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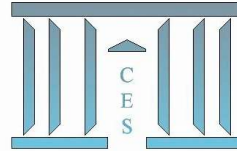
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## Can geography lock a society in stagnation?

Nguyen Thang DAO, Julio DAVILA

2013.37



# Can geography lock a society in stagnation?

Nguyen Thang DAO<sup>1</sup> and Julio DÁVILA<sup>2</sup>

## Abstract

We extend Galor and Weil (2000) by including geographical factors in order to show that under some initial conditions, an economy may be locked in Malthusian stagnation and never take off. Specifically, we characterize the set of geographical factors for which this happens, and this way we show how the interplay of the available “land”, its suitability for living, and its degree of isolation, determines whether an economy can escape stagnation.

**Keywords:** Geographical factors, loss of technology, human capital.

**JEL Classification:** O11, O33.

## 1. Introduction

Galor and Weil (2000) advanced a unified growth model to explain the transition to modern growth as the result of the interaction between population, technology, and output. In their model, the authors show that the transition from stagnation to sustained growth is an *inevitable* outcome when the driving forces for technological progress are the education and size of the population. Specifically, in Galor and Weil (2000), technological progress is assumed to appear even for zero education investments and arbitrarily small populations so that, eventually, Malthusian stagnation vanishes endogenously, leaving the arena to modern growth forces and letting thus the economy take off and converge to a modern steady state growth. In this paper we study conditions under which take-off is not inevitable, but rather stagnation is.

In order to understand how a society can be locked in stagnation it is useful to identify what exactly drives a take-off in Galor and Weil (2000). A key ingredient to the mechanism proposed there and that takes the economy out of stagnation is the positive dependence of technological progress on population. Still it is worth noting that this dependence is, nevertheless, not indispensable for an explanation. In effect, for instance Galor and Moav (2002) show that a society can still take off without having to assume a positive effect of population on technological progress. In this case, it is the composition of the population

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(in terms of the households' preferences about quality vs. quantity of their offspring) rather than the size of the population that matters in order to spur technological progress. In effect, the appearance of a fraction (even a tiny one) of "quality-loving mutant" households suffices for a society to take off in the long-run, regardless of population size, by triggering a change in the composition of the population. Still, if the population size does not matter in Galor and Moav (2002) it is because of their explicit assumption according to which the costs (not related to education) of rearing a child do not depend on the population size. Nevertheless, population density is known to have an impact on the childrearing costs that are unrelated to education. Specifically, evidence shows that when households have small dwellings, child production is more costly and households have fewer children (see De la Croix and Gosseries (2012), citing evidence from Goodsell (1937) and Thompson (1938)). It is precisely this kind of interplay between a population and its environment —and its impact on growth— what our model aims at capturing.<sup>3</sup>

In this paper, we build on Galor and Weil (2000), introducing geographical factors instead, in order to show that, under some initial conditions, an economy may be locked in stagnation, with a small population, a basic technology, and no education, *even if population size has, per se, a positive effect on technological progress* (so that the economy should eventually take off instead according to Galor and Weil (2000)). In order to show this, we take into account too the often overlooked role of technology losses in the determination (along with education investments and population size) of the technological level of the society.<sup>4</sup> The key mechanism is, in this case, that recurrent technology losses allow for technological progress only if the population size is large enough to offset them. When this is the case, the level of technology will increase until it reaches a threshold beyond which the returns to education are high enough to trigger investment in human capital, the tipping point where education kicks in and from which sustained growth obtains. Nonetheless, societies whose geographical factors cannot support a sufficiently large population never escape stagnation. This paper therefore makes stand out clearly the role of geographical factors —such as the amount of available land or, more generally, environmental resources, its suitability for living and production, and its degree of isolation— in the creation of a stagnation trap.

It is interesting to note an alternative mechanism that De la Croix and Dottori (2008) propose to explain the road to stagnation followed in Easter island in particular. In that paper the authors argue that the population collapse in Easter island was the result of a population race —that played the role of an arms race given the labor-intensive warring technology— triggered by the non-cooperative bargaining between clans about the allocation of the society's total output (in case of disagreement, a war would break out whose outcome would be determined by relative population sizes of the belligerent clans, so that in order to improve their bargaining power, each clan would increase its size to the point of jointly depleting natural resources and leading eventually the society to collapse). Therefore, in De la Croix and Dottori (2008) the conflict-driven population race is the prime cause of

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<sup>3</sup>Whether the Galor and Moav (2002) population-composition mechanism allows too for a stagnation trap when population composition itself affects the childrearing costs remains an open question.

<sup>4</sup>Diamond (1997) provides evidence that some societies show no sign of escaping stagnation on their own due to losses of technology and culture, in particular small and isolated societies. An extreme case, but by no means the only documented one (see Aiyar *et al.* (2008) for technological losses driven by population shocks and, more generally, footnote 7 below), took place on the Tasmania island. Aborigines in Tasmania were separated from mainland Australians due to rising sea level around 10.000 years ago. With a stable population of 4.000, Tasmanians had, at the time of arrival of Europeans, the simplest material culture and technology of any people in the modern world. Like mainland Aborigines, they were hunter-gatherers but they lacked many technologies and artifacts widespread on the mainland. Some technologies were brought to Tasmania when it was still a part of the Australian mainland, and were subsequently lost in Tasmania's cultural isolation. For example, the disappearance of fishing, and of awls, needles, and other bone tools, around 1500 BC (Diamond 1997, pp. 312–13). Diamond argues that a small population of 4.000 was able to survive for 10.000 years, but was not enough to prevent significant losses of technology and culture, as well as the failure to invent new technology, leaving it with a uniquely simplified material culture.

stagnation in an environment whose resources are bounded but not necessarily insufficient for sustaining take-off in the absence of conflict. On the contrary, in this paper, the cause of stagnation is a geography unable to support a population large enough to offset technology losses.<sup>5</sup>

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes its equilibria. Geographical factors under which an economy is unable to escape stagnation are studied in section 4. Specifically, we show that a society for which (i) the population level guaranteeing technological progress, and (ii) the level of technology guaranteeing education investment, imply a high enough *effective population density*, never escapes stagnation, under some initial conditions. Section 5 makes a summary and concludes the paper.

## 2. The model

### 2.1. Geographical factors

We refer by “land” to the set of geographical and environmental conditions supporting the life and economic activity of a society (obviously, living and production conditions depend on how suitable for that the ecosystem around us is). How much of this land can be put to productive use depends on the interplay of its intrinsic suitability for that purpose and the level of technology. The suitability of land captures its adequacy for people to live and work in the ecosystem as a whole, such as temperature, humidity, orography, river density, bio-diversity, etc. Typically, suitability and technological constraints prevent people to make the most of their environment, i.e. the available land. For instance, people may just occupy the part of their geographical territory that is most suitable for their lives, or may be unable to tap certain resources with the current technology. We refer to the fraction  $X_t$  of the available land  $X$  that is put to productive use at period  $t$  as “productive land” and its size depends positively on its suitability  $\theta$  and (moreover concavely) on the technological level  $A_t \geq 0$ , i.e.

$$X_t = \chi(\theta, A_t)X \tag{1}$$

with  $\chi(\theta, A_t) \in (0, 1)$ ,  $\chi_\theta(\theta, A_t) > 0$ ,  $\chi_A(\theta, A_t) > 0$ ,  $\chi_{AA}(\theta, A_t) < 0$ .

### 2.2. Production and technology

The productivity of each household in period  $t$  is determined by its human capital  $h_t \geq 0$  and the technological level  $A_t$ , so that the output per household in period  $t$  is

$$y_t = f(A_t)h_t$$

where  $f(A_t) > 0$ ,  $f'(A_t) > 0$ .<sup>6</sup>

The technological level in period  $t + 1$  is

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<sup>5</sup>Mariani et al. (2010) address a related topic noting the possibility of an environmental poverty trap in a set-up in which environmental quality and life expectancy are jointly determined. In such a setup multiple equilibria are possible according to which agents either invest in both environmental quality and longevity, or do not, which may lock an economy in an environmental poverty trap in the latter case. However, the mechanism there is basically a coordination problem where population growth and its interaction with the environment is abstracted.

<sup>6</sup>In order to make stand out clearly the importance of the interplay between population, education, and environment, we abstract from land as an input of the production function. Introducing land in the production function does not change the qualitative analysis.

$$A_{t+1} = [1 - \lambda(\omega)][1 + g_t]A_t \quad (2)$$

where  $g_t$  is the rate of technological progress in  $t$ , and  $\lambda(\omega) \in (0, 1)$  is a rate of loss of technology that depends *positively* on the degree of isolation  $\omega$  of the society, i.e.  $\lambda'(\omega) > 0$  (a higher  $\omega$  corresponds to a higher isolation),<sup>7</sup> so that  $[1 - \lambda(\omega)][1 + g_t]$  is technological growth rate between periods  $t$  and  $t + 1$ . As in Galor and Weil (2000), we assume that  $g_t \geq 0$  depends positively on the average education  $e_t \geq 0$  and the size  $L_t \geq 0$  of the working generation in period  $t$ , i.e.

$$g_t = g(e_t, L_t) \quad (3)$$

with  $g_e(e_t, L_t), g_L(e_t, L_t) > 0$ , as well as  $g(0, L_t) > 0$  and  $\lim_{L_t \rightarrow 0^+} g(0, L_t) = 0$ , so that technological progress (before losses) is positive as long as population is too.

From (??) and (??) we know that if the education of the working generation  $t$  is zero, then the economy has positive technological growth if, and only if, the population size is large enough, i.e.

$$[1 - \lambda(\omega)][1 + g(0, L_t)] > 1 \quad \Leftrightarrow \quad g(0, L_t) > \frac{\lambda(\omega)}{1 - \lambda(\omega)}$$

which implies that for positive technological growth to exist it must hold  $L_t > \underline{L}$ , where  $\underline{L}$  is the smallest population able to sustain technological progress given  $\omega$ , i.e. satisfying

$$g(0, \underline{L}) = \frac{\lambda(\omega)}{1 - \lambda(\omega)} \quad (4)$$

Applying the implicit function theorem to (??),  $\underline{L}$  is implicitly defined to be a function  $\mathcal{L}$  of  $\omega$ , i.e.

$$\underline{L} = \mathcal{L}(\omega) \quad \text{such that} \quad \mathcal{L}'(\omega) = \frac{\lambda'(\omega)}{[1 - \lambda(\omega)]^2 g_L(0, \underline{L})} > 0.$$

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<sup>7</sup>Diamond (1997) addresses the role of geography in losses of technology. He argues that technologies must not only be acquired but maintained too, which may depend on unpredictable factors, among them cultural prejudices and fads that see economically useful technologies become devalued. A famous example (see Diamond (1997), pp. 257–258) is the loss of gun production technologies in isolated Japan under the Shogunate, when the Samurai class worked against the acceptance of firearms because a cultural preference of swords as class symbols as well as works of art. Such a phenomenon could not happen lastingly (for obvious reasons) in less isolated European countries, even though similar attitudes towards guns occasionally happened. Another instance of technology losses due to isolationist attitudes is the abandonment of oceangoing navigation techniques by the Chinese under the Ming after the explorations by Zheng He. A non isolated society that temporarily turns against useful technology can reacquire it easily by diffusion from neighboring societies, while its chances to reacquire it are increasingly hampered by higher degrees of isolation.

In this paper we make abstraction of the causes of technology losses and, for the sake of simplicity, assume a rate (not necessarily large) of recurrent technology losses  $\lambda(\omega)$  that depends positively on the degree  $\omega$  of isolation of the society (alternatively this losses came made, more realistically, random, but this would not change qualitatively the results). This rate may also depend on the society's population size, education level, technological level, etc. Indeed, a larger and better educated population may be better at maintaining technological knowledge due to dissemination scale and interaction of people. These effects can however be captured in technological progress factor  $g(e, L)$  in (??) below. A high technological level itself may help a society from losing technologies in two ways: (i) through better storage devices in which to save technologies, and (ii) better communications and transportation to offset isolation. This paper, however, focuses on societies in very early stages of development without widespread literacy and modern communications.

Aiyar et al. (2008) focus on a different phenomenon of technology regress based on external shocks reducing the population in societies in which the transmission of technology is embodied in the human capital instead of recorded. They argue that, when the population shrinks, aggregate demand falls, leading to some technologies to become unprofitable at the margin. As a consequence, those out-of-use technologies are not transmitted to the next generation, and hence lost until rediscovered by chance.

In other words, the more isolated a society is, the bigger the population needed to offset technology losses and generate technological growth. For less isolated societies a smaller population suffices to offset technology losses because the latter are partially offset by technology diffusion from neighboring societies too.

An illustration of the effect of population size in offsetting technology losses is provided by the divergence in technological level between Easter island and Hawaii at the time of the arrival of Europeans in 1770s. Both islands are extremely isolated and had the same cultural background of Polynesian colonizers, but Hawaii had a bigger population (due to more resources available for living and production), which had resulted in a more advanced technology than Easter island by the time Europeans arrived.

### 2.3. Households

In each period  $t$  there is a generation of  $L_t$  identical working households. Each household lives for two periods. In the first period (say childhood)  $t - 1$  it uses up a fraction of its parent's time. In the second period (say parental)  $t$  it is endowed with one unit of time which it allocates between child-rearing and production. The preferences of the household born in period  $t - 1$  are defined over the number and quality (i.e. human capital) of its (household-)children,  $n_t$  and  $h_{t+1}$  respectively, as well as from its consumption  $c_t$  in period  $t$  as follows

$$u_t = \gamma \ln(n_t h_{t+1}) + (1 - \gamma) \ln c_t \quad (5)$$

Each household chooses a number and quality of children under the constraint of the unit of time available for child-rearing and production. The only input required to produce both child quantity and quality is time. We assume that the time to raise children physically, regardless of education investment, is decreasing in per household resources  $X_t/L_t$ .<sup>8</sup> For simplicity, we assume that the cost in time for raising  $n_t$  children physically is  $(\frac{L_t}{X_t})^\beta n_t$ , where  $\beta \in (0, 1)$ . We define  $\frac{L_t}{X_t}$  as the *effective population density*, i.e. the density of the population with respect to *productive* land only. So the opportunity cost of raising  $n_t$  children with education  $e_{t+1}$  is  $y_t n_t [(\frac{L_t}{X_t})^\beta + e_{t+1}]$ . Hence, the agent born at date  $t - 1$  maximizes at date  $t$  its utility (??) under the following budget constraint

$$y_t n_t [(\frac{L_t}{X_t})^\beta + e_{t+1}] + c_t \leq y_t \quad (6)$$

Galor and Weil (2000) assume that human capital formation of children born at date  $t$ ,  $h_{t+1}$ , depends positively on education investment  $e_{t+1}$  they receive, and negatively on the growth rate of technological progress  $g_t$  from period  $t$  to period  $t + 1$ , their rationale being that education lessens the obsolescence of human capital due to a changing technology. As a consequence, households have incentives to invest in education whenever technological progress is high enough, regardless the level of technology. It can be argued, however, that the incentives to educate their offspring depend not on technological progress, i.e. the rate of change  $g_t$  of the technological level, but on the level of technology  $A_{t+1}$  itself. In effect, in an economy with a high enough level of technology agents have incentives to educate their offspring in order to able them to make use of the technology, even in the absence

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<sup>8</sup>This idea is introduced in Goodsell (1937) and Thompson (1938), recently cited by de la Croix and Gosseries (2012) to take into account that when households have small dwellings, child production is more costly and households have fewer children.

of technological progress. Hence, we assume that human capital at  $t + 1$  is eroded by an increasing technology  $A_{t+1}$ , but it increases with education  $e_{t+1}$  with a return that is higher the higher the level of technology, i.e.

$$h_{t+1} = h(e_{t+1}, A_{t+1}) \quad (7)$$

with  $h_e(e, A) > 0$ ,  $h_A(e, A) < 0$ , and  $h_{eA}(e, A) > 0$  at interior points. It is moreover assumed that households are endowed with some human capital even in the absence of education and any technological sophistication, i.e.  $h(0, 0) > 0$ , so that production can take place. Also it is assumed (i)  $h_e(0, 0) = 0$ , i.e. education does not increase human capital in the absence of technology, (ii)  $\lim_{A \rightarrow +\infty} h(0, A) = 0$  so that an unbounded increase in the level of technology wipes out human capital in the limit, and (iii) the return to education remains bounded away from zero as technology grows unboundedly, i.e.  $\lim_{A \rightarrow +\infty} h_e(0, A) > 0$ .

### **Household's optimization**

Each household  $t$  chooses the quantity  $n_t$  and quality  $h_{t+1}$  of its offspring, as well as consumption  $c_t$ , so as to maximize its utility. From (??), (??), and (??), the optimization problem is

$$\max_{n_t, e_{t+1}} \gamma \ln [n_t h(e_{t+1}, A_{t+1})] + (1 - \gamma) \ln \left[ (1 - n_t \left[ \left( \frac{L_t}{X_t} \right)^\beta + e_{t+1} \right]) y_t \right]$$

The first-order condition (FOC) with respect to  $n_t$  gives us

$$n_t = \frac{\gamma}{\left( \frac{L_t}{X_t} \right)^\beta + e_{t+1}} \quad (8)$$

And the FOC with respect to  $e_{t+1}$  requires the following relationship between  $e_{t+1}$  and  $A_{t+1}$ ,  $\frac{L_t}{X_t}$  to hold:

$$G \left( e_{t+1}, A_{t+1}, \frac{L_t}{X_t} \right) \equiv h_e(e_{t+1}, A_{t+1}) \left[ \left( \frac{L_t}{X_t} \right)^\beta + e_{t+1} \right] - h(e_{t+1}, A_{t+1}) \begin{cases} = 0 & \text{if } e_{t+1} > 0 \\ \leq 0 & \text{if } e_{t+1} = 0 \end{cases} \quad (9)$$

**Proposition 1:** *In the economy set up above, there exists, for each effective population density, a technological level  $\hat{A}_{t+1} = \hat{A} \left( \frac{L_t}{X_t} \right) > 0$ , such that households educate their offspring if, and only if, technology exceeds that level, i.e.*

$$e_{t+1} = e \left( A_{t+1}, \frac{L_t}{X_t} \right) \begin{cases} = 0 & \text{if } A_{t+1} \leq \hat{A} \left( \frac{L_t}{X_t} \right) \\ > 0 & \text{if } A_{t+1} > \hat{A} \left( \frac{L_t}{X_t} \right) \end{cases}$$

Moreover,  $\hat{A}' \left( \frac{L_t}{X_t} \right) < 0$ .



**Proof:** We prove that, for each  $\frac{L_t}{X_t}$  there exists a unique  $\hat{A}_{t+1}$  such that  $G(0, \hat{A}_{t+1}, \frac{L_t}{X_t}) = 0$ . From the assumptions on  $h(e_{t+1}, A_{t+1})$  and the equation (??), we find that  $G(0, A_{t+1}, \frac{L_t}{X_t})$  is monotonically increasing in  $A_{t+1}$ ,

$$\frac{\partial G(0, A_{t+1}, \frac{L_t}{X_t})}{\partial A_{t+1}} = h_{eA}(0, A_{t+1})\left(\frac{L_t}{X_t}\right)^\beta - h_A(0, A_{t+1}) > 0$$

Furthermore, from the assumptions made on  $h$  it follows that  $G(0, 0, \frac{L_t}{X_t}) < 0$  while  $\lim_{A_{t+1} \rightarrow +\infty} G(0, A_{t+1}, \frac{L_t}{X_t}) > 0$ . So, there exists a unique  $\hat{A}_{t+1} > 0$ , given  $\frac{L_t}{X_t}$ , such that  $G(0, \hat{A}_{t+1}, \frac{L_t}{X_t}) = 0$ , and therefore, as it follows from (??),  $e_{t+1} = 0$  for  $A_{t+1} \leq \hat{A}_{t+1}$ .

Applying the implicit function theorem to  $G(0, \hat{A}_{t+1}, \frac{L_t}{X_t}) = 0$ , we get  $\hat{A}_{t+1} = \hat{A}(\frac{L_t}{X_t})$ , and

$$\hat{A}'\left(\frac{L_t}{X_t}\right) = \frac{-\beta h_e(0, \hat{A}_{t+1})\left(\frac{L_t}{X_t}\right)^{\beta-1}}{h_{eA}(0, \hat{A}_{t+1})\left(\frac{L_t}{X_t}\right)^\beta - h_A(0, \hat{A}_{t+1})} < 0.$$

Q.E.D.

### 3. Equilibria

We look for the initial conditions and geographical factors preventing a given economy to escape stagnation, that is to say, such that the technological level stays always below the threshold triggering education investment. The corresponding equilibria are therefore characterized by  $e_t = 0$ , for all  $t$ , on top of (i) the households' utility maximization under constraints, (ii) the determination of output, (iii) the population dynamics, (iv) the technological progress dynamics, and (v) the determination of productive land. Therefore, a competitive equilibrium is fully determined by the following system of equations (??)-(??), given  $\beta, \gamma, \theta, \omega, X, L_0$ , and  $A_0$ :

$$\text{FOCs :} \quad n_t = \gamma \left(\frac{X_t}{L_t}\right)^\beta \quad (10)$$

$$e_{t+1} = 0 \quad (11)$$

$$\text{Production :} \quad y_t = f(A_t)h_t \quad (12)$$

$$\text{Population :} \quad L_{t+1} = n_t L_t \quad (13)$$

$$\text{Technology :} \quad A_{t+1} = [1 - \lambda(\omega)][1 + g(0, L_t)]A_t \quad (14)$$

$$\text{Land :} \quad X_t = \chi(\theta, A_t)X \quad (15)$$

The competitive equilibrium system above are characterized by the reduced equilibrium dynamics of the population  $L_t$  and technology  $A_t$ :

$$L_{t+1} = \gamma (\chi(\theta, A_t)X)^\beta L_t^{1-\beta} \quad (16)$$

$$A_{t+1} = [1 - \lambda(\omega)][1 + g(0, L_t)]A_t \quad (17)$$

for a given initial conditions  $L_0, A_0$  (and  $e_0 = 0$ ).

## 4. Stagnation trap

This section studies the conditions on geographical factors  $(X, \theta, \omega)$ , i.e. the amount of “land” available, its suitability, and its degree of isolation, under which an economy starting from specific initial conditions never escapes stagnation. Specifically we characterize the set of geographical factors that do not allow an economy to reach the critical population size  $\mathcal{L}(\omega)$  guaranteeing technological growth. As a consequence, the technological level will remain below the take-off threshold, locking the economy at zero-education. Zero-education associated with small population cannot guarantee a technological progress able to offset the losses of technology, so that the economy cannot expand its productive land to enhance fertility and reach a bigger population. This negative feedback loop prevents the economy from escaping stagnation.

**Proposition 2:** *An economy with land  $X$ , a given suitability  $\theta$  for it, and a degree of isolation  $\omega$  such that*

$$\frac{\mathcal{L}(\omega)}{\chi(\theta, \hat{A}(\mathcal{L}(\omega)/X))X} \geq \gamma^{1/\beta},$$

*i.e. with too high effective population density (specifically above  $\gamma^{1/\beta}$ ) —at the population level  $\mathcal{L}(\omega)$  guaranteeing technological progress, and the level of technology triggering education investment  $\hat{A}(\mathcal{L}(\omega)/X)$ — will be locked in a stable steady state with  $\tilde{e} = 0$ , i.e. no education investment, the lowest level of technology  $\tilde{A} = 0$ , and a small population  $\tilde{L}$ , with*

$$\tilde{L} = \gamma^{1/\beta} \chi(\theta, 0)X < \mathcal{L}(\omega)$$

*for initial conditions  $L_0 < \mathcal{L}(\omega)$ ,  $e_0 = 0$ , and  $A_0 \leq \hat{A}\left(\frac{\mathcal{L}(\omega)}{\chi(\theta, 0)X}\right)$ , i.e. for an initial population not big enough to guarantee technological progress, no initial education investment, and a level of technology smaller than the one triggering education investment under zero technology.*

**Proof:** The claim follows from the fact that, starting from  $e_0 = 0$  and  $L_0 < \mathcal{L}(\omega)$ , if  $L_t < \mathcal{L}(\omega)$ , then  $L_{t+1} < \mathcal{L}(\omega)$ , for all  $t$ . In effect, assume  $L_t < \mathcal{L}(\omega)$ . Given that  $\chi$  is increasing in  $A$ ,  $L_t < \mathcal{L}(\omega)$ , and  $\hat{A}' < 0$ ,

$$A_t \leq \hat{A}\left(\frac{\mathcal{L}(\omega)}{\chi(\theta, 0)X}\right)$$

implies

$$A_t < \min \left\{ \hat{A} \left( \frac{L_t}{\chi(\theta, A_t)X} \right), \hat{A} \left( \frac{\mathcal{L}(\omega)}{X} \right) \right\}$$

Moreover  $L_t < \mathcal{L}(\omega)$  implies

$$A_{t+1} = [1 - \lambda(\omega)][1 + g(0, L_t)]A_t < A_t$$

and hence

$$A_{t+1} < \hat{A} \left( \frac{L_t}{\chi(\theta, A_t)X} \right)$$

so that  $e_{t+1} = 0$  and

$$L_{t+1} = \gamma \left( \frac{\chi(\theta, A_t)X}{L_t} \right)^\beta L_t < \gamma \left( \frac{\chi(\theta, \hat{A}(\mathcal{L}(\omega)/X))X}{\mathcal{L}(\omega)} \right)^\beta \mathcal{L}(\omega) \leq \mathcal{L}(\omega)$$

(where the last inequality comes from the assumed excessive effective population density). As a consequence,  $L_{t+1} < \mathcal{L}(\omega)$  and  $e_t = 0$ , for all  $t = 0, 1, 2, \dots$ . Moreover

$$A_{t+1} = [1 - \lambda(\omega)]^{t+1} \prod_{i=0}^t [1 + g(0, L_i)] A_0$$

with  $[1 - \lambda(\omega)][1 + g(0, L_t)] < 1$ , for all  $t$ , so that the technological level converges monotonically to  $\tilde{A} = 0$ , and the population converges, according to (??), to the level  $\tilde{L}$  solution to

$$\gamma \left( \frac{\chi(\theta, 0)X}{L} \right)^\beta = 1$$

that is to say

$$\tilde{L} = \gamma^{1/\beta} \chi(\theta, 0)X$$

The economy will, therefore, be locked in the stable steady state characterized by

$$(\tilde{L}, \tilde{A}, \tilde{e}) = (\gamma^{1/\beta} \chi(\theta, 0)X, 0, 0).$$

Q.E.D.

## 5. Conclusion

In early stages of development, i.e. with a small population and a low technological level giving households no incentive to educate their children (as in the first case in Proposition 1), if the geographical factors  $(X, \theta, \omega)$  —i.e. land, its suitability, and its degree of isolation— do not allow for a sufficiently large population (i.e.  $L < \mathcal{L}(\omega)$ ), there will be no technological growth in the long run, as well as no education investment to enhance technological progress. As a consequence, the economy will be locked in stagnation (as stated in Proposition 2).

If, on the contrary, the geographical factors allow for a sufficiently large population able to offset losses of technology, then the mechanism for the economy to take off is similar to the one in Galor and Weil (2000): for a large enough population, technological growth appears, the increase in the level of technology over time increases the returns to education,

households educate their children, and this triggers sustained technological progress and growth, forcing the economy to take off and leave Malthusian stagnation.

By showing how in a set up close to Galor and Weil (2000) —complemented with geographical factors and the possibility of technology losses— an economy can get trapped in Malthusian stagnation, this paper makes stand out the role of some geographical and environmental conditions for the development process.

## References

**Aiyar, S. et al. (2008)**, “Technological progress and regress in pre-industrial times”, *Journal of Economic Growth*, 13, 125 - 144.

**De la Croix, D. and Dottori, D. (2008)**, “Easter Island’s collapse: a tale of a population race”, *Journal of Economic Growth*, 13, 27 - 55.

**De la Croix, D. and Gosseries, A. (2012)**, “The Natalist Bias of Pollution Control”, *Journal of Environmental Economics and Management*, 63, 271 - 287.

**Diamond, J. (1997)**, “*Guns, Germs, and Steel: The Fates of Human Societies*”, New York: W. W. Norton.

**Galor, O. and Moav, O. (2002)**, “Natural Selection and the Origin of Economic Growth”, *Quarterly Journal of Economics*, Vol. CXVII, 1133 - 1191.

**Galor, O. and Weil, D.N. (2000)**. “Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond”. *American Economic Review*, Vol. 90, 806 - 828.

**Goodsell, W. (1937)**. “Housing and birth rate in Sweden”. *American Sociological Review*, 2 (6), 850 - 859.

**Mariani, F. et al. (2010)**, “Life expectancy and the environment”, *Journal of Economic Dynamics and Control*, 34, 798 - 815.

**Thompson, W.S. (1938)**. “The effect of housing upon population growth”. *The Milbank Memorial Fund Quarterly*, 16 (4), 359 - 368.