



# CANNON WAS RIGHT BUT INCOMPLETE: FRANKEL WAS A NEGLECTED EARLY CONTRIBUTION TO GROWTH THEORY

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**« CANNON WAS RIGHT BUT INCOMPLETE:  
FRANKEL WAS A NEGLECTED EARLY  
CONTRIBUTION TO GROWTH THEORY »**

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# CANNON WAS RIGHT BUT INCOMPLETE: FRANKEL WAS A NEGLECTED EARLY CONTRIBUTION TO GROWTH THEORY

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## ABSTRACT

This paper reexamines an important early contribution to the growth literature highlighted in 2000 by Cannon in the American Economic Review: “The production function in allocation and growth: A synthesis” by Marvin Frankel in 1962. First this contribution provides a formal proof in the continuous time framework of trajectories that are not provided by Frankel and Cannon. Second it shows that in this framework poverty traps are possible while it was not detected by these authors.

## RESUME

Ce document revient sur une contribution ancienne en théorie de la croissance remise au goût du jour en 2000 par Cannon dans l’American Economic Review: “The production function in allocation and growth: A synthesis” de Marvin Frankel en 1962. Ce document présente d’une part la preuve d’existence de solutions en temps discret absente dans les deux contributions. Et, d’autre part, démontre que dans ce cadre il existe un piège à pauvreté qui n’a pas été détecté par ces deux auteurs.

*Keywords:* Economic growth, Poverty traps.

*JEL classification numbers:* O41.

## I. INTRODUCTION

In 2000, Cannon concluded his contribution by “Frankel (1962) anticipated ideas in use in the modern literature and deserves wider recognition”. Indeed the model developed by Frankel belongs to the AK family models such as the seminal work of Romer (1986, 1987, 1990), Lucas (1988), Rebelo (1991), Abe (1995). This document reexamines the contribution of Frankel and Canon in providing proof of solutions in the continuous time framework. Second it will be shown that this model also belongs of the family of few models where poverty traps may exist (see Azariadis and Stachurski, 2005 for a survey of these models). The following section presents the model and proves the existence of a poverty trap. The last section concludes.

## II THE MODEL

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The framework presented by Frankel is very similar to Solow (1956). However the main divergence in Frankel is that the production function, assumed to be a Cobb Douglas homogenous of degree one, is indexed by a development modifier. Then:

$$Q_t = aH_t \widehat{K}_t^\alpha \widehat{L}_t^\beta, \alpha + \beta = 1,$$

where  $\widehat{L}_t$  stands for labor,  $\widehat{K}_t$  is the stock of capital and  $H_t$  is the development modifier. Frankel emphasizes the fact that the modifier can be education, technical change or any other indicator. In his contribution Frankel suggests capital deepening. In this contribution the framework is more general and a ratio of a function of capital over a function of labor is considered. The development modifier is:

$$H_t = \frac{\widehat{K}_t^\nu}{\widehat{L}_t^\omega},$$

replacing in the production function one has:

$$Q_t = aH_t \widehat{K}_t^\alpha \widehat{L}_t^\beta = a \frac{\widehat{K}_t^\nu}{\widehat{L}_t^\omega} \widehat{K}_t^\alpha \widehat{L}_t^\beta = a \widehat{K}_t^{\alpha+\nu} \widehat{L}_t^{\beta-\omega},$$

The production function is not any longer homogeneous of degree one unless  $\nu = \omega$ . If the development modifier is replaced by a deterministic trend then the model is similar to Solow with exogenous technical progress. One can see that another hypothesis is possible rather than introducing capital deepening one may assume that the development modifier is a government spending financed by a tax  $H_t = G_t^\varepsilon$  and the fiscal constraint is  $G_t = \tau Q_t$ , substituting in the production function one has:

$$Q_t = (a \tau^\varepsilon \widehat{K}_t^\alpha \widehat{L}_t^\beta)^{\frac{1}{1-\varepsilon}},$$

One can see that the two formulations of the production function are similar and the framework prove to be very general and can encompass several frameworks. Changing notation the production function is:

$$Q_t = b \widehat{K}_t^\theta \widehat{L}_t^\phi.$$

At each period of time the budget constraint is:

$$Q_t = C_t + I_t,$$

where  $C_t$  is consumption and  $I_t$  investment. As in Solow and Frankel, let  $s$  be the constant fraction of their income saved and  $\delta$  the depreciation rate. Then one as:

$$Q_t = (1-s)Q_t + \widehat{K}_{t+1} - (1-\delta)\widehat{K}_t,$$

replacing output by its expression and rearranging term one has:

$$\widehat{K}_{t+1} - (1-\delta)\widehat{K}_t = sb \widehat{K}_t^\theta \widehat{L}_t^\phi.$$

Only balanced growth path are under investigation then  $\widehat{K}_t = (1+g)^t \widehat{K}_0$  and let  $n$  be the exogenous growth rate of labor then replacing in the previous equation one has:

$$\widehat{K}_0 (1+g)^{t+1} - (1-\delta)(1+g)^t \widehat{K}_0 = sb(1+g)^{\theta t} \widehat{K}_0^{\theta} \widehat{L}_0^{\phi} (1+n)^{\phi t}$$

this identity can only be true if and only if:

$$(1+g) = (1+n)^{\frac{\phi}{1-\theta}}.$$

Let the capital stock without the trend be:

$$K_t = \frac{\widehat{K}_t}{(1+g)^t}.$$

Replacing in the expression above and using the condition on the trend, rearranging terms, the evolution of capital stock across time is given by:

$$K_{t+1} = \frac{sbL_0^{\phi}}{1+g} K_t^{\theta} + \frac{1-\delta}{1+g} K_t.$$

The long run trajectories of capital and in due course of output are defined by the shape of the function:

$$h(K) = \frac{sbL_0^{\phi}}{1+g} K^{\theta} + \frac{1-\delta}{1+g} K.$$

The resolution is standard in the framework of models a la Solow. The first and second order derivatives are:

$$h'(K) = \theta \frac{sbL_0^{\phi}}{1+g} K^{\theta-1} + \frac{1-\delta}{1+g}$$

$$h''(K) = \theta(\theta-1) \frac{sbL_0^{\phi}}{1+g} K^{\theta-2}$$

It is obvious that the first order derivative is positive, then function is increasing but the sign of the second order derivative depends on the value of  $\theta$ . In the first case if  $\theta < 1$  the second order derivative is negative and the function is concave. This is the standard Solow model and the capital stock converges towards a steady state whose value is:

$$K^s = \left( \frac{g+\delta}{sbL_0^{\phi}} \right)^{\frac{1}{\theta-1}}.$$

But, if  $\theta > 1$  the second order derivative is positive and the function is convex. And it is easy to show that there are two fixed points depending on the value of the

parameter characterizing the economy. The first point is zero and the second is defined by:

$$h(K) = K$$

Then:

$$h(K) = \frac{sbL_0^\phi}{1+g} K^\theta + \frac{1-\delta}{1+g} K = K$$

And the fixed point is:

$$K = \left( \frac{\delta + g}{sbL_0^\phi} \right)^{\frac{1}{\theta-1}} > 0$$

Then there exist a poverty trap such that for any initial capital stock below the fixed point the economy converges towards zero.

### III CONCLUSION

This document emphasizes the pedagogical value of the model developed by Frankel (1962). It introduces in an easy way three main families of growth models: models a la Solow, AK endogenous growth models and poverty traps. In addition this note provides a complete characterization of long run trajectories and fixed points.

### REFERENCES

Abe, N. (1995). 'Poverty trap and growth with public goods', *Economics Letters*, Elsevier, 47, pp.361-366.

Azariadis, C. and Stachurski, J. (2005). 'Poverty Traps', Handbook of Economic Growth, in: Philippe Aghion and Steven Durlauf (ed.), Handbook of Economic Growth, edition 1, volume 1, chapter 5 Elsevier.

Frankel, M. (1962). 'The production function in allocation and growth: A synthesis', *The American Economic Review*, 52, pp.996-1022.

Lucas, R. (1988). 'On the mechanics of economic development', *Journal of Monetary Economics*, Elsevier, 22, pp.3-42.

Rebelo, S. (1991). 'Long-Run Policy Analysis and Long-Run Growth', *Journal of Political Economy*, University of Chicago Press, 99, pp.500-521.

Romer, P.M. (1986). 'Increasing Returns and Long-run Growth', *Journal of Political Economy*, University of Chicago Press, 94, pp.1002-37.

Romer, P.M. (1987). 'Growth Based on Increasing Returns Due to Specialization', *American Economic Review*, American Economic Association, 77, pp.56-62.

Romer, P.M. (1990). 'Are Nonconvexities Important for Understanding Growth?', *American Economic Review*, American Economic Association, 80, pp.97-103.

Solow, R. (1956). 'A contribution to the theory of economic growth', *Quarterly Journal of Economics*, 70, pp.65-94.