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► **To cite this version:**

Marion Davin, Karine Gente, Carine Nourry. Social Optimum in an OLG Model with Paternalistic Altruism. 2011. halshs-00644094

**HAL Id: halshs-00644094**

**<https://shs.hal.science/halshs-00644094>**

Preprint submitted on 23 Nov 2011

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**Social Optimum in an OLG Model with  
Paternalistic Altruism**

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**November 2011**

**DT-GREQAM**

# Social Optimum in an OLG Model with Paternalistic Altruism\*

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November 20, 2011

## Abstract

There is no consensus yet on the correct way to write the social utility function in presence of paternalistic altruism. This note shows that the specification of the central planner objective is crucial for optimal capital intensity and optimal growth in a one and a two-sector models. In a one-sector model, optimal growth depends on preferences when paternalistic altruism enters the social utility function; otherwise it does only depend on the capital share as in the standard golden rule. In a two-sector model, optimal growth depends on preferences and relative capital intensities when paternalistic altruism enters the social utility function; otherwise it does only depend on the capital share of the investment good sector. Moreover, both in a one and a two sector model, the optimal growth rate tends to be higher when warm-glow altruism enters the social utility function.

**Keywords:** Optimal balanced growth path, social welfare function, paternalistic altruism

*JEL Classification Number:* D64; D90; O41

## 1 Introduction

How does the inclusion of paternalistic altruistic feelings in the social planner's objective affect the optimal growth rate? According to a strict definition, the social utility is the discounted sum of individual utility functions, namely a utilitarian social function. However, Harsanyi (1995) recommends to exclude all external preferences from the social utility function, what we will call an Harsanyi social function. Thus there is no consensus yet on the correct way to write this social utility function. On one side, considering an overlapping generations one-sector model with consumption separable utility function, Cremer and Pestieau (2006) underline that optimal policy depends on the specification of the social utility function. Nevertheless, they do not clearly examine the implications on the optimal balanced growth path. On the other side, de la Croix and Monfort (2000) do not include the "joy of giving" term in the welfare function. In this paper, we show, in both a one and a two-sector model, that the way to write the central planner objective is crucial for the optimal growth path. In this purpose, we will consider the example of a paternalistic altruism where agents are concerned by the level of human capital of their children. We

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\*We thank T. Seegmuller for useful comments and suggestions.

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also assume that human capital is a simple function of parents investment in their child's education. Our contribution is double. First, excluding paternalistic altruism (Harsanyi social function), we show that the optimal growth rate is determined only by the technological parameters, whereas if the central planner maximizes the discounted sum of individual utilities (utilitarian social function), the optimal growth rate depends also on the preference parameters. Second, in a two-sector model, we obtain that optimal growth in the Harsanyi case only depends on the investment good sector technological parameters. Whereas in the utilitarian case, relative capital intensities do matter.

These results come from the fact that as long as education is ignored in the social welfare function, the only way to increase welfare is to maximize consumption. Conversely, when child's education provides direct welfare to parents, there is an arbitrage in the social utility function between consumption and education. This is the reason why preference parameters enter the optimal growth rate with the utilitarian social function. In the two-sector case, welfare depends only on the consumption good with the Harsanyi social function whereas it depends also on the investment good through education in the utilitarian social function. Thus, in the Harsanyi social function, the optimal capital intensity is given by a standard modified golden rule where matters the physical capital share of the investment good sector only; whereas in the utilitarian social function, the central planner objective is to produce efficiently over the two sectors since both types of goods (investment good through education and consumption good) matter for welfare.

Moreover, the optimal growth rate tends to be higher with a utilitarian social welfare function than Harsanyi social welfare function.

## 2 Social Optimum in a one-sector framework

Consider a perfectly competitive economy in which the final output is produced using physical capital  $K$  and human capital  $H$ . The production function of a representative firm is an homogeneous function of degree one:  $F(K, H)$ . We assume for simplicity a complete depreciation of the capital stock within one period. Denoting, for any  $H \neq 0$ ,  $k = K/H$  the physical to human capital ratio, we define the production function in intensive form as  $f(k)$ .

**Assumption 1**  $f(k)$  is defined over  $\mathbb{R}_+$ ,  $\mathbf{C}^r$  over  $\mathbb{R}_{++}$  for  $r$  large enough, increasing ( $f'(k) > 0$ ) and concave ( $f''(k) < 0$ ). Moreover,  $\lim_{k \rightarrow 0} f'(k) = +\infty$  and  $\lim_{k \rightarrow +\infty} f'(k) = 0$ .

We can also compute the share of capital in total income:

$$s(k) = \frac{kf'(k)}{f(k)} \in (0, 1) \quad (1)$$

As in Michel and Vidal (2000), we consider a three-period overlapping generations model. In their first period of life, individuals are reared by their parents. In the second period they work and receive a wage, consume, save and rear their own children. In their last period of life, they are retired and consume their saving returns. Following Glomm and Ravikumar (1992), we consider a paternalistic altruism according to which parents value the quality of education received by their children. Thus, the preferences of a paternalistic agent born in  $t$  are represented by:

$$U_t = u(c_t, d_{t+1}) + v(h_{t+1}) \quad (2)$$

where  $c_t$  and  $d_{t+1}$  represent adult and old aggregate consumption, and  $h_{t+1}$  child's human capital.

**Assumption 2.**

i)  $u(c, d)$  is  $\mathbf{C}^2$ , increasing with respect to each argument ( $u_c(c, d) > 0$ ,  $u_d(c, d) > 0$ ), concave and homogeneous of degree  $a$  with  $a \in (0, 1)$ . Moreover, for all  $d > 0$ ,  $\lim_{c \rightarrow 0} u_c(c, d) = +\infty$ , and for all  $c > 0$ ,  $\lim_{d \rightarrow 0} u_d(c, d) = +\infty$ .

ii)  $v(h)$  is  $\mathbf{C}^2$ , increasing ( $v'(h) > 0$ ), convex ( $v''(h) > 0$ ) and satisfies  $\lim_{h \rightarrow 0} v'(h) = +\infty$ .

Parents devotes  $e_t$  to his child's education, so human capital in  $t + 1$  is given by:

$$h_{t+1} = G(e_t) \quad (3)$$

**Assumption 3.** The human capital production function  $G(e)$  is  $\mathbf{C}^2$ , strictly increasing with  $e$ .

Grandparents' expenditures in education generate a positive intergenerational external effect in human capital accumulation. Indeed, parents decide for their child's education but do not consider the impact of this decision on their grand child's education. We assume that population is constant over time and is normalized to 1, i.e  $N_t = N = 1$ . Moreover, clearing condition on the labor market gives  $H_t = Nh_t = h_t$ .

The social planner maximizes the discounted sum of the life-cycle utilities of all current and future generations under the resource constraint of the economy and the human capital accumulation equation.

$$\max_{c_t, d_t, K_{t+1}, H_{t+1}} \sum_{t=-1}^{\infty} \delta^t (u(c_t, d_{t+1}) + \epsilon v(H_{t+1})) \quad (4)$$

$$\begin{aligned} \text{subject to: } \quad \forall t \geq 0 \quad & F(K_t, H_t) = c_t + d_t + K_{t+1} + e_t \\ & H_{t+1} = G(e_t) \\ & K_0, H_0 \text{ and } c_{-1} \text{ given} \end{aligned} \quad (5)$$

with  $\delta \in (0, 1)$ , and  $\epsilon$  taking alternatively the extreme values 0 (Harsanyi social function) and 1 (utilitarian social function).

The Lagrange function is:

$$\begin{aligned} \mathcal{L} = & \delta^{-1} [u(c_{-1}, d_0) + \epsilon v(H_0)] \\ & + \sum_{t=0}^{\infty} \delta^t (u(c_t, d_{t+1}) + \epsilon v(H_{t+1}) + q_t (F(K_t, H_t) - c_t - d_t - G^{-1}(H_{t+1}) - K_{t+1})) \end{aligned} \quad (6)$$

where  $q_t$  the Lagrange multiplier associated with the constraint. First order conditions for all  $t \geq 0$  are:

$$u_c(c_t, d_{t+1}) = q_t \quad (7)$$

$$u_d(c_{t-1}, d_t) = \delta q_t \quad (8)$$

$$\delta^{t+1} F_K(K_{t+1}, H_{t+1}) q_{t+1} = \delta^t q_t \quad (9)$$

$$\delta^t \epsilon v'(H_{t+1}) + \delta^{t+1} q_{t+1} F_H(K_{t+1}, H_{t+1}) = \delta^t q_t G^{-1}'(H_{t+1}) \quad (10)$$

For initial conditions  $K_0$  and  $H_0$  and for all  $t \geq 0$ , optimal solutions satisfy (7), (9) and (10). From (8) and (9) we can rewrite the condition that gives optimal physical capital accumulation:

$$f'(k_{t+1}) = \frac{u_c(c_t, d_{t+1})}{u_d(c_t, d_{t+1})} = \frac{q_t}{\delta q_{t+1}} \quad (11)$$

From (7), (9) and (10), we obtain the following expression that determines optimal human capital accumulation:

$$MRS_{e/c} = G^{-1'}(H_{t+1}) - k_{t+1} \left( \frac{1}{s(k_{t+1})} - 1 \right) \quad (12)$$

with  $MRS_{e/c} \equiv \frac{\epsilon v'(H_{t+1})}{u_c(c_t, d_{t+1})}$  the marginal rate of substitution between education and first period consumption.

**Definition 1** *Along the balanced growth path per-capita variables grow at a constant rate  $g$ .*

As marginal utility of consumption is homogeneous of degree  $(a - 1)$ , from (11), we have  $\delta f'(k) = (1 + g)^{1-a}$ , where  $k$  the physical to human capital ratio along the balanced growth path, is determined by equation (12). Proposition 1 summarizes the different cases.

**Proposition 1** *Under assumptions 1, 2 and 3, the balanced growth path depends on the definition of the social welfare function.*

*If  $\epsilon = 0$  (Harsanyi social function) optimal physical to human capital ratio,  $k$ , and optimal growth rate  $g$  are directly determined both by technological parameters and the education technology, as  $MRS_{e/c} = 0$ .*

*If  $\epsilon = 1$  (utilitarian social function) optimal physical to human capital ratio,  $k$ , and optimal growth rate  $g$  are determined by both technological parameters and preference parameters as  $MRS_{e/c} > 0$ .*

Proposition 1 emphasizes the importance of paternalistic altruism to design balanced growth path. With Harsanyi social function ( $\epsilon = 0$ ), equation (12) becomes:

$$f'(k_{t+1}) = (f(k_{t+1}) - k_{t+1}f'(k_{t+1})) \frac{dH_{t+1}}{de_t} \quad (13)$$

The optimal return on investment in human capital (through education) is equal to the return on physical capital since the central planner does not differentiate between physical and human capital accumulation. The welfare increases only with consumption. Thus optimal  $k$  corresponds to the standard Modified Golden Rule.

Conversely, with utilitarian social function ( $\epsilon = 1$ ), from equation (12) we get:

$$f'(k_{t+1}) > (f(k_{t+1}) - k_{t+1}f'(k_{t+1})) \frac{dH_{t+1}}{de_t} \quad (14)$$

The optimal return on investment in human capital (through education) is lower than the one on physical capital because human capital accumulation provides direct welfare. There is a trade-off between consumption and education. Then, we depart from the Modified Golden Rule through the  $MRS$  term.

### 3 Social Optimum in a two-sector framework

We introduce a two-sector setting *à la Galor* (1992). There are a consumption good, produced by sector 0, and an investment good, produced by sector 1. Denoting respectively by  $K^i$  and  $H^i$ , the  $i$ -

sector physical and human capital. Production functions are  $Y_t^0 = F^0(K_t^0, H_t^0)$  and  $Y_t^1 = F^1(K_t^1, H_t^1)$ , with  $F^i(K_t^i, H_t^i)$  homogeneous of degree one. Production factors are mobile between sectors and fully employed:  $K = K^0 + K^1$  and  $H = H^0 + H^1$ . Let  $k^i = K^i/H^i$  be the physical to human capital ratio in sector  $i$ , then  $H^0 k^0 + H^1 k^1 = k$ . We define the production functions in intensive forms as  $f^0(k^0)$  and  $f^1(k^1)$ .

**Assumption 4**  $f^0(k^0)$  and  $f^1(k^1)$  are defined over  $\mathbb{R}_+$ ,  $\mathbf{C}^r$  over  $\mathbb{R}_{++}$  for  $r$  large enough, increasing ( $f^{i'}(k^i) > 0$ ) and concave ( $f^{i''}(k^i) < 0$ ). Moreover,  $\lim_{k^i \rightarrow 0} f^{i'}(k^i) = +\infty$  and  $\lim_{k^i \rightarrow +\infty} f^{i'}(k^i) = 0$ .

We can also compute the share of capital in total production for each sector:

$$s^i(k^i) = \frac{k^i f^{i'}(k^i)}{f^i(k^i)} \in (0, 1) \quad (15)$$

The social planner objective becomes:

$$\max_{c_t, d_t, K_{t+1}, H_{t+1}} \sum_{t=-1}^{\infty} \delta^t (u(c_t, d_{t+1}) + \epsilon v(H_{t+1})) \quad (16)$$

$$\begin{aligned} \text{subject to: } \quad \forall t \geq 0 \quad & F^0(K_t^0, H_t^0) = c_t + d_t \\ & F^1(K_t^1, H_t^1) = K_{t+1}^0 + K_{t+1}^1 + e_t \\ & H_{t+1}^0 + H_{t+1}^1 = G(e_t) \\ & K_0, H_0 \text{ and } c_{-1} \text{ given} \end{aligned} \quad (17)$$

And the Lagrange function:

$$\begin{aligned} \mathcal{L} = & \delta^{-1} (u(c_{-1}, d_0) + \epsilon v(H_0)) + \sum_{t=0}^{\infty} \delta^t (u(c_t, d_{t+1}) + \epsilon v(H_{t+1}) + q_t^0 (F^0(K_t^0, H_t^0) - c_t - d_t)) \\ & + \sum_{t=0}^{\infty} \delta^t (q_t^1 (F^1(K_t^1, H_t^1) - K_{t+1}^0 - K_{t+1}^1 - G^{-1}(H_{t+1}^0 + H_{t+1}^1))) \end{aligned} \quad (18)$$

with  $q_t^0$  and  $q_t^1$  the Lagrange multipliers. First-order conditions for all  $t \geq 0$  are:

$$u_c(c_t, d_{t+1}) = q_t^0 \quad (19)$$

$$u_d(c_{t-1}, d_t) = \delta q_t^0 \quad (20)$$

$$\delta^{t+1} F_K^0(K_{t+1}^0, H_{t+1}^0) q_{t+1}^0 = \delta^t q_t^1 \quad (21)$$

$$\delta^{t+1} F_K^1(K_{t+1}^1, H_{t+1}^1) q_{t+1}^1 = \delta^t q_t^1 \quad (22)$$

$$\delta^t \epsilon v'(h_{t+1}) + \delta^{t+1} q_{t+1}^0 F_H^0(K_{t+1}^0, H_{t+1}^0) = \delta^t q_t^1 G^{-1}'(H_{t+1}^0 + H_{t+1}^1) \quad (23)$$

$$\delta^t \epsilon v'(h_{t+1}) + \delta^{t+1} q_{t+1}^1 F_H^1(K_{t+1}^1, H_{t+1}^1) = \delta^t q_t^1 G^{-1}'(H_{t+1}^0 + H_{t+1}^1) \quad (24)$$

$$F_K^0(K_0^0, H_0^0) q_0^0 = q_0^1 F_K^1(K_0^1, H_0^1) \quad (25)$$

$$F_H^0(K_0^0, H_0^0) q_0^0 = q_0^1 F_H^1(K_0^1, H_0^1) \quad (26)$$

$$K_0 = K_0^0 + K_0^1 \quad (27)$$

$$H_0 = H_0^0 + H_0^1 \quad (28)$$

We obtain the following Proposition:

**Proposition 2** *In a two-sector setting, under assumptions 2, 3 and 4, optimal capital intensities for each sector depend on the definition of the social welfare function.*

- i) *If  $\epsilon = 0$  (Harsanyi social function) in each sector  $i$ , optimal sectoral physical to human capital ratio  $k^i$  is determined by both technological parameters of this sector  $i$  and the education technology, as  $MRS_{e/c} = 0$ .*  
ii) *If  $\epsilon = 1$  (utilitarian social function) optimal physical to human capital ratios,  $k^1$  and  $k^2$ , are determined by both sectors technological parameters and preference parameters as  $MRS_{e/c} > 0$ .*

Proof:

From equations (19), (21), (22) and (23), we obtain the following expression that determines optimal human capital in the consumption sector:

$$\frac{\epsilon v'(H_{t+1})}{f^{0'}(k_{t+1}^0) \delta u_c(c_{t+1}, d_{t+2})} = G^{-1'}(H_{t+1}) - k_{t+1}^0 \left( \frac{1}{s^0(k_{t+1}^0)} - 1 \right) = G^{-1'}(H_{t+1}) - k_{t+1}^1 \left( \frac{1}{s^1(k_{t+1}^1)} - 1 \right) \quad (29)$$

Thus when  $\epsilon = 0$ , we obtain i) from:

$$k_{t+1}^0 \left( \frac{1}{s^0(k_{t+1}^0)} - 1 \right) = k_{t+1}^1 \left( \frac{1}{s^1(k_{t+1}^1)} - 1 \right) = G^{-1'}(H_{t+1}) \quad (30)$$

From (21) and (22) we obtain the following expression that determines optimal physical capital accumulation:

$$\delta f^{0'}(k_{t+1}^0) = \frac{q_t^1}{q_{t+1}^0} \quad \delta f^{1'}(k_{t+1}^1) = \frac{q_t^1}{q_{t+1}^1} \quad (31)$$

Using equation (19) and (31), we can write:

$$\frac{u_c(c_{t+1}, d_{t+2})}{u_c(c_t, d_{t+1})} = \frac{f^{0'}(k_t^0)}{f^{0'}(k_{t+1}^0)} \frac{1}{\delta f^{1'}(k_{t+1}^1)} \quad (32)$$

Finally, using equations (30) and (32), we obtain ii) from:

$$MRS_{e/c} = \frac{f^{0'}(k_t^0)}{f^{1'}(k_{t+1}^1)} \left( G^{-1'}(H_{t+1}) - k_{t+1}^0 \left( \frac{1}{s^0(k_{t+1}^0)} - 1 \right) \right) = \frac{f^{0'}(k_t^0)}{f^{1'}(k_{t+1}^1)} \left( G^{-1'}(H_{t+1}) - k_{t+1}^1 \left( \frac{1}{s^1(k_{t+1}^1)} - 1 \right) \right) \quad (33)$$

□

From equation (32), the growth rate along the balanced path is given by  $1 + g = \left( \delta f^{1'}(k^1) \right)^{\frac{1}{1-a}}$ . This optimal growth rate only depends on the marginal productivity of capital in the sector producing the investment good. From Proposition 2, optimal capital intensity determinants vary with the specification of the social utility function, and so does the optimal growth rate.

Considering an Harsanyi social function ( $\epsilon = 0$ ), optimal  $k$  does not depend on agents preferences (impatience, altruism) nor on sector 0 technology. In this two-sector framework, producing the investment good (good 1) is the only way to optimally allocate wealth between periods and generations. Thus the fact that sector 0 technology does not matter for the two-sector Modified Golden Rule is rather intuitive. Conversely, with a utilitarian social function, both education (good 1) and consumption good (good 0) are now welfare improving. Thus the central planner has to produce efficiently over the two sectors and optimal  $k$  is directly linked to preferences and relative capital intensities.



As regards the growth effect of including paternalistic altruism in the social welfare function, we can deduce the following proposition:

**Proposition 3** *When  $G(\cdot)$  is linear, under assumptions 2, 3, and 1 or 4, along the balanced growth path, the optimal growth rate is higher in the case with utilitarian social function than in the case with Harsanyi social function.*

Proof:

Along the balanced growth path, equation (12) can be rewritten:

$$k \left( \frac{1}{s(k)} - 1 \right) = G^{-1'}(H_{t+1}) - MRS_{e/c} \quad (34)$$

The left hand side of this equality is an increasing function of  $k$ . As  $G^{-1'}(H_{t+1})$  is independent of  $k$ , and  $MRS_{e/c} \geq 0$ , then  $k$  is decreasing with  $MRS_{e/c}$ . The growth rate is increasing with  $MRS_{e/c}$ .

In the two-sector framework, equation (33) becomes:

$$k^1 \left( \frac{1}{s^1(k^1)} - 1 \right) = G^{-1'}(H_{t+1}) - \frac{f^{1'}(k^1)}{f^{0'}(k^0)} MRS_{e/c} \quad (35)$$

As  $\frac{f^{1'}(k^1)}{f^{0'}(k^0)} MRS_{e/c} \geq 0$ ,  $k^1$  is decreasing with  $MRS_{e/c}$  and hence the growth rate is increasing with  $MRS_{e/c}$ .  $\square$

## References

- [1] Cremer, H., and Pestieau, P. (2006): "Intergenerational Transfer of Human Capital and Optimal Education policy," *Journal of Public Economic Theory* 8, 529-545.
- [2] De la Croix, D., and Monfort, P. (2000): "Education Funding and Regional Convergence," *Journal of Population Economics* 13, 403-424.
- [3] Galor, O. (1992): "A Two-Sector Overlapping-Generations Model: a Global Characterization of the Dynamical System," *Econometrica* 60, 1351-1386.
- [4] Glomm, G., and Ravikumar, B. (1992): "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy* 4, 818-833.
- [5] Harsanyi, J.C (1995): "A theory of prudential values and a rule utilitarian theory of morality," *Social Choice and Welfare* 12, 319-333.
- [6] Michel, P., and Vidal, JP. (2000): "Economic Integration and Growth under Intergenerational Financing of Human-Capital formation," *Journal of Economics* 3, 275-294.