



**HAL**  
open science

# Government Spending, Monetary Policy, and the Real Exchange Rate

Aurélien Eyquem, Hafedh Bouakez

► **To cite this version:**

Aurélien Eyquem, Hafedh Bouakez. Government Spending, Monetary Policy, and the Real Exchange Rate. 2012. halshs-00655972v1

**HAL Id: halshs-00655972**

**<https://shs.hal.science/halshs-00655972v1>**

Preprint submitted on 3 Jan 2012 (v1), last revised 15 Mar 2012 (v2)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

W P 1139

**Government Spending, Monetary Policy,  
and the Real Exchange Rate**

Hafedh Bouakez, Aurélien Eyquem

Décembre 2011

**GATE Groupe d'Analyse et de Théorie Économique Lyon-St Étienne**

93, chemin des Mouilles 69130 Ecully – France

Tel. +33 (0)4 72 86 60 60

Fax +33 (0)4 72 86 60 90

6, rue Basse des Rives 42023 Saint-Etienne cedex 02 – France

Tel. +33 (0)4 77 42 19 60

Fax. +33 (0)4 77 42 19 50

Messagerie électronique / Email : [gate@gate.cnrs.fr](mailto:gate@gate.cnrs.fr)

Téléchargement / Download : <http://www.gate.cnrs.fr> – Publications / Working Papers

# Government Spending, Monetary Policy, and the Real Exchange Rate\*

Hafedh Bouakez<sup>†</sup>

Aurélien Eyquem<sup>‡</sup>

November 2011

## Abstract

A robust prediction across a wide range of open-economy macroeconomic models is that an unanticipated increase in public spending in a given country appreciates its currency in real terms. This result, however, contradicts the findings of a number of recent empirical studies, which instead document a significant and persistent depreciation of the real exchange rate following an expansionary government spending shock. In this paper, we rationalize the findings of the empirical literature by proposing a small-open-economy model that features three key ingredients: incomplete and imperfect international financial markets, sticky prices, and a not-too-aggressive monetary policy. The model predicts that in response to an unexpected increase in public expenditures, the risk-adjusted long-term real interest rate falls, causing the real exchange rate to depreciate. We establish this result both analytically, within a special version of the model, and numerically for the more general case.

*JEL classification:* F31, F41

*Keywords:* Real exchange rate, public spending shocks, small open economy, sticky prices, monetary policy.

---

\*Financial support from SSHRC is gratefully acknowledged. We thank Federico Ravenna and Nicolas Vincent for helpful comments and suggestions.

<sup>†</sup>*Corresponding author.* HEC Montréal and CIRPÉE, 3000 chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada H3T 2A7. Tel.: 1-514-340-7003; Fax: 1-514-340-6469; E-mail: hafedh.bouakez@hec.ca.

<sup>‡</sup>Université de Lyon, Lyon, F-69007, France ; Ecole Normale Supérieure de Lyon, Lyon, F-69007, France ; CNRS, GATE Lyon St Etienne, Ecully, F-69130, France ; GREDI, Canada. Phone: +33 04.37.37.62.81. E-mail: aurelien.eyquem@ens-lyon.fr

# 1. Introduction

A robust prediction across a wide range of open-economy macroeconomic models is that an unanticipated increase in public spending in a given country appreciates its currency in real terms. This result holds both in the traditional Mundell-Fleming-Dornbusch framework and in standard dynamic general-equilibrium models that assume complete financial markets. In the former, the appreciation of the real exchange rate results from the rise in aggregate demand induced by the increase in public spending, which falls more heavily on domestically produced goods, raising their relative price with respect to foreign goods. In dynamic general-equilibrium models with complete markets, the real appreciation ensues from a risk-sharing condition that relates the real exchange rate to the ratio of marginal utilities of consumption across countries. Because the increase in public spending must ultimately be financed through taxes, it lowers households' permanent income, which in turn raises the shadow value of wealth in the domestic country and appreciates its currency in real terms.

This prediction, however, appears to be at odds with the data. Several empirical studies indeed find that the real exchange rate depreciates persistently in response to an unexpected exogenous increase in government expenditures (Corsetti and Müller [2006], Kim and Roubini [2008], Müller [2008], Monacelli and Perotti [2010], Enders, Müller and Scholl [2011], Ravn, Schmitt-Grohé and Uribe [2011], and Bouakez, Chihi and Normandin [2011]). This result holds across different countries, sample periods, and identification schemes, and is thus increasingly becoming a generally accepted stylized fact.

In this paper, we rationalize the findings of the empirical literature by proposing a small-open-economy model that combines three key features: *(i)* incomplete and imperfect international financial markets, *(ii)* sticky prices, and *(iii)* a not-too-aggressive monetary policy. Under incomplete financial markets, the real exchange rate is no longer pinned down by the ratio of marginal utilities of consumption, but instead by (the negative of) the long-term real interest rate net of a risk premium. The latter captures financial market imperfection and is assumed to be an increasing function of the economy's foreign debt. By raising demand for both domestically produced and foreign goods, an increase in public spending raises domestic inflation and deteriorates the current account. When prices are sticky and monetary policy does not react too aggressively to (expected) inflation, the resulting increase in the long-term real interest rate is smaller than the increase in

the risk premium, causing the real exchange rate to depreciate.

We illustrate this mechanism within a special version of our model in which we restrict the real interest rate to remain constant at its steady state value at all times. This is achieved by assuming that the monetary authority changes the nominal interest rate one for one with expected inflation. The constancy of the real interest rate, which is only possible under sticky prices, implies that consumption also remains constant in response to transitory shocks. We can then solve the model analytically (up to a first-order approximation) and show that an increase in government purchases leads to a depreciation of the real exchange rate and to a fall in net foreign assets, consistent with the empirical findings. We also show that the real depreciation reflects a larger rightward shift of the aggregate supply curve (in the output-real exchange rate space) than that of the aggregate demand curve. Importantly, in this special case, the rise in aggregate supply occurs despite the fact that the labor supply curve remains unchanged.

We then proceed to the analysis of the general case via numerical simulations in order to quantify the relationship between the aggressiveness of monetary policy and the real exchange rate response to a public spending shock. We find that the values of the policy-rule parameter that generate a real depreciation fall in the range of empirically plausible numbers. Finally, we perform a sensitivity analysis by varying the values of key parameters within the range of available estimates and by assuming an alternative specification of consumer preferences. We find our main conclusion to be extremely robust to these perturbations.

Three closely related papers to ours are those by Kollmann [2010], Corsetti, Meier and Müller [2011], and Ravn et al. [2011]. Kollmann [2010] considers a two-country model with incomplete financial markets and flexible prices, and shows that an increase in public spending in one country can depreciate its real exchange rate, provided that labor supply is highly elastic. The values of the elasticity of labor supply needed to generate a real depreciation, however, seem to be hard to reconcile with available empirical estimates. In contrast, our explanation does not require such an extreme assumption about the elasticity of labor supply.

Corsetti et al. [2011] develop a two-country model with complete markets, sticky prices and wages, and spending reversals. The latter feature refers to the assumption that debt-financed increases in government spending will cause subsequent spending to fall below its steady state level for some time. In turn, this lowers long-term real interest rates and appreciates the currency in

real terms. However, while empirical studies based on structural vector autoregressions (SVAR) invariably find that the path of government spending in the U.S. is “self-correcting”, increasing for six to twelve quarters before eventually falling below trend, results for other countries do not display such a reversal, especially for long sample periods (e.g., Perotti [2005] and Bouakez et al. [2011]).

Finally, Ravn et al. [2011] also consider a two-country model with complete financial markets, but depart from time-separable preferences by assuming that consumers form deep habits, i.e., habits at the level of individual varieties of goods. The presence of deep habits induces firms to lower their markups in markets in which aggregate demand rises. Thus, in response to an increase in government spending in the domestic economy, markups on domestically sold goods will be lower than markups abroad, making those goods relatively cheaper in the domestic economy, and implying a depreciation of the real exchange rate. While our explanation involves a completely different mechanism than those discussed above, we regard these different mechanisms as complementary, rather than competing or mutually exclusive, explanations for the “puzzling” real depreciation caused by an expansionary public spending shock.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the effects of a government spending shock first analytically within a simplified version of the model and then numerically for the general case. It also performs an extensive sensitivity analysis with respect to key parameters of the model. Section 4 concludes.

## **2. The Model Economy**

### **2.1 The environment**

The world consists of a continuum of identical small open economies that trade in final goods and financial assets. The size of each economy is small relative to the rest of the world. Each economy is populated by a continuum of identical, infinitely lived, households, and has a continuum of monopolistically competitive firms that produce a differentiated final good. The number of households and firms is normalized to one. Firms do not price discriminate between the domestic and export market, which means that the law of one price holds for all good. Financial markets are assumed to be incomplete as households can only trade one-period non-state-contingent domestic and international bonds. Financial markets are also assumed to be imperfect in that households

need to pay a portfolio-adjustment cost to acquire (or to sell) foreign bonds. In what follows, we describe a representative small open economy, which we call “home country”, and denote by an asterisk variables pertaining to the rest of the world.

**Households.** The representative household in the home country maximizes its expected lifetime utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \ln c_s - \omega \frac{\ell_s^{1+1/\zeta}}{1+1/\zeta} \right), \quad (1)$$

subject to the budget constraint

$$\varepsilon_t B_t^* + B_t + P_t (c_t + \gamma_t) = \varepsilon_t R_{t-1}^* B_{t-1}^* + R_{t-1} B_{t-1} + P_t w_t \ell_t + D_t - \tau_t, \quad (2)$$

and to the appropriate no-ponzi-game conditions. In expression (1), the parameters  $0 < \beta < 1$  and  $\zeta > 0$  are, respectively, the subjective discount factor and the Frisch elasticity of labor supply,  $c_t$  is a consumption index, and  $\ell_t$  is the quantity of labor competitively supplied to the firms. In equation (2),  $B_t^*$  and  $B_t$  are the household’s nominal holdings of, respectively, international and domestic bonds at the end of period  $t$ ;  $\varepsilon_t$  is the nominal exchange rate, defined as the price of a unit of the foreign currency in terms of the domestic currency;  $\gamma_t$  is an adjustment cost on foreign bonds holdings, expressed in units of the domestic consumption index, and paid to the government;  $R_{t-1}^*$  and  $R_{t-1}$  are the nominal interest rates paid on, respectively, foreign and domestic bonds between periods  $t-1$  and  $t$ ;  $P_t$  is the consumption-based price index (CPI hereafter),  $w_t$  is the real wage rate,  $D_t = \int_0^1 D_t(i) di$  denotes the household’s claims on firms’ (indexed by  $i$ ) nominal profits, and  $\tau_t$  denotes lump-sum taxes paid to the government. Denoting by  $b_t^* \equiv B_t^*/P_t^*$  the real value of nominal holdings of foreign bonds, and by  $q_t \equiv \varepsilon_t P_t^*/P_t$  the real exchange rate, it is assumed that the portfolio adjustment cost,  $\gamma_t$ , has the following functional form:

$$\gamma_t = \frac{\gamma}{2} (q_t b_t^* - q b^*)^2, \quad (3)$$

where variables without time subscripts denote steady-state values. The household’s first order conditions imply

$$\omega c_t \ell_t^{1/\zeta} - w_t = 0, \quad (4)$$

$$\beta R_t E_t \left[ \frac{P_t c_t}{P_{t+1} c_{t+1}} \right] - 1 = 0, \quad (5)$$

$$E_t \left[ \frac{1}{P_{t+1} c_{t+1}} \left( R_t (1 + \gamma (q_t b_t^* - q b^*)) - \frac{q_{t+1}}{q_t} \frac{P_{t+1}}{P_t} \frac{P_t^*}{P_{t+1}^*} R_t^* \right) \right] = 0, \quad (6)$$



in addition to the usual transversality conditions. Equation (4) is a standard labor supply equation, which equates the marginal rate of substitution between consumption and leisure to the real wage. Equation (5) is the Euler equation, relating the expected growth rate of consumption to the real interest rate. Equation (6) is a no-arbitrage condition that equates the expected rates of return on foreign and domestic bonds. A log-linear approximation of this equation yields a modified uncovered interest rate parity condition that includes a risk premium.

The consumption index,  $c_t$ , is an aggregate of consumption of goods produced in the home country ( $c_{H,t}$ ) and goods produced in the rest of the world ( $c_{F,t}$ )

$$c_t = \left( (1 - \alpha)^{\frac{1}{\mu}} (c_{H,t})^{\frac{\mu-1}{\mu}} + \alpha^{\frac{1}{\mu}} (c_{F,t})^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}}, \quad (7)$$

where the parameter  $\alpha \in [0, 1]$  measures the share of imported goods in total consumption, thus reflecting the degree of openness of the small open economy,<sup>1</sup> and the parameter  $\mu$  is the elasticity of substitution between domestic and foreign goods. The associated CPI is

$$P_t = \left( (1 - \alpha) (P_{H,t})^{1-\mu} + \alpha (P_{F,t})^{1-\mu} \right)^{\frac{1}{1-\mu}}, \quad (8)$$

where  $P_{H,t}$  and  $P_{F,t}$  are the price sub-indexes associated with  $c_{H,t}$  and  $c_{F,t}$ , respectively. The optimal allocation of consumption between home and foreign goods gives rise to the following demand functions:

$$c_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\mu} c_t, \quad c_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\mu} c_t. \quad (9)$$

The consumption sub-indexes  $c_{H,t}$  and  $c_{F,t}$  are bundles of differentiated varieties produced domestically and abroad, and are given by, respectively

$$c_{H,t} = \left( \int_0^1 c_{H,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad c_{F,t} = \left( \int_0^1 c_{F,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad (10)$$

where  $c_{H,t}(j)$  (respectively  $c_{F,t}(j)$ ) is consumption of a typical variety  $i$  produced in the home country (respectively in the rest of the world) and  $\theta > 1$  is the elasticity of substitution between varieties originating in the same country. The price sub-indexes associated with  $c_{H,t}$  and  $c_{F,t}$  are given by, respectively

$$P_{H,t} = \left( \int_0^1 P_{H,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad P_{F,t} = \left( \int_0^1 P_{F,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad (11)$$

---

<sup>1</sup>The fraction  $(1 - \alpha)$  therefore measures the degree of home bias in consumption.

where  $P_{H,t}(j)$  (respectively  $P_{F,t}(j)$ ) is the domestic-currency price of a typical variety  $i$  produced in the home country (respectively in the rest of the world). Demand functions for domestically produced and foreign varieties are given by

$$c_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta} c_{H,t}, \quad c_{F,t}(i) = \left( \frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\theta} c_{F,t}. \quad (12)$$

Note that because the law of one price holds for all goods, we have  $P_{F,t}(i) = \varepsilon_t P_{F,t}^*(i) \forall i$ , and thus  $P_{F,t} = \varepsilon_t P_{F,t}^*$ .

**Firms.** Each monopolistically competitive firm is the supplier of a single variety  $i$ , produced with the following linear technology:

$$y_t(i) = n_t(i), \quad (13)$$

where  $n_t(i)$  is the quantity of labor hired by firm  $i$ . Since firms do not segment markets by country, the export price of a typical firm  $i$ ,  $P_{H,t}^*(i)$ , will be equal to  $P_{H,t}(i)/\varepsilon_t$ , which also implies that  $P_{H,t}^* = P_{H,t}/\varepsilon_t$ . Prices are set à la Calvo [1983]. That is, each period, a fraction  $(1 - \eta)$  of firms are randomly selected to set new prices while the remaining fraction  $\eta$  of firms keep their prices unchanged. Let  $\bar{P}_{H,t}$  denote the price set by a typical firm at period  $t$ .<sup>2</sup> Assuming that the government consumption indexes are analogous to those of private consumption, the total demand facing this firm in period  $s$  ( $s \geq t$ ) is

$$\begin{aligned} \bar{y}_s &= \left( \frac{\bar{P}_{H,t}}{P_{H,s}} \right)^{-\theta} (c_{H,s} + c_{H,s}^* + g_{H,s} + g_{H,s}^* + \gamma_s) \\ &= \left( \frac{\bar{P}_{H,t}}{P_{H,s}} \right)^{-\theta} \left( \frac{P_{H,s}}{P_s} \right)^{-\mu} ((1 - \alpha)(c_s + g_s + \gamma_s) + \alpha q_s^\mu (c_s^* + g_s^*)). \end{aligned} \quad (14)$$

The optimal price,  $\bar{P}_{H,t}$ , is chosen to maximize

$$\bar{D}_t = E_t \sum_{s=t}^{\infty} (\eta\beta)^{s-t} \lambda_{t,s} (\bar{P}_{H,t} - P_s w_s) \bar{y}_s, \quad (15)$$

where  $\lambda_{t,s} = \frac{P_t c_t}{P_s c_s}$ . The first-order condition for this problem yields

$$\bar{P}_{H,t} = \frac{\theta}{\theta - 1} \frac{\sum_{s=t}^{\infty} (\eta\beta)^{s-t} E_t [w_s \bar{y}_s / c_s]}{\sum_{s=t}^{\infty} (\eta\beta)^{s-t} E_t [\bar{y}_s / (P_s c_s)]}. \quad (16)$$

---

<sup>2</sup>The optimal price does not depend on the firm index because all the firms that have the opportunity to change their prices at a given time choose the same price.

Aggregating across all firms, the price index for domestically produced goods is given by

$$P_{H,t} = \left( (1 - \eta) \bar{P}_{H,t}^{1-\theta} + \eta P_{H,t-1}^{1-\theta} \right)^{\frac{1}{1-\theta}}. \quad (17)$$

**Government.** Since Ricardian equivalence holds in this model, we abstract from public debt and assume that the government finances its expenditures through lump-sum taxes and the portfolio-adjustment fees. That is,

$$P_t g_t = \tau_t + P_t \gamma_t. \quad (18)$$

Public spending is exogenous and follows the autoregressive process

$$\ln \left( \frac{g_t}{g} \right) = \rho_g \ln \left( \frac{g_{t-1}}{g} \right) + \epsilon_t^g, \quad (19)$$

where  $0 \leq \rho_g \leq 1$  and  $\epsilon_t^g$  is a serially uncorrelated disturbance with zero mean.

**Monetary authority.** The monetary authority in the small open economy sets the nominal interest rate according to the following rule:

$$\ln \left( \frac{R_t}{R} \right) = \rho_r \ln \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_r) \phi_\pi E_t \left( \frac{\pi_{t+1}}{\pi} \right), \quad (20)$$

where  $\pi_t = P_t/P_{t-1}$  is the CPI inflation rate between periods  $t - 1$  and  $t$ ,  $0 \leq \rho_r < 1$  captures the degree to which the monetary authority smooths out changes in the nominal interest rate, and  $\phi_\pi \geq 1$  measures the aggressiveness with which it responds to expected inflation. The assumption that the monetary authority responds to expected, rather than current inflation, is only made for tractability as it allows us to obtain analytical results in Section 3.1, but it is not essential to the general point made in this paper.

## 2.2 Equilibrium, steady-state and log-linearized model

**Equilibrium.** Given that we consider a small open economy, we assume that domestically produced goods receive a negligible weight in the world consumption bundle. This implies that  $P_{F,t}^* = P_t^*$ . For the purpose of deriving the market clearing conditions, let  $y_t \equiv \left( \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$  denote aggregate output. The goods market clearing condition is then given by

$$y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\mu} \left( (1 - \alpha) (c_t + g_t + \gamma_t) + \alpha q_t^\mu (c_t^* + g_t^*) \right). \quad (21)$$

Furthermore, aggregating the production functions of all domestic firms (equation 13) yields

$$\Delta_t y_t = \int_0^1 y_t(i) di = \int_0^1 n_t(i) di \equiv n_t, \quad (22)$$

where  $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} di$  measures the dispersion of domestic producer prices.<sup>3</sup> Labor-market clearing implies that

$$n_t = \ell_t. \quad (23)$$

Since households in the domestic economy are identical, the domestic bonds market is in zero net supply, i.e.,  $B_t/P_t \equiv b_t = 0$ . Imposing this condition and consolidating the households' and government budget constraints yields the following balance of payments equation:

$$q_t b_t^* = R_{t-1}^* q_t b_{t-1}^* + \frac{P_{H,t}}{P_t} y_t - c_t - g_t, \quad (24)$$

where we have substituted the firms's current dividends, used the fact that  $P_{H,t} y_t = \int_0^1 P_{H,t}(i) y_t(i) di$ , and imposed the labor-market clearing condition.

Conditionally on both monetary policy and the exogenous variables, a symmetric equilibrium for this economy is a collection of 9 sequences  $(c_t, n_t, y_t, b_t^*, w_t, q_t, P_t, P_{H,t}, \text{ and } \bar{P}_{H,t})_{t=0}^{\infty}$  satisfying the households' first-order conditions (equations 4–6, with equation 23 substituted in for  $\ell_t$ ), the definition of the CPI (equation 8), the pricing condition (equation 16), the definition of the price index for domestically produced goods (equation 17), the aggregate production function (equation 22), the goods market clearing condition (equation 21, with equation 3 substituted in for  $\gamma_t$ ), and the balance of payments (equation 24).

**Steady state.** We define a symmetric flexible-price initial steady state in which exogenous variables take identical values in the small open economy and in the rest of the world, and net foreign assets are zero, i.e.,  $b^* = 0$ . This ensures that  $c^* = c$  and  $q = 1$ . Moreover, we assume without loss of generality that all prices are equal to unity in steady state. Denoting by  $\kappa$  the steady-state ratio of government spending to output, the steady state allocations are

$$y = n = \left( \frac{\theta - 1}{\omega (1 - \kappa) \theta} \right)^{\frac{\zeta}{1+\zeta}}, \quad c = (1 - \kappa) y. \quad (25)$$

---

<sup>3</sup>The variable  $\tilde{y}_t \equiv \int_0^1 y_t(i) di = \Delta_t y_t$  is the appropriate measure of GDP in this model. However, as shown by Galí and Monacelli [2005],  $\Delta_t$  is of second order, which implies that GDP and aggregate output are equivalent up to a first order approximation.

**Log-linearization.** In order to solve the model, we log-linearize its equilibrium conditions around the steady state. Denoting by a circumflex the log-deviation of a variable from its steady-state value, the log-linearized version of the model is given by (see the Appendix for the derivation)

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - (\widehat{R}_t - E_t \widehat{\pi}_{t+1}), \quad (26)$$

$$\widehat{q}_t = E_t \widehat{q}_{t+1} + (\widehat{R}_t^* - E_t \widehat{\pi}_{t+1}^*) - (\widehat{R}_t - E_t \widehat{\pi}_{t+1}) - \gamma y \widehat{b}_t^*, \quad (27)$$

$$\widehat{b}_t^* = \beta^{-1} \widehat{b}_{t-1}^* + \widehat{y}_t - \left( (1 - \kappa) \widehat{c}_t + \kappa \widehat{g}_t + \frac{\alpha}{1 - \alpha} \widehat{q}_t \right), \quad (28)$$

$$\widehat{\pi}_{H,t} = \beta E_t \widehat{\pi}_{H,t+1} + \frac{(1 - \eta)(1 - \eta\beta)}{\eta} \left( \widehat{w}_t + \frac{\alpha}{1 - \alpha} \widehat{q}_t \right), \quad (29)$$

$$\widehat{w}_t = \widehat{c}_t + \zeta^{-1} \widehat{n}_t, \quad (30)$$

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{\alpha}{1 - \alpha} (\widehat{q}_t - \widehat{q}_{t-1}), \quad (31)$$

$$\widehat{y}_t = \widehat{n}_t, \quad (32)$$

$$\widehat{y}_t = (1 - \alpha) ((1 - \kappa) \widehat{c}_t + \kappa \widehat{g}_t) + \mu \alpha \left( \frac{2 - \alpha}{1 - \alpha} \right) \widehat{q}_t + \alpha ((1 - \kappa) \widehat{c}_t^* + \kappa \widehat{g}_t^*), \quad (33)$$

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1 - \rho_r) \phi_\pi E_t \widehat{\pi}_{t+1}, \quad (34)$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_t^g. \quad (35)$$

### 3. Public Spending Shocks and the Real Exchange Rate

Assume that all foreign variables remain at their steady state values ( $\widehat{c}_t^* = \widehat{g}_t^* = \widehat{\pi}_t^* = \widehat{R}_t^* = 0$ ), an assumption that we shall maintain in the rest of the paper. Combining the balance of payments (equation 28) and the goods market clearing condition (equation 33) yields:

$$\alpha \kappa \widehat{g}_t = -\widehat{b}_t^* + \beta^{-1} \widehat{b}_{t-1}^* - \alpha (1 - \kappa) \widehat{c}_t + \frac{\alpha}{1 - \alpha} (\mu (2 - \alpha) - 1) \widehat{q}_t. \quad (36)$$

This equation states that for given paths of private consumption and net foreign assets, an increase in government spending implies a depreciation of the real exchange rate, provided that  $\mu (2 - \alpha) >$

1. How plausible is this parametric restriction? Available estimates of the elasticity of substitution between domestic and foreign goods indicate that  $\mu$  is slightly above 1, although values as large as 1.5 or 2 are frequently used in calibrated open-economy models. Estimates of the degree of trade openness suggest that plausible values for  $\alpha$  are in the range of (0.1, 0.4). Clearly, even the most conservative values of  $\mu$  and  $\alpha$  imply that the condition  $\mu (2 - \alpha) > 1$  is met. Therefore, the rest of the analysis will assume that this condition holds.

Of course, whether the real exchange rate appreciates or depreciates in equilibrium will depend on the general-equilibrium adjustment of consumption and net foreign assets. In this paper, we show that a real depreciation occurs as long as consumption does not fall too much in response to the increase in government spending, which implies that the long term real interest rate,  $E_t \sum_{i=0}^{\infty} (\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i})$ , does not increase too much.<sup>4</sup> Below, we illustrate this point by focusing on a special case of the model in which the real interest rate, and hence consumption, remain constant in equilibrium. This version allows us to solve analytically for the equilibrium path of the real exchange rate. We then turn to the analysis of the more general version of the model using numerical simulations.

### 3.1 A special case

Consider a simplified version of our model in which  $\rho_r = 0$  and  $\phi_\pi = 1$ . In this case, the monetary authority changes the nominal interest rate one for one with expected inflation, thereby implying that the short and long-term real interest rates remain constant in every period, i.e.,  $\widehat{R}_t - E_t \widehat{\pi}_{t+1} = E_t \sum_{i=0}^{\infty} (\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i}) = 0$  for all  $t$ . This in turn implies that consumption remains constant in equilibrium if the economy is affected by transitory shocks, i.e.,  $\widehat{c}_t = 0$  for all  $t$ . Note that this equilibrium exists only if prices are sticky, as the real interest rate is independent of monetary policy and must adjust in a flexible-price equilibrium.

In the absence of intertemporal substitution (stemming from the constancy of the real interest rate), it can be easily seen that the dynamics of net foreign assets and the real exchange rate can be solved for independently of the rest of the model. More specifically, these two variables are determined by the following two-equation dynamic system:

$$\widehat{b}_t^* - \beta^{-1} \widehat{b}_{t-1}^* = \frac{\alpha}{1-\alpha} (\mu(2-\alpha) - 1) \widehat{q}_t - \alpha \kappa \widehat{g}_t, \quad (37)$$

$$E_t \widehat{q}_{t+1} - \widehat{q}_t = \gamma y \widehat{b}_t^*. \quad (38)$$

A graphical solution of this system is shown in Figure 1, which depicts the phase diagram in the  $(\widehat{b}_{t-1}^*, \widehat{q}_t)$  space.<sup>5</sup> The loci  $\Delta \widehat{b}_t^* = 0$  and  $\Delta \widehat{q}_{t+1} = 0$  are characterized by the following equations,

<sup>4</sup>Indeed, iterating equation (26) forward yields:

$$\widehat{c}_t = \lim_{T \rightarrow \infty} E_t \widehat{c}_{t+T} - E_t \sum_{i=0}^{\infty} (\widehat{R}_{t+i} - \widehat{\pi}_{t+1+i}),$$

which shows that the reponse of consumption to a transitory shock (so that  $\lim_{T \rightarrow \infty} E_t \widehat{c}_{t+T} = 0$ ) is entirely determined by the (negative of) response of the long term real interest rate.

<sup>5</sup>This graphical solution abstracts from uncertainty.

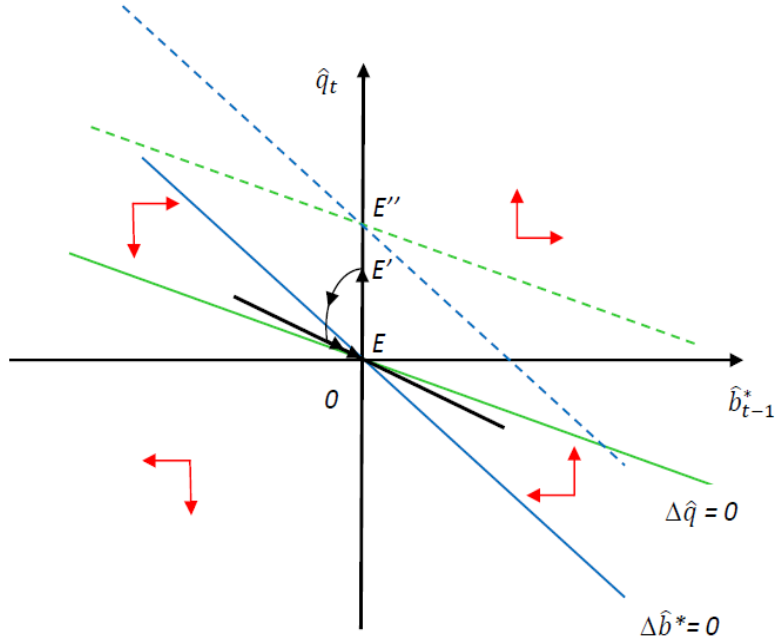


Figure 1: Illustrating the effects of an increase in government spending using the phase diagram.

respectively:

$$\hat{q}_t = -\frac{(\beta^{-1} - 1)(1 - \alpha)}{\alpha(\mu(2 - \alpha) - 1)}\hat{b}_{t-1}^* + \frac{\kappa(1 - \alpha)}{\mu(2 - \alpha) - 1}\hat{g}_t, \quad (39)$$

$$\hat{q}_t = -\frac{\beta^{-1}(1 - \alpha)}{\alpha(\mu(2 - \alpha) - 1)}\hat{b}_{t-1}^* + \frac{\kappa(1 - \alpha)}{\mu(2 - \alpha) - 1}\hat{g}_t. \quad (40)$$

The two loci are downward sloping, but the  $\Delta\hat{q}_{t+1} = 0$  locus has a steeper slope. Assume that the economy is initially at the equilibrium denoted by  $E$ , and that there is an unexpected temporary increase in government spending. The change in  $\hat{g}_t$  shifts both loci upward by exactly the same amount (see equations 39 and 40). At the time of the shock, the real exchange rate depreciates immediately, jumping to point  $E'$  in Figure 1. As the economy moves to point  $E''$ , the real exchange rate appreciates and net foreign assets fall for some time (inducing a current-account deficit) before starting to increase. Both variables then move down the saddle path until the economy returns to its initial equilibrium.

We now provide an analytical solution to system (37)-(38). To do so, we use the method of undetermined coefficients. That is, we conjecture that the solution takes the form

$$\hat{q}_t = \psi_1\hat{b}_{t-1}^* + \psi_2\hat{g}_t, \quad (41)$$

and determine  $\psi_1$  and  $\psi_2$  as a function of the deep parameters. By straightforward algebra, it can be shown that the coefficients  $\psi_1$  and  $\psi_2$  solve the following system:

$$\frac{\psi_1}{\psi_1 - \gamma y} = \beta^{-1} + \frac{\psi_1 \alpha}{1 - \alpha} (\mu (2 - \alpha) - 1), \quad (42)$$

$$\frac{\psi_2 (1 - \rho_g)}{\psi_1 - \gamma y} = \frac{\psi_2 \alpha}{1 - \alpha} (\mu (2 - \alpha) - 1) - \alpha \kappa. \quad (43)$$

The portfolio-adjustment-cost parameter,  $\gamma$ , is typically calibrated to a small value, usually of the order of  $10^{-3}$ . For example, Schmitt-Grohé and Uribe [2003] and Devereux, Lane and Xu [2006] set  $\gamma$  to 0.0007. When  $\gamma$  tends towards 0, it is straightforward to show that  $\psi_1$  and  $\psi_2$  converge to the following values:

$$\psi_1 = -\frac{(\beta^{-1} - 1)(1 - \alpha)}{\alpha(\mu(2 - \alpha) - 1)} < 0, \quad (44)$$

$$\psi_2 = \frac{\kappa(\beta^{-1} - 1)(1 - \alpha)}{(\beta^{-1} - \rho_g)(\mu(2 - \alpha) - 1)} > 0. \quad (45)$$

Note that  $\psi_2$  is unambiguously positive, which means that the real exchange rate depreciates following a positive government spending shock. One can also easily check that the current account deteriorates following such a shock. Intuitively, given that the real interest rate is constant, the balance of payments equilibrium cannot be restored with too large a decline in net foreign assets, as such a scenario would violate the transversality condition. The increase in government spending must therefore be accompanied by a real depreciation (see equation 36).

The effects of government spending shocks in this special case with a constant real interest rate can also be understood using the traditional aggregate demand-aggregate supply framework. The aggregate demand for the domestic good is given by

$$\hat{y}_t = \mu \alpha \left( \frac{2 - \alpha}{1 - \alpha} \right) \hat{q}_t + \kappa (1 - \alpha) \hat{g}_t. \quad (46)$$

Note that this curve is increasing in  $\hat{q}_t$ , i.e., the relative price of foreign goods. To derive the aggregate supply, we start by using (41) to solve for  $E_t \hat{q}_{t+1}$  in (38). This yields (assuming that  $\gamma$  is sufficiently small)  $\hat{b}_t^* = \frac{1}{\psi_1} \hat{q}_t - \frac{\psi_2 \rho_g}{\psi_1} \hat{g}_t$ . Inserting this expression into the balance of payments equation and substituting for  $\psi_1$  and  $\psi_2$ , we obtain

$$\hat{y}_t = -\beta^{-1} \hat{b}_{t-1}^* - \frac{\alpha}{1 - \alpha} \left( \frac{\mu(2 - \alpha) - \beta^{-1}}{\beta^{-1} - 1} \right) \hat{q}_t + \kappa \left( \frac{\alpha + \beta^{-1} - \rho_g}{\beta^{-1} - \rho_g} \right) \hat{g}_t. \quad (47)$$



The aggregate supply is decreasing in  $\hat{q}_t$  (increasing in the relative price of domestic goods) provided that  $\mu(2 - \alpha) > \beta^{-1}$ , which holds for plausible values of  $\mu$  and  $\alpha$ .

Equations (46) and (47) show that an increase in  $\hat{g}_t$  shifts both the aggregate demand and aggregate supply curves outward (see Figure 2). The shift in aggregate demand is maximal given that it is not attenuated by a decline in private consumption (recall that  $\hat{c}_t = 0$ ). The shift in aggregate supply reflects the fact that more output is needed, for a given level of the real exchange rate, to prevent too large a decline in net foreign assets that would put the economy on an equilibrium path that violates the transversality condition. Note that the shift in aggregate supply is not caused by a shift in labor supply and is therefore distinct from the mechanism highlighted by Kollmann [2010]. Because the supply curve shifts more than does the demand curve (note that  $\kappa(\alpha + \beta^{-1} - \rho_g) / (\beta^{-1} - \rho_g) > \kappa(1 - \alpha)$ ), the rise in government spending leads to an increase in output and to a depreciation of the real exchange rate.

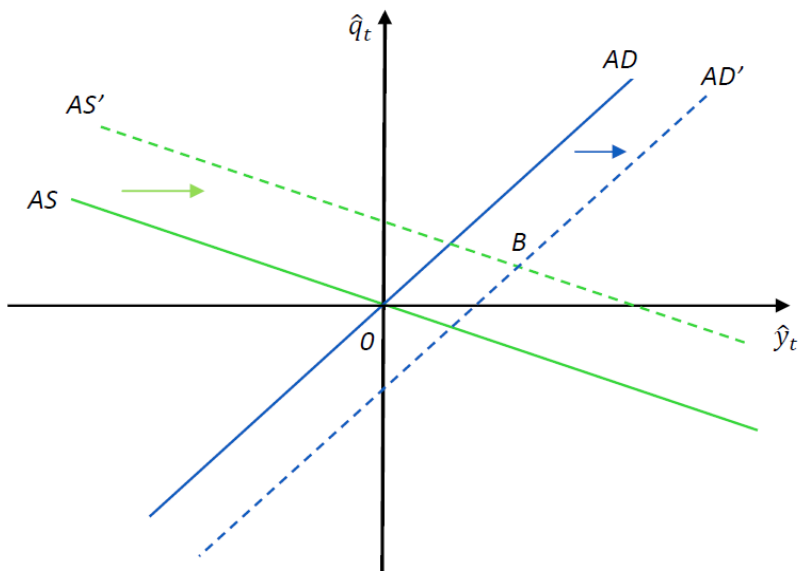


Figure 2: Illustrating the effects of an increase in government spending using the AD-AS framework.

Recall, however, that the depreciation of the real exchange rate is necessary in this special case because the equilibrium cannot be restored through a change in consumption, or alternatively, a change in the long-term real interest rate. In the general case where monetary policy changes the nominal interest rate more than one for one with expected inflation, the response of the real

exchange rate will depend on the magnitude of the adjustment of the long-term real interest rate. We explore this matter in the next section.

### 3.2 The general case

In this section, we show that the mechanism highlighted in the special case discussed above holds under less restrictive assumptions about monetary policy. In particular, we show that the real exchange rate continues to depreciate in response to a positive public spending shock if monetary policy adjusts the nominal interest rate more than one for one to expected inflation, without being overly aggressive. When  $\phi_\pi$  takes on values that are strictly larger than 1, however, the linearized model no longer admits an analytical solution and we have to proceed with numerical simulations. To do so, we need to assign values to the structural parameters.

*Parameter values.* We calibrate the model at a quarterly frequency. We start by discussing the parameters whose values are relatively conventional in the literature. The discount factor,  $\beta$ , is set to 0.99 so that the implied steady state real interest rate is 4.1% annually. The preference parameter  $\omega$  is a scaling parameter and is set to 1 in the benchmark calibration. When performing the sensitivity analysis, however, this value is adjusted to maintain the same level of labor in steady state as in the benchmark model. We choose the elasticity of substitution across domestic varieties of final goods,  $\theta$ , to equal 6, which yields a steady state markup of 20%, as in Rotemberg and Woodford [1997]. The steady-state share of public spending in total output,  $\kappa$ , is set to 0.25. The parameter governing trade openness,  $\alpha$ , is calibrated to 0.3 to match the average share of imports over GDP in industrialized economies. The smoothing parameter,  $\rho_r$ , is set to 0.8. This value can be considered as conservative, as Smets and Wouters [2005] report estimates of this coefficient of 0.85 for the Euro Area and 0.9 for the U.S. Finally, based on the discussion above, we set the portfolio-adjustment-cost parameter,  $\gamma$ , to 0.001.

Next, we turn to the parameters whose values are relatively more controversial. For each of those parameters, our strategy is to choose a benchmark value that lies roughly in the middle of the range of available estimates and then to perform a sensitivity analysis. Starting with the elasticity of substitution between domestic and foreign goods,  $\mu$ , available micro estimates tend to be much larger than macro estimates (see Ruhl [2004] for an extensive review of the literature). We choose to follow the macroeconomic literature and set  $\mu = 1.5$  in the benchmark calibration (as in Backus

and Kehoe [1994], for example). In the sensitivity analysis, we vary  $\mu$  between 1 and 2. Merz [1995] reports that existing estimates of the Frisch elasticity of labor supply,  $\zeta$ , vary between 0.03 and 3. In general, micro estimates converge towards low values of  $\zeta$ , whereas macro estimates tend to be large. Following Galí, Gertler and López-Salido [2007], we consider a benchmark value of 0.5 and investigate the sensitivity of the results to values of  $\zeta$  between 0.2 and 1. We set the Calvo probability of not changing prices,  $\eta$ , to 0.75, which implies an average duration of price spells of 4 quarters. In Section 3.3, we let this probability span the entire range of possible values, i.e.,  $(0, 1)$ . Finally, we calibrate the autocorrelation coefficient of the government spending process,  $\rho_g$  to 0.9, and study the sensitivity of the results when this parameter varies between 0.6 and 1. The benchmark values assigned to the parameters are summarized in Table 1.

Table 1: Parameter values

Discount factor	$\beta = 0.99$
Preference parameter	$\omega = 1$
Elasticity of substitution of between varieties	$\theta = 6$
Steady state ratio of public spending to output	$\kappa = 0.25$
Trade openness	$\alpha = 0.3$
Nominal-interest-rate smoothing parameter	$\rho_r = 0.8$
Portfolio-adjustment-cost parameter	$\gamma = 0.001$
Elasticity of substitution of between goods	$\mu = 1.5$
Frisch elasticity of labor supply	$\zeta = 0.5$
Calvo probability of not changing prices	$\eta = 0.75$
Autocorrelation of public spending shocks	$\rho_g = 0.9$

**Impulse response functions.** We now discuss the impulse responses generated by the model under the benchmark calibration when the economy is hit by a 1 percent positive government spending shock. To shed light on the importance of monetary policy for the response of the real exchange rate, we study two scenarios: one in which the monetary authority fights inflation aggressively ( $\phi_\pi = 1.5$ ) and one in which monetary policy is moderately aggressive ( $\phi_\pi = 1.1$ ). The results are depicted in Figure 3.

The responses of output, consumption, net foreign assets, the nominal interest rate and the CPI inflation rate are qualitatively similar across the two cases. The increase in public spending increases aggregate demand, thus putting pressure on production capacities and input prices and inducing higher domestic-price inflation. The resulting increase in CPI inflation leads the monetary

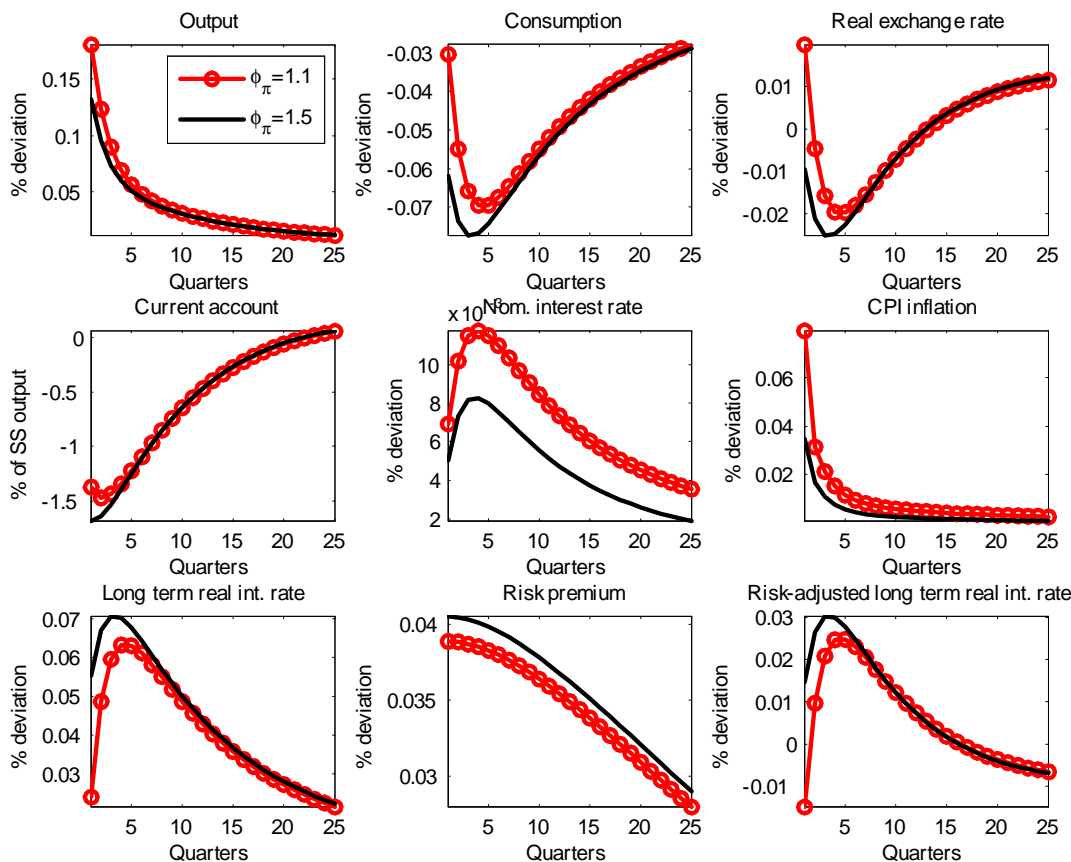


Figure 3: Impulse responses to a 1 percent increase in public spending under aggressive ( $\phi_\pi = 1.5$ ) and moderately aggressive ( $\phi_\pi = 1.1$ ) monetary policy.

authority to raise the nominal interest rate, which, under the assumption that  $\phi_\pi > 1$  raises the long-term real interest rate,  $E_t \sum_{i=0}^{\infty} (\hat{R}_{t+i} - \hat{\pi}_{t+1+i})$ . This in turn causes consumption to fall. As in the special case discussed above, net foreign assets decline. Increasing  $\phi_\pi$  from 1.1 to 1.5 only changes the magnitude of these responses, but not their sign.

In contrast, the real exchange rate exhibits very different responses under the two scenarios, appreciating when  $\phi_\pi = 1.5$  and depreciating when  $\phi_\pi = 1.1$ . In order to understand this result, it is useful to iterate the modified uncovered interest rate parity condition (equation 27) forward to

express the real exchange rate as

$$\hat{q}_t = \lim_{T \rightarrow \infty} E_t \hat{q}_{t+T} - E_t \sum_{i=0}^{\infty} (\hat{R}_{t+i} - \hat{\pi}_{t+1+i}) - \gamma y E_t \sum_{i=0}^{\infty} \hat{b}_{t+i}^*. \quad (48)$$

If the shock is temporary, the term  $\lim_{T \rightarrow \infty} E_t \hat{q}_{t+T}$  must be equal to zero, and the response of the real exchange rate will depend on the difference between the response of the long-term real interest rate and that of the long-term risk premium,  $-\gamma y E_t \sum_{i=0}^{\infty} \hat{b}_{t+i}^*$ , or, equivalently, on the response of the risk-adjusted long-term real interest rate,  $E_t \sum_{i=0}^{\infty} (\hat{R}_{t+i} - \hat{\pi}_{t+1+i} + \gamma y \hat{b}_{t+i}^*)$ . Figure 3 shows that under aggressive monetary policy ( $\phi_\pi = 1.5$ ), the rise in the long-term real interest rate is larger in magnitude than the increase in the long-term risk premium. Therefore the risk-adjusted long-term real interest rate increases, causing the real exchange rate to appreciate. Under less aggressive monetary policy ( $\phi_\pi = 1.1$ ), the risk premium dominates, the risk-adjusted rate falls initially, and the real exchange rate depreciates.

Figure 5 depicts the relationship between the initial response of the real exchange rate to a positive government spending shock and the parameter  $\phi_\pi$ , conditional on our benchmark calibration. The figure shows that this response is monotonically decreasing  $\phi_\pi$ , and that a real depreciation is obtained with empirically plausible values of  $\phi_\pi$ .

Figure 4:

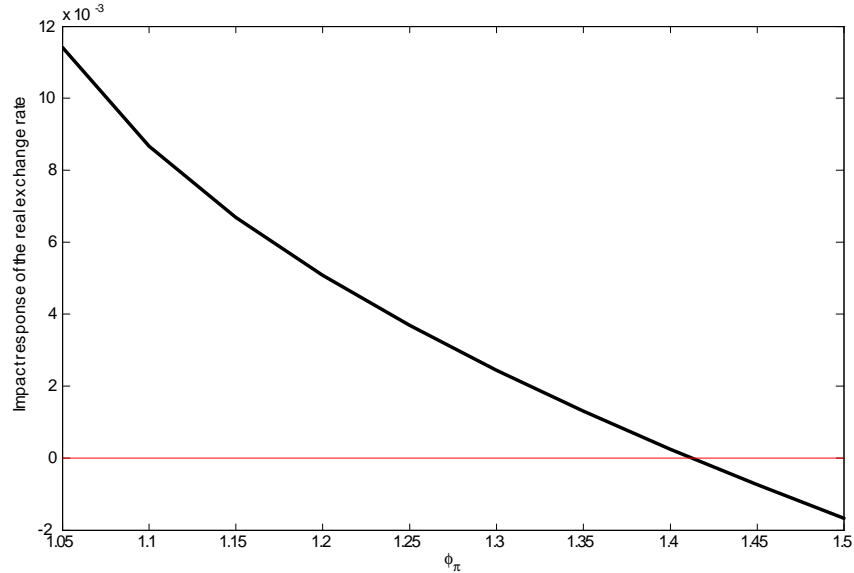


Figure 5: Initial response of the real exchange rate as a function of  $\phi_\pi$ .

### 3.3 Sensitivity analysis

We now study the sensitivity of our results to alternative values of key parameters, namely, the elasticity of substitution between domestic and foreign goods, the Frisch elasticity of labor supply, the Calvo probability of not changing prices, and the autocorrelation of the government spending shock. In each case, we focus on the initial response of the real exchange when both the parameter of interest and the policy parameter,  $\phi_\pi$ , vary. We also consider an alternative specification of households' preferences whereby government spending enters the utility function.

***Elasticity of substitution between domestic and foreign goods*** ( $\mu$ ). As stated above, the necessary condition for the real exchange rate to depreciate in response to an expansionary government spending shock, i.e.,  $\mu(2 - \alpha) - 1 > 0$ , holds for any sensible value of  $\mu$ . But how sensitive is this depreciation across the range of plausible values of  $\mu$ ? Figure 6 depicts the initial response of the real exchange rate for values of  $\mu$  ranging from 1 to 2, and values of  $\phi_\pi$  ranging from 1 to 1.5. The figure shows the magnitude of the exchange rate response is essentially insensitive to the value of  $\mu$  (within the range considered), regardless of the degree of aggressiveness of monetary policy.

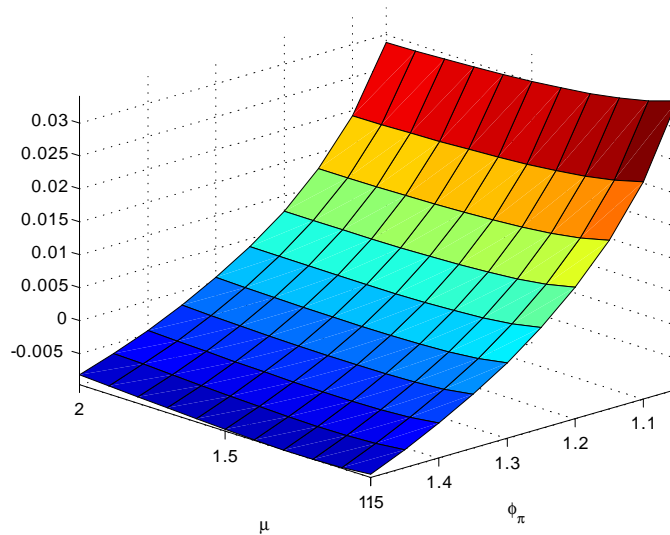


Figure 6: Initial response of the real exchange rate in the  $(\mu, \phi_\pi)$  space.

***Frisch elasticity of labor supply*** ( $\zeta$ ). In the special case of no intertemporal substitution, discussed in Section 3.1, the absence of a consumption response implies that  $\zeta^{-1}\hat{n}_t = \hat{w}_t$ , which

means that the labor supply curve does not shift after an increase in public spending. In that version of the model, the parameter  $\zeta$  plays virtually no role in the determination of the real exchange rate. In the general case, however, changes in consumption shift the labor supply curve. An expansionary government spending shock will therefore shift the aggregate supply curve rightward through its effect on labor supply, and the resulting effect on the exchange rate will be larger the larger is  $\zeta$ . As pointed out by Kollmann [2010], however, under flexible prices, generating a real depreciation requires very large values of  $\zeta$ , which lie far outside the range of available estimates.<sup>6</sup> In contrast, as shown in Section 3.2, our model generates a real depreciation using a value of  $\zeta$  that is in line with empirical estimates. Interestingly, Figure 7 below shows that, as  $\zeta$  tends towards the lower bound of existing estimates, a real depreciation is still possible to obtain, but it requires less and less aggressive monetary policy.

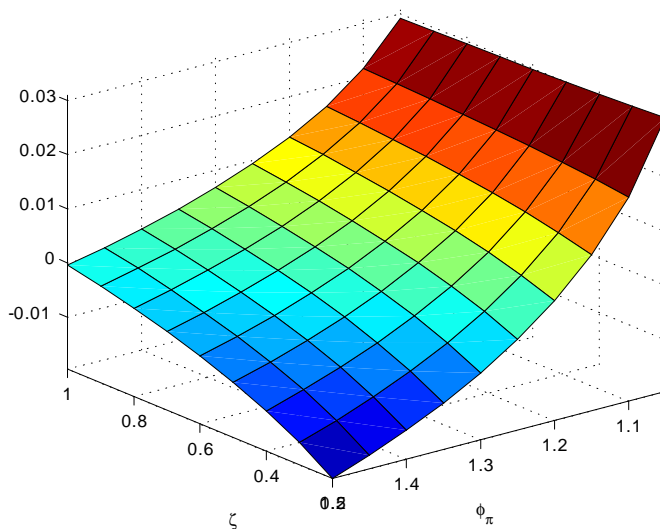


Figure 7: Initial response of the real exchange rate in the  $(\zeta, \phi_\pi)$  space.

*Calvo probability of not changing prices* ( $\eta$ ). While the equilibrium path of the exchange rate is independent of the degree of price rigidity in the special case with constant real interest rate, this no longer holds in the more general version of the model. Figure 8 indeed shows that the magnitude of the exchange rate response to a government spending shock increases exponentially

<sup>6</sup>Kollmann [2010] considers values of the Frisch elasticity equal to 2, 5 and infinity.

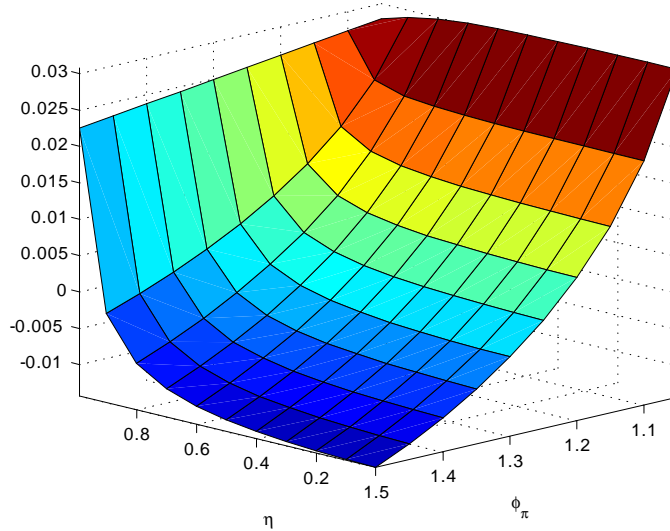


Figure 8: Initial response of the real exchange rate in the  $(\eta, \phi_\pi)$  space.

with  $\eta$ , and that the model can generate a real depreciation even with an aggressive monetary policy if prices are sufficiently rigid.

***Autocorrelation of the government spending shock*** ( $\rho_g$ ). Figure 9 shows that the way in which the persistence of the public spending shock affects the exchange rate response depends on the policy parameter,  $\phi_\pi$ . For relatively small values of  $\phi_\pi$ , this response increases monotonically with  $\rho_g$ ,<sup>7</sup> but for larger values of  $\phi_\pi$ , the relationship becomes non-monotonic, being negative for values of  $\rho_g$  that are less than 0.9 and positive for larger values. Again, this means that one can obtain a depreciation of the real exchange rate in response to a positive public spending shock even with an aggressive monetary policy, provided that the shock is sufficiently persistent.

***Alternative specification of the utility function.*** The model presented above predicts that consumption is crowded out following an increase in public spending, except in the special case where the real interest rate is held constant. Several empirical papers, however, find that consumption increases following an expansionary government spending shock. In what follows, we show that allowing for an alternative specification of consumer preferences whereby public spending increases

<sup>7</sup>In the special case of no intertemporal substitution ( $\phi_\pi = 1$ ), this can be clearly seen from equation (41) and the expression of  $\psi_2$ .



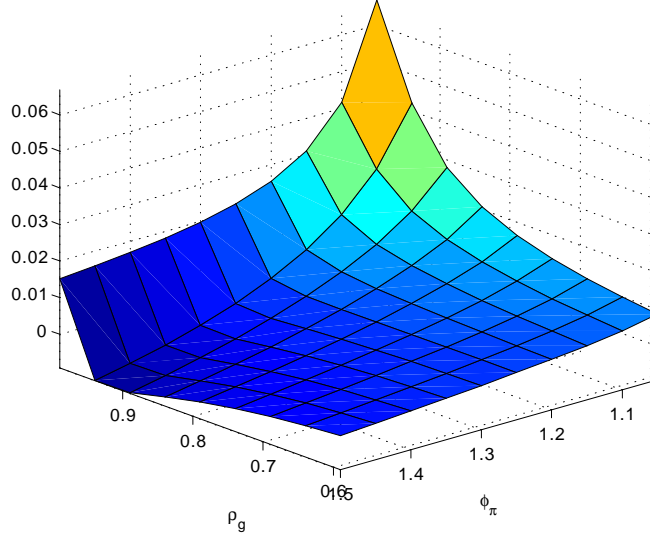


Figure 9: Initial response of the real exchange rate in the  $(\rho_g, \phi_\pi)$  space.

the marginal utility of private consumption (i.e., public and private goods are Edgeworth complements) can generate a crowding-in of private consumption in response to a positive government spending shock,<sup>8</sup> without altering the mechanism that gives rise to a real depreciation.

To see this, assume that the representative consumer maximizes the following lifetime utility:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \ln(c_s - \nu g_s) - \omega \frac{\ell_s^{1+1/\zeta}}{1+1/\zeta} \right), \quad (49)$$

where  $\nu$  is a parameter satisfying the condition  $0 < \nu < \frac{1-\kappa}{\kappa}$ . The fact that  $\nu$  is positive ensures that the marginal utility of consumption is increasing in  $g$  (i.e.,  $c$  and  $g$  are edgeworth complements). In this case, equations (26) and (30) are replaced by, respectively

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{\nu\kappa}{1-\kappa} (\hat{g}_t - E_t \hat{g}_{t+1}) - \frac{1-\kappa(1+\nu)}{1-\kappa} (\hat{R}_t - E_t \hat{\pi}_{t+1}), \quad (50)$$

$$\hat{w}_t = \frac{1-\kappa}{1-\kappa(1+\nu)} \hat{c}_t - \frac{\nu\kappa}{1-\kappa(1+\nu)} \hat{g}_t + \zeta^{-1} \hat{n}_t, \quad (51)$$

whereas equations (27)–(29) and (31)–(35) remain unchanged. In the special case where the monetary authority changes the nominal interest rate one for one with expected inflation, the resulting

<sup>8</sup>The idea that Edgeworth complementarity between private and public spending can cause private consumption to rise in response to an expansionary government spending shock has been suggested by Bouakez and Rebei [2007].

constancy of the real interest rate implies that the marginal utility of consumption remains constant, which by equation (50) implies that

$$\widehat{c}_t = \frac{\nu\kappa}{1-\kappa}\widehat{g}_t. \quad (52)$$

Since  $\frac{\nu\kappa}{1-\kappa} > 0$ , an increase in government spending raises private consumption by an amount that increases linearly with  $\nu$ . Equations (27), (28), (33) and (52) yield again a two-equation system that allows us to solve for net foreign assets and the real exchange rate independently of the rest of the variables

$$\widehat{b}_t^* - \beta^{-1}\widehat{b}_{t-1}^* = \frac{\alpha}{1-\alpha}(\mu(2-\alpha)-1)\widehat{q}_t - \alpha\kappa(1+\nu)\widehat{g}_t, \quad (53)$$

$$E_t\widehat{q}_{t+1} - \widehat{q}_t = \gamma y\widehat{b}_t^*. \quad (54)$$

The equilibrium path of the real exchange rate is again given by equation (41), where  $\psi_2$  is now given by (under the assumption that  $\gamma$  is sufficiently small)

$$\psi_2 = \frac{\kappa(1+\nu)(\beta^{-1}-1)(1-\alpha)}{(\beta^{-1}-\rho_g)(\mu(2-\alpha)-1)} > 0.$$

That is, the real exchange rate depreciates following an expansionary government spending shock, and this depreciation is larger the larger is  $\nu$ . Intuitively, as  $\nu$  increases (that is, as edgeworth complementarity between private and public expenditures becomes stronger), the increase in public spending shifts aggregate supply by a larger amount relative to aggregate demand.<sup>9</sup> This leads to a larger increase in output and a stronger depreciation of the real exchange rate, which are both necessary to accommodate the increase in private consumption.

## 4. Conclusion

Empirical studies conclude that the real exchange rate depreciates sharply after an expansionary public spending shock. This paper has shown that a sticky-price small-open-economy model with

---

<sup>9</sup>Note that aggregate demand and aggregate supply are now characterized by the following two equations, respectively:

$$\begin{aligned} \widehat{y}_t &= \mu\alpha\left(\frac{2-\alpha}{1-\alpha}\right)\widehat{q}_t + \kappa(1+\nu)(1-\alpha)\widehat{g}_t, \\ \widehat{y}_t &= -\beta^{-1}\widehat{b}_{t-1}^* - \frac{\alpha}{1-\alpha}\left(\frac{\mu(2-\alpha)-\beta^{-1}}{\beta^{-1}-1}\right)\widehat{q}_t + \kappa(1+\nu)\left(\frac{\alpha+\beta^{-1}-\rho_g}{\beta^{-1}-\rho_g}\right)\widehat{g}_t. \end{aligned}$$

incomplete and imperfect financial markets can account for this empirical regularity, at least qualitatively, provided that monetary policy is not too aggressive in fighting inflation. Admittedly, however, the magnitude of the real depreciation implied by the model is small in comparison with that documented in the empirical literature. Nonetheless, the mechanism highlighted in this paper can be viewed as a step forward towards a successful quantitative account of the effects of fiscal policy shocks on the real exchange rate.

## References

- Backus, D. K. and Kehoe, P. J. [1994], ‘Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?’, *American Economic Review* **84**(1), 84–103.
- Bouakez, H., Chihi, F. and Normandin, M. [2011], Fiscal Policy and External Adjustment: New Evidence, CIRPÉE Working Paper N. 11-23.
- Bouakez, H. and Rebei, N. [2007], ‘Why Does Private Consumption Rise After a Government Spending Shock?’, *Canadian Journal of Economics* **40**, 954–979.
- Calvo, G. [1983], ‘Staggered Prices in a Utility-maximizing Framework’, *Journal of Monetary Economics* **12**(3), 383–398.
- Corsetti, G., Meier, A. and Müller, G. J. [2011], ‘Fiscal Policy Transmission with Spending Reversals’, *Review of Economics and Statistics* **Forthcoming**.
- Corsetti, G. and Müller, G. J. [2006], ‘Twin Deficits: Squaring Theory, Evidence and Common Sense’, *Economic Policy* **21**(48), 597–638.
- Devereux, M. B., Lane, P. R. and Xu, J. [2006], ‘Exchange Rates and Monetary Policy in Emerging Market Economies’, *Economic Journal* **116**(511), 478–506.
- Enders, Z., Müller, G. J. and Scholl, A. [2011], ‘How Do Fiscal and Technology Shocks Affect Real Exchange Rates?: New Evidence for the United States’, *Journal of International Economics* **83**(1), 53–69.
- Galí, J., Gertler, M. and López-Salido, J. D. [2007], ‘Markups, Gaps, and the Welfare Costs of Business Fluctuations’, *The Review of Economics and Statistics* **89**(1), 44–59.
- Galí, J. and Monacelli, T. [2005], ‘Monetary Policy and Exchange Rate Volatility in a Small Open Economy’, *Review of Economic Studies* **72**(3), 707–734.
- Kim, S. and Roubini, N. [2008], ‘Twin Deficit or Twin Divergence? Fiscal Policy, Current Account, and Real Exchange Rate in the U.S’, *Journal of International Economics* **74**(2), 362–383.
- Kollmann, R. [2010], ‘Government Purchases and the Real Exchange Rate’, *Open Economies Review* **21**(1), 49–64.

- Merz, M. [1995], ‘Search in the Labor Market and the Real Business Cycle’, *Journal of Monetary Economics* **36**(2), 269–300.
- Monacelli, T. and Perotti, R. [2010], ‘Fiscal Policy, the Real Exchange Rate and Traded Goods’, *The Economic Journal* **120**(544), 437–461.
- Müller, G. J. [2008], ‘Understanding the Dynamic Effects of Government Spending on Foreign Trade’, *Journal of International Money and Finance* **27**(3), 345–371.
- Perotti, R. [2005], Estimating the Effects of Fiscal Policy in OECD Countries, Federal Reserve Bank of San Francisco Proceedings.
- Ravn, M. O., Schmitt-Grohé, S. and Uribe, M. [2011], Explaining the Effects of Government Spending Shocks on Consumption and the Real Exchange Rate, mimeo.
- Rotemberg, J. and Woodford, M. [1997], An Optimization–based Econometric Framework for the Evaluation of Monetary Policy, *in* B. S. Bernanke and J. J. Rotemberg, eds, ‘NBER Macroeconomics Annual’.
- Ruhl, K. J. [2004], The International Elasticity Puzzle, mimeo.
- Schmitt-Grohé, S. and Uribe, M. [2003], ‘Closing Small Open Economy Models’, *Journal of International Economics* **61**(1), 163–185.
- Smets, F. and Wouters, R. [2005], ‘Comparing Shocks and Frictions in US and Euro Area Business Cycles: A Bayesian DSGE Approach’, *Journal of Applied Econometrics* **20**(2), 161–183.

## Appendix

### A Log-linearization

Equations (4), (5), (6), (8), (16), (17), (22), (21), (24), as well as the monetary policy rule (20) and the government spending shock process (19) are log-linearized around the initial steady state defined in Section 2.2.

The log-linearized versions of equations (4)–(6) are, respectively

$$\widehat{c}_t + \zeta^{-1}\widehat{n}_t = \widehat{w}_t, \quad (\text{A1})$$

$$E_t\widehat{c}_{t+1} - \widehat{c}_t = \widehat{R}_t - E_t\widehat{\pi}_{t+1}, \quad (\text{A2})$$

$$\widehat{q}_t = E_t\widehat{q}_{t+1} + (\widehat{R}_t^* - E_t\widehat{\pi}_{t+1}^*) - (\widehat{R}_t - E_t\widehat{\pi}_{t+1}) - \gamma y\widehat{b}_t^*, \quad (\text{A3})$$

which are, respectively, equations (30), (26), (27) in the main text.

Replacing  $P_{F,t}$  by  $\varepsilon_t P_t^*$  in equation (8) and dividing both sides by  $P_{t-1}$  yields

$$1 = (1 - \alpha)(p_{H,t})^{1-\mu} + \alpha(q_t)^{1-\mu}, \quad (\text{A4})$$

where  $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$ . Log-linearizing equation (A4) yields

$$\widehat{p}_{H,t} = -\frac{\alpha}{1-\alpha}\widehat{q}_t, \quad (\text{A5})$$

Noting that  $\widehat{\pi}_{H,t} = \widehat{p}_{H,t} - \widehat{p}_{H,t-1} + \widehat{\pi}_t$ , and using equation (A5) to substitute for  $\widehat{p}_{H,t}$  and  $\widehat{p}_{H,t-1}$ , we obtain

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{\alpha}{1-\alpha}(\widehat{q}_t - \widehat{q}_{t-1}). \quad (\text{A6})$$

which is equation (31) in the main text.

The optimal pricing condition (16) can be rewritten as

$$\begin{aligned} \frac{\overline{P}_{H,t}}{P_{H,t}} &= \frac{\theta}{\theta-1} \frac{k_t}{h_t}, \quad (\text{A7}) \\ k_t &= \sum_{s=t}^{\infty} (\eta\beta)^{s-t} E_t [w_s \overline{y}_s / c_s], \\ h_t &= \sum_{s=t}^{\infty} (\eta\beta)^{s-t} E_t [P_{H,t} \overline{y}_s / (P_s c_s)]. \end{aligned}$$

Expressing  $k_t$  and  $h_t$  recursively yields

$$k_t = \eta\beta E_t k_{t+1} + w_t \bar{y}_t / c_t, \quad (\text{A8})$$

$$h_t = \eta\beta E_t (h_{t+1} / \pi_{H,t+1}) + (P_{H,t} \bar{y}_t) / (P_t c_t). \quad (\text{A9})$$

Log-linearizing these equations yields

$$\widehat{k}_t = \eta\beta E_t \widehat{k}_{t+1} + (1 - \eta\beta) \left( \widehat{y}_t + \widehat{w}_t - \widehat{c}_t \right), \quad (\text{A10})$$

$$\widehat{h}_t = \eta\beta E_t (\widehat{h}_{t+1} - \widehat{\pi}_{H,t+1}) + (1 - \eta\beta) \left( \widehat{y}_t - \widehat{c}_t - \frac{\alpha}{1 - \alpha} \widehat{q}_t \right), \quad (\text{A11})$$

where we have used the fact that  $\pi_H = 1$ . Further, dividing both sides of equation (17) by  $P_{H,t}$  implies

$$(1 - \eta) \left( \frac{\theta}{\theta - 1} \frac{k_t}{h_t} \right)^{1 - \theta} + \eta \pi_{H,t}^{\theta - 1} = 1. \quad (\text{A12})$$

Using the fact  $k/h = w = (\theta - 1) / \theta$ , equation (A12) is log-linearized as

$$\widehat{\pi}_{H,t} = \frac{(1 - \eta)}{\eta} (\widehat{k}_t - \widehat{h}_t), \quad (\text{A13})$$

Combining equations (A10), (A11) and (A13), we obtain

$$\widehat{\pi}_{H,t} = \beta E_t \widehat{\pi}_{H,t+1} + \frac{(1 - \eta)(1 - \eta\beta)}{\eta} \left( \widehat{w}_t + \frac{\alpha}{1 - \alpha} \widehat{q}_t \right), \quad (\text{A14})$$

which is equation (29) in the main text.

The log-linearized versions of the remaining equations (22, 21, 24, 20 and 19) are, respectively

$$\widehat{b}_t^* = \beta^{-1} \widehat{b}_{t-1}^* + \widehat{y}_t - \left( (1 - \kappa) \widehat{c}_t + \kappa \widehat{g}_t + \frac{\alpha}{1 - \alpha} \widehat{q}_t \right), \quad (\text{A15})$$

$$\widehat{y}_t = \widehat{n}_t, \quad (\text{A16})$$

$$\widehat{y}_t = (1 - \alpha) \left( (1 - \kappa) \widehat{c}_t + \kappa \widehat{g}_t \right) + \mu \alpha \left( \frac{2 - \alpha}{1 - \alpha} \right) \widehat{q}_t + \alpha \left( (1 - \kappa) \widehat{c}_t^* + \kappa \widehat{g}_t^* \right), \quad (\text{A17})$$

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1 - \rho_r) \phi_\pi E_t \widehat{\pi}_{t+1}, \quad (\text{A18})$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_t^g, \quad (\text{A19})$$

which are equations (28), (32), (33), (34), and (35) in the main text.