



Bhāskara I's versified solutions of a linear indeterminate equation

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Bhāskara I's versified solutions of a linear indeterminate equation^{*}

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Abstract

At the end of Bhāskara's 7th century commentary on the mathematical part of the *Ārya-bhaṭṭya* 91 verses provide solutions of linear indeterminate equations using astral parameters.

The number of civil days and the number of revolutions elapsed since the beginning of the

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Kali Yuga are computed knowing specific mean positions of the Sun, the Moon, the Moon's Apogee, the Moon's ascending node, Mars, Mercury, Jupiter, Venus, Saturn, and the Moon's Anomaly. Other computed elements concern intercalary months, intercalary days, omitted days and the sun's declination. This article provides an English translation of the verses, an explanation and exploration of the data provided. It shows how the verses are used to tabulate data in a compact form. I propose these tables had two aims: first to be used as a tool when solving more elaborate equations; and second as an exploration of the ratios between planet positions and time elapsed, if not a more general exploration of the numbers involved in these ratios.

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Abbreviations and Notations

Sources

Ab The *Āryabhaṭīya* of Āryabhaṭa (K. S. Shukla; K. V. ; Sharma, 1976).

BAB The *Āryabhaṭīyabhāṣya* of Bhāskara (K. S. Shukla, 1976); (Keller, 2006).

BAB.2 corresponds to the *Āryabhaṭīyabhāṣya*'s commentary on the second chapter of the *Āryabhaṭīya*, the one devoted to mathematics. Thus BAB.2.32 corresponds to Bhāskara's commentary on verse 32 of the second chapter of the *Āryabhaṭīya*.

BAB.Ku corresponds to the portion of *Āryabhaṭīyabhāṣya* containing the 91 verses studied here. Thus BAB.Ku.21 corresponds to verse 21 of this series.

Conventions

I've used the usual transliteration of Sanskrit, when I wanted to make words and phrases accessible to non-sanskritists. In general then, when quotations are made for the benefit of sanskritists alone, i've used devanagari transcriptions. The only non-conventional transliteration and transcription of Sanskrit I used was for the term Kali Yuga- and not *Kali-yuga* or कलियुग- which I believe belongs now to English.

In translations of Bhāskara's text, [] stand for addendums by the editor, K. S. Shukla, ⟨⟩ stand for my own addendums, () give specifications.

Mathematical Notation

KU(a, b, c)[x,y]

Pulverizer (*kuṭṭaka* or *kuṭṭākāra*) for $a, b, x, y \in \mathbb{N}^*, c \in \mathbb{Z}^*, a, b, c$ known, x, y unknown.

a notation for the problem and procedure to solve, $y = \frac{ax+c}{b}$.

(α, β) notes a specific solution of this equation.

With an algorithmic vocabulary: the input is (a,b,c), the output are all given values of (α, β) .

In the verses studied here (a,b) are implicit, $c = -1$ and (α, β) is stated using standard names for sanskrit numbers and word numerals.

To Aurelia, who happily is not a number in a table!

1 Introduction

At the end of Bhāskara's 7th century commentary on the mathematical part of the *Āryabhaṭīya* (499 CE), 91 verses provide the solutions of linear indeterminate equations using astronomical constants. These verses exist in all the manuscripts used by K. S. Shukla's 1976 edition of Bhāskara's *Āryabhaṭīyabhāṣya*¹. They are translated at the end of this article, in Appendix A.

They constitute, to our knowledge², the only collection of equation solutions in Sanskrit mathematical and astral literature.

Why and how then were these solutions collected? Who could have used them, and why did they not have any posterity? These are the questions that we will try to elucidate in the following.

The *Āryabhaṭīya* is made of four³ parts (*pāda*): the first coins parameters and is named after the meter in which it is composed, the *gītikā*. The second part is on mathematics (*gaṇita*), the third on time computation (*kāla-krīya*) and the fourth on the celestial sphere (*gola*). Bhāskara's commentary follows the order of Āryabhaṭa's text, commenting his verses more or less one after the other⁴.

The verses studied here are provided within Bhāskara's 7th century commentary. They are situated at the end of the commentary on the mathematical part and before the commentary on time computations. They are in this way situated at the interface between mathematics (*gaṇita*) and astral science (*jyotiṣa*).

¹(K. S. Shukla, 1976, 156-171).

²This is already noted by Shukla, (K. S. Shukla, 1976, Introduction, lxxxii).

³If we accept Shukla's analysis of the text which is sometimes transmitted without the first part considered by some as an independant text. See (K. S. Shukla, 1976, Introduction, 6.2, xxv-xxvi).

⁴He can comment on half a verse, or two verses at times, etc.

1.1 Betwixt mathematics and astronomy

Indeed, Bhāskara at the opening of his commentary stresses the complexity of relationships between the *Āryabhaṭīya*'s different parts⁵: planetary computations can be considered a sub-part of mathematics; but here mathematics is a small part of a treatise devoted to planetary and time computations⁶. Thus examining these collected solutions is also a way of reflecting on the paradoxical relationship of mathematics and astral science for Bhāskara.

Actually, these verses concern a kind of problem and a method of resolution, which later in Sanskrit literature, will be famed for its ambiguous position in classifications of mathematics⁷ and astral science: the pulverizer (*kuṭṭaka*⁸).

In Bhāskara's commentary, the pulverizer is classified with proportion (*anupāta*) among specific topics of mathematics (*gaṇita-viśeṣa*)⁹. Planetary computations (*graha-gaṇita*) are similarly characterized as belonging to (*gaṇita-viśeṣa*). In this part of the commentary Bhāskara does not elucidate how both could be related. However, in practice, with trigonometry, the procedural form of the Pythagorean Theorem and the Rule of Three, pulverizers appear as a type of problem and procedure useful both in mathematical astronomy and in mathematics. The pulverizer is treated by Āryabhaṭa at the end of the mathematical part. In Bhāskara's commentary of these last verses of the mathematical part, examples both purely numerical and using astronomical parameters are solved¹⁰.

This article then will explore the mathematical and astral interpretation of the numbers coined in these verses, in an effort to understand what was their aim.

⁵(K. S. Shukla, 1976, 5-7).

⁶(Keller, 2007).

⁷Between arithmetics and algebra, (Plofker, 2009, 296).

⁸Bhāskara uses the orthography *kuṭṭākāra*, "what makes a pulverization". Once in his commentary (K. S. Shukla, 1976, 135, line 1) *kuṭṭaka* is used in a compound-*nir-agra-kuṭṭaka*. Manuscripts should be checked to be sure that this is not an editorial misprint. Later authors indeed seem to have privileged the orthography *kuṭṭaka*, an action noun derived from the verb *kuṭ-*, "to pulverize", whose literal meaning is "a tool to pulverize", hence a "pulverizer".

⁹(K. S. Shukla, 1976, 44), (Keller, 2007).

¹⁰See (K. S. Shukla, 1976, 136-156) and (Keller, 2006, 135-166). Note that topics at the interface between mathematics and astral science may have been used by Āryabhaṭa to make transitions from one chapter to another. Thus the 13th and last verse of the *gīṭikā* part deals with Sines, another topic which is treated in both domains. However, the procedure derived from the Pythagorean theorem is stated in the middle of the part on mathematics, in verse 17.

1.2 Can verses be numerical tables?

However, another question is raised by these verses: Is this but another instance of the kind of numerical table used in India before the tabular format with lines and columns took over? Do these verses testify to a way of stocking data in a condensed form that provides tabular material? Additionally, can the verses be seen also as a sort of formal table, a syntactical equivalent of rows and columns?

Before turning to these questions we will first explore how these solutions were found. A first section evokes briefly the mathematical procedure and the astral backdrop enabling the computation of these values. A second section reflects on the tabular nature of the data and verses considered. The last section speculates on what could have been the aim of these verses.

2 Understanding the mathematics, Understanding the astronomy

This section aims at explaining first how the numbers provided in these verses were obtained, how they could have been used mathematically, and finally what they could have meant from an astral point of view.

The numerical data recorded in all 91 verses, translated in Appendix A, is tabulated in Appendix B at the end of this article.

2.1 Mathematics

In verses 32 and 33, the last of the mathematical part of the *Āryabhaṭīya*, a rule is given to solve a set of problems grouped by Bhāskara under the name *kuṭṭākāra*. The 7th century commentator reads these verses as providing two equivalent procedures for two equivalent types of pulverizer problems and methods¹¹. The pulverizer without remainder (*nir-agra-*

¹¹(K. S. Shukla, 1976); (K. S. Shukla K. V. ; Sharma, 1976); (Keller, 2006). Other commentators will do so as well, along very similar lines. See for instance (?, ?). Much of the mathematical and astral data developed here can be found in this publication.

kuṭṭakāra) solves the indeterminate equation, for $a, b, x, y \in \mathbb{N}^*, c \in \mathbb{Z}^*$:

$$y = \frac{ax \pm c}{b} \quad (1)$$

x and y are unknown, a, b and c are given fixed constants. The problem as presented in (1), its solution, and the method used to solve it will be noted KU(a, b, c)[x, y]¹².

For (i) the problem, (ii) the general method used to solve the problem, and (iii) the first solution, α , of the couple (α, β) , Bhāskara uses the expression *kuṭṭakāra*. In the verses studied here x is more often called *guṇakāra* “multiplier”. Several synonyms of multiplier or multiplicand are subsequently used, such as *guṇa*, *gunya*, etc. y is usually called “the result” (*labdha*, *lābha*, *labdhaka* etc.), translated here as “quotient” because it is the result of a division.

Studies of the different steps of Āryabhaṭa’s pulverizer, as understood by Bhāskara, have been published several times already¹³.

The verses studied here contain ordered pairs of solutions of what in secondary sources is generally called a “permanent pulverizer” (*sthira-kuṭṭaka*). A “permanent pulverizer”, rests on the fact that if (α, β) is a solution of KU($a, b, -1$)[x, y], then (α_0, β_0) are a solution of KU($a, b, -c$)[x, y], for $c \in \mathbb{N}^*$, were α_0 and β_0 are respectively the remainders of the division of $c\alpha$ by b and $c\beta$ by a ¹⁴. Thus with solutions of KU($a, b, -1$)[x, y], solutions of KU($a, b, -c$)[x, y] are easily generated. So that computing solutions of KU($a, b, -1$)[x, y] can be seen as a short-cut to computing solutions of KU($a, b, -c$)[x, y]. This is one of the key ideas behind the use of the data coined in the 91 verses.

Bhāskara does not have a specific name for the “permanent pulverizer”, which is included as a sub procedure in the larger category of “pulverizer without remainder”. He introduces this short-cut in his commentary on example 7 of BAB.2.32-33 and in many subsequent examples¹⁵. A general rule concerning the use of this short-cut is given in the *Mahābhāskarīya*

¹²Using the notation developed in Takao Hayashi’s edition of the *Bījagaṇita*, (Hayashi, 2009, Appendix A, p. 117 sqq).

¹³(Datta, 1932), (K. S. Shukla, 1976), (Keller, 2006), among others.

¹⁴In other words, $\frac{c\alpha}{b} = q + \frac{\alpha_0}{b}$ and $\frac{c\beta}{a} = p + \frac{\beta_0}{a}$.

¹⁵(K. S. Shukla K. V. ; Sharma, 1976, 137) (Keller, 2006, 135-136).

(I.45-46)¹⁶. Thus, we can deduce that Bhāskara considered this “short-cut” as a convenient sub-procedure while solving pulverizer type problems, with non trivial values of c .

The numerical couples transmitted in the 91 verses studied here are solutions (α, β) of $KU(a, b, -1)[x, y]$, for implicit astronomical parameters a and b .

Let us take the data given in BAB.Ku.9. The verse runs as follows¹⁷:

When pulverized [by the revolutions] of the moon, (the multiplier is made of the numbers)
seven-three-eight-six-seven-seven [776 837]|| A heap which is a quotient quantity is laid
out as (the following numbers) three-three-four-eight-two [24 833]||9||

We will set aside here how the numbers are expressed in the verses- this will be treated in the next section. This verse provides two values, those forming the couple solution (α, β) : (776 837, 24 833). The values of a and b are not stated explicitly. But for reasons that will be explained later, we know that these (α, β) are solutions of the following equation: $y = \frac{78\ 898x-1}{2\ 155\ 625}$. They are actually the smallest positive solutions possible for this equation, since a standard *kuttaka* procedure derives such smallest solutions.

Later literature¹⁸ notes that if $KU(a, b, c)[\alpha_0, \beta_0]$, then $KU(a, b, c)[\alpha_0 + bk, \beta_0 + ak]$, for $k \in \mathbb{Z}$. This property enables one to derive the infinite solutions of such indeterminate problems. This property is not stated by Āryabhaṭa or Bhāskara. However it seems to be used in one case here. Indeed, verse 14 provides two values, [6 518, 18 550 831 927], solutions of $y = \frac{9\ 816\ 173\ 568x-1}{3\ 449}$.

¹⁶[Shukla 1960; p. 32-33]:

“रूपं एकम पास्यापि कुट्टाकारः प्रसाध्यते॥
गुणकारोऽथ लब्धं च राशी स्यातामुपर्यधः॥४५॥
इष्टेन शेषम भिहत्य भजेदृच्छदाभ्यां शेषमिदानीमभगणादि च कीर्त्यतेऽत्र॥

Mbh.I.45-46ab. Alternatively, the pulverizer is solved by subtracting one. The upper and lower quantities are the multiplier and quotient. Having multiplied the remainder by each, one should divide the respective products by the abraded (divisor and dividend). The remainders obtained here are the days etc. and the revolutions etc.”

¹⁷

शीतरश्मि[भगणैः]प्रकुट्टिते सप्तारामवसुषट्स्वराद्रयः।
लब्धराशिनिचयः व्यवस्थितः पुष्कराग्निकृतनागबाहवः॥९॥

. Note that in translations of Bhāskara’s text, [] stand for addendums by the editor, K. S. Shukla, ⟨⟩ stand for my own addendums, () give specifications.

¹⁸Notable in the *Līlāvati* (verse 256) and the *Bījagaṇita* (verse 36 ab), see (Hayashi, 2009, 106).
¹⁹

तत्परेषु धृतिभूतषट्क्रकाः निर्दिष्टि गुणकारसंख्यया।

BAB.Ku.14 In thirds, eighteen-five-six [6 518] indicates by means of a number which is the multiplier| The following (e.g. the quotient) is ⟨made of the digits⟩ twenty seven-nine-one-three-eight-zero-five-five-eighteen [18 550 831 927]||14||

However the smallest positive solution of $y = \frac{9\ 816\ 173\ 568x-1}{3\ 449}$ is the couple $(3\ 069, 8\ 734\ 658\ 359) = (\alpha_0, \beta_0)^{20}$.

Since

$$\alpha_0 + b = 3\ 069 + 3\ 449 = 6\ 518$$

and

$$\beta_0 + a = 8\ 734\ 658\ 359 + 9\ 816\ 173\ 568 = 18\ 550\ 831\ 927$$

Thus the information contained in verse 14, can be formalized as:

$$Ku(9\ 816\ 173\ 568, 3\ 449, -1)[6\ 518, 18\ 550\ 831\ 927] = K(a, b, -1)[\alpha_0 + b, \beta_0 + a].$$

This is the only case that cannot be ascribed to scribal error in which the couple solution is not the smallest possible.

How then do we know the values of a , b , and c that are implicit in theses verses? The answer is to be found in the astral backdrop within which these couples of numbers are given.

2.2 Planetary Pulverizers

Problems solved by a pulverizer can be derived from constant ratios between circular planetary movements and passing time.

Indeed, in the verses considered here, as when it is introduced in the *Āryabhaṭīyabhāṣya*, the *Mahābhāskarīya*, or the *Laghubhāskarīya*, the *kuṭṭakāra* is used with astronomical parameters. Bhāskara provides a name for such a category of pulverizers: a “planet-pulverizer” (*grahakuṭṭakāra*)²¹. The 91 verses can be seen as belonging to this category of pulverizers.

ऋक्षनन्दशशिरामकुञ्जरव्योमबाणशरधार्तवोऽपरः॥१४॥

²⁰This can be checked in the spreadsheets I am putting online.

²¹Note that in the introductory sentence to the versified table, the expression *grahakuṭṭakāra* is used ambiguously: it could be the couple of solutions given in each verse as well as the totality of given solutions. But it does not seem to designate either the method, nor just the xs obtained.

The astral meaning of the parameters are represented in Table 1.

Table 1: Data in Verses (1)

Verses	Theme
1-2	Introduction
3-8	Revolutions of the Sun since the beginning of the Kali Yuga
9-16	Revolutions of the Moon
17-24	Revolutions of the Moon's Apogee
25-32	Revolutions of the Moon's Ascending node
33-40	Revolutions of Mars
41-48	Revolutions of (the <i>śighrocca</i> of) Mercury
49-56	Revolutions of Jupiter
57-64	Revolutions of (the <i>śighrocca</i> of) Venus
65-72	Revolutions of Saturn
73-80	Revolutions of the Moon's Anomaly
81	Intercalary Days since the beginning of the Kali Yuga
82	Omitted Days "
83	Sun's Declination
84-91	Intercalary Months since the beginning of the Kali Yuga

The astronomical basis of these planetary pulverizers is quite standard and has been often described²². For Āryabhaṭa all *yugas* have a same time length, 4 320 000 years, Kali Yuga included. Each planet makes a fixed integer number of revolutions in a *yuga*. This number is usually called in the secondary literature, the planet's "revolution number". A given planet g (*graha*)- during a certain time x - evaluated in terms of civil days - does not make an integer number of revolutions but has, in addition to an integral value, a fractional part, or residue (*śeṣa*). If a_g is the number of revolutions of g , during the time cycle of the Kali Yuga, and b the number of civil days in the Kali Yuga (these are fixed parameters), and if y_g is the number of revolutions made by a given planet in x days. The assumption is that

$$\frac{y_g}{a_g} = \frac{x}{b}$$

Usually only a fractional part of y_g is known: λ_g , the mean longitude of the planet. $y_g = y + \lambda_g$

The remaining part, y , is unknown, together with x .

²²(Sengupta, 1927), (Pingree, 1970-1995),(Sen, 1971), (K. S. Shukla K. V. ; Sharma, 1976), (Pingree, 1981), (Rao, 2000),(Plofker, 2009).

Thus

$$\frac{y + \lambda_g}{a_g} = \frac{x}{b}$$

and therefore

$$y = \frac{a_g x - b \lambda_g}{b}$$

in which we recognize equation (1).

Note that if $c = -b\lambda_g$, then $\lambda_g = -\frac{c}{b}$.

We see then that theoretically an astral interpretation of the couple solutions of a pulverizer is possible: knowing that a_g revolutions are performed in b days, when g is in λ_g ²³, then x days have elapsed, and y_g revolutions have been performed by g .

Ab.1.3-4 gives the numbers of revolutions of the sun, moon, earth etc. in a *yuga*, and the date of the beginning of the Kali Yuga²⁴.

Civil days are defined in Ab.3.5c: “The conjunctions of the Sun and the Earth are (civil) days”²⁵. So that the number of civil days in a *yuga* (b) is equal to the number of revolutions of the Sun in a *yuga* minus the number of revolutions of the earth in a *yuga* : $b = 1\,582\,237\,500 - 4\,320\,000 = 1\,577\,917\,500$.

Table 2 lists these parameters (usually called in the secondary litterature “the revolution number of the planets”, or “period revolutions”) which are subsequently used by Bhāskara in the solved examples of his commentary of Ab.2.32-33 and in the 91 verses.

Take for instance the parameters used in verses 9 to 16 concerning the revolutions of the

²³The negative values that can be taken by c , and thus by λ accounting for retrograde motions of g , if any.

²⁴The numbers in these verses are given with Āryabhaṭa’s special device for noting numbers. (K. S. Shukla K. V. ; Sharma, 1976, 6-8), provides edition and translation here:

युगरविभगणः खयुघु, शशीचयगियिङ्शुछल, कुडिशिबुगलुब्व प्राक्॥
 शनिदुडिवध्व, गुरुसिच्युम्, कुजभदिलभ्रुवमभुगुबुधसौराः॥३॥
 चन्द्रोच्चर्जुषिध, बुध सुगुशित्थन, भृगुजषविखुछ, शेषारकाः॥
 बुफिनचपातविलोमा, बुधाह्न्यजाकोर्दयाच्च लङ्कायाम्॥४॥

In a *yuga*, the eastward revolutions of the Sun are 43,20,000; of the Moon, 5,77,53,336; of the Earth, 1, 58,22,37,500; of Saturn, 1,46,564; of Jupiter, 3, 64224; of Mars 22,96,824; of Mercury and Venus the same as those fo the Sun; of the Moon’s apogee, 4,88,219; of (the *śighrocca* of) Mercury, 1, 79, 37, 020; of (the *śighrocca* of) Venus, 70, 22,388; of (the *śighrocca* of) the other planets, the sam as those of the Sun; of the Moon’s ascending node in the opposite direction (i.e. westward), 2,32,226. There revolutions commenced at the beginning of the sign Aries on Wednesday at sunrise at Lañkā (when it was the commencement of the current *yuga*).

Note, that these verses stock numerical data, that has one non-numerical increment, an astral body -real or theoretical- and one output, the number of revolutions in a *yuga*.

²⁵[Sharma-Shukla 1976; p. 91]:रविभुयोगादिवसाः.

Table 2: Astral parameters used by Bhāskara I

Parameters for whole Yuga	Notation	Values as in Ab.1.3-4; Ab.1.6
Civil days	b	1 577 917 500
Sun's revolutions	a_s	4 320 000
Moon's revolutions	a_c	57 753 336
Moon's Apogee revolutions	a_{cu}	488 219
Moon's Ascending Node's revolutions	a_p	232 226
Mars' revolutions	a_b	2 296 824
(<i>śighrocca</i> of) Mercury's revolutions	a_{bu}	17 937 020
Jupiter's revolutions	a_{gu}	364 224
(<i>śighrocca</i> of) Venus's revolutions	a_{bh}	7 022 388
Saturn's revolutions	a_{sa}	146 564
Moon's Anomaly's revolutions	a_{ck}	57 265 117
Intercalary Days	a_{ad}	47 800 080
Solar months	b_{ad}	51 840 000
Omitted days	a_{av}	25 082 580
Lunar Days	b_{av}	1 603 000 080
Maximum of the sun's declination	a_{apa}	1397
Radius of the sky	b_{apa}	3438
Intercalary months	a_{am}	1 593 336

Moon during the Kali Yuga.

By Ab.1.3, we know that the revolutions of the moon (*candra*), a_c , during a whole *yuga*, in particular during the Kali Yuga has the value 57 753 336. And as we have seen above, the number of civil days during the Kali Yuga, b , is equal to 1 577 917 500. These are the two underlying parameters of the pulverizers solved in the series of verses, 9 to 16.

Values of a_g can vary according to the measuring units considered for a revolution. These measuring units, as seen in Table 3, naturally recover the different subdivisions of a circle in mathematics.

Therefore in verses 9 to 15 the verses provide solutions for pulverizers considering values of a_c in revolutions (verse 9), signs (verse 10), degrees (verse 11), minutes (verse 12), seconds (verse 13), thirds (verse 14) and fourths (verses 15).

Note that the use of such fine subdivisions as thirds (*tatpara*) and fourths (*pratatpara*) is an indication of how accurate the values considered in these verses are.

However, values of a_g are further transformed because before entering the pulverizer

Table 3: Subdivisions of a Revolution

Sanskrit	English	Amounts in a Revolution	Powers of 60
<i>maṇḍala</i>	Revolutions	1	$\frac{1}{60} \times 60$
<i>rāśi</i>	Signs	12	$\frac{1}{5} \times 60$
<i>bhāga</i>	Degrees	360	6×60
<i>lptā</i>	Minutes	21 600	6×60^2
<i>vikālā</i>	Seconds	1 296 000	6×60^3
<i>tatpara</i>	Thirds	77 760 000	6×60^4
<i>pratatpara</i>	Fourths	4 665 600 000	6×60^5

process, common divisors of a_g and b are used to reduce these two numbers. Thus, the values considered for a_c are actually reduced by divisors that are listed in the last verse of the series, in verse 16²⁶:

BAB.Ku.16 Starting with the moon's degrees the chosen divisor is five [5] five [5] and five after | In due order here (it) is completely thirty-two-seven [732]||16||

As shone in Table 7, in verse 9, both a_c and b are divided by 732. Since $\frac{a_c}{732} = \frac{57\,753\,336}{5} = 78\,898$, and $\frac{b}{732} = \frac{1\,577\,917\,500}{732} = 2\,155\,625$, then the numerical data coined in verse 9 can thus be stated as $KU(78\,898, 2\,155\,625, -1)[776\,837, 24\,833]$.

In other words, (776 837, 24 833) is one (actually the smallest positive) couple solution of $y = \frac{78\,898x-1}{2\,155\,625}$.

But from the point of view of the astral parameters used here, these results also have a meaning, which is the following: Knowing that during 2 155 625 days, the moon performs 78 898 revolutions, when $-\frac{1}{b} = \frac{1}{2\,155\,625}$ revolutions have been performed by the Moon, then 776 837 days have elapsed since the beginning of the Kali Yuga and $24\,833 + \frac{1}{2\,155\,625}$ revolutions have already been completed by the Moon.

In other words, in these verse $KU(a_c, b, -1)[x, y_c]$ are considered. $y_c + \frac{1}{b}$ corresponds to the number of revolutions (signs, degrees, minutes, seconds, thirds and fourths) of the moon since the beginning of the Kali Yuga and x the number of civil days elapsed since the beginning of the Kali Yuga.

²⁶अंशातारभ्य शीतांशोः पञ्चपञ्चगुणः परः।
छेदः कल्प्यः क्रमातत्र दन्तशैलसमन्वितः॥

This information is the one that enables us to unravel the data on which the verses considered rest. Further it can give us a clue to the way these values were interpreted and hypothetically used. But before further speculating on the aim and use of such stocked data, let us come back to the relations these verses and the data they contain have to numerical tables.

3 Numbers in Verses and Numerical Tables

The 91 verses that end Bhāskara’s commentary on the mathematical part of the *Āryabhaṭīya* are referred to by Shukla as “tables”²⁷. Is any string of numbers a kind of numerical table? In the following we will suggest some characterizations of numerical data in relation to tables, before looking at the kind of textual object with numbers is created by these verses.

3.1 Numerical Tabular data

Inspired by an algorithmic and functional definition of what is a numerical table, in the following we will call “tabular data” any set of two varying increments producing one or several outputs. This may be distinct from a “tabular alinement” or a “numerical array” involving lines and columns²⁸.

As seen in Figure 1, $KU(a,b,c)[x,y]$ has three increments a , b and c , producing two outputs, x and y .

In practice, values of b and c are previously fixed in the verses here.

Indeed, c has a constant value: -1.

b is the constant number of civil days in a *yuga*, 1 577 917 500. Nonetheless, as we have seen, the value of b is constantly changing because before solving the equation, a and b are reduced by common divisors.

a varies in three ways:

a , in most of the cases- this nuance will be discussed below-, is the number of revolutions of a planet. It varies then every set of seven verses, as seen in Table 1, with values presented

²⁷(K. S. Shukla, 1976, lxxxii, titles in Appendix II:335-339).

²⁸These characterizations I owe to Dominique Tournes, Karine Chemla and the larger HTN group.

Figure 1: Inputs and Outputs

$Ku(a,b,-l)[x,y]$

Implicit input (a,b,-l)

Explicit output $[\alpha,\beta]$

a varies at each input according to the
 -astral data considered;
 -the measuring unit considered
 -if it has a common divisor with b

b varies at each input according to the common
 divisor it has or not with a

in Table 2. Thus the 91 verses can be seen as containing 11 or 12 subsets of numerical tabular data. Indeed, for each astral parameter incremented in a , two other variations take place. a also varies because the measuring unit used to evaluate the number of revolutions of a given planet changes from revolutions, to signs, degrees, minutes, seconds, thirds and fourths, in this regular order, as shown in Table 25. Finally, each sub-set of a also varies because it can be reduced with b by a common divisor, d . d can vary with every new value incremented into a .

These many variations of a and b which in turn produce different couples $[\alpha, \beta]$ are fully exposed in numerical arrays in Appendix B following the verse's translations. They explain why we can consider that different sets of *tabular data* are coined in these verse: two varying increments produce two outputs.

However I would like to argue that something else is at play here as well. Indeed, in the following, in an attempt to explore whether this numerical tabular material is transmitted through "syntactical tables" two questions will be raised: First, is the data given in a strict order, which could be the syntactical equivalent of the order of rows and columns? Second is the data retrieved without requiring necessarily a linear "reading"?

BAB.Ku.2 states²⁹:

I indicate here with delight for those who know them the pulverizer (*kuṭṭākāra*) and

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कुट्टाकारः च लाभः च द्वन्द्वतः भगणादितः निर्दिश्यते क्रमात् त्रतद्विदाम्प्रीतयेमया॥२॥॥

quotient (*lābha*) ⟨derived⟩ two by two from revolutions and so forth in due order ||2||

Verse 2 provides once and for all information both on the order (*kramāt*) in which the data (*kuṭṭākāra* and quotient) is given in the verse and more generally on how the data is arranged in respect to the implicit ordered variations of the parameters.

3.2 Order

The verses indeed lists ordered pairs of numbers, they thus list couples of solutions (with our notations (α, β) of $KU(a, b, -1)[x, y]$). Take for instance, verse 9 again³⁰:

When pulverized [by the revolutions] of the moon, ⟨the mulitplier is made of the numbers⟩
seven-three-eight-six-seven-seven [776 837]|| A heap which is a quotient quantity is laid
out as ⟨the following numbers⟩ three-three-four-eight-two [24 833].||

As indicated by the brackets, the fact that the first value given here is a “multiplier” (α) is not recalled here, but assumed to be known, since this was already specified in Verse 2. This *rathoddhata* verse provides the two numbers α and β in this order. All verses always state these value in this order, whatever the meter.

The text itself refers regularly to this order. Indeed, the data is enumerated “in due order” (*kramāt*, *kramena*). Thus, in verse 14 the element coming after the multiplier is called *apara*, “the subsequent”. Its place in the order is thus referred to rather than its nature as a quotient.

Further, verse 2 alludes to the variations of a : first as one of the two components enabling the derivation of α and β (*dvandva*). This order is integrated in the larger order of the subsets of numerical data provided in the verses. Thus, as seen in Table 1 in most cases, groups of seven verses, give solution couples for a certain astral component of a . Thus verse 9 to 16 list elements concerning revolutions of the moon. The seventh verse, lists the divisors used to reduce a and b for specific equations, as verse 16 lists the reducers of a_c s.

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शीतरश्मि[भगणैः] प्रकुट्टिते सप्तरामवसुषट्स्वराद्रयः।
लब्धराशिनिचयः व्यवस्थितः पुष्कराग्निकृतनागबाहवः॥

Note that the order of the planets used for the classification of these sub-sets of solutions is not the one of the enumeration of Ab.1.3-4.

Verse 2 also alludes to another variations of a by suggesting that its measuring will vary in a given standard order (from revolutions and so forth, *bhaganādi*). In verse 15, the measuring unit (fourths coming after thirds), and the quotient (comming after the multiplier) are referred to by the fact that they come after what precedes, if it is not that the units of the number enumerated need to be stated in the right order (*kramāt*) as well³¹.

That the order of the measuring units for a , is considered known is particularly clear in the *śloka* which provide the reducers according to measuring unit. Thus, for example, in verse 91, after signs, all the other measuring units are referred to as ‘the remaining’ (*śeṣa*).

However, there is a certain amount of inaccuracy in these *ślokas*, since the loose enumeration does not always apply to all its terms. For instance, in verse 48 dealing with the divisors used for computing solutions from astral data concerned with (the *śighrocca* of) Mercury, five is announced as a divisor for “degrees etc.”³². As seen in Table 16 however only degrees and minutes are reduced by five; seconds, thirds and fourths are left untouched.

As noted previously, values of a and b are not explicitly stated in the verses considered here. In this sense, as in other versified tables known in Sanskrit literature, the increment of the tables is implicit.

Similarly, the order in which the measuring unit for a varies, is overall standard and implicit. It is but sometimes specified in the passing. This order is however key to the ordering of the sub-sets of solutions by varying astral parameters.

Therefore the order in which the data is provided is structured by its implicit data. The structure of the successive verses rests on regular variations of its implied increments.

Not only does the order provide a structure for the data it also enables part of it to be implicit: order and conciseness are closely intertwined. More, the order of the data is also

³¹तत्परेषु परतः च कीर्तिताः रामनन्दयमदस्रकाः क्रमात् ।

रुद्रबानगिरिदस्रसागराः भूतषट्कतिथिनन्दविक्रमाः ॥

In what follows thirds (e.g fourths, *tatpareṣu parataḥ*) three-nine-two-two are mentioned [2 293] in due order| In due order, eleven-five-seven-two-four five-six-fifteen-nine-three [3 91 565 427 511] ||15||

³²भागादीनाम्क्रमात्पञ्च [5] प्रवदन्तिमनीषिनः॥ Wise ones indicate for degrees etc. in due order five [5]||48||.

structured by the verses and its meter.

This great regularity which uses the versification is encouraging if we want to consider these verses as not only containing “tabular data” but also as a kind of “syntactical table”. Indeed, with a regular order, emplacement and versification, one can imagine that the values learnt by heart could be retrieved without a statement or reading of all the verses.

3.3 Arranging Data within verses

Contrary to what is announced in the introductory sentence, most of the verses considered here are not *śloka*s³³. They are in their great majority the 11 syllable per quarter *indravajrā*. The *śloka*, with 8 syllables per quarter is used to indicate values of d , the common divisor of a and b , every set of seven verses.

Thus, as seen in Table 4, the meter change after each seven groups, provides a syntactical form indicating the structure of the data, isolating each group of seven verses.

The first twenty verses display a great variety actually. Verse 17 is a sort of hybrid, starting as an 11 syllable *indravajrā* and ending as a 14 syllable *uddarṣiṇī*. It marks a clear turning point within the metre structure of the verses, because until the case points of verses 81-83, this will be a regular alternance of six *indravajrā* and a *śloka*.

The author of these verses gives some indication that the format used is specific. Thus, in verse 9, the numbers are considered to be layed down (*vyavasthita*) almost as when they are noted on a working surface before being computed on . Further, in several verses the fact that the numbers have been learned by heart, and are then recalled (*smṛti*) is evoked.

However, the accuracy with which this order is used can be wanting at times: Indeed, no two verses are the same: they all contain changing words and expletives. Thus, as seen in Table 4 not all verses follow the pattern of one value per half-meter.

It is difficult to comment on the non regularity of the meter in the beginning and in the end of the set. Is this the sign that these verses were compiled? Were such variations used as mnemonic devices to remember the data?

Further, as seen in Table 1 the last 10 verses of the set do not concern directly planetary

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These planet pulverizers (*graha-kutṭakāra*) are indeed condensed (*upanibādh-*) with *śloka* ⟨meters⟩.

Table 4: Data in Verses (2)

Verses	Theme	Meter
1-2	Introduction	<i>śloka</i>
3	Revolutions of the Sun in revolutions and signs	<i>praharṇī</i>
4-5	Revolutions of the Sun in degrees, minutes, seconds	<i>uddarṣiṇī</i>
6	Revolutions of the Sun in thirds	<i>rathoddhata</i>
7	Revolutions of the Sun in fourths	<i>uddarṣiṇī</i>
8	Common divisors for a_s and b	<i>śloka</i>
9-15	Revolutions of the Moon	<i>rathoddhata</i>
16	Common divisors of a_c and b	<i>śloka</i>
17	Revolutions of the Moon's Apogee	half <i>uddarṣiṇī</i> , half <i>indravajrā</i>
18-23	Revolutions of the Moon's Apogee in signs, etc.	<i>indravajrā</i>
24	Common divisors of a_{cu} and b	<i>śloka</i>
25-31	Revolutions of the Moon's Ascending node in signs, etc.	<i>indravajrā</i>
32	Common divisors of a_p and b	<i>śloka</i>
33-39	Revolutions of Mars	<i>indravajrā</i>
40	Common divisors of a_b and b	<i>śloka</i>
41-47	Revolutions of (the <i>śīghrocca</i> of) Mercury	<i>indravajrā</i>
48	Common divisors of a_u and b	<i>śloka</i>
49-55	Revolutions of Jupiter	<i>indravajrā</i>
56	Common divisors of a_{gu} and b	<i>śloka</i>
57-63	Revolutions of (the <i>śīghrocca</i> of) Venus	<i>indravajrā</i>
64	Common divisors of a_{bh} and b	<i>śloka</i>
65-71	Revolutions of Saturn	<i>indravajrā</i>
72	Common divisors of a_{sa} and b	<i>śloka</i>
73-80	Revolutions of the Moon's Anomaly	<i>indravajrā</i>
81	Intercalary Days since the beginning of the Kali Yuga	<i>śloka</i>
82	Omitted Days	<i>śloka</i>
83	Sun's Declination	<i>śloka</i>
84-90	Intercalary Months since the beginning of the Kali Yuga	<i>indravajrā</i>
91	Common divisors of a_{am} and b_{am}	<i>śloka</i>

matter. The three first verses of this set (verses 81-83) are like loose strings in this regular ordered structure. Of course, one can always argue that surprises in a regular form may help the data to be impressed in the mind³⁴. However, since we have no testimony concerning the use of this data, this hypothesis remains very speculative.

It is at this stage difficult to decide whether this set of verses provide an example of syntactical table. If its structure is quite regular, it isn't absolutely so (a certain number of verses don't exactly follow the given pattern). However, one can further note that these verses, like other texts stocking numbers, are not quite exactly formulated in natural language.

Thus the layered orders in which the numbers are stated, the use of versification, and the technical language, all concur to make of such a text, a very specific object that shares some properties with tabular displays. The 91 verses open with a short introductive sentence:

एते ग्रगकुट्टाकाराः श्लोकैरप्युपनिबाध्यन्ते।

These planet pulverisers (*graha-kuttakāra*) are indeed condensed (*upanibādh-*) with *śloka* ⟨meters⟩.

What exactly is meant by *upanibādh-* here? Indeed, *nibādh-* which means “to compress and oppress”, may refer to the compact form of these verses. But the addition of the preverb *upa*, seems to convey to it a technical meaning, as if it was referring to the particular, ordered and compact text it introduced. However, the key to assessing if it should be considered as a table, lies in the way such a text was used.

3.4 A side effect: how quantities are expressed

Before turning to how such verses could have been used, we can note that the long successions of numerical compounds in verses, gives rises to peculiar expressions concerning quantities and numbers. In most cases the numbers are given in *dvandva* compounds, using regular number names and word-numerals (*bhūtasarikhyā*)³⁵. An appendix lists the names used for

³⁴(Severi, 2007)

³⁵Studies on this way of expressing numbers are innumerable and were published as early as the XVIIIth century. For the latest insight, see (Sarma, 2009)

each number³⁶. Full numbers are sometimes stated as well. More generally, the enumeration is not restricted to units smaller than nine.

It is clearly stated, such as in verse 12 (*gaṇitena labhyate*), several times that the numbers are obtained through computation.

In many cases, as this is standard with such expressions, the number compounds are in a plural form, as if they consisted in a heap (*nicaya*, *nivaha*, *samūha*) of units. This is sometimes made explicit, as in the much quoted verse 9³⁷:

A heap which is a quotient quantity (*labdharāśīnicaya*) is laid out as ⟨the following numbers⟩ three-three-four-eight-two [24 833]||9||

The reference to a quantity ⟨made of (the numbers)⟩, is the code I have used in the translation, to indicate that the compound listing the units was in the plural form while the quantity (*rāśī*) it refers to is in singular³⁸. The term used for quantity, *rāśī*, has literally the meaning of “heap”, so that the expression *labdharāśīnicaya* used above can seem redundant. Indeed, the term translated as “heap” here, is also sometimes used as a synonym for quantity, as in the subsequent verse 10³⁹:

Here the quotient quantity (*labdhanicaya*) is separately announced as eleven-three-six-nine-eight [896 311]||10||

In the next verse again, the term is used in a plural compound maybe as representing the digits as “heaps”⁴⁰:

The quotient is ⟨made of the numbers⟩ five-five-five-one-eight and heaps (*nivaha*) ⟨that are ⟩ four and four [4 481 555]||11||

³⁶I note here, having not seen this remarked elsewhere, that a standard name for nine, can be seen as a meta-name. Indeed, the cardinal of a sub-set of numbers is used to name a number: *anika*, the nine units to indicate the number nine.

³⁷लब्धराशिनिचयः व्यवस्थितः पुष्कराग्निकृतनागबाहवः॥९॥

³⁸Of course there need not be a conceptual consistency between the way syntactically numbers and quantities are related and the way they are thought of. But the close study of syntactical expressions, may when they show variations, yield some information in this respect.

³⁹तत्र लब्धनिचयः विकथयते रुद्रवह्निस्सनन्दपन्नगाः ॥१०॥

⁴⁰भूतबाणशरचन्द्रकुञ्जराः सागराम्बुनिवहाः ॥११॥

A same verse, can contain unit compounds in the singular and in the plural, as in verse 4⁴¹:

For minutes seven-four-six-one-eight [81647], then
one-nine-two-eight-two-eight-four [4 828 291]||4||

In verse 10 and subsequently others, several compounds are used for a same number⁴²:

For signs also the multiplier is six-two-seven zero-four-zero-two [2 040 726]

More generally the link between the quantity designated and its stated amount is linked to the operation of measuring (*sammita*, *pramāna*⁴³). Thus, for instance in verse 51⁴⁴:

Indeed what is called a multiplier produced for degrees is the quantity nine-seven-eight-eight-zero seven-four [4 708 879] Those who know indicate that the measure of the quotient is equal to five-nine-twelve-nine-five [391 295]||51||

Multipliers and quotients are quantities (*rāśi* or any word meaning heap- note that *rāśi* is also word for “signs”) to which a number (*saṅkhyā*), when it is not called a measure, is associated. Thus, in verse 63⁴⁵:

For fourths the multiplier quantity is the number six-seven-zero-nine [9 076]

Although the convolution, shortness or spinning of these phrases rests certainly on meter requirement, they nonetheless open a door on the differences that could have existed in between numbers, measures and quantities, for Bhāskara.

In general the multiplier or quotient seems to be a quantity (*rāśi*, or any equivalent name for a heap), whose measure (*pramāna*) provides an amount, a number (*saṅkhyā*) given in long dvandva compounds or in successions.

⁴¹लेसो(sic)ऽद्विवेदरसरूपमतंगगजो ऽधो रूपांकदस्रभुजगद्विकनागवेदाः॥४४॥

⁴²राशितो ऽपि रसदस्रतापसा व्योमवेदगनाशविनो गुणः.

⁴³Note that if etymologically *sam-Mī* is related to the act of measuring, its current meaning can also be “to fasten things together”, thus a quantity can also be understood here as being expressed by a set of units/numbers fastened together.

⁴⁴नन्दाद्रिवस्वष्टनभो ऽद्विवेदा राशिर्गुणाख्यः खलु भागजातः।
भूतांकतिगमांशुनवाग्निदुल्यं लाभप्रमाणां प्रवदन्ति तजजाः॥५१॥

⁴⁵प्रत्पराणां गुणकारराशिः अंगक्षामभृगगणांकसंख्यः।

As we will now see, this underlying reflexion on the links between quantity, number and measure may be one door to the intended aim of these verses.

4 Speculating on the uses and meaning of the values obtained

4.1 Solving other equations

Most of the solution-couples presented here, as noted above, can be seen as intermediary resolutions for a problem of the form $KU(a, b, \pm c)[x, y]$. Indeed, this is how such a pulverizer method is introduced by Bhāskara in his commentary on these rules. In this sense, and because the verses appear in the commentary of the mathematical part of the *Āryabhaṭīya*, this set of verses can be seen as a mathematical tool to be used when computing solutions of other equations.

However, most of these solutions, can have an astronomical interpretation, linking the position of a planet to the time elapsed. They may provide the number of civil days and the number of revolutions elapsed since the beginning of the Kali Yuga (taken standardly as having began on February 18, 3102 B.C.E at sunrise at the intersection of the equator and the meridian in Ujjain) for the Sun, the Moon, the Moon's Apogee, the Moon's ascending node, Mars, Mercury, Jupiter, Venus, Saturn, the Moon's Anomaly. Other computed elements could concern, as we will see, intercalary months, intercalary days, omitted days and the sun's declination for the same time period. However, as we will see, they could also just be involving familiar large numbers, in a more purely mathematical endeavor.

4.2 Linking Time and Space

As we have seen, the ratios linking periodical planetary movements in the sky and passing time- a topic traditionally explored by ancient mathematical astronomy- enabled the formulation of problems that could be solved by a pulverizer.

Indeed, the astral interpretations of the pulverizer concern both time and position: Thus,

when (α, β) is a solution of $KU(a_g, b, -1)[x, y]$ is considered, we can understand it as meaning: Knowing that g , makes a_g revolutions (or signs, degrees, etc.) in a *yuga* of b civil days, when its position is seen as being a longitude of $\frac{-1}{b}$ revolutions (or signs, degrees and so forth), then this means that α days have elapsed and β revolutions (or signs, degrees, etc.) have elapsed.

With such an interpretation in mind, $KU(a_g, b, c)[x, y]$ can be seen as aiming at determining for a given position of g at λ_g (which depends on the value of y), how many days will have been elapsed, and how many revolutions of g will be necessary. Subdivisions of a revolution, serving then as providing an increasingly accurate value.

Some of the entities considered are not planets per se, but theoretical points associated to the movement of planets. They can however be considered as planet entities. This is the case for Mercury and Venus's *śighrocca*, the Moon's apogee and the Moon's ascending node, all noted in Ab.1.3 and Ab.1.4. The Moon's anomalistic revolutions, similarly can be treated in this way.

In all of these cases then, whether what is computed can be interpreted as real planetary movements or theoretical points in the sky, related to planetary movements, an interpretation of the results of the pulverizer is possible, although the accuracy of the values questions how "practical" such values could be.

Other cases seem more difficult to interpret, thus Verse 83 concerns the proportions linking the Rsine of the sun's declination with the Rsine of the sun's tropical longitude: how to interpret the results are not clear to me.

In some even more touchier cases, the verses seem to compute proportional timings for timings that may not be proportional. Thus verse 81 computes solutions of $KU(a_{ad}, b_{ad}, -1)[x, y]$, where a_{ad} is the number of intercalary days (*adhika*) in a *yuga*, and b_{ad} is the number of solar months in a *yuga*. If what is taken in account is the astral meaning of these phenomenas, they should thus be simply proportional for the pulverizer process to make sense, but this actually leads to absurd results:

Similarly, verse 82 computes a solution of $KU(a_{av}, b, -1)[x, y]$, where a_{av} is the number of omitted lunar days (*avama*) in a *yuga* and b_{av} the number of lunar days in a *yuga*. Supposing that these values are proportional as in the case examined above leads to absurd

statements.

In a consistent theory all timings should be consistent with one another. Could such explorations then have to do with verifying the consistency of the parameters employed in Āryabhaṭa's theory?

In the end, the computations for the planets (in a general sense) have a specific astral meaning, since the measure of time (civil days, solar years, lunar months) are computed according to their positions in the sky. Another hypothesis could be that this accumulation of numbers is a reflection on how time and space are computed and linked. These proportions may reflect disparities in different ways of computing dates for instance, or mean longitudes.

Does it make sense then to imagine that the authors or users of this table had these interpretations in mind?

If they did, maybe were they touching to the limits of Āryabhaṭa's parameters or theory of proportion?

Another, hypothesis, could be of course that such astral parameters were used for exploring the numbers they had at hand, outside of their astral meaning.

5 Conclusion

To end this study, let us come back to the questions raised in the introduction: Why and how then were these solutions collected?

Who could have used them, and why did they not have any posterity?

Of course answers to why such verses had no posterity in sanskrit astral and mathematical literature are bound to be very speculative. To attempt an answer, we would need to know for what kind of computations were more general pulverizers used? Were they “practical” tools for making almanacs and calendars (*pacāṅga*)? Were they mathematicians' games? We can note here the accuracy of the numbers undertaken here from the measuring units in thirds and fourths to the use of the “theoretical” epoch of the Kali Yuga. Together with the fact that this is found in the commentary of a *siddhānta* all point to the fact that such a repository of data may have been used for more contingent aims: explorations, verifications, fun?

Such questions also bring us to the place where mathematics and astral science meet in astral literature: precisely where procedures are ascertained, grounded, and playfully explored. Finally, the puzzle of these verses, can thus be seen as highlighting a key element of the historical study of numerical tables: it is by observing not only how they were derived but how they were used, that we can unravel more properly their context and function.

A Translation

This section presents a translation of the verses that end Bhāskara's commentary of the mathematical part of the *Āryabhaṭīya*. Shukla's 1976 edition of the Sanskrit text, which is not reproduced here, was used for this purpose.

In this translation, [] stand for addendums by the editor, K. S. Shukla, ⟨⟩ stand for my own addendums, () give specifications.

[Result of a *kuṭṭākāra* (pulverizer) with one as decrease]

These planet pulverizers (*graha-kuṭṭākāra*) are indeed condensed (*upanibādh-*) with *śloka* ⟨meters⟩. It is as follows:

BAB.Ku.1 Hommage to he who is the body of the ⟨asterisms⟩ starting with the sun (*Bhāskara*), whose vital spirit is ten thousand ⟨times that⟩ of the sun, to Śiva (as Śaṃbhu, “the benevolent”) cause of the destruction and creation of the earth||1||

p.156,

BAB.Ku.2 I indicate here with delight for those who know them the pulverizer (*kuṭṭākāra*) and quotient (*lābha*) ⟨derived⟩ two by two from revolutions and so forth in due order ||2||

line 10

[For the Sun]

BAB.Ku.3 For the sun, [two] -zero-six-four-nine [9 4602]. That quotient derived from the revolution ⟨number⟩ (*bhagaṇa*) is ⟨made of the numbers⟩ nine-five-two [259].⁴⁶ And for signs (*rāśi*), ⟨the multiplier is made of the numbers⟩ eight-seven-zero-three-eleven [113 078]

15

The quotient should be the numbers five-one-seven-three [3 715]||3||⁴⁷

BAB.Ku.4 For degrees (*samāṃśakas*) ⟨the multiplier is made of the numbers⟩ three-seven-eight-nine-five [59 873] the quotient is taught as ⟨made of the

⁴⁶The derivation of these values is explained in Example 7 of BAB.2.32-33, (K. S. Shukla, 1976, 137-138), (Keller, 2006, 135-136).

⁴⁷BAB.2.32-33 ex.9 gives the derivation of these values. (K. S. Shukla, 1976, 138-139), (Keller, 2006, 138-139).

numbers⟩ [eleven]-zero-nine-fives [59011]]⁴⁸

For minutes seven-four-six-one-eight [81647], then one-nine-two-eight-two-eight-
four [4 828 291]⁴⁹||4||

BAB.Ku.5 [For seconds], the multiplier (*guṇakāra*) [is indeed produced as] ⟨the
numbers⟩ one-seven-nine-zero-six [60 971], and then here the quotient is the
number which is one-nine-four-five-three-three-six-one two [21 633 5491]. And
after that, for thirds||5||

BAB.Ku.6 Here two-eight-[seven-four-four-one 144 782] are wished as multiplier.
And subsequently (*adhas*) one-nine-four-one-eight-six-two-two-eight-zero-three
[30 822 671 491]||6||⁵⁰

BAB.Ku.7 The multiplier quantity (*guṇakārārāśi*) [produced for fourths] should
be known as a number equal to twenty-six-four-nine [9 426]. Then here the
quotient ⟨is made of the numbers⟩ one-nine-four-nine-one-two-zero-four-zero-
twelve [1 20 402 191491]||7||

BAB.Ku.8 Just one is the recalled denominator for the sun, for revolutions and
so forth until fourths, zero-zero-five-seven [7500]⁵¹||8||

[For the Moon]

BAB.Ku.9 When pulverized (*prakuṭṭita*) [by the revolutions] of the moon, ⟨the
multiplier is made of the numbers⟩ seven-three-eight-six-seven-seven [776 837]||
A heap which is a quotient quantity (*labdharāśinicaya*) is laid out (*vyavasthita*)
as ⟨the following numbers⟩ three-three-four-eight-two [24 833]||9||

BAB.Ku.10 For signs also the multiplier is ⟨made of the numbers⟩ six-two-seven

⁴⁸The derivation of these values is explained in BAB.2.32-33.ex.10 (with maybe a misprint of 51 011 rather than 59 011, in the solved equation in both Shukla's edition, and my translation), (K. S. Shukla, 1976, 140), (Keller, 2006, 140-141).

⁴⁹These values are computed by Bhāskara in BAB.2.32-33 ex11, (K. S. Shukla, 1976, 141-142), (Keller, 2006, 141-143).

⁵⁰Note that in Shukla's table for the sun, the printed value misses the zero.

⁵¹This last number is the greatest common divisor of the revolution number of the sun, and the number of civil days in a yuga, that is the GCD(4 320 000, 1 577 917 500). Despite the formulation it serves here only as a divisor for the number of revolutions of the sun in the Kali Yuga, all other values remain unchanged (or "divided by one").

- zero-four-zero-two [2 040 726]| Here the quotient quantity (*labdhanicaya*) is separately announced as eleven-three-six-nine-eight [896 311]||10|| 20
- BAB.Ku.11 What is called the multiplier for the remaining degrees is ⟨made of the numbers⟩ one-twelve-zero-four-three [340 121]| The quotient is ⟨made of the numbers⟩ five-five-five-one-eight and heaps (*nivaha*) ⟨that are⟩ four and four [4 481 555]||11||
- BAB.Ku.12 In minutes, having computed (*vigaṇayya*), ⟨the multiplier⟩ is measured (*sammīta*) as eight-five-eleven-two [21 158]| Nine-eleven-seven two-seven [sixteen 16 727 119]⁵² ⟨are the numbers⟩ obtained [here]||12|| p. 158 line 1
- BAB.Ku.13 ⟨The multiplier⟩ produced with the moon's seconds is measured as four-thirteen-six-one [16 134]| The next is recalled as ⟨made of the digits⟩ five-one-three-fourteen-three five-six-seven [765 314 315]||13|| 5
- BAB.Ku.14 In thirds, with a number which is the multiplier eighteen-five-six [6 518] indicate ⟨the multiplier⟩| The following (e.g. the quotient) is ⟨made of the digits⟩twenty seven-nine-one-three-eight-zero-five-five-eighteen [18 550 831 927]||14|| 10
- BAB.Ku.15 In what follows thirds (e.g fourths) three-nine-two-two are mentioned [2 293] in due order| In due order, eleven-five-seven-two-four five-six-fifteen-nine-three [3 91 565 427 511]||15|| 15
- BAB.Ku.16 Starting with the moon's degrees the chosen divisor is five [5] five [5] and five after | In due order here ⟨it⟩ is completely thirty-two-seven [732]||16||
- [For the Moon's Apogee]
- BAB.Ku.17 The number equal (*samāna*) to nine-seven-eight/seven-seven/eight-six six-eight-one [718 667 879]| is [here the multiplier of] the moon's apogee (*indūcca*) which is shown with all the asterisms (*bhacakraḍṣṣṭa*) and the quotient is one-six-three-two-two-two [222 361]||17|| 20

⁵²This Shukla correction of अष्टि to rectify the manuscripts readings as सप्ति, seven, respects the verse's meter, accounts for a correct result. Nonetheless the manuscript error has been noted in Tables 7 and Table 8.

BAB.ku.18 The multiplier quantity measures (*pramāṇa*) four-eight [two-two-zero two-one] six [61 202 254] Here in due order for signs the computed quotient is seven-three-two-sever-two-two [227 237]||18||

BAB.Ku. 19 Four-eight-five-six-six is stated with a number in due order nine-eight-one [18 966 584] In due order for degrees, with a computation the quotient is ⟨made of (the numbers)⟩ three-two-six-twelve-one-two [2 112 623]||19||

BAB.Ku. 20 When that is elapsed minutes, the multiplier quantity is thirty-two-zero-seven-eight-zero-five [5 087 032] Here the told number which is the quotient is twenty-seven-six-seven-nine-nine-three-three [33 997 627]||20||

BAB.Ku.21 The multiplier number for seconds should be one-seven-thirty two-seven [73 271] Those who know three-six-ten-eight-three-nine two [29 381 063] command over the quotient||21||

BAB.Ku.22 What indeed is here derived from thirds is equal in numbers to eight-six-seven-three-nine [93 768] | And the quotient is five-nine-three-one-one-zero-six-five-two-two [2 256 011 395]||22||

BAB.ku.23 In due order, what is [derived from] fourths is equal in numbers to four-seven-eight-nine-six-one [169 874] And the quotient is eleven-eight-one-three-zero-five-two-two-five-four-two [245 225 031 811]||23||

BAB.ku.24 For signs etc in due order [here the divisors should be obtained as told] Twelve [12] and then five [5], [five 5 and upto] fourths||24||

[For the Moon's Node]

BAB.Ku.25 They say the multiplier which is the number seven-seven-one-six-zero-six-five-two-six [625 606 177]

[And the divisor of the node] concerning revolutions equals the measure two-seven-zero-two-nine [92 072]||25||

BAB.Ku.26 Those who know that declare one-zero-four-three-six-five-five-eleven [1 155 63 401] |

[Here] the quotient which brings together numerous quantities in due order is

three-nine-zero-four-zero and two [204 093]||26||

BAB.ku.27 One-seven-six-three-four-six-three-two [23 643 671] whose ⟨digits⟩ make the multiplier| Alongside the quotient is stated with a number whose parts are in due order one-nine-six-two-five-twelve [1 252 691] ||27||

For minutes, the multiplier's numbers are eight-five-eight-one-six-one-four [4 161 858]|| They proclaim a quotient equal to ⟨the numbers⟩ five-three-two-zero-three-two-thirteen [13 230 235]||28||

BAB.Ku.28 With a reasoning (*yukti*) on the multiplier for seconds which is a quantity four-nine-seven-two-seven-eight [872 794] ⟨is found⟩| In due order here the divisor was seven-zero-eight-two-seven-four-six-six-one [166 472 807]||29||

BAB.Ku.29 That which is a multiplier quantity in thirds is here eight-six-six-seven-three [37 668]|| The quotient also is measured as seven-zero-[seven]-seven-zero-one-three-four [431 077 507]||30||

BAB.Ku.30 They say for fourths the number nine-three-nine-eight-six-one [168 939] which is a multiplicand| And the quotient is one-four-five-seven-nine-five-one-zero-zero-six-eleven [116 001 597 571]||31|| BAB.Ku.32 Concerning the divisors two, for the sign's divisor twelve is stated| For the assemblage of degrees, etc five is stated by the wise one's||32||

[For Mars]

BAB.Ku.33 For Mars (*bhauma*) concerning revolutions of the asterisms, the produced multiplier was thirteen-twelve-ten-sixteen [16 101 213]|| Here the quotient is in due order told to measure seven-three-four-three-two [23 437]||33||

BAB.Ku.34 In that case, the measure of the multiplier is nine-four-zero-five-one-two-four-three [34 215 049]|| For signs in due order, the amount of the quotient is specified to be three-four-six-seven-nine-five [597 643]||34||

BAB.Ku.35 They say the multiplier for measures in degrees, four-zero-four-nine-one-thirteen [1 319 404]|| Here the ordered number of the quotient is equal to one-nine-three-one-nine-six [691 391]||35||

BAB.Ku.36 With known multipliers [which are minutes], the multiplier is two-nine-one-three-six-eight-one [1 863 192]|| Here the quotient is seen to be equal to one-three-[seven-zero-eight-five]-eight-five [58 580 731]||36||

BAB.Ku.37 For seconds, the multiplier is recognized as ⟨made of (the numbers)⟩ six-six-two-five-five-ones [155 266]

The quotient, they said was equal to five-five-six-three-zero-nine-two-nine-two [292 903 655]||37||

BAB.Ku.38 In that case, for a number in thirds, in due order, the multiplicand measures three-six-two-eight-eighteen [188 263]

And the quotient is ⟨made of (the numbers)⟩ five-three-six-seven-seven-zero-nine-zero-three-one-two [21 309 077 635]||38||

BAB.Ku.39 For fourths the multiplier's amount measures two-two-seven-eight-four [48 722]

And the quotient is ⟨made of (the numbers)⟩ one-five-six-three-zero-two-four-eight-eight-zero-three-three [330 884 203 651]||39||

BAB.Ku.40 For revolutions (*maṇḍala*) and signs (*grha*) the memorized divisor (*cheda*) is twelve

For degrees etc. ⟨it is⟩ repeatedly (*pareṣām*) five and five indeed continuously (*sthiti*) indeed||40||

[For the *śighrocca* of Mercury]

BAB.Ku.41 The multiplier [for mercury] is the [number] six-seven-two-[seven-eight-five-three 23 587 276]|| And for revolutions the quotient in due order is the number nine-twelve-eight-six-two [268 129]||41||

BAB.Ku.42 With signs in due order the multiplier should be equal to [nine]-one-eight-six-nine-eight-five [5 896 819]|| And the quotient ⟨obtained⟩ with a method (*vidhi*) is the indicated number seven-eight-three-four-zero-eight [804 387]||42||

BAB.Ku.43 In degrees, they stated the multiplier ⟨made of (the numbers)⟩ four-

two-four-nine-five-eight-one [1 859 424]| There the quotient indeed measures the number nine-three-three nine-zero-six-seven [7 609 339]||43||

BAB.Ku.44 They stated the established multiplier for minutes two-five-nine-four-five- one [154 952]| The quotient whose number is five-nine-six-six-four-zero-eight-three [38 046 695] is specified by mathematicians||44||

BAB.Ku.45 The quantity called the multiplier for seconds measures two-seven-one-three-fifteen [153 172]| Here, the quotient ⟨obtained⟩ with a method (*vid-dhi*) is nine-seven-one-eight-seven-five-six-five-two-two [2 256 578 179]||45||

BAB.Ku.46 The told multiplier whose number for thirds is here they say zero-nine-eighteen [184 890]| And the quotient is equal to one-nine-zero-five-six-four-one-three-four-three-sixteen [163 431 465 091]||46||

BAB.Ku.47 For fourths the multiplier is six-seven-two-eight-zero and one [108 276]. Now the quotient,| one-nine-eight-five-six-one-two-six-five-two-four fives [5 742 562 165 891]||47||

BAB.Ku.48 Twenty [20] and then sixty [60] is the divisor producing revolutions and signs| Wise ones indicate for degrees etc. in due order five [5]||48||

[For Jupiter]

BAB.Ku.49 The produced multiplier is collectively told to be eight three-zero-three-five-zero-six-seven [76 053 038]| The quotient for Jupiter, in due order for revolutions, will be five-five-five-seven-one [17 555]||49||

BAB.Ku.50 The multiplier seen with a method for signs is four-seven-two-three-five-two-eight-two [28 253 274]| The quotient for an established computation is said to be in this case the measure nine-twenty five-eight-seven [78 259]||50||

BAB.Ku.51 Indeed what is called a multiplier produced for degrees is the quantity nine-seven-eight-eight-zero seven-four [4 708 879]| Those who know indicate that the measure of the quotient is equal to five-nine-twelve-nine-five [391 295]||51||

BAB.Ku.52. Those who know will the multiplier in due order for minutes to be ⟨made of (the numbers)⟩ seven-one-seven-zero-three-eight [830 717]| The quotient for minutes, having computed ⟨it⟩ is told to be nine-one-eighteen-four-one-four [4 141 819]||52||

BAB.Ku.53 For seconds, the multiplier is produced by means of ⟨the numbers⟩ one-six-eight-two-eight [682 861] | In that case, ⟨the numbers⟩ one-nine-nine-seven-seven-two-four-zero-two [204 277 991] are said to be the quotient||53||

BAB.Ku.54 And for thirds here indeed the number thirty-two-six-nine-seven-one [179 632] is the multiplier| Seven-zero-seven-sixteen-two-four-twos connected with thirty two [3 224 216 707] are told to be the quotient||54||

BAB.Ku.55 Here the produced multiplier for fourths is measured as one-three-three-five eighteen [185 331]. The quotient is| one-seven-one-seven-eight-four-zero-nine-five-nine-nine-one [199 590 487 171]||55||

BAB.Ku.56 For revolutions and signs the divisor is remembered as twelve| The divisors of the quantities (*rāśi*) in degrees, minutes etc are told to be precisely five ||56||

[For the *śighrocca* of Venus]

BAB.Ku.57 For revolutions, the multiplier for the apogee of Venus is nine-four-zero-six-four-zero-seven [70 046 049]| The quotient is said to be equal to four-three-seven-eleven-three [311 734] according to the order of numbers of a numerical rule||57||

BAB.Ku.58 The multiplier for signs is pointed out to be two-five-four-zero-one-seven-eight-threes [38 710 452]| The quotient is an amount equal to one-three-three-seven-six-zero-two [2 067 331]||58||

BAB.Ku.59 The multiplier for degrees is equal to [the number] two-four-seven-one [five-four six 6 451 742]| Then here the quotient is the quantity five-five-six-six-three-three-zero-one [10 336 655]||59||

BAB.Ku.60 For minutes the produced multiplier is the quantity six-six-two-four-one-fourteen[1 414 266] Here the quotient is equal to [one]-three-nine-one-five-nine-five-three-one [135 951 931]||60||

BAB.Ku.61 In seconds they say the result called “multiplier” equal to eight-two-eight-three-four-six [643 828] And the quotient should be one-seven-twelve-three-four-three-one-seven-three [3 713 431 271]||61||

BAB.Ku.62 In due numerical order the multiplier which is a quantity for thirds is the number two-eight-seven-three-twelve [123 782] Here the quotient is equal to three-eight-two-nine-two-five-six-three-eight-two-four [42 836 529 283]||62||

BAB.Ku.63 For fourths the multiplier quantity is the number six-seven-zero-nine [9 076] And the quotient is one-five-eight-six-six-seven-two with five-four-eight-eighteen [188 452 766 851]||63||

BAB.Ku.64 For revolutions together with signs the divisor is recalled as twelve| And they have told five [5] just five [5] for the degrees and so forth for Venus||64||

[For Saturn]

BAB.Ku.65 In revolutions, the mul[tiplier is indica]ted by those who know] to be eleven-two-[five-six-zero-zero-three-eleven 113 065 211] The quotient quantity of the one born from self (and) yoked to the sun (Saturn) is two-zero-five-zero-one [10 502]||65||

BAB.Ku.66 In this case, the multiplier quantity is four-eight-five-nine-three-one-one-six [61 139 584] But according to signs the number seven-four-one-eight-six [68 147] is declared equal to the quotient||66||

BAB.Ku.67 What is called multiplier appears (*prabhava*) as nine-three-one-six-five-nine-eight-one[18 956 139] For up to degrees (*aṃśāvadhi*) the produced quotient is the quantity (*samūha*) three-six-eight-three-three-six [633 863]||67||

BAB.Ku.68 The seen quantity (*samūha*) produced as the multiplier is seven-four-

seven-four-six-two [264 747]| And in due order for minutes here, computing,
the quotient measures three-six-eleven-three-five [531 163]||68||

BAB.Ku.69 The quantity one-two-zero-eleven-eight [811 021] is produced for seconds for the multiplier| Here the quotient bestowed measures seven-eight-two-nine-two-six-seven-nine [97 629 287]||69||

BAB.Ku.70 The multiplier for amounts of thirds is indicated as twelve-three-zero-nine[-one 190 312]| The quotient is seven-zero-five-[five-six]-five-four-seven-three-ones [1 374 565 507]||70||

BAB.Ku.71 For fourths, the multiplier is nine-zero-five-five-eight- one [185 509].
Now the quotient is| eleven-two-four-nine-four-two-nine-three-zero joined with eight [80 392 494 211]||71||

BAB.Ku.71 The divisor of revolutions is four [4], and just twelve [12] for signs|
In due order the exact divisor is said to be five for Saturn (*saurasya lavata* “a tiny bit of the sun”) ||72||

[For the Moon’s Anomaly]

BAB.Ku.73 The multiplier produced for revolution is three-five-nine-three-one-twelve-four-threes [341 213 953]| And the quotient for the moon’s anomaly is three-nine-one-three-eight-three-twelve [12 383 193]||73||

BAB.Ku.74 The multiplier for its signs is equal to three-zero seven-seven-two-two-eight-seven [78 227 703]| The quotient is proclaimed by mathematicians to be two-eight-zero-eight-six zero-four- three [34 068 082]||74||

BAB.Ku.75 The multiplier born from the degrees should amount to three-six-two-seven-eight-one-six two [26 187 263]| The proclaim the quotient amounts to one-six-seven-five-three-one-two-four-three [342 135 761]||75||

BAB.Ku.76 In this <case>, the multiplier quantity produced from minutes is four-two-eight-three-seven-three-four [4 373 824]| Indeed there the quotient is five-one-three-two-three[-six-eight-two-four]-three [3 428 632 315]||76||

BAB.Ku.77 The multiplier quantity for seconds is equal to seven-three-eight-three-one [13 837]| Here the quotient is equal to nine-seven-eight-seven-zero-eight-zero-five-six [650 807 879]||77||

BAB.Ku.78 The seen multiplier has a measure of zero-eight-eight-three-twelve [123 880]| And the quotient, the quotient which is a quantity in thirds is eleven-zero-seven-six-four-three-nine-five-nine-four and with three [349 593 467 011]||78||

BAB.Ku.79 Here the multiplier for fourths is twenty-four-three-two-fourteen [142 324], but the quotient quantity is| one-five-two-nine-eight-seven-two-eight-five-eight-nine-zero and twenty-four [24 098 582 789 251] ||79||

BAB.Ku.80 The reducer which is a denominator should be known for signs to be twelve [12]| In due order for degrees and so forth the denominators are seen in due order by the enlightened ones to be five [5]||80||

[For Intercalary Days]

BAB.Ku.81 The ⟨number of⟩ intercalary ⟨days⟩ which is a multiplier is equal to nine-zero-five-two-seven-six [672 509]| The denominator quantity in due order is zero-[seven-six]-zero two [20 670]||81||

[For Omitted Days]

BAB.Ku.82 The ⟨number of⟩ omitted ⟨days⟩ which is a multiplier is one-three-zero-seven-four-two-six[6 247 031]| The quotient also is mentioned as nine-four-seven-seven-nine [97 749]||82||

For the Sun's Declination

BAB.Ku.83 A multiplier in due order for the declination is seven-four-three [347]| In due order, the quotient quantity is seen to be [one-four]-one [141]||83||

For Intercalary Months

BAB.Ku.84 For the intercalary months in a *yuga*, the multiplier is seen as seven-one-three-zero-zero-nine and eighteen [18 900 317]|| Here also, in this case, the quotient is constantly for revolutions five-eight-zero-nine-one [19 085]||84||

BAB.Ku.85 The quantity which is a multiplier is equated with <the numbers> one-nine-three-zero eleven-two-twelve [122 110 391]|| Here the quotient indeed born by signs is the quantity which is the number three-four-six-nine-seven-fourteen [1 479 643]||85||

BAB.Ku.86 The quantity which is a multiplier should be six-three-eight-four-three-seven-four two [24 734 836] | In due order the quotient established with a method for degrees is one-five-five-one-nine-nine-eight [8 991 551]||86||

BAB.Ku. 87 In minutes, the quantity for the multiplier seen is the number eight-seven-four-fourteen-eight-three [3 814 478]||

[The numbers which give] the quotient are said to measure nine-one- eight-seven-nine-one-three-eight [83 197 819]||87||

BAB.Ku.88 The established multiplier for seconds is seen to measure nine-four-five-two- fourteen [142 549]|| The [numbers which make] the quotient are said to be eleven-seven-forty-eight-five-six-eight-one [186 548 711]||88||

BAB.Ku.89 And also for thirds the number [for the multiplier] is six, zero six-four-three one [134 606]|| And the quotient is three-eight-six-nine-three-two-nine-six-five with ten [10 569 239 683]||89||

BAB.Ku.90 The quantity obtained for multiplier is declared to be for fourths twelve-four-three-nine [93 412]|| Here the quotient is equal to one-three-five-eight [three-sixteen]-nine-zero-zero-four-four [440 081 638 531]||90||

BAB.Ku.91 The reducer for revolutions with signs is exactly twelve [12]|| For the remaining, when reducing the divisor is five [5] and also five [5]||91||

This being the case of Bhāskara's commentary on the *Āryabhaṭatantra*, thus ends the mathematical quarter.

B Tabular data in the verses

Tabular displays of the values provided in these verses have already been published by K. S. Shukla⁵³. Here, I would like to enhance his tables by explaining and commenting on the values found. In all these tables, for resolutions of $KU[a, b, 1](x, y)$, the columns for a and b are implicit. To mark their implicit status they are tabulated in italic. They use data more or less explicited by Bhāskara or Āryabhaṭa in other parts of the text, which are referenced when they have been traced. Although the values for a and b are not specified in the verses, each section of versified data, ends with a list of divisors used to reduce simultaneously a and b . These divisors are listed in a separate column, labeled d .

I've recomputed all the solutions of these pulverizers. I've done this using spreadsheets, that enable one to see and check all the intermediary values computed in a pulverizer process. These spreadsheets which actually enable one to compute any pulverizer problem are available at hal-shs-website address⁵⁴.

In most cases the tabulated solutions are the smallest possible. There is one notable exception, noted in bold. A rectified value of a manifest scribal errors has in the same way been underlined.

B.1 Standard Values

In most cases $KU(a_g, b, -1)[x, y_g]$ is the data coined implicitly and explicitly in a given verse, were:

a_g is the revolution number of planet g , as given in Ab.1.3-4

b is the number of civil days in the Kali Yuga (1 577 917 500)

y_g is the number of revolutions of g since the beginning of the Kali Yuga

x is the number of civil days elapsed since the beginning of the Kali Yuga.

⁵³(K. S. Shukla, 1976, 335-339).

⁵⁴I am ready to improve their readability on a simple electronic request.

B.2 Sun

The tables here, corresponds to the data stored in verses 3-9. $KU(a_s, b, 1)[x, y_s]$ considered. a_s is the number of revolutions of the sun in the Kali Yuga, 4 320 000. BAB.2.32-33 ex 7, also underlines that 7500 is a common divisor of a_s and b , and works with the reduced values:

$$\frac{a_s}{b} = \frac{4320000}{1577917500} = \frac{576}{210389}$$

Table 5: Tabulated values for the Sun

Measuring Units	d	a_s	b in days	x	y
Revolutions	7500	<i>576</i>	<i>210 389</i>	94602	259
Signs	1	<i>6 912</i>	<i>210 389</i>	113078	3715
Degrees	1	<i>207 369</i>	<i>210 389</i>	59873	59011
Minutes	1	<i>12 441 600</i>	<i>210 389</i>	81 647	4 828 291
Seconds	1	<i>746 496 000</i>	<i>210 389</i>	60 971	216 335 491
Thirds	1	<i>4 478 976 000</i>	<i>210 389</i>	144 782	30 822 671 491
Fourth	1	<i>2 687 385 600 000</i>	<i>210 389</i>	9426	120 402 19[1] 491

In Table 6, the data is presented as tabulated solutions of these indeterminate equations

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Table 6: Tabulated values for the Sun (2)

Equation	Couple solution
$y = \frac{576x-1}{210389}$	(94 602, 259)
$y = \frac{6912x-1}{210389}$	(113 078, 3 715)
$y = \frac{207360x-1}{210389}$	(59 873, 59 011)
$y = \frac{12441600x-1}{210389}$	(81 647, 4 828 291)
$y = \frac{746496000x-1}{210389}$	(60 971, 216 335 491)
$y = \frac{44789760000x-1}{210389}$	(144 782, 30 822 671 491)
$y = \frac{2687385600000x-1}{210389}$	(9426, 120 402 19[1] 491)

B.3 Moon

The tables here, corresponds to the data stored in verses 9-16. $Ku(a_c, b, 1)[x, y_c]$ is considered, where, a_c is the number of revolutions of the Moon in the Kali Yuga, 57 753 336.

In one instance, the values for thirds, the smallest solution $Ku(a, b, -1)[\alpha, \beta]$ is not stated, but $Ku(a, b, -1)[\alpha + b, \beta + a]$. This is explained in subsection 2.1.

Further, Shukla has corrected for seconds faulty readings from manuscripts, as underlined in the translations of these texts.

Table 7: Tabulated values for the Moon (1)

Measuring Units	d	a_c	b in days	x	y
Revolutions	732	78 898	2 155 625	776 837	24 833
Signs		946 776	2 155 625	2 040 726	896 311
Degrees	5	5 680 656	431 125	340 121	4 481 555
Minutes	5	68 167 872	86 225	21 158	16 727 119
Seconds	5	818 014 464	17 245	<u>16 134</u>	<u>765 314 315</u>
Thirds	5	9 816 173 568	3449	6 518	18 550 831 927
Fourths		588 970 414 080	3449	2 293	3 91 565 427 511

Table 8: Tabulated values for the Moon (2)

Equation	Couple solution
$y = \frac{78898x-1}{2155625}$	(776 837, 24 833)
$y = \frac{946776x-1}{2155625}$	(2 040 726, 896 311)
$y = \frac{5\ 680\ 656x-1}{431\ 125}$	(340 121, 4 481 555)
$y = \frac{68167872x-1}{17245}$	(21 158, 16 727 119)
$y = \frac{818014464x-1}{3449}$	(16 134, 765 314 315)
$y = \frac{9816173568x-1}{3449}$	(6 518, 18 550 831 927)
$y = \frac{588970414080x-1}{3449}$	(2 293, 3 91 565 427 511)

B.4 Moon's Apogee

The tables here, corresponds to the data stored in verses 17-24.

KU($a_{cu}, b, 1$)[x, y_{cu}] is considered. a_{cu} is the number of revolutions of the Moon's Apogee since the beginning of the Kali Yuga, 488 219.

Table 9: Tabulated values for the Moon's Apogee (1)

Measuring Units	d	a_{cu}	b	x	y
Revolutions		<i>488 219</i>	<i>1 577 917 500</i>	718 667 879	222 361
Reduced signs	12	<i>488 219</i>	<i>131 493 125</i>	61 202 254	227 237
Reduced degrees	5	<i>2 929 314</i>	<i>26 298 625</i>	18 966 584	2 112 623
Reduced minutes	5	<i>35 151 768</i>	<i>5 259 725</i>	5 087 032	33 997 627
Reduced Seconds	5	<i>421 821 216</i>	<i>1 051 945</i>	73 271	29 381 063
Thirds	5	<i>5 061 854 592</i>	<i>210 389</i>	93 768	2 256 011 395
Fourths		<i>303 711 275 520</i>	<i>210 389</i>	169 874	245 225 031 811

Table 10: Tabulated values for the Moon's Apogee (2)

Equation	Couple solution
$y = \frac{488219x-1}{1577917500}$	(718 667 879, 222 361)
$y = \frac{488219x-1}{131493125}$	(61 202 254, 227 237)
$y = \frac{2\ 929\ 314x-1}{26\ 298\ 625}$	(18 966 584, 2 112 623)
$y = \frac{35151768x-1}{5259725}$	(5 087 032, 33 997 627)
$y = \frac{421821216x-1}{1051985}$	(73271, 29 381 063)
$y = \frac{5061854592x-1}{210389}$	(93 768, 2 256 011 395)
$y = \frac{303711275520x-1}{210389}$	(169 874, 245 225 031 811)

B.5 Moon's Ascending Node

Verses 25-32 tabulate solutions of $KU(a_p, b, -1)[x, y_p]$, where a_p is the number of revolutions of the moon's node in an opposite direction (eg. ascending, *pātaviloma*) in a *yuga* (Ab.1.4 gives 232 226 revolutions).

Table 11: Tabulated values for the Moon's Ascending Node(1)

Measuring Units	d	a	b in days	x	y
Reduced Revolutions	2	116 113	788 958 750	625 606 177	92 072
Reduced signs, divisor	12	232 226	131 493 125	1 155 63 401	204 093
Reduced degrees	5	1 393 356	26 298 625	23 643 671	1 252 691
Reduced minutes	5	16 720 272	5 259 725	872 794	166 472 807
Reduced Seconds	5	200 643 26	1 051 945	73 271	29 381 063
Thirds	5	2 407 719 168	210 389	37 668	431 077 507
Fourths	5	144 463 150 080	210 389	168 939	116 001 597 571

Table 12: Tabulated values for the Revolutions of the Moon's Ascending node (2)

Equation	Couple solution
$y = \frac{116113x-1}{788958750}$	(625 606 177, 92 072)
$y = \frac{232 \ 226x-1}{131493125}$	(1 155 63 401, 204 093)
$y = \frac{1 \ 393 \ 356x-1}{26 \ 298 \ 625}$	(23 643 671, 1 252 691)
$y = \frac{16 \ 720 \ 272x-1}{5259725}$	(4 161 858, 13 230 235)
$y = \frac{200 \ 643 \ 264x-1}{1 \ 051 \ 945}$	(872 794,166 472 807)
$y = \frac{2 \ 407 \ 719 \ 168x-1}{210 \ 389}$	(37 668,431 077 507)
$y = \frac{144 \ 463 \ 150 \ 080x-1}{210 \ 389}$	(168 939, 116 001 597 571)

B.6 Mars

Verses 33-40 tabulate solutions of $KU(a_b, b, -1)[x, y_b]$, where a_b is number of revolutions of Mars (*bhaumya*) in a *yuga*. According to Ab.1.3, $a_b = 2296824$ revolutions.

Table 13: Tabulated values for Mars (1)

Measuring Units	d	a	b in days	x	y
Revolutions	12	191 402	131 493 125	16 101 213	23 437
Signs		2 296 824	131 493 125	34 215 049	597 643
Degrees	5	137 80 944	26 298 625	1 319 404	691 391
Minutes	5	165 371 328	5 259 725	1 863 192	58 580 731
Seconds	5	1 984 455 936	1 051 945	155 266	292 903 655
Thirds	5	23 813 471 232	210 389	188 263	431 077 507
Fourths		1 428 808 273 920	210 389	48 722	330 884 203 651

Table 14: Tabulated values for the Revolutions of Mars (2)

Equation	Couple solution
$y = \frac{191 \ 402x-1}{131 \ 493 \ 125}$	(16 101 213, 23 437)
$y = \frac{2296824x-1}{131493125}$	(34 215 049, 597 643)
$y = \frac{13780944x-1}{26 \ 298 \ 625}$	(1 319 404, 691 391)
$y = \frac{165 \ 371 \ 328x-1}{5259725}$	(1 863 192, 58 580 731)
$y = \frac{1984455936x-1}{1051945}$	(155 266, 292 903 655)
$y = \frac{23813471232x-1}{210389}$	(188 263, 21 309 077 635)
$y = \frac{1428808273920x-1}{210 \ 389}$	(48 722, 330 884 203 651)

B.7 (*śīghrocca* of) Mercury

Verses 41-48 tabulate solutions of $KU(a_u, b, -1)[x, y_u]$. a_{bu} is the number of revolutions of (the *śīghrocca* of) Mercury (*bhudha*) in a *yuga* ccording to Ab.1.3 $a_{bu} = 17937020$.revolutions⁵⁵

Table 15: Tabulated values for (the *śīghrocca* of) Mercury (1)

Measuring Units	d	<i>a</i>	<i>b</i> in days	x	y
Revolutions	2	896 851	788 958 750	23 587 276	268 129
Signs	30	3 587 404	26 298 625	5 896 819	804 387
Degrees	5	21 524 424	5 259 725	1 859 424	7 609 339
Minutes	5	258 293 088	38 046 695	154 952	58 580 731
Seconds	5	3 099 517 056	210 389	153 172	2 256 578 179
Thirds		185 971 023 360	210 389	184 890	163 431 465 091
Fourths		1 428 808 273 920	210 389	108 276	5 742 562 165 891

⁵⁵Shukla and Sarma add- (K. S. Shukla K. V. ; Sharma, 1976, 6-7), that it is is actually a mean value in which the mean revolutions of Mercury around the earth are equated to the mean revolutions of Mercury around the sun (*śīghrocca*). But this is not at stake in the present data considered.

Table 16: Tabulated values for the Revolutions of (the *śighrocca* of) Mercury (2)

Equation	Couple solution
$y = \frac{896 \ 851x-1}{788958750}$	(23 587 276, 268 129)
$y = \frac{3 \ 587 \ 404x-1}{26 \ 298 \ 625}$	(5 896 819, 804 387)
$y = \frac{21 \ 524 \ 424x-1}{5259725}$	(1 859 424, 7 609 339)
$y = \frac{258 \ 293 \ 088x-1}{1051945}$	(154 952, 38 046 695)
$y = \frac{3 \ 099 \ 517 \ 056x-1}{210 \ 389}$	(153 172, 2 256 578 179)
$y = \frac{185 \ 971 \ 023 \ 360x-1}{210389}$	(184 890, 163 431 465 091)
$y = \frac{226 \ 576 \ 465 \ 920x-1}{210389}$	(184 890, 163 431 465 091)

B.8 Jupiter

Verses 49-56 tabulate solutions of $KU(a_{gu}, b, -1)[x, y_{gu}]$, where a_{gu} is the number of revolutions of Jupiter (*guru*) in a *yuga*. According to Ab.1.3 $a_{gu} = 364224$ revolutions.

Table 17: Tabulated values for Jupiter

Measuring Units	d	a	b in days	x	y
Revolutions	12	30 352	131 493 125	76 053 038	17 555
Signs	12	364 224	131 493 125	28 253 274	78 259
Degrees	5	2 185 344	26 298 625	4 708 879	391 295
Minutes	5	26 224 128	5 259 725	830 717	4 141 819
Seconds	5	314 689 536	1 051 945	682 861	204 277 991
Thirds	5	3 776 274 432	210 389	179 632	3 224 216 707
Fourths	5	226 576 465 920	210 389	185 331	199 590 487 171

Table 18: Tabulated values for the Revolutions of Jupiter (2)

Equation	Couple solution
$y = \frac{30 \ 352x-1}{131 \ 493 \ 125}$	(76 053 038, 17 555)
$y = \frac{364 \ 224x-1}{131 \ 493 \ 125}$	(28 253 274, 78 259)
$y = \frac{2 \ 185 \ 344x-1}{26 \ 298 \ 625}$	(4 708 879, 391 295)
$y = \frac{26 \ 224 \ 128x-1}{5259725}$	(830 717, 4 141 819)
$y = \frac{314 \ 689 \ 536x-1}{1051945}$	(682 861, 204 277 991)
$y = \frac{3 \ 776 \ 274 \ 432x-1}{210389}$	(179 632, 3 224 216 707)
$y = \frac{226 \ 576 \ 465 \ 920x-1}{210389}$	(185 331, 199 590 487 171)

B.9 (*śīghrocca* of) Venus

Verses 57-64 tabulate solutions of $KU(a_{bh}, b, -1)[x, y_{bh}]$, where a_{bh} is the number of revolutions of (the *śīghrocca* of) Venus (*bhrgu*) in a *yuga*. According to Ab.1.4, $a_{bh} = 7022388$ revolutions.

Table 19: Tabulated values for (the *śīghrocca* of) Venus (1)

Measuring Units	d	<i>a</i>	<i>b</i> in days	x	y
Revolutions	12	585 199	131 493 125	70 046 049	311 734
Signs	12	7 022 388	131 493 125	38 710 452	2 067 331
Degrees	5	42 134 328	26 298 625	6 451 742	10 336 655
Minutes	5	505 611 936	5 259 725	1 414 266	135 951 931
Seconds	5	6 067 343 232	1 051 945	643 828	3 713 431 271
Thirds	5	72 808 118 784	210 389	123 782	42 836 529 283
Fourths	5	4 368 487 127 040	210 389	9 076	188 452 766 851

Table 20: Tabulated values for the Revolutions of (the *śighrocca* of) Venus (2)

Equation	Couple solution
$y = \frac{585 \ 199x-1}{131 \ 493 \ 125}$	(70 046 049, 311 734)
$y = \frac{7 \ 022 \ 388x-1}{131 \ 493 \ 125}$	(38 710 452, 2 067 331)
$y = \frac{42 \ 134 \ 328x-1}{26 \ 298 \ 625}$	(6 451 742, 10 336 655)
$y = \frac{505 \ 611 \ 936x-1}{505 \ 611 \ 936}$	(1 414 266, 135 951 931)
$y = \frac{6 \ 067 \ 343 \ 232x-1}{1051945}$	(643 828, 3 713 431 271)
$y = \frac{72 \ 808 \ 118 \ 784x-1}{210389}$	(643 828, 42 836 529 283)
$y = \frac{4 \ 368 \ 487 \ 127 \ 040x-1}{210389}$	(9 076, 188 452 766 851)

B.10 Saturn

Verses 65-72 tabulate solutions of $KU(a_{sa}, b, -1)[x, y_{sa}]$., where a_{sa} is the number of revolutions of Saturn (*śani*) in a *yuga*. According to Ab.1.3 $a_{sa} = 146564$ revolutions.

Table 21: Tabulated values for the revolutions of Saturn (1)

Measuring Units	d	a	b in days	x	y
Revolutions	4	<i>36 641</i>	<i>394 479 375</i>	113 065 211	10 502
Signs	3	<i>146 564</i>	<i>131 493 125</i>	61 139 584	68 147
Degrees	5	<i>879 384</i>	<i>26 298 625</i>	18 956 139	633 863
Minutes	5	<i>10 552 608</i>	<i>5 259 725</i>	264 747	531 163
Seconds	5	<i>126 631 296</i>	<i>1 051 945</i>	811 021	97 629 287
Thirds	5	<i>1 519 575 552</i>	<i>210 389</i>	190 312	1 374 565 507
Fourths	5	<i>91 174 533 120</i>	<i>210 389</i>	185 509	80 392 494 211

Table 22: Tabulated values for the Revolutions of Saturn (2)

Equation	Couple solution
$y = \frac{36641x-1}{394479375}$	(113 065 211, 10 502)
$y = \frac{146564x-1}{131493125}$	(61 139 584, 68 147)
$y = \frac{879384x-1}{26\ 298\ 625}$	(18 956 139, 633 863)
$y = \frac{510552608x-1}{5259725}$	(264 747, 531 163)
$y = \frac{126631296x-1}{5259725}$	(811 021, 97 629 287)
$y = \frac{1519575552x-1}{210389}$	(190 312, 1 374 565 507)
$y = \frac{91174533120x-1}{210389}$	(185 509, 80 392 494 211)

B.11 Moon's Anomaly

Ab.3.4 defines the number of anomalistic revolutions of a planet during the Kali Yuga as the difference between its revolution number and the revolutions of its apogee⁵⁶. It thus provides a value for a_{ck} the number of anomalistic revolutions of the moon (*candrakendra*) during the Kali Yuga. $a_{ck} = 57753336 - 488219 = 57265117$.

Verses 73-80 tabulate solutions of $KU(a_{ck}, b, -1)[x, y_{ck}]$, where a_{ck} is number of anomalistic revolutions of the moon in a *yuga*.

⁵⁶The verse actually gives a more general rule, this is but one of its interpretations, see (K. S. Shukla K. V. ; Sharma, 1976, 87)

Table 23: Tabulated Values for the Moon's Anomaly

Measuring Units	d	<i>a</i>	<i>b</i>	x	y
Revolutions		<i>57 265 117</i>	<i>1 577 917 500</i>	341 213 953	12 383 193
Signs	12	<i>57 265 117</i>	<i>131 493 125</i>	78 227 703	34 068 082
Degrees	5	<i>343 590 702</i>	<i>26 298 625</i>	26 187 263	342 135 761
Minutes	5	<i>41 23 088 424</i>	<i>5 259 725</i>	4 373 824	3 428 632 315
Seconds	5	<i>49 477 061 088</i>	<i>1 051 945</i>	13 837	650 807 879
Thirds	5	<i>593 724 733 056</i>	<i>210 389</i>	123 880	349 593 467 011
Fourths	5	<i>35 623 483 983 360</i>	<i>210 389</i>	142 324	24 098 582 789 251

Table 24: Tabulated values for the Moon's Anomaly (2)

Equation	Couple solution
$y = \frac{57265117x-1}{1577917500}$	(341 213 953, 12 383 193)
$y = \frac{57265117x-1}{131\ 493\ 125}$	(78 227 703, 34 068 082)
$y = \frac{343590702x-1}{26298625}$	(26 187 263, 342 135 761)
$y = \frac{4123088424x-1}{5259725}$	(4 373 ,3 428 632 315)
$y = \frac{49477061088x-1}{1\ 051\ 945}$	(13 837, 650 807 879)
$y = \frac{593724733056x-1}{210\ 389}$	(123 880, 349 593 467 011)
$y = \frac{35623483983360x-1}{210\ 389}$	(142 324, 24 098 582 789 251)

B.12 Intercalary days

According to Ab. 3.6ab:

आधिमाषका युगे ते रविमासेभयोऽधिास्तु ये चान्द्रः *adhimāsakā yuge te ravimāseb-*
hyo'dhikās tu ye cāndrāḥ | The intercalary months in a *yuga* are the lunar
 ⟨months⟩ that are in excess of the solar months⁵⁷

A month is defined in Ab.3.1. as being of thirty days, a year of twelve months. Therefore we know from Ab.1.3 that there are $4320000 \times 12 = 51840000$ solar months in a *yuga*. We know from Ab.1.3 and AB.3.6 that there are $57753336 - 4320000 = 53433336$ lunar months in a *yuga*. And there for deduce that there are $53433336 - 51840000 = 15933336$ intercalary months in a *yuga*. Therefore there are $15933336 \times 30 = 47800080$ intercalary days in a *yuga*.

These values are also found in Mbh.VII.6-8. Adapting what we know of planetary pulverizers here, we can deduce that in verse 81 computes $KU(a_{ad}, b_{ad}, -1)[x_{ad}, y_{ad}]$, were a_{ad} is the number of intercalary days (*adhika*) in a *yuga*, that is 47 800 080, and b_{ad} is the number of solar months in a *yuga*, 51 840 000. a_{ad}, b_{ad} are reduced by 24:

$$\frac{a_{ad}}{b_{ad}} = \frac{47800080}{51840000} = \frac{66389}{2160000}$$

The assumption is that $\frac{Y_{ad}}{a_{ad}} = \frac{x_{ad}}{b_{ad}}$, where x_{ad} is the number of intercalary days elapsed since the beginning of the Kali Yuga, and Y_{ad} the number of solar months elapsed.

Only a fractional part of Y_{ad} is known: $\lambda_{ad}, Y_{ad} = y_{ad} + \lambda_{ad}$ The remaining part, y_{ad} , is unknown. In this case $\lambda_{ad} = \frac{1}{b_{ad}} = \frac{1}{2160000}$, a very small number indeed.

(672 509, 20 670) is the smallest positive solution of $y = \frac{66389x_{ad}-1}{2160000}$.

In theory then one could try to spell out an astronomical interpretation to this solution, which would be as follows: when $\frac{1}{2160000}$ solar months are known to have elapsed, then 672 509 intercalary days have elapsed, and further 20 670 solar months have passed. But written in this way, it is obviously absurd, revealing that the proportion on which the equation seems to rest is problematic.

⁵⁷(K. S. Shukla K. V. ; Sharma, 1976, p.91) translates: The lunar months (in a *yuga*) which are in excess of the solar months (in a *yuga*) are (known as) the intercalary months in a *yuga*.

B.13 Omitted Days

According to Ab.3.6.cd:

शशिवसा विज्ञेया बूदिवसोनास्तिथिप्रलयाः॥

śaśidivasā vijñeyā būdivasonāstithipralayāḥ||

lunar days decreased by solar days should be known as omitted *tithis*||

The number of civil days in a *yuga* is $1582237500 - 57753336 = 15779175000$ The number of lunar days in a *yuga* is $53433336 \times 30 = 1603000080$ Therefore the number of omitted days is $1603000080 - 15779175000 = 25082580$ Verse 82 computes a solution of $KU(a_{av}, b_{av}, -1)[x_{av}, y_{av}]$, where a_{av} is the number of omitted lunar days (*avama*) in a *yuga*, 25 082 580 and b_{av} , 1 603 000 080, is the number of lunar days in a *yuga*. y_{av} represents the number of omitted lunar days since the beginning of the Kali Yuga

a_{av} and b_v are additionally reduced by 60:

$$y = \frac{25082580x - 1}{1603000080} \frac{418043x - 1}{26716668}$$

By a reasoning analogous to the one used for Planetary Pulverizers and intercalary days, one can attempt to interpret the results found, according to the assumption that the number of omitted lunar days during the Kali Yuga is proportional to the number of lunar days. But, as for intercalary days, the result voiced in this way provides an absurd result.

(6 247 031, 97 749) is the smallest positive solution of $y = \frac{418043x_{av}-1}{26716668}$.

With a reasoning attempting to make “sense” of the result, one would interpret the solution as follows, when $\frac{1}{b_{av}} = \frac{1}{1603000080}$ lunar days are known to have been omitted, then 6 247 031 lunar days have elapsed, and 97 749 have additionally been omitted since the beginning of the Kali Yuga. An obviously absurd statement.

B.14 Sun’s declination

Verse 83 provides a solution of $KU(a_{apa}, b_{apa}, -1)[x, y]$, where a_{apa} is the Rsine of the maximum value of the sun’s declination in a *yuga*, 1397, and b_{apa} corresponds to the Radius

of the celestial sphere in minutes, 3438. According to Ab.4.24⁵⁸, if λ is the sun's tropical longitude, and δ , the sun's declination then

$$\frac{R\sin\delta}{1397} = \frac{R\sin\lambda}{3438}$$

Verse 83 provides (347, 141) as the least positive solution of $y = \frac{1397x-1}{3438}$. Using the proportion provided by Ab.2.4, an astral interpretation of this result might be that if $\frac{1}{3438}$ of $R\sin\delta$ is known, one can derive its total value, and also the corresponding value of $R\sin\lambda$. My knowledge of Hindu astronomy does not allow me to realize if such a conclusion makes any sense.

B.15 Intercalary months

Verses 84-91 tabulate solutions of $KU(a_{am}, b, -1)[x, y]$, where a_{am} is the number of intercalary month (*adhimāsa*) in a *yuga*, that is 1 593 336 (according to Ab.1.3 and AB.3.4) and b the number of civil days in the Kali Yuga.

A possible astral interpretation of this rule would thus be, knowing that $\frac{1}{b}$ part of an intercalary month is known to have elapsed, the total number of intercalary months from the beginning of the Kali Yuga and the number of days elapsed can be produced.

Verses 84-90 evoke the measuring units for the movements of the planets to evoke what theoretically is here the subdivisions of months. Ab.3.2 however specifies the fact that there is an equivalence: having spelled out the subdivisions of time, as shown in Table 25, it notes⁵⁹:

क्षेत्रविभागस्तथा भगनात्

kṣetra-vibhāgas tathā bhaganāt

The division of a circle (*lit. the ecliptic*) proceeds in a similar manner from the revolution.

Note however that the names given time do not go as far as the fourths considered here.

⁵⁸According to (K. S. Shukla K. V. ; Sharma, 1976, 132-133), it derives from this rule.

⁵⁹(K. S. Shukla K. V. ; Sharma, 1976, 85).

Table 25: Subdivisions of Time

Sanskrit	English	Amounts in a Year	Powers of 60
<i>varṣa</i>	Years	1	$\frac{1}{60} \times 60$
<i>māsa</i>	Months	12	$\frac{1}{5} \times 60$
<i>divasa</i> <i>nāḍī</i>	Days 21 600	360 6×60^2	6×60
<i>vināḍī</i>		1 296 000	6×60^3
<i>gurvākṣara</i>	(time taken to pronounce) a long syllable	77 760 000	6×60^4

Table 26: Tabulated Values for Intercalary Months (1)

Measuring Units	d	<i>a</i>	<i>b</i> in days	x	y
Months	12	132 778	131 493 125	18 900 317	19 085
Days	12	1 593 336	131 493 125	122 110 391	1 479 643
	5	9 560 016	26 298 625	24 734 836	8 991 551
<i>nāḍī</i>	5	114 720 192	5 259 725	3 814 478	83 197 819
<i>vināḍī</i>	5	1 376 642 304	1 051 945	142 549	186 548 711
<i>gurvākṣara</i>	5	16 519 707 648	210 389	134 606	10 569 239 683
Fourths	5	991182458880	210 389	93 412	24 098 582 789 251

Table 27: Tabulated values for Intercalary months (2)

Equation	Couple solution
$y = \frac{132\ 778x-1}{131\ 493\ 125}$	(18 900 317, 19 085)
$y = \frac{1593336x-1}{131\ 493\ 125}$	(122 110 391, 1 479 643)
$y = \frac{9560016x-1}{26298625}$	(24 734 836, 8 991 551)
$y = \frac{1147201922x-1}{5259725}$	(3 814 478, 83 197 819)
$y = \frac{1376642304x-1}{1\ 051\ 945}$	(142 549, 186 548 711)
$y = \frac{16519707648x-1}{210\ 389}$	(134 606, 10 569 239 683)
$y = \frac{991182458880x-1}{210\ 389}$	(93 412, 440 081 638 531)

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The reference in parentheses indicates the first occurrence of the expression in the verse.

“And also” indicates other synonyms used.

Zero ⁶⁰ *nabhas* void, sky (verse 3) and also *kha* (verse 3) *viyad*, *viyat* (verse 4), *vyoma*, *gagana* (verse 9), *abhra* (verse 57).

One *śasin*, “the one who has a rabbit” the moon (verse 3) and also *śītāṃśu* “cold-rayed” (verse 5) *prāleyaraśmi* “frosty rayed” (verse 17), *himāṃśu* (verse 27), *sudhāmayūkha* (verse 49) *niśānātha* “lord of the night/dreams” (verse 58), *rātrināthā* (verse 65), *mṛ-tasanmayūkha?* (verse 70).

Two *dasra*, a name of the *aśvins* (verse 3), and also *yama*, *nayana* (according to a Shukla inference in verse 3) *bhuja*, the arms (verse 4), and also *bahu* (verse 9).

Three *Rāma*, there are three famous Rāmas: the hero of the *Rāmayaṇa*, Balarāma (Kṛṣṇa’s brother) and Parasurāma (verse 3). *Guṇa* as the three qualities of all created things (truth/goodness for gods (*sattva*), matter/passions for men (*rajas*), darkness/ignorance for demons (*tamas*) (verse 3); *vahni*, or *agni* as the three sacrificial fires (verse 9), and *kṛśānu* (verse 21), *dahana* (verse 62) and *vikrama* as the third astrological mansion. And also *puṣkara* (verse 9, 83)- I do not know why.

Four *abdhi*, ocean, there are four oceans in puranic cosmology (verse 3), and also *sāgara*, *ambu* (verse 11), *samudra* (verse 33) *apagānātha* (?) (verse 59), *udadhi* (verse 60); *veda*, like the four *ṛg*, (verse 4); *kṛta* as the lucky age in the *vedas*, or the lucky throw in a dice game showing the number four (verse 30).

Five *iṣu*, arrow, as the five arrows of Kāma, the god of love, (verse 3) and also *śara* (verse 3) *bāna* (verse 11) *bhūta*, the five elements (earth, air, fire, water and stone) (verse 4), but also (*tanmātra*) (verse 22), *artha* (verse 37).

Six *rasa*, perfume, taste. There are six tastes: *kaṭu* (acrid), *amla* (sour, acid), *madhura* (sweet), *lavaṇa* (saline), *tikta* (bitter) and *kaṣāya* (astringent, fragrant) (verse 3); *aṅga*, as the six *Vedāṅgas*.(verse 3); *ṛtu* as six seasons (verse 66).

⁶⁰I’m not too sure if some words meaning “void”, should not be understood as other names for zero, such as *kha*.

Seven *naga*, “that which does not move”, a mountain, there are seven chains of mountains according to puranic mythology (verse 3) and also *adri* (verse 3), *śaila* (verse 16), *aga* (verse 17), *paravata* (verse 20), *kṣoṇādhārā* (verse 21), *kṣitibhṛt* (verse 41- partly a Shukla addendum) *bhūbhṛt* (verse 62), *śiloccaya*; *tāpasa* “one who practices austerities”, as a synonym of *muni*, designating the seven stars of Ursa Major (verse 9) , *ṛṣi* (verse 17), *aśva* “horse” as the number of horses of the sun (verse 25); *svara* “notes of the musical scale” (verse 77).

Eight *vasu*, a class of eight deities (verse 3); *nāga* elephant; there are eight elephants symbolizing the eight cardinal directions (East, West, South, North, South-east, South-west, North-east, North-west) (verse 3) and also *gaja*, *kuñjara*; *bhujaga*; *bhujāṅga* for snakes; serpent demons, would there be 8 serpent demons? and also *pannaga* (verse 10), *nāga* (vers 17).

Nine *nanda* either the nine treasures of Kubera or the nine brother-kings called “Nanda” (verse 3), *anka*, the nine units (verse 4) *randhra*, orifice; the nine orifices of the human body are: the two eyes, the two nostrils, the mouth , the two ears, the sex, the anus. And also *chidra* (verse 26).

Ten *diś* as the ten directions: the four cardinal directions, North, South, East, West, and North west, North-east, South-west, South-east, below, above.

Eleven *śiva*, as the head of a group of eleven gods called collectively *rudra* (verse 3).

Twelve The sun, as the twelve signs of the zodiac, or the twelve months of the solar year?, *Sūrya* (verse 11) *vivasvat* (verse 61).

Thirteen *Viśva* as a class of gods containing 1. Vasu , 2. Satya , 3. Kratu , 4. Daksha , 5. Kāla , 6. Kāma , 7. Dhṛiti , 8. Kuru , 9. Purūravas , 10. Mādravas, 11. Rocaka 12. Dhvani 13. Dhūri (according MW) (verse 28).

Fourteen *Indra* (verse 85), *śakra* (as a class of gods related to indra) (vers 60), *manu* (verse 7).

Fifteen *Tithi* as the fifteen tithis of the moons increase (and decrease) (verse 45).

Sixteen *bhūpa* expression introduced by K. S. Shukla.

Eighteen *Dhṛtia* class of metres consisting of 4 X 18 syllables (according to Colebrooke) (verse 38) and also *saṃskāradhṛti* (verse 46).

Twenty-four *jina* 24 Jinas are supposed to flourish in each of the 3 Avasarpinīs?? (hence) the number “” 24 ”” (verse 79).

Twenty-Five *Tattva* (verse 50).

Twenty-seven *ṛkṣa* as a synonym of *nakṣatra* (verse 14).

Thirty-two *danta* (verse 20).

Forty-Eight *saṃskāra* (verse 87)- should this account for the number of ceremonies, or the different qualities?

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