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To cite this version:

André Lapied, Thomas Rongiconi. Ambiguity as a Source of Temptation: Modeling Unstable Beliefs. 2013. halshs-00797631

HAL Id: halshs-00797631
https://shs.hal.science/halshs-00797631v1

Preprint submitted on 6 Mar 2013

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WP 2013 - Nr 16

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Ambiguity as a Source of Temptation: Modeling Unstable Beliefs.

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March 2013

Abstract

The "General-Self-Control-Preference" model introduced by Noor and Takeoka (2010) allows to take into account non linear costs of Self-Control. In this paper we extend this theory to situations in which a decision-maker faces ambiguity. We focus on the fact that lack of information is a potential source of temptation. Indeed lack of information doesn't allow the decision-maker to put a probability measure on uncertain events. Our basic hypothesis is that, in ambiguous situation, individuals are not confident enough about their beliefs and could therefore be tempted to use other beliefs to evaluate the alternatives in the second period. We study a two period model where ex ante dominated choice may tempt the decision-maker in the second period. Individuals have preferences over sets of alternatives that represent second period choices. We provide a Choice-Theoretic model where the ex ante belief is a probability measure whereas ex post belief is a Choquet-capacity, in order to take into account individual attitudes towards ambiguity in the second period.
KEYWORDS: Temptation, Self-control, Ambiguity, Choquet-Expected-Utility, Comonotonic-Temptation-Independence.
JEL CLASSIFICATION: D81.

1 Introduction

In most economic situations, individuals take their decisions in an ambiguous environment. As shown by the Ellsberg's Paradox, Savage's theory cannot model preferences with a non neutrality towards ambiguity. During the last two decades, the decision theorists have provided many models to rationalize behaviors under ambiguity. Loosely speaking, the main explanation from these theories is that the lack of objective information prevents a precise measurement of the likelihood of an event. However, while beliefs are based on an imprecise measure, they are stable across time. We provide an axiomatic foundation for the preferences of the agents with unstable beliefs under ambiguity.

As noted by Epstein and Kopylov (2007), aversion to ambiguity in the standard model is static and doesn't capture the notion of a belief that can change over time. Yet, an ambiguous context suggests that individuals could doubt the reliability of their beliefs and so could not have a definitive opinion upon the realization of the states of the world. To take into account the variation of beliefs even in the absence of new information,

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Epstein and Kopylov (2007) combine the model of ambiguity with the model of temptation of Gul and Pesendorfer (2001, henceforth GP). In the GP model, the choice is not determined by an unique preference ordering: the choices are the aggregated result of two conflicting orderings: a temptation preference that captures the agent's desires, and a normative preference[1] that captures his view of what choices he *should* make. This duality is also presents for choices under uncertainty. Since there are not enough objective information to describe likelihoods on the states of the world, we can think that individuals are not influenced solely by their rational beliefs to make a choice. Indeed emotions such as fear, anxiety, or excitation cannot be completely eliminated by the beliefs based upon an imprecise measure. Hence, it seems that beliefs are constituted by two conflicting components: a part analyzing the uncertainty of the situation and the other part representing a more immediate perception of reality. The first part is the normative component of beliefs while the second part can be understood as an emotional component.

We must still define what we mean by normative beliefs. As shown by Noor (2011), the temporal distance between the time of choice and the time of the consequences of this choice is fundamental in order to separate the normative preference and temptation preferences. So, under ambiguity the normative belief could be revealed when an agent is some time away from the consequences of his choice[2]. Inversely the temporal proximity increases the weight of tempting beliefs in the decision-making.

In order to capture the effect of temporal proximity on choices under ambiguity we adopt the theoretical framework defined in Epstein and Kopylov (2007): uncertainty is described by a finite states space S, and the decision problem occurs in three steps. A menu of Anscombe-Aumann acts is chosen in the first step. In the second step the agent chooses an act in the menu selected in the initial step. In the third step, a state of the world is realized and payoffs are received. For example, this framework allows to interpret the following situation: In the the first period, the agent can choose between the option a and the menu $\{a, b\}$. If the option a is selected, then when the second period comes, the decision maker have no other choice but to conserve the option a. Hence the pression of temporal proximity is canceled out since the decision cannot deviate of her initial choice. In the other hand, if the menu $\{a, b\}$ is choosed, then the decision maker still have the possibility to choose a or b in the second period, therefore the decision is made while the pression of temporal proximity is strong. Under ambiguity, if we observe the ranking $\{a\} \succ \{a, b\}$, then we can interpret this choice in the following way: during the first period the *reasonable* belief's of the decision maker implies that the option a is preferable to the option b, nevertheless if he chooses the menu $\{a, b\}$, then, during the second period the tempting belief's leads the decision maker to choose option b. In other words, the choice of $\{a, b\}$ implies a risk to deviate from the initial choice. Now, we can provide an answer to the following legitimate question: why individuals tend to modify their beliefs between the ex-ante and the ex post choice,

[1] For a behavioral fundation of this interpretation see Noor 2011.

[2] We don't provide a behavioral foundation of this interpretation in our formal analyse, however most experimental evidences confirm this interpretation (see e.g Armor and Taylor (2002) and Trope and Liberman (2003))

while the uncertainty upon the situation does not change? In an ambiguous situation, a decision-maker is not confident enough about his belief and could therefore be tempted to use other beliefs in order to evaluate the alternatives in the second period. For instance, a well know stylized fact in psychology is that individuals tend to lose confidence in their prospects when they approach the "moment of truth" (see e.g Gilovich and al. 1993). We focus on the fact that lack of information is a potential source of temptation.

By mixing the temptation model of GP et Maxmin-Expected-Utility, Epstein and Kopylov axiomatize the following utility function:

$$W(A) = \max_{f \in A} \left\{ EU(f) - k\left(\max_{g \in A} MEU(\cdot) - MEU(f) \right) \right\}$$

where

$$MEU(f) := \min_{q \in Q} \int_S u(f(s))dq(s) \quad \text{and} \quad EU(f) = \int_S u(f)dp$$

where Q is a closed convex set of probability, and p is a probability. The function EU represents the *normative* preference while $MEU(f)$ represents the tempting ranking. The function

$$c(g, f) = k\left(\max_{g \in A} MEU(\cdot) - MEU(f) \right)$$

describe the function of self-control, that is, $c(f, g)$ measure, in utility term, the frustration caused by the adherence to the initial choice in the presence of an tempting alternative.

Although our model has an undeniable affiliation with the model of Epstein and Kopylov, we differ on two key points:

1) Self-Control Cost

In our model, the cost of Self-Control is not necessarily linear. To achieve this, we extend the General-Self-Control-Preference of Noor and Takeoka under a framework allowing to take into account the presence of ambiguity. The non linearity of Self-Control-Cost allows to capture a broader pattern of preferences during the *ex-post* choice, namely what act chooses an agent in the menu selected *ex-ante*. For instance, as shown by Noor and Takeoka (2010), non linear Self-Control explains some form of Menu-Dependence. Under ambiguous context, this assumption seems relevant. Indeed the agents could have a reaction towards ambiguity which depends of the available choices.

2) The tempting Utility

In our model, the tempting preferences is represented by the Choquet-Expected-Utility Model (henceforth CEU) allowing to take into account individual attitudes towards ambiguity in the second period. Formally: let f and g two AA acts such that f is prefered *ex-ante* to g, then g is tempting for f if and only if $CEU(g) > CEU(f)$. An interesting particular case of CEU-temptation ranking is equal to the Hurwicz criterion's:

$$CEU(f) = \alpha \min_{s \in S} u(f(s)) + (1 - \alpha) \max_{s \in S} u(f(s))$$

where $\alpha \in [0,1]$. In other words, the temptation-ranking puts focus uniquely on the extreme outcomes, suggesting that the agent has in mind an over reaction to uncertainty during the ex-post choice. Hence the Choquet-Expected-Utility is not limited to an extreme pessimism in the second period of choice.

The article starts with the description of the Setup in section 2. In Section 3 we dress the design of the value of the menus and we propose and discuss an axiomatic foundation for the choice-theoretic model in section 3.2. In section 4 we announce the representation results. We conclude this paper in section 5. The proof are relayed in appendix.

2 Setup

Consider a finite set S of states of nature, 2^S is the set of subsets of S called events, and $Y = \triangle(X)$ is the set of probability measures on the Borel σ-algebra of X, where X is a compact metric space of outcomes. We assume that Y is embedded with the weak convergence topology, hence Y is compact and metrizable[3]. We denote by $\mathcal{F}$ the set of all Ascombe-Aumann (AA) acts: the set of all finite valued 2^S-measurable functions $f : S \to Y$. Given any $y \in Y$, with the usual abuse of notation denote $y \in \mathcal{F}$, the constant act such that $y(s) = y$ for all $s \in S$, thus we identify Y with the subset of constant acts in $\mathcal{F}$. For every $f,\ g \in \mathcal{F}$ and $\alpha \in [0,1]$ as usual we denote by $\alpha f + (1-\alpha)g$ ($f\alpha g$ for short) the act in $\mathcal{F}$, which yields $\alpha f(s) + (1-\alpha)g(s) \in Y$, for every $s \in S$. Hence we can derive the topology of $\mathcal{F}$ from the Cartesian product of metric spaces Y^S,

$$d(f,\ g) = \sup_{s \in S} d_Y(f(s), g(s)) \tag{1}$$

with $f, g \in \mathcal{F}$ and where $d_Y(f(s), g(s))$ is the metric generated by the weak convergence topology on Y. Since Y is a compact space, the Tychonoff theorem implies that $\mathcal{F}$ is also a compact space. The objects of our analyze are the compact subsets of $\mathcal{F}$. Let $\mathcal{M}$ be the set of nonempty compact subsets of $\mathcal{F}$. We endow $\mathcal{M}$ with the topology generated by the Haussdorf metric

$$d_{\mathcal{M}}(A,B) := \max\left\{\sup_{f \in A}\inf_{g \in B} d(f,g), \sup_{g \in B}\inf_{f \in A} d(f,g)\right\}. \tag{2}$$

Futhermore since $\mathcal{F}$ is a compact space with the metric d, $\mathcal{M}$ is also a compact space with the metric $d_{\mathcal{M}}$.[4]

3 Model

3.1 Utility

The General-Self-Control-Preference model accommodates temptation with not necessarily linear Self-Control costs by keeping Set-Betweenness. The main motivation of this weakening is to capture *uphill self-control* retaining the essential idea of the GP model: in situations where there exists tempting alternatives an

[3]See Aliprantis and Border 2006, Theorem 15.11
[4]See Kopylov (2009)

individual could deviate of her "normative preference". The agent want to resist to temptation must to use her self-control, but this resistance implies a frustration which is represented by an opportunity costs. Therefore if the agent maintains her initial choice, the utility of the best alternative is decreased by the cost of self-control. If the frustration due to the temptation resistance is too strong, the agent is constrain to cede to temptation and chooses a dominated alternative. In order to avoid costs of self-control caused by temptation, the agent chooses during first period, a menu whitout tempting alternatives. This explains a preference for commitment in the ex-ante choice. Hence the utility of menu A is write in GP model as follows

$$U(A) = \max_{f \in A}\{u(f) - (\max_{g \in A} v(g) - v(f))\}$$

where u is a von Neuman utility representing the "normative preference" and v is a von Neuman utility representing the temptation ranking. The cost of self-control is evaluated according to the most tempting alternative in the menu A: $\max_{g \in A} v(g) - v(f)$. Noor and Takeoka allow to keep this interpretation about the preference commitment, but in their theory the value of a menu A is represented by an "indirect utility" function:

$$U(A) = \max_{f \in A}\{u(f) - c(f, \max_{g \in A} v(g))\}$$

where $c(f, \max_{g \in A} v(g))$ is strictly increasing and positive function in its second argument. Moreover the function c satisfied two minimal features of Self-Control cost:

- *a)* $c\,(f, v(g)) > 0 \Rightarrow v(g) > v(f)$. This means that the self- control cost is positive only when self-control is exerted.

- *b)* $u(f) > u(g)$ and $v(g) > v(f) \Rightarrow c\,(f, v(g)) > 0$. This means that the self-control cost of resisting temptation is strictly positive.

To better capture the impact of ambiguity on *ex-post* decisions, we represent Temptation-utility with Choquet-Expected-Utility criterion. Although it is most often as modeling non neutrality ambiguity, the Choquet-Expected-Utility (CEU) is sometimes interpreted in terms of over or under reaction to uncertainty, since the agent chooses an action with a subjective perception of uncertainty represented by a capacity. As well know, a capacity is a normalized monotone set function. Since S is finite, CEU can be write as:

$$\int_S u(f) d\upsilon = \sum_{j=1}^{m} (u(x_j) - u(x_{j+1}))\, \upsilon \left(\bigcup_i^j E_i \right) \tag{3}$$

for all acts $f \in \mathcal{F}$ such that $u(x_1) \geqslant \cdots \geqslant u(x_m) \geqslant 0$.

In this paper we extend the Noor and Takeoka model's of temptation to situations where a Decision Maker is facing ambiguity. Namely in our model the value of menu of Anscombe-Aumann acts A is formulated as follows:

$$U(A) = \max_{f \in A}\{EU(f) - c(f, \max_{g \in A} CEU(\cdot))\}$$

where $EU(\cdot)$ is the classical SEU-Model, and the function c sastified $a)$ and $b)$. We can remark that both subjective and objective probabilities are present in the model but they are treated differently: for simplicity we assume that objective lotteries are not a source of temptation. Therefore the function of Self-Control cost admits a supplementary property:

- $c)$ $\{f(s)\} \sim \{f'(s)\}$ and $\{g(s)\} \sim \{g'(s)\}$ for all $s \in S \Rightarrow c\,(f, CEU(g)) = c\,(f', CEU(g'))$.

This simply says that if f, f' are equivalents from lotteries point of view, and identically for g, g', then there is not additionally cost of Self-Control. To resume, our model of utility has the form:

Definition 1 (Choquet General Self Control Preference)**.** We say that $\succcurlyeq$ has Choquet General Self-Control Preference if $A \succcurlyeq B \Leftrightarrow W(A) \geqslant W(B)$ where $W : \mathcal{M} \to \mathbb{R}$ is defining in the following way

$$W(A) = \max_{f \in A} \left\{ \sum_{s \in S} \mu(s) u(f(s)) - c\left(f, \max_{g \in A} \int_S u(g(s)) d\upsilon(s) \right) \right\}$$

where $\mu : 2^S \to [0,1]$ is a probability, $\upsilon : 2^S \to [0,1]$ is a capacity, $u : Y \to \mathbb{R}$ is a continuous function and $c : \mathcal{F} \times CEU\,(\mathcal{F}) \to \mathbb{R}_+$ is a continuous function that is weakly increasing in its second argument and satisfies $a), b)$ and $c)$.

3.2 Axioms

We suppose that the DM is sophisticated, namely the agent understands when choosing a menu at the ex post stage, she will choose an act from that menu, moreover, she anticipates that the passage of time will have an effect on her belief. Hence the menu A is chosen according to $\succcurlyeq$ on $\mathcal{M}$ in *ex-ante*. The first axiom is standard:

A1 (Weak Order): *$\succcurlyeq$ on $\mathcal{M}$ is complete, transitive and non-trivial.*

A2 (Continuity): *For each $A \in \mathcal{M}$, $\{B \in \mathcal{M} : B \succcurlyeq A\}$ and $\{B \in \mathcal{M} : A \succcurlyeq B\}$ are closed sets.*

A3 (Commitment-Independence): *For all $f, g, h \in \mathcal{F}$, and for all $\alpha \in (0,1)$,*

$$\{f\} \succcurlyeq \{g\} \Leftrightarrow \{\alpha f + (1-\alpha)h\} \succcurlyeq \{\alpha g + (1-\alpha)h\}$$

A2 provides the continuity of preferences in the classical sense. Axioms **A1**-**A3** imply by a standard result an Expected Utility representation over singletons, in other words the DM is ambiguity neutral for singletons.

A4 (Set-Betweenness): *For all $A, B \in \mathcal{M}$, $A \succcurlyeq B \Rightarrow A \succcurlyeq A \cup B \succcurlyeq B$*

The interpretation of this axiom is standard when it is assumed that the DM is sophisticated. For instance if the menu A is preferred to menu B, then $A \succ A \cup B$ reveals that B contains some acts which are source of temptation. In this case, the commitment in ex-ante period allows to avoid either the cost of

self-control or either to cede at the tempting alternatives. Following the above interpretation (suggest e.g by Noor and Takeoka) for the particular case in which $A = \{f\}$, $A \cup B = \{f, g\}$ and $B = \{g\}$, we can infer if g is a tempting act when f is the best choice with respect to the normative preference. Since the ranking of singletons $\{f\} \succcurlyeq \{g\}$ reveals the normative preference, the ranking $\{f\} \succ \{f, g\}$ suggests that the presence of g could lead the decision maker to deviate of her optimal choice. Inversely $\{f\} \sim \{f, g\}$ suggests that g doesn't represents an tempting alternative for the DM. Noor and Takeoka (2010) show that in this context, a supplementary axiom must be imposed: there is no *temptation-reversal* when two acts are mixed with a common act. During the *ex-ante* choice, if the DM reveals that g tempts f, then $\alpha g + (1-\alpha)h$ should be also an alternative which tempts $\alpha f + (1-\alpha)h$. More formally:

Definition 2 (*Temptation-Independence*)**.** For all acts $f, g, h \in \mathcal{F}$, if $\{f\} \succ \{g\}$ then for any $\alpha \in (0, 1)$, (i) and (ii) holds:

(i) $\{f\} \succ \{f, g\} \Rightarrow \{\alpha f + (1-\alpha)h\} \succ \{\alpha f + (1-\alpha)h, \alpha g + (1-\alpha)h\}$

(ii) $\{f\} \sim \{f, g\} \Rightarrow \{\alpha f + (1-\alpha)h\} \sim \{\alpha f + (1-\alpha)h, \alpha g + (1-\alpha)h\}$.

This assumption needs more discussion. The main goal of our model is to emphasize the link between ambiguity perception and time. However, Temptation-Independence is not appealing under ambiguity. To illustrate how the anticipation of ex-post ambiguous-belief can lead to a violation of Temptation Independence, we focus on two example.

EXAMPLE 1: ex-post Optimism.

Suppose the DM anticipates that she will attracted by ambiguity when she chooses an act in the menu selected ex-ante. For concreteness, imagine that the DM is face to an Ellsberg urn with 30 red balls and 60 green or yellows balls. There is two bets: $f = (100, R; 0, G; 0, Y)$ and $g = (0, R; 110, G; 0, Y)$. She must choose *ex-ante* between $\{f\}, \{g\}$ or $\{f, g\}$. If $\{f\} \succ \{g\}$, we can conclude that f is better than g with respect to her normative belief's. Clearly in our example f is an unambiguous act whereas g is ambiguous act. So, the expected payoffs of f is based on precise probability, while the expected payoffs of g are by definition unknown. Since the DM is attracted by ambiguity in the second period, it's seems plausible that $\{f\} \succ \{f, g\}$. Because the payoff of $g(G)$ is more important than $f(R)$, she could be tempted to overestimate the probability of the event G, and to deviate of her best choice in the second period if the menu $\{f, g\}$ is chosen during first period. In order to cancel the effect of temptation she commits in her best choice $\{f\}$. Now suppose the two following bets: $f' = (50, R; 5, G; 60, Y)$ and $g' = (0, R; 60, G; 60, Y)$. We can remark that $f' = \frac{1}{2}f + \frac{1}{2}h$ and $g' = \frac{1}{2}g + \frac{1}{2}h$, with $h = (0, R; 10, G; 120, Y)$. From Singleton-Independence we have $\{f'\} \succ \{g'\}$, but the dominated bets g' is not ambiguous. Therefore the commitment towards $\{f'\}$ *ex-ante* is superfluous and the following ranking should be observed: $\{f'\} \sim \{f', g'\}$.$\diamondsuit$

EXAMPLE 2: ex-post Pessimism.

Suppose the DM anticipates that she will averse towards ambiguity when she chooses an act in the menu selected ex-ante. There is two bets: $f = (100, R; 0, G; 0, Y)$ and $g = (0, R; 0, G; 110, Y)$. As in Example 1, she must choose *ex-ante* between $\{f\}, \{g\}$ or $\{f, g\}$. If $\{g\} \succ \{f\}$, we can conclude that g is better than f with respect to her normative belief's. Clearly in our example f is an unambiguous act whereas g is ambiguous act. Since the DM is averse towards ambiguity in the second period, it's seems plausible that $\{g\} \succ \{g, f\}$: thought the payoff of $g(Y)$ is more important than $f(R)$, she could be tempted to underestimate the probability of the event Y, leading the DM to choose f in the second period in order to avoid the ambiguous act. Now suppose the two following bets: $f' = (50, R; 60, G; 5, Y)$ and $g' = (0, R; 60, G; 60, Y)$. We can remark that $f' = \frac{1}{2}f + \frac{1}{2}h$ and $g' = \frac{1}{2}g + \frac{1}{2}h$, with $h = (0, R; 120, G; 10, Y)$. From Singleton-Independence we have $\{g'\} \succ \{f'\}$, but the best choice g' is not ambiguous. Therefore f' is not attractive for the DM, and the commitment towards $\{g'\}$ *ex-ante* is not necessarly. In other word we get $\{g'\} \sim \{g', f'\}$.$\diamondsuit$

In Example 1 and 2, *temptation reversal* arises because h is not comonotone to f and g. To better capture the impact of *ex-post* ambiguity on decisions, it's seems more realistic to restrict Temptation-Independence for comonotonic acts.[5] More formally:

A5 (Comonotonic-Temptation-Independence): *If $f, g, h \in \mathcal{F}$ are pairwise comonotonic and $\{f\} \succ \{g\}$, then Temptation-Independence holds.*

A5 highly suggests that temptation can be represented by an CEU criterion. By analogy with respect to a classical preference relation, it appears that we can interpret temptation as a binary relation on $\mathcal{F}$:

Definition 3 (*Temptation relation*)**.** Let $\mathbf{T}$ and $\mathbf{NT}$ two binary relations on $\mathcal{F}$. We say that for all $f, g \in \mathcal{F}$

- $g\mathbf{T}f$ (g tempts f) if $\{f\} \succ \{f, g\} \succcurlyeq \{g\}$

- $g\mathbf{NT}f$ (g no tempts f) if $\{f\} \sim \{f, g\} \succ \{g\}$.

Nevertheless, we can note that temptation relation is not complete: if $\{f\} \sim \{g\}$ by definition we cannot say if g tempts or doesn't tempts f. Moreover, **A4** doesn't implies transitivity:

$$g\mathbf{T}f \text{ and } h\mathbf{T}g \not\Rightarrow h\mathbf{T}f$$

In fact, if $g\mathbf{T}f$ and $h\mathbf{T}g$, then $\{f\} \succ \{h\}$, therefore from **A4** we get

$$\{f\} \succ \{f, h\} \succcurlyeq \{h\} \text{ or } \{f\} \sim \{f, h\} \succ \{h\}.$$

In other words if g tempts f and h tempts g, then necessarily $\{f\} \succcurlyeq \{f, h\}$. Thus it's difficult to draw an analogy between the temptation relation and a classical preference relation. In the absence of the transitivity property on temptation relation, Noor and Takeoka strengthens Temptation-Independence in order to achieve a binary relation more structured.

[5] Two acts f and g are say comonotonics if and only if $(f(s) - f(s'))(g(s) - g(s')) \geqslant 0$ for all $s, s' \in S$.

Definition 4 (*Temptation-Convexity*)**.** For $f, g, h \in \mathcal{F}$ if $\{f\} \succ \{g\}, \{g'\}$, then for any $\alpha \in [0,1]$, (i) and (ii) holds:

(i) $\{f\} \succ \{f, g\}$ and $\{f\} \succ \{f, g'\} \Rightarrow \{f\} \succ \{f, \alpha g + (1-\alpha)g'\}$

(ii) $\{f\} \sim \{f, g\}$ and $\{f\} \sim \{f, g'\} \Rightarrow \{f\} \sim \{f, \alpha g + (1-\alpha)g'\}$.

This axiom simply says that if g and g' tempt f, then any mixture of g and g' tempts f. A stronger form of this property is central to capture temptation ranking: it can be formulated as follows:

Definition 5 (*Strong-Temptation-Convexity*)**.** For $f, f', g, g' \in \mathcal{F}$ if $\{f\} \succ \{g\}, \{f'\} \succ \{g'\}$ then for any $\alpha \in (0,1)$ (i) and (ii) holds:

(i) $\{f\} \succ \{f, g\}$ and $\{f'\} \succ \{f', g'\} \Rightarrow \{\alpha f + (1-\alpha)f'\} \succ \{\alpha f + (1-\alpha)f', \alpha g + (1-\alpha)g'\}$.

(ii) $\{f\} \sim \{f, g\}$ and $\{f'\} \sim \{f', g'\} \Rightarrow \{\alpha f + (1-\alpha)f'\} \sim \{\alpha f + (1-\alpha)f', \alpha g + (1-\alpha)g'\}$.

Noor and Takeoka (2011) show that if Temptation-Independence holds then Temptation-Convexity and Strong-Temptation-Convexity are in fact equivalent. According to the weakening of Temptation-Independence we must modifying also Temptation-Convexity and Strong-Temptation-Convexity for they be adapted to an ambiguous context. More precisely, we define an comonotonic cone by the following set:

$$C_\rho := \left\{ f \in \mathcal{F} : f(s_{\rho(1)}) \preccurlyeq f(s_{\rho(2)}) \preccurlyeq \ldots \preccurlyeq f(s_{\rho(S)}) \right\}$$

where ρ is any permutation of S. We maintain Temptation-Convexity only if f, g, g' lie in the same comonotonic cone, and Strong-Temptation-Convexity holds only if

- f and f' are comonotonic

- g and g' are comonotonic

- f, f' doesn't lie to the same comonotonic cone that g and g'.

However, with our weakening of Temptation-Independence and of Temptation-Convexity we cannot derive the Strong-Temptation-Convexity axiom as in the paper of Noor and Takeoka. Hence we must postulate the following axiom:

A6 (Comonotonic-Temptation-Convexity): *For f, f' comonotonic, and g, g' comonotonic if $\{f\} \succ \{g\}, \{f'\} \succ \{g'\}$ then for any $\alpha \in (0,1)$ (i) and (ii) holds:*

(i) $\{f\} \succ \{f, g\}$ *and* $\{f'\} \succ \{f', g'\} \Rightarrow \{\alpha f + (1-\alpha)f'\} \succ \{\alpha f + (1-\alpha)f', \alpha g + (1-\alpha)g'\}$.

(ii) $\{f\} \sim \{f, g\}$ *and* $\{f'\} \sim \{f', g'\} \Rightarrow \{\alpha f + (1-\alpha)f'\} \sim \{\alpha f + (1-\alpha)f', \alpha g + (1-\alpha)g'\}$.

A6' (Comonotonic-Temptation-Convexity): *Let $f, f', g, g' \in \mathcal{F}$.*

(i) If $\{f\} \succ \{g\}, \{g'\}$ and $f, g, g' \in C_\rho$, then Temptation-Convexity holds

(ii) Let $\{f\} \succ \{g\}$, $\{f'\} \succ \{g'\}$ such that $f, f' \in C_\rho$ and $g, g' \in C_{\hat{\rho}}$, if and $\rho \neq \hat{\rho}$, then Strong-Temptation-Convexity holds.

From a Behavioral point of view, **A6** is not very different to **A5**. We can interpreted **A6** as follows: in the classical model of temptation, if Independence holds for any menus, namely

$$A \succcurlyeq B \Rightarrow \alpha A + (1-\alpha)C \succcurlyeq \alpha B + (1-\alpha)C$$

then Kopylov (2009.b)[6] show that $A \sim co(A)$, where $co(\cdot)$ is the convex hull of any set. This means that a menu is indifferent with respect to it properly randomization. In a ambiguous context, we can understood **A6** as a suitably restriction of randomization to comonotone acts: there is no *temptation reversal* if the randomization is suitably effected according to (i) and (ii) of **A6**.

The next axiom means that subjective acts and lotteries are treated differently: while the agent chooses new beliefs ex post about her subjective uncertainty, she does not modify her preference on lotteries during the three period. We consider lotteries as an component of taste and for simplicity we suppose that taste are constant across time. This leads to the following axioms:

A7 (Monotonic-Strategic-Rationality): *If $f, g \in \mathcal{F}$ and $f(s) \succcurlyeq g(s)$ for all $s \in S$ then $\{f\} \sim \{f, g\} \succcurlyeq \{g\}$. Moreover if for all $i = 1, \ldots, n < \infty$, $f_i(s) \sim f_i'(s)$ for all $s \in S$, then $\bigcup_{i=1}^n \{f_i\} \sim \bigcup_{i=1}^n \{f_i'\}$.*

If the evaluation of a lottery does not depend on the state, then a dominating act should be preferred under commitment. Similarly, if f dominates g, we would not expect f to be tempted by g. The second part of axiom say that if $\{f(s)\} \sim \{f'(s)\}$, $\{g(s)\} \sim \{g'(s)\}$ for all $s \in S$, then $\{f, g\} \sim \{f', g'\}$. This property holds for menus A and B that have same number of acts.

A8 (Temptation-Aversion): *For any $f, g, h \in \mathcal{F}$ if $\{f\} \succ \{f, g\} \succ \{g\}$ then if $\{h\} \succ \{h, g\}$ or $\{g\} \sim \{g, h\} \succ \{h\}$ implies that $\{f, h\} \succcurlyeq \{f, g\}$.*

This axiom means that if g is more tempting than h, then agents prefer the menu with the less tempting alterntive: $\{f, h\} \succcurlyeq \{f, g\}$.

4 Representation Results

4.1 Capture commitment under Ambiguity

In this section, we introduce the representation of the temptation relation by an Choquet-Expected-Utility.

[6] See also Dekel and al (2001)

Theorem 1. *If $\succcurlyeq$ on $\mathcal{M}$ satisfies **A1-A4** and **A7** with $\{f\} \succ \{g\}$ then (i) and (ii) are equivalents:*

*(i) $\succcurlyeq$ satisfies **A5** and **A6***

(ii) There exists a unique capacity $\upsilon : 2^S \to [0,1]$ and a Choquet integral $I : \mathcal{F} \to \mathbb{R}$ such that

$$\{f\} \succ \{f,g\} \Leftrightarrow I(g) = \int_S u(g)d\upsilon > I(f) = \int_S u(f)d\upsilon \tag{4}$$

where $u : Y \to \mathbb{R}$ is increasing, continuous, mixture linear and unique up to affine transformation.

Proof. See Appendix □

A brief outline of the proof of theorem 1:

We adopt a geometrical proof for this theorem. Since S is finite we use the method of piecewise linear integral on comonotonic cone to built an Choquet integral: firstly we represent the temptation relation by a linear form on all comonotonic cones, secondly the representation are extended entire domain. In order to represent the temptation relation on each comonotonic cone, we use the Hyperplan Theorem. Hence we identify $\mathbf{T}_{\mathcal{F}}$ on $\mathcal{F}$ to $\mathbf{T}_{B(u)}$ on $B(u)$, where $B(u)$ is the set of all function $\varphi : S \to \mathbb{R}$ such that $u(y_*) \leqslant \varphi(s) \leqslant u(y^*)$[7]. Clearly, $B(u)$ generates $\mathbb{R}^S$. Therefore we can apply the Hyperplan Separation Theorem, because $B(u)$ is a subset of vectorial space. Next, the proof take places in three stages:

• Applying the Hyperplan Separation to each comonotonic cone denoted C_ρ where ρ is a permutation of S, we prove the existence of probability vector representing the temptation relation for all act. The proof of the unicity of the probability is slightly different as usual. Firstly we show that for any act f in C_ρ and g in C_ρ, there exists a unique probability such that $\{f\} \succ \{f,g\}$ if and only if $E_{\pi^\rho_f}(g) > E_{\pi^\rho_f}(f)$. Secondly we show that for any $f, g \in C_\rho$, we have necessarly $\pi^\rho_f = \pi^\rho_g$. Hence for each C_ρ, the temptation-relation is well represented by

$$\{f\} \succ \{f,g\} \Leftrightarrow E_{\pi^\rho}(g) > E_{\pi^\rho}(f),$$

where $f, g \in C_\rho$.

• In the classical representation of comonotone preferences, since any constant act lies in all comonotonic cones, with a standard argument of transitivity, we can easily link all the comonotone cones between them, and obtain an Choquet Integral. However, in our case, the transitivity of temptation relation doesn't holds, for instance if $f \in C_\rho$ and $g, h \in C_{\hat{\rho}}$, then $g\mathbf{T}f$ and $h\mathbf{T}g \not\Rightarrow h\mathbf{T}f$. Hence we must built an Choquet Integral without transitivity. We argue as follow: let C_ρ and $C_{\hat{\rho}}$ and fix any $f \in C_\rho$. We define the two following sets:

$$\{g \in C_{\hat{\rho}} : g\mathbf{T}f \quad \text{or} \quad f\mathbf{NT}g\} \quad \text{and} \quad \{g \in C_{\hat{\rho}} : g\mathbf{NT}f \quad \text{or} \quad f\mathbf{T}g\}.$$

[7] y_* and y^* are the worst and the best loteries respectively.

From **A6** we show that this sets are convex and disjoint. So we can apply the Hyperplan Theorem, an deduce the existence of probability vector $\gamma_f^{\hat{\rho}}$ which represents the temptation relation at any $f \in C_\rho$, namely there exists $a \in \mathbb{R}$ such that:

$$\{f\} \succ \{f, g\} \Leftrightarrow E_{\gamma_f^{\hat{\rho}}}(g) > a,$$

where $g \in C_{\hat{\rho}}$. By a similar proof of stage 1 we show the unicity of the probability vector, namely $\gamma_f^{\hat{\rho}} = \gamma_{f'}^{\hat{\rho}}$ for all $f, f' \in C_\rho$. Next we show that $a = E_{\pi^\rho}(f)$. Hence we get

$$\{f\} \succ \{f, g\} \Leftrightarrow E_{\gamma^{\hat{\rho}}}(g) > E_{\pi^\rho}(f).$$

- Now, we must show that

$$\{f\} \succ \{f, g\} \Leftrightarrow E_{\pi^{\hat{\rho}}}(g) > E_{\pi^\rho}(f). \tag{5}$$

The mainly argument is to show that we have necessarily $\pi^{\hat{\rho}} = \gamma^{\hat{\rho}}$, and we obtain the desired result. The building of the Choquet integral with (6) derive from standard arguments (see e.g Ryan 2009; Chateauneuf, Kast and Lapied 2001).$\diamondsuit$

Because we have assumed that the individual is neutral towards ambiguity for singleton in the ex ante stage, the preference for commitment or equivalently anticipation of temptation in the ex post stage can be interpreted as a increasing sensitivity towards ambiguity when the third stage approaches. We specify the temptation utility as follow:

Definition 6. Let $f, g \in \mathcal{F}$, we say that f and g are correlated on Extreme outcomes (CEO) if $\overline{C}(f) = \overline{C}(g)$ and $\underline{C}(f) = \underline{C}(g)$, where

$$\begin{aligned} \overline{C}(f) &:= \{s \in S : \{f(s')\} \succcurlyeq \{f(s)\},\ \forall s' \in S\} \\ \underline{C}(f) &:= \{s \in S : \{f(s')\} \preccurlyeq \{f(s)\},\ \forall s' \in S\}. \end{aligned}$$

Definition 7. We say that υ is Hurwicz capacity if $\upsilon(\varnothing) = 0$, $\upsilon(S) = 1$ and for any $E \subset S$, $\upsilon(E) = \alpha$.

A5' (H-Temptation): *For $f, g \in \mathcal{F}$, if f, g, h are CEO then*

(i) $\{f\} \succ \{f, g\} \Leftrightarrow \{\alpha f + (1-\alpha)h\} \succ \{\alpha f + (1-\alpha)h, \alpha g + (1-\alpha)h\}$

(ii) $\{f\} \sim \{f, g\} \Leftrightarrow \{\alpha f + (1-\alpha)h\} \sim \{\alpha f + (1-\alpha)h, \alpha g + (1-\alpha)h\}$

Proposition 1. *If $\succcurlyeq$ on $\mathcal{M}$ satisfies **A1-A7** then (i) and (ii) are equivalents:*

*(i) $\succcurlyeq$ satisfies **A5'***

(ii) υ is a Hurwicz capacity

Proof. See Appendix. $\square$

A5" (Ambiguity aversion-Temptation): *For $f, g, h \in \mathcal{F}$ if $\{f\} \sim \{f, g\}$ and $\{f\} \sim \{f, h\}$ then for any $\alpha \in (0, 1)$, $\{f\} \succcurlyeq \{f, \alpha g + (1 - \alpha)h\}$.*

Proposition 2. *If $\succcurlyeq$ on $\mathcal{M}$ satisfies **A1-A7** then (i) and (ii) are equivalents:*

*(i) $\succcurlyeq$ satisfies **A5"***

(ii) υ is a convex capacity

Proof. See Appendix. □

A5"' (Ambiguity Loving-Temptation): *For $f, g, h \in \mathcal{F}$, if $\{f\} \succ \{f, g\}$ and $\{f\} \succ \{f, h\}$ then for any $\alpha \in (0, 1)$, $\{f\} \succcurlyeq \{f, \alpha f + (1 - \alpha)g\}$.*

Proposition 3. *If $\succcurlyeq$ on $\mathcal{M}$ satisfies **A1-A7** then (i) and (ii) are equivalents:*

*(i) $\succcurlyeq$ satisfies **A5"'***

(ii) υ is a concave capacity

Proof. See Appendix. □

4.2 Representation Theorem

Our main result is that the axioms on menus characterize the functional form described in Section 3.1. We say $\succcurlyeq$ is non degenerate if there exists $f, g \in \mathcal{F}$ such that $\{f\} \succ \{f, g\} \succ \{g\}$.

Theorem 2. *The binary relation $\succcurlyeq$ on $\mathcal{M}(\mathcal{F})$ is represented by a Choquet-General-Self-Control-Preference if and only if $\succcurlyeq$ satisfies **A1-A8**. Suppose that (u, μ, υ, c) and $(u', \mu', \upsilon', c')$ are both representation of a non degenerate Choquet-General-Self-Control. Then there exists $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $u' = \alpha u + \beta$, $\mu = \mu'$, and $\upsilon = \upsilon'$. Moreover, $c'(f, l) = \alpha c(f, (l - \beta)/\alpha)$ on the set:*

$$\left\{(f, l) \middle| \int_S u'(f) d\upsilon \geqslant l \text{ or } \{f\} \succ \{f, g\} \succ \{g\} \text{ for some } g \text{ with } \int_S u'(g) d\upsilon = l\right\}.$$

Proof. See appendix 3.3 □

While the Choquet-General-Self-Control-Preference is a representation for an *ex-ante* preference over menus, it suggests that *ex-post* choice is given by the choice correspondance defined by

$$\mathcal{C}(A) := \arg\max_{f \in A} \left\{ \sum_{s \in S} \mu(s) u(f(s)) - c\left(f, \max_{g \in A} \int_S u(g(s)) d\upsilon(s)\right)\right\}.$$

5 Concluding remarks

To conclude, we describe some variation of the above model.

Ambiguity sensitivity in ex-ante stage

We can think that to represent the singletons-preference by a SEU is not very realistic. Indeed why don't the agents have Ellsberg preferences when the consequences of choice are delayed? We can easily replace the singleton-independence by

$$\{f\} \succ \{g\} \Leftrightarrow \alpha\{f\} + (1-\alpha)h \succ \alpha\{g\} + (1-\alpha)\{h\}$$

when f, g and h are pairwise comonotonic. Hence we should get:

$$W(A) = \max_{f \in A} \left\{ \int_S u(f) d\upsilon - c\left(f, \max_{f \in A} \int_S u(g(s)) d\rho \right) \right\}$$

where υ, ρ are capacities. We could argue that *ex-ante*, the agents have a small sensitivity to ambiguity and a huge sensitivity to ambiguity during the *ex-post* choice. A generalization to a broader class of criterion, such as the MEU for singletons is, in our opinion also possible.

Random-Strotz-Utility under Ambiguity

Chatterjee and Krishna (2008) have axiomatized under risk the following model:

$$U(A) := \alpha \max_{f \in A} u(f) + (1-\alpha) \max_{g^* \in \mathcal{B}_v(A)} u(g^*)$$

where $\mathcal{B}_v(A) := \{f \in A \mid f \in \arg\max_{g \in A} v(\cdot)\}$. The theorem of representation of temptation in Section 4.1 is independent of the General-Self-Control model and could be used to model a Random-Strotz-Utility under ambiguity, where the temptation-utility is a CEU criterion and not a classical von Neuman-utility.

Appendix 1: Related Materials

1.1 Hyperplan separation theorem

Theorem (Aliprentis and Border, p.276)**.** *In a finite dimensional vector space, any two disjoint convex sets can be properly separated by a nonzero linear functional.*

Theorem (Aliprentis and Border, p.279)**.** *In a finite dimensional vector space two nonempty convex sets can be properly separated if and only if their relative interiors are disjoints.*

Theorem (Aliprentis and Border, p.279)**.** *Let C be a nonempty convex subset of a finite dimensional Hausdorff space and let x belong to C. Then there is a linear functional properly supporting C at x if and only if $x \notin ri\,(C)$.*

1.2 Hausdorff metric

Recall that $\mathcal{M}(\mathcal{F})$ is the set of compact menus therefore we can write the Hausdorrf metric in the following way

$$d_{\mathcal{M}}(A,B) := \max\left\{\max_{x\in B} d(x,A), \max_{x\in A} d(x,B)\right\} \tag{6}$$

where $d(f,A) = \min_{g\in A} d(f,g)$. Denote by $N_\epsilon(g)$ and $N_\epsilon(\{f,g\})$ the ϵ-neighborhood of g and $\{f,g\}$ respectively where $\epsilon > 0$.

Claim 1. If $\lim_{n\to\infty} g_n = g$, then $\{f,g_n\} \to \{f,g\}$ when $n \to \infty$.

Proof: We say that $A_n \to A$ in Hausdorff metric if $d_{\mathcal{M}}(A_n, A) \to 0$ when $n \to \infty$. Pick any binary menus in $\mathcal{M}(\mathcal{F})$, say $\{f,g\}$. Hence we have to show that if $\lim_{n\to\infty} g_n = g$ then $d_{\mathcal{M}}(\{f,g\}\}, \{f,g_n\}) \to 0$ when $n \to \infty$. We have for all n

$$\max_{x\in\{f,g_n\}} d(x,\{f,g\}) = \max\{\min\{d(f,f), d(f,g)\}, \min\{d(g_n,g), d(g_n,f)\}\}.$$

By definition $\min\{d(f,f), d(f,g)\} = 0$ and $\min\{d(g_n,f), d(g_n,g)\} \to 0$ when $n \to \infty$. Therefore when $n \to \infty$ we get $\max_{x\in\{f,g_n\}} d(x,\{f,g\}) \to 0$. The same result holds for $\max_{x\in\{f,g\}} d(x,\{f,g_n\})$. Hence when $n \to \infty$, we get

$$d_{\mathcal{M}}(\{f,g\},\{f,g_n\}) = \max\left\{\max_{x\in\{f,g_n\}} d(x,\{f,g\}) \to 0, \max_{x\in\{f,g\}} d(x,\{f,g'\}) \to 0\right\} \to 0,$$

as desired.

Appendix 2: Proof of Theorem 1

2.1 Preliminary results

Lemma 1. *If $\succcurlyeq$ satisfies A1-A3, there exists an affine functions $u : Y \to \mathbb{R}$ such for all $x, z \in Y$, $x \succcurlyeq z \Leftrightarrow u(x) \geqslant u(z)$.*

Proof. This is an immediate consequence of von-Neumann theorem since the independence axiom for constant act is implied by Commitment-Independence. □

Lemma 2. *If $\succcurlyeq$ on $\mathcal{M}$ satisfies A1-A4 and A7 then given $u : Y \to \mathbb{R}$ of lemma 1, there exists a continuous function $T : \mathcal{M}(\mathcal{F}) \to \mathbb{R}$ such that*
(i) For $A \in \mathcal{M}(Y)$, $T(A) = \max_{y_i\in A} u(y_i)$ and $\{y_i\} \succcurlyeq \{y_j\} \Leftrightarrow T(\{y_i\}) \geqslant T(\{y_j\})$.
(ii) For all $A, B \in \mathcal{M}$ $A \succcurlyeq B \Leftrightarrow T(A) \geqslant T(B)$, and for all $f, g \in \mathcal{F}$ $\{f\} \succcurlyeq \{g\} \Leftrightarrow T(\{f\}) \geqslant T(\{g\})$.

Proof. For all constant act **A7** holds, hence $\{y_i\} \succcurlyeq \{y_j\} \Rightarrow \{y_i\} \sim \{y_i, y_j\}$. By lemma 1 with $T(\{y_i\}) = u(y_i)$ we can put $T(\{y_i\}) = T(\{y_i, y_j\}) \geqslant T(\{y_j\})$ and by a recurrence argument (i) holds. Turn to (ii), by **A4**

and **A7** $\{y^*\} \succcurlyeq A \succcurlyeq \{y_*\}$[8], and by **A2** $\{\{f\} \in \mathcal{M} : \{f\} \succcurlyeq A\}$ and $\{\{f\} \in \mathcal{M} : A \succcurlyeq \{f\}\}$ are closed set. Thus there exists a unique $\alpha \in [0,1]$ such that $\{\alpha_A y^* + (1-\alpha_A)y_*\} \sim A$. Put $T(A) = \alpha_A$ and (ii) holds. Moreover T is continuous because $\succcurlyeq$ satisfies continuity and hence the set $\{A \in \mathcal{M} : T(A) \geqslant \gamma\}$ and $\{A \in \mathcal{M} : T(A) \leqslant \gamma\}$ are closed for all $\gamma \in \mathbb{R}$. □

We let B denote the set of real-valued 2^S-measurable functions, or equivalently the vector space generated by characteristics functions 1_A of the events $A \in 2^S$. If $f \in \mathcal{F}$ and $u : Y \to \mathbb{R}$, we denote by $\varphi = u(f)$ the element of B defined by $u(f)(s) = u(f(s))$ for all $s \in S$. Given a compact interval K in the real line, we denote by $B(K)$ the subset of the functions in B taking values in K. Clearly, $B = B(\mathbb{R})$. Since S is finite we can identify B to $\mathbb{R}^S$. Define $\mathcal{M}(B,K)$ as the set of closed and compact subset of $B(K)$. In other words $\mathcal{M}(B,K)$ is the set of menus on $B(K)$. We denote by $\mathcal{M}_0(B,u)$ the set of finite menus in $\mathcal{M}(B,K)$. We let that $0 \in int(u)$, hence without loss of generality we put $u = [-a,a]$ where $a \in \mathbb{R}_+$.

Lemma 3. *If $\succcurlyeq$ on $\mathcal{M}$ satisfies A1-A4 and A7 then there exists a continuous function $W : \mathcal{M}(B,u) \to \mathbb{R}$ such that (i), (ii) holds*

(i) For all $f \in \mathcal{F}$, there exists a probability vector $\mu \in \mathbb{R}^S$ such that $W(\{u(f)\}) = T(\{f\}) = E_\mu u(f)$.

(ii) For all $\cup_{i=1}^n \{f_i\} \in \mathcal{M}_0$, $W(\cup_{i=1}^n \{u(f_i)\}) = T(\cup_{i=1}^n \{f_i\})$.

Proof. (i) this is an immediate consequences of Ascombe-Aumann Theorem because all conditions for singletons hold. Turn to (ii): by lemma 2 we have $A \succcurlyeq B$ if and only if $T(A) \succcurlyeq T(B)$. Let $\Psi : \mathcal{M}_0 \to \mathcal{M}_0(B,u)$ where $\Psi(\cup_{i=1}^n \{f_i\}) = \cup_{i=1}^n \{u \circ f_i\}$ and such that for $A \sim B$ with $A, B \in \mathcal{M}_0$ we have $\Psi(A) = \Psi(B)$. From **A7**, if $\{f_i(s)\} \sim \{g_i(s)\}$ for all $s \in S$ and for all $i = 1, \ldots, n$ then $\cup_{i=1}^n \{f_i\} \sim \cup_{i=1}^n \{g_i\}$, hence $\Psi(\cup_{i=1}^n \{f_i\}) = \Psi(\cup_{i=1}^n \{g_i\})$. Therefore we get a weak order $\succcurlyeq'$ on $\mathcal{M}_0(B,u)$ such that

$$\cup_{i=1}^n \{f_i\} \succcurlyeq \cup_{i=1}^n \{g_i\} \Leftrightarrow \Psi(\cup_{i=1}^n \{f_i\}) \succcurlyeq' \Psi(\cup_{i=1}^n \{g_i\}).$$

Clearly, $\succcurlyeq'$ satisfies **A1-A7**, hence $W(\cup_{i=1}^n \{u \circ f_i\}) = T(\cup_{i=1}^n \{f_i\})$ is well defined. □

By abus of notation we write $\succcurlyeq' = \succcurlyeq$.

2.2 Proof of Theorem 1

We define the comonotonic cone of $\varphi \in B(u) \subset B$ by the set of all acts that is ordered by identix way each of their componments. We denote this comonotonic cone by

$$C_\rho := \{\varphi \in B(u) : \varphi(s_{\rho(1)}) \leqslant \varphi(s_{\rho(2)}) \leqslant \ldots \leqslant \varphi(s_{\rho(S)})\} \tag{7}$$

where ρ is inclued in the set of all bijective mapping from S to S.

Step 1: We prove that the temptation relation is represented by a linear function $I_\rho : B \to \mathbb{R}$ such that

[8] Where y^* and y_* are the constant acts such that for all $f \in \mathcal{F}$, $\{y^*\} \succcurlyeq \{f\} \succcurlyeq \{y_*\}$

for all $\varphi, \psi \in C_\rho$ we have $\{\varphi\} \succ \{\varphi, \psi\}$ if and only if $I_\rho(\psi) > I_\rho(\varphi)$, where $I_\rho(\varphi) = \sum_{s \in S} \pi^\rho(s)\varphi(s)$ with $\pi^\rho \in \mathbb{R}^S$ is a unique probability vector.

Step 2: Let $\varphi \in C_\rho$ and $\psi \in C_{\hat{\rho}}$, we show that there exists a unique non zero linear form $Q : B \to \mathbb{R}$ such that $\{\varphi\} \succ \{\varphi, \psi\}$ if and only if $Q(\psi) > I_\rho(\varphi)$.

Step 3: We show that for any $\psi \in C_{\hat{\rho}}$, $Q(\psi) = \sum_{s \in S} \pi^{\hat{\rho}}(s)\psi(s)$.

Step 4: We construct a capacity $\upsilon : 2^S \to [0, 1]$ such that for all $\varphi, \psi \in B(u)$ we have

$$\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \int_S \psi(s)d\upsilon(s) > \int_S \varphi(s)d\upsilon(s).$$

Proof of Step 1: Take any $\varphi \in C_\rho$: If $\{\varphi\} \sim \{y_*\}$ then by definition $\{\psi\} \succcurlyeq \{\varphi\}$ for all $\psi \in C_\rho$, therefore there is nothing to prove. If there don't exists $\psi \in C_\rho$ such that $\{\varphi\} \succ \{\varphi, \psi\}$, then putting $\pi^\rho(s) = \mu(s)$ for all $s \in S$ we get from lemma 3, $I_\rho(\varphi) = W(\{\varphi\})$, hence **A4** implies that $W(\{\varphi\}) = W(\{\varphi, \psi\}) > W(\{\psi\})$ for all $\varphi, \psi \in C_\rho$ such that $\{\varphi\} \succ \{\psi\}$, and we obtain the desired results. Turn now at the case where the temptation relation is nontrivial on C_ρ, namely there exists $\varphi, \psi \in C_\rho$ such that $\{\varphi\} \succ \{\varphi, \psi\}$. Let the two sets

$$L_\rho(\varphi) := \Big\{\psi \in C_\rho : \{\varphi\} \succ \{\varphi, \psi\}\Big\} \text{ and } K_\rho(\varphi) := \Big\{\psi \in C_\rho : \{\varphi\} \sim \{\varphi, \psi\} \succ \{\psi\}\Big\}.$$

By non triviality of temptation $L_\rho(\varphi)$ is nonempty. It is the same for $K_\rho(\varphi)$. Indeed, pick $\lambda \in \mathbb{R}_+$ such that $\varphi - \lambda 1_S > y_*$, obviously φ and $\varphi - \lambda 1_S$ are comonotonics and $\varphi - \lambda 1_S \in B(u)$. Hence **A7** is satisfied and we get $\{\varphi\} \sim \{\varphi, \varphi - \lambda 1_S\} \succ \{\varphi - \lambda 1_S\}$, this implies that $\varphi - \lambda 1_S \in K_\rho(\varphi)$. Thus $K_\rho(\varphi) \neq \varnothing$. It's easy to show that $K_\rho(\varphi)$ and $L_\rho(\varphi)$ are disjoints and it's follow directly from **A6**, that are convex sets. Moreover B is a finite dimensional vector space hence all conditions for the application of the separation Hyperplan Theorem[9] hold, $K_\rho(\varphi)$ and $L_\rho(\varphi)$ can be separated by a nonzero linear functional $J_\varphi : B \to \mathbb{R}$ and $c \in \mathbb{R}$ such that

$$J_\varphi(\psi) \leqslant c \leqslant J_\varphi(\psi') \quad \text{where} \quad \psi \in K_\rho(\varphi) \text{ and } \psi' \in L_\rho(\varphi). \tag{8}$$

Since $B \equiv \mathbb{R}^S$, from Riesz Representation Theorem we can put $J_\varphi(\cdot) = \langle q_\varphi, \cdot \rangle$ where $q_\varphi \in \mathbb{R}^S$ and $q_\varphi \neq 0_S$. Now we will show that $J_\varphi(\varphi) = c$. For prove this we show that φ is a suppport point of $cl(K_\rho(\varphi))$ and $cl(L_\rho(\varphi))$.

Lemma 4. *φ is a support point of $cl\,(K_\rho(\varphi))$ and $cl\,(L_\rho(\varphi))$.*

Proof. $\varphi \notin ri\,(L_\rho(\varphi))$[10] since by definition $\varphi \notin L_\rho(\varphi)$. Hence we have to show that φ is an boundary point of $L_\rho(\varphi)$. To show this we prove that for all open neighborhood $\mathcal{O} \in \mathbb{R}^S$ of φ we have $\mathcal{O} \cap L_\rho(\varphi) \neq \varnothing$. For any Neighborhood $\mathcal{O}$ of φ by definition there exists an open ball centered at φ noted $\mathcal{B}_\epsilon(\varphi) \subset \mathcal{O}$ with $\epsilon > 0$. Pick $\psi \in L_\rho(\varphi)$, by **A5** we get $\{\varphi\} \succ \{\varphi, \alpha\varphi + (1 - \alpha)\psi\}$ for all $\alpha \in (0, 1)$, putting $\alpha < \epsilon$ we have

[9] See Appendix 1

[10] $ri(L_\rho(\varphi)) \neq \varnothing$ because $L_\rho(\varphi)$ is convex.

$\alpha\varphi + (1-\alpha)\psi \in \mathcal{B}_\epsilon(\varphi)$[11]. Therefore $\mathcal{B}_\epsilon(\varphi) \cap L_\rho(\varphi) \neq \varnothing$, implying that $\varphi \in cl\,(L_\rho(\varphi))$. By Theorem of support points[12], φ is a support points of $cl\,(L_\rho(\varphi))$. By similar arguments with $\psi \in K_\rho(\varphi)$ we can show that φ is a support point of $cl\,(K_\rho(\varphi))$. □

By above lemma, J_φ is a supporting Hyperplan of $cl\,(K_\rho(\varphi))$ and $cl\,(L_\rho(\varphi))$ at φ. Hence we can put $J_\varphi(\varphi) = c$ and we get

$$J_\varphi(\psi) \leqslant J_\varphi(\varphi) \leqslant J_\varphi(\psi') \tag{9}$$

where $\psi \in K_\rho(\varphi)$ and $\psi' \in L_\rho(\varphi)$. Moreover $J_\varphi(\cdot)$ is a positive linear functionnals. Indeed if not we have $J_\varphi(\varphi) \leqslant J_\varphi(\varphi - \lambda 1_S)$ for all $\lambda \in \mathbb{R}_+$ such that $\varphi - \lambda 1_S \in B(u)$. If $J_\varphi(\varphi) = J_\varphi(\varphi - \lambda 1_S)$, then $J_\varphi(\cdot) = 0$ for all $\psi \in B$, contradicting the theorem of separation. If $J_\varphi(\varphi) < J_\varphi(\varphi - \lambda 1_S)$ then equation (10) implies that $\varphi - \lambda 1_S \in L_\rho(\varphi)$, but from **A7** we get $\varphi - \lambda 1_S \in K_\rho(\varphi)$: contradiction, because $K_\rho(\varphi)$ and $L_\rho(\varphi)$ are disjoints.

Lemma 5. *If $\{\varphi\} \succ \{\psi\}$ and $J_\varphi(\psi) = J_\varphi(\varphi)$ then $\{\varphi\} \sim \{\varphi, \psi\} \succ \{\psi\}$.*

Proof. If $\psi = y_*$ then it's follows directly from **A7** that $\{\varphi\} \sim \{\varphi, \psi\} \succ \{\psi\}$. Now we suppose that $\psi \neq y_*$. By **A2** $\{A \in \mathcal{M}(B, u) : \{\varphi\} \sim A\}$ is a closed set, thus by claim 2, if $\{\varphi\} \sim \{\varphi, \psi_n\}$ for all $n \in \mathbb{N}$ and that $\lim_{n\to\infty} \psi_n = \psi$ we have $\{\varphi\} \sim \{\varphi, \psi\}$. Pick $\psi_n = \psi - \frac{1}{n+1}\delta 1_S$ where $\delta > 0$ is such that $\psi - \delta 1_S \in B(u)$. This ensure that $\psi_n \in B(u)$ for all n. Hence **A7** is satisfied and we get $\{\psi\} \succ \{\psi - \frac{1}{n+1}\delta 1_S\}$. Since $J_\varphi(\cdot)$ is a positive nonzero linear functional we have $J_\varphi(\psi - \frac{1}{n+1}\delta 1_S) < J_\varphi(\psi)$ for all n. Moreover for any ψ', if $\{\varphi\} \succ \{\psi'\}$ and $J_\varphi(\psi') < J_\varphi(\varphi)$ then $\{\varphi\} \sim \{\varphi, \psi'\}$. Indeed suppose not: if $\{\varphi\} \succ \{\varphi, \psi'\}$, then we have by equation (10) that $J_\varphi(\psi') \geqslant J_\varphi(\varphi)$: in contradiction to $J_\varphi(\psi') < J_\varphi(\varphi)$. Therefore we get $\{\varphi\} \sim \{\varphi, \psi - \frac{1}{n+1}\delta 1_S\} \succ \{\psi - \frac{1}{n+1}\delta 1_S\}$ for all n. Clearly $\lim_{n\to\infty} \psi_n = \psi$ and by **A2** we have $\{\varphi\} \sim \{\varphi, \psi\} \succ \{\psi\}$ as desired. □

By above lemma we can conclude that $J_\varphi(\psi) \leqslant J_\varphi(\varphi) < J_\varphi(\psi')$ where $\psi \in K_\rho(\varphi)$ and $\psi' \in L_\rho(\varphi)$. In other words $\{\varphi\} \succ \{\varphi, \psi\}$ if and only if $J_\varphi(\varphi) < J_\varphi(\psi)$.

Lemma 6. *There exists a unique probability vector $\pi^\rho_\varphi \in \mathbb{R}^S$ such that $J_\varphi(\cdot) = \langle \pi^\rho_\varphi, \cdot \rangle$*

Proof. We know that $q_\varphi \neq 0$ and because J_φ is positive there are at least a $s \in S$ such that $q_\varphi(s) > 0$. Hence $\sum_{s\in S} q_\varphi(s) > 0$. Moreover $\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \langle \alpha q_\varphi, \psi \rangle > \langle \alpha q_\varphi, \varphi \rangle$ for all $\alpha > 0$. Putting $\alpha = 1/\sum_{s\in S} q_\varphi(s)$ we normalize q_φ by a probability vector of $\mathbb{R}^S$ denoted by π^ρ_φ and it is the unique probability vector representing temptation as above. □

Now we have to show that for any φ on C_ρ, π^ρ_φ is the unique probability vector representing the temptation relation. Namely we must show that $\pi^\rho_\varphi = \pi^\rho_{\varphi'}$ for any $\varphi, \varphi' \in C_\rho$. For this we let

[11] Indeed $\mathbb{R}^S$ is a locally convex topological vector space.
[12] See Appendix 1

$$V_\varphi := \{\psi \in C_\rho : \langle \pi^\rho_\varphi, \psi\rangle = \langle \pi^\rho_\varphi, \varphi\rangle\} \cap \{\psi \in C_\rho : \{\varphi\} \succ \{\psi\}\},$$

and we define the binary relation $\asymp$ on $C_\rho \times C_\rho$ as follows:

$$\{\varphi\} \asymp \{\varphi, \psi\} \Leftrightarrow \psi \in V_\varphi.$$

The lemma 6 assure the unicity of a probability π^ρ_φ for each $\varphi \in C_\rho$, hence the set of acts $\psi \in C_\rho$ such that $\{\varphi\} \asymp \{\varphi, \psi\}$ is characterized by $\psi \in V_\varphi$. In order to demonstrate the unicity of the probability we first show that V_φ is an affine subspace of dimension $S-1$ for any $\varphi \in C_\rho$ and secondly we show that for any φ and φ', the sets V_φ and $V_{\varphi'}$ are parallel. Hence the linear forms associated to V_φ and $V_{\varphi'}$ are such that $\pi^\rho_{\varphi'} = \beta\pi^\rho_\varphi$ for $\beta \in \mathbb{R}$. Since $\pi^\rho_{\varphi'}$ is a probability we have necessarily $\beta = 1$, hence $\pi^\rho_{\varphi'} = \pi^\rho_\varphi$. So we can conclude that if V_φ and $V_{\varphi'}$ are parallel affine subspaces then $\pi^\rho_\varphi = \pi^\rho_{\varphi'}$.

Definition 8. A subset A of $\mathbb{R}^S$ is an affine subspace if and only if for all $m \in \mathbb{N}^*$, for all $(\alpha_1, \ldots, \alpha_m) \in \mathbb{R}^m$ such that $\alpha_1 + \ldots + \alpha_m = 1$ and for all $\psi_1, \ldots, \psi_m \in A$, $\alpha_1\psi_1 + \ldots + \alpha_m\psi_m \in A$.

Lemma 7. *V_φ and $\alpha V_\varphi + (1-\alpha)\varphi'$ are affine subspaces. Moreover $dim(V_\varphi) = dim(\alpha V_\varphi + (1-\alpha)\varphi') = S-1$.*

Proof. First we show that V_φ is an affine subspace. Take $m \in \mathbb{N}^*$, $\psi_1, \ldots, \psi'_m \in V_\varphi$, and any $(\alpha_1, \ldots, \alpha_m) \in \mathbb{R}^m$ such that $\alpha_1 + \ldots + \alpha_m = 1$. Then by definition, $\{\varphi\} \succ \{\psi_i\}$ and $\langle \pi^\rho_\varphi, \varphi\rangle = \langle \pi^\rho_\varphi, \psi_i\rangle$ for all $i = 1, \ldots, m$. From **A3**

$$\{\varphi\} \succ \{\psi\} \Leftrightarrow \{\alpha\varphi + (1-\alpha)\varphi\} \succ \{\alpha\varphi + (1-\alpha)\psi\}$$

$$\{\varphi\} \succ \{\psi'\} \Leftrightarrow \{\alpha\varphi + (1-\alpha)\psi\} \succ \{\alpha\psi' + (1-\alpha)\psi\}$$

and from transitivity we get $\{\varphi\} \succ \{\alpha\psi' + (1-\alpha)\psi\}$. By repeating **A3** and transitivity we get $\{\varphi\} \succ \{\alpha_1\psi_1 + \ldots + \alpha_m\psi_m\}$, and by linearity of $\langle \pi^\rho_\varphi, \cdot\rangle$: $\langle \pi^\rho_\varphi, \sum_{i=1}^m \alpha_i\psi_i\rangle = \sum_{i=1}^m \alpha_i\langle \pi^\rho_\varphi, \psi_i\rangle = \langle \pi^\rho_\varphi, \psi_i\rangle = \langle \pi^\rho_\varphi, \varphi\rangle$. Hence $\sum_{i=1}^m \alpha_i\psi_i \in V_\varphi$.
Now we show that $dim(V_\varphi) = S-1$. Since V_φ is an affine subspace and that $0_S \in V_\varphi - \{\psi'\}$ for any $\psi' \in V_\varphi$, it's follows that $V_\varphi - \{\psi'\}$ is a vector subspace of $\mathbb{R}^S$ for any $\psi' \in V_\varphi$. Moreover $\langle \pi^\rho_\varphi, \psi - \psi'\rangle = 0$ since $\psi, \psi' \in V_\varphi$. Therefore $V_\varphi - \{\psi'\} = ker(H_\varphi)$ for any $\psi' \in V_\varphi$, thus $dim(V_\varphi) = S-1$. The proof for $\alpha V_\varphi + (1-\alpha)\varphi$ is the same. □

Lemma 8. *V_φ and $\alpha V_\varphi + (1-\alpha)\varphi'$ are parralel affine subspaces.*

Proof. Let $V_\varphi + \{\mathbf{x}_0\}$ where $\{\mathbf{x}_0\} = (\alpha-1)\varphi + (1-\alpha)\varphi'$. Hence by definition we get $\alpha\varphi + (1-\alpha)\varphi' \in V_\varphi + \{\mathbf{x}_0\}$. Obviously V_φ and $V_\varphi + \{\mathbf{x}_0\}$ are parralel. Denote by $H(V_\varphi)$ the hyperplan generated by V_φ. Its follows directly that $H(V_\varphi + \{\mathbf{x}_0\}) = H(V_\varphi) + \{\mathbf{x}_0\}$. So we get:

$$H(V_\varphi + \{\mathbf{x}_0\}) := \{\psi \in B(2^S) : \langle \pi^\rho_\varphi, \psi\rangle = \langle \pi^\rho_\varphi, \alpha\varphi + (1-\alpha)\varphi'\rangle\}$$

We can observe that for any $\psi \in V_\varphi$ we have $\alpha\psi + (1-\alpha)\varphi' \in \alpha V_\varphi + (1-\alpha)\varphi'$ if and only if any we have

$$\langle \pi^\rho_\varphi, \psi' \rangle = \langle \pi^\rho_\varphi, \alpha\varphi + (1-\alpha)\varphi' \rangle.$$

Moreover we have $\alpha V_\varphi + (1-\alpha)\varphi' \cap V_\varphi + \{\mathbf{x}_0\} \neq \varnothing$. From this remark its comes that $\alpha V_\varphi + (1-\alpha)\varphi' \subset H(V_\varphi + \{\mathbf{x}_0\})$. Thanks to lemma 7, $dim(\alpha V_\varphi + (1-\alpha)\varphi') = S-1$, therefore $H(\alpha V_\varphi + (1-\alpha)\varphi') = H(V_\varphi + \{\mathbf{x}_0\})$, this conclude the proof. □

Lemma 9. *V_φ and $V_{\alpha\varphi+(1-\alpha)\varphi'}$ are parallel affine subspaces.*

Proof. We argue by contradiction: suppose that V_φ and $V_{\alpha\varphi+(1-\alpha)\varphi'}$ are not parallel. Take $V_\varphi + \{\mathbf{x}_0\}$, the translated affine subspace of V_φ such that $\varphi + \{\mathbf{x}_0\} = \alpha\varphi + (1-\alpha)\varphi'$. Therefore $V_\varphi + \{\mathbf{x}_0\}$ is parallel to V_φ and $V_\varphi + \{\mathbf{x}_0\} \cap V_{\alpha\varphi+(1-\alpha)\varphi'} \neq \varnothing$ since $\alpha\varphi + (1-\alpha)\varphi' \in V_\varphi + \{\mathbf{x}_0\} \cap V_{\alpha\varphi+(1-\alpha)\varphi'}$. Clearly we have $\alpha V_\varphi + (1-\alpha)\varphi' \subset V_\varphi + \{\mathbf{x}_0\}$. From lemma 8 we have that $dim(\alpha V_\varphi + (1-\alpha)\varphi') = S-1$, therefore the hyperplan generates by $\alpha V_\varphi + (1-\alpha)\varphi'$ and by $V_\varphi + \{\mathbf{x}_0\}$ are the same, and we can deduce that $\alpha V_\varphi + (1-\alpha)\varphi'$ and V_φ are parallel affine subspaces. Moreover from **A5** we have by taking $\varphi, \psi \in V_\varphi$:

$$\{\varphi\} \asymp \{\varphi, \psi\} \Leftrightarrow \{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \alpha\psi + (1-\alpha)\varphi'\},$$

and by linearity of $\langle \pi^\rho_\varphi, \cdot \rangle$, we have

$$\langle \pi^\rho, \varphi \rangle = \langle \pi^\rho, \psi \rangle \Leftrightarrow \langle \pi^\rho_\varphi, \alpha\varphi + (1-\alpha)\varphi' \rangle = \langle \pi^\rho, \alpha\psi + (1-\alpha)\varphi' \rangle.$$

Therefore we can deduce that

$$\{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \alpha\psi + (1-\alpha)\varphi'\} \Leftrightarrow \langle \pi^\rho_\varphi, \alpha\varphi + (1-\alpha)\varphi' \rangle = \langle \pi^\rho_\varphi, \alpha\psi + (1-\alpha)\varphi' \rangle,$$

in other words

$$\psi \in \alpha V_\varphi + (1-\alpha)\varphi' \Rightarrow \{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \psi\}. \tag{10}$$

However by hypothesis $\alpha V_\varphi + (1-\alpha)\varphi'$ and $V_{\alpha\varphi+(1-\alpha)\varphi'}$ are not parralel, so we have $\alpha V_\varphi + (1-\alpha)\varphi' \neq V_{\alpha\varphi+(1-\alpha)\varphi'}$ and since $\alpha V_\varphi + (1-\alpha)\varphi' \cap V_{\alpha\varphi+(1-\alpha)\varphi'} \neq \varnothing$ we get by definition that $dim(\alpha V_\varphi + (1-\alpha)\varphi' \cap V_{\alpha\varphi+(1-\alpha)\varphi'}) = S-2$. So there exists at least a $\psi \in \alpha V_\varphi + (1-\alpha)\varphi'$ such that $\psi \notin V_{\alpha\varphi+(1-\alpha)\varphi'}$. In other words by (11) $\{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \psi\}$ and $\psi \notin V_{\alpha\varphi+(1-\alpha)\varphi'}$, this is a contradiction because $\{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \psi\}$ if and only if $\psi \in V_{\alpha\varphi+(1-\alpha)\varphi'}$. □

With the same type of argument we proove that $\pi^\rho_{\alpha\varphi+(1-\alpha)\varphi'} = \pi^\rho_{\varphi'}$. We have showed that $\pi^\rho_\varphi = \pi^\rho_{\varphi'}$ for any $\varphi, \varphi' \in C_\rho$. Now it's easy to see that

$$\{\varphi\} \sim \{\varphi, \psi\} \Leftrightarrow \begin{cases} \{\varphi\} \asymp \{\varphi, \psi\} \Leftrightarrow \langle \pi^\rho, \varphi \rangle = \langle \pi^\rho, \psi \rangle \\ \{\varphi\} \simeq \{\varphi, \psi\} \Leftrightarrow \langle \pi^\rho, \varphi \rangle > \langle \pi^\rho, \psi \rangle \end{cases}$$

Thus unicity of π^ρ with regard to $\asymp$ implies the unicity of π^ρ for the temptation relation on $C_\rho \times C_\rho$. Hence for any $\varphi, \psi \in C_\rho$ such that $\{\varphi\} \succ \{\psi\}$ we have $\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \langle \pi^\rho, \psi \rangle > \langle \pi^\rho, \varphi \rangle$. This conclude the proof of step 1.

Proof of Step 2: In this step we show that there exists a linear functional $Q_\varphi(\cdot) : B \to \mathbb{R}$ such that

$$\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow Q_\varphi(\psi) > I_\rho(\varphi) \tag{11}$$

Where $\varphi \in C_\rho$, $\psi \in C_{\hat{\rho}}$ and $\{\varphi\} \succ \{\psi\}$. Let the following sets:

$$NT^+ := \left\{ \psi \in C_{\hat{\rho}} : \{\psi\} \sim \{\psi, \varphi\} \succ \{\varphi\} \right\} \text{ and } T^+ := \left\{ \psi \in C_{\hat{\rho}} : \{\varphi\} \succ \{\varphi, \psi\} \right\}$$

$$T^- := \left\{ \psi \in C_{\hat{\rho}} : \{\psi\} \succ \{\psi, \varphi\} \right\} \text{ and } NT^- := \left\{ \psi \in C_{\hat{\rho}} : \{\varphi\} \sim \{\varphi, \psi\} \succ \{\psi\} \right\}$$

We have two case: $\{y^*\} \sim \{\varphi\} \succ \{y_*\}$ or $\{y^*\} \succ \{\varphi\} \succ \{y_*\}$.

First we analyse the case where $\{y^*\} \succ \{\varphi\} \succ \{y_*\}$. Let $Z^+(\varphi) := NT^+ \cup T^+$ and $Z^-(\varphi) := T^- \cup NT^-$. First $Z^+(\varphi)$ and $Z^-(\varphi)$ are nonempty since from **A7**, $C_{\hat{\rho}} \cap \{\psi : \psi(s) \succ \varphi(s), \forall s \in S\} \cap \{\psi : \varphi(s) \succ \psi(s), \forall s \in S\}$ is nonempty, thus NT^+ and NT^- are nonempty. We denote by $co(\cdot)$ the convex hull of any set in $B(u)$.

Lemma 10. *$ri(coZ^+(\varphi)) \cap ri(coZ^-(\varphi)) = \varnothing$.*

Proof. Since NT^+, T^+, NT^- and T^- are convex from **A6**, we get $coZ^+(\varphi) = \lambda NT^+ + (1-\lambda)T^+$ and $coZ^-(\varphi) = \lambda NT^- + (1-\lambda)T^-$ with $\lambda \in [0,1]$. Since $\psi_0 \in NT^+ \Rightarrow \{\psi_0\} \succ \{\varphi\}$ and $\psi_1 \in T^+ \Rightarrow \{\varphi\} \succ \{\psi_1\}$, there exists $\lambda^* \in (0,1)$ such that $\lambda^*\psi_0 + (1-\lambda^*)\psi_1 \in coZ^+(\varphi)$ and $\{\lambda^*\psi_0 + (1-\lambda^*)\psi_1\} \sim \{\varphi\}$. Hence for $\lambda < \lambda^*$ we get $\lambda\psi_0 + (1-\lambda)\psi_1 \in T^+$ and for $\lambda > \lambda^*$ we get $\lambda\psi_0 + (1-\lambda)\psi_1 \in NT^+$. In other words

$$coZ^+(\varphi) = \left\{ \psi \in C_{\hat{\rho}} : \{\psi\} \sim \{\varphi\} \text{ or } \psi \in NT^+ \text{ or } \psi \in T^+ \right\}$$

$$coZ^-(\varphi) = \left\{ \psi \in C_{\hat{\rho}} : \{\psi\} \sim \{\varphi\} \text{ or } \psi \in NT^- \text{ or } \psi \in T^- \right\}$$

Suppose by way of contradiction that there exists $\psi \in ri(coZ^+(\varphi)) \cap ri(coZ^-(\varphi))$:

Case 1: If $\{\psi\} \succ \{\varphi\}$, then we get $\{\psi\} \sim \{\psi, \varphi\}$ and $\{\psi\} \succ \{\psi, \varphi\}$: contradiction.

Case 2: If $\{\varphi\} \succ \{\psi\}$, then we get $\{\varphi\} \sim \{\varphi, \psi\}$ and $\{\varphi\} \succ \{\varphi, \psi\}$: contradiction.

Case 3: If $\{\psi\} \sim \{\varphi\}$, then there exists an open ball centered at ψ with $\epsilon > 0$ such that for all $\psi' \in B_\epsilon(\psi)$ we have $\psi' \in ri(coZ^+(\varphi)) \cap ri(coZ^-(\varphi))$. Take $\psi' = \psi - \lambda 1_S$ with $\lambda < \epsilon$. Clearly $\psi' \in B_\epsilon(\psi)$ and from **A7**, $\{\psi\} \succ \{\psi'\}$. Hence $\{\varphi\} \succ \{\psi'\}$, from case 2 we get a contradiction. Thus we have to shown that $ri(co(Z^+(\varphi)) \cap ri(coZ^-(\varphi)) = \varnothing$ □

Hence we can apply Hyperplan Theorem, there exists a nonull linear functional $Q_\varphi : B_0 \to \mathbb{R}$ and $a \in \mathbb{R}$ such that

$$Q_\varphi(\psi) \leqslant a \leqslant Q_\varphi(\psi') \text{ where } \psi \in co(Z^-(\varphi)) \text{ and } \psi' \in co(Z^+(\varphi)). \tag{12}$$

Since by hypothesis $\{y^*\} \succ \{\varphi\} \succ \{y_*\}$ NT^+ and NT^- are nonempty, we have $C_{\hat{\rho}} \cap \{\psi : \psi(s) \succ \varphi(s), \forall s \in S\} \subset NT^+$ and $C_{\hat{\rho}} \cap \{\psi : \varphi(s) \succ \psi(s), \forall s \in S\}$. It follows that $Q_\varphi(\cdot)$ is a positive linear functional. Indeed suppose not, then this implies that for $y1_S$, $y'1_S$ such that $\{y1_S\} \succ \{\varphi(s)\} \succ \{y'1_S\}$ for all $s \in S$ we have $Q_\varphi(y'1_S) \geqslant Q_\varphi(y1_S)$. If $Q_\varphi(y'1_S) = Q_\varphi(y1_S)$ then $Q_\varphi(\cdot)$ is null, this is impossible. If $Q_\varphi(y'1_S) > Q_\varphi(y1_S) \geqslant a$, then this implies that $y'1_S \in Z^+(\varphi)$, in contradiction with **A7**.

Lemma 11. *If $\psi \in C_{\hat{\rho}}$ and $Q_\varphi(\psi) = a$, then $\{\varphi\} \sim \{\varphi, \psi\}$ if $\{\varphi\} \succ \{\psi\}$, and $\{\psi\} \sim \{\psi, \varphi\}$ if $\{\psi\} \succ \{\varphi\}$.*

Proof. Since $Q_\varphi(\cdot)$ is a nonull positive linear functional, this lemma holds with the same types of arguments as in lemma 5. □

Therefore we get for $\varphi \in C_\rho$ and $\psi \in C_{\hat{\rho}}$:

$$\{\varphi\} \succ \{\varphi, \psi\} \quad \Leftrightarrow \quad Q_\varphi(\psi) > a \tag{13}$$

$$\{\psi\} \sim \{\psi, \varphi\} \succ \{\varphi\} \quad \Leftrightarrow \quad Q_\varphi(\psi) \geqslant a \tag{14}$$

$$\{\varphi\} \sim \{\varphi, \psi\} \succ \{\psi\} \quad \Leftrightarrow \quad Q_\varphi(\psi) \leqslant a \tag{15}$$

$$\{\psi\} \succ \{\psi, \varphi\} \quad \Leftrightarrow \quad Q_\varphi(\psi) < a \tag{16}$$

Hence we can deduce the following lemma:

Lemma 12. *There exists a unique probability vector $\gamma^{\hat{\rho}} \in \mathbb{R}^S$ such that $Q_\varphi(\cdot) = \langle \gamma^{\hat{\rho}}, \cdot \rangle$*

Proof. By (14)-(17), the proof is the same of lemma 6. □

Lemma 13. $I_\rho(\varphi) = a$.

Proof. By definition any constant act $y1_S \in C_\rho \cap C_{\hat{\rho}}$. Suppose by contradiction that $I_\rho(\varphi) \neq a$. If $I_\rho(\varphi) = c > a$. Then there exists $b \in \mathbb{R}$ such that $a > b > c$. We have two case. If $\{b1_S\} \succ \{\varphi\}$, then from step 1 one we get $\{b1_S\} \sim \{b1_S, \varphi\}$, and from (17) we get $\{b1_S\} \succ \{b1_S, \varphi\}$: contradiction. If $\{\varphi\} \succ \{b1_S\}$, then from Step 1 we get $\{\varphi\} \succ \{\varphi, b1_S\}$ and from (16) we get $\{\varphi\} \sim \{\varphi, b1_S\}$: contradiction. Now suppose $I_\rho(\varphi) = c < a$, pick $b \in \mathbb{R}$ such that $a < b < c$, we have again two cases. If $\{b1_S\} \succ \{\varphi\}$, then from Step 1 $\{b1_S\} \succ \{b1_S, \varphi\}$ and from (15) we get $\{b1_S\} \sim \{b1_S, \varphi\}$: contradiction. If $\{\varphi\} \succ \{b1_S\}$, then from Step 1 $\{\varphi\} \sim \{\varphi, b_S\}$ and from (14) we get $\{\varphi\} \succ \{\varphi, b1_S\}$: contradiction. Hence $I_\rho(\varphi) = a$. □

Now we analyse the case where $\{\varphi\} \sim \{y^*\}$. We have two possibilities $T^+ \neq \varnothing$ or $T^+ = \varnothing$. By definition of y^* we have $\{y^*\} \succcurlyeq \{\varphi\}$ for all $\varphi \in B(u)$. In other words $\varphi(s) \leqslant y^*$ for all $s \in S$, hence $\{\varphi\} \sim \{y^*\}$ if and only if $\varphi(s) = y^*$ for all $s \in S$. Therefore $\{\varphi\} \sim \{y^*\}$ if and only if φ is a constant act.
If $T^+ \neq \varnothing$. Since $\varphi \in C_\rho \cap C_{\hat{\rho}}$ the results follow directly from step 1.
If $T^+ = \varnothing$. Put $Q_\varphi(\cdot) = \langle \mu, \cdot \rangle$, and we have the desired results.

Now we want to show that for any $\varphi \in C_\rho$, $\gamma^{\hat{\rho}}_\varphi$ is the unique probability vector representing the temptation relation betweenn C_ρ and $C_{\hat{\rho}}$. Namely we show that $\gamma^{\hat{\rho}}_\varphi = \gamma^{\hat{\rho}}_{\varphi'}$. For this we define the following sets:

$$\hat{V}^1_\varphi := \{\psi \in C_{\hat{\rho}} : \langle \pi^\rho, \varphi \rangle = \langle \gamma^{\hat{\rho}}_\varphi, \psi \rangle\} \cap \{\psi \in C_{\hat{\rho}} : \{\psi\} \succ \{\varphi\}\}$$

$$\hat{V}^2_\varphi := \{\psi \in C_{\hat{\rho}} : \langle \pi^\rho, \varphi \rangle = \langle \gamma^{\hat{\rho}}_\varphi, \psi \rangle\} \cap \{\psi \in C_{\hat{\rho}} : \{\varphi\} \succ \{\psi\}\}$$

and let the binary relation $\asymp$ on $B(u) \times B(u)$ such that:

$$\{\varphi\} \asymp \{\varphi, \psi\} \Leftrightarrow \psi \in \hat{V}_\varphi$$

where $\varphi \in C_\rho$ and $\psi \in C_{\hat{\rho}}$. The lemma 12 assure the unicity of a probability $\gamma^{\hat{\rho}}_\varphi$ for each $\varphi \in C_\rho$, hence the set of acts $\psi \in C_{\hat{\rho}}$ such that $\{\varphi\} \asymp \{\varphi, \psi\}$ is characterized by $\psi \in \hat{V}_\varphi$. We can remark that $\asymp$ satisfied **A6**.

By similar arguments of lemma 7 and 8 we can show that $\hat{V}_\varphi$ and $\alpha\hat{V}_\varphi + (1-\alpha)\varphi'$ are parallel affine subspaces of dimension of $S-1$.

Lemma 14. *$\hat{V}_\varphi$ and $\hat{V}_{\alpha\varphi+(1-\alpha)\varphi'}$ are parallel affine subspaces.*

Proof. We argue by contradiction: suppose that V_φ and $V_{\alpha\varphi+(1-\alpha)\varphi'}$ are not parallel. Take $V_\varphi + \{\mathbf{x}_0\}$, the translated affine subspace of V_φ such that $\varphi + \{\mathbf{x}_0\} = \alpha\varphi + (1-\alpha)\varphi'$. Therefore $V_\varphi + \{\mathbf{x}_0\}$ is parallel to V_φ and $V_\varphi + \{\mathbf{x}_0\} \cap V_{\alpha\varphi+(1-\alpha)\varphi'} \neq \varnothing$ since $\alpha\varphi + (1-\alpha)\varphi' \in V_\varphi + \{\mathbf{x}_0\} \cap V_{\alpha\varphi+(1-\alpha)\varphi'}$. Clearly we have $\alpha V_\varphi + (1-\alpha)\varphi' \subset V_\varphi + \{\mathbf{x}_0\}$. From lemma 8 we have that $dim(\alpha V_\varphi + (1-\alpha)\varphi') = S-1$, therefore the hyperplan generates by $\alpha V_\varphi + (1-\alpha)\varphi'$ and by $V_\varphi + \{\mathbf{x}_0\}$ are the same, and we can deduce that $\alpha V_\varphi + (1-\alpha)\varphi'$ and V_φ are parallel affine subspaces. Moreover from **A6** we have

$$\{\varphi\} \asymp \{\varphi, \psi\} \Leftrightarrow \{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \alpha\psi + (1-\alpha)\varphi'\},$$

and by linearity of $\langle \pi^\rho_\varphi, \cdot \rangle$, we have

$$\langle \pi^\rho, \varphi \rangle = \langle \pi^\rho, \psi \rangle \Leftrightarrow \langle \pi^\rho_\varphi, \alpha\varphi + (1-\alpha)\varphi' \rangle = \langle \pi^\rho, \alpha\psi + (1-\alpha)\varphi' \rangle.$$

Therefore we can deduce that

$$\{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \alpha\psi + (1-\alpha)\varphi'\} \Leftrightarrow \langle \pi^\rho_\varphi, \alpha\varphi + (1-\alpha)\varphi' \rangle = \langle \pi^\rho_\varphi, \alpha\psi + (1-\alpha)\varphi' \rangle,$$

for any $\psi \in V_\varphi$. In other words

$$\psi \in \alpha V_\varphi + (1-\alpha)\varphi' \Rightarrow \{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \psi\}. \tag{17}$$

However by hypothesis $\alpha V_\varphi + (1-\alpha)\varphi'$ and $V_{\alpha\varphi+(1-\alpha)\varphi'}$ are not parralel, so we have $\alpha V_\varphi + (1-\alpha)\varphi' \neq V_{\alpha\varphi+(1-\alpha)\varphi'}$ and since $\alpha V_\varphi + (1-\alpha)\varphi' \cap V_{\alpha\varphi+(1-\alpha)\varphi'} \neq \varnothing$ we get by definition that $dim(\alpha V_\varphi + (1-\alpha)\varphi' \cap V_{\alpha\varphi+(1-\alpha)\varphi'}) = S-2$. So there exists at least a $\psi \in \alpha V_\varphi + (1-\alpha)\varphi'$ such that $\psi \notin V_{\alpha\varphi+(1-\alpha)\varphi'}$. In other words by (11) $\{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \psi\}$ and $\psi \notin V_{\alpha\varphi+(1-\alpha)\varphi'}$, this is a contradiction because $\{\alpha\varphi + (1-\alpha)\varphi'\} \asymp \{\alpha\varphi + (1-\alpha)\varphi', \psi\}$ if and only if $\psi \in V_{\alpha\varphi+(1-\alpha)\varphi'}$. □

With the same type of argument we proove that $\pi^{\rho}_{\alpha\varphi+(1-\alpha)\varphi'} = \pi^{\rho}_{\varphi'}$. We have showed that $\pi^{\rho}_{\varphi} = \pi^{\rho}_{\varphi'}$ for any $\varphi, \varphi' \in C_\rho$. Now it's easy to see that

$$\{\varphi\} \sim \{\varphi, \psi\} \Leftrightarrow \begin{cases} \{\varphi\} \asymp \{\varphi, \psi\} \Leftrightarrow \langle \pi^\rho, \varphi \rangle = \langle \pi^\rho, \psi \rangle \\ \{\varphi\} \simeq \{\varphi, \psi\} \Leftrightarrow \langle \pi^\rho, \varphi \rangle > \langle \pi^\rho, \psi \rangle \end{cases}$$

Thus unicity of π^ρ with regard to $\asymp$ implies the unicity of π^ρ for the temptation relation on $C_\rho \times C_\rho$. Hence for any $\varphi, \psi \in C_\rho$ such that $\{\varphi\} \succ \{\psi\}$ we have $\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \langle \pi^\rho, \psi \rangle > \langle \pi^\rho, \varphi \rangle$. This conclude the proof of step 2.

Proof of Step 3: We show that $\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \langle \pi^{\hat{\rho}}, \psi \rangle > \langle \pi^\rho, \varphi \rangle$. From step 1 we know that $\{\varphi\} \succ \{\varphi, \psi\}$ if and only if $\langle \pi^\rho, \psi \rangle > \langle \pi^\rho, \varphi \rangle$ for $\varphi, \psi \in C_\rho$, and, $\{\varphi'\} \succ \{\varphi', \psi'\}$ if and only if $\langle \pi^{\hat{\rho}}, \psi' \rangle > \langle \pi^{\hat{\rho}}, \varphi' \rangle$ for any $\varphi', \psi' \in C_{\hat{\rho}}$. On the other hand, from step 2 $\{\varphi\} \succ \{\varphi, \psi'\}$ if and only if $\langle \gamma^{\hat{\rho}}, \psi \rangle > \langle \pi^\rho, \varphi \rangle$ for any $\varphi \in C_\rho$ and any $\psi \in C_{\hat{\rho}}$. Pick y an any constant act, then by definition $y \in C_\rho \cap C_{\hat{\rho}}$, since by step 1 there exists a unique probability representing the temptation relation on $C_{\hat{\rho}}$ we have necessarily $\gamma^{\hat{\rho}} = \pi^{\hat{\rho}}$. Moreover by step 2 the unicity of probability representing the temptation relation between C_ρ and $C_{\hat{\rho}}$ implies $\gamma^{\hat{\rho}}_{\varphi} = \gamma^{\hat{\rho}}_{y} = \pi^{\hat{\rho}}$, hence for any $\varphi \in C_\rho$ and any $\psi \in C_{\hat{\rho}}$

$$\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \langle \pi^{\hat{\rho}}, \psi \rangle > \langle \pi^\rho, \varphi \rangle.$$

Proof of Step 4: Take 1_E where $E \subset S$. If $1_E \in C_\rho \cap C_{\hat{\rho}}$ then by Step 3 we get $\pi^\rho(E) = \pi^{\hat{\rho}}(E)$. If not define $\upsilon : 2^S \to [0, 1]$ a follows:

$$\upsilon(E) = \pi^\rho(E)$$

for any ρ such that $1_E \in C_\rho$. We remark that υ is monotone, indeed let $E \subseteq F$ there exists some $\hat{\rho}$ such that $1_E, 1_F \in C_{\hat{\rho}}$. Thus $\upsilon(E) = \pi^{\hat{\rho}}(E) \leqslant \pi^{\hat{\rho}}(F) = \upsilon(F)$. By definition $\upsilon(\varnothing) = 0$ and $\upsilon(S) = 1$. Hence υ is a capacity. Suppose that $F = E \cup \{\hat{s}\}$ for some $\hat{s} \notin E$. Then for any $\hat{\rho}$ with $1_E, 1_F \in C_{\hat{\rho}}$, we have $\upsilon(F) - \upsilon(E) = \pi^{\hat{\rho}}(\hat{s})$. Thus for any $\varphi \in B_0(u)$ and any ρ such that $\varphi \in C_\rho$,

$$\langle \pi^\rho, \varphi \rangle = \varphi(s_{\rho(1)})\upsilon(E^\rho_1) + \sum_{k=2}^{|S|} \varphi(s_{\rho(k)}) \Big[\upsilon(E^\rho_k) - \upsilon(E^\rho_{k-1}) \Big]$$

where $E^\rho_k := \{s_{\rho(1)}, \ldots, s_{\rho(k)}\}$. This expresion is the Choquet-Expected value of φ with the capacity υ. Hence by Step 3 we have for $\varphi \in C_\rho$ and $\psi \in C_{\hat{\rho}}$, $\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \langle \pi^{\hat{\rho}}, \psi \rangle > \langle \pi^\rho, \varphi \rangle$, thus ψ tempts φ if and only if:

$$\psi(s_{\hat{\rho}(1)})\upsilon(E^{\hat{\rho}}_1) + \sum_{k=2}^{|S|} \psi(_{\hat{\rho}(k)}) \Big[\upsilon(E^{\hat{\rho}}_k) - \upsilon(E^{\hat{\rho}}_{k-1}) \Big] > \varphi(s_{\rho(1)})\upsilon(E^\rho_1) + \sum_{k=2}^{|S|} \varphi(s_{\rho(k)}) \Big[\upsilon(E^\rho_k) - \upsilon(E^\rho_{k-1}) \Big].$$

By lemma 3 we know that $\{\varphi\} \succ \{\varphi, \psi\} \Leftrightarrow \{f\} \succ \{f, g\}$ for $f, g \in \mathcal{F}$ such that $u(f)(s) = \varphi(s)$ and $u(g)(s) = \psi(s)$ for all $s \in S$, thus

$$\{f\} \succ \{f, g\} \Leftrightarrow I(g) = \int_S u(g) d\upsilon > I(f) = \int_S u(f) d\upsilon,$$

as desired, moreover $I(\cdot)$ is continuous. This concludes the proof of theorem 1.

Appendix 3: Proof of Proposition 1-3

Proof of proposition 1:

Take f and g which are CEO and such that $\sum_{s\in S} u(f) > \sum_{s\in S} u(g)$, hence we have $\{f\} \succ \{g\}$. For simplicity assume first that $\max_{s\in S} f(s) \sim \max_{s\in S} g(s)$ and $\min_{s\in S} f(s) \sim \min_{s\in S} g(s)$. We get

$$\{f\} \asymp \{f, \max_{s\in S} f(s)1_s + \min_{s\in S} f(s')1_{s'}\}$$

$$I(f) = \upsilon(s)u(f(s)) + (1-\upsilon(s'))u(f(s'))$$

Proof of proposition 2:

$(i) \Rightarrow (ii)$: From Theorem 1 $\asymp$ on $\mathcal{F}\times\mathcal{F}$ is equivalent to $\{f\} \succ \{g\}$ and $I(f) = I(g)$. Clearly if the temptation relation satisfied **A5"** then $\asymp$ satisfied **A5"**. Therefore from Theorem 1 we get $I(f) = I(g) = I(h)$ and $I(\alpha g + (1-\alpha)h) \geqslant I(f) = I(g) = I(h)$. In particular since $I(g) = I(h)$ we have

$$I(g+h) = I(2(\frac{1}{2}g + \frac{1}{2}h)) = 2I(\frac{1}{2}g + \frac{1}{2}h) \geqslant I(g) + I(h) \tag{18}$$

Let E, F two subsets of S, and assume *w.l.o.g* that $\upsilon(E) \geqslant \upsilon(F)$. Then there exists $\theta \geqslant 1$ such that $\upsilon(E) = \theta\upsilon(F)$. Therefore $I(1_E) = \upsilon(F) = \theta\upsilon(F) = I(\theta 1_F)$. From (19) we have $I(1_E+\theta 1_F) \geqslant I(1_E)+I(\theta 1_F)$. However we remark that $1_E + \theta 1_F = 1_{E\cap F} + (\theta-1)1_F + 1_{E\cup F}$. Moreover $1_{E\cap F}, (\theta-1)1_F$ and $1_{E\cup F}$ are comonotonic, so we obtain

$$\begin{aligned}
I(1_{E\cap F} + (\theta-1)1_F + 1_{E\cup F}) = I(1_E + \theta 1_F) &\geqslant I(1_E) + I(\theta 1_F) \\
I(1_{E\cap F}) + I((\theta-1)1_F) + I(1_{E\cup F}) &\geqslant I(1_E) + I(\theta 1_F) \\
\upsilon(E\cap F) + (\theta-1)\upsilon(F) + \upsilon(E\cup F) &\geqslant \upsilon(E) + \theta\upsilon(F) \\
\upsilon(E\cap F) + \upsilon(E\cup F) &\geqslant \upsilon(E) + \upsilon(F)
\end{aligned}$$

So υ is a convex capacity. The proof of $(ii) \Rightarrow (i)$ is obvious.

Proof of proposition 3:

The proof is similar to proof of proposition 2.

Appendix 4: Proof of Theorem 2

Lemma 15. *For all $f, g, h \in \mathcal{F}$ with $\{f\} \succ \{f, g\} \succ \{g\}$ then $CEU(h) \leqslant CEU(g) \Rightarrow \{f, h\} \succcurlyeq \{f, g\}$*

Proof. The first case where $\{h\} \succcurlyeq \{g\}$, by Theorem 1 we have $EU(f) > EU(g)$ and $CEU(g) > CEU(f)$. Take h' in the comonotonic cone of h (denoted $C(h)$). We have two case: there exists h' comonotonic to h such that $\{h'\} \succ \{h', h\}$, or there don't exists h' in comonotonic cone of h who is tempted by h.

Case 1: There exists $h' \in C(h)$ such that $\{h'\} \succ \{h', h\}$.

By Theorem 1 $CEU(h') < CEU(h)$ and $EU(h') > EU(h)$. By hypothesis we get $EU(h'\alpha h) > EU(g)$ and $CEU(g) > CEU(h'\alpha h)$ and by Theorem 1 $\{h'\alpha h\} \succ \{h'\alpha h, g\}$. Therefore by Temptation-Aversion axiom $\{f, h'\alpha h\} \succcurlyeq \{f, g\}$ for all $\alpha \in (0, 1)$, by continuity as $\alpha \to 0$, $\{f, h\} \succcurlyeq \{f, g\}$[13].

Case 2: There don't exists $h' \in C(h)$ such that $\{h'\} \succ \{h', h\}$.

This implies that $CEU(h) = EU(h)$ on $C(h)$. Suppose that $\{h\} \succ \{g\}$ and take h' such that $\{h\} \succ \{h'\} \succ \{g\}$, hence we get $CEU(h') < CEU(g)$, moreover $EU(h'\alpha h) > EU(g)$ and $CEU(g) > CEU(h'\alpha h)$, therefore Theorem 1 $\{h'\alpha h\} \succ \{h'\alpha h, g\}$ and Temptation-Aversion implies $\{f, h'\alpha h\} \succcurlyeq \{f, g\}$, as $\alpha \to 0$ we get $\{f, h\} \succcurlyeq \{f, g\}$. Now suppose that $\{h\} \sim \{g\}$, Take h' comontonic to h and such that $\{h\} \succ \{h'\}$, then $CEU(h) > CEU(h')$, thus $CEU(h\alpha h') < CEU(g)$ and $EU(g) > EU(h\alpha h')$ and theorem 1 implies that $\{g\} \sim \{g, h\alpha h'\} \succ \{h\alpha h'\}$, by temptation aversion we get $\{f, h\alpha h'\} \succcurlyeq \{f, g\}$. Take $\alpha \to 1$ and we obtain $\{f, h\} \succcurlyeq \{f, g\}$.

Next suppose that $\{g\} \succ \{h\}$, if $\{g\} \succ \{g, h\}$ we have by Theorem 1 $CEU(h) > CEU(g)$ which contradict the assumption, hence Set-Betweenness implies $\{g\} \sim \{g, h\} \succ \{h\}$. By Temptation-Aversion $\{f, h\} \succcurlyeq \{f, g\}$. □

Define the correspondance $L : I(\mathcal{F}) \rightsquigarrow \mathcal{F}$ by $L(l) := \{g : I(g) \leqslant l\}$. By continuity of I, $L(l)$ is nonempty and compact set for each l. Define the self-control cost function by

$$c(f, g) = \max\left[0, \max_{g \in L(l)} \left\{W(\{u(f)\}) - W(\{u(f), u(g)\})\right\}\right]$$

Lemma 16. *The conditions $(i) - (vi)$ hold:*

(i)For any f, l, if $\{f\} \succ \{f, g\} \succ \{g\}$ for some g with $CEU(g) = l$, then $c(f, l) = u(f) - W(\{f, g\})$

(ii) For any f, l, if $\{f\} \succ \{f, g\}$ for some $g \in L(l)$, then $c(f, l) > 0$

(iii) For any f, l, if $l \leqslant CEU(f)$, then $c(f, l) = 0$

(iv) If $u(f) > u(g)$ and $l = \max_{\{f,g\}} CEU$, then $CEU(f) < CEU(g) \Leftrightarrow c(f, g) > 0$

(v) For any f, $c(f, \cdot)$ is weakly increasing

(vi) The function c is continuous

Proof. See Lemma 3 of Noor and Takeoka (2010) □

[13] See claim 2 in Appendix 1

Lemma 17. *For all $f, g \in \mathcal{F}$,*

$$W(\{f,g\}) = \max_{h \in \{f,g\}} \left\{ u(h) - c\left(h, \max_{\{f,g\}} CEU\right) \right\}. \tag{19}$$

Proof. See Lemma 4 of Noor and Takeoka (2010) □

Lemma 18. *For all finite menus $A \in \mathcal{M}(\mathcal{F})$,*

$$W(A) = \max_{h \in A} \left\{ u(h) - c\left(h, \max_{A} CEU\right) \right\}. \tag{20}$$

Proof. See Lemma 5 of Noor and Takeoka (2010) □

Lemma 19. *For all $A \in \mathcal{M}(\mathcal{F})$, W can be written as the desired form.*

Proof. By lemma 0 of GP (2001), there exists a sequence of subset A^n of A such that each A^n is finite and $A^n \to A$ in the Hausdorff metric. By lemma 8,

$$W(A^n) = \max_{h \in A^n} \left\{ u(h) - c\left(h, \max_{A^n} CEU\right) \right\}. \tag{21}$$

Since by (vi) of lemma 5 c is continuous, the maximum theorem implies that the right-hand side of (18) converge to

$$\max_{h \in A} \left\{ u(h) - c\left(h, \max_{A} CEU\right) \right\}. \tag{22}$$

On the other hand, by (ii) of lemma 1, $W(A^n) \to W(A)$. This conclude the proof of Theorem 2. □

Appendix 5: Proof of Theorem 3

Lemma 20. $c(f, \max_{g \in A} v(g)) = c(f + \theta 1_S, \max_{g \in A} v(g + \theta 1_s))$

Proof. By lemma ?? $W(f + \theta 1_S, g + \theta 1_S) = W(f, g) + u(\theta 1_S)$, so we have

$$u(f + \theta 1_S) - c(f + \theta 1_S, \max_{g \in A} v(g + \theta 1_S)) = u(f) - c(f, \max_{g \in A} v(g)) + u(\theta 1_S),$$

by the additivity of u we get $u(f + \theta 1_S) = u(f) + u(\theta 1_S)$, therefore

$$c(f + \theta 1_S, \max_{g \in A} v(g + \theta 1_S)) = c(f, \max_{g \in A} v(g))$$

□

Moreover the function $c(f, v(g)) = c(v(f), v(g))$. Indeed suppose that $CEU(f) \leqslant CEU(f')$, then by lemma 15 we get $\{f', h\} \succcurlyeq \{f, h\}$.

$$c(f + \theta 1_S, v(h + \theta 1_S)) = c(f, h)$$

, thus

$$c(f + \theta 1_S, v(h)) \leqslant c(f + \theta 1_S, v(h + \theta 1_S)) = c(f, h),$$

therefore we can rewrite $c(f, v(h))$ as $c(v(f), v(h))$.

Lemma 21. *Let $\{f\} \succ \{f, g\} \succ \{g\}$ and $\{f'\} \succ \{f', g'\} \succ \{g'\}$. (i) and (ii) holds*

(i) $v(g') \geqslant v(g) \Rightarrow c(v(f), v(g')) \geqslant c(v(f), v(g))$

(ii) $v(f) \geqslant v(f') \Rightarrow c(v(f'), v(g)) \geqslant c(v(f), v(g))$

Proof. (i) follows directly of the definition of the function $c(f, v(g))$. Turn to (ii) by lemma 20 we know that

$$c(v(f'), v(g)) = c(v(f') + \theta 1_S, v(g) + \theta 1_S).$$

Since $v(f) \geqslant v(f')$, putting $\theta = [v(f) - v(f')]1_S$ we get $v(f) = v(f') + \theta 1_S$. So,

$$c(v(f'), v(g)) = c(v(f') + \theta 1_S, v(g) + \theta 1_S) = c(v(f), v(g) + \theta 1_S).$$

Since $v(f) \geqslant v(f')$ we have that $\theta \geqslant 0$, so $v(g) + \theta 1_S \geqslant v(g)$. By (i) it's comes

$$c(v(f'), v(g)) = c(v(f), v(g) + \theta 1_S) \geqslant c(v(f), v(g)).$$

□

Lemma 22. *Let $\{f\} \succ \{f, g\} \succ \{g\}$ and $\{f'\} \succ \{f', g'\} \succ \{g'\}$, then*

$$v(g') - v(f') \geqslant v(g) - v(f) \Rightarrow c(v(f'), v(g')) \geqslant c(v(f), v(g))$$

Proof. Clearly we have $v(g') - v(f') \geqslant v(g) - v(f) \Leftrightarrow v(g' + \theta 1_S) - v(f' + \theta 1_S) \geqslant v(g) - v(f)$. Putting $\theta 1_S$ such that $v(g') + \theta 1_S = v(g)$ then $v(f') + \theta 1_S \leqslant v(f)$, we have by (ii) of lemma 21:

$$c(v(f' + \theta 1_S), v(g' + \theta 1_S)) = c(v(f' + \theta 1_S), v(g)) \geqslant c(v(f), v(g))$$

□

We can conclude the proof of theorem 3. By lemma 22 we have

$$v(f') - v(g') = v(f) - v(g) \Leftrightarrow c(v(f'), v(g')) = c(v(f), v(g))$$

So the following function:

$$c(f, v(g)) = \phi(v(g) - v(f))$$

is well defined.

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