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[“Questions to Danielle Macbeth on Frege’s logical notation and related topics” is a slightly revised version of the questions that were asked at the 2006 Logic Colloquium. Modifications of a stylistic nature have been introduced in november 2020.]

## **LOGIC COLLOQUIUM 2006**

Raboud University, The Netherlands  
July 29, 2006

**Special Symposium with  
Norma Goethe, Fabrice Pataut and Sanford Shieh  
On Danielle Macbeth’s *Frege’s Logic***

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## **QUESTIONS TO DANIELLE MACBETH ON FREGE’S LOGICAL NOTATION AND RELATED TOPICS**

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Danielle Macbeth’s purpose in *Macbeth 2005* is threefold. Her monograph proposes “to provide a logical justification for all aspects of Frege’s peculiar notation, to motivate and explain the developments in Frege’s views over the course of his intellectual life, and to explicate his most developed, critically reflective conception of his *Begriffsschrift*, his formula language of pure thought” (p. vii). I shall focus here on a few selected aspects of the first and third points and leave on the side the discussion of the historical development of Frege’s views on logic, semantics and mathematics.

The book's underlying claim is that although Frege's logical language, "can of course be read as a language of quantificational logic, [it] can also be read very differently" (p. vii), so much so, as a matter of fact, that although Frege's logic is usually thought of as a "notational variant of standard quantificational logic," it is nevertheless "unknown to us" (p. 1).

This naturally prompts three questions: "What is Frege's logic?", "What is quantificational logic?", and given the way *we* understand formal languages for first and higher order predicate logic, most notably the notions of argument place (for a predicate symbol), of scope and of binding (of variables – either individual or predicate of first or higher order): "How come Frege's logic says nothing about such notions, or something so different from what we say?"

Obviously, the claim that Frege's formula language of pure thought can and indeed *should* be read otherwise than as a notational variant of the logic of quantification runs contrary to the understanding of Frege's contribution to logic which has been advocated by most historians and philosophers of the discipline.

As far as the propositional calculus is concerned, there is no genuine disagreement regarding what Frege has achieved, and what his contribution to the development of logic consists in. As is well known, Frege presents a system of 9 axioms in the second part of the *Begriffsschrift* (Frege [1874] 1972), corresponding to his formulas (1), (2), (8), (28), (31), (41), (52), (54) and (58), and claims completeness for this set of axioms. One may argue that Frege's notion of completeness [*Vollständigkeit*] is insufficiently specified when it is claimed that we can only attain completeness by looking for a kernel [*Kern*] of judgements of pure thought which, together with the rules for the application of the symbols, must *potentially* (Frege's emphasis) imply all the others. It remains, as Łukasiewicz subsequently showed, that the first six axioms do indeed form a complete set for the propositional calculus in the

following sense: they imply, using only *modus ponens* and substitution, all the tautologies of the classical propositional calculus (Łukasiewicz [1934] 1967).

It is all very different with the claim that Frege's formula language isn't quantificational, or even with the somewhat weaker albeit related claim that its non linear two-dimensional notation is unable to "trace" the truth-conditions of its formulas, or to trace them uniquely (p. 2). I shall restrict myself to a discussion of Frege's means of expressing generality, and to the so-called truth-conditional account of meaning, because both points are crucial to Macbeth's project of offering an alternative to the received view of Frege's achievements.

With respect to the first point, it seems everyone will disagree. Quine (see the historical notes of chapters 20, 25 and 36 in *Quine 1950*, and *Quine 1952*), Dummett (in *Dummett (1959) 1978*: 66, and *Dummett 1981*: ch. 2, 15), Bocheński (see *Bocheński 1962*: 268), Kneale and Kneale (*Kneale and Kneale 1962*: 478-493), Hunter (*Hunter 1971*: 259) *et al*, all point to the fact that what separates Frege from his predecessors, most notably Boole, is indeed: (i) the use of operators which bind variables and may be nested (and those are quantifiers or, at any rate, do just what quantifiers are supposed to do), and (ii) the use of a formal system in which generality may be expressed non ambiguously with the help of the notation of quantifiers and variables.

On the standard reading of the *Begriffsschrift*, Frege's concavity symbol introduced in the horizontal content stroke, and containing a German (i.e. Gothic) letter, is a notational variant for, respectively, the universal quantifier and the variables bound by that quantifier.

Macbeth has two arguments against this view. The first is that Frege *also* uses Latin italic letters in the expression of generality, whereas in quantificational logic, we wouldn't use two different sorts of letters because it would obscure the fact that it is free variables which end up being bound when

a quantifier is attached to them (p. 8). But this fact is not significant. The means of distinguishing between the two is nevertheless available in Frege: we can always tell whether a given variable is free or bound.

Macbeth then claims in chapter 3 (especially in sect. 3.3, at pp. 90-93) that the use of Latin *and* German letters is as a matter of fact justified from Frege's point of view, because these symbols play distinct logical roles. A *Begriffsschrift* hypothetical formula expressed with the use of Latin letters takes on the form of a subordination of concepts. When the same formula is expressed using the concavity notation with the German letters, it takes on the form of a subsumption of first level concepts under a second level concept. In this second case, what is asserted is that a concept stands in a *second level relation* to another concept.

It remains, as Shieh points out (*Shieh 2005: 7*), that in the *Grundgesetze* (*Frege [1893] 1964: §8, p. 42*), the notion of generality: (i) is introduced by using Latin letters and without involving the subordination of concepts (or relations of a given level) to concepts of a higher order, only (ii) to be rejected for the reason that "by this stipulation, the scope of the generality would not be well enough demarcated." If we were to express generality in this way, we wouldn't be able to distinguish between the negation of a generality and the generality of a negation. In order to express this difference, we must introduce the concavity sign with German letters. The definition of generality which is then obtained is such that we get an expression which denotes the True if, for every argument, the value of the relevant function is the True, and which denotes the False otherwise.

Now, this definition or explanation of generality by way of stipulating a scope is, on the face of it, quantificational.

Macbeth's second argument (p. 4) is that since Frege never indicates that the concavity with German letters is interdefinable with the concavity flanked

by negation strokes, the first one isn't related to the second in the way the universal quantifier is related to the existential quantifier in (genuine) quantificational logic. So they can't be identified in the way proposed by the standard account. But the fact that Frege's logical symbolism contains only one quantifier might be accounted for by the fact that his logic is classical, so that the existential quantifier is expressible in terms of the universal quantifier with negation, i.e., in terms of the concavity symbol in the horizontal stroke containing a German letter, with negation (see *Dummett 1981* : 513).

With respect to the second point: why should the distinction between *Sinn* and *Bedeutung* be “deeply problematic on the assumption that Frege's logic is a quantificational logic” (p. 4)? One of the reasons given by Macbeth is that if meaning is to be understood directly in terms of truth, i.e., if the meaning or content of a formula is to be given by what is the case for it to be true, then a sentence might express a thought, yet fail to have either the truth-value True, or the truth-value False.

Frege explicitly advocates only once the idea that to understand the meaning of a sentence (or of a formula), i.e., the thought expressed by the sentence (or the formula) is to know its truth-conditions, or whether or not these conditions are fulfilled, namely in *Grundgesetze*, at § 32.

Frege says :

“Every [...] name of a truth-value *expresses* a sense, a thought [*drückt einen Sinn, einen Gedanken aus*]. Namely, by our stipulations, it is determined under what conditions [*Bedingungen*] the name denotes [*bedeutet*] the True. The sense of this name – *the thought* – is the *thought* that these conditions are fulfilled [*dass diese Bedingungen erfüllt sind*].

*Frege [1893] 1964: § 32*

In this context, sentences are “names of truth-values.” Frege applies the word “sentence” (*Satz*) only to formulas prefixed with the assertion sign.

Unprefixed formulas may not properly be ascribed either a *Sinn* or a *Bedeutung*. Either may be ascribed only to the formulas to which the assertion sign is prefixed, indicating the attachment of assertoric force. Such an expression *names* a truth-value, but doesn't say or expresses that it is the truth-value true, or that it has that truth-value as a denotation. In this sense, it doesn't say or express anything at all.

This remark of Frege is closely connected to the criticism of psychologistic definitions put forward in the *Grundlagen* (see *Frege [1884] 1950*: Introduction, pp. iii, vi, viii-ix, and Part II, §§ 26, 27). If you *define* a mathematical concept in terms of the mental operations needed (by us, or by any other rational being) to grasp the concept, you cannot use the definition to prove anything, i.e., to acknowledge the truth of any proposition of arithmetic. So when can you use a definition in a proof? When it tells you under what conditions a sentence (or formula) involving the defined term (for the concept) would be true.

This suggests that the notion of *Sinn* is indeed cognitive. The fact that the *Sinn/Bedeutung* distinction in *Frege [1892] 1984* is an "extralogical theory of the cognitive aspect of language use" (p. 110), as Dummett and Evans have insisted it is, hardly conflicts with Frege's own assessment of his discovery as a logical advance, as a "thoroughgoing development of [his] logical views" (*Frege [1893] 1964*: 6-7). "Logical," as Macbeth uses the expression, should mean something which would exclude considerations pertaining to language use. But this doesn't seem to fit in Frege's conception. Even if it did, it wouldn't thereby follow that meaning cannot be understood in terms of truth, or that the Fregean notation has particular problems with what, after all, is one of Frege's most innovative insights.

To conclude, I shall very briefly comment on another puzzling aspect of Macbeth's reading of Frege's notation. Macbeth believes that Frege's use of

the judgment stroke, introduced in §2 of the *Begriffsschrift*, raises a special difficulty. The difficulty is that “[i]n our logics the truth of the premisses is irrelevant to the correctness of an inference ; what matters is only whether the conclusion is true on the assumption that the premisses are true” (p. 3). Frege’s well known argument in §2 is that with the content stroke alone, the judgement is a mere combination of ideas [*blasse Vorstellungsverbindung*]. It evokes an idea, or triggers a mental representation and ties the symbols which follow into a whole. Two very different claims are at stake here. One is that the premisses of an inference expressed in the conceptual notation must be true ; another is that the premisses of an inference expressed in the conceptual notation must be *acknowledged* to be true. It is clear that the role of the judgement stroke is to have not just ideas or representation but thoughts, *Gedanken*, or propositions and this also why, in Frege’s view, the truth of the premisses matter greatly.



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