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A Theoretical Interpretation**

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Juillet 2012

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# On the Price Effects of Horizontal Mergers: A Theoretical Interpretation\*

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## Abstract

Horizontal mergers are usually under the scrutiny of antitrust authorities due to their potential undesirable effects on prices and consumer surplus. Ex-post evidence, however, suggests that not always these effects take place and even relevant mergers may end up having negligible price effects. The analysis of mergers in the context of non-localized spatial competition may offer a further interpretation to the ones proposed in the literature: in this framework both positive and zero price effects are possible outcomes of the merger activity.

JEL code: D43, L11, L13.

Keywords: horizontal mergers, price effects, spokes model.

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# 1 Introduction

Antitrust authorities worldwide are highly concerned with the price effects of mergers (Whinston, 2007). When two or more firms operating in the same market merge, the concentration of the market increases and this may drive to undesirable increases in prices with a consequent damage for consumers. According to Weinberg (2008), “the agencies review mergers in an effort to identify and block mergers if they would increase prices”. For these reasons several approaches have been developed in the literature and by the practitioners to evaluate the price effects: "event studies" based on the behaviour of the stock markets evaluation of the companies involved, "simulation studies" based on theoretical models of mergers and appropriately parametrized and "direct studies" of the price effects within a specific industry (Pautler, 2001; Weinberg, 2008). Although the literature mainly focuses on price increasing mergers, as they affect consumers and society’s welfare, a number of papers also provide ex-post evidence of little or no price effect of merging activity: a common trait of the retrospective evidence available is that not always anti-competitive effects arise (Ashenfelter et al., 2009). Ashenfelter and Hosken (2010) find that in two of the five mergers that they consider, no substantial price effect was registered. In particular, the Aurora-Kraft syrup merger had almost no effect while the Marathon-Ashland joint venture in the gasoline sector had negative (although non-significant) medium term effects on prices; the latter case is also discussed by Taylor and Hosken (2007) with similar conclusions. Ashenfelter et al. (2011) document Whirlpool’s acquisition of Maytag, a merger of two large manufacturers of appliances. They find no change in the prices of washing machines and sensible price increase only for some categories of products. Simpson and Taylor (2008) analyse the effects of MAP’s 1999 acquisition of Ultraman Diamond Shamrock’s terminaling assets on retail prices in Michigan: they analyse gasoline prices in six cities for a period of five years and find no evidence that the transaction led to higher prices. In the same sector, Csorba et al. (2011) obtain economically negligible price effects studying two almost

simultaneous mergers in the Hungarian retail gasoline market. In Spain, the acquisition by DISA of Shell's activities leads to no significant effect on pricing (Jiménez and Perdiguero, 2012). Neumann and Sanderson (2007) analyse the Corus and WIC merger by collecting information from market participants through interviews, other studies and from public sources about post-merger conditions. Although their evidence might have a limited value, they conclude that price increases are mostly attributed to inflation.

Several explanations for no-price effect of mergers have been proposed: the efficiency gains argument is the most common and it is used by most firms in defending a merger's proposal; another factor may be collusion in the market before a merger takes place (Jiménez and Perdiguero, 2012); or the competitive effect of actual (Ashenfelter and Hosken, 2010; Taylor and Hosken, 2007) or potential substitutes (Ashenfelter et. al., 2009). However, the retrospective evidence is mixed and does not always support the proposed explanations. Connor et al. (1998), for example, analyse general hospitals' mergers: they find a 5% price decrease in merging hospitals relative to non-merging ones. In addition, they find that costs also decrease by about 5% in the merging hospitals, indicating an almost perfect pass-through of the cost-savings on prices. Csorba et al. (2011) believe that no price effect derives from managerial decisions to stick with the pre-merger pricing policies, while Neumann and Sanderson (2007) and Simpson and Taylor (2008) provide no interpretation at all. This paper proposes a simple theoretical analysis of mergers in differentiated product industries which may provide a further explanation for the ex-post evidence on the price effects of a merger. In our framework, in fact, some horizontal mergers induce substantial price effects while others do not have any effect at all.

The rest of the paper is organized as follows. Section 2 locates our contribution in the literature. Section 3 briefly reviews the spokes model and its properties. Mergers are introduced in section 4. Section 5 discusses the price effects of mergers, comparing the *pre-merger* with the *post-merger* equilibria. Concluding remarks follow in section 6.

## 2 Related literature and contribution

We study the effects of horizontal mergers in the context of "non-localized" competition. The "traditional" approach to spatial competition uses the circular city model of Vickrey (1964), also referred to as the Salop (1979) model: one of its limits is to only allow to address "localized" spatial competition (Rothschild, 2000). Chen and Riordan (2007a) develop a new tool to analyse spatial differentiation which naturally fits to the idea of "non-localized" competition, the spokes model. In this model firms are located at the extreme of a market constituted of several spokes all linked at a common centre. There may be more spokes than firms (Chen and Riordan, 2007a) or as many spokes as firms (Caminal and Clatici, 2007). The model has two main properties. First, it allows to study multi-firm spatial competition with no neighbouring effects; second, when the number of firms tends to infinity, it captures the idea of monopolistic competition *à la Chamberlin*. The model is particularly useful to analyse markets in which consumers have a strong preference for a specific brand while being rather indifferent among alternatives: natural examples are markets for composite goods that require original parts to be completed or repaired. In a merger context, the spokes approach has a desirable feature: competition is non-localized and, in equilibrium, all firms are competing against each other. The effect of a merger is then to reduce the intensity of competition. The price effect, however, depends on which segment of the market firms are targeting when setting prices. The demand for the firms' product is composed of several segments characterized by different elasticities. There exist, then, equilibria in which mergers have important price effects: these mergers also imply demand effects that lead to increased transportation costs and, consequently, negative welfare effects. However, if firms target a "kink" of the demand function, equilibria with no price effect arise as in Economides (1989, 1993). These results can be interpreted as a further possible explanation of the mixed evidence on the price effects of mergers in the empirical literature.

The contribution of this paper is also related to the literature on en-

ogenous mergers. The goal of many papers on this topic is to solve the paradoxes posed by the game theoretical analysis of mergers and coalition formation. One of the puzzles is that under price competition and differentiated products, mergers are always profitable for insiders (Deneckere and Davidson, 1985). However, the equilibrium displays free-riding properties: "outsiders" earn higher profits than "insiders". This property also characterizes our analysis: in the regions where prices increase following a merger, outsiders' prices and profits raise more than the ones of insiders'. In other instances, however, prices do not increase. Brito (2003) considers mergers in the context of the circular city. He shows that even if market power is the motivation for a merger, firms may want to be insiders (preemptive merger) and the impact of the merger on the rival firms depends on their locations. Firms may prefer to be insiders even if some outsiders benefit more (but others less). In this context, he finds that mergers have relevant impacts on prices only if one of the neighbouring firms takes part into it. On the contrary, our zero-price effect result is not related to the proximity of firms, which plays no relevant role in the spokes model. Braid (1999) considers two-dimensional competition between firms located on a grid: the price effects of a merger are lower due to a reduction in the benefits of merging for insiders. As opposed to standard single dimension models, this feature implies that the incentives to raise prices are spread in different directions.

Our results contribute to the rapidly flourishing literature on the spokes model as a tool for analysis of multi-firm differentiated product competition.<sup>1</sup> In this context Caminal and Granero (2012), who analyse the role of multi-product firms in supplying variety, is the closest paper to ours. They consider a continuous approximation of the model in which a multi-product firm competes against a fringe. In our paper, a merger also results in consti-

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<sup>1</sup>The spokes model is used, between others, by Caminal and Claiici (2007) in the context of loyalty rewarding schemes; Germano (2009) and Germano and Meier (2010) to analyze the information bias; Caminal (2010) on the supply of content in a bilingual environment; Ganuza and Hauk (2005) to address allocation of ideas in tournaments; Mantovani and Ruiz-Aliseda (2011) to analyse innovation in presence of complements and Chen and Riordan (2007b) on vertical integration.



tuting a multi-product firm; however, differently from Caminal and Granero (2012), we consider all four equilibrium regions that characterise the spokes model. Moreover, we focus on the price effects of a merger rather than on the market provision of variety. Finally, the multi-product firm resulting from the merger competes against other firms of comparable size rather than a fringe.

Taking into account asymmetries may be quite complex in other models of product differentiation as the circular city model (Brito, 2003; Borla, 2012; Syverson, 2004; Vogel, 2008; Alderighi and Piga, 2012). In our approach the non-localized nature of competition avoids complex feedback effects on the prices of neighbouring firms; this property may strengthen the case for the spokes model as a convenient and reasonable alternative to the circular city in addressing spatial competition between several firms.

### 3 The Framework

Consider the model introduced by Chen and Riordan (2007a). The market has a spatial structure made up of  $N$  spokes of constant length  $1/2$ , with a common centre;  $n$  firms are on the market, with  $n \leq N$  exogenously given. Each firm is located at the extreme of its own spoke and supplies an homogeneous good: transportation costs are the only source of differentiation. Customers are uniformly distributed over all the  $N$  spokes. Consumers have unit demand and their evaluation of the good is  $v$ , the transport cost is normalized to one and the marginal cost to zero. Tractability requires that consumers only like the brand located on their spoke and a finite subset of the  $N - 1$  alternative brands: as Chen and Riordan (2007a) it is assumed that any random consumer on a spoke likes only one alternative brand. Notice that if  $n < N$ , a consumer may not find available the first or the second brand or both: in that case the market is not fully covered.

The profit function of a given firm  $i$  is:

$$\pi_i(p_i, p_{-i}) = p_i D_i(p_i, p_{-i})$$

From firm  $i$ 's viewpoint there are different types of customers:<sup>2</sup>

1. Customers on  $i$ -th firm's spoke that have one of the remaining firms as an alternative. The demand from this group is defined by identifying the location  $\hat{x}$  of the consumers who are indifferent between buying from  $i$  or buying from the rival firm  $\alpha$ :

$$\hat{x} = \max \left\{ \min \left\{ \frac{1}{2} + \frac{p_\alpha - p_i}{2}; 1 \right\}, 0 \right\}$$

The constraints are imposed to ensure that the consumer is located on either of the spokes and not outside.

2. Customers on the  $i$ -th firm's spoke who do not have an existing alternative brand *and* customers who do not have a first favorite brand but have  $i$  as a second favorite. The marginal consumer in the set of these two types is identified by:

$$\tilde{x} = \max \{ \min \{ v - p_i, 1 \}, 0 \}$$

Simplifying the constraints, the demand function is defined by the following segments:

$$D_i(p_i, p_{-i}) = \begin{cases} \frac{2}{N} \frac{1}{N-1} \sum_{\alpha \neq i} \left( \frac{1}{2} + \frac{p_\alpha - p_i}{2} \right) + \frac{2}{N} \frac{N-n}{N-1} (v - p_i) & \text{if } \frac{1}{2} \leq v - p_i \leq 1 \\ \frac{2}{N} \frac{1}{N-1} \sum_{\alpha \neq i} \left( \frac{1}{2} + \frac{p_\alpha - p_i}{2} \right) + \frac{2}{N} \frac{N-n}{N-1} & \text{if } v - p_i > 1 \end{cases} \quad (1)$$

where  $2/N$  is the mass of consumers on each spoke,  $1/(N-1)$  is the probability of firm  $\alpha$  being a customers' second favorite brand and  $(N-n)/(N-1)$  is the probability of a consumer having no first or no second favorite brand available. The following regularity conditions need to be satisfied:  $|p_\alpha - p_i| < 1$ ,  $\forall \alpha \neq i$ , and  $v - p_i \geq 1/2$  to ensure that competition between firms occurs. The first order conditions identifying the equilibrium prices are given by:

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<sup>2</sup>Despite being aware of the existence of different types of customers, firms use a unique price and do not price discriminate.

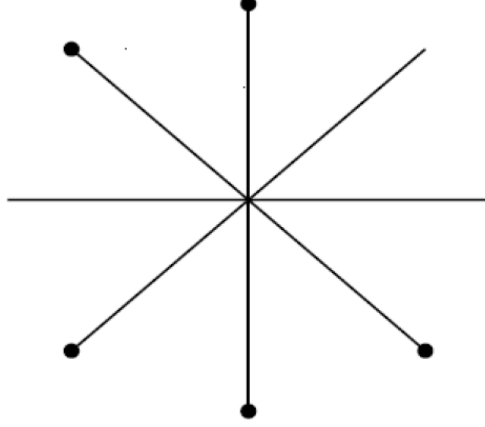


Figure 1: The Spokes Model, Chen-Riordan (2007a),  $n = 5$ ,  $N = 8$ .

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}) + p_i \frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = 0$$

Given the definition of the demand and profit functions, it can be checked that there exist four possible equilibrium regions.<sup>3</sup> The equilibrium regions are characterized depending on  $v$ , the parameter capturing consumers' evaluation of the good; the equilibrium prices before a merger takes place,  $p_{bm}^*$ , are defined as in Table 1:

**Table 1. Before Merger Equilibrium Prices.**

Region	Range	$p_{bm}^*$
1bm	$v_{1D}^{bm} = \frac{2(N-1)}{n-1} < v \leq 1 + \frac{(2N-n-1)^2}{2(N-n)(n-1)} = v_{1U}^{bm}$	$\frac{(2N-n-1)}{n-1}$
2bm	$v_{2D}^{bm} = 2 \leq v \leq \frac{2(N-1)}{n-1} = v_{2U}^{bm}$	$v - 1$
3bm	$v_{3D}^{bm} = \frac{1}{2} \frac{4N-n-3}{2N-n-1} < v < 2 = v_{3U}^{bm}$	$\frac{2v(N-n)+(n-1)}{4N-3n-1}$
4bm	$v_{4D}^{bm} = 1 \leq v \leq \frac{1}{2} \frac{4N-n-3}{2N-n-1} = v_{4U}^{bm}$	$v - \frac{1}{2}$

<sup>3</sup>The details of the derivations can be found in Proposition 1 of Chen and Riordan (2007a).

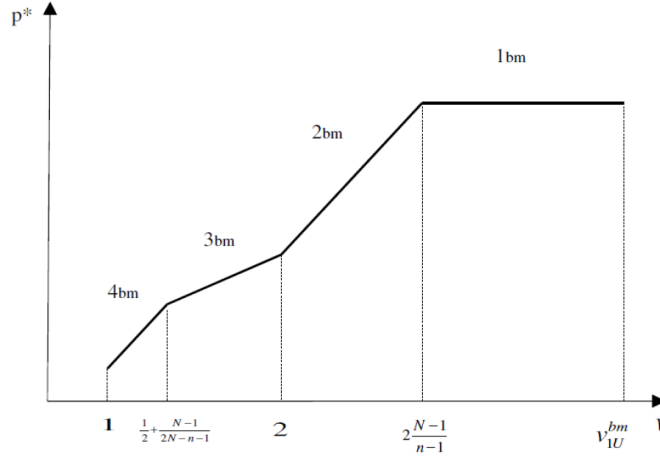


Figure 2: The before merger equilibrium prices, Chen-Riordan (2007a).

As illustrated in Figure 2 the price is a non-decreasing function of the value of the good  $v$ . For values of  $v$  above  $v_{1U}^{bm} = 1 + \frac{(2N-n-1)^2}{2(N-n)(n-1)}$  a pure strategy equilibrium of the game does not exist: a too large valuation of the good implies firms have a unilateral incentive to raise their price to  $p = v - 1$  which is however not an equilibrium either. Our analysis will focus on the equilibrium Regions 1bm-4bm: more details on the features of these equilibria are provided in Chen and Riordan (2007a) and are also discussed in Section 5.2. For even lower values of  $v$  an equilibrium would exist but all firms would be local monopolists serving only part of the consumers located on their spoke.

## 4 Horizontal Mergers

Following the literature, it is assumed that the merging firms maximize their joint profits. The after merger profits are split in equal parts between the participating firms and we abstract from bargaining considerations: in other words, the only effect of a merger is to create a multi-product firm.

#### 4.1 The Effects of a Merger

Suppose that a merger of  $k$  of the  $n$  firms,  $k < n$ , takes place creating  $M$ . Denote by  $i \in I = \{1, \dots, k\}$  a firm belonging to  $M$ . All other firms are symmetric and indexed by  $i \in O = \{k + 1, \dots, n\}$ . Let us denote by  $S$  the set of all firms ( $S = I \cup O$ ). In a market featuring  $N$  spokes, the number of active firms reduces to  $m = n - k + 1$ .

Focus first on the merged firms who constituted  $M$ . The following equation:

$$v - p_i - x = v - p_j - (1 - x) \quad \forall i \in I, \forall j \in S, \forall j \neq i$$

still identifies the indifferent customers who have an alternative brand existing on the market and the set of indifferent consumers is described by:

$$\hat{x}_{ij} = \max \left\{ \min \left\{ \frac{1}{2} + \frac{p_j - p_i}{2}; 1 \right\}, 0 \right\} \quad \forall i \in I, \forall j \in S, \forall j \neq i$$

Notice, however, that now there are two subsets of indifferent consumers: consumers whose other brand is supplied by one of the other firms taking part to the merger ( $j \in M, j \neq i$ ) and consumers whose other brand is supplied by one of the outsiders ( $j \in O$ ). Indifferent customers with no kind of alternative brand are still identified by:

$$\check{x}_i = \max \{ \min \{ v - p_i, 1 \}, 0 \} \quad \forall i \in I$$

To sum up, from the perspective of one of the firms who took part to the merger and constituted firm  $M$  there are three types of customers after the merger; hence, a randomly drawn customer is of a certain type with the following probabilities:

1.  $\frac{k-1}{N-1}$ : probability of a customer that has its second favourite brand supplied by firms located on other spokes but belonging to  $M$ ;
2.  $\frac{n-k}{N-1}$ : probability of a customer that has an alternative brand not supplied by other factories affiliated to  $M$ ;
3.  $\frac{N-n}{N-1}$ : probability of a customer that does not have a second favourite brand.

The merger does not necessarily imply a market expansion effect: the agents that do not have a first or a second favourite brand available are still excluded: the fraction of this type of consumers is unaffected by the merging activity.

The demand function of the merger  $M$  is defined by the sum of the segments served by the  $k$  firms. Proceeding in a similar way as in the benchmark case, the demand for each of the  $k$  segments is given by:

$$D_i(p_i, p_{-i}) = \begin{cases} \frac{2}{N} \frac{1}{N-1} \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^k \left( \frac{1}{2} + \frac{p_\alpha - p_i}{2} \right) + \frac{2}{N} \frac{1}{N-1} \sum_{\alpha=k+1}^n \left( \frac{1}{2} + \frac{p_\alpha - p_i}{2} \right) + \frac{2}{N} \frac{N-n}{N-1} (v - p_i) \\ \text{if } \frac{1}{2} \leq v - p_i \leq 1 \\ \\ \frac{2}{N} \frac{1}{N-1} \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^k \left( \frac{1}{2} + \frac{p_\alpha - p_i}{2} \right) + \frac{2}{N} \frac{1}{N-1} \sum_{\alpha=k+1}^n \left( \frac{1}{2} + \frac{p_\alpha - p_i}{2} \right) + \frac{2}{N} \frac{N-n}{N-1} \\ \text{if } v - p_i > 1 \end{cases} \quad (2)$$

The first term between square brackets represents consumers with both favourite brands being supplied by  $M$ . The second term represents consumers whose second favourite brand is supplied by one of the outsider firms. The third term identifies the demand of the consumers whose only desired brand is supplied by firm  $i$ . The demand of each of these segments is weighted by the respective probabilities of a given consumer being one of the three possible types described above.

Turning to outsider firms, their demand is:

$$D_j(p_j, p_{-j}) = \begin{cases} \frac{2}{N} \frac{1}{N-1} \sum_{\alpha=1}^k \left( \frac{1}{2} + \frac{p_\alpha - p_j}{2} \right) + \frac{2}{N} \frac{1}{N-1} \sum_{\substack{\alpha=k+1 \\ \alpha \neq j}}^n \left( \frac{1}{2} + \frac{p_\alpha - p_j}{2} \right) + \frac{2}{N} \frac{N-n}{N-1} (v - p_j) \\ \text{if } \frac{1}{2} \leq v - p_j \leq 1 \\ \\ \frac{2}{N} \frac{1}{N-1} \sum_{\alpha=1}^k \left( \frac{1}{2} + \frac{p_\alpha - p_j}{2} \right) + \frac{2}{N} \frac{1}{N-1} \sum_{\substack{\alpha=k+1 \\ \alpha \neq j}}^n \left( \frac{1}{2} + \frac{p_\alpha - p_j}{2} \right) + \frac{2}{N} \frac{N-n}{N-1} \\ \text{if } v - p_j > 1 \end{cases} \quad (3)$$

For each case, the three terms represent, respectively, the demand faced from consumers who have, as other favourite, a brand supplied by firms in  $M$ , consumers who have, as other favourite, a brand supplied by another non-merged firm and consumers whose only desired brand is supplied by the firm. The profit functions for the merged entity and for each outsider are respectively:

$$\pi_M = \sum_{\alpha=1}^k p_\alpha D_\alpha(p_\alpha, p_{-\alpha})$$

$$\pi_j = p_j D_j(p_j, p_{-j}) \quad \forall j \in O$$

The first order conditions for the merged and the non-merged firms are, respectively:

$$\frac{\partial \pi_i}{\partial p_i} = D_i(p_i, p_{-i}) + p_i \frac{\partial D_i(p_i, p_{-i})}{\partial p_i} + \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^k p_\alpha \frac{\partial D_\alpha(p_\alpha, p_{-\alpha})}{\partial p_i} = 0 \quad (4)$$

$$\frac{\partial \pi_j}{\partial p_j} = D_j(p_j, p_{-j}) + p_j \frac{\partial D_j(p_j, p_{-j})}{\partial p_j} = 0 \quad \forall i \in I, \forall j \in O \quad (5)$$

Comparing (4) and (5), the effect of the merger is to lead each of the participating firms to internalize the externalities imposed by one's own price choices on the demand for other brands in the merger. This property, first illustrated by Deneckere and Davidson (1985), plays an important role in determining the results and it is further discussed in Section 5. As in the benchmark, regularity conditions, i.e.  $|p_\alpha - p_i| < 1, \forall \alpha \in I, \forall i \in I, \alpha \neq i$ , have to be imposed.

## 4.2 The After-Merger Equilibrium

The after merger equilibrium regions and corresponding prices for the merged and non-merged firms, denoted by  $p_m^*$  and  $p_{nm}^*$ , are characterized in Proposition 1.

**Proposition 1** *In presence of an horizontal merger of  $k$  firms, constituting  $M$ , four equilibrium regions can be identified; they are characterized by the*

following prices and hold for the following ranges of values of  $v$ :

Region	Range	$p_m^*$	$p_{nm}^*$
1am	$]v_{1D}^{am}; v_{1U}^{am}]$	$\frac{(2N-n-1)(2n-1)}{(n-k)(2n+k-2)}$	$\frac{(2N-n-1)(2n-k)}{(n-k)(2n+k-2)}$
2am	$[v_{2D}^{am}; v_{2U}^{am}]$	$v - 1$	$v - 1$
3am	$]v_{3D}^{am}; v_{3U}^{am}[$	$\frac{(4N-2n-1)[2v(N-n)+n-1]}{\Psi}$	$\frac{(4N-2n-k)[2v(N-n)+n-1]}{\Psi}$
4am	$[v_{4D}^{am}; v_{4U}^{am}]$	$v - \frac{1}{2}$	$v - \frac{1}{2}$

where:  $\Psi = 16N^2 - 20Nn - 4Nk - 4N + 6n^2 + 3nk + 2n - k^2 + 2k$  and  $v_{lD}^{am} = \max\{v_{lD}^m, v_{lD}^{nm}\}$ ,  $v_{lU}^{am} = \min\{v_{lU}^m, v_{lU}^{nm}\}$ ,  $\forall l = 1, 2, 3, 4$ , provided that:  $v_{4D}^{am} \leq v_{4U}^{am} \leq v_{3D}^{am} \leq v_{3U}^{am} \leq v_{2D}^{am} \leq v_{2U}^{am} \leq v_{1D}^{am} \leq v_{1U}^{am}$ .

**Proof** see Appendix A.

All the expressions for the threshold values of  $v$  are reported in the Appendix. The derivation of the price expressions is quite straightforward using the demand functions identified in (2)-(3) and the corresponding first-order conditions (4)-(5); finding the values for which the after merger equilibrium regions exist is more elaborate: the steps closely follow the Proof of Proposition 1 in Chen and Riordan (2007a), and are reported in Appendix A.

Proposition 1 implies that four equilibrium regions, Regions 1am-4am, can be identified also in case a merger takes place; these are the analogous of Regions 1bm-4bm in the benchmark case. As a result of the merger, insider and outsider firms choose different prices and, consequently, face potential deviations with different profitability; hence, the equilibrium prices hold for different values of the parameter  $v$  and Regions 1am-4am are defined as the intersection of these regions for both insiders and outsiders. It can be noticed, then, that one effect of the merger is to reduce the overall size of values for which the four equilibrium regions are defined: as discussed in more details in Section 5.2, however, other equilibrium regions exist between the four defined by Proposition 1.



## 5 Results and Discussion

### 5.1 The main result

Bringing together the results of the pre-merger benchmark situation in Section 3 and the post-merger equilibrium in Section 4, we identify *four equilibrium regions*.

**Definition 1** *Four equilibrium regions can be identified:*

$$\begin{aligned}
 \text{Region 1} \quad v_{1D} &= \max\{v_{1D}^{am}, v_{1D}^{bm}\} < v \leq \min\{v_{1U}^{am}, v_{1U}^{bm}\} = v_{1U} \\
 \text{Region 2} \quad v_{2D} &= \max\{v_{2D}^{am}, v_{2D}^{bm}\} \leq v \leq \min\{v_{2U}^{am}, v_{2U}^{bm}\} = v_{2U} \\
 \text{Region 3} \quad v_{3D} &= \max\{v_{3D}^{am}, v_{3D}^{bm}\} < v < \min\{v_{3U}^{am}, v_{3U}^{bm}\} = v_{3U} \\
 \text{Region 4} \quad v_{4D} &= \max\{v_{4D}^{am}, v_{4D}^{bm}\} \leq v \leq \min\{v_{4U}^{am}, v_{4U}^{bm}\} = v_{4U}
 \end{aligned}$$

and they are characterized by the following before and after merger prices for both insiders and outsiders:

Region	$p_{bm}^*$	$p_m^*$	$p_{nm}^*$
1	$\frac{(2N-n-1)}{n-1}$	$\frac{(2N-n-1)(2n-1)}{(n-k)(2n+k-2)}$	$\frac{(2N-n-1)(2n-k)}{(n-k)(2n+k-2)}$
2	$v - 1$	$v - 1$	$v - 1$
3	$\frac{2v(N-n)+(n-1)}{4N-3n-1}$	$\frac{(4N-2n-1)[2v(N-n)+n-1]}{\Psi}$	$\frac{(4N-2n-k)[2v(N-n)+n-1]}{\Psi}$
4	$v - \frac{1}{2}$	$v - \frac{1}{2}$	$v - \frac{1}{2}$

where  $\Psi = 16N^2 - 20Nn - 4Nk - 4N + 6n^2 + 3nk + 2n - k^2 + 2k$ .

The four regions identified in Definition 1 can be seen as the analogous of the ones analysed by Chen and Riordan (2007a) in the context of a merger: the values of parameters corresponding to each region are given by the appropriate boundary between the ones defining the before and after merger equilibria.<sup>4</sup> As we shall see in Section 5.2, equilibria in these regions also share many properties of the ones in Chen and Riordan (2007a), presented in Section 3. Having identified the equilibrium regions, we now provide conditions on the parameters that guarantee that these regions exist.

<sup>4</sup>Formally, Regions 1 to 4 are identified as the intersection of Regions 1bm-4bm and Regions 1am-4am, as defined in Section 3 and Proposition 1. The relevant prices are derived from Table 1 ( $p_{bm}^*$ ) and Proposition 1 ( $p_m^*, p_{nm}^*$ ).

**Lemma 1** *Regions existence: (1) Region 1 exists provided that either: (i)*

$v_{1D} = v_{1D}^{am} = v_{1D}^m < v_{1U} = v_{1U}^{am} = v_{1U}^m$  which requires:

$$N \leq \frac{12n^3 - 8n^2 - 8kn^2 - 3k^2n + 10kn - k^2 + k^3 - k}{2(2n-1)(2n-k)}$$

*or: (ii)  $v_{1D} = v_{1D}^{am} = v_{1D}^m < v_{1U} = v_{1U}^{nm}$  which requires either: (ii.a)*

$$\frac{12n^3 - 8n^2 - 8kn^2 - 3k^2n + 10kn - k^2 + k^3 - k}{2(2n-1)(2n-k)} \leq N \leq \frac{6n^2 - 2n - 3kn + 2k - k^2}{2(2n-k)}$$

and

$$n > \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$$

*or: (ii.b)*

$$\frac{12n^3 - 8n^2 - 8kn^2 - 3k^2n + 10kn - k^2 + k^3 - k}{2(2n-1)(2n-k)} \leq N < \frac{4n^3 - 8n^2 - k - 3k^2n + 8kn}{2(k-1)(2n-k)}$$

and

$$k < n < \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$$

*or: (iii)  $v_{1D} = v_{1D}^{am} = v_{1D}^m < v_{1U} = v_{1U}^{bm}$  which requires either: (iii.a)*

$$N > \frac{6n^2 - 2n - 3kn + 2k - k^2}{2(2n-k)}$$

and

$$n > \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$$

*or: (iii.b)*

$$N > \frac{2n^3 - 6n^2 + 4n + k^2n + kn^2 + k^2 - kn - 2k}{2(k-1)(k+n-1)}$$

and

$$k < n < \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$$

**(2)** *Region 2 exists provided that  $v_{2D} \leq v_{2U}$  which requires:  $N \geq n + \frac{k-1}{2}$ ;*

**(3)** *Region 3 exists provided that  $v_{3D} < v_{3U}$  which requires:  $N > \frac{5n+3k-1}{8} + \frac{\sqrt{n^2+6n+9+21k^2-6kn-30k}}{8}$ ; **(4)** *Region 4 always exists as:  $v_{4D} \leq v_{4U}$ .**

*Proof* See Appendix B.

As Regions 1-4 are identified by the intersection between the before and after merger regions, they do not necessarily need to exist: the lower and upper boundaries may overlap and the equilibrium prices hold for no value of  $v$ . Lemma 1 identifies conditions on the number of spokes and firms for which all four equilibrium regions exist. Section 5.2 discusses what happens if the conditions provided in Lemma 1 do not hold; in the remaining, however, we shall assume that these conditions are satisfied.

Our analysis focuses on the price effects of a merger and now equilibrium prices before and after the merger can be compared.

**Proposition 2** *A merger leads to an increase in the market price of all firms in Region 1 and Region 3; it has no price effect in Region 2 and Region 4.*

*Proof* See Appendix C.

The results in Proposition 2 are graphically illustrated in Figure 3 using a specific parametrization. Before and after-merger prices are plotted in the four equilibrium regions.<sup>5</sup> As it can be seen, in Regions 1 and 3 the after-merger market price dominates the before merger price; in Regions 2 and 4 the two prices coincide.

## 5.2 Discussion

Proposition 2 states that in two of the equilibrium regions prices increase as a consequence of the merger while this is not the case in the remaining two regions. We shall first describe the mechanisms that lead to this result before providing an interpretation.

In Region 1, "standard" oligopolistic competition takes place: as the best response functions are upward sloping, both insiders and outsiders raise

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<sup>5</sup>The after merger price is defined as the average of the prices of insider and outsider firms, with the market shares as weights.

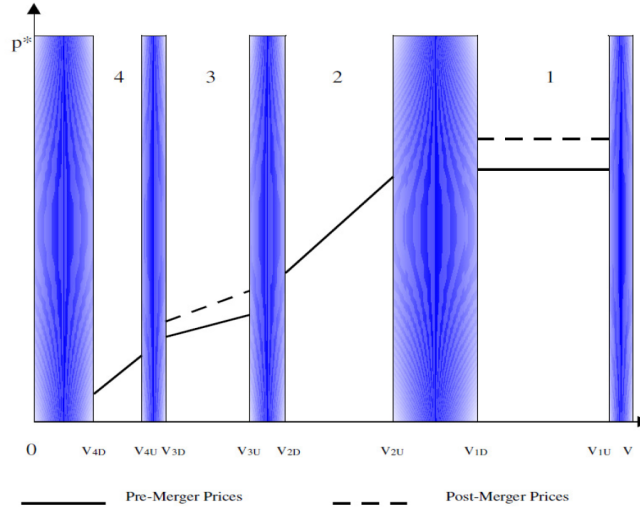


Figure 3: The before and after merger equilibrium price comparison.

their prices, creating an overall increase of prices and earning higher profits compared with the benchmark situation. The mechanisms described by Deneckere and Davidson (1985) apply to mergers in most price competition models and the spokes model is no exception: an outsider, firm  $j$ , faces competition both from the firms constituting  $M$  and from all other outsiders. Then, firm  $j$  shares with a given insider, firm  $i$ ,  $n - 2$  competitors; but both firms face also another competitor. For the outsider firm  $j$  this competitor is a member of the merger  $M$ , i.e. a firm charging a higher price. The insider firm, on the other hand, faces competition of another outsider firm, which is charging a lower price. Hence, the outsider  $j$  faces less fierce price competition and, consequently, its profits are higher than the ones of firms in  $M$ .

In Region 2 a "kinked equilibrium" takes place as firms concentrate on extracting surplus from consumers who do not have a second favourite brand and are indifferent between buying or not. For this reason, the merger does not affect prices and profits. The same amount of consumers takes part to the market and prices are unaffected.

Region 3 features a price increase similar to Region 1: the reason, however, is less intuitive in this case. Competition in this region implies extracting all surplus from the marginal consumer who lacks a second alternative brand. A consequence of this feature is that the elasticity of demand is larger on the monopolistic segment; this leads to price increasing competition. Despite this, the mechanisms as in Region 1 are in operation: the best response functions are still upward sloping so that prices of both insiders and outsiders increase, leading also to an increase in profits with respect to the benchmark situation.

In Region 4 the "kinked equilibrium" is of a different type: firms focus their attention on the indifferent consumers who have its brand as a first choice. As in Region 3, also in this region not all consumers with at least one favourite brand are served. However, as in Region 2, the kinked nature of the equilibrium implies that the prices remain unchanged even when a merger takes place.

The analysis of the four equilibrium regions has shown how several economic mechanisms operate in the spokes model. These mechanisms determine the price effects of an horizontal merger between firms. The results can be interpreted as follows: when genuine price competition is in operation, as in Regions 1 and 3, then the "free-riding" property of the equilibrium takes place and the classical price increase result is confirmed. Such price effects imply that mergers have a negative impact not only on consumers but also on overall welfare. Asymmetric price increases are reflected on different equilibrium demand shares for different firms: this leads to an increase of overall transport costs, impacting negatively on welfare. "Kinked equilibria", however, take place in Regions 2 and 4 and the equilibrium prices are independent of whether a subset of firms merge. In other words, when firms target a specific key segment of the market, mergers do not have any price effect. The results, then, seem to provide a relevant policy implication: if a market is characterized by product differentiation and non-localized price competition, then a merger may not have detrimental effects for consumers and welfare. In fact, it was shown that in two of the four regions considered,

mergers do not have any price effect at all. The results of the model seem consistent with the "ex-post" empirical evidence that not all mergers have important price effects, even when cost synergies, substitute goods, collusion or other factors do not play a key role.

The regions in which no price effect takes place do not capture a peculiar or special case. First of all, the size of the sub-space of parameters for which such equilibria take place may be non negligible as witnessed, for a specific parametrization, by Figure 3. Secondly, in Region 4, firms focus on the marginal consumer who does not have an alternative brand and hence they only serve consumers on their own spoke; however, in Region 2, firms serve all types of consumers apart from the ones which would be cut out of the market in any case because none of the brands they like is available. In this sense competition between firms is fully in operation.

The four regions discussed do not, of course, exhaust the space of parameters; further regions can be identified as, for example, in the shadowed areas in Figure 3. In this paper we focused on Regions 1-4 for comparability with Chen and Riordan (2007a). The results in the shaded regions are quantitatively different from what we presented in this paper; qualitatively, however, they are similar and do not add to the understanding and interpretation of the price effects. In those regions, in fact, a price increase takes place following a merger in a similar fashion and for the same reasons as in Region 1 and Region 3 presented above. This further implies that if any of the Regions 1-3 does not exist, as by Lemma 1, regions with similar characteristics still exist.

## 6 Conclusions

Antitrust authorities focus much of their attention on the price effects of mergers: when two or more firms operating in the same market merge, the concentration of the market increases and this may drive to undesirable price effects. This paper provides a simple theoretical interpretation of the price effects of horizontal mergers which seems consistent with the

empirical evidence: many mergers have an important impact on prices but cases of negligible price effects are not rare either. Negligible price effects may have several explanations: the outcome of cost synergies between the merged firms, prior collusion between firms or others that the literature has suggested; however, negligible price effects may simply result from market interactions: our paper provides a theoretical underpinning for the latter possibility. The results are provided through the analysis of horizontal mergers in a context of spatial but non-localized competition: two key features of the spokes model, a recently introduced tool to address non-localized competition, are that proximity between firms plays no role and that not all spokes may feature a firm located on it. These properties allow to identify four types of equilibria with different features. In two of the equilibria, sensible price effects take place and the "free riding" mechanism described by Deneckere and Davidson (1985) is in operation. The merger modifies firms' reaction functions and drives both insiders and outsiders to raise their prices, with consequent harm to consumers' and overall welfare. The latest effect operates through increased transportation costs due to the asymmetric demand effects of price increases. The two remaining equilibria, however, have different properties: in those, firms focus only on a specific type of consumer. Mergers then have no effects on prices, which are simply determined by the "kink" in the demand function that is targeted by firms. The results, then, suggest that a merger will not necessarily imply a sharp increase in the price level: whether this is the case or not will actually depend on the type of consumers that firms are targeting when setting their prices.

The results provided do not exhaust the applicability of the spokes model for the analysis of the effects of horizontal mergers. First, in our framework free entry may change firms' incentives to merge: a more relaxed pricing environment might induce further entry on the empty spokes with the possibility of further reducing merged firms' profit. The interaction between mergers and entry constitutes an interesting direction to extend the model. Secondly, the spokes model can be used to address some of the questions posed by the endogenous mergers literature. Firms might differ in two di-

mensions: marginal cost and the number of varieties produced. In such a setting, one could evaluate which firms would merge and with what consequences. For example, as in Pita Barros (1998) and Socorro (2004), a merger between some firms with asymmetric production costs may imply that the merged firms can produce at a cost equal to the most efficient of its participants. Thus a merger entails a rationalisation gain since production can be re-allocated from a high-cost to a low-cost plant. The evaluation of the effects of similar mergers in the context of the spokes model is an interesting topic for future research.

## References

- [1] Alderighi M. and C. Piga (2012), Localized Competition, Heterogeneous Firms and Vertical Relations, *Journal of Industrial Economics*, vol. 60 (1), 46-74.
- [2] Ashenfelter O. and D. Hosken (2010), The effect of mergers on consumer prices: Evidence from five mergers on the enforcement margin, *Journal of Law and Economics*, vol. 53 (3), 417-466.
- [3] Ashenfelter O., Hosken D. and M. Weinberg (2009), Generating Evidence to Guide Merger Enforcement, NBER Working Paper 14798.
- [4] Ashenfelter O., Hosken D. and M. Weinberg (2011), The price effects of a large merger of manufacturers: a case study of Matyag-Whirlpool, SSRN Working paper.
- [5] Borla S. (2012), Spatial Competition and Merging Incentives when Firms Produce Complements, *Regional Science and Urban Economics*, vol. 42 (1-2), 221-229.
- [6] Braid R.M. (1999), The Price and Profit Effects of Mergers in Two Dimensional Spatial Competition, *Economics Letters*, vol. 62, 113-119.



- [7] Brito D. (2003), Preemptive Mergers under Spatial Competition , International Journal of Industrial Organization, vol. 21, 1601-1622.
- [8] Caminal R. (2010), "Markets and linguistic diversity", Journal of Economic Behavior & Organization, vol. 76 (3), 774-790.
- [9] Caminal R. and A. Clatici (2007), Are Loyalty Rewarding Schemes Anti-Competitive?, International Journal of Industrial Organization, vol. 25 (4), 657-674.
- [10] Caminal R. and L.M. Granero (2012), Multi-Product Firms and Product Variety, *Economica*, vol. 79 (314), 303-328.
- [11] Chen Y. and M. Riordan (2007a), Price and Variety in the Spokes Model, *Economic Journal*, vol.117, 897-921.
- [12] Chen Y. and M. Riordan (2007b), Vertical Integration, Exclusive Dealing and Ex-Post Cartelization, *Rand Journal of Economics*, vol.38, 1-21.
- [13] Connor R., Feldman R. and B. Dowd (1998), The Effects of Market Concentration and Horizontal Mergers on Hospital Costs and Prices, *International Journal of Economics Business*, vol.159, 180.
- [14] Csorba G., Koltay G. and D. Farkas (2011), Separating the ex post effects of mergers: an analysis of structural changes on the Hungarian retail gasoline market, *IEHAS Discussion Papers* 1118.
- [15] Deneckere R. and C. Davidson (1985), Incentives to Form Coalitions with Bertrand Competition, *Rand Journal of Economics*, vol.16, 473-486.
- [16] Economides N. (1989), Symmetric Equilibrium Existence and Optimality in Differentiated Product Markets, *Journal of Economic Theory*, vol. 47 (1), 178-194.
- [17] Economides N. (1993), Hotelling's "Main Street" with More than Two Competitors, *Journal of Regional Science*, vol. 33 (3), 303-319.

- [18] Ganuza J.J. and E. Hauk (2005), Allocating Ideas: Horizontal Competition in Tournaments, Universitat Pompeu Fabra, Working Paper.
- [19] Germano F. (2009), "On commercial media bias", Economics Working Papers 1133, Universitat Pompeu Fabra
- [20] Germano F. and Meier (2010), "Concentration and self-censorship in commercial media," Economics Working Papers 1256, Universitat Pompeu Fabra.
- [21] Jiménez J. and J. Perdiguero (2012), Mergers and difference-in-difference estimator: why firms do not increase prices?, Research Institute of Applied Economics, Working Paper.
- [22] Mantovani A. and F. Ruiz Aliseda (2011), "Equilibrium Innovation Ecosystems: The Dark Side of Collaborating with Complementors", NET Institute Working Paper 11-31.
- [23] Neumann M. and M. Sanderson (2007), Ex post merger review : an evaluation of three competition bureau merger assessments, Final Report, CRA International.
- [24] Pautler P.A. (2001), Evidence on Mergers and Acquisitions, Federal Trade Commission Working Paper.
- [25] Pita Barros P. (1998), Endogenous mergers and size asymmetry of merger participants, Economics Letters, vol.60, 113-119.
- [26] Rothschild R. (2000), Merger under spatial competition, Urban Studies, vol. 37, 443-449.
- [27] Salop S.C. (1979), Monopolistic Competition with Outside Goods, Bell Journal of Economics, vol. 10, 141-156.
- [28] Simpson J. and C. Taylor (2008), Do gasoline mergers affect consumer prices ? The Marathon Ashland Petroleum and Ultraman Diamond Shamrock transaction, The Journal of Law and Economics, vol. 51, 135-152.

- [29] Socorro P. (2004), Mergers and the importance of fitting well, *Economics Letters*, vol. 82, 269-274.
- [30] Syverson C. (2004), Market Structure and Productivity: A Concrete Example, *Journal of Political Economy*, vol.112, 1181-1222.
- [31] Taylor C. and D. Hosken (2007), The economic effects of the Marathon-Ashland joint venture: the importance of industry supply shocks and vertical market structure, *Journal of Industrial Economics*, vol.55, 419-451.
- [32] Vickrey W. (1964), *Microstatics*, Harcourt-Brace and World, New York.
- [33] Vogel J. (2008), Spatial Competition with Heterogeneous Firms, *Journal of Political Economy*, vol. 116 (3), 423-466.
- [34] Weinberg M. (2008), The Price Effects of Horizontal Mergers, *Journal of Competition Law and Economics*, vol. 4 (2), 433-447.
- [35] Whinston M. (2007), Antitrust Policy Towards Horizontal Mergers. In: M. Armstrong, R. Porter (Eds.). *Handbook of Industrial Organization*, Vol. III. Amsterdam, North Holland, 2369-2440.

## A Proof of Proposition 1

As reported in the proposition, there are two types of firms (firms constituting  $M$  and outsiders  $j$ ) and four candidate equilibrium prices for each. Following similar steps as the Proof of Proposition 1 in Chen and Riordan (2007a), this proof identifies the equilibrium prices and the values of  $v$  for which each candidate prices constitute an equilibrium.

*Region 1am*

*Merged entity M*: from the system of the first order conditions (4) and (5), evaluated using the second expressions in (2) and (3), the candidate

equilibrium price for all firms constituting  $M$  is:  $p_m^* = \frac{(2N-n-1)(2n-1)}{(n-k)(2n+k-2)}$ . From (2), the equilibrium requires  $v - p_m^* > 1$ , hence:

$$v > v_{1D}^m = 1 + \frac{(2N-n-1)(2n-1)}{(n-k)(2n+k-2)}. \quad (6)$$

As both the first order and second order conditions are satisfied at the candidate equilibrium, the most profitable possible deviation for firm  $M$  is to increase the price so that it reaches the demand kink at  $p^D = v - 1$ ; in that case, it is most profitable for the firm to change all  $k$  prices rather than only a subset of them. If all  $k$  prices change, the deviation is not profitable if:  $\Pi_M^* \geq \Pi_M^D = \sum_{i=1}^k (v-1)D_i(p^D, p_{nm}^*)$ . The latter implies that the possible deviation is not profitable for firm  $M$  provided that:

$$v \leq v_{1U}^m = 1 + \frac{(2n-1)^2}{(n-k)} \frac{(2N-n-1)^2}{(2n+k-2)[4n(N-n) - 2N + n(k+1) + (k-1)^2]} \quad (7)$$

It follows that the candidate prices are an equilibrium for  $M$  for the following values of  $v$ :  $v_{1D}^m < v \leq v_{1U}^m$ .

*Outsider firm  $j$* : from the system of the first order conditions (4) and (5), evaluated using the second expressions in (2) and (3), the candidate equilibrium price for a representative non-merged firm  $j$  ( $j \in O$ ) is:  $p_{nm}^* = \frac{(2N-n-1)(2n-k)}{(n-k)(2n+k-2)}$ . From (2), the equilibrium requires  $v - p_{nm}^* > 1$ , hence:

$$v > v_{1D}^{nm} = 1 + \frac{(2N-n-1)(2n-k)}{(n-k)(2n+k-2)}. \quad (8)$$

As both the first order and second order conditions are satisfied at the candidate equilibrium, the most profitable possible deviation for firm  $j$  is to increase the price so that it reaches the demand kink at  $p^D = v - 1$ ; the deviation is not profitable if  $\pi_j^* \geq \pi_j^D = (v-1)D_i(p_m^*, p^D, p_{nm}^*)$ . From the latter, the possible deviation is not profitable for firm  $M$  provided that:

$$v \leq v_{1U}^{nm} = 1 + \frac{(2n-k)^2}{(n-k)} \frac{(2N-n-1)^2}{(2n+k-2)[2(N-n)(2n-k) + k(k-1)]} \quad (9)$$

It follows that the candidate price is an equilibrium for outsider firms for the following values of  $v$ :  $v_{1D}^{nm} < v \leq v_{1U}^{nm}$ .

By comparing (6) and (8),  $v_{1D}^m$  is larger than  $v_{1D}^{nm}$  and, as such, constitutes the stricter of the two constraints to guarantee that no firm has an incentive to deviate. A similar comparison on (7) and (9) is not conclusive. Hence, the candidate prices constitute an equilibrium for the following values of  $v$ :

$$v_{1D}^{am} = \max\{v_{1D}^m, v_{1D}^{nm}\} = v_{1D}^m < v \leq \min\{v_{1U}^m, v_{1U}^{nm}\} = v_{1U}^{am}$$

*Region 2am*

*Merged entity M*: the candidate equilibrium price is  $p_m^* = v - 1$ . The merged firm  $M$  may consider to change  $h$  of its  $k$  prices ( $1 \leq h \leq k$ ). The two possible deviations are to higher prices:  $p_i > v - 1$  or to a lower prices:  $p_i < v - 1$ . Suppose first  $M$  considers to increase  $h$  of its prices to  $p_i > v - 1$ . In that case they face a demand given by the second expression in (2). All other firms stick to their equilibrium prices. Such potential deviation is not profitable provided that:

$$\frac{d\Pi_M}{dp} = \sum_{i=1}^k \frac{\partial \pi_i}{\partial p_i} dp_i \Big|_{p_i > v-1, p_m^*, p_{nm}^*} \leq 0, \forall i \in I$$

where  $dp_i = 0$  for all the remaining  $(k - h)$  prices that are not changed. As the deviation profit increases in  $h$ , the most profitable possible deviation takes place as  $h = k$ . Hence, to ensure that the possible deviation is not guaranteeing more profits than at the equilibrium requires:

$$v \geq v_{2D}^m = \frac{4N - 2n - k - 1}{2N - n - k} \quad (10)$$

Suppose now that  $M$  considers to change  $h$  prices to  $p_i < v - 1$ . In that case they face a demand given by the first expression in (2). All other firms stick to their equilibrium prices. Such potential deviation is not profitable

provided that:

$$\frac{d\Pi_M}{d\bar{p}} = \sum_{i=1}^k \frac{\partial \pi_i}{\partial p_i} dp_i \Big|_{p_i < v-1, p_m^*, p_{nm}^* \leq 0, \forall i \in I}$$

where  $dp_i = 0$  for all the remaining  $(k - h)$  prices that are not changed. As the deviation profit increases in  $h$ , the most profitable possible deviation takes place as  $h = k$ . The possible deviation should not lead to higher profits than at the candidate equilibrium; this requires the following condition:

$$v \leq v_{2U}^m = \frac{2N - k - 1}{n - k} \quad (11)$$

It follows that the candidate prices are an equilibrium for  $M$  for the following values of  $v$ :  $v_{2D}^m \leq v \leq v_{2U}^m$ .

*Outsider firm  $j$* : the candidate equilibrium price for a representative non-merged firm  $j$  ( $j \in O$ ) is  $p_{nm}^* = v - 1$ ; an analogous reasoning can be used to rule out possible deviations. Suppose that firm  $j$  considers raising its price to  $p_j > v - 1$ . In that case the demand faced is given by the second expression in (3). Such potential deviation is not profitable provided that:

$$\frac{\partial \pi_j}{\partial p_j} \Big|_{p_m^*, p_{nm}^*, p_j > v-1} = p_j \frac{\partial D_j}{\partial p_j} + D_j \Big|_{p_m^*, p_{nm}^*, p_j > v-1} \leq 0, \forall j \in O$$

which is equivalent to require:

$$v \geq v_{2D}^{nm} = 2 \quad (12)$$

Suppose firm  $j$  consider decreasing the price to  $p_j < v - 1$ . In that case the demand faced is given by the first expression of (3). Such potential deviation is not profitable provided that:

$$\frac{\partial \pi_j}{\partial p_j} \Big|_{p_m^*, p_{nm}^*, p_j < v-1} = p_j \frac{\partial D_j}{\partial p_j} + D_j \Big|_{p_m^*, p_{nm}^*, p_j < v-1} \leq 0, \forall j \in O$$

which is equivalent to:

$$v \leq v_{2U}^{nm} = 2 \frac{N - 1}{n - 1} \quad (13)$$

It follows that the candidate price is an equilibrium for outsider firms for the following values of  $v$ :  $v_{2D}^{nm} \leq v \leq v_{2U}^{nm}$ .

By comparing (10) and (12),  $v_{2D}^m$  is larger than  $v_{2D}^{nm}$ ; a similar comparison on (11) and (13) leads to conclude that  $v_{2U}^m$  is smaller than  $v_{2U}^{nm}$  so that the candidate prices constitute an equilibrium for:

$$v_{2D}^{am} = \max\{v_{2D}^m, v_{2D}^{nm}\} = v_{2D}^m \leq v \leq \min\{v_{2U}^m, v_{2U}^{nm}\} = v_{2U}^m = v_{2U}^{am}.$$

*Region 3am*

*Merged entity M*: the system of the first order conditions (4) and (5), evaluated using the first expressions in (2) and (3) provide the candidate equilibrium prices for the merged firm  $M$ :  $p_m^* = \frac{(4N-2n-1)[2v(N-n)+n-1]}{\Psi}$ . From (2), the equilibrium requires  $\frac{1}{2} < v - p_m^* < 1$ , implying:

$$v > v_{3D}^m = \frac{16N^2 - 4Nk - 12Nn - 12N - k^2 + 3kn + 2k + 2n^2 + 4n + 2}{16N^2 - 8Nk - 16Nn - 4N - 2k^2 + 6kn + 4k + 4n^2} \quad (14)$$

and:

$$v < v_{3U}^m = \frac{16N^2 - 4Nk - 16Nn - 8N - k^2 + 3kn + 2k + 4n^2 + 3n + 1}{8N^2 - 4Nk - 8Nn - 2N - k^2 + 3kn + 2k + 2n^2} \quad (15)$$

It can be verified that the second order conditions are satisfied at the candidate equilibrium prices. It follows that the candidate prices are an equilibrium for  $M$  for the following values of  $v$ :  $v_{3D}^m < v < v_{3U}^m$ .

*Outsider firm j*: consider now a representative non-merged firm  $j$  ( $j \in O$ ); from the system of the first order conditions (4) and (5), evaluated using the first expressions in (2) and (3), the candidate equilibrium price is:  $p_{nm}^* = \frac{(4N-2n-k)[2v(N-n)+n-1]}{\Psi}$ . By (2), the equilibrium requires  $\frac{1}{2} < v - p_{nm}^* < 1$ , implying:

$$v > v_{3D}^{nm} = \frac{16N^2 - 4Nk - 12Nn - 12N - k^2 + kn + 4k + 2n^2 + 6n}{16N^2 - 4Nk - 16Nn - 8N - 2k^2 + 2kn + 4k + 4n^2 + 4n} \quad (16)$$

and:

$$v < v_{3U}^{nm} = \frac{16N^2 - 4Nk - 16Nn - 8N - k^2 + 2kn + 3k + 4n^2 + 4n}{8N^2 - 2Nk - 8Nn - 4N - k^2 + kn + 2k + 2n^2 + 2n} \quad (17)$$

It can be verified that the second order conditions are satisfied at the candidate equilibrium price. It follows that  $p_{nm}^*$  is the equilibrium price for firm  $j$  for:  $v_{3D}^{nm} < v < v_{3U}^{nm}$ .

Comparing the thresholds,  $v_{3D}^m$  is larger than  $v_{3D}^{nm}$  and  $v_{3U}^{nm}$  is smaller than  $v_{3U}^m$ . Hence, the candidate prices constitute an equilibrium for the following values of  $v$ :

$$v_{3D}^{am} = \min\{v_{3D}^m, v_{3D}^{nm}\} = v_{3D}^m < v < \min\{v_{3U}^m, v_{3U}^{nm}\} = v_{3D}^{nm} = v_{3U}^{am}$$

#### *Region 4am*

*Merged entity M*: the candidate equilibrium prices are in this case  $p_m^* = v - \frac{1}{2}$ . The merged firm  $M$  may consider to change  $h$  of its  $k$  prices ( $1 \leq h \leq k$ ). The two possible deviations are to higher prices:  $p_i > v - \frac{1}{2}$  or to a lower prices:  $p_i < v - \frac{1}{2}$ . Suppose first  $M$  considers to change  $h$  prices to  $p_i > v - \frac{1}{2}$ . In this case, firms face a demand as if they were local monopolist on their market segment. As all other firms stick to the candidate equilibrium prices, such a potential deviation is not profitable provided that:

$$\frac{d\Pi_M}{d\bar{p}} = \sum_{i=1}^k \frac{\partial \pi_i}{\partial p_i} dp_i \Big|_{p_i > v - \frac{1}{2}, p_m^*, p_{nm}^*} \leq 0, \forall i \in I$$

where  $dp_i = 0$  for all the remaining  $(k - h)$  prices that are not changed. The profits obtained from the deviation do not depend on  $h$ ; hence:

$$v \geq v_{4D}^m = 1$$

Suppose now that  $M$  considers to change  $h$  prices to  $p_i < v - \frac{1}{2}$ . In that case they face a demand given by the second expression in (2). The potential deviation is not profitable provided that:

$$\frac{d\Pi_M}{d\bar{p}} = \sum_{i=1}^k \frac{\partial \pi_i}{\partial p_i} dp_i \Big|_{p_i < v - \frac{1}{2}, p_m^*, p_{nm}^*} \leq 0, \forall i \in I$$

where  $dp_i = 0$  for all the remaining  $(k - h)$  prices that are not changed. As the deviation profit increases in  $h$ , the most profitable possible deviation



takes place as  $h = k$ . The possible deviation should not lead to higher profits than at the candidate equilibrium, requiring:

$$v \leq v_{4U}^m = \frac{4N - n - k - 2}{2(2N - n - 1)}. \quad (18)$$

It follows that the candidate prices are an equilibrium for  $M$  for the following values of  $v$ :  $v_{4D}^m \leq v \leq v_{4U}^m$ .

*Outsider firm  $j$* : the candidate equilibrium price for a representative non-merged firm  $j$  ( $j \in O$ ) is  $p_{nm}^* = v - \frac{1}{2}$ ; an analogous reasoning can be used to rule out possible deviations. Suppose that firm  $j$  considers raising its price to  $p_j > v - \frac{1}{2}$ . In this case, the firm would face a demand as if it was a local monopolist. The potential deviation is not profitable provided that:

$$\frac{\partial \pi_j}{\partial p_j} \Big|_{p_m^*, p_{nm}^*, p_j > v - \frac{1}{2}} = p_j \frac{\partial D_j}{\partial p_j} + D_j \Big|_{p_m^*, p_{nm}^*, p_j > v - \frac{1}{2}} \leq 0, \forall j \in O$$

which, as in the previous case, implies:

$$v \geq v_{4D}^{nm} = 1.$$

Suppose firm  $j$  consider decreasing the price to  $p_j < v - 1$ . In that case the demand faced by the firm is given by the second expression in (3). The potential deviation is not profitable provided that:

$$\frac{\partial \pi_j}{\partial p_j} \Big|_{p_m^*, p_{nm}^*, p_j < v - \frac{1}{2}} = p_j \frac{\partial D_j}{\partial p_j} + D_j \Big|_{p_m^*, p_{nm}^*, p_j < v - \frac{1}{2}} \leq 0, \forall j \in O$$

which implies:

$$v \leq v_{4U}^{nm} = \frac{4N - n - 3}{2(2N - n - 1)}. \quad (19)$$

It follows that the candidate prices are an equilibrium for  $M$  for the following values of  $v$ :  $v_{4D}^{nm} \leq v \leq v_{4U}^{nm}$ .

By comparing (18) and (19),  $v_{4U}^{nm}$  is smaller than  $v_{4U}^m$  so the candidate prices constitute an equilibrium for the following values of  $v$ :

$$v_{4D}^{am} = \max\{v_{4D}^m, v_{4D}^{nm}\} \leq v \leq \min\{v_{4U}^m, v_{4U}^{nm}\} = v_{4U}^m = v_{4U}^{am}.$$

*Q.E.D.*

## B Proof of Lemma 1

1) Existence of Region 1 requires:  $\max\{v_{1D}^m, v_{1D}^{nm}, v_{1D}^{bm}\} = v_{1D} < v \leq v_{1U} = \min\{v_{1U}^m, v_{1U}^{nm}, v_{1U}^{bm}\}$ . It is simple to show that:  $v_{1D} = v_{1U}^m$ .

(i) A first scenario implies:  $v_{1U} = v_{1U}^m = \min\{v_{1U}^m, v_{1U}^{nm}, v_{1U}^{bm}\}$ . This requires  $v_{1U}^m \leq v_{1U}^{nm}$  and  $v_{1U}^m \leq v_{1U}^{bm}$ ; both of these are satisfied provided that:  $N \leq \frac{12n^3 - 8n^2 - 8kn^2 - 3k^2n + 10kn - k^2 + k^3 - k}{2(2n-1)(2n-k)}$ . At the same time, this also implies:  $v_{1D}^m < v_{1U}^m$ .

(ii) Another scenario is:  $v_{1U} = v_{1U}^{nm} = \min\{v_{1U}^m, v_{1U}^{nm}, v_{1U}^{bm}\}$ . This is the case if  $v_{1U}^{nm} \leq v_{1U}^m$  and  $v_{1U}^{nm} \leq v_{1U}^{bm}$ , implying:  $\frac{12n^3 - 8n^2 - 8kn^2 - 3k^2n + 10kn - k^2 + k^3 - k}{2(2n-1)(2n-k)} \leq N \leq \frac{6n^2 - 2n - 3kn + 2k - k^2}{2(2n-k)}$ .

However, for Region 1 to exist we should also have  $v_{1D}^m < v_{1U}^{nm}$  which requires  $N < \frac{4n^3 - 8n^2 - k - 3k^2n + 8kn}{2(k-1)(2n-k)}$ . The latter is not binding if:  $n > \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$ . However, if  $k < n < \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$  then:  $\frac{12n^3 - 8n^2 - 8kn^2 - 3k^2n + 10kn - k^2 + k^3 - k}{2(2n-1)(2n-k)} \leq N < \frac{4n^3 - 8n^2 - k - 3k^2n + 8kn}{2(k-1)(2n-k)}$ .

(iii) Finally, the last scenario is:  $v_{1U} = v_{1U}^{bm} = \min\{v_{1U}^m, v_{1U}^{nm}, v_{1U}^{bm}\}$ . This is the case if  $v_{1U}^{bm} \leq v_{1U}^m$  and  $v_{1U}^{bm} \leq v_{1U}^{nm}$  implying:  $N \geq \frac{6n^2 - 2n - 3kn + 2k - k^2}{2(2n-k)}$ . However, for Region 1 to exist we should have  $v_{1D}^m < v_{1U}^{bm}$  which requires  $N > \frac{2n^3 - 6n^2 + 4n + k^2n + kn^2 + k^2 - kn - 2k}{2(k-1)(k+n-1)}$  which is binding if:  $k < n < \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$ . However, it is not binding if:  $n > \frac{2k+1}{4} + \frac{\sqrt{12k^2 - 20k + 9}}{4}$ .

2) Existence of Region 2 requires:  $\max\{v_{2D}^m, v_{2D}^{nm}, v_{2D}^{bm}\} = v_{2D} \leq v \leq v_{2U} = \min\{v_{2U}^m, v_{2U}^{nm}, v_{2U}^{bm}\}$ .

A simple comparison leads to establish that:  $v_{2D} = \max\{v_{2D}^{bm}, v_{2D}^{nm}, v_{2D}^m\} = v_{2D}^m$  and  $v_{2U} = \min\{v_{2U}^{bm}, v_{2U}^{nm}, v_{2U}^m\} = v_{2U}^{bm} = v_{2U}^{nm}$ . Moreover, it needs to be checked that:  $v_{2D} \leq v_{2U}$  implying:  $N \geq n + \frac{k-1}{2}$ .

3) Existence of Region 3 requires:  $\max\{v_{3D}^m, v_{3D}^{nm}, v_{3D}^{bm}\} = v_{3D} < v < v_{3U} = \min\{v_{3U}^m, v_{3U}^{nm}, v_{3U}^{bm}\}$ .

A simple comparison leads to establish that:  $v_{3D} = \max\{v_{3D}^m, v_{3D}^{nm}, v_{3D}^{bm}\} = v_{3D}^m$  and  $v_{3U} = \min\{v_{3U}^m, v_{3U}^{nm}, v_{3U}^{bm}\} = v_{3U}^{bm}$ . Moreover, it needs to be checked that:  $v_{3D} < v_{3U}$  implying:  $N > \frac{5n+3k-1}{8} + \frac{\sqrt{n^2+6n+9+21k^2-6kn-30k}}{8}$ .

4) Existence of Region 4 requires:  $\max\{v_{4D}^m, v_{4D}^{nm}, v_{4D}^{bm}\} = v_{4D} \leq v \leq v_{4U} = \min\{v_{4U}^m, v_{4U}^{nm}, v_{4U}^{bm}\}$ .

A simple comparison leads to establish that:  $v_{4D} = \max \{v_{4D}^{bm}, v_{4D}^{nm}, v_{4D}^m\} = v_{4D}^m = v_{4D}^{nm} = v_{4D}^m$  and  $v_{4U} = \min \{v_{2U}^{bm}, v_{2U}^{nm}, v_{2U}^m\} = v_{4U}^m$ . As  $k < n$ ,  $v_{4D} \leq v_{4U}$  is surely satisfied.

*Q.E.D.*

## C Proof of Proposition 2

In Region 1 the difference between the pre- and post-merger equilibrium prices are:

$$\begin{aligned} p_m^* - p_{bm}^* &= \frac{(k-1)(n+k-1)(2N-n-1)}{(n-1)(n-k)(2n+k-2)} \\ p_{nm}^* - p_{bm}^* &= \frac{k(k-1)(2N-n-1)}{(n-1)(n-k)(k+2n-2)} \end{aligned}$$

As  $k < n$ , both are strictly positive. Then  $p_m^* > p_{bm}^*$ .

In Region 2 it is immediate to verify that  $p_m^* = p_{nm}^* = p_{bm}^* = v - 1$ .

In Region 3, the price differentials are:

$$p_m^* - p_{bm}^* = (k-1) \frac{4N - 3n + k - 1}{4N - 3n - 1} \frac{2Nv - 2nv + n - 1}{16N^2 - 4Nk - 20Nn - 4N - k^2 + 3kn + 2k + 6n^2 + 2n}$$

and

$$p_{nm}^* - p_{bm}^* = \frac{k(k-1)}{4N - 3n - 1} \frac{2Nv - 2nv + n - 1}{16N^2 - 4Nk - 20Nn - 4N - k^2 + 3kn + 2k + 6n^2 + 2n}$$

As  $k < n \leq N$ , both  $p_m^*$  and  $p_{nm}^*$  are higher than  $p_{bm}^*$ .

In Region 4,  $p_m^* = p_{nm}^* = p_{bm}^* = v - \frac{1}{2}$ .

*Q.E.D.*