining a firm's management and improving its performance. The equal opportunity rule (EOR) is seen as protecting investors in the event of a transfer of control. The basic question is how these two preoccupations combine themselves. Whatever the procedure or the form of transfer (public tender offer or private block trade), a new controlling shareholder substitutes for an old one. As the third party, outside investors face a new economic story and the firm in which they had previously invested becomes another economic project. In order to develop protection for outside investors in such a situation, European countries have introduced legal provisions such as the equal opportunity rule or a mandatory bid price mechanism. Both result in a put option, allowing investors to exit at the same selling price as the paid acquisition price. These rules modify the equilibrium in the reallocation of control. For instance, with the mandatory bid rule, the buyer of a control block may receive an unknown and larger stake of capital from the target firm. This makes the buyer's choice different from situations of unconstrained acquisition of control. This question combines with the fact that the initially acquired block may be different from the stake of capital desired by the new controller..

The EOR rule has particular importance in the European context where firm's ownership is often concentrated (Berglof and Burkart, 2003; Goergen et al., 2005). Beside the mechanisms used for it, the transfer of control should refer to concentrated/diluted ownership systems (Burkart et al., 2000). The presence of a controlling shareholder is associated with potential opportunistic behavior towards minority or outside shareholders. These outside

shareholders suffer expropriation of private benefits from controlling shareholders. Although a large number of standard company law techniques exist to resolve conflicts between the controlling shareholder and minority shareholders, the equal opportunity rule is a key provision of corporate governance. In contrast to the blockholder system, the dispersed shareholder system is almost unaffected by the introduction of the mandatory bid rule. The consequences of the requirement to make an unconditional public tender offer are less important than in a situation of more concentrated ownership because of the absence of private benefits. Within an equal opportunity rule framework, when the takeover is initiated the outside shareholders have to choose between keeping their shares and selling them at an acquisition price that is always above the market price. Why, in an apparently irrational way, do outside shareholders, who may benefit from a price guarantee, not systematically sell their shares? Although tax reasons may explain it, many investors will refuse to accept an open unconditional bid. This puzzly question has not been extensively analyzed in the literature.

Therefore, the valuation of the firm develops in a joint economic valuation framework between the new ruling shareholders and the outside investors. The problem is made complex because outside investors are not passive and will act to optimize the percentage of shares they sell back to the initiator at the acquisition price. This can be analyzed as an implicit contract mixing agency problems of future private benefits and signaling problems of delivering private information. An equilibrium is established through the two key parameters of an offer: the size of the block of control and the acquisition price.

In the model presented in this paper, both parties need to design the characteristics of an implicit contract in order to share the firm's ownership. Through a signaling process, outside investors will integrate an expected level of future private benefits into their valuation schedule.

We show that new controlling shareholder issues a signal to the outside shareholders to deliver private information on two key variables: the firm's future economic return and the rate of private appropriation. As in the Leland and Pyle (1977) framework, we show that ownership is a good signal: the higher the share of capital held by the controlling shareholder, the better the prospects of future economic return as perceived by outsiders. Moreover, a high capital stake underlines a decline in the appropriation rate of private benefits. Another signaling effect results from the premium embedded in the acquisition price: the future profitability of the target under the buyer's ownership is increasing in the acquisition price. The rate of buyer's expropriation of future benefits also appears to the investors to be decreasing in the share the buyer acquires (with a 100% ownership there is no need to expropriate). The buyer maximizes his return by his choice of the acquisition price and his share of the target's capital. In an EOR context, we highlight that the buyer takes into account the inferences the outside shareholders draw from his choices. These inferences are not taken into account with no EOR rules. The setting of EOR rule enforces a constraint because the buyer is no longer completely free to set the share of capital he wants to purchase: the investors can force that share up by tendering their stocks to the buyer.

The setting of an EOR rule will specifically draw two consequences (i) It initiates a self limitation mechanism in future expropriation. The value of outside investor's shares is decreasing in the amount of expropriation the investors predicts to engage in. Since the rate of expropriation is decreasing in the share the buyer acquires, he is encouraged to take a larger stake. The EOR exaggerates this process by giving the investors an exit option at the purchase price. The buyer can be forced to increase his stake involuntarily. The buyer's best response, since he does not want to buy the entire firm, is to reduce the rate of benefits appropriation. (ii) The bid price and the purchased share of the target company as signals are lower quality signals

and convey less information to the market after the EOR: the buyer's ability to signal is reduced by the exogenous constraint induced by the EOR. Moreover, we also explain why the exit option may be partially used by rational outside investors in an EOR system. As a result, the legal environment appears important because it modifies and improves the equilibrium contract locus and the value of the firm in comparison with the absence of a price guarantee mechanism.

This paper is divided into three parts. A review of the literature is presented in the first section. Section 2 presents an analysis of information asymmetries, future private benefits and expected economic return in a signaling equilibrium model, which introduces outside shareholder choices. A conclusion follows.

1. Review of the Literature

The possibility and conditions of transfers of control have been extensively analyzed in the academic literature. In a well-known paper, Grossman and Hart (1980) showed that tender offers are rare because an informed seller will ask for a price at least equivalent to the future value of the firm under a new management. The only possibility of stimulating buyers is the appropriation of a rent leading to the offer of a lower public offer price. Subsequently, if takeover bids occur, it will be because the regulation context allows the controlling buyer to appropriate directly all or part of the value of minority shareholders' rights. Hirshleifer and Titman (1988) analyzed the impact of previous shareholding by the buyer, or toehold. His wealth will increase due to the increase in value of the shares he held before the takeover bid, even if he realizes no gain on the shares bought during the process of acquisition. Bebchuk (1989) reconsidered the main hypothesis made by Grossman and Hart predicting that the only successful takeover bids are those in which the future gains can be determined with certainty. In fact, the author shows that a bid can be successful only if it is an unconditional offer. The

reason for such takeover bids is that each individual shareholder wants his shares to be bought. Therefore, an unconditional tender offer can be successful because of its possibility of failure. Just like Bagnoli and Lipman (1988), Bebchuk (1989) showed that, even without any private appropriation by the buyer, the buyer initiates a takeover bid if he is able to increase the value of the firm. Like Grossman and Hart, Bebchuk emphasizes the importance of the incentive resulting from the gains in appropriation obtained by the buyer.

Introducing corporate governance rules may affect the welfare and the efficiency of corporate control and market discipline (Davies and Hopt, 2004; Goergen et al., 2005). These rules may discourage a takeover bid, as well as possibly discouraging the current blockholders from accepting an offer. Consequently, the equal opportunity principle is an additional barrier to a well-functioning market for corporate control in a blockholder-based governance regime. Introducing an equal opportunity/mandatory bid rule has some implications for the ownership and control structure in a blockholder system. First, it makes the blockholder system less efficient, as it reduces the occurrence of trade in controlling blocks, which is the dominant way to transfer control (Burkart et al., 1998; Köke and Renneboog, 2005). Consequently, control may remain in the hands of inefficient blockholders. Second, it restricts the size of the stake a blockholder is allowed to acquire without triggering a tender offer. Third, the higher the bid price in a mandatory tender offer, the lower the acquirer's incentive to make a bid, so ownership and control in the blockholder system are likely to remain concentrated.

In a controlling shareholder system, the possibility of the appropriation/reallocation of the economic net cash flow within groups of firms is at the heart of the question insofar as takeover or block trade will develop in groups of firms (Faccio et al., 2003). Diversion by the holding company or by the controlling shareholder is the first eventuality. Setting internal transfer prices or imposing global costs of structure are well-known ways to reallocate the cash

flow within groups. Even if the subsidiary is not economically integrated, different ways still exist for “tunneling” part of the new cash flow that has been generated following a reorganization of the acquired company. Without going as far as diversion of the existing cash flow and its legal dangers, the allocation within a group of the new extra cash flow is legitimate. Takeovers can generate gains in synergy within a group or between firms. Therefore, reduction in costs, economies of scale, higher market power can benefit the whole group when firms are economically integrated. Attributing the entire gain to the last bought firm is like stating that it is the last drop of water that makes the glass overflow. It is not surprising that the extra cash flow, which is the consequence of the better efficiency of the whole group, is not totally allocated to the controlled firm. We will use the terms appropriation rate or private benefits to describe the part of the economic cash flow that is directly levied by the controlling shareholder. Outside investors will then value the firm considering only the net reported cash flow after appropriation.

In the controlling shareholder system, the dominant agency conflict that develops is the one with outside investors. Private benefits are levied by the controlling shareholder (Shleifer and Vishny, 1986, or La Porta, et al., 1999). The problem is estimating private benefits that are concealed from outside investors and that often result from negative management decisions (not to do something rather than doing it). This leads Hofstetter (2006) to ask why not make private benefits an explicit part of the corporate contract. An instantaneous measure of private benefits is made when a change in ownership implies a transfer of control to a new buyer. The transaction price logically integrates the rent of control. This is the discounted value of the controlling shareholder’s private benefits. These private benefits should be considered as an agency variable in the controlling-outside investors’ relationship. In a controlling ownership

system, some efficient level of private benefits balancing monitoring costs may exist (Burkart et al., 2000).

Many empirical studies have tried to estimate the value of control in the acquisition price by separating the part that corresponds to private benefits. Barclay and Holderness (1989), in examining takeovers in the USA, pointed out that large blockholders possess abnormally large benefits. The acquisition price of a share in a block trade compared with its value in the market before the transaction date is an approximate estimate of the private benefits. Considering 63 block trades of NYSE listed firms, they outlined a positive premium in 80% of the transfers of control. The average premium was 20% and represented 13% of the global transaction price. In a later empirical study, Barclay et al. (1991) analyzed 106 block acquisitions. The average size of the block of shares represented 27% of the capital. In most cases (90%), a new controlling buyer replaced the incumbent dominant stockholder. In France, Schatt and Roy (2004) considered 80 block trades during the period 1996-2002. The size of the block represented an average stake of 60.6% of capital. A total of 63 trades involved more than 50% of the vote rights. In 17 operations, the initiator was already in the firm's capital with a "toehold" participation of 22%. They wanted to obtain control by increasing their stake with an average block of 43.8%, resulting in a controlling ownership after the transaction of 66% of the capital. In a majority block trade transaction, French (and more globally European) regulation imposes a mandatory bid takeover aimed at minority shareholders. The authors showed that, during the period studied, the average stake of capital sold by outside investors in that framework represented an average stake of 25.7% of capital. In the end, the initiator obtained an average 90.8% of capital of the target firms. It should be noted that the authors found that the price guarantee mechanism attracted only three-quarters of outside shareholders, who will only partially use the exit opportunity.

Only a few studies are devoted to the technical difference between the two procedures of block trade and public tender offer within a context of private benefits. Transfer of control can be achieved by block trade acquisitions or by public takeover bids. Bolton and von Thadden (1998) argue that the advantage of monitoring by blockholders is that it takes place on an ongoing basis. However, a block trade means that a situation of control or dominant influence pre-exists. Another technique is a tender offer in a diluted market framework. Holmen and Novorozhkin (2007) analyzed empirically the difference between tender offers and block trade in the Swedish market. Their basic hypothesis is that a tender offer indicates a larger future performance improvement and solves the Grossman and Hart (1980) free rider problem in convincing outside investors to subscribe. Therefore, in the case of large ownership, a controlling shareholder will be inclined to use a public tender offer. Burkart, Gromb and Pannuzi (2000) consider that the new dominant shareholder will improve target firm performance whatever the mode of transfer. However, a block trade leads to a less concentrated ownership (in a no equal opportunity rule context). Therefore, this supports the incentive to continue to extract private benefits. Conversely, public tender will lead to larger ownership of the dominant shareholder and will then lessen the appropriation of private benefits. Considering Swedish firms over the period 1986-2001, Holmen, et al. (2007) confirmed that the choice of transfer mode depends on the size of private benefits. Block trades (tender offers) are privileged when the controlling shareholder has a smaller (larger) stake and when private benefits are larger (smaller). However, this empirical study did not seem to take into account the fact that the mandatory bid rule had been introduced in Sweden in 1999 (with a threshold of 40%) and apparently block trades are not submitted to the equal opportunity mechanism, giving outside investors an exit option at the trade price. However, the two procedures are financially identical for outside investors. La Bruslerie and Deffains (2004) considered the two market techniques of

simplified tender offers (“OPA simplifiée”) and market price guarantee after a block trade (“Garantie de cours”) used in the French market to implement the equal opportunity rule once the transfer of control is known. Only between half and three-quarters of the outside investors will exercise their “in-the-money” exit option. This illustrates the empirical importance of the puzzle linked with the equal opportunity rule offered to outside investors. La Bruslerie and Deffains (2004) developed a contingent claim analysis of the equal treatment right given to outside shareholders.

The analysis of the efficiency of the transfer of control is made by comparing the situation with and without an equal opportunity rule. An efficient transfer of control is an operation that creates new economic value. Does the equal opportunity rule, which benefits outside shareholders, help or hurt the efficient allocation of control? In comparison with a system with no specific outside investor protection, which one works better? Bebchuk (1994, 1999) shows that, in a pure market system with no protective regulation, the system efficiently protects outside shareholders under two conditions: (i) no private benefits before or after the transfer of control, and (ii) the asymmetry of information at the inception of the transaction disappears with an accounting system that efficiently reports the true economic profit of the firm. Burkart and Pannunzi (2004) introduced into this framework the future private benefits levied by the new controlling shareholder. They show that the condition for a transfer of control under the EOR is more demanding than the simple economic efficiency constraint. It leads to the ruling out of some efficient transfers of control. The EOR system reallocates some part of the gain ensuing from the transfer of control to minority shareholders. Moreover, it protects them from inefficient transfers of control. Even if new controlling blockholders continue to appropriate privately a proportion of the benefits, they must offer a higher price to the former controlling shareholder in order to satisfy the constraint imposed by the EOR. The above analysis is developed from a macro behavioral point of view. It is open to several criticisms:

- The size of the control block is supposedly fixed. This is considered as an endogenous variable. No flexibility is offered. The new controlling shareholder may aim at an optimal percentage participation stake, which is not the same as the size of the block trade.

- We need to explore further the asymmetry of information between the buyer and the seller. Does the new controlling blockholder accurately estimate private benefits levied by the previous one?

- According to the analysis of Bebchuk (1994) or Burkart and Panunzi (2004), the EOR exit option is supposedly totally exercised by minority shareholders. The above analysis does not explain the puzzle of a partial exercise of this option. How to justify this?

- Linked with the above point, there will remain some minority shareholders after the takeover. They know rationally that they will be exposed to a new uncertain hazard: the future expropriation of private benefits by the new controlling shareholder. In a framework of agents' rational anticipations, their choices will *ex ante* take this risk into account. The above analysis focuses on the calculus of the buyers and sellers of control. Nothing is said about the behavior of rational current and future minority shareholders.

2 Information Asymmetries, Future Private Benefits and Economic Return in a Signaling Equilibrium Model

A - Setting of the model

This paper will analyze the equal opportunity rule for outside shareholders in a non-hostile takeover context, which is the most usual one encountered in the financial markets. During the remaining time of the offer, outside shareholders make their choices by considering that the tender offer has been a success. Similarly, they know that the controlling block has been bought. The shareholder knows that he has to consider different value for the newly

controlled firm. They look forward to the future uncertain profitability of the newly controlled firm. These situations are frequent, corresponding to friendly takeover bids where success is known or quasi-certain from the beginning. This is also the case for many raider bids. Very often, institutional investors make public their decision to accept the public offer. Therefore, it is possible to know the evolution of the part of the capital that accepts the bid. With a raider's bid, outside shareholders benefit from a price guarantee when the success is rapidly known, which means before the end of the procedure. It is only if the offer is hostile and uncertain up to the end of the procedure that shareholders do not know whether they will benefit from a price guarantee mechanism. Such situations are uncommon.

This analysis considers mandatory bid price mechanisms for outside shareholders who are aware that control of the firm has just moved toward a new controlling shareholder. Outside shareholders know that the offer is successful and that they benefit from an equal opportunity rule for one of the following two reasons: (i) there is an explicit price guarantee or (ii) there is an implicit price guarantee because the success of the tender offer is quasi-certain and is a mandatory unconditional 100% equity acquisition. Under such conditions, the partial use of the exit price opportunity appears as the result of a rational economic calculation, which needs to be analyzed. Outside investors will question their optimal ownership according to the future economic profitability of the firm or to the appropriation policy of the new controlling blockholder. Looking at signals is a way to reach ownership equilibrium.

In order to analyze better the consequences of the equal opportunity rule, we need to take into account the information asymmetry existing between the new controlling shareholders and outside investors. The outside investors want to know more about the future prospects of the firm.

Private benefits have to be considered as important in our framework. They are not only European or Asiatic features. As identified by Bebchuk (1999), acquisition premium is a way to assess the rent of control of the exiting controlling shareholder and to buy it out. The market for the transfer of control is also a market in which to exchange private benefits. Traditionally, the acquisition price is presented as the payment for lost benefits to the exiting controlling shareholder. On theoretical grounds, takeovers would be rare without private benefits. Introducing a mandatory equality of opportunity gives outside investors access to that price. Nevertheless, looking at the past is not the key point in a deal. Outside investors with an exit option at the acquisition price are facing a double question: What will be the future economic return of the newly controlled firm and what will be the future private benefits levied by the new controlling shareholders? Investors have to be considered as risk averse because the future economic return is different from the old one and is uncertain.

We need to explain the partial use of the exit opportunity given to outside shareholders in an EOR system. We acknowledge that the acquirer will not buy 100% of the capital of the target firm as assumed in the Bebchuk or Burkart and Panunzi frameworks. The final stake in the capital should be considered as an endogenous variable resulting from the minority shareholders' choice. However this choice is also conditioned by the acquirer, who may adapt his offer and his information to aim at an optimal participation stake at the end of the takeover.

The gap in information between the new controlling shareholder, who knows the future prospects of the firm better than do outside shareholders (who may opt out according to the information they are given), is crucial. It explains the *ex post* sharing of capital of the target firm after the takeover. In this section, we develop a joint equilibrium model within a two-party signaling game based on *ex ante* expectations. The new controlling shareholder will aim at an optimal participation stake in the target firm. He uses signaling variables to influence the

outside investors and, beyond them, the market. Following Leland and Pyle (1977), we know that the fraction of capital targeted by the controlling investor is a sound signal of the profitability of the investment projects of the firm. Minority shareholders will use the exit opportunity following the information they infer from the signal about the future prospects of the firm under its new management.

A situation of asymmetry of information exists because outside investors do not know the true future economic profitability of the firm after takeover and they ignore the amount of private benefits levied by the new controller. A joint signaling model highlights the importance of information and will focus on the existence of some auto limitation mechanisms. We conjecture that these mechanisms are specific to the EOR system

We use the following variables:

A : bid offer price

α_0 : initial controlling block bought from the former controlling shareholder

α : percentage of shares acquired by the new controller when the transfer of control is completed

t : appropriation rate of private benefits

V_S : value of the firm before the takeover

V_e : invested economic capital

B_S : value of the private benefits of the selling controlling shareholder

k : risk adjusted cost of capital

The value of the firm before the transfer of control V_S is the sum of the wealth of the incumbent controlling shareholder, W_S , and the outside investors' wealth, W_O . Before the deal, it is equal to the invested economic capital V_e multiplied by the economic return on invested

capital, r_e^S . This return is an uncertain variable forecasted from the former controlling shareholder's strategic choices. We identify t_S with the forecast appropriation rate levied by the former blockholder¹:

$$V_S = W_S + W_O = (V_S - B_S) + B_S = \underbrace{\frac{V_e \cdot (r_e^S - t_S)}{k}}_{\text{public market value}} + \underbrace{\frac{V_e \cdot t_S}{k}}_{\text{private benefits}} \quad (1)$$

The initiator looks for control through a target participation stake of α by proposing a bid price A . This price integrates the buy out of the selling shareholder's private benefits. It should be above the minimum condition for the previous controlling shareholder to accept to sell. In a context without an EOR rule, the new shareholder buys an initial stake of capital α_0 . The incentive for the incumbent blockholder to sell is:

$$\alpha_0 \cdot A \geq \alpha_0 \cdot (V_S - B_S) + B_S \quad (2)$$

This condition means that the price paid to the incumbent controlling shareholder covers (at least) his share of the public value of the firm and his private benefits. It is supposed satisfied and the transfer of control occurs. The logic of signaling and the implicit contract between the new controlling shareholder and outside investors is only exogenously constrained by values of A and α satisfying (2). The target stake of capital of the new controlling shareholder and the acquisition price may thus be endogenously set in the equilibrium model. We introduce λ as the increase of value captured by the previous controlling shareholder so that he is paid above the minimum binding price including his former private benefits. From (2):

¹ This formula apparently means that benefits extraction entails no specific costs borne by the controlling shareholder. These costs are in a trade-off with the amount of private benefits. They are a function of the investor's stake of capital (Burkart et al., 2000). Without any loss of generality, we assume them to be zero in our comparison between the "without" and "with" EOR context. A more complete model introducing marginally increasing specific costs borne by the controlling shareholder leads to similar results. It is available to the author. We recognize that, looking at (1), the absence of specific cost favours a maximum appropriation behaviour by the controlling blockholder.

$$\alpha_0.A = \lambda[\alpha_0.(V_s - B_s) + B_s] \quad (3)$$

The new controlling shareholder has to buy out the previous owner's private benefits. The incentive condition holds for any $\lambda > 1$. This parameter expresses the part of future profits of the firm captured by the previous controlling shareholder. In an EOR context, the new controlling shareholder extends that price to any other shareholders. The setting of the initial transfer of control, particularly the price A, is left undefined and depends on the free variable λ . The only condition for a signaling equilibrium between the new controlling shareholder and the outside investors is (2). Particularly, the acquirer has partial discretion on A and the targeted participation stake α is not set equal to α_0 because the control changed. The control premium which is paid over the previous public market value of the firm is:

$$A = \underbrace{(V_s - B_s)}_{\text{Market value}} + \underbrace{\left[(\lambda - 1)(V_s - B_s) + \frac{\lambda}{\alpha_0} B_s \right]}_{\text{Control premium}} \quad (4)$$

For $\lambda=1$, the minimum incentive condition of the seller is just satisfied and if $\alpha=1$ the new controlling shareholder will minimize the control premium which is then equal to B_s . The minimum acceptable value for the acquisition price is then V_s , the global economic value before the takeover.

The wealth of the new controlling shareholder will depend on the average future economic return on invested capital ensuing from the new management, r_e^* , and from the future appropriation rate t_A^* . It also depends on his equity stake α^2 . The value of r_e^* is private information of the new controlling shareholder.

² The controlling shareholder bears some specific agency costs. They should be subtracted from the private benefits. Without any loss of generality, our model assumes that these costs are null. A complete version introducing costs marginally increasing with benefits extraction leads to similar conclusions. See footnote (1).

$$\tilde{W}_A = \alpha \frac{V_e \cdot (r_e^* + \tilde{x} - t_A^*)}{k} + \frac{V_e \cdot t_A^*}{k} - \alpha \cdot A \quad (5)$$

The new controlling shareholder knows the average value r_e but the economic profitability is exposed to an error term \tilde{x} such that $E(x)=0$. We assume the economic uncertainty to be normal with standard deviation σ_x . Controlling and outside investors share the same information on σ_x . Outside investors remaining in the firm do not know the true value t_A^* . However, they know that t_A^* is lower than r_e , which means that no default is considered in the setting. The new controlling blockholder looks for appropriating private benefits in a long-term (infinite) perspective and does not want the firm to go bankrupt. The parameter t_A^* is set by the new blockholder. The outside shareholders will forecast the future values of the economic profitability and of the appropriation rate and will receive information from the controlling shareholder. We define their forecast as functions of parameters of the delivered information, $r_e(\cdot)$ and $t_A(\cdot)$. The value of the outside investors' stake in the firm is:

$$\tilde{W}_0 = (1 - \alpha) \cdot \frac{V_e (r_e(\cdot) + \tilde{x} - t_A(\cdot))}{k} \quad (6)$$

Outside shareholders remaining after the takeover know that private appropriation exists and that the rate t_A is a function of α and of A . These two signals are identified by the market and are integrated into the valuation by market participants. The market valuation scheme of the firm by outside shareholders depends on the perceived future economic return and expropriation. The market value is only set by outside investors, who will consider two signals in order to build their forecast. The acquirer chooses two values as signals and outside investors set their $r_e(\alpha, A), t_A(\alpha, A)$. The future economic return is uncertain and represents from the outside investors' point of view a hazard due to economic noise such that $E(x)=0$.

The acquirer will choose the values of α and A , which will maximize his expected utility of wealth. He knows that his optimal choice will induce a market valuation based on implicit values for r_e and t_A :

$$\begin{aligned}\alpha^* &= \alpha^*(r_e^*, t_A^*) \\ A^* &= A^*(r_e^*, t_A^*)\end{aligned}\tag{7a}$$

A joint equilibrium schedule (Leland and Pyle, 1977) will imply that the valuation is based on a fair appraisal by the market of the true values of r_e and t_A . This valuation is done using the public signal values α and A . Reversing the equations at market equilibrium gives:

$$\begin{aligned}r_e[\alpha^*(\cdot, \cdot), A^*(\cdot, \cdot)] &= r_e^* \\ t_A[\alpha^*(\cdot, \cdot), A^*(\cdot, \cdot)] &= t_A^*\end{aligned}\tag{7b}$$

If, for instance, the value $r_e(\alpha, A)$ used by the outside investors and the market were greater than the true value of r_e (only known by the new controlling shareholder), the stock market value would be over-valued and, in the end, outside investors would receive less than the required expected risk adjusted return on the market (Leland and Pyle, 1977, p.374). In an infinite constant cash-flow valuation, at equilibrium, the numerator of the market public value is the equalized net economic return: $r_e(\cdot) - t_A(\cdot) = r_e^* - t_A^*$. The valuation of the firm in the market is the same considered by each category of shareholder in order to optimize their wealth.

B - Without EOR

In a system without the equal opportunity rule, we maximize the controlling shareholder's net wealth. The existence of economic uncertainty enhances the investor's risk aversion. We introduce a utility function of wealth $U(\cdot)$ and consider the expected utility of wealth. Using equation (5) the net wealth of the new controlling shareholder is with $r_e(\cdot)$ and $t_A(\cdot)$ set at their optimum:

$$\tilde{W}_A = \alpha \frac{V_e \cdot (r_e(\cdot) + \tilde{x} - t_A(\cdot))}{k} + \frac{V_e \cdot t_A(\cdot)}{k} - \alpha \cdot A \quad (8)$$

In order to optimize, we set the first derivative to zero with respect to the two signals.

Deriving versus the acquisition price A , we obtain:

$$\frac{dE[U(W_A)]}{dA} = E \left\{ U'(W_A) \cdot \left[\frac{\alpha \cdot V_e}{k} \cdot \left(\frac{dr_e(\cdot)}{dA} - \frac{dt(\cdot)}{dA} \right) + \frac{V_e}{k} \cdot \frac{dt(\cdot)}{dA} - \alpha \right] \right\} = 0 \quad (9)$$

Equivalently:

$$\alpha \frac{V_e}{k} \cdot \left(\frac{dr_e(\cdot)}{dA} \right) + (1 - \alpha) \frac{V_e}{k} \cdot \left(\frac{dt(\cdot)}{dA} \right) = \frac{\alpha \cdot E[U'(W_A)]}{E[U'(W_A)]} = \alpha > 0$$

And:

$$\left(\frac{dr_e(\cdot)}{dA} \right) + \frac{(1 - \alpha)}{\alpha} \cdot \left(\frac{dt(\cdot)}{dA} \right) = \frac{k}{V_e} > 0 \quad (10)$$

If $\alpha=1$, equation (10) simplifies and we get $dr_e(\cdot)/dA=k/V_e$. This expression is positive: if the acquirer proposes a high acquisition price A he issues a positive signal on the future economic return of the target firm. From that binding limit, equation (10) simplifies because $dt(\cdot)/dA$ should be null in order to be equal to k/V_e , whatever the value of α . As a consequence, the expropriation rate forecasted by outside investors does not depend on the value of the acquisition price A . Economically, considered from the new controlling shareholder's point of view, the acquisition price A pays for the past private benefits and is a signal for the future economic return; it does not depend on his future private expropriation.

If we set the acquisition price as equal to the minimum acceptable value V_s for a transfer of control to occur, we get: $A = V_s = V_e \cdot r_e^S / k$, with r_e^S being the economic profitability of the firm under the former controlling blockholder's management. As a result, the previous shareholder will receive only the market value before the transfer and with a control premium equal to the former private benefits. We stand at the limit condition where $\lambda=1$ in equations (3).

In that eventuality, the new controlling shareholder does not issue any signal about the future economic return. The price just satisfies the transfer condition and nothing more is given because the future economic return of the firm is the same as the economic return considered before the transfer of control: $r_e=r_e^S$. We now integrate the restricted right hand side of (10), $dr_e=(k/V_e).dA$, over the two variables r_e and A and use the previous limit condition $r_e=r_e^S$ (for $A=V_S$). We get a relation where the future economic return of the firm after a transfer of control is a linear form of the acquisition premium $(A-V_e.r_e^S/k)$. This last variable is the acquisition premium.³ It is defined as the difference between the acquisition price and the minimum acquisition price to initiate a transfer, which is equal to the former economic value of the firm (see Figure 1):

$$r_e = \frac{k}{V_e} \cdot (A - \frac{V_e \cdot r_e^S}{k}) + r_e^S \quad (11)$$

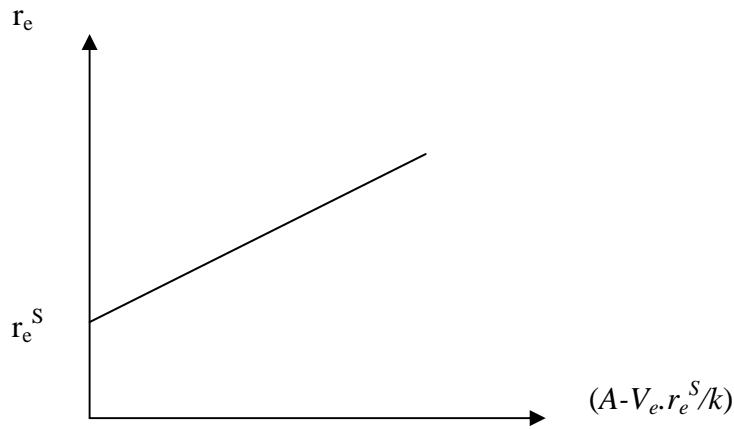


Figure 1: Relationship between the expected economic return and the acquisition premium

³ Using (4) we get the acquisition premium: $A - V_S = \left[(\lambda - 1)(V_S - B_S) + \frac{\lambda - \alpha_0}{\alpha_0} B_S \right]$

The relationship $r_e(\alpha, A,)$ increases linearly with the acquisition premium. The higher the acquisition price, the higher the expected future economic return as perceived by outside shareholders. Moreover, the acquisition price is a useless piece of information to anticipate the future expropriation rate of the new controlling shareholder. Outside investors know that the acquisition price is a pure signal in the sense that it will only reveal future economic profitability after a transfer of control.

The acquirer's optimization with regard to the ownership stake α in the target firm gives:

$$\frac{dE[U(\tilde{W}_A)]}{d\alpha} = E\left\{U'(\tilde{W}_A) \cdot \left[\frac{V_e}{k} \cdot (r_e(\cdot) + \tilde{x} - t_A(\cdot)) - A + \alpha \cdot \frac{V_e}{k} \left(\frac{dr_e(\cdot)}{d\alpha} - \frac{dt_A(\cdot)}{d\alpha} \right) + \frac{V_e}{k} \frac{dt_A(\cdot)}{d\alpha} \right] \right\} = 0 \quad (12)$$

In the expected value, we identify a product because W_A and x are random variables. It leads to a covariance term⁴:

$$\alpha \cdot \frac{V_e}{k} \left(\frac{dr_e(\cdot)}{d\alpha} \right) + (1 - \alpha) \frac{V_e}{k} \left(\frac{dt_A(\cdot)}{d\alpha} \right) = - \frac{E[U''(\tilde{W}_A)] \text{cov}\left(\tilde{W}_A, \frac{V_e}{k} \cdot (\tilde{x})\right)}{E[U'(\tilde{W}_A)]} + \left[\frac{V_e}{k} (r_e(\cdot) - t_A(\cdot)) - A \right]$$

Introducing (8) and manipulating the covariance term gives:

$$\text{cov}\left[\tilde{W}_A, \frac{V_e}{k} \cdot (\tilde{x})\right] = \text{cov}\left[\frac{\alpha \cdot V_e}{k} \cdot (\tilde{x}), \frac{V_e}{k} \cdot (\tilde{x})\right] = \alpha \left(\frac{V_e}{k}\right)^2 \cdot \sigma_x^2$$

Using μ as a (positive) equivalent risk aversion coefficient, we state that $E[U''(\cdot)]/E[U'(\cdot)] = -1/\mu$.

$$\alpha \cdot \left(\frac{dr_e(\cdot)}{d\alpha} \right) + (1 - \alpha) \left(\frac{dt_A(\cdot)}{d\alpha} \right) = \frac{1}{\mu} \cdot \alpha \left(\frac{V_e}{k} \right) \cdot \sigma_x^2 - \left[(r_e(\cdot) - t_A(\cdot)) - \frac{k}{V_e} A \right] \quad (13)$$

⁴ Remembering that for normal variables: $\text{COV}(f(x), y) = f'(x) \cdot \text{COV}(x, y)$

In order to solve the differential equation (13) we posit an additional hypothesis on the relationship between t and α . We assume a negative linear relationship. The idea is that the controlling shareholder is less incited to expropriate as he gains more capital ownership (see Lemma 1, Burkart et al., 1998, 2000). At the limit, if he owns 100% of the equity he is indifferent to a choice between private and public benefits⁵. We set:

$$t_A = \gamma_0 - \gamma_1 \cdot \alpha \tag{14}$$

The appropriation relationship (14) is only defined for values of ownership α higher than the threshold level α_{min} to take over control of the firm (and for values lower than 100%). For $\alpha=100\%$, the appropriation rate is zero, so: $\gamma_0 = \gamma_1 = \gamma > 0$ (see Figure 2).

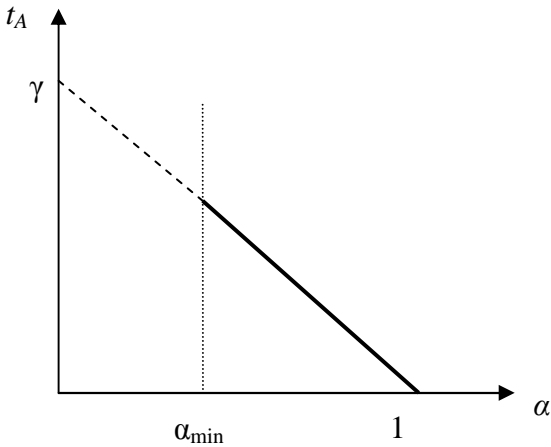


Figure 2: Relationship between appropriation rate and ownership stake of capital of the controlling blockholder

⁵ Assuming no difference in tax treatment.

Applying (14) to (13) gives:

$$\frac{dr_e(\cdot)}{d\alpha} = \frac{2(1-\alpha)}{\alpha} \cdot \gamma - \frac{\left(r_e(\cdot) - \frac{A.k}{V_e}\right)}{\alpha} + \frac{1}{\mu} \cdot \left(\frac{V_e}{k}\right) \cdot \sigma_x^2$$

Equivalent to:

$$\frac{dr_e(\cdot)}{d\alpha} + \frac{r_e(\cdot)}{\alpha} = \frac{\left(2\gamma + \frac{A.k}{V_e}\right)}{\alpha} - 2\gamma + \frac{1}{\mu} \cdot \left(\frac{V_e}{k}\right) \cdot \sigma_x^2 \quad (15)$$

The right hand side of (15) is positive (with α set between 0 and 1). Therefore, the left hand side is also positive. Because of $r_e(\cdot)/\alpha$ being positive, we get an unknown sign of the derivative between an increase in the ownership stake of the controlling blockholder and the evolution of the future economic return of the target firm. The solution of the differential equation (15) is (see Annex):

$$r_e(\alpha) = \left(2\gamma + \frac{A.k}{V_e}\right) + \left[\frac{1}{\mu} \cdot \left(\frac{V_e}{k}\right) \cdot \sigma_x^2 - 2\gamma\right] \cdot \frac{\alpha}{2} + \frac{c}{\alpha} \quad (16)$$

Equation (16) defines a family of curves according the values of the integration constant c (see Figure 3). Recalling that the derivative should be positive in order to deliver a sound signal of increasing profitability with the stake in capital belonging to the controlling

shareholder, the sufficient following condition needs to be satisfied: $\left[\frac{1}{\mu} \cdot \left(\frac{V_e}{k}\right) \cdot (\sigma_x^2) - 2\gamma\right] > 0$

(with c negative). The last part of the equation is not binding, since c can take any value. The first part of the right hand side of equation (16) imposes an upside limit on the appropriation ratio.

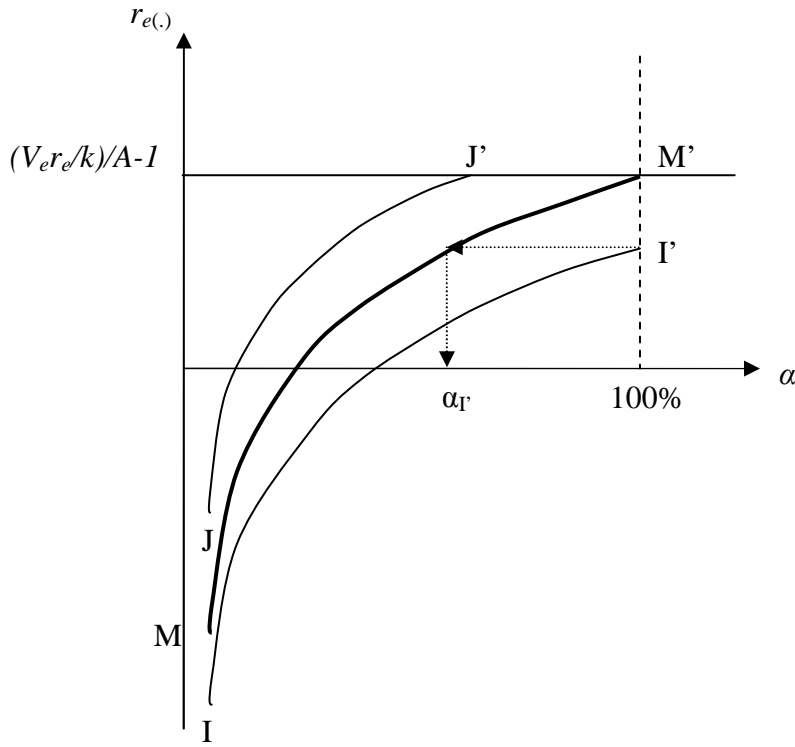


Figure 3: Situations of the new controlling shareholder

For the limit value $\alpha = 1$, equation (16) gives:

$$r_e(1, A) = \left(2\gamma + \frac{A.k}{V_e} \right) + \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) \sigma_x^2 - 2\gamma \right] \frac{1}{2} + c. \text{ This case corresponds to a total ownership of}$$

the firm by the acquirer. It determines the locus of point in Figure 3 defined by the intersection of the family of curves (16) and the vertical line from $\alpha=100\%$. Among all the equilibrium curves, the curve MM' cuts across that vertical line at point M' . Moreover, we know that if the ownership stake is 100%, the controlling shareholder will not expropriate. The value of the firm under his total control and ownership is $V_e r_e^*/k$ for the new shareholder. His required future return can be calculated using the acquisition price as the investment cost. His return is obtained from the ratio of the firm's values after takeover divided by the acquisition price minus 1:

$$\frac{V_e r_e^*/k}{A} - 1. \text{ Substituting, in } r_e(1, A), \text{ we define the point } M' \text{ and the curve } MM' \text{ by setting the}$$

constant value equal to: $c = \frac{V_e r_e^* / k}{A} - 1 - \gamma - \frac{A.k}{V_e} - \frac{1}{2} \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) (\sigma_x^2) \right]^6$. As a result, the acquirer's

announced future return is effectively: $r_e(1, A) = \frac{V_e r_e^* / k}{A} - 1$, and the return on his investment is

equal to the return of the economic project. The MM' curve is the best equilibrium locus for the controlling shareholder. If, for instance, a 100% shareholder wants to sell a marginal fraction of capital, he is better off there because, by selling at an announced economic profitability $M' - \varepsilon$, he will sell at a higher price than if he stays on a curve below MM'.

According to different values of c , we can set, for instance, $c < \frac{V_e r_e^* / k}{A} - 1 - \gamma - \frac{A.k}{V_e} - \frac{1}{2} \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) (\sigma_x^2) \right]$. We then define a point I', which gives

$r_e(1,.) < M'$. The acquirer may launch a total acquisition with a disclosed $r_e(.) < M'$; he will receive a return on his investment lower than M' . If he wants to disclose a future economic return r_e lower than M' , he is better off staying on the equilibrium curve MM', holding an equity stake α_I and letting a stake of capital $(1 - \alpha_I)$ go to outside shareholders. Even if the controlling blockholder issues a weaker future economic profitability by setting $r_e(.) < M'$, the private benefits he appropriates will account for the difference.

If the new controlling shareholder privileges an equilibrium curve above MM', he may not announce economic returns higher than M' . If he stays at point J', he announces $r_e(.) = M'$, but he wants to hold only a stake α of the capital. He will not find outside investors in the market to buy the complementary $(1 - \alpha)$ percentage, because minority shareholders will

⁶ The sufficient negative condition on c is therefore equivalent to $\frac{r_e^*}{k} < \frac{A}{V_e} (1 + \gamma) + \left(\frac{A}{V_e} \right)^2 k + \frac{1}{2} \frac{A}{V_e} \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) (\sigma_x^2) \right]$.

The right hand side of this inequality is positive. It compares the takeover premium A/V_e and the ratio of the expected profitability of the new management divided by the market risk adjusted valuation rate: This condition is easily satisfied except for extremely large values of r_e^* . Therefore, the relationship between the announced profitability and the ownership stake may turn negative.

integrate future expropriation into their valuation. The new controlling shareholder is therefore better off setting $r_e=M'$ because then he will not suffer from a discount in the market price of his shares resulting from his 100% ownership.

As a consequence, the MM' curve is the only feasible equilibrium locus set for the new controlling shareholder. Its equation is:

$$r_e = \left(2\gamma + \frac{Ak}{V_e} \right) + \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) \sigma_x^2 - 2\gamma \right] \frac{\alpha}{2} + \frac{1}{\alpha} \left[\frac{V_e r_e^* / k}{A} - 1 - \gamma - \frac{Ak}{V_e} - \frac{1}{2} \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) (\sigma_x^2) \right] \right] \quad (17)$$

Through the implicit profitability as forecasted by the market, the new blockholder's stake influences the market value of the firm and his global wealth. The new controller has a large set of choice in a non-EOR context. He can balance, for instance, a low market valuation due to a small controlling block α with a higher private appropriation through relation (14). Particularly, the targeted stake of equity can be different from α_0 .

If we move to the simplified context of Leland and Pyle (1977), where private appropriation does not exist (i.e. $t_A=0$ or equivalently $\gamma=0$). The ownership stake of the dominant shareholder is the only signal to outside investors. So, relationship (15) simplifies to:

$$\frac{dr_e}{d\alpha} + \frac{r_e(\cdot)}{\alpha} = \frac{\left(\frac{Ak}{V_e} \right)}{\alpha} + \frac{1}{\mu} \left(\frac{V_e}{k} \right) \sigma_x^2 \quad (18)$$

Using (17) and setting γ to zero, the solution curve is:

$$r_e = \left(\frac{Ak}{V_e} \right) + \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) \sigma_x^2 \right] \frac{\alpha}{2} + \frac{1}{\alpha} \left[\frac{V_e r_e^* / k}{A} - 1 - \frac{Ak}{V_e} - \frac{1}{2} \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) (\sigma_x^2) \right] \right] \quad (19)$$

For a total ownership $\alpha=1$, we are in the same situation as previously discussed. The equilibrium curves between the situation with and without private benefits will share the same equilibrium point M' which gives full 100% ownership. The equilibrium curve ZM' without appropriation (as in a standard dispersed ownership system) defined by equation (19) is located above the equilibrium curve MM' in a situation of expropriating controlling shareholders (see Figure 4).

The equilibrium curve with appropriation as defined by equation (17) is logically below that without private benefits. The gap with the curve defined by (19) is explained by the appropriation rate γ , which enters negatively into the formula (17) in the linear slope term and in the inverse term. Using (17), we see that $dr_e/d\gamma$ is strictly negative. This means that, for a given value of α (inferior to 1), the announced future economic profitability r_e is lower in a context of private appropriation. Outside shareholders discount the signal by a forecast of private benefits. The quality of the signal increasingly deteriorates as controlling ownership falls. A similar way to express it is to say that, in order to issue a future economic profitability, r_e^* , being identical to the one in a no private benefits environment, the controlling shareholder located on MM' should hold a higher stake in capital in order to compensate for the suspicion of private benefit and to give more strength to the signal coming from α (see Figure 4).

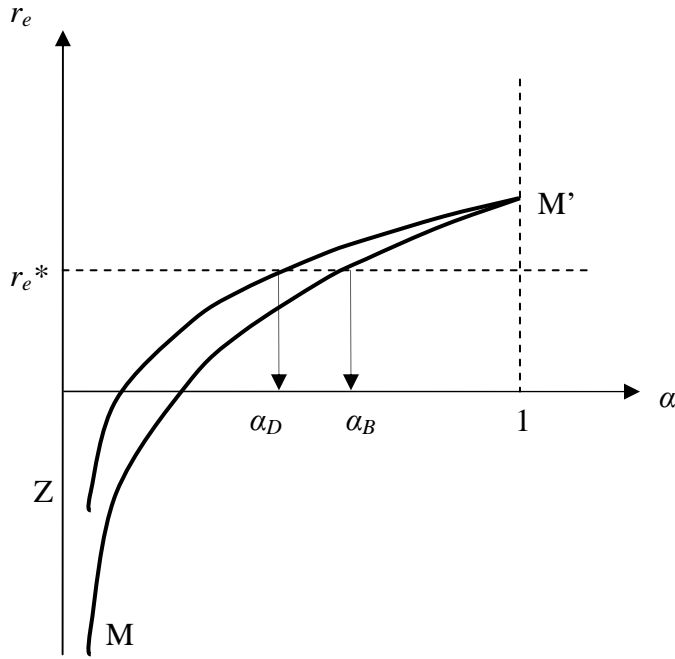


Figure 4: Comparison of the equilibrium curves for acquiring shareholder in dispersed ownership and concentrated ownership

C - With EOR

We refer now to a situation with an equal opportunity rule. The wealth function of the acquirer should include a new variable α_g , which is the part of the capital bought by the acquirer as a result of the mandatory bid rule.

$$\tilde{W}_A = (\alpha + \tilde{\alpha}_g) \frac{V_e(r_e(\cdot) + \tilde{x} - t_A(\cdot))}{k} + \frac{V_e t_A}{k} - (\alpha + \tilde{\alpha}_g)A \quad (20)$$

The new controlling shareholder will optimize the number of shares bought by outside investors through the mandatory bid procedure or the price guarantee mechanism set into force during the takeover. However, he has to take into account the choices of outside investors, who

want to optimize their participation in the firm, comparing this with the exit option granted by the EOR.

1) Outside shareholder behavior with EOR

The wealth of outside investors increases with their opportunity to sell out shares at the acquisition price A .

$$\tilde{W}_o = (1 - \alpha - \alpha_g) \frac{V_e(r_e(\cdot) + \tilde{x} - t_A(\cdot))}{k} + \alpha_g A \quad (21)$$

Minor investors can react and optimize from their point of view the value of α_g . We first need to solve the outside shareholders' problem because the controlling shareholder knows that the proportion of capital he should buy following the EOR depends on the future economic return and on the appropriation rate, both of which are anticipated by minority shareholders through the publicly observed values α and A . Setting to zero the derivative of the expected utility with regard to α_g yields:

$$\frac{dE[U(\tilde{W}_o)]}{d\alpha_g} = E \left\{ U'(\tilde{W}_o) \left[-\frac{V_e}{k} (r_e(\cdot) + \tilde{x} - t_A(\cdot)) + A - \alpha_g \frac{V_e}{k} \left(\frac{dr_e(\cdot)}{d\alpha_g} - \frac{dt_A(\cdot)}{d\alpha_g} \right) \right] \right\} = 0 \quad (22)$$

Manipulating:

$$\begin{aligned} \left[\alpha_g \frac{V_e}{k} \left(\frac{dr_e(\cdot)}{d\alpha_g} - \frac{dt_A(\cdot)}{d\alpha_g} \right) \right] &= \frac{E \left\{ U'(\tilde{W}_o) \left[-\frac{V_e}{k} (r_e(\cdot) + \tilde{x} - t_A(\cdot)) + A \right] \right\}}{E[U'(\tilde{W}_o)]} \\ &= \frac{E[U''(W_o)] \text{cov} \left(W_o, -\frac{V_e}{k} \tilde{x} \right)}{E[U'(W_o)]} + \left[-\frac{V_e}{k} (r_e(\cdot) - t_A(\cdot)) + A \right] \end{aligned}$$

Referring to $E(U'')/E(U') = -1/\mu$, this is equivalent to:

$$\left[\alpha_g \left(\frac{dr_e(\cdot)}{d\alpha_g} - \frac{dt_A(\cdot)}{d\alpha_g} \right) \right] = \frac{1}{\mu} (1 - \alpha - \alpha_g) \left(\left(\frac{V_e}{k} \right) \sigma_x^2 \right) - \left[(r_e(\cdot) - t_A(\cdot)) - A \frac{k}{V_e} \right]$$

We look at $(r_e - t_A)$, which is the net profitability announced by the controlling shareholder and used by the outside investors to value their wealth in the market. We get:

$$\frac{d(r_e(\cdot) - t_A(\cdot))}{d\alpha_g} + \frac{(r_e(\cdot) - t_A(\cdot))}{\alpha_g} = \frac{1}{\alpha_g} \left[\frac{1}{\mu} (1 - \alpha) \left(\frac{V_e}{k} \right) \sigma_x^2 + \left(\frac{Ak}{V_e} \right) \right] - \frac{1}{\mu} \left(\frac{V_e}{k} \right) \sigma_x^2 \quad (23)$$

Integrating in a similar way as above gives:

$$(r_e(\cdot) - t_A(\cdot)) = \left[\frac{1}{\mu} (1 - \alpha) \left(\frac{V_e}{k} \right) \sigma_x^2 + \left(\frac{Ak}{V_e} \right) \right] - \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) \sigma_x^2 \right] \frac{\alpha_g}{2} + \frac{c}{\alpha_g} \quad (24)$$

The two limit conditions to be satisfied by (24) are $\alpha_g = (1 - \alpha)$, meaning the outside investor can only sell the available shares not initially bought by the new controlling blockholder, and $\alpha_g = 0$ for very high values of $(r_e - t_A)$. Equation (24) defines a family of decreasing curves as long as the integration constant is positive (see Figure 5). These curves have a simple economic meaning: if the prospect of net future economic profitability of the firm is high, the stocks are a good investment and outside shareholders will only use the mandatory bid exit for a small part of their investment. At the limit for extremely good prospects of profitability, outside shareholders will keep all their shares. That gives an asymptotic upward oriented shape to the locus of their choices.

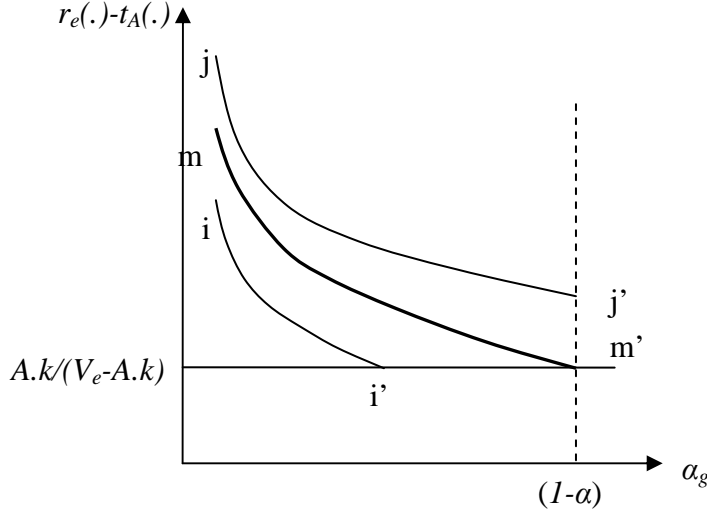


Figure 5: Equilibrium curves of outside shareholders

A specific choice of the announced net public profitability ($r_e - t_A$) will result in a return of the investment strictly equal to $(r_e - t_A)$. Therefore $r_e(.) - t_A(.) = \frac{V_e \cdot r_e(.) - t_A(.)}{k} - 1$. This defines a horizontal line $r_e(.) - t_A(.) = A.k/(V_e - A.k)$ in Figure 5. The outside shareholder will not consider any net return below that line. The reverse would mean that the outside investors accept that they will hold shares in the firm with a lower value rather than take the opportunity to sell and exit the firm. There exists an intercept point m' of that minimum horizontal line with one of the curves defined by equation (24) located at the specific value $\alpha_g = (1-\alpha)$. That point will identify the optimal curve mm' for the choices of minor investors. That curve is their only set of rational choices. It allows us to determine the value of the integration constant

$$c = (1-\alpha) \left[\frac{A.k}{V_e - A.k} - \frac{A.k}{V_e} \right] - \frac{1}{\mu} (1-\alpha)^2 \frac{V_e}{k} \frac{\sigma_x^2}{2}.^7$$

For instance, the curve jj' is not acceptable: staying at point j' would mean that outside investors would bring all their shares ($\alpha_g = (1-\alpha)$) at

⁷ This is positive for acquisition A prices such that: $\left[\frac{A.k}{V_e - A.k} - \frac{A.k}{V_e} \right] > \frac{1}{\mu} (1-\alpha)^2 \frac{V_e}{k} \frac{\sigma_x^2}{2}$

the exit opportunity, selling them at an acquisition price that is below the market valuation based on the future prospect of profitability they forecast. The equilibrium locus ii' is cut across on its downward curve because of the floor value imposed by the acquisition price. It is also dominated by the set of choices resulting from the jj' curve above it. The equation of the mm' curve is:

$$(r_e(.) - t_A(.)) = \left[\frac{1}{\mu} (1 - \alpha) \left(\frac{V_e}{k} \right) \sigma_x^2 + \left(\frac{A.k}{V_e} \right) \right] - \left[\frac{1}{\mu} \left(\frac{V_e}{k} \right) \sigma_x^2 \right] \frac{\alpha_g}{2} \\ + \frac{(1 - \alpha)}{\alpha_g} \left[\frac{A.k}{V_e - A.k} - \frac{A.k}{V_e} \right] - \frac{1}{\mu} (1 - \alpha)^2 \frac{V_e}{k} \frac{\sigma_x^2}{2} \quad (25)$$

As an input to equation (25), outside shareholders need to identify the controlling shareholder's targeted stake α .

2) Situation of the new controlling shareholder

Knowing the set of possible rational choices of the outside shareholder, the controlling blockholder will try to optimize his situation. He chooses with regard to the acquisition price, A , and the stake of capital he aims to buy on his own, α . Setting the derivative of his wealth with regard to A to zero:

$$\frac{dE[U(\tilde{W}_A)]}{dA} = E \left\{ U'(\tilde{W}_A) \left[\frac{(\alpha + \alpha_g)V_e}{k} \left(\frac{dr_e(.)}{dA} - \frac{dt(.)}{dA} \right) + \frac{V_e}{k} \frac{dt(.)}{dA} - (\alpha + \alpha_g) \right] \right\} = 0 \quad (26)$$

We get a differential equation close to relationship (10) but introducing explicitly the expected shares to be bought back through the EOR in the acquirer's calculus:

$$\left(\frac{dr_e(.)}{dA} \right) + \frac{(1 - \alpha - \alpha_g)}{(\alpha + \alpha_g)} \left(\frac{dt(.)}{dA} \right) = \frac{k}{V_e} > 0 \quad (27)$$

The analysis we developed earlier regarding the equilibrium locus of choices ensuing from relationship (10) also applies here. The derivative $dr_e./dA = k/V_e$ is still positive. The

existence of an equal opportunity mechanism does not alter the positive relationship between A and the anticipated economic profitability of the target firm after the takeover. The acquisition premium is a simple and direct signal of the future economic return of the firm. The linear relation (11) is still valid. The derivative $dt(\cdot)/dA$ remains equal to zero, so the acquisition price does not signal anything about the future appropriation rate.

Turning now to the optimization of the new controlling shareholder's wealth with regard to α , we get:

$$\frac{dE[U(W_A)]}{d\alpha} = E \left\{ U'(W_A) \left[\frac{V_e}{k} \cdot (r_e(\cdot) + \tilde{x} - t_A(\cdot)) - A + (\alpha + \alpha_g) \frac{V_e}{k} \left(\frac{dr_e(\cdot)}{d\alpha} - \frac{dt_A(\cdot)}{d\alpha} \right) + \frac{V_e}{k} \frac{dt_A(\cdot)}{d\alpha} \right] \right\} = 0$$

After some manipulation:

$$\left(\frac{dr_e(\cdot)}{d\alpha} \right) + \frac{(1 - \alpha - \alpha_g)}{(\alpha + \alpha_g)} \left(\frac{dt(\cdot)}{d\alpha} \right) = \frac{1}{\mu} \left(\frac{V_e}{k} \right) \cdot \sigma_x^2 - \frac{1}{\alpha} \left[(r_e(\cdot) - t_A(\cdot)) - \frac{k}{V_e} A \right] \quad (28)$$

In comparison with a situation without EOR, the coefficient of the $dt(\cdot)/d\alpha$ term is $(1 - \alpha - \alpha_g)/(\alpha + \alpha_g)$; this is lower than the one in equation (13), which was $(1 - \alpha)/\alpha$. For a given increase in r_e , the new controlling shareholder is driven to expropriate less. *Ceteris paribus*, for given increases of r_e and α at equilibrium, $dt(\cdot)/d\alpha$ is negative. However, in order to compensate, the absolute value of the derivative $dt(\cdot)/d\alpha$ will be higher with EOR than without it. At equilibrium, the appropriation rate t_A will decrease more for an increase in r_e . Consequently, the EOR system is more disciplinary with regard to expropriation.

In order to solve the differential equation (28), we need to add the hypothesis that $dt(.)/d\alpha$ is a negative constant, which implies a linear decreasing function similar to (14) between t_A and α . We get the differential equation versus α (remembering that α_g is a fixed parameter):

$$\frac{dr_e(.)}{d(\alpha + \alpha_g)} + \frac{r_e(.)}{(\alpha + \alpha_g)} = \frac{\left(2\gamma + \gamma\alpha_g + \frac{Ak}{V_e}\right)}{(\alpha + \alpha_g)} - 2\gamma + \frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2 \quad (29)$$

The solution of (29) is a family of increasing curves:

$$r_e(\alpha) = \left(2\gamma + \gamma\alpha_g + \frac{Ak}{V_e}\right) + \left[\frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2 - 2\gamma\right] \frac{(\alpha + \alpha_g)}{2} + \frac{c}{(\alpha + \alpha_g)} \quad (30)$$

As previously, we identify a point M' for total ownership of capital: $(\alpha + \alpha_g) = 1$. This point is the same as the one in the situation without EOR. The only acceptable equilibrium curve is $M_{EOR}M'$, shown in Figure 6, and is defined by the equation:

$$r_e = \left(2\gamma + \gamma\alpha_g + \frac{Ak}{V_e}\right) + \left[\frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2 - 2\gamma\right] \frac{(\alpha + \alpha_g)}{2} + \frac{1}{(\alpha + \alpha_g)} \left[\frac{V_e r_e^* / k}{A} - 1 - \gamma - \gamma\alpha_g - \frac{Ak}{V_e} - \frac{1}{2} \left[\frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2\right] \right] \quad (31)$$

From the optimization of minority shareholders, α_g is either positive or zero. A direct comparison between (31) and (17) shows that the $M_{EOR}M'$ curve in the “with EOR” case is below the MM' curve without EOR. The difference of implicit economic return $r_e(.)$ for a given set of parameters is $\Delta r_e(.) = -\gamma\alpha_g(1 - \alpha - \alpha_g)/(\alpha + \alpha_g)$, which is negative. The system with EOR leads to lower quality information than in a situation without price guarantee protection. A given signal α is more trustworthy and is better perceived by outside shareholders in a no equal opportunity system. For a given value of the signal, The EOR rule weakens the informative situation of outside shareholders in a blockholding system. In order to balance this weakness,

the controlling shareholder should issue a stronger or a different signal. His behavior is modified: if he wants to signal a given level of future economic profitability, he needs to buy or to bid for a more important ownership stake within EOR than he does without EOR. We should obviously add that the signaled $r_e(.)$ needs to be higher than the risk adjusted cost of capital, k , otherwise the new controlling shareholder would not be encouraged to launch the takeover (see Figure 6).

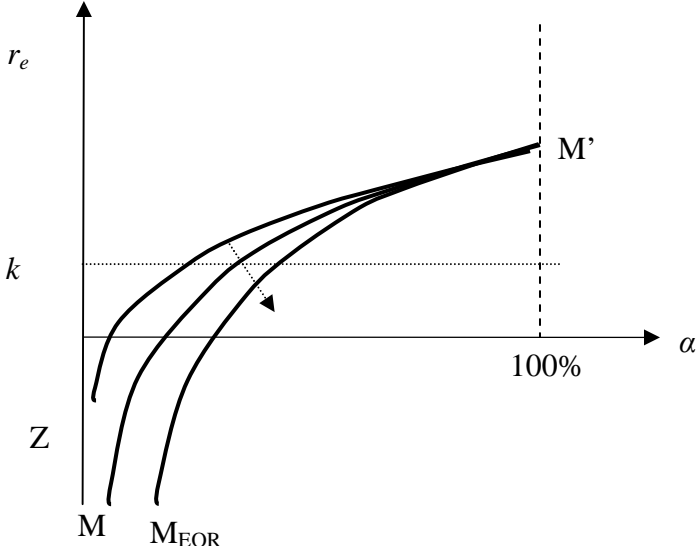


Figure 6: New controlling shareholder equilibrium choices with and without EOR system

The EOR system changes the location of the optimal choice curve of the controlling shareholder by moving it away from the situation ZM' , corresponding to the absence of private benefits. The situation is more complex insofar as private benefits play a more important role in setting the equilibrium curve with EOR. The derivative or $r_e(.)$ with regard to γ is more sensitive in such a situation. For a given value of new economic profitability, the controlling shareholder would own a larger stake of capital, which would lead him to expropriate less. In that sense, the EOR system leads to enhanced disciplinary pressure. At the limit, for low values of γ , the choices converge more quickly toward the no private benefits case.

3) Joint equilibrium setting

The final equilibrium between the two acting parties depends on the choice of α_g set by outside shareholders. For these outside shareholders, the equilibrium is a function of the net economic profitability of the firm after subtraction of private benefits. The final setting by the controlling shareholder will integrate the number of shares he will buy following the EOR procedure.

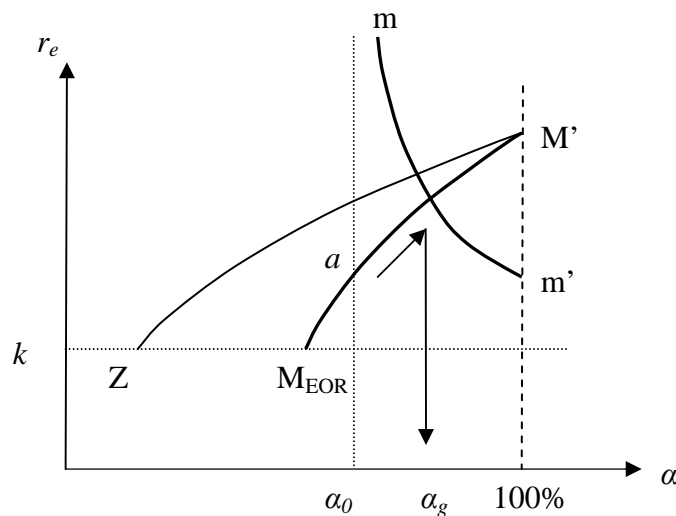


Figure 7: Equilibrium choice of controlling shareholder and outside investors

The acquirer who buys a control block α_0 may think that it is enough for him to locate on his equilibrium curve $M_{EOR}M'$ at point a (see figure 7) and to issue the signal α , which corresponds to a profitability r_e^* optimal for him. Receiving that information on the block size, the optimal ownership of minor investors is located on the curve mm' at the same vertical level as that of point a . The implied r_e^* is so weak that all outside investors will stand at point m' , exercise their exit options, and bring their shares to the new controlling shareholder. The latter will get $(\alpha + \alpha_g) = 100\%$ and will be pushed away from his equilibrium curve $M_{EOR}M'$. The only common equilibrium contract is the point defined by the intersection of the two curves $M_{EOR}M'$ and mm' . Figure 7 mixes the equilibrium sets of the controlling shareholder (Figure 6) and of

the minor investors (Figure 5). The intersection point means an announcement of higher future economic profitability, the selling of a fraction α_g of the capital through the EOR mechanisms and/or, on the minor investors' side, a lesser fear of appropriation.

The above analysis assumes an endogenous implicit value t_A because the equilibrium curve mm' set for the outside shareholders refers in fact to the net economic return ($r_e - t_A$). A complementary logic for convergence is to condition the delivered information to attract outside investors toward the desired global participation stake α^* , corresponding to an economic future profitability $r_e(\alpha^*)$ lower than $r_e(\alpha + \alpha_g)$. A bias of exaggeration or undervaluation of r_e cannot be envisaged in this framework because the controlling shareholder delivers a signal through objective variables on the economic profitability of the firm and he does not manipulate information (see equation (5)). The private information on the true value of r_e is exogenous and conditions the process. It is disseminated in the setting of the quantity equilibrium. In order to modify the shape and the location of the curve mm' , other possibilities exist. The first way could be to modify the acquisition price A . This may be an eventuality in a tender offer where the problem of the acquirer is at the same time to gain control and to forecast the consequences of the mandatory bid rule. In that case, α_g is the percentage of shares bought above the controlling shareholder's initially targeted control stake. This eventuality is not opened up after a block trade. The price A results from a negotiation between the previous and the new controlling blockholders. The second way is to publicly announce that the desired stake of capital is $(\alpha + \alpha_g)$. This integrates the demand function for the stocks of the outside investors. The third way is to reduce t_A , which is under the new shareholder's responsibility. This is signaled to outside investors through an increase in α . The controlling shareholder may modify the slope parameter of the expropriation rate γ and then, for a given value α_0 of a control block bought in a deal, set the implicit t_A^* . Reducing the appropriation rate shifts downward the

equilibrium locus mm' of outside shareholders and consequently moves the equilibrium point with $M_{EOR}M'$ to the left, which results in a lower value α_g for the shares brought through the mandatory bid procedure (see Figure 8).

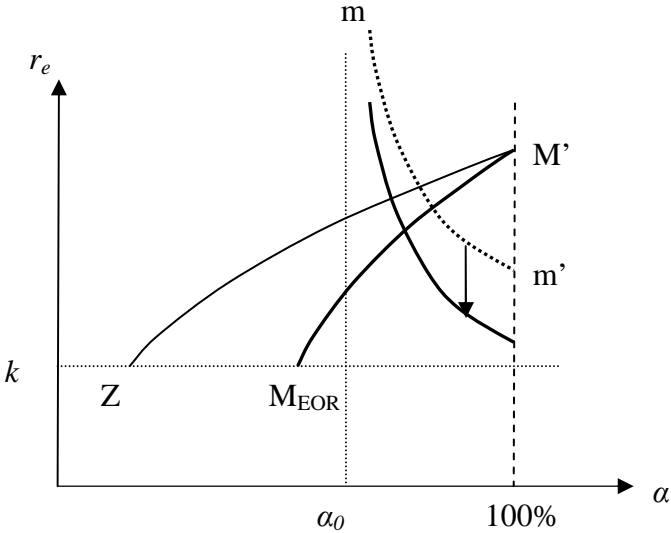


Figure 8: Equilibrium choices of controlling shareholder and outside investors with lower appropriation rate

The joint equilibrium framework recognizes that the two parties interact. That equilibrium mixes directly quantity and information. The price is constrained by legal rules and the takeover context. So, only the quantities can adjust, and the total stake of capital after EOR of the controlling shareholder must be equal to the ownership percentage wished by minor investors. This Cournot equilibrium finds its own regulation with the future economic profitability that is announced by the new controller and what he intends to achieve. In a rational equilibrium schedule, this equilibrium defines an implicit contract between the new blockholder and the outside shareholders. They exchange quantities and information on future profits. Rational behavior pushes the acquirer to disclose the true economic value and to share this information with other investors. In an EOR system, the percentage of capital is an economically valuable signal and minor investors are not passive. In a concentrated blockholder

ownership system, it also signals the existence of private benefits. Those are the adjustment variables in the hands of the controlling shareholder.

As a consequence, the EOR modifies the behaviors in a situation of appropriation of private benefits. For a targeted ownership stake in capital, it puts pressure on the appropriation rate. The economic calculus of the rational new blockholder is more complex. He is pushed to issue a signal through this ownership percentage on the true future economic profitability of the target firm r_e . The quality of information is better and the discipline stronger. The regulating mechanism is simple: in order to avoid the risk of owning a final stake of capital higher than the one aimed at, the controlling shareholder will lessen his private appropriation of benefits.

The mandatory bid rule underlines the nature of private benefits, which are uncertain and contingent on the new controlling shareholder. They cannot be considered as given and exogenous in a transfer of control, but rather as a regulation variable in the controlling-outside investors' agency problem. It is for this reason that we need to analyze separately the past private benefits paid back to the former controlling shareholder and the future private benefits. The normative solution to issue preferred shares to compensate controlling shareholders in a concentrated ownership system, as proposed by Hofstetter (2006), is not adapted in the sense that it sets once and forever the amount of accepted private benefits. The mandatory bid rule appears as a common and mandatory opportunity to "negotiate" a sort of implicit contract between the new controlling shareholders and the outside investors and to question the accepted ex ante level of private benefits. It helps to make explicit what is implicit.

The quality of information may also improve in the sense that the signal given to outside investors should be stronger. We highlight the fact that equal opportunity rules lower the

prospects of private benefits. We also outline that in a controlling ownership system, the equal opportunity rule modifies the information delivered to outside investors: the same signal lead to lower forecasts of profitability. To balance this effect, we analyzed the possibility for the buyer to modify his choices. However, the acquirer may also issue directly private information about his willingness to expropriate less or disclose the true perspectives of future returns. In financial markets, rules may also force the buyer to disclose information. In addition, analysts may provide the market with information about the transactions. Outside investors are informed by other sources than the information delivered through the two channels used by the acquirer and identified in the model. The buyers may enter in a signaling game because otherwise they will not be at optimum and not maximize of their profit.

Conclusion

The existence of the equal opportunity rule appears as far more important in a context of concentrated stock ownership and private benefits of control than in a framework of dispersed ownership. The buyer can use the bid acquisition price and the target participation rate as signals. We noticed that a more general setting of the choices should also take into account the possibility of a direct appropriation of the cash flow by the new controlling investor, which is a source of risk for minor investors.

This paper develops an analysis with a double asymmetry of information within a quasi-process of negotiation. The equal opportunity rule is not an explicit contract, but it leads to an interaction between the new controlling shareholder and outside investors that characterizes an implicit contract. The terms have to be jointly settled regarding common variables: the number of shares brought to the exit opportunity option, and the appropriation rate of private benefits. The joint equilibrium framework recognizes that the two parties interact. A partial use of the

exit option given to outside investors finds a rational explanation in our model. The characteristics of the joint equilibrium mixes directly quantity and information. The price is constrained by legal rules and is set in the takeover context. Therefore, only the quantities can adjust and the total stake of capital after EOR of the controlling shareholder must be equal to the ownership percentage wished for by minor investors. They exchange quantities and information on future profits. In an EOR system, the percentage of capital is an economically valuable signal and minor investors are not passive. In a concentrated blockholder ownership system, it also signals the existence of private benefits. Those are the adjustment variables in the hands of the controlling shareholder. As a consequence, the EOR modifies the behaviors in a situation of appropriation of private benefits. For a targeted ownership stake in capital, this puts pressure on the appropriation rate. The regulating mechanism is simple: in order to avoid the risk of owning a final stake of capital higher than the one aimed at, the controlling shareholder will lessen his private appropriation of benefits. The economic calculus of the rational new blockholder is more complex. Outside investors interact and participate in the setting of the equilibrium. They may profit from a better discipline on the forecasted private appropriation. However, the signals at the disposal of the controlling investor are less effective and the quality of the information delivered to the market is lower in comparison with a situation without an equal opportunity rule.

The analysis of the EOR system also highlights that regulation has consequences in the delivery of better information. When one party obtains new information, its risk against uncertainty lowers. A collateral effect of law, regulation or contracts occurs on the reduction or the management of situations of asymmetry of information. By making things more complex, regulation may lower the quality of a given signal and make the transactions or the negotiation processes more difficult. The examples of the equal opportunity rule or mandatory bid

mechanisms in the case of a takeover illustrate this feature. It puts a pressure to buyer's choices to balance it.

Regulation thus initiates a constrained contractual process, which in itself may create economic value. The progress toward an agreement is not a "take it or leave it" situation. Both parties will, to some extent, exchange possibly biased information. In uncertainty about future returns, more of the price is based on information as compared with an immediate transfer of goods against payment. Information asymmetry is part of the deal and a disclosure of private information has the same effect as a price discount. The contractual process in itself is important, so allowing contractual freedom has value. Moreover, beside explicit contracts, numerous implicit contracts do appear, for instance between controlling and outside investors. The existence of a process of negotiation is more frequent than typically seen. The weaker party is not an automatic loser. In an EOR system, the exit option given to outside shareholders is a tool to curb possible future private expropriation. Reciprocal interaction allows the development of an exchange of information in order to find a joint agreement. The contractual process creates value when it leads to better *ex ante* information, at least for one party. The equal opportunity rule as an external regulation is an important example of an implicit bilateral equilibrium contract, which develops in a framework of asymmetric information and imposes an overall disciplinary pressure.

From a social public choice perspective, the welfare property of EOR has to be analyzed. In a world without EOR a buyer is free to create an EOR by announcing he wants to purchase all shares at the bid price. Buyer usually seems to reject this option, which implied that EOR is not privately optimal. Then the EOR makes the buyers worse off. However, we show that the rule makes the investors well off by increasing the information they would otherwise have and by reducing private benefit expropriation. We have not analyzed if the EOR

effect is only distributional and. The question of the welfare efficiency of EOR is still opened and not addressed in the paper.

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Annex

To solve the differential equation (A1), we call $g(\alpha)$ the left hand side:

$$g(\alpha) = \frac{dr_e(\cdot)}{d\alpha} + \frac{r_e(\cdot)}{\alpha} = \frac{\left(2\gamma + \frac{Ak}{V_e}\right)}{\alpha} - 2\gamma + \frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2 \quad (\text{A1})$$

The solution has the form:

$$r_e(\alpha) = K(\alpha)e^{-\ln(\alpha)} = \frac{1}{\alpha} K(\alpha) \quad (\text{A2})$$

Deriving (A2) with regard to α gives:

$$\frac{dr_e(\cdot)}{d\alpha} + \frac{r_e(\cdot)}{\alpha} = \frac{1}{\alpha} K'(\alpha) = g(\alpha)$$

Using (A1):

$$K'(\alpha) = \left(2\gamma + \frac{Ak}{V_e}\right) + \left[\frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2 - 2\gamma\right] \alpha \quad (\text{A3})$$

Integrating (A3):

$$K(\alpha) = \left(2\gamma + \frac{Ak}{V_e}\right) \alpha + \left[\frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2 - 2\gamma\right] \frac{\alpha^2}{2} + c \quad (\text{A4})$$

Where c is an integration constant and using (A2), we finally get:

$$r_e(\alpha) = \left(2\gamma + \frac{Ak}{V_e}\right) + \left[\frac{1}{\mu} \left(\frac{V_e}{k}\right) \sigma_x^2 - 2\gamma\right] \frac{\alpha}{2} + \frac{c}{\alpha} \quad (\text{A5})$$