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**Oskar Lange and the influence of Walrasian theory
during the Keynesian revolution**

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1. Introduction

The growing reference to the Walrasian theory is a striking feature of the history of contemporary macroeconomics. This is particularly paradoxical as far as Keynesian theory is concerned. Keynes was trained as a Marshallian economist and, except for its general equilibrium perspective, the model of his 1936 *General Theory* had nothing to do with Walras' work. Yet, a few years after the publication of Keynes' book, a number of economists began to present his model as a particular case of the Walrasian model. This idea appeared in Klein (1947) or in Modigliani (1944) for instance. As I showed in a preceding paper (Rubin, 2004), if the authors of the neoclassical synthesis widely believed that *IS-LM* was linked in some way to the Walrasian model, they hardly went further than general statements remaining unaware of the non-Walrasian character of their macro models. Only Patinkin's *Money, Interest and Prices* (1956) systematically attempted to relate *IS-LM* and the Walrasian model. Patinkin's option had no sequel in the 1960's but the route he opened was taken later. In the 1970's, "disequilibrium" theorists (e.g., Barro and Grossman or Benassy) returned to the task of constructing a Keynesian model starting from the Walrasian one. This attempt was short-lived, as it was dethroned by the new classical revolution that marked the real takeoff of a purely Walrasian macroeconomics.

This paper examines the way Oskar Lange related Keynes' *General Theory* and The Walrasian general equilibrium theory during his American period from 1936 to 1944. Our aim is to show that his works were instrumental in imposing the belief that the foundations of Keynes' theory were to be found in the general equilibrium theory. In order to proceed with our subject we will first clarify the role attributed to the Walrasian approach in Lange's contributions prior to 1938 (section 2). In 1938, Lange published "The rate of interest and the optimum propensity to consume". In this paper he presented an *IS-LM* type interpretation of the *General Theory* and maintained that Keynes' theory was a simplified version of Walras' theory (section 3). Yet, Lange did not pursue this line of thought in the following years. In "Say's law, a restatement and criticism" (1942) and in *Price Flexibility and Employment* (1944), he developed a way of wedding Keynes' contribution and the general equilibrium theory inspired by Hicks' *Value and Capital* (1939). This piece of work can be read as a proposed alternative to *IS-LM* macroeconomics (section 4). In my last section (section 5) I try to assess the influence of Lange's contributions on the generation of economists who elaborated mainstream macroeconomics in the 1950s and 1960s. This leads me to emphasize the connection between Lange and Patinkin.

2. The place of Walrasian theory in Lange's fight for socialism (1932-1937)

In studying Lange's contribution to the birth of macroeconomics one cannot abstract from his political stance and from the questions motivating his works in the period preceding the establishment of a communist regime in Poland. Starting from Lange's commitment to socialism, we will clarify the role he attributed to Walrasian theory in his theoretical contributions prior to 1938.

2.1. Vision and Utopia

Lange's vision of capitalism was made clear in a text co-authored by Marek Breit and published in Polish in 1934 and in Lange (1935). Capitalism was based on technical progress. The latter was necessary to maintain the "reserve army of unemployed" and preserve low wages and high profits (1935: 198). But technical progress led to the concentration of industry. Imperfect competition in turn introduced "rigidities and inelasticities" destroying the

adjustment mechanisms prevailing under perfect competition. Whereas crises were transitory during the 19th century, capitalism was now prone to protracted crises as evidenced by the events of the 1930s:

“Capitalist crises are becoming more severe and protracted, and the way out of the crisis is increasingly difficult using the means appropriate to the capitalist economy. In this way, in monopoly capitalism, capitalist crisis are turning into a crisis of capitalism, an inability of capitalism to function anymore” (1934: 52).

In “Marxian economics and economic theory” (1935) Lange claimed that the Marxian analysis of the long run tendencies in the development of the capitalist system was a clear element of superiority with regards to modern (marginalist) economics. In the list of tendencies put forward by Lange, “the increase of economic instability in the capitalist system, which by destroying the economic and social security of the population of capitalist countries, causes them to rebel against the existing economic system, whatever the ideology and program underlying this rebellion (Socialism or Fascism)” (1935: 190) seems the most relevant.

Lange’s commitment to the establishment of a socialist organization of the economy illustrated by his participation to the socialist calculation debates is famous. This was his utopia. In one of his last American papers, Lange defined the socialist economy as “an economic organization where production is done by public services operated for the satisfaction of the wants of the community” (1945-6: 28). In a capitalist system, the pursuit of money profits dominates the satisfaction of collective wants. For this reason, a socialist system should enhance the welfare of the masses:

“Can socialist therefore increase the economic welfare of the masses? Yes it can.” (Breit and Lange, 1934:66)

At the scientific level, Lange’s vision and utopia led to two different issues. The first issue was the explanation of the malfunctioning of a capitalist system in order to justify its abandonment. How could one account for the increasing instability of the capitalist system? The second issue was the economic organization of a socialist system.

2.2. From Marx to marginalism

In order to deal with his main issues, Lange had to choose his scientific tools. At this stage, Lange departed from his Marxian standpoint. According to him, the best tools were offered by “modern economics”. This claim was the subject of “Marxian Economics and Modern Economic Theory”.

The paper was an answer to Shibata, a Japanese Marxist. Shibata contended that Walrasian general equilibrium theory failed on two accounts. First, it could not “make clear” the present organization of capitalism. Second, it could not explain the laws of its development. Lange conceded the validity of the second criticism. Because bourgeois economics did not specify the institutional data of his economic systems, it could not grasp the tendencies deriving from the specific institutions of capitalism. Yet, Lange rejected firmly Shibata’s first criticism. Actually, it was Marxian economics and not “bourgeois” economics which could not “grasp the phenomena of the every-day life of a capitalist economy”. And even worse, it could not be used to conceive the organization of a socialist system:

« What can Marxian economics say about monopoly prices? What has it to say on the fundamental problems of monetary and credit theory? What apparatus has it to offer for analyzing the incidence of a tax, or the effect of a certain technical innovation on wages? And (irony of Fate!) what can Marxian economics contribute to the problem of the optimum distribution of productive resources in a socialist economy? (...). On the other hand,

'bourgeois' economics is able to grasp the phenomena of the everyday life of a capitalist economy in a manner far superior to anything the Marxists can produce » (1935: 191).

Lange's position seems to stem from his appreciation of Marx' theory of value. Like marginalist price theories, the labour theory of value was a "static theory of general economic equilibrium". But, unlike more modern theories, it was only valid when applied to a perfectly competitive economy of independent producers or Smith "primitive state". Once capital and profits entered the scene, the theory stumbled on the transformation issue: "In fact, the adherence to an antiquated form of the theory of economic equilibrium is the cause of the inferiority of Marxian economics in many fields" (1935: 195-6). Besides, efforts to explain the difficulties involved in the accumulation of capital developed by Marxian economists using Marx' reproduction schemes had not "reached any definite conclusions" (Lange, 1937, note 2: 130)¹.

2.3. An eclectic marginalist

Lange was an eclectic marginalist. Modern economic theory was not tantamount to the Lausanne School in his view. In 1935 he referred to "the Austrian School, Pareto and Marshall" or "Wieser, Boehm-Bawerk, Pareto or even Marshall". Yet, Lange drew mainly on the contribution of the Marshallian and the Walrasian Schools. As stated by Lendgel (2001), Lange saw the tools provided by these schools as complementary. Now we need to clarify how Lange divided the labor between Marshall and Walras until 1937.

Considering the papers published by Lange until 1937, one can see that he referred to the Walrasian theory only insofar as stable systems and the organization of a socialist economy were at stake. In other words, the Walrasian framework was relevant to discuss the properties of well-functioning or stable systems. The general equilibrium theory was fundamental for Lange because it showed the efficiency of an ideal market system, that is to say a perfectly competitive one. This idea was the starting point of Lange's discussion in "On the theory of socialism I" (1936). Walras' tâtonnement, in particular, was mentioned in relation to a stable system:

« Such is the theoretical solution of the problem of equilibrium on a competitive market. Now let us see how the problem is solved actually by trial and error. (...). As Walras has so brilliantly shown this is done by a series of successive trials (tâtonnements) » (1936: 59).

When Lange dealt with the defects of the capitalist system, at this stage, he referred to the works of Marshallian economists. This is particularly apparent in the second part of his essay "On the theory of socialism". In the first part, Lange showed that the outcomes of a well organized socialist economy would be entirely similar to the outcomes of a perfectly competitive market system. In the second part, he explained why a socialist economy would be superior to actual market systems. In this respect he mentioned Pigou's concept of externality, the works of Chamberlin and Robinson on imperfect competition and, finally, the contribution of Robertson (1926) and Keynes (1936):

« Further, as Mr Robertson has already shown, and Mr Keynes has elaborated in his analysis of the factors determining the total volume of employment, in a capitalist economy the public's attempt to save may be frustrated by not being followed by an appropriate rate of investment, with the result that poverty instead of increased wealth results from the people's propensity to save » (1937: 127)

This division of labor between Marshall and Walras before 1937 shows no overlap. At this stage one cannot see how Keynes could meet Walras within the works of Lange.

¹ On the limits of the theories to which Lange referred see Tutin (2000).

3. Keynes' model and the Walrasian system

Immediately after the publication of the *General Theory* there were many attempts to restate Keynes' theory under the form of a simultaneous equations system, a fact that came to be obscured by the success of Hicks (1937) and of his *SI-LL* model². Among the economists engaged in this quest, Lange (1938) was probably the first to link Keynes *General Theory* and Walras' general equilibrium theory (at least in print). Actually, he went further since what he contended was that Keynes' "apparatus" was Walrasian.

3.1. No interpretation of Keynes as a Walrasian economist before 1938

Before dealing with Lange's contribution, something must be said of other *IS-LM* interpretation of Keynes and in particular of Hicks' paper, "Mr Keynes and the Classics". For this contribution has often been accused of reading Keynes through the lens of the Walrasian theory.

Most of the economists who tried to translate the *General Theory* in mathematical form were trained in the Marshallian tradition and had no Walrasian background. They simply used Keynes' book or the lectures he gave in 1933. The perfect example is the Australian economist Reddaway. In 1936, Reddaway published a review of the *General Theory* in an Australian journal, *The Economic Record*, containing an equational representation of Keynes' central message. This came out of his reading of a copy of the book he had been given by Keynes' himself and that he had brought with him on the boat taking him back to Australia. As he told in an interview with Young: "I wanted to make sure that the thing was the right number of equations for the right number of unknowns and so I scribbled this thing down" (Young, 1987: 76). Reddaway illustrates the fact that the equations of the *IS-LM* model were taken from Keynes' book³ and had nothing to do with the works of the Lausanne school.

Hicks differed from Reddaway or Harrod. He knew Walras and was in the process of writing *Value and Capital*, a book considered of having introduced the anglo-saxon world to the general equilibrium theory. Nevertheless, there was no reference to Walras or to the Lausanne school in "Mr Keynes and Classics". Hicks mentioned Wicksell, a reader of Walras, but only to remark that the *IS-LM* framework could account for cumulative inflation when the *IS* and *LM* curves were both horizontal. The paper explicitly identified Keynes' theory as a contribution to the Marshallian tradition. Hicks' paper compared Keynes' model to a Classical model. The latter was said to descend from "Ricardo and Marshall" and was the result of further qualifications by the "successors of Marshall". Keynes' model now was only a qualified version of the Classical model (1937: 150). According to Hicks, as soon as Keynes introduced income into the liquidity preference equation, "his theory became hard to distinguish from the revised and qualified Marshallian theories" (1937: 153). Hicks maintained this position in *A Contribution to the Theory of the Trade Cycle* (1950):

"[O]ne must never forget that the General Theory is in essentials a formalization (and sometimes over-formalization) of the great Cambridge tradition in monetary economics, which descends from Marshall to Keynes, not without significant contributions from Pigou and Lavington, Robertson and Kahn." (1950: 4)

In his 1937 paper, Hicks extracted the simultaneous system of equation contained in the *General Theory* and rightly emphasized its Marshallian roots. But he failed to stress the novelty of Keynes' general equilibrium perspective. In opposition to what Hicks suggested there was no mathematical model expressing the interaction between the money and the goods

² See Warren Young (1987).

³ Dimand (2007) recalls that Keynes was the first to present his views using a simultaneous equations system in a lecture given in 1933.

markets in the works of Marshallians before 1936. The originality of this general equilibrium perspective was the characteristic that captured Lange's attention.

Leontief reading of Keynes must also be considered. In "The fundamental assumption of Mr. Keynes' monetary theory of unemployment" (1936), he assessed Keynes' assault on orthodoxy. For him, orthodoxy meant "the framework of a classical Walrasian system" (1936: 196). According to Leontief, Keynes' new theory differed from the orthodox theory mainly insofar as it repudiated the homogeneity postulate or the assumption that "all supply and demand functions, with prices taken as independent variables and quantity as a dependent one, are homogeneous functions of the degree zero" (1936: 1993). Yet, Keynes offered no evidence that the homogeneity postulate was invalid and thus his case had yet to be proven. Leontief's reading of the *General Theory* through the lens of the Walrasian framework seems to imply that he considered Keynes' theory as a special case of the Walrasian orthodoxy. But in latter papers he distinguished Keynes' approach and the Walrasian approach, the former being weakened by "the artificial conventions of Marshallian short run analysis" (1947: 240). Keynes' theory could not be subsumed under the Walrasian approach because of its Marshallian methodological traits. The main point of divergence was the treatment of dynamics which, in the Marshallian tradition, involved "selected omission of some of the relevant relationships, on the one hand, and treatment as independent of some of the really dependent variables, on the other" (1947: 239). In this respect, concluded Leontief, Keynes' "short-run analysis [was] related to a truly dynamic approach in the same way as the, also Marshallian, partial equilibrium theory stands in respect to the Walrasian general equilibrium analysis" (ibidem).

3.2. *IS-LM* as the aggregate version of a Walrasian system

I begin with a presentation of Lange's version of *IS-LM*. Like Hicks, Lange developed a general version of the model that he called "a general theory of interest". The main specificities of this model were that income was measured in real terms ("in terms of wage-units, or of any other numéraire") and that investment depended on consumption. Its equations were the following:

$$M = L(i, Y) \quad (1)$$

$$C = \varphi(Y, i) \quad (2)$$

$$I = F(i, C) \quad (3)$$

$$Y \equiv C + I \quad (4)$$

$$Q = wM \quad (5)$$

The equations and the symbols are very similar to the textbook fixed-price version of *IS-LM*. Note that M is the real supply of money and Q the quantity of money. If everything is measured in wage-units, M is the quantity of money Q divided by the money wage w . Lange used this model to show that, in the general case, a rise in the marginal efficiency of capital or investment would raise the rate of interest and a decline of the propensity to consume (a rise of savings) would lower the rate of interest. Then he introduced Mr. Keynes' theory and the traditional theory characterised respectively by equations (1a) and (1b):

$$M = L(i) \quad (1a)$$

$$M = L(Y) \quad (1b) \text{ (or } M = kY)$$

In the Keynesian case variations of I and S had no effect on i , hence equilibrium was obtained through variations of Y alone. The reverse was the case with the "traditional theory". Just like Hicks, Lange concluded:

“Thus both the Keynesian and the traditional theory of interest are but two limiting cases of what may be regarded to be the general theory of interest” (Lange, 1938: 20).

Lange recognized explicitly that his general model came from Keynes:

“By introducing liquidity preference into the theory of interest Mr. Keynes has provided us with an analytical apparatus of great power to attack problems which hitherto have successfully resisted the intrusion of the economic theorist” (Lange, 1938: 12).

But then, he asserted boldly that the main ingredients of *IS-LM* had been previously discovered by Walras:

“It is a feature of great historical interest that the essentials of this general theory are contained already in the work of Walras” (Lange, 1938: 20).

This contention was argued with precise references to the *Etudes d'économie politique appliquée* (1898) and to two editions of the *Eléments d'économie politique pure* (1874, 1900). At this stage, the parallel between *IS-LM* and Walras works was perfect. Lange noted that the demand for money in Walras did not depend on real income or that the determinants of investment were not as explicit as in equation (3). But the relation between Keynes and Walras was not a question of priority.

Lange contented that Keynes' model was the “Walrasian system” presented in a simplified way. More precisely, there was a relation of correspondence between the *IS-LM* model and an underlying “Walrasian system”. This idea appeared as Lange tried to clarify his notion of real magnitudes. Income and cash balances were measured in terms of wages units or in terms of a numéraire but:

“This presupposes, of course, that the ratio of the price of each commodity or service to the price of the commodity or service which is chosen as the numéraire is given. These ratios may be thought of as determined by the Walrasian or Paretian system of equation of general equilibrium. Thus index numbers are not involved in this procedure” (1938: 13).

Suppose that the underlying “Walrasian or Paretian system” contains N commodities or services and that the first one is labor, if Y is the real income in terms of wage units, in accordance with the preceding quotation it can be defined by the following equation:

$$Y = \frac{p_2}{w} q_2 + \dots + \frac{p_N}{w} q_N \quad (6)$$

As long as the ratios $p_2/w, \dots, p_N/w$ are given, a variation of Y reflects a variation of the quantities q . Of course it is difficult to see how quantities can vary independently from relative prices in a Walrasian setting. Anyhow, Lange used the assumption of an underlying Walrasian system to define the aggregate magnitudes of his *IS-LM* model. The equilibrium of this Walrasian system provided a vector of “given” relative prices.

Lange also used the underlying system to clarify the nature of the equality between aggregate income and the sum of consumption and investment. According to him this equation was an identity resulting from the aggregation of the “budget equations” of the Walrasian system:

“This is done by our identity (4) which corresponds to the sum of the budget equations in the Walrasian system and shows how expenditures on consumption and investment *determine* the total income” (Lange, 1938: 23).

Lange's last contention about the relation between the two models concerned the adjustment mechanism. In addition to its greater simplicity, an advantage of the *IS-LM* model was that it revealed how aggregate income variations adjusted investment and savings, an aspect of the tâtonnement mechanism hidden by the Walrasian presentation:

“In our system, as in the theory of Mr. Keynes, equation (4) is an identity. Whatever the investment and saving *decisions* are, the volume of total income always adjusts itself so as to equalise saving and investment *actually performed*. (...). Walras, however, treats the equality of investment and saving not as an identity but as a genuine

equation which holds true only in a position of equilibrium. Hence his investment (value of the *capitaux neufs*) and saving (excess of income over consumption) are to be interpreted as decisions which finally are brought into equilibrium by a change in the rate of interest and in total income.⁴ But this equation does not show how total income changes so as to bring saving actually performed always into equality with investment” (Lange, 1938: 22).

The last sentence illustrates my point. Lange considered the fact that income appeared as an adjustment variable in the *IS-LM* framework as an advantage over the Walrasian presentation. The assumed consistency of the two models in this respect was further clarified in footnote 4 on page 22:

“In the process of *tâtonnements* [sic] described by Walras all the prices change and thus total income changes, too” (Lange, 1938, note 4: 22).

In other words, the variations of aggregate income in the *IS-LM* model reflected the variations of commodity prices in the Walrasian models. Aggregate income was the substitute for the price vector in the aggregate behavioural functions:

“By introducing the prices of all commodities [Walras] brings income indirectly into the equation expressing the propensity to save” (1938: 21-22).

3.3. Pondering upon the consistency of Lange’s 1938 viewpoint

Lange’s viewpoint raised an obvious problem. Keynes’s theory aimed to show the possible coexistence of equilibrium and involuntary unemployment. If involuntary unemployment is defined as a situation in which trade occurs even though there is an excess supply of labor, this outcome is barely compatible with the Walrasian theory and its *tâtonnement* hypothesis. But Lange was not inconsistent on this score. On the last page of his article, he offered a definition of involuntary unemployment compatible with labor market clearing:

“If involuntary unemployment of a factor is defined by its supply being elastic, it is absent whenever the elasticity of supply is finite” (1938: 31).

Lange assumed that the supply curve of labor was horizontal for a certain level of money wage and up to a certain level of employment. As long as the demand curve cut the supply curve on its horizontal portion there was involuntary unemployment. This meant that involuntary unemployment coincided with market clearing. Hence it was compatible with a state of general equilibrium in the Walrasian meaning⁴.

How could Lange relate this underlying system with *IS-LM*? Suppose the demand for certain goods increases in the Walrasian framework. This will raise their prices. The producers will demand more labor in order to increase output. A new equilibrium will be reached with a higher level of employment but the same money wage. In terms of *IS-LM*, the rise of prices increases Y . w remains even but the p_i and the q_i increase in equation (6) above. Consumption increases but not as much as Y . M or Q/w is constant and the rate of interest increases. Hence, a new equilibrium will be reached.

The problem with this rationalization is that it refers to ingredients that are absent from Lange’s system of equations, namely the supply side of the model. If the model combined “rigid” wages and flexible prices, it should have featured an aggregate supply function depending on the price level until the “full employment” level was reached. Lange’s arguments concerning the link between and a Walrasian system and his definition of involuntary unemployment implied the assumption that prices were flexible. But the

⁴ See below, section 4.2, for Patinkin’s accurate critique of the way Lange interpreted Keynes concept of involuntary unemployment. Using De Vroey’s classification (De Vroey, 2004) one may say that Lange dealt with underemployment and not with involuntary unemployment. Employment was not at its maximum level yet unemployment implied no departure from agent’s optimizing plans.

equations of his model and his contention that “the ratio of the price of each commodity or service to the price of the commodity or service which is chosen as the numéraire is given” seem to imply that prices are fixed. Since the numéraire is labor and since its price is constant (in the underemployment zone at least), all other prices must also be constant. Lange’s definition of equation (4) as the sum of “budget equations” is even more puzzling. This statement is based on the definition of budget constraints as the equality between “individuals’ incomes” and “the sum of expenditure on consumption and investment”. Where are the incomes derived from property shares and bond holdings? What about the expenditures on bonds and cash balances? The sum of budget constraints should give Walras’ law. But where are the excess demands for labor, bonds and money? Finally, Lange’s overall interpretation of *IS-LM* was marred by a number of inconsistencies.

What was the role of Lange’s reference to the Walrasian theory in his 1938 paper? Lange presented his point concerning the connexion between *IS-LM* and the Walrasian general equilibrium theory as an anecdotal aside (“a feature of great historical interest”) to the main thread of the inquiry. But it was more important than that. First, he used this reference to assess the originality of Keynes’ *General Theory*. For Lange, the most important contribution of the book was to indicate how to develop a simplified presentation of the Walrasian system that clarified the equilibrating role of aggregate income. And this “apparatus” could be used “to attack problems which hitherto have successfully resisted the intrusion of the economic theorist” (1938: 12). Second, Lange used the reference to the Walrasian model in order to clarify the micro-foundations of his *IS-LM* model. This could have led him to formulate a research program. But Lange (erroneously) considered that the search for micro-foundations was not on the agenda since these were already available. In his view, *IS-LM* was a market clearing system just like the Walrasian system. Yet, as we have shown, the consistency between his *IS-LM* system and a Walrasian system was far from obvious. The seeds of a research program were there waiting for a critical reader to pick them up.

4. Lange’s alternative to Keynesian macroeconomics: approaching chapter 19 from a Walrasian perspective

In his 1942 and 1944 contributions, Lange developed a different approach. His 1938 paper “explored the internal logic”⁵ of Keynes’ model and used it to answer a question raised by Malthus. In “Say’s law: a restatement and criticism” and in *Price Flexibility and Employment*, he used a Walrasian approach to tackle questions raised by Keynes. I will explain why this approach may be considered as an alternative to Keynesian macroeconomics. This new step in Lange’s thinking can be attributed to his discovery of Hicks’ *Value and Capital* and Samuelson’s approach of dynamics (1941).

4.1. The *Value and Capital* hypothesis

Our conjecture is that Lange’s reading of *Value and Capital* came as a shock to him. As discussed in Rubin (2011), the last chapters of Hicks’ book considered the conditions leading to point in time instability of a capitalist economy. Hicks argued that when price expectations were elastic and the rate of interest was sufficiently low, the economic system could collapse. Hicks was actually quite uneasy concerning the place that his conclusion should occupy within a general representation of the capitalist system. Could the actual system be represented as an unstable equilibrium? Finally, he concluded that although the collapse was a very extreme case, this possibility should remain opened in a good theory of the business

⁵ Kowalik (1987).

cycle. Reading Hicks, Lange must have realized that he could use the Walrasian framework to develop a systematic discussion of the inherent instability of the capitalist system. Lange would take up Hicks discussion of unstability conditions leaving aside all of the latter's scruples concerning the realism of the unstable scenario.

Value and Capital contained more than a verbal discussion of stability conditions. It also provided a tentative mathematical demonstration of the stability of a static system. This was the promise that a truly rigorous treatment of tâtonnement was possible. This hope was enhanced by Samuelson's article on "The Stability of Equilibrium: Comparative Statics and Dynamics" (1941). Through Patinkin (1995), we know that the appendix of *Value and Capital* and Samuelson's paper were the basis of the course on mathematical economics taught by Lange at the University of Chicago in 1944.

4.2. "Say's Law a Restatement and Criticism" (1942)

The title of the 1942 essay obviously echoed Keynes rebuttal of Say's law. Yet, if, by the forties, the theme had become Keynesian, the link with the approach followed in the *General Theory* was tenuous. The main aim of the paper was to establish the dismissal of Say's law. Lange used Walras' law to define a universal glut as a situation in which a general excess supply of commodities (products and factors alike) was associated with an excess demand for money. Say's law, stating that "the total demand for commodities (exclusive of money) is identically equal to their total supply", eliminated this possibility. But this proposition, Lange showed, also implied that money prices were indeterminate. Moreover, the quantity theory of money offered no escape since its equation and Say's law were contradictory. The conclusion was straightforward:

"We have seen that Say's law precludes any monetary theory. The theory of money must, therefore, start with a rejection of Say's law" (1942, 66).

The main objective of the theory of money, as defined by Lange, was to study the "stability of the monetary equilibrium" or the conditions under which a general oversupply would be eliminated by a fall of the price level.

In the paper, Lange referred explicitly to Keynes only two times. The first time he pointed out the correspondence between one of his concepts and the concept of "user cost". The second time, he clarified the difference between his definition of unemployment and Keynes' concept of involuntary unemployment. For him, unemployment meant "an excess supply of primary factors and direct services" whereas he still interpreted Keynes involuntary unemployment as "an equilibrium position obtained by intersection of a demand and of a supply curve" (Lange, 1942: 61). This meant that general equilibrium was tantamount to full employment from his own perspective: "[Unemployment] requires, as we have seen, absence of monetary equilibrium" (ibidem). But, following Hicks (1939), Lange bypassed the existence issue. For him a theory of chronic unemployment was about the stability of general equilibrium, a serious breach with Keynes unemployment equilibrium approach:

"The objective of the theory of money is then to study the conditions under which equilibrium of total demand and of total supply of commodities (or, instead, equilibrium of total demand for cash balances and the quantity of money available) obtains and the processes by which such equilibrium is attained" (Lange 1942: 66).

Lange stressed implicitly the difference between his definition of Say's law (quoted above) and Keynes' definition when he wrote:

"Say's law, however, does not imply that the total demand and the total supply of products are identically equal. Neither does it imply an identity of the total demand and the total supply of primary factors and direct services. (...). Under Say's law an excess supply of primary factors and direct services always implies an excess demand of equal amount for products, and vice versa. This tends directly to restore equilibrium" (1942: 57-8).

Remind that Keynes defined Say's law as the identity between the aggregate supply for goods and the aggregate demand. Lange did not discuss explicitly Keynes' attack on Say's law but referred instead to the debates between Malthus and Say. His inspiration came in part from his knowledge of the early 19th century Classics⁶.

4.3. *Price Flexibility and Employment* (1944)

In the 1942 paper, the dismissal of Say's law was meant to open the way for a theory of chronic unemployment interpreted as a stability issue. This was the subject of the 1944 monograph but there it was explicitly presented as a problem rose by Keynes:

"This monograph presents a systematic investigation of the effect of price flexibility, particularly flexibility of prices of factors of production, upon employment and economic stability. According to traditional economic doctrine, unemployment is entirely due to rigidity of factor prices. Hence flexibility of these prices is regarded as desirable and is advocated as a norm of an economic policy which aims at full employment and proper allocation of resources. This view has been subject to serious criticism. Lord Keynes maintains that, under certain conditions, changes in money wage rates have no effect upon employment but influences only the level of product prices" (1944: 1).

In chapter 19 of the *General Theory*, Keynes used the model developed in previous chapters to practise a comparative static experiment. If the exogenous wage is lowered, will the new equilibrium feature more or less unemployment? The adjustment of wage rates was then represented as a sequence of unemployment equilibria with the money wage level gradually decreasing but no diminution of unemployment. Inspired by chapter 20 of *Value and Capital*, his experiment started with a system in a state of general equilibrium where all markets were cleared and all factors were fully employed. Then he introduced a rise in the supply of one factor and asked whether the adjustment of prices would restore general equilibrium.

Lange centred his analysis on the "monetary effect". This was defined as the effect of price level variation on the excess demand for money. Along the lines of his 1942 article, Lange used Walras' law to state that the value of the excess demand of money was equal to the value of the excess supply of goods⁷. A variation of the excess demand of money implied an inverse variation of the excess supply of goods. If a reduction of the price level reduced the excess demand of money it would decrease the excess supply of goods. In this case Lange called the monetary effect "positive". The monetary effect was "negative" if the excess demand for money increased; the effect was "neutral" if the excess demand for money remained the same. The monetary effect described the substitution between goods and money resulting from a change in the price of money in terms of goods. A positive effect occurred when a rise in the price of money caused agents to increase their excess demand for goods and to decrease their excess demand for money. Lange claimed that his emphasis on substitution between money and goods had been inspired by Keynes:

"The interest in the problem and the recognition of the crucial importance of substitution between money and goods were inspired by Lord Keynes" (1945: vii).

But he immediately added that "for the tools of analysis" he was heavily indebted to Hicks. Again, Keynesian issues were treated with un-Keynesian methods. Lange neither used Keynes' toolbox, nor the *IS-LM* framework directly inspired by the *General Theory*.

In the first stage of the inquiry, Lange showed that stability depended on the sign of the monetary effect. He supposed that one factor was in excess supply so that its price diminished. This triggered a substitution of other factors for this factor and an increase of output of all products using this factor, the expansion effect. If all goods were substitutable

⁶ Jonsson (1999) has pointed out the hidden connexions between Lange's 1942 contribution and his Marxian background, Neisser (1934) and Taylor's *Principles of Economics* (1909).

⁷ Lange defined goods as the sum of commodities (products and factors) and securities (stock and bonds).

“enough” all prices were led to decrease. Equilibrium, said Lange, could be restored only if some prices diminished in smaller proportion than the price of the unemployed factor. Then the substitution and the expansion effects would increase the demand for the unemployed factor. This was the case if the reduction of the price level triggered a substitution of goods for money. The demand for certain goods would increase so that their price would rise relative to the price of others.

After that, Lange turned his attention to the parameters determining the sign of the monetary effect and to its transmission to the goods market. According to Lange, the real demand for money was independent of the price level as long as the elasticity of expectations was equal to one. But if the elasticity was greater than one it could increase when the price level decreased due to intertemporal substitution effects. So, if the money supply was constant, and if there was no intertemporal substitution, the monetary effect was positive. According to Lange this was the position of the “traditional theory of equilibrium”. But the supply of money could decrease with prices and expectations could be elastic. Lange argued that even when the monetary effect was positive it may not affect the goods markets. An unequal distribution of cash balances could lead to a pure substitution between money and bonds. If uncertainty was high, Lange showed that a reduction of the short rate of interest would not affect the long rate hence investment would not increase. Then Lange enlarged his analysis to account for international trade, imperfect competition and innovations.

Lange’s conclusions were very similar to the conclusions of Keynes’ chapter 19 but with a socialist flavour added. Like Keynes, Lange concluded that, in the post world war I context, flexibility of money wages would not “result in the automatic maintenance or restoration of equilibrium of demand and supply of factors of production” (1944: 83), the “capitalist economy” had become unstable. The Marxian tonality of Lange’s historical analysis lies in his emphasis on the results of the rise of oligopolies. Oligopolies were a major source of instability. They were also responsible for the fact that innovation now resulted in technological unemployment. He also insisted on the problem raised by the “exhaustion of investment opportunities” associated with a decline of the marginal productivity of investment goods. For all these reasons, Lange proposed a policy of fixing the price of labor or of some other “important commodity”. But he also advocated the creation of “public agencies” designed to regulate oligopolistic firms or even the “socialization of the respective industries” with due reference to his essay *On the Economic Theory of Socialism* (1938). He also defended a strict control of the money supply and public interventions to increase the demand for commodities (subsidies to consumers and public investment). The book ended with a mathematical appendix using the tools put forward by Samuelson (1941) to demonstrate that fixing certain prices could restore the stability of an otherwise unstable system.

4.4. A Walrasian approach

Lendjel (2001) has shown that Lange’s classic papers “On the Economics Theory of Socialism” (1936-7) developed a conception of the formation of equilibrium that drew more on Marshall than on Walras. Lendjel notes that, in an early work (Lange, 1932), Lange stressed the sterility of the method of the Lausanne school for the analysis of concrete problems and considered that Marshall’s method of “isolation” was superior in this respect. For each problem one had to identify the best method or even try a synthesis. In brief, Lange was not a purely Walrasian economist but an eclectic marginalist with a Marxian background⁸.

Notwithstanding, in the first pages of *Price Flexibility and Employment* Lange explained that in order to disentangle the controversies surrounding the effect of wage flexibility upon

⁸ This fact appears also in Lange’s methodological writings. See Lange 1935 and 1945-6.

employment, a general framework was required. And the name of Hicks who had “provided the most up-to-date formulation of the theory of general economic equilibrium” was put forward. The main reference was no longer Walras, like in 1938, since he had been outdated by Hicks’ *Value and Capital*. Lange must have considered that the issues raised by Keynes were general and abstract enough to justify the use of the Lausanne school method. Yet this does not tell us what made his approach Walrasian.

Lange’s analysis of the formation of equilibrium is what betrays the Walrasian nature of his 1944 approach. In book V of the *Principles of Economics*, Marshall theorized the formation of the “temporary equilibrium of demand and supply” assuming that “everyone will try to guess the state of the market and govern his action accordingly”. Sellers and buyers anticipate the supply and the demand curves and, as a result, the equilibrium price of the current period. If expectations are perfect no-one trades until this price is established⁹. This contrasts with the Walrasian approach in which buyers and sellers are price takers. They cannot anticipate current prices since they have no information concerning the supply and demand curves of others. In Lange, expectations only concerned future prices and not current prices and agents were price takers. The hands of the auctioneer appear in the working of the law of supply and demand¹⁰. Even if he did not use the word in 1944 (he did in 1938), Lange treated the issue of chapter 19 as a tâtonnement issue.

4.5. Macroeconomics

According to De Vroey (1) macroeconomics belongs to general equilibrium theory but (2) it is concerned with simplified general equilibrium models; (3) it consists of mathematical models and (4) it is concerned with providing practical policy advice; (5) its models are geared towards a confrontation with the data.

For Lange, the general theory of a monetary economy defined a number of adjustment scenarios possible in unemployment states. Given its parameters a system could be stable or not. Confrontation with the data would show whether the actual economic system was stable and point the sources of instability. Once the latter were identified, the economist had to define policies geared toward modifying the parameters or designed to compensate their effects in order to stabilize the economy. Lange macroeconomics were a sub-field of the stability analysis of general equilibrium models with money.

One difference between Lange’s 1944 and macroeconomics is his insistence on the need to analyze complex systems and not simplified ones like *IS-LM* in order to settle the controversies raised by Keynes. But the macro-dimension of Lange’s work appeared in his attempt to organize the analysis thanks to the monetary effect concept and the division of goods in three categories (factors, products and bonds). In the end, Lange’s model came close to the classical version of *IS-LM* that is to say a four market model. A lot of his analysis could be cast into the language of *IS-LM* though bringing a clearer understanding of the model. This would be shown by Patinkin in *Money, Interest and Prices* (1956). The most important feature of his analysis in this respect was Lange’s treatment of the money market and of its relations with goods markets (factors, products and securities). Keynes’ theory appeared as a special case of Lange’s model in which the monetary effect was restricted to the bonds market so that a disequilibrium on the money market resulted in a variation of the rate of interest only. But his analysis also put forward factors of instability related to the heterogeneity of goods and agents. For instance, Lange insisted on the possible lack of substitutability between

⁹ “The price of 36s. has thus some claim to be claimed to be called the true equilibrium price [...] because every dealer who has a perfect knowledge of the circumstances of the market expects that price to be established” (Marshall, 1920: 278).

¹⁰ A more exhaustive test can be done using De Vroey’s distinctions (De Vroey, 1999, 2006) between the Walrasian and the Marshallian schools.

factors. A positive monetary effect could increase the demand for engineers. But the fact that their wages were higher would not lower the unemployment of unskilled workers.

Probably related to the emphasis on complex systems was the lack of a mathematical model supporting Lange's analysis. The model in the appendix had no room for elasticities of price expectations, uncertainty and the time horizon of agents, the rate or rates of interest or money. Lange did not model his temporary equilibrium system. He did not clarify agents' decision programs and actually made errors on this ground. This was probably the main weakness of his approach compared to *IS-LM* macroeconomics.

5. Influence of a discontinued research program

5.1. The uncertain reception of Lange's contributions

The confrontation between Lange and Keynes' *General Theory* resulted in the definition of two research programs. The first one was dormant in the 1938 paper and proposed to clarify the relation between the *IS-LM* apparatus and an underpinning Walrasian model. The 1942-1944 research program, in turn, offered to develop Walrasian macroeconomics defined as the analysis of the causes and remedies to the instability of a temporary general equilibrium.

The influence of the latter program was probably limited. The fact is documented by Solow:

"'Price Flexibility and Employment' was an important book for my generation of graduate students. It did not count as much as *Value and Capital* or *The Foundations of Economic Analysis* even then; and it has not worn nearly as well. But at the time it seemed more important than its later eclipse would suggest. It held out the promise of using the Hicksian version of general equilibrium theory and the dynamics of Samuelson to say things of absolutely central importance to macroeconomics. (...).

The effect wore off quickly, however, perhaps largely as a result of Milton Friedman's adverse review." (Solow, 1982)

As a matter of fact, Friedman (1946) wrote a disparaging review of *Price Flexibility and Employment* for the *American Economic Review*. Friedman called Lange's approach "taxonomic". Beginning with very general assumptions he sought to "enumerate all possible economic systems to which these functions could give rise". The result was a set of imaginary worlds of which stable systems appeared as special cases. But the fact that stable systems were special cases within the bound of theory did not mean that they were special case in reality. Harrod (1946) wrote another adverse review for *The Economic Journal*. He was actually quite sympathetic to Lange's policy conclusions and to his endorsement of Keynes' views concerning the effects of wage flexibility but he heavily criticised Lange's basic framework. The monetary effect was conceptually and theoretically flawed so Harrod proposed to revert to "Lord Keynes or some other theory" as far as monetary analysis was concerned. He also rejected Lange's dynamics.

Friedman and Harrod were both Marshallians¹¹. Their negative reviews do not explain why young economists like Solow, who were trained with *Value and Capital*, did not try to develop Lange's approach. The review by Timlin (1946), for the *Canadian Journal of Economics and Political Sciences* offers some indications. Timlin praised Lange's general framework and completely endorsed his research program:

« Further redefinition and exploration of Dr. Lange's models to allow for distribution changes and their effects might cast light on some very important problems. Manipulation of the models under varying assumptions regarding interventions could be used to form hypotheses about the relative desirability of financing public works, subsidies, or other anti-deflationary projects by various methods, and also the relative merits of various

¹¹ Consider Harrod's review of Jaffé's edition of Walras: 'Almost all those general qualities that made Marshall's *Principles* a great classic, despite the fact that its original contributions to pure theory are admittedly limited, are lacking in Walras.' (Harrod, 1956: 311-2)

methods of inflation control. More fundamentally, such manipulation could be used to form hypotheses respecting the relative desirability of monetary systems based on varying proportions of credit money and primary money and the relation of various states of expectations to stability of prices and employment in these systems.” (Timlin, 1946: 212)

Yet, she stressed the need to develop the framework and to look carefully to the data before formulating policy proposal. Like Friedman she was sceptical with regard to the way Lange jumped from theory to policy prescription. But his method of exploring theoretical scenarios could be fruitful. However, developing the kind of theory sketched by Lange was not an easy task. *IS-LM* was available and an easier way to macroeconomic analysis.

The belief in an organic link between *IS-LM* and the Walrasian model put forward in the 1938 essay had more success with the founders of the neoclassical synthesis. The idea that *IS-LM* was a simplified version of the Walrasian general equilibrium model can be found in Modigliani (1944: 46) and in Klein (1947: 56-7). But, like Lange, neither Klein nor Modigliani tried to clarify the relation between *IS-LM* and the Walrasian model. Only Patinkin, the “true heir” of Lange according to Solow (1982), really drew on the Polish economist’s legacy.

5.2. Lange influence on Patinkin’s doctoral dissertation

During his training at the University of Chicago, from 1941 to 1943, Don Patinkin discovered mathematical economics in Lange’s classes, a fact he stressed in an autobiographical paper:

“In a sequence of systematic courses, Lange brought us to what were then the frontiers of micro- and macroeconomic theory.” (Patinkin, 1995: 371)

The supervisors of Patinkin’s Ph.D. thesis were Marschak and Lewis, but the main influence was Lange. The whole work can be read as an internal criticism of Lange’s 1944 book¹².

In the first part of the thesis, Patinkin attempted to clarify the issue raised by Lange in “Say’s law a restatement and criticism” and found that *Price Flexibility and Employment* was flawed in at least two respects. Firstly, Lange assumed that the real demand for money was homogeneous of degree zero with respect to money prices (1944: 13) and extended this assumption to all excess demand functions (1944: 10). This assumption implied that the price level was indeterminate in Lange’s framework. Secondly, like Walras and Pareto, Lange failed to put money into the utility function. In order to justify the homogeneity of degree one of the bonds demand with respect to money prices, Lange wrote a maximization program in which money did not appear at all.

At this stage Lange’s research program was undermined but not condemned. Patinkin considered several solutions in order to escape the indeterminacy of the price level. One was the solution of incorporating “the real value of assets” A/P in excess demand functions. This implied that “a proportionate change in all prices causes a substitution between money and goods” (1947: 48). Here he referred to Lange. It was possible to develop the analysis undertaken in *Price Flexibility and Employment* on a sounder basis. But this approach was inconsistent with the Keynesian views developed in the second part of the thesis. The wealth effect would have undermined Patinkin’s explanation of unemployment. This solution was not acceptable at this stage¹³.

The aim of Patinkin, in the second part of his thesis, was to build a theory of involuntary unemployment. For him this amounted to clarify the relation between the *IS-LM* model and an

¹² For a detailed presentation of Patinkin’s dissertation see Rubin (2012).

¹³ In his doctoral dissertation, Patinkin presented unemployment as the result of a situation in which the full employment rate of interest was negative and could not be reached. This approach neglected the Pigou effect considered to be empirically insignificant. See Rubin (2005).

underlying Walrasian model. In other words, Patinkin took up the research program implicit in Lange's 1938 paper:

"Our attention will be concentrated on the microsystem lying behind this [macrosystem]." (1947: 88)

To begin with, he would write down a disaggregated model with a number of goods, of sectors, of firms within this sectors and a number of households. Then, he would write down the maximizing plans of firms and households from which demand and supply functions were derived in full employment states as in involuntary unemployment equilibrium states. Patinkin did not go very far along this road, to say the least, but his intention was clear.

But why did he adopt a research program that Lange would have deemed irrelevant? The reason was that he rejected Lange's conception of the relation between IS-LM and the Walrasian model. Involuntary unemployment could not be defined as a labor market clearing phenomenon. Hence, a Keynesian equilibrium and a Walrasian equilibrium were not one and the same thing.

In order to clarify the foundations of the "Keynesian macro-models", one had to modify the Walrasian system to account for the possibility of trade in markets not cleared. For involuntary unemployment implied a disequilibrium on the labor market. This led to the elaboration of a theory of choice with "additional constraints" and a "theory of compromise" showing how the volume of trade was determined whenever desired supplies and demands did not coincide. One should add that Patinkin assumed flexible prices and wages all the way. Since Keynes wanted to show that unemployment was not a consequence of wage rigidity it was better to dispense with the assumption. On this point he followed Lange's interpretation of Keynes and Lange's 1944 approach.

By rejecting a basic component of Lange's interpretation of the Keynesian theory Patinkin undermined his conception of the relation between *IS-LM* and Walras' model and activated the research program still dormant in the 1938 paper. Doing so, he anticipated the work developed in the 60's and 70's by fixed-price equilibrium theorists. Yet, Patinkin's results were disappointing. He did not find how to use his additional constraint concept and his Keynesian general equilibrium concept was heavily criticized by the members of the Cowles Commission.

After the completion of his PhD thesis, Patinkin began to work on the manuscript of *Money, Interest and Prices*. To some extent, it represented a return to the inspiration of *Price Flexibility and Employment* with its emphasis on stability issues analyzed from the perspective of the substitution between goods and money. The book appeared in 1956 and became a landmark of the neoclassical synthesis. Patinkin developed the vision of the economic system at work in Lange's 1942 and 1944 texts to analyze an *IS-LM* framework¹⁴. But for Patinkin, unemployment still implied trading and producing out of equilibrium. His analysis of this issue in terms of spill over effect would inspire Clower (1965) and Barro and Grossman (1971). Besides, Patinkin never accepted to transform macroeconomics into instability analysis. Instability was a possible yet an extreme scenario. It could not account for the normal functioning of the economy. Furthermore, Lange's demonstration of instability gave a very important role to expectations. For Patinkin very little could be said on this subject and, conversely, any result could be obtained once expectations entered the scene:

"Once the Pandora box of expectations and interest and price uncertainty is opened upon the real world of economic analysis, anything can happen." (1956: 180)

¹⁴ See Rubin (2004).

6. Conclusion

The works of Lange defined two ways of relating Walrasian general equilibrium theory and macroeconomics. His 1938 article defined macroeconomics as the use of a simplified version of the Walrasian model, for this was how Lange considered *IS-LM*. In 1942 and 1944, he developed a different kind of approach. This time a disaggregate or complex Walrasian model in which money was directly used to tackle basic issues of Keynesian macroeconomics (the validity of Say's law and of the quantity theory of money, the effect of wage variations on employment) and to diagnose the sources and the remedies of macroeconomic malfunctioning (unemployment, inflation). If the kind of Walrasian macroeconomics defended in *Price Flexibility and Employment* was not followed, Lange did influence the evolution of macroeconomics. This influence ran mainly through the contribution of Patinkin, his student at Chicago University. Our study of the relation between their works clarifies the reasons why Lange's alternative to *IS-LM* was not taken up, it shows its weaknesses, but it also reveals the insights that made their ways up to the 1970's. The defects of Lange's second approach were numerous and Patinkin was highly aware of them. First of all, Lange did not provide a mathematical model as tractable and operational as *IS-LM*. The general framework used in *Price Flexibility and Employment* was too complex to be formalized. As a result, young Keynesians all opted for the *IS-LM* model. Lange's framework also suffered from a defective integration of money. The last weakness highlighted by Patinkin was the definition of macroeconomics as the study of instability. Like Patinkin, many economists would be reluctant to picture the normal functioning of a market economy as an instable system in the Walrasian meaning, either for political reasons or, more basically, because, like Hicks, they would find that "not realistic". Besides, Lange's demonstration of instability relied on factors that receded to the back of the theoretical scene during the era of the neoclassical synthesis like the Hicksian theory of expectations and imperfect competition. Nonetheless, Lange did participate to the formation of modern macroeconomics. A major contribution was his idea of wedding *IS-LM* to a Walrasian model. Retrospectively, the wedding seems unnatural, but it proved fruitful. Patinkin's criticism of Lange's conception of the connexion between the macro and the micro models, his rejection of the horizontal supply curve of labor, led to the foundations of the disequilibrium theory developed in the 1970's. The method inspired by *Price Flexibility and Employment* helped Patinkin clarify the properties of the *IS-LM* model and showed the relevance of the Walrasian approach. Finally and ironically, Lange's influence played an important role in the contribution that convinced Lucas of the necessity of a Walrasian approach to macroeconomics¹⁵.

7. Bibliography

- Barro, R. J. Grossman, H. I. (1971). "A General Disequilibrium Model of Income and Employment". *American Economic Review* 61:82-93.
- Breit, M. Lange, O. (1934)[2003] "The Way to the Socialist Planned Economy", translated by Jan Toporowski, *History of Economics Review*: 51-70.
- Clower, R. (1965). "The Keynesian Counter-Revolution: a Theoretical Appraisal". In *The Theory of Interest Rate* edited by F. H. Hahn. F. P. R. Brechling. London: Macmillan. 103-25.
- De Vroey, M. (1999) "The Marshallian Market and the Walrasian Economy. Two Incompatible Bedfellows". .
- De Vroey, M. (2004). *Involuntary Unemployment: The elusive quest for a theory*, London, New York: Routledge.

¹⁵ See Lucas (2004: 15).

- De Vroey, M. (2006) "Getting De Vroey, M. *Involuntary Unemployment: The elusive quest for a theory*, (London, New York: Routledge 2004). rid of Keynes? A reflection on the history of macroeconomics", *Discussion paper, Département des sciences économiques de l'université de Louvain*, 51.
- Dimand, R. (2007) "Keynes, IS-LM, and the Marshallian Tradition", *History of Political Economy*, 39 (1): 81-95.
- Friedman, M. (1946) "Lange on Price Flexibility and Employment: a Methodological Criticism". *The American Economic Review*. 36 (4): 613-631.
- Harrod, R. (1946) "Price Flexibility and Employment by Oscar Lange". *The Economic Journal*. 56 (221): 102-7.
- Harrod, R. (1956) "Walras: a Re-Appraisal". *The Economic Journal*. 66 (262): 307-316.
- Hicks, J. (1937) "Mr Keynes and the Classics: a suggested interpretation". *Econometrica* 5:147-59.
- Hicks, J. (1939) [1946]. *Value and Capital. An Inquiry Into Some Fundamental Principles of Economic Theory*. Oxford: Clarendon Press.
- Jonsson, P. O. (1999) "On the origins and the legacy of Langes' laws", communication to the annual meeting History of Economics Society in Greensboro.
- Keynes, J. M. (1936) [1991]. *The General Theory of Employment, Interest and Money*. San Diego, New York, London: Harvest/Harcourt Brace.
- Klein, L. [1947a] 1966. *The Keynesian Revolution*. Second edition, New York, London: Macmillan.
- Kowalik, T. (1987) "Lange, Oskar Ryszard", *The New Palgrave Dictionary of Economics*, vol 3. London: Macmillan.
- Lange, O. (1935) "Marxian Economics and Modern Economic Theory", *The Review of Economic Studies*, 2 (3): 189-201.
- Lange, O. (1936) "On the Economic Theory of Socialism Part I". *Review of Economic Studies*. 4 (1): 53-71.
- Lange, O. (1937) "On the Economic Theory of Socialism Part II". *Review of Economic Studies*. 4 (2): 123-42.
- Lange, O. (1938) "The rate of interest and the optimum propensity to consume", *Economica*, 5: 12-32.
- Lange, O. (1942) "Say's law: a restatement and criticism", in Lange O. et al (eds), *Studies in Mathematical Economics and Econometrics*, Chicago.
- Lange, O. (1945) *Price Flexibility and Employment*, San Antonio: Principia Press of Trinity University.
- Lange, O. (1945-1946) "The Scope and Method of Economics", *The Review of Economic Studies*, 13 (1): 19-32.
- Lange, O. Taylor F. (1938). *On the Economic Theory of Socialism*. Mineapolis: University of Minnesota Press.
- Lendjel, E. (2001) ' Le tâtonnement marshallien dans les premiers écrits d'Oskar Lange '. *Cahiers d'économie politique*, 38 : .
- Lucas, R. E. (2004) "Keynote Address to the 2003 HOPE conference: My Keynesian Education". HOPE, annual supplement to volume 36: 12-24.
- Modigliani, F. 1944. "Liquidity Preference and the Theory of Interest and Money". *Econometrica* XII:45-88.
- Neisser, H. (1934) "General overproduction: a study of Say's law of markets", *Journal of Political Economy*, 42 (4): 433-465.
- Patinkin, D. (1947). *On the Consistency of Economic Models: A Theory of Involuntary Unemployment*. Doctoral dissertation. University of Chicago, August.
- Patinkin, D. (1956) *Money, Interest and Prices*. Row, Evanston: Peterson and Compagny.
- Patinkin, D. (1995) "The training of an economist". *Banco Nazionale Del Lavoro Quarterly Review*. 195: 359-95.
- Rubin, G. (2004). "Patinkin on IS-LM: An alternative to Modigliani", *History of Political Economy*, annual supplement to vol. 36: 57-91.
- Rubin, G. (2005). "Patinkin and the Pigou effect: or how a Keynesian came to accept and anti-Keynesian argument", *The European Journal of the History of Economic Thought*, 12(1): 47-72.

- Rubin, G. (2008). "Patinkin's interpretation of Keynesian economics: a genetic approach", in Leeson, R. (ed.) *Archival Insights into the Evolution of economics*. Vol. 1 *The Keynesian tradition*, Routledge: 99-125.
- Rubin, G. (2011) « Hicks et l'économie de la dépression », *Recherches Economiques de Louvain*, 77 (4), forthcoming.
- Rubin, G. [2012] « Don Patinkin's Ph.D dissertation as the prehistory of disequilibrium theories », *History of Political Economy*, 44 (1), forthcoming.
- Samuelson, P. A. (1941). "The Stability of Equilibrium: Comparative Statics and Dynamics". *Econometrica*. 9: 97-120.
- Solow, R. M. (1982) "Cowles and the Tradition of Macroeconomics", Cowles Fiftieth Anniversary Volume.
- Young, W. (1987) *Interpreting Mr Keynes The IS-LM enigma*, Cambridge: Polity Press.
- Timlin, M. (1946). "Price Flexibility and Employment. Review article". *The Canadian Journal of Economics and Political Science*. 12 (2): 204-213.