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## On the Role of External Financing Costs in Optimal Investment Decisions

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# On the role of external financing costs in optimal investment decisions

Mohamed Belhaj \*      Nataliya Klimenko †

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## Abstract

This paper brings into focus a link between the investment and financing decisions of a firm which has an access to costly debt financing. Our analysis shows that *lump-sum debt issuance costs* play a prominent role in a determination of the optimal investment strategy. Faced with larger lump-sum debt issuance costs, a firm will optimally set up a higher-scale investment project in order to "compensate" dead-weight financing costs by higher return. Moreover, in the presence of lump-sum debt issuance costs, the optimal investment scale of financially constrained firms exhibits an inverted U-shaped relationship with the firm's borrowing capacity, so that relatively more/less constrained firms will realize smaller investment projects, whereas firms with an intermediate borrowing capacity will undertake larger investment.

**Keywords:** real options, investment intensity, debt issuance costs, credit constraints

**JEL classification:** G31, G32, G33

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# 1 Introduction

Recognizing the fact that the Modigliani and Miller irrelevance theorem (1958) is not robust in the context of any frictions, a corporate finance literature doesn't cease looking for a consistent explanation of the connection between financing and investment decisions. Given a substantial difference between the costs of external and internal funding caused by information asymmetry problems and eventual transaction costs, a question of particular interest is how the optimal choices of investment time and the scale of investment are affected by the tightness of financial constraints?

While many studies unanimously advocate for the U-shaped relationship between the investment time and the tightness of financial constraints (see, for instance, *Belhaj and Djembissi (2009)*, *Hirth and Uhrig-Homburg (2010)*, *Wong(2010)*, *Shibata and Nishihara (2012)*), there is a lack of consensus concerning the impact of financial constraints on the optimal choice of investment scale. Earlier theoretical works (*Fazzari et al. (1988)*, *Hoshi et al. (1988)*, *Whited (1992)*, *Gilchrist and Himmelberg (1995)*, *Hubbard (1998)*) argue that a sensitivity of investment expenditures to cash flows are much stronger for the firms which are likely to be financially constrained. This fact suggest about a negative monotonic relationship between the tightness of financial constraints and the scale of investments chosen by firms. *Kaplan and Zingales (1997)* provide a conflicting evidence, showing that less financially constrained firms exhibit higher investment cash-flow sensitivities. Recent studies of *Cleary, Povel and Raith (2007)* and *Guariglia (2008)* report a U-shaped relation between investment size and internal funds, explaining this form by a trade-off between a higher risk of default and a higher expected investment return of the levered firm. A theoretical model of *Wong (2010)* contributes to this puzzle, showing that a negative effect of higher default risk and a positive effect of higher investment return resulted from larger investment will offset each other. As a result, the optimal investment scale will be independent on the firm's credit constraints, remaining equal to that of the all-equity financed firm.

However, it seems to be misleading to discuss the optimal choice of in-

vestment scale without taking into account external financing costs. Allowing for external financing costs and the dynamic aspect of investment, our model uncovers *an inverted U-shaped relationship between the scale of investment and the firm's borrowing capacity*. A striking evidence we obtain is that this relationship emerges only in the presence of *lump-sum* debt issuance costs. Without *lump-sum* debt issuance costs, a scale of investment would be completely independent on the firm's borrowing capacity, so that a neutrality of debt in investment intensity established by *Wong (2010)* would hold even under significant variable debt issuance costs.

It is worth noting that several recent studies have attempted to integrate external financing costs into the analysis of optimal investment decisions. *Lyandres (2007)* documents a hump-shaped relationship between the magnitude of external financing costs and investment, explaining this form by the interplay between two opposite effects of external financing costs on investment timing. Indeed, external financing costs reduce the attractiveness of current investment, but at the same time also reduce the value of the option to wait, whereas a scale of investment is found to be monotonically increasing on the investment time. The study of *Hirth and Uhrig-Homburg (2010)* examines the impact of variable external financing costs on the optimal investment time in the context of a fixed-scale investment model. They find that low-liquidity firms will delay investment to avoid present financing costs, whereas high-liquidity firms will speed up investment to avoid even higher financing costs in future. They associate earlier investment with a higher investment volume, arguing that the optimal investment scale would exhibit a reversed U-shaped pattern when plotted against liquid funds. *Nishihara and Shibata (2012)* allow for two levels of investment scale and show that relatively low and relatively high levels of cash reserves imply larger investment scale, whereas the intermediate levels of cash correspond to the lower investment scale.

While the above mentioned studies use the firm's current level of cash holdings as a proxy for financial constraints and consider a discrete choice of investment scale, we conduct our analysis in the context of explicit credit constraints, allowing for a continuous choice of investment scale. Indeed,

in practice firms often face a quantity credit rationing, so that borrowed funds represent only a fraction of a required investment amount, whereas a remaining part has to be financed by internal funds. Such a representation of credit constraints makes it possible to capture endogenous changes in credit capacity following the changes of investment scale.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the optimal investment decisions of an all-equity financed firm. In Section 4 we examine the optimal financing and investment decisions of a levered firm when it faces debt issuance costs. Section 5 concludes.

## 2 The model

We consider an owner-managed firm protected by limited liability and endowed with a perpetual investment option. An investment project can be undertaken at the irreversible cost  $I(q)$  which is positively related with investment intensity,  $q > 0$ . Moreover, we assume that  $I''(q) > 0$ ,  $I(0) \geq 0$  and  $I'(0) = 0$ . Investment intensity affects the scale of stochastic earnings before interest and taxes (EBIT),  $qX_t$ , where  $X_t$  denotes EBIT per unit of investment intensity and follows a stochastic process:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad (1)$$

where  $W = \{W_t, \mathcal{F}_t, 0 \leq t < \infty\}$  is a standard Brownian motion on a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . Investors are risk neutral and discount the future at a constant rate  $r > \mu$ .

The firm operating profits are taxed at the rate  $\theta$ , so that equity holders collect a residual operating cash-flow  $(1 - \theta)qX_t$ .

A firm has an access to the external financing. In order to finance a part of investment costs and to benefit from tax shields, at the investment date  $\tau$  the firm's equity holders can raise an amount  $b \leq \psi I(q)$  of perpetual debt, where  $\psi$  reflects an exogenously given borrowing capacity of the firm. The remaining fraction of investment costs,  $I(q) - b > 0$ , is financed by equity

capital.<sup>1</sup> However, issuing debt is costly because of information asymmetries inherent to external financing. As *Belhaj and Djembissi (2007)*, we consider a linear form of debt issuance costs,  $K(b) = kb + K_0$ , with both variable and fixed components.<sup>2</sup>

A debt contract implies that a constant coupon  $c$  will be continuously paid to debt holders until the firm's default, which occurs when shareholders cease injecting cash in order to serve the debt. The optimal default trigger maximizing the firm's equity value is given by the standard formula:

$$x_L(q) = \frac{\beta_1}{\beta_1 - 1} \frac{r - \mu}{r} \frac{c}{q} \equiv \delta \frac{c}{q} \quad (2)$$

where  $\beta_1 < 0$  is a root of  $\sigma^2/2\beta(\beta - 1) + \mu\beta = r$ .

Given that after the default the firm will be run by new owners as an unlevered concern, we model bankruptcy costs as a fraction  $\gamma$  of the all-equity financed firm's value.

Equityholders optimally decide about the timing, the scale and the financial structure of investment, maximizing ex-ante equity value under the exogenously given credit constraint. For any coupon  $c$  and investment intensity  $q$ , let  $D(X_\tau, q, c)$  denote debt value at the investment time  $\tau$ , given that the firm will be liquidated at the time  $\tau_L = \inf\{t \geq 0 \text{ s.t. } X_t = x_L(q)\}$ . Since, by the absence of arbitrage, we have  $b = D(X_\tau, c, q)$ , the equity holders' maximization program can be formalized as follows:

$$\begin{aligned} \text{Sup}_{\tau, q, c < \infty} \mathbb{E} [e^{-r\tau} (V(X_\tau, q, c) - I(q) - kD(X_\tau, q, c) - K_0)] \\ \text{s.t. } D(X_\tau, q, c) \leq \psi I(q) \end{aligned} \quad (3)$$

where  $V(X_\tau, q, c)$  denotes a value of the firm at the investment time.<sup>3</sup>

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<sup>1</sup>In the case when  $I(q) - b < 0$  (which is possible only if  $\psi > 1$ ), equity value at the investment time increases by  $b - I(q)$ .

<sup>2</sup>Issuing equity might be costly as well. However, it is commonly recognized that the cost of external financing exceed the cost of internal financing, so we assume the latter to be zero in order to focus on the impact of external financing costs.

<sup>3</sup>For the expressions of contingent claims used in the model see Appendix A.

### 3 Investment decisions of an all-equity financed firm: a benchmark

Consider first a benchmark case where  $\psi = 0$ , so that the investment project is fully financed by equity and the firm never goes bankrupt. In this case the equity holders' problem takes the following form:

$$Sup_{\tau, q < \infty} \mathbb{E} \left[ e^{-r\tau} \left( \frac{(1-\theta)qX_\tau}{r-\mu} - I(q) \right) \right] = Sup_{x_I^e, q} \left[ ((1-\theta)\nu qx_I^e - I(q)) \left( \frac{X_t}{x_I^e} \right)^{\beta_2} \right] \quad (4)$$

where  $x_I^e$  is the investment trigger such that  $\tau = \inf\{t \geq 0 \text{ s.t. } X_t = x_I^e\}$ ,  $\beta_2 > 0$  is a root of  $\sigma^2/2\beta(\beta-1) + \mu\beta = r$  and  $\nu = (r-\mu)^{-1}$ .

The corresponding optimal investment intensity satisfies:

$$I'(q^e) = \frac{\beta_2}{\beta_2 - 1} \frac{I(q^e)}{q^e} \quad (5)$$

whereas the optimal investment trigger is given by:

$$x_I^e = \frac{\beta_2}{\beta_2 - 1} \frac{I(q^e)}{q^e} \frac{1}{(1-\theta)\nu} \quad (6)$$

The optimal investment intensity in (5) is chosen in a way to balance the marginal investment cost and the average investment cost adjusted by the option value multiple  $\beta_2/(\beta_2 - 1)$ . Thus, the optimal investment scale of an all-equity financed firm is constant and completely independent on the optimal choice of investment time.

### 4 Investment and financing decisions of a levered firm

#### 4.1 The case of non-binding credit constraints

First, we are going to consider a solution of the maximization problem in the case when  $\psi$  is large enough, so that the firm can raise a required



amount of debt without binding credit constraints. In this case, the credit constraint in (3) can be omitted and the program can be resolved in two steps: (i) first, for any fixed investment parameters  $q$  and  $x_I$  such that  $\tau = \inf\{t \geq 0 \text{ s.t. } X_t = x_I\}$ , we define the optimal coupon  $c^*(q, x_I)$ ; (ii) then, the problem is solved for the remaining parameters of investment policy and the optimal coupon is recovered.

For any given  $q$  and  $x_I$ , the optimal  $c^*(q, x_I)$  maximizing the firm's value net of the variable debt issuance costs is given by:

$$c^*(q, x_I) = \frac{h}{\delta} q x_I, \quad (7)$$

where

$$h = \left[ \frac{\theta - k}{(1 - \beta_1)(\theta - k) - \beta_1(1 - \theta)(\gamma + k(1 - \gamma))} \right]^{\frac{1}{-\beta_1}} \quad (8)$$

Given the optimal coupon  $c^*(q, x_I)$ , the maximization problem of equity holders can be rewritten as follows:

$$\text{Sup}_{x_I, q < \infty} ((1 - \theta)\nu q x_I + (\theta - k)h\nu q x_I - I(q) - K_0) \left( \frac{X_t}{x_I} \right)^{\beta_2} \quad (9)$$

The solution of the problem is given by the optimal investment trigger

$$x_I^f = \frac{\beta_2}{\beta_2 - 1} \frac{I(q) + K_0}{q} \frac{1}{(1 - \theta)\nu + (\theta - k)h\nu} \quad (10)$$

and the optimal investment intensity  $q^f$  which satisfies the following equation:

$$q^f I'(q^f) = \frac{\beta_2}{\beta_2 - 1} (I(q^f) + K_0) \quad (11)$$

The optimal coupon will be given by  $c^f = c^*(q^f, x_I^f)$ .

**Lemma 1**  $q^f$  is increasing on  $K_0$ .

□ Let  $f_1(q)$  and  $f_2(q, K_0)$  denote respectively the left and the right side of the equation (11). Both  $f_1(q)$  and  $f_2(q, K_0)$  monotonically increase on  $q$ . Given that  $f_2(0, K_0) > f_1(0)$ , the equation (11) has a unique solution  $q^*(K_0)$ . Since  $\frac{\partial f_2(q, K_0)}{\partial K_0}$  is increasing on  $K_0$ , we conclude that  $q^*(K_0)$  is increasing on  $K_0$ . ■

Thus, similar to the benchmark case with all-equity financing, the optimal investment intensity is set independently on both the optimal investment time and the firm's borrowing capacity, remaining constant.<sup>4</sup> However, under non-zero fixed debt issuance costs, the levered firm will realize higher investment as compared to the all-equity financed firm. At the same time, given higher investment intensity, the investment trigger of the levered firm would be lower than the investment trigger of the equity-financed firm when fixed debt issuance costs are quite small.

To illustrate the impact of debt issuance costs on the firm's investment and financing decisions in the context of non-binding credit constraints, we resort to numerical simulations (see Table 1.a-1.b in Appendix B) and obtain the following evidence:

- The optimal investment intensity,  $q^f$ , and thus the optimal scale of investment,  $I(q)^f$ , are independent on  $k$  but increase on  $K_0$ , since the firm will tend to "compensate" higher lump-sum debt issuance costs by a higher asset return.
- The optimal investment trigger,  $x_I^f$ , is increasing on both  $k$  and  $K_0$ . Indeed, a higher investment trigger reduces a current discounted value of fixed debt issuance costs, increasing a value of the firm's option.
- The optimal coupon value,  $c^f$ , and thus the firm's leverage is increasing with  $K_0$ , since the firm will tend to reduce the average cost of debt issuance by raising more debt. However, the amount of debt decreases on  $k$ , since the variable debt issuance cost reduces the value of tax shields. As a result, the firm's leverage ratio  $D(x_I^f, q^f, c^f)/I(q)^f$  tends to zero when  $k \rightarrow \theta$ .
- The value of investment option calculated at the current moment of time is decreasing on both  $k$  and  $K_0$ .

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<sup>4</sup>This is in line with the *Almeida and Campello (2002)* who suggest that the investment of unconstrained firms is insensitive to cash flow, taken the latter as a proxy for credit constraints.

## 4.2 The case of binding credit constraints

Consider now the optimal investment and financing decisions of the firm in the context of binding credit constraints. For any given investment intensity  $q$  and coupon rate  $c$ , the binding credit constraint provides the following investment trigger:

$$x_I(q, c) = \left[ \frac{1 - \psi r \frac{I(q)}{c}}{1 - (1 - \gamma)(1 - \theta)\nu\delta r} \right]^{\frac{1}{\beta_1}} x_L(q), \quad (12)$$

where  $x_L(q)$  is given by (2).

Then, the equity holders' problem (3) can be rewritten as follows:

$$Sup_{q,c} (V(q, c) - I(q)(1 + k\psi) - K_0) \left( \frac{X_t}{x_I(q, c)} \right)^{\beta_2}, \quad (13)$$

where  $V(q, c)$  is a firm value under the investment threshold  $x_I(q, c)$ .

The optimal investment and financial strategies are determined by the optimal values of  $q$  and  $c$  which solves the above problem.

In order to track the impact of debt issuance costs on the investment intensity, we use the Lagrangian method to obtain the following optimality condition:

$$qI'(q) = \frac{\beta_2}{\beta_2 - 1} \left[ I(q) + K_0 \left( 1 - \lambda\psi \left( \frac{x_I}{X_0} \right)^{\beta_2} \right)^{-1} \right], \quad (14)$$

where  $X_0$  is a current value of the state variable and  $\lambda > 0$  is the optimal Lagrange multiplier, independent on  $X_0$ .

**Proposition 1** *Given non-zero fixed debt issuance costs, the optimal investment intensity of a financially constrained firm will be affected by its borrowing capacity.*

Numerical simulations suggest the existence of an inverted U-shaped relationship between the optimal investment intensity  $q$  and the firm's borrowing capacity  $\psi$  that also reflects the firm's leverage (see Table 2.c in Appendix

B). In other words, a firm with relatively low or relatively high borrowing capacity will set up smaller investment project, whereas a firm with intermediate borrowing capacity will undertake larger investment project. In fact, larger investment scale implies two opposite effects on the firm's perspectives. On the one hand, larger investment implies a higher return, which would increase the equity holders' ability to serve the debt and thus would reduce default risk. On the other hand, under the binding credit constraint, larger investment is associated with a larger volume of debt, thereby, increasing default risk. Thus, when a firm's borrowing capacity is relatively low, the first effect dominates, so that the optimal investment scale is increasing with  $\psi$ . However, as  $\psi$  reaches some critical level, the second effect becomes stronger, so that the optimal scale of investment goes down. It is important to note that *fixed debt issuance costs play a crucial role in the trade-off between two described effects*. In fact, in the presence of fixed debt issuance costs, larger investment will reduce a marginal cost of debt issuance, which makes the first effect more pronounced. Without fixed debt issuance costs, two effects would offset each other and the optimal investment intensity would be constant, as was shown by *Wong (2010)*.<sup>5</sup>

Expression (14) also shows that, in contrast to the case of the non-binding credit constraint, the optimal choice of investment intensity will be affected not only by fixed debt issuance costs, but by variable costs as well. This is due to the fact that a choice of investment intensity under the binding credit constraint becomes inseparable from the choice of investment trigger, whereas the latter is affected by variable debt issuance costs. Numerical simulations show that the optimal investment intensity is decreasing with  $k$ .

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<sup>5</sup>Examining a simultaneous choice of investment timing, investment scale and leverage in the presence of variable investment costs, *Sarkar (2011)* finds that, generally, investment scale is first decreasing and then increasing on the amount of debt raised, but for high levels of a tax rate and bankruptcy costs this pattern is reversed. However, his model doesn't allow for credit rationing and lump-sum financing costs.

## 5 Conclusion

In this paper we contribute to the ongoing investigations of the link between a firm's investment and financing decisions in the context of external financing costs. We show that, for a firm which is not financially constrained, a choice of investment intensity will be unrelated to the firm's debt capacity and will remain constant. However, in the presence of non-zero lump-sum debt issuance costs, a financially constrained firm will realize larger investment projects for intermediate levels of credit capacity and will undertake investment projects of lower scales when credit capacity is relatively low/high.

## Appendix A

### A.1. Equity value

The firm's equity value is given by:

$$E(X_t, q, c) = (1 - \theta) \left[ \frac{qX_t}{r - \mu} - \frac{c}{r} + \left( \frac{c}{r} - \frac{qx_L(q)}{r - \mu} \right) \left( \frac{X_t}{x_L(q)} \right)^{\beta_1} \right], \quad (15)$$

where  $\beta_1 < 0$  is a root of  $\sigma^2/2\beta(\beta-1) + \mu\beta = r$  and the optimal liquidation rule  $x_L(q)$  such that  $E'_{x_L}(X_t, q, c) = 0$  is given by:

$$x_L(q) = \frac{\beta_1}{\beta_1 - 1} \frac{r - \mu}{q} \frac{c}{r} \equiv \delta \frac{c}{q} \quad (16)$$

### A.2. Debt value

The value of the firm's debt is given by:

$$D(X_t, q, c) = \frac{c}{r} - \left( \frac{c}{r} - (1 - \gamma)(1 - \theta) \left( \frac{qx_L(q)}{r - \mu} \right) \right) \left( \frac{X_t}{x_L(q)} \right)^{\beta_1} \quad (17)$$

### A.3. A firm's value

The value of the firm is given by the sum of equity and debt:

$$V(X_t, q, c) = \frac{(1 - \theta)qX_t}{r - \mu} + \frac{\theta c}{r} - \left( \theta + \gamma(1 - \theta) \frac{\beta_1}{\beta_1 - 1} \right) \frac{c}{r} \left( \frac{X_t}{x_L(q)} \right)^{\beta_1} \quad (18)$$

## Appendix B

We use the following parameter values: market interest rate  $r = 8\%$ , the expected growth rate of operating profit  $\mu = 1\%$ , operating profit volatility  $\sigma = 30\%$ , the default cost coefficient  $\gamma = 0.3$ , the tax rate  $\theta = 0.15$ , the investment function  $I(q) = 10 + 5q^3$ . A current value of the state variable is taken as  $X_0 = 1$ .

The optimal investment strategy in the benchmark case (all-equity financing) is given by  $q^e = 1.8566$  and  $x_I^e = 4.2582$ .

### B.1. Optimal investment and financing decisions when the credit constraint is not binding

**Table 1.a** displays simulation results for  $k = 0.1$  and different values of  $K_0$ . **Table 1.b** displays results obtained for  $K_0 = 5$  and different values of  $k < \theta$ . We denote  $F$  the current value of the firm investment option, whereas  $\rho = D(x_I, q, c)/I(q)$  represents the firm's leverage ratio.

**Table 1.a**

	$k = 0.1$						
$K_0$	$q^f$	$x_I^f$	$c^f$	$I(q^f)$	$D(x_I^f, q^f, c^f)$	$\rho$	$F$
0	<b>1,8566</b>	4,2282	2,1644	42	24,7629	0,5896	4,1613
1	1,9166	4,5056	2,3809	45,2	27,2392	0,6026	4,0885
2	1,9730	4,7746	2,5973	48,4	29,7155	0,6140	4,0232
3	2,0263	5,0363	2,8138	51,6	32,1917	0,6239	3,9640
4	2,0770	5,2914	3,0302	54,8	34,6680	0,6326	3,9100
5	2,1253	5,5405	3,2467	58	37,1443	0,6404	3,8603

**Table 1.b**

	$K_0 = 5$						
$k$	$q^f$	$x_I^f$	$c^f$	$I(q^f)$	$D(x_I^f, q^f, c^f)$	$\rho$	$F$
0	2,1253	5,3258	6,9924	58	70,8100	1,2209	4,1413
0,02	2,1253	5,3769	6,4445	58	66,5943	1,1482	4,0716
0,04	2,1253	5,4250	5,8118	58	61,3926	1,0585	4,0077
0,06	2,1253	5,4691	5,0799	58	54,9698	0,9478	3,9504
0,08	2,1253	5,5080	4,2318	58	47,0209	0,8107	3,9008
0,1	2,1253	5,5405	3,2467	58	37,1443	0,6404	3,8603
0,12	2,1253	5,5645	2,0987	58	24,8034	0,4276	3,8307
0,14	2,1253	5,5780	0,7563	58	9,26800	0,1598	3,8143

## B.2. Optimal investment and financing decisions when the credit constraint is binding

**Table 2.a** displays simulation results for  $\psi = 0.7$ ,  $k = 0.1$  and different values of  $K_0$ . **Table 2.b** displays results obtained for  $\psi = 0.7$ ,  $K_0 = 5$  and different values of  $k < \theta$ . **Table 2.c** contains simulation outcomes for  $K_0 = 5$ ,  $k = 0$  and different values of  $\psi \in [0.1, 1]$ .

**Table 2.a**

	$\psi = 0.7, k_0 = 0.1$					
$K_0$	$q$	$x_I$	$c$	$I(q)$	$D(x_I, q, c)$	$F$
0	<b>1,8566</b>	4,2507	2,6192	42	29,4003	4,1591
1	1,9161	4,5242	2,8097	45,1751	31,6225	4,0869
2	1,9723	4,7903	3,0011	48,3585	33,8510	4,0220
3	2,0254	5,0494	3,1928	51,5445	36,0812	3,9631
4	2,0760	5,3025	3,3852	54,7372	38,3161	3,9093
5	2,1243	5,5499	3,5778	57,9305	40,5514	3,8598



**Table 2.b**

$\psi = 0.7, K_0 = 5$						
$k$	$q$	$x_I$	$c$	$I(q)$	$D(x_I, q, c)$	$F$
0	2,1408	5,2565	3,6762	59,0559	41,3391	4,0640
0,02	2,1373	5,3151	3,6554	58,8190	41,1733	4,0212
0,04	2,1340	5,3737	3,6352	58,5892	41,0125	3,9795
0,06	2,1307	5,4323	3,6155	58,3642	40,8549	3,9387
0,08	2,1274	5,4911	3,5964	58,1443	40,7010	3,8988
0,1	2,1243	5,5499	3,5778	57,9305	40,5514	3,8598
0,12	2,1212	5,6087	3,5599	57,7229	40,4061	3,8217
0,14	2,1182	5,6676	3,5423	57,5193	40,2635	3,7844

**Table 2.c**

$k_0 = 0, K_0 = 5$						
$\psi$	$q$	$x_I$	$c$	$I(q)$	$D(x_I, q, c)$	$F$
<b>0</b>	<b>1,8566</b>	<b>4,2582</b>		<b>42</b>		<b>0,7387</b>
0,1	2,1287	5,5165	0,4717	58,2309	5,82310	3,8527
0,2	2,1319	5,4576	0,9595	58,4466	11,6893	3,8924
0,3	2,1348	5,4041	1,4646	58,6451	17,5935	3,9309
0,4	2,1372	5,3561	1,9881	58,8124	23,5250	3,9678
0,5	2,1391	5,3147	2,5308	58,9418	29,4709	4,0026
0,6	2,1404	5,2811	3,0934	59,0260	35,4156	4,0349
0,7	2,1408	5,2565	3,6763	59,0561	41,3393	4,0640
0,8	2,1403	5,2421	4,2789	59,0210	47,2168	4,0893
0,9	2,1387	5,2393	4,9003	58,9125	53,0213	4,1103
1	2,1359	5,2497	5,5386	58,7221	58,7221	4,1263

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