Optimal Licensing of Uncertain Patents in the Shadow of Litigation

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Abstract

This paper investigates the choice of a licensing mechanism by the holder of a patent whose validity is uncertain. Focusing first on weak patents, i.e. patents that have a high probability of being invalidated by a court if challenged, we show that the patent holder finds it optimal to use a per-unit royalty contract if the *strategic* effect of an increase in a potential licensee's unit cost on the equilibrium industry profit is positive. The latter condition ensures the superiority of the per-unit royalty mechanism independently of whether the patent holder is an industry insider or outsider, and is shown to hold in a Cournot (resp. Bertrand) oligopoly with homogeneous (resp. differentiated) products under general assumptions on the demands faced by firms. We then examine the optimal licensing of patents that are uncertain but not necessarily weak. As a byproduct of our analysis, we contribute to the oligopoly literature by offering some new insights of independent interest regarding the effects of cost variations on Cournot and Bertrand equilibria.

Keywords: Licensing mechanisms, Uncertain patents, Patent litigation, Cost comparative statics.
JEL Classification: D45, L10, O32, O34.

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1 Introduction

Since the seminal contribution by Arrow (1962), analyzing the licensing contracts whereby patent holders sell the right to use protected technologies has become an important topic in the economics of innovation and technology diffusion. Arrow compared the revenues that an outside innovator obtains from licensing a cost-reducing innovation to a competitive industry and to a monopolistic industry. He showed that when a per-unit royalty is charged, a perfectly competitive industry generates higher licensing revenues than a monopolistic one.[1] Subsequently, Katz and Shapiro (1985, 1986) and Kamien and Tauman (1984, 1986) analyzed different licensing mechanisms (fixed fee, auction and per-unit royalty) when the potential licensees are members of an *oligopoly*. A key insight of the theoretical literature that has built on those seminal papers is that the optimal licensing mechanism depends on many factors, including the type of downstream competition, the degree of differentiation between products and whether the patent holder is active or not in the downstream market.[2] These three factors have been shown to be critical in the sense that predictions regarding the optimal mechanism can be completely overturned by varying any of them.[3]

A common feature of the existing papers on the comparison of different licensing mechanisms is that patents are viewed as certain or "ironclad" rights, the validity of which is unquestionable. This clearly contradicts what we observe in practice: about half of the patents that are challenged before US courts are invalidated (Allison & Lemley, 1998).[4] It is now largely recognized that a patent is not a perfectly enforceable right, as are other forms of property. Patents correspond much more to "uncertain or probabilistic rights because they only give a limited right to try to exclude by asserting the patent in court" (Ayres and Klemperer, 1999; Shapiro, 2003; Lemley and Shapiro, 2005). Moreover, this uncertainty is strengthened by the fact that many applications are granted patent protection by the patent office (PO) even though they probably do not meet one or several of the statutory requirements: belonging to the patentable subject matters, utility, novelty and non-obviousness (or inventiveness). Such patents are *weak* in the sense that they have a high probabilty of being invalidated by a court if challenged by a third party.[5]

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[1]This is a consequence of the well known *replacement effect*, according to which the willingness to pay for an innovation is larger for an entrant in a competitive industry than for an incumbent firm.

[2]Empirically, royalties seem to be more often used than fixed fees (Taylor and Silberstone, 1973; Rostoker, 1984). However, available data on patent licensing is very limited, because most firms elect not to disclose their private licensing contracts. Most empirical investigations emphasize the factors that affect the likelihood of firms to engage in licensing agreements but are less informative on the licensing scheme (Anand and Khanna, 2000; Vonortas and Kim, 2004, Zuniga and Guellec, 2009)

[3]For instance, it has been shown that under Cournot competition with homogeneous products, fixed fees dominate per-unit royalties when the licensor is an industry outsider (Kamien and Tauman, 1984, 1986; Kamien *et al.*, 1992). However, the reverse result holds if the licensor is an industry insider (Shapiro, 1985; Wang, 1998; Kamien and Tauman, 2002; Sen, 2002; Sen and Tauman, 2007, 2012). Furthermore, under price competition with differentiated products, per-unit royalties dominate fixed fees when the products are close substitutes or, if not, when the size of the cost reduction is small, while the reverse holds if the products are weak substitutes and the innovation is large (Muto, 1993). The results are quite different if one considers the same differentiated product environment but assumes that firms compete in quantities instead of prices (Wang, 2002).

[4]This concerns the patent disputes that are not settled prior to the court judgement.

[5]The notion of "weak patent" has at least two different meanings in the literature (Ginarte and Park, 1997, van

The proliferation of uncertain patents can be explained by several reasons. First, the major patent offices (USPTO, EPO and JPO) have insufficient resources to ensure an effective review process for the huge and growing number of patent applications (Friebel *et al.*, 2006). Second, mistakes are unavoidable because the patentability requirements, in particular novelty and non-obviousness, are difficult to assess especially for newly patentable subject matters, such as software, business methods and research tools. Third, the incentives provided to the examiners are inadequate for making them fully prosecute and reject the applications that do not meet the standards (Farrell and Merges, 2004; Langinier and Marcoul, 2009; Lei and Wright, 2010). Finally, the continuing debate on what subject matters are patentable throws additional uncertainty into the validity of many patents (Guellec and van Pottelsberghe, 2007).

In this paper we investigate the optimal licensing mechanism from the perspective of a licensor holding an *uncertain* patent and facing the *threat of patent litigation*. We thus extend three strands of literature:

1. *The vast literature that compares various licensing mechanisms in the context of ironclad patents.*[6] Our contribution is to extend this comparison to patents whose validity is uncertain. We believe that such an extension is warranted in light of the growing proliferation of uncertain patents and, in particular, weak ones.[7] We thus investigate first the case of weak patents and provide a sufficient condition under which the holders of such patents prefer to charge a per unit royalty rather than a fixed fee.[8] This condition has a very natural economic interpretation: it states that the strategic effect of an increase in an oligopolist's unit cost on industry profits is positive. This result is very general since the type of competition between the potential users of the technology is not specified in our model except for assuming the existence and uniqueness of a Nash equilibrium to the competition game and some properties that are shown to be broadly satisfied in usual imperfect competition models. Then we show that the sufficient condition above holds and, therefore the per-unit royalty mechanism is optimal from the patent holder's perspective, in a wide range of settings.[9] A key difference with the literature considering ironclad patents is that the superiority

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Pottelsberghe de la Potterie, 2010). First, a patent can be said to be weak if it gives its holder a low protection against imitators and other potential infringers, either because the patent's scope is badly defined or because the protection extends to countries in which the enforcement of intellectual property rights is low. Second, a patent can be qualified as weak when it is likely that it does not satisfy at least one of the patentability requirements (patentable subject matter, utility, novelty, non obviousness or inventiveness in the European terminology). In this sense, a weak patent is likely to be invalidated by a court if challenged by a third party. The present paper focuses on the latter meaning.

[6] See Bhattacharya *et al.* (2012) for a recent survey on the licensing of ironclad patents and other R&D arrangements.

[7] Bessen and Meurer (2008) argue that the inclusion of software and business methods in the patentable subject matters in the US has resulted in an increase in the share of weak patents among all the patents issued by the USPTO.

[8] This condition is shown to be also sufficient for the optimal two-part tariff contract to be a (pure) per-unit royalty contract.

[9] This is a novel justification, based on the uncertainty over patent validity, for the use of per-unit royalties instead of fixed fees in licensing. Various other reasons have been explored in the literature on ironclad rights, including risk aversion (Bousquet et al., 1998), asymmetry of information (Gallini and Wright, 1990; Macho-Stadler and Pérez-Castrillo, 1991; Beggs, 1992; Sen 2005a), moral hazard (Macho-Stadler et al., 1996; Choi, 2001), production differentiation (Muto, 1993; Wang and Yang, 1999; Caballero-Sanz et al., 2002; Poddar and Sinha, 2004; Stamatopoulos and Tauman, 2007), strategic delegation (Saracho, 2002), integer nature of the number of licensees (Sen, 2005b), variation in the quality of

of per-unit royalties for the licensing of weak patents is a very robust result: it is independent of the type of downstream competition, the degree of differentiation between products and whether the patent holder is active or not in the downstream market. We then extend our analysis to the licensing of patents that are uncertain, but not necessarily weak, and show that our finding about the optimality of the per-unit royalty mechanism may hold even for patents that have a relatively low probability of being invalidated by a court if challenged.

2. *The burgeoning literature on the licensing of uncertain patents* (Farrell and Shapiro, 2008; Encaoua and Lefouili, 2005, 2009; Choi 2010). This literature has been mostly concerned with the inefficiencies stemming from the low private incentives to litigate a weak patent. The social harm of weak patents also depends on *how* they are licensed out and, therefore, we argue that it is crucial to get a better understanding of the licensing schemes the holders of those problematic patents use. In Farrell and Shapiro (2008) and Encaoua and Lefouili (2009), the licensor is assumed to offer two-part tariff licensing contracts. However, when considering the effects of the uncertainty over patent validity on the licensor's profits and/or social welfare when negative fixed fees cannot be used, each of these two papers relies on a technical *ad hoc* assumption on the shape of the (endogenous) licensing revenue function[10] that substantially simplifies the analysis by immediately guaranteeing that pure per-unit royalty contracts are optimal for the licensors of weak patents in the class of contracts deterring litigation. In sharp contrast to the former papers, our result that the holder of a weak patent finds it optimal to use a per-unit licensing contract is implied by a mild condition which has a natural economic interpretation and is shown to hold with broad generality in standard oligopoly models with general demand functions (as are all the assumptions made in our model). This finding is therefore arguably robust, especially as it is shown to hold as well in various extensions of our baseline model. Furthermore, we show that a social planner always prefers the license of an uncertain patent deterring litigation to take the form of a fixed fee contract, which implies that the holder of a weak patent chooses a socially suboptimal licensing mechanism. A policy implication is that welfare gains may be realized by encouraging or constraining the holders of weak patents to license them by means of fixed fees.

3. *The extensive literature on oligopoly.* We contribute to this literature by providing new insights regarding the effects of cost variations on oligopolists' profits (Seade, 1985; Kimmel, 1992; Février and Linnemer, 2004). Since our result that the licensor of a weak patent finds it optimal to use a per-unit royalty contract, holds whenever the strategic effect of a unilateral cost increase on industry profits is positive, we examine the latter condition and show that it holds (i) for Cournot competition with homogeneous products under complete generality, and (ii) for Bertrand competition with differentiated products under strategic complementarity. In other words, no restrictions on the demand systems are needed beyond those commonly invoked to establish existence and uniqueness of

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innovation (Rockett, 1990).

[10] Farrell and Shapiro (2008) assume that the licensing revenue function where the fixed fee is set to its optimal level (for a given per-unit royalty) is single-peaked in the per-unit royalty while Encaoua and Lefouili (2009) assume that the same function is concave in the per-unit royalty (see assumption A6 in their paper).

a pure-strategy equilibrium. An ancillary result is that we provide a lower bound on the effect of a unilateral cost increase on industry profits.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the optimal per-unit royalty (resp. fixed fee) license from the patent holder's perspective. In Section 4 we compare the two licensing mechanisms for weak patents. In Section 5, we extend the analysis by successively (i) including (small) litigation costs, (ii) allowing the patent holder to offer two-part tariff licensing contracts, and (iii) considering a patent holder that is an industry insider. In Section 6, we show that the general assumptions made on the equilibrium profits in our reduced-form model of competition and the (sufficient) condition under which the per-unit royalty mechanism is optimal to the patent holder (be it an industry outsider or an insider) hold under general conditions on the demand functions for both a Cournot oligopoly with homogenous goods and a Bertrand oligopoly with differentiated goods. In Section 7, we extend our analysis to uncertain patents of any strength. Welfare implications are discussed in Section 8. Section 9 concludes.

2   The Model

We consider an industry consisting of $n \geq 2$ symmetric risk-neutral firms producing at a marginal cost $\bar{c}$ (fixed production costs are assumed to be zero). A firm P outside the industry holds a patent covering a technology that, if used, allows a firm to reduce its marginal cost from $\bar{c}$ to $\bar{c} - \epsilon$ where $\epsilon \in]0, \bar{c}[$.

We consider the following three-stage game:

First stage: The patent holder P proposes to all firms a licensing contract[11] whereby a licensee can use the patented technology against the payment of a per-unit royalty $r \in [0, \epsilon]$ or a fixed fee $F \geq 0$.[12]

Second stage: The n firms in the industry simultaneously and independently decide whether to purchase a license. If a firm does not accept the license offer, it can challenge the patent's validity before a court.[13] The outcome of such a trial is uncertain: with probability $\theta > 0$ the patent is upheld by the court and with probability $1 - \theta$ it is invalidated. Hence, the parameter θ may be interpreted as the patent's quality or the patent's strength. If the patent is upheld, then a firm that does not purchase the license uses the old technology, thus producing at marginal cost $\bar{c}$ whereas a firm that accepted the license offer uses the new technology and pay the per-unit royalty r or the fixed fee F to the patent holder. If the patent is invalidated, all the firms, including those that accepted the license offer can use for free the new technology and their common marginal cost is $\bar{c} - \epsilon$.

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[11] Following Farrell and Shapiro (2008) and Encaoua and Lefouili (2009), we focus on take-it-or-leave-it license offers.

[12] In Section 5.2, we show that allowing the patent holder to offer two-part tariff contracts does not affect our central result.

[13] For patents granted by the European Patent Office (EPO), the timing is slightly different. Indeed, any patent issued by the EPO can be opposed by a third party and the notice of opposition must be filed in writing at the EPO within nine months from the publication of the mention of the grant of the European patent.

Third stage: The n firms produce under the cost structure inherited from the second stage. We do not specify the type of competition that occurs. We only assume that there exists a unique equilibrium of the competition game for any cost structure and we set some general assumptions on the equilibrium profit functions.[14] For this purpose, denote $\pi^e(k,c)$ (respectively $\pi^i(k,c)$) the equilibrium profit function, gross of any potential fixed cost, e.g. a fixed license fee, of a firm producing with marginal cost $c \leq \bar{c}$ (respectively with marginal cost $\bar{c}$) when $k \leq n$ firms produce at marginal cost c and the remaining $n - k$ firms produce at the marginal cost $\bar{c}$.

We now make the following general assumptions for any given n and $k = 1, ..., n$.

A1. The equilibrium profits of an efficient firm and an inefficient firm, i.e. $\pi^e(k,c)$ and $\pi^i(k,c)$ respectively, are both continuously differentiable in c over the subset of $[0, \bar{c}]$ in which $\pi^i(k,c) > 0$. Furthermore, the output function $c \to q^e(n,c)$ when the n firms produce at marginal cost c is continuously differentiable and strictly positive over $[0, \bar{c}]$.

A2. If the firms are symmetric (in terms of efficiency), an identical increase in all firms' marginal costs leads to a decrease in each firm's equilibrium profit: $\frac{\partial \pi^e}{\partial c}(n,c) < 0$.

A3. An inefficient firm's equilibrium profit is increasing in the efficient firms' marginal cost: If $\pi^i(k,c) > 0$ then $\frac{\partial \pi^i}{\partial c}(k,c) > 0$ and if $\pi^i(k,c) = 0$ then $\pi^i(k,c') = 0$ for any $c' < c$.

A4. A firm's profit is decreasing in the number of efficient firms in the industry: for any $c < \bar{c}$ and any $k < n$ it holds that $\pi^e(k,c) > \pi^e(k+1,c)$ and $\pi^i(k,c) \geq \pi^i(k+1,c)$.

A5. A firm's profit increases as it moves from the subgroup of inefficient firms to the subgroup of efficient firms: for any $c < \bar{c}$ and any $k < n$ it holds that $\pi^i(k,c) < \pi^e(k+1,c)$.

As we shall argue in precise detail in Section 6, all these assumptions are satisfied with broad generality in the standard oligopoly models with general demand functions. In particular, these assumptions are clearly satisfied for instance for the widely used settings of Cournot competition with homogeneous goods and linear demand and Bertrand competition with differentiated goods and linear demands.

3   Licenses deterring litigation

If litigation occurs then, with probability θ, the patent is upheld by the court (thus becoming an ironclad right) and, with probability $1 - \theta$, it is invalidated and the technology can be used for free by all firms. Thus, if the patent holder expects its license offer to trigger litigation, it should make an offer that maximizes its revenues should the patent be ruled valid by the court. The patent holder would then essentially act as if the patent were ironclad, and the determination of the terms on which the technology is patented under each licensing mechanism would amount to the analysis of licensing offers for ironclad patents, which has already been done extensively in the literature. We therefore consider in what follows only the class of license offers deterring litigation, for which the uncertainty

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[14] A similar general approach was previously adopted by Reinganum (1982) in a strategic search model, and by Amir and Wooders (2000), Boone (2001) and Encaoua and Lefouili (2009) in different R&D models.

over patent validity does matter. In doing so, we follow Farrell and Shapiro (2008) who also focus on license offers such that litigation is avoided because they aim to investigate the social costs of the uncertainty over patent validity (which is resolved if litigation occurs).[15]

Since we look for the subgame perfect equilibria of the game in which litigation is deterred, we start our analysis by determining, under each mechanism, the license offers that do not induce litigation at the second stage of the game.

3.1 Per-unit royalty mechanism

Let us first examine a firm's incentives to challenge the patent's validity when the patent holder makes a license offer involving the payment of a per-unit royalty r. A firm that decides not to purchase a license is always (weakly) better off challenging the patent's validity: If no other firm challenges the patent's validity it gets a payoff $\theta \pi^i(n-1, \bar{c} - \epsilon + r) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon)$ which is strictly greater than the profit $\pi^i(n-1, \bar{c} - \epsilon + r)$ it would get by not challenging the patent, and if some other firm challenges the patent's validity then it is indifferent between challenging and not. Thus, a situation where one or more firms do not buy a license and no firm challenges the patent's validity can never be an equilibrium of the second stage subgame. It follows that a license offer deters litigation if and only if it is accepted by all firms.

Assume that the patent holder makes a license offer (in the first stage) involving the payment of a per-unit royalty $r < \epsilon$. Let us show that in this case, any outcome with $k \leq n-2$ licensees cannot be an equilibrium. We have already shown that a situation where not all firms buy a license and no firm challenges the patent cannot be an equilibrium so we can focus on situations with $k \leq n-2$ licensees and at least one non-licensee challenging the patent. Any of the other $n-k-1 \geq 1$ non-licensees has an incentive to unilaterally deviate by buying a license since it would get an expected profit of $\theta \pi^e(k+1, \bar{c} - \epsilon + r) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon)$ instead of the strictly lower expected profit $\theta \pi^i(k, \bar{c} - \epsilon + r) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon)$ if it remains a non-licensee. Therefore, any equilibrium of the second stage subgame involves at least $n-1$ firms if $r < \epsilon$. The latter result extends to the case $r = \epsilon$ if it is assumed, as will be the case from now on, that a firm which is indifferent between getting a license and not buying a license purchases a license.

Let us now write the condition under which all firms accepting the license offer r is an equilibrium of the second stage subgame. A firm anticipating that all other firms will purchase a license gets a profit equal to $\pi^e(n, \bar{c} - \epsilon + r)$ if it accepts the license offer. If it does not and challenges the patent's validity then with probability θ, the patent is upheld by the court and the challenger gets a profit equal to $\pi^i(n-1, \bar{c} - \epsilon + r)$ and, with probability $1 - \theta$, the challenger gets a profit of $\pi^e(n, \bar{c} - \epsilon)$ (and so do all other firms). Thus, a firm challenging the patent's validity when all other firms accept the license offer, gets an expected profit of $\theta \pi^i(n-1, \bar{c} - \epsilon + r) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon)$. Therefore, all

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[15] Moreover, there is empirical evidence that the vast majority of patent disputes are settled using licensing agreements before the court decides whether the patent is valid or not (Allison and Lemley, 1998 and Lemley and Shapiro, 2005).

firms accepting the license offer is an equilibrium if and only if:

$$\pi^e(n, \bar{c} - \epsilon + r) \geq \theta \pi^i(n - 1, \bar{c} - \epsilon + r) + (1 - \theta)\,\pi^e(n, \bar{c} - \epsilon). \tag{1}$$

The next proposition characterizes the values of the per-unit royalty set by the licensor that induce all firms to buy a license (thus deterring any litigation).

Proposition 1 *Define $r(\theta)$ as the unique solution in r to the following equation:*

$$\pi^e(n, \bar{c} - \epsilon + r) = \theta \pi^i(n - 1, \bar{c} - \epsilon + r) + (1 - \theta)\,\pi^e(n, \bar{c} - \epsilon).$$

Then all firms accepting to pay a per-unit royalty r is an equilibrium if and and only if $r \leq r(\theta)$. Moreover, this is the unique equilibrium of the second stage subgame when $r \leq r(\theta)$.

Proof. See Appendix. ∎

3.2 Fixed fee mechanism

The previous observation that a license offer deters litigation if and only if it is accepted by all firms remains true when the licensor uses a fixed fee scheme. For a license offer involving the payment of a fixed fee F to be accepted by all firms, the following condition must hold:

$$\pi^e(n, \bar{c} - \epsilon) - F \geq \theta \pi^i(n - 1, \bar{c} - \epsilon) + (1 - \theta)\,\pi^e(n, \bar{c} - \epsilon)$$

which can be rewritten as:

$$F \leq \theta\left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)\right]. \tag{2}$$

The next proposition is the counterpart of Proposition 1 for the fixed fee mechanism.

Proposition 2 *All firms accepting to pay the fixed fee F to use the patented technology is an equilibrium if and only if $F \leq F(\theta) = \theta[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)]$. Moreover, for sufficiently weak patents, i.e. for sufficiently small values of θ, all firms accepting the license offer is the unique equilibrium when $0 \leq F \leq F(\theta)$.*

Proof. See Appendix. ∎

3.3 Optimal licensing of weak patents under each mechanism

The patent holder's licensing revenues are given by $nrq^e(n, \bar{c} - \epsilon)$ if a per-unit royalty contract is used and by nF if the contract takes the form of a fixed fee payment. The next proposition characterizes the license that maximizes the patent holder's revenues subject to the constraint that no firm challenges the patent's validity.

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Proposition 3 *For sufficiently weak patents, the optimal per-unit royalty license deterring litigation involves the payment of the royalty rate $r = r(\theta)$ and the optimal fixed fee license deterring litigation is such that $F = F(\theta)$.*

Proof. See Appendix. ∎

4 Optimal licensing mechanism for weak patents

The licensing revenues from the optimal per-unit royalty contract deterring litigation are given by

$$\tilde{P}_r(\theta) = n r(\theta) q^e(n, \bar{c} - \epsilon + r(\theta))$$

and the licensing revenues from the optimal fixed fee licensing contract deterring litigation are

$$\tilde{P}_F(\theta) = n F_n(\theta) = n\theta \left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon) \right]$$

which, combined with $\pi^e(n, \bar{c} - \epsilon + r(\theta)) = \theta \pi^i(n - 1, \bar{c} - \epsilon + r(\theta)) + (1 - \theta)\pi^e(n, \bar{c} - \epsilon)$, yields

$$\tilde{P}_r(\theta) - \tilde{P}_F(\theta) = A(\theta) + B(\theta)$$

where

$$A(\theta) = n \left[\pi^e(n, \bar{c} - \epsilon + r(\theta)) + r(\theta) q^e(n, \bar{c} - \epsilon + r(\theta)) - \pi^e(n, \bar{c} - \epsilon) \right]$$

and

$$B(\theta) = -n\theta \left[\pi^i(n - 1, \bar{c} - \epsilon + r(\theta)) - \pi^i(n - 1, \bar{c} - \epsilon) \right].$$

The term $A(\theta)$ is the difference between the industry profits generated under the optimal per-unit royalty contract and those under the optimal fixed fee contract. The term $B(\theta)$, which is always non-positive[16], captures the fact that a potential licensee's outside option (when all other firms buy a license) is weakly higher under the per-unit royalty contract: in the event where the patent is invalidated, a challenger gets the same profit $\pi^e(n, \bar{c} - \epsilon)$ under both mechanisms, but in the event where the patent is upheld the challenger's profit is weakly higher under the per-unit royalty mechanism since $\pi^i(n - 1, \bar{c} - \epsilon + r(\theta)) \geq \pi^i(n - 1, \bar{c} - \epsilon)$ by **A3**. The latter implies that each licensee gets a higher profit under the optimal per-unit royalty contract deterring litigation than under its fixed fee counterpart. A corollary of this observation is that a *necessary* condition for the optimal per-unit royalty contract deterring litigation to be preferred over its fixed fee counterpart by the patent holder is that it generates (weakly) higher industry profits. It turns out that this condition,

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[16] Two scenarios have to be distinguished according to whether the set $\{c' > \bar{c} - \epsilon, \pi^i(n - 1, c') = 0\}$ is empty or not. If it is then $\pi^i(n - 1, \bar{c} - \epsilon + r(\theta)) > \pi^i(n - 1, \bar{c} - \epsilon)$ for any $\theta > 0$, and therefore $B(\theta) > 0$ for any $\theta > 0$. If it is not then it is easily shown that there exists $\theta_0 > 0$ such that $\pi^i(n - 1, \bar{c} - \epsilon + r(\theta)) = \pi^i(n - 1, \bar{c} - \epsilon) = 0$ for $\theta \leq \theta_0$ and $\pi^i(n - 1, \bar{c} - \epsilon + r(\theta)) > \pi^i(n - 1, \bar{c} - \epsilon)$ for $\theta > \theta_0$ by **A3**. It then follows that $B(\theta) \geq 0$ with the inequality being strict if and only if $\theta > \theta_0$.

when taken as a strict inequality, is actually *sufficient* when the patent is sufficiently weak. The reason for this is that $B(\theta)$ is, at most, a second-order term for θ sufficiently small (since $B(0) = 0$ and $B'(0) = 0$) while $A(\theta)$ is generally a first-order term. Therefore, for θ sufficiently small, the sign of $\tilde{P}_r(\theta) - \tilde{P}_F(\theta)$ will be the same as $A(\theta)$ whenever $A'(0) \neq 0$. In words, the holder of a (sufficiently) weak patent will find it optimal to choose the licensing mechanism that maximizes industry profits under the constraint that all firms accept the license offer. The next proposition provides a sufficient condition for the per-unit royalty mechanism to be preferred over the fixed fee mechanism by the holder of a (sufficiently) weak patent.

Proposition 4 *For a sufficiently weak patent, the optimal per-unit royalty contract deterring litigation provides higher licensing revenues than the optimal fixed fee contract deterring litigation if*

$$\frac{\partial \pi^e}{\partial c}(n, \bar{c} - \epsilon) > -q^e(n, \bar{c} - \epsilon). \tag{3}$$

Moreover, the reverse holds if the reverse strict inequality is satisfied.

Proof. See Appendix. ∎

Condition (3) means that the *strategic* effect of an identical increase in the marginal cost of all (symmetric) firms on the firms' profits is positive.[17] This interpretation results from the following decomposition (where we use the generic variable c instead of $\bar{c} - \epsilon$ as the second argument of the considered functions):

$$\frac{\partial \pi^e}{\partial c}(n, c) = \underbrace{-q^e(n, c)}_{direct\ effect} + \underbrace{q^e(n, c)\frac{\partial p^e}{\partial c}(n, c) + (p^e(n, c) - c)\frac{\partial q^e}{\partial c}(n, c)}_{strategic\ effect}.$$

An increase in all firms' marginal cost c affects their equilibrium profits $\pi^e(n, c)$ through two channels. First, it yields an increase in each firm's production costs (for a given output). Second, it entails an adjustment of their outputs and/or prices. The first effect, captured by the term $-q^e(n, c)$, can be interpreted as a *direct* effect of a common cost increase on equilibrium profits while the second effect, captured by the term $q^e(n, c)\frac{\partial p^e}{\partial c}(n, c) + (p^e(n, c) - c)\frac{\partial q^e}{\partial c}(n, c)$, can be interpreted as the *strategic* effect of a cost increase on profits.[18]

To the best of our knowledge, Condition (3) has not been studied in the literature on the effects of cost variations on oligopolists' profits which has mainly focused on the *overall* effect of cost changes on profits (e.g. Seade, 1985, Kimmel, 1992; Février and Linnemer 2004). In Section 6 we show the mildness of this condition by establishing that it holds under general assumptions in two of the

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[17]Another interpretation of this condition can be given in terms of the Lerner index. The condition holds iff the Lerner index is below the ratio of the price and quantity elasticities with respect to marginal cost.

[18]This strategic effect can be further split into a *price effect* captured by the term $q^e(n, c)\frac{\partial p^e}{\partial c}(n, c)$ and an *output effect* captured by the term $(p^e(n, c) - c)\frac{\partial q^e}{\partial c}(n, c)$. In usual models of competition, the latter is negative while the former is positive. Therefore the strategic effect is positive if the price effect outweighs the output effect.

most usual competition models, namely Cournot competition with homogenous goods and Bertrand competition with differentiated goods.

5 Extensions

5.1 Litigation costs

Let us assume in this section that a firm that challenges the patent's validity before a court has to incur some legal costs $C \geq 0$.[19] It is straightforward that the higher those costs the higher the licensing revenues the patent holder can extract from the licensees without triggering litigation. This qualitative observation holds under both mechanism. However, we show in what follows that on the quantitative side, the marginal effect of litigation costs on the patent holder's licensing revenues is higher under the per-unit royalty mechanism than under the fixed fee mechanism if condition (3) holds. This implies that the result in Proposition 4 remains true - and is actually strenghtened - if the model is extended to include (small) legal costs that any challenger must incur.

Suppose first that the patent holder makes a license offer involving the payment of a per-unit royalty $r \in [0, \epsilon[$. Note that the inclusion of legal costs in our setting does not affect the fact that the strategy "not buy a license *and* not challenge the patent's validity" is always dominated by the strategy "buy a license". Therefore, the only way a patent holder can deter litigation is to make a license offer that is accepted by all firms. This will be the case if and only if

$$\pi^e(n, \bar{c} - \epsilon + r) \geq \theta\pi^i(n-1, \bar{c} - \epsilon + r) + (1-\theta)\,\pi^e(n, \bar{c} - \epsilon) - C.$$

It is easily shown the latter constraint is met if and only if $r \leq r(\theta, C)$ where $r(\theta, C)$ is the solution in r to the equation

$$\pi^e(n, \bar{c} - \epsilon + r) = \theta\pi^i(n-1, \bar{c} - \epsilon + r) + (1-\theta)\,\pi^e(n, \bar{c} - \epsilon) - C$$

and that, for θ and C sufficiently small, the optimal per-unit royalty license deterring litigation involves the payment of the royalty $r(\theta, C)$ (i.e. the constraint is binding). Note also that $r(\theta, C)$ is strictly increasing in both its arguments.

Suppose now that the patent holder makes a license offer involving the payment of a fixed fee F. Such a license offer is accepted by all firms if and only if

$$\pi^e(n, \bar{c} - \epsilon) - F \geq \theta\pi^i(n-1, \bar{c} - \epsilon) + (1-\theta)\,\pi^e(n, \bar{c} - \epsilon) - C$$

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[19] If we consider the opposition procedure at the European Patent Office (EPO) instead of litigation before a court, the cost C can be interpreted as the administrative fee a challenger of a patent has to pay to the EPO for the patent to be reexamined.

and, therefore, the optimal fixed fee license deterring litigation involves the payment of the fee

$$F\left(\theta, C\right) = \theta\left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)\right] + C.$$

Let us now compare the licensing revenues derived by the patent holder under the two mechanisms. Under the optimal per-unit royalty contract, they are given by

$$\tilde{P}_r\left(\theta, C\right) = nr\left(\theta, C\right)q^e(n, \bar{c} - \epsilon + r\left(\theta, C\right))$$

and under the optimal fixed fee contract, they are given by

$$\tilde{P}_F\left(\theta, C\right) = nF_n\left(\theta, C\right) = n\theta\left[\pi^e\left(n, \bar{c} - \epsilon\right) - \pi^i\left(n - 1, \bar{c} - \epsilon\right)\right] + nC.$$

Since $\tilde{P}_r\left(0, 0\right) = \tilde{P}_F\left(0, 0\right)$, a sufficient condition for the existence of $\tilde{\theta} > 0$ and $\tilde{C} > 0$ such that the inequality $\tilde{P}_r\left(\theta, C\right) > \tilde{P}_F\left(\theta, C\right)$ holds for any $\theta < \tilde{\theta}$ and $C < \tilde{C}$ is that

$$\frac{\partial \tilde{P}_r}{\partial \theta}\left(0, 0\right) > \frac{\partial \tilde{P}_F}{\partial \theta}\left(0, 0\right)$$

and

$$\frac{\partial \tilde{P}_r}{\partial C}\left(0, 0\right) > \frac{\partial \tilde{P}_F}{\partial C}\left(0, 0\right)$$

The former inequality has already been shown to be equivalent to condition (3). Surprisingly enough, the latter inequality is equivalent to condition (3) too. Indeed,

$$\frac{\partial \tilde{P}_F}{\partial C}\left(0, 0\right) = n$$

and

$$\frac{\partial \tilde{P}_r}{\partial C}\left(0, 0\right) = n\frac{\partial r}{\partial C}\left(0, 0\right)q^e(n, \bar{c} - \epsilon).$$

Differentiating with respect to C the equation defining $r\left(\theta, C\right)$ at point $\left(\theta, C\right) = \left(0, 0\right)$, we get: $\frac{\partial r}{\partial C}\left(0, 0\right) = -\frac{1}{\frac{\partial \pi^e}{\partial c}(n, \bar{c} - \epsilon)}$. Thus,

$$\frac{\partial \tilde{P}_r}{\partial C}\left(0, 0\right) = -\frac{nq^e\left(n, \bar{c} - \epsilon\right)}{\frac{\partial \pi^e}{\partial c}\left(n, \bar{c} - \epsilon\right)}.$$

Hence

$$\frac{\partial \tilde{P}_r}{\partial C}\left(0, 0\right) > \frac{\partial \tilde{P}_F}{\partial C}\left(0, 0\right) \iff \frac{\partial \pi^e}{\partial c}(n, \bar{c} - \epsilon) > -q^e(n, \bar{c} - \epsilon).$$

Therefore, the result in Proposition 4 is robust - and is actually strengthened - in the presence of relatively small legal costs.

5.2 Two-part tariff contracts

Assume in this section that the patent holder can offer a two-part tariff licensing contract involving the payment of a fixed fee $F \geq 0$ *and* a per-unit royalty $r \geq 0$. The optimal two-part tariff contract $\left(\tilde{F}(\theta), \tilde{r}(\theta) \right)$ deterring litigation is a solution to the following maximization program

$$\max_{F \geq 0, \; r \in [0,\epsilon]} n \left(F + r q^e (n, \bar{c} - \epsilon + r) \right)$$

$$s.t. \; \pi^e(n, \bar{c} - \epsilon + r) - F \geq \theta \pi^i(n - 1, \bar{c} - \epsilon + r) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon).$$

Since the objective function is increasing in F, the constraint has to be binding at the optimum. Therefore, the optimal two-part tariff contract $\left(\tilde{F}(\theta), \tilde{r}(\theta) \right)$ is such that $\tilde{F}(\theta) = \pi^e(n, \bar{c} - \epsilon + \tilde{r}(\theta)) - \theta \pi^i(n - 1, \bar{c} - \epsilon + \tilde{r}(\theta)) - (1 - \theta) \pi^e(n, \bar{c} - \epsilon)$ and $\tilde{r}(\theta)$ is a solution to

$$\max_{r \in [0,\epsilon]} \left[r q^e (n, \bar{c} - \epsilon + r) + \pi^e(n, \bar{c} - \epsilon + r) - \theta \pi^i(n - 1, \bar{c} - \epsilon + r) - (1 - \theta) \pi^e(n, \bar{c} - \epsilon) \right]$$

$$s.t. \; \pi^e(n, \bar{c} - \epsilon + r) \geq \theta \pi^i(n - 1, \bar{c} - \epsilon + r) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon).$$

Note that the latter constraint could be rewritten as $r \leq r(\theta)$, and that $\tilde{F}(\theta) = 0$ if and only if $\tilde{r}(\theta) = r(\theta)$, i.e. the constraint is binding. The next proposition shows that our main finding, i.e. that the holder of a sufficiently weak patent finds it optimal to use a per-unit royalty contract whenever Condition (3) holds, extends to a setting where the larger set of two-part tariff contracts is considered.

Proposition 5 *For sufficiently weak patents, the optimal two-part tariff licensing contract deterring litigation is a pure per-unit royalty contract if:*

$$\frac{\partial \pi^e}{\partial c}(n, \bar{c} - \epsilon) > -q^e(n, \bar{c} - \epsilon).$$

Proof. See Appendix. ∎

5.3 Internal patentee

Let us consider the case where the patent holder is active in the (downstream) market. More specifically, we assume that one of the n firms operating in the market, say firm 1, gets a patent on a technology that lowers the unit production cost from $\bar{c}$ to $\bar{c} - \epsilon$. We assume that $n \geq 3$ (as we want to have at least two potential licensees).

Here again, we assume that there exists a unique equilibrium of the competition game for any cost structure (with identical profits for firms producing at the same unit cost) and we set some general assumptions on the equilibrium profit functions. We focus on industry cost structures that can emerge following the licensing game, that is, situations in which: one firm - the patent holder

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- produces at unit cost $\bar{c} - \epsilon$, a number $k \leq n - 1$ of firms - the licensees - produce at a unit cost $c \in [\bar{c} - \epsilon, \bar{c}]$ and the remaining $n - k$ firms - the non-licensees - produce at unit cost $\bar{c}$. We denote by $\pi^p(k, c)$, $\pi^l(k, c)$ and $\pi^n(k, c)$ the equilibrium market profits of the patent holder, a licensee producing at an effective unit cost c and a non-licensee respectively.

Given the new environment we consider, we need to replace the assumptions **A1-A5** made in our baseline model with the following assumptions:

A1'. The equilibrium profits $\pi^p(k, c)$, $\pi^l(k, c)$ and $\pi^n(k, c)$ are continuously differentiable in c over $[0, \bar{c}]$ over the subset of $[0, \bar{c}]$ in which $\pi^n(c, k) > 0$. Furthermore, the function $c \rightarrow q^l(n, c)$ is continuously differentiable over the subset of $[0, \bar{c}]$ in which it is strictly positive.

A2'. An identical increase in the costs of all firms but the patent holder decreases each one of those firms' equilibrium profit: $\frac{\partial \pi^l}{\partial c}(n - 1, c) < 0$.

A3'. A non-licensee's equilibrium profit is increasing in the licensees' unit cost: If $\pi^n(k, c) > 0$ then $\frac{\partial \pi^n}{\partial c}(k, c) > 0$ and if $\pi^n(k, c) = 0$ then $\pi^n(k, c') = 0$ for any $c' < c$.

A4'. A firm's market profit is decreasing in the number of licensees in the industry: for any $c < \bar{c}$ and any $k < n - 1$ it holds that $\pi^p(k, c) > \pi^p(k + 1, c)$; $\pi^l(k, c) > \pi^l(k + 1, c)$ and $\pi^n(k, c) \geq \pi^n(k + 1, c)$.

A5'. A firm's market profit increases as it moves from the subgroup of non-licensees to the subgroup of licensees: for any $c < \bar{c}$ and any $k < n - 1$ it holds that $\pi^n(k, c) < \pi^l(k + 1, c)$.

The comparison of the innovator's *overall* profit, i.e. the sum of its market profit and licensing revenues, under the two licensing mechanisms, yields the following result:

Proposition 6 *For sufficiently weak patents, the optimal per-unit royalty contract deterring litigation generates higher overall profit for the patent holder than its fixed fee counterpart if*

$$\frac{\partial \pi^p}{\partial c}(n - 1, \bar{c} - \epsilon) + (n - 1)\frac{\partial \pi^l}{\partial c}(n - 1, \bar{c} - \epsilon) > -(n - 1)q^l(n, \bar{c} - \epsilon). \tag{4}$$

Moreover, the reverse holds if the reverse strict inequality is satisfied.

Proof. See Appendix. ∎

To see how Condition (4) compares to its counterpart when the patent holder is not active on the market, i.e. Condition (3), let us rewrite both of them with the same notations. For that purpose, let us denote by $\Pi^*(c_1, c_2, ..., c_n)$ the sum of all firms' equilibrium market profits, i.e. the equilibrium (downstream) industry profit, and $q_i^*(c_1, c_2, ..., c_n)$ firm i's output when each firm $j = 1, 2, ..., n$ produces at unit cost c_j.

The sufficient condition for a patent holder who is an industry outsider to prefer the per-unit royalty mechanism for sufficiently weak patents can be rewritten as (replacing $\bar{c} - \epsilon$ with the generic variable c):

$$\sum_{i=1}^{n}\frac{\partial \Pi^*}{\partial c_i}(c, c, ..., c) > -\sum_{i=1}^{n}q_i^*(c, c, ..., c). \tag{5}$$

The sufficient condition for a patent holder who is an industry insider to prefer the per-unit royalty mechanism for sufficiently weak patents can be rewritten as (replacing again $\bar{c} - \epsilon$ by the generic variable c, and denoting P the patent holder):

$$\sum_{\substack{i=1 \\ i \neq P}}^{n} \frac{\partial \Pi^*}{\partial c_i} (c, c, ..., c) > -\sum_{\substack{i=1 \\ i \neq P}}^{n} q_i^* (c, c, ..., c). \tag{6}$$

The two inequalities have very close interpertations: Condition (5) means that the *strategic* effect of an identical increase in all firms' (common) unit cost on the industry profit is positive and Condition (6) means that the strategic effect of an increase in the costs of all firms but one on the industry profits is positive (firms being equally efficient initially). Note also that both conditions are implied by the following inequality when it holds for any $i = 1, 2, ..., n$:

$$\frac{\partial \Pi^*}{\partial c_i} (c, c, ..., c) > -q_i^* (c, c, ..., c). \tag{7}$$

This condition means that when firms are equally efficient initially, the strategic effect of an increase in one firm's unit cost on the aggregate profit is positive.

We show in the next section that Condition (7) holds (i) for Cournot competition with homogeneous products under complete generality, and (ii) for Bertrand competition with differentiated products under strategic complementarity, provided existence an uniqueness of a pure-strategy equilibrium holds in these two environments. It then follows that both Condition (5) and Condition (6) hold since they are implied by Condition (7).

6 Two standard oligopoly applications

In this section, we provide sufficient conditions of a general nature on the primitives of the two most widely used models of imperfect competition, which lead to Assumptions **A1-A5** and **A1'-A5'** and Condition (7) being verified. Since some of the results below are new to the oligopoly literature, and of some independent interest, we derive them for fully asymmetric versions of the Cournot and Bertrand oligopolies with linear costs. Accordingly, we also change the notation as needed, relative to the other parts of the paper.

6.1 Cournot competition with homogeneous products

Consider an industry consisting of n firms competing in Cournot fashion. Firm i's marginal cost is denoted c_i (fixed production costs are assumed to be zero or otherwise sunk). Suppose the firms face an inverse demand function $P(\cdot)$ satisfying the following minimal conditions:

C1 $P(\cdot)$ is twice continuously differentiable and $P'(\cdot) < 0$ whenever $P(\cdot) > 0$.
C2 $P(0) > c_i > P(Q)$ for Q sufficiently high, $i = 1, 2, ..., n$.

C3 $P'(Q) + QP''(Q) < 0$ for all $Q \geq 0$ with $P(\cdot) > 0$.

These assumptions are quite standard. **C3** is the familiar condition used by Novshek (1985) to guarantee downward-sloping reaction curves (for any cost function). It is equivalent to the statement that each firm's marginal revenue is decreasing in rivals' output (see Amir, 1996 for an alternative condition).

Firm i's profit function and reaction correspondence are (here, $Q_{-i} = \sum_{j \neq i} q_j$)

$$\pi_i(q_i, Q_{-i}) = q_i \left[P(q_i + Q_{-i}) - c_i \right] \text{ and } r_i(Q_{-i}) = \arg\max_{q_i \geq 0} \pi_i(q_i, Q_{-i}).$$

The next proposition provides general conditions under which Assumptions **A1-A5** and **A1'-A5'** hold for a Cournot oligopoly.

Proposition 7 *Under Assumptions **C1-C3**, the following holds:*

(a) There exists a unique Cournot equilibrium.

(b) Firm i's equilibrium profit π_i^* is differentiable in c_i and in c_j for any $j \neq i$.

(c) Firm i's equilibrium profit π_i^* is decreasing in c_i and increasing in c_j for any $j \neq i$.

If in addition, the game is symmetric (with c denoting the unit cost), then

(d) The unique Cournot equilibrium is symmetric.

(e) The equilibrium output q^* strictly decreases in c.

(f) Per-firm equilibrium profit π^* decreases in c.

Proof. See Appendix. ∎

It is straightforward to relate the different parts of Proposition 7 to Assumptions **A1-A5** and **A1'-A5'**. Part (a) is needed to avoid vacuous statements. Assumptions **A1** and **A1'** are implied by part (b) and the proof of part (e). Assumption **A2** follows from part (f) and Assumption **A2'** follows from combining part (f) with part (c). Assumptions **A3** and **A3'** are implied by part (c). Assumptions **A4** and **A4'** follow from repeated applications of part (c), with one rival firm's cost decreasing at a time. Assumptions **A5** and **A5'** follow from part (c).

We now discuss the scope of these assumptions, focusing on parts (f) and (c), the other assumptions being well known for existence and uniqueness of Cournot equilibrium. Though intuitive, part (f) actually has a less universal scope than one might think. Indeed, there is an extensive literature dealing with taxation in oligopolistic industries and one of its key insights is that a common cost increase can lead to some firms benefiting at the expense of others (Seade, 1985, Kimmel, 1992, and Février and Linnemer, 2004). More surprisingly, in a symmetric setting, a cost increase may be beneficial to all firms, when the inverse demand function is sufficiently convex. In light of this result, part (f) may be viewed as giving sufficient conditions for this counter-intuitive effect of taxation not to arise.

Since the cost paradox literature considers cost increases that are common to all firms, it does not deal directly with the profit effects of a unilateral cost change, as in the results of part (c).

Nevertheless, in light of the cost paradox, it is intuitive that the intuitive findings of part (c) need not hold for sufficiently convex inverse demand functions, as is confimed via an example below. In light of this discussion, it emerges that ruling out the cost paradox and its plausible implications is one of the most restrictive requirements of the present setting. All in all, Assumption C3 is then clearly appropriate here since it guarantees both the existence and uniqueness of Cournot equilibrium (with constant unit costs) and the abscence of the cost paradox.[20]

As an instructive illustration, we provide a class of convex (hyperbolic) demand functions that violates C3.

Example. Consider a duopoly industry with an iso-elastic inverse demand function, $P(Q) = Q^{-1/b}$, $\frac{1}{2} < b < 1$, which clearly fails assumption **C3**. The profit functions are $\pi_i(q_i, q_j) = [(q_i + q_j)^{-1/b} - c_i]q_i$. The equilibrium outputs and profits are, with $i, j \in \{1, 2\}$, $i \neq j$,

$$q_i^* = \frac{(2b-1)^b}{b^b(c_i + c_j)^{b+1}}[c_i(1-b) + c_j b] \text{ and } \pi_i^*(c_i, c_j) = \frac{(2b-1)^{b-1}}{b^b}\frac{[c_i(1-b) + c_j b]^2}{(c_i + c_j)^{b+1}}.$$

It is easily verified that, in violation of our basic assumptions **A2 - A5**,
(i) $\frac{\partial \pi_i^*(c_i, c_j)}{\partial c_i} > 0$ for some values of the parameters. In particular $\frac{\partial \pi_i^*(c_i, c_j)}{\partial c_i}\mid_{c_1 = c_2} > 0$ if $\frac{1}{2} < b < \frac{3}{5}$.
(ii) $\frac{\partial \pi_i^*(c_i, c_j)}{\partial c_j} < 0$ for some values of the parameters.
(iii) In the n-firm symmetric version of this example, $\frac{\partial \pi^*}{\partial c} > 0$ (see Kimmel, 1992, and Février and Linnemer, 2004).

Note that the above counter-intuitive results pertain to the unique interior Cournot equilibrium. Indeed, this example gives rise to two Cournot equilibria, one of which has each firm producing zero output.

As illustrated by this simple example, an insightful perspective on Proposition 7 is that, by imposing one of the commonly used conditions to guarantee existence of Cournot equilibrium via the property of strategic substitutes (Novshek, 1985 and Amir, 1996), namely **C3**, one also obtains as a byproduct that the counter-intuitive results on the effects of uniform or unilateral cost increases (or uniform or individual taxation) on firms' profits do not hold.

Proposition 8 *Under Assumptions **C1-C3**, Condition (7) is verified.*

Proof. See Appendix. ∎

As can easily be seen in the proof, this result actually only requires that total equilibrium output decreases with a unilateral unit cost increase, which holds universally in Cournot competition with linear costs. Note also that this result provides a lower bound on the effect of a unilateral cost increase on industry profits, an issue not considered in the related literature.

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[20]Interestingly, while the alternative condition given in Amir (1996) – that inverse demand be log-concave – is sufficient for existence and uniqueness of equilibrium, we were unable to prove that it guarantees the absence of the cost paradox.

Thus, we can conclude that under the general assumptions **C1-C3**, the holder of a weak patent finds it optimal to license it out using a per-unit royalty licensing contract. This result holds whether the patent holder is active in the (downstream) market or not.

6.2   Bertrand competition with differentiated products

Consider an industry consisting of n single-product firms, with constant unit costs $c_1, c_2, ..., c_n$. Assume that the goods are imperfect substitutes. Denoting $D_i(p_1, p_2, ..., p_n)$ the demand for the good produced by firm i, its profit function and reaction correspondence are defined as usual by

$$\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i}) \text{ and } r_i(p_{-i}) = \arg\max_{p_i} \pi_i(p_i, p_{-i}).$$

We will say that the Bertrand oligopoly is symmetric if the demand functions are symmetric and $c_1 = c_2 = ... = c_n \triangleq c$.

Let $S_i \triangleq \left\{ (p_1, p_2, ..., p_n) \in R^n_+ \mid D_i(p_1, p_2, ..., p_n) > 0 \right\}$. We assume throughout that for every firm i:

B1 D_i is twice continuously differentiable on S_i.
B2 (i)$\frac{\partial D_i}{\partial p_i} < 0$, (ii)$\frac{\partial D_i}{\partial p_j} > 0$ and (iii) $\sum_{k=1}^n \frac{\partial D_i(p,p,...,p)}{\partial p_k} < 0$ over the set S_i.
B3 $D_i \frac{\partial^2 \log D_i}{\partial p_j \partial p_i} - \frac{\partial \log D_i}{\partial p_j} \frac{\partial \log D_i}{\partial p_i} > 0$ over the set S_i, for $j \neq i$.
B4 $\sum_{j=1}^n \frac{\partial^2 D_i(p_1, p_2,...,p_n)}{\partial p_i \partial p_j} < 0$ over the set S_i.

These conditions are quite general, and are commonly invoked for differentiated-good demand systems. They have the following meanings and economic interpretations. For **B2,** part (i) is just the ordinary law of demand; part (ii) says that goods i and j are substitutes; and part (iii) is a dominant diagonal condition for the Jacobian of the demand system, which is required to hold only at equal prices (see e.g., Vives, 1999). It says that, along the diagonal, own price effect on demand exceeds the total cross-price effects. **B3** says that each demand has (differentiably) strict log-increasing differences in own price and any rival's price. The exact economic interpretation is that the price elasticity of demand strictly increases in any rival's price, which is a very natural assumption (Milgrom and Roberts, 1990). **B4** says that the Hessian of the demand system has a dominant diagonal, which is a standard assumption invoked to guarantee uniqueness of Bertrand equilibrium (Milgrom and Roberts, 1990 or Vives, 1999). **B2(iii)** and **B4** hold that own effects of price changes dominate cross effects, for the level and the slope of demand, respectively.

The following proposition provides sufficient conditions for Assumptions **A1-A5** and **A1'-A5'** to hold in this framework.

Proposition 9 *Under Assumptions **B1-B4**,*
(a) The Bertrand game is of strict strategic complements, and has a unique Bertrand equilibrium.
(b) Firm i's equilibrium price p_i^* is increasing in c_j for any j.

(c) Firm i's equilibrium profit π_i^* is differentiable in c_i and c_j for any $j \neq i$.

(d) Firm i's equilibrium profit π_i^* is increasing in c_j for any $j \neq i$.

In addition, if the game is symmetric, then

(e) the unique Bertrand equilibrium is symmetric.

(f) the equilibrium price increases in c.

(g) per-firm equilibrium profit π_i^* is differentiable in c, and decreasing in c.

Proof. See Appendix. ∎

We leave to the reader the task of matching the different parts of Proposition 9 to Assumptions **A1-A5** and **A1'-A5'**, as this step is quite similar to the Cournot case. Note here that in order to fit Assumptions A1-A5 and A1'-A5', one needs to assume a symmetric form of product differentiation, as is often done in the literature.

Anderson, DePalma and Kreider (2001) extends the analysis of the effects of taxation to Bertrand competition with differentiated products, and report analogous findings as in the Cournot case. Since Proposition 9 contains only intuitive results on the effects of cost changes on profits, one concludes that the standard assumptions for existence and uniqueness of Bertrand equilibrium preclude any counter-intuitive effects of exogenous cost increases.

Proposition 10 *Under Assumptions **B1-B4**, Condition (7) is verified.*

Proof. See Appendix. ∎

As can be seen from the proof, this result only requires that each firm's equilibrium price increases with a unilateral unit cost increase, which holds in Bertrand oligopoly with linear costs whenever the game is supermodular (i.e, **B3** holds). This result also provides a lower bound on the effect of a unilateral cost increase on industry profits, an issue not considered in the related literature.

We can then conclude that under the general assumptions **B1-B4**, the holder of a weak patent prefers to license it out using a per-unit royalty licensing contract, both for the cases of an industry insider and outsider.

7 Uncertain patents of any strength

Our analysis has focused so far on "sufficiently weak" patents. One might wonder how weak patents need to be for our results to hold and, relatedly, whether there exists a critical value of patent strength below (above) which a patent holder finds it optimal to use a per-unit royalty (fixed fee) contract. To provide insights into these issues, we investigate in what follows the optimal licensing mechanism for uncertain patents of any strength $\theta \in\ ]0, 1[$ considering in turn (i) a setting with the general form of competition described in Section 2 and "sufficiently small" innovations, (ii) a setting where the competition between the potential licensees is intense, and (iii) a linear Cournot oligopoly with homogeneous products.

7.1 Licensing uncertain patents covering small innovations

In this subsection we consider the general setting described in Section 2 and characterize the optimal mechanism for the licensing of small innovations[21] for *any* value of the patent strength, as long as deterring litigation is optimal. For the sake of simplicity we will assume that the litigation deterrence constraint is binding for the optimal per-unit royalty license deterring litigation, for any $\theta \in]0,1[$ (this has been shown to hold for sufficiently low values of θ in Proposition 3). In others words, when using a royalty contract, the licensor finds it optimal to ask for the maximum per-unit royalty $r(\theta)$ that deters litigation.[22]

Proposition 11 *(i) If $\frac{\partial \pi^e}{\partial c}(n,\bar{c}) + q^e(n,\bar{c}) \geq \frac{\partial \pi^i}{\partial c}(n-1,\bar{c})$, then for any $\theta \in]0,1[$, there exists $\tilde{\epsilon}_\theta > 0$ such that for any $\epsilon \in]0,\tilde{\epsilon}_\theta[$ the patent holder prefers to offer a per-unit royalty contract.*

(ii) If $0 < \frac{\partial \pi^e}{\partial c}(n,\bar{c}) + q^e(n,\bar{c}) < \frac{\partial \pi^i}{\partial c}(n-1,\bar{c})$, then, denoting $\tilde{\theta} = \frac{\frac{\partial \pi^e}{\partial c}(n,\bar{c}) + q^e(n,\bar{c})}{\frac{\partial \pi^i}{\partial c}(n-1,\bar{c})}$, we have the following:

- If $\theta \in \left]0,\tilde{\theta}\right[$ then there exists $\tilde{\epsilon}_\theta > 0$ such that for any $\epsilon \in]0,\tilde{\epsilon}_\theta[$ the patent holder prefers to offer a per-unit royalty contract.

- If $\theta \in \left]\tilde{\theta},1\right[$ then there exists $\hat{\epsilon}_\theta > 0$ such that for any $\epsilon \in]0,\hat{\epsilon}_\theta[$ the patent holder prefers to offer a fixed fee contract.

Proof. See Appendix. ∎

This proposition shows that the strategic effect of an increase in all the licensees' marginal cost on their profits, i.e. $\frac{\partial \pi^e}{\partial c}(n,\bar{c}) + q^e(n,\bar{c})$, is still driving the choice of the licensing mechanism when we extend the analysis to patents of any strength covering small innovations.[23] However, in contrast to the analysis focusing on sufficiently weak patents, what matters now is the *magnitude* of this strategic effect and not merely its sign.[24] If that magnitude is sufficiently large, in the sense that $\frac{\partial \pi^e}{\partial c}(n,\bar{c}) + q^e(n,\bar{c}) \geq \frac{\partial \pi^i}{\partial c}(n-1,\bar{c})$, then the per-unit royalty mechanism is preferred over the fixed fee mechanism *whatever* the patent's strength (as long as deterring litigation is optimal to the patent holder). However, if it is moderate, in the sense that $\frac{\partial \pi^e}{\partial c}(n,\bar{c}) + q^e(n,\bar{c}) < \frac{\partial \pi^i}{\partial c}(n-1,\bar{c})$, then the optimal licensing scheme depends on the patent's strength: there exists a threshold $\tilde{\theta}$, which is increasing in the magnitude of the strategic effect, such that patents whose strength is lower than $\tilde{\theta}$ are licensed by means of per-unit royalties while those whose strength is higher than $\tilde{\theta}$ are licensed

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[21] Many empirical studies conclude that the distribution of patent values is highly rightward-skewed (see e.g. Lanjouw et al., 1996) which suggests that the majority of innovations are indeed "small".

[22] Note that under the usual setting of Cournot competition with homogeneous products and linear demand, this holds whenever the innovation size is not too large, which is consistent with our focus on small innovations in this subsection.

[23] As we have already established that this strategic effect is positive in the standard oligopoly models with broad generality (see Section 6), we excluded from the proposition the (very unlikely) scenario where this effect would be negative. Note however that it is straightforward to include it.

[24] More precisely, what matters is the relative magnitude of this strategic effect with respect to the marginal loss a firm would incur if all its rivals' marginal costs uniformly fall, i.e. the ratio $\frac{\frac{\partial \pi^e}{\partial c}(n,\bar{c}) + q^e(n,\bar{c})}{\frac{\partial \pi^i}{\partial c}(n-1,\bar{c})}$.

by means of fixed fees. Therefore, the lower the patent's strength the higher the propensity to use per-unit royalties. Moreover, the lower the magnitude of the identified strategic effect, the more likely the holder of an uncertain patent will license it using a per-unit royalty contract.

7.2 Licensing uncertain patents when competition is intense

In this subsection, we show that the licensor of an uncertain patent will find it optimal to use a per-unit royalty contract whenever competition between the potential licensees is intense enough to drive an unsucessful challenger out of the market, and a global condition on the strategic effect of a cost variation on equilibrium profits holds. We then esatblish that the latter condition, while being more restrictive than Condition (3), is satisfied under Cournot competition with homogeneous products and Bertrand competition with differentiated products under the same general conditions considered in Section 6, whenever the innovation size is not too large in a sense that we specify.

We consider again the general setting described in Section 2 and assume that the litigation deterrence constraint is binding for the optimal per-unit royalty license deterring litigation, for any $\theta \in]0, 1[$. It follows from Section 4 that:

$$
\begin{aligned}
\tilde{P}_r(\theta) - \tilde{P}_F(\theta) &= n\left[r(\theta)q^e(n, \bar{c} - \epsilon + r(\theta)) - \pi^e(n, \bar{c} - \epsilon) + \pi^e(n, \bar{c} - \epsilon + r(\theta))\right] \quad (8) \\
&\quad - n\theta\left[\pi^i(n-1, \bar{c} - \epsilon + r(\theta)) - \pi^i(n-1, \bar{c} - \epsilon)\right].
\end{aligned}
$$

The first term between brackets is positive if the following condition, which is a global version of the local Condition (3), holds :

$$
\pi^e(n, c') - \pi^e(n, c) > -(c' - c)q^e(n, c') \text{ for any } c, c' \in [\bar{c} - \epsilon, \bar{c}] \text{ such that } c < c'. \quad (9)
$$

In what follows we argue that the second term between brackets in (8) is zero if competition is intense enough, which will ensure that the per-unit royalty scheme is optimal from the patent holder's perspective whenever the competitive environment is such that Condition (9) is met. For this purpose, let us denote $\gamma \in [0, 1[$ a measure of competition intensity satisfying the following natural assumptions:

(i) For any $\gamma \in [0, 1[$ there exists a threshold $\tilde{c}(\gamma) < \bar{c}$ such that $\pi^i(n-1, c) > 0 \iff c > \tilde{c}(\gamma)$,

(ii) $\tilde{c}(\gamma)$ is (weakly) increasing in γ and $\tilde{c}(\gamma) \to \bar{c}$ as $\gamma \to 1$.

Denote $\bar{\gamma} \in [0, 1[$ the level of competition intensity such that $\pi^i(n-1, \bar{c} - \epsilon) = 0$ if and only if $\gamma \geq \bar{\gamma}(\epsilon)$. For any $\gamma \geq \bar{\gamma}$, let $\bar{\theta}(\gamma)$ be the unique value of θ such that $\bar{c} - \epsilon + r(\theta) = \tilde{c}(\gamma)$. Then, from (i) it follows that for any $\theta \in [0, \bar{\theta}(\gamma)]$, it holds that $\pi^i(n-1, \bar{c} - \epsilon + r(\theta)) = 0$ and, *a fortiori*, $\pi^i(n-1, \bar{c} - \epsilon) = 0$. Moreover, using (ii) we get that $\bar{\theta}(\gamma)$ is increasing in γ and $\bar{\theta}(\gamma) \to 1$ as $\gamma \to 1$. We can therefore state the following:

Proposition 12 *If Condition (9) holds then there exists a level of competition intensity $\bar{\gamma} < 1$ and a threshold $\bar{\theta}(\gamma)$ increasing in γ such that, for any $\gamma \in [\bar{\gamma}, 1[$, the licensor of a patent of strength*

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$\theta \in \left[0, \bar{\theta} \left(\gamma \right) \right]$ *prefers to use a per-unit royalty contract. Moreover* $\bar{\theta} \left(\gamma \right) \to 1$ *as* $\gamma \to 1$.

Alternatively, this proposition can be formulated as follows: If Condition (9) is satisfied, then for *any* patent strength value $\theta \in [0, 1[$ the licensor will find it optimal to use a per-unit royalty contract if the intensity of competition γ is above some threshold $\tilde{\gamma} \left(\theta \right)$ which is increasing in θ and goes to 1 when θ goes to 1. It is therefore crucial to assess the generality of Condition (9). Note first that it implies the local Condition (3) (while the converse does not hold) and that the interpretations for the two conditions are very close. Rewritting (9) as $\pi^e(n, c) - \pi^e(n, c') < (c' - c)q^e(n, c')$ for any $c, c' \in [\bar{c} - \epsilon, \bar{c}]$ such that $c < c'$ shows that it means the following: the strategic effect of a *decrease* in the common marginal cost (from c' to c) on profits is *negative*.[25]

In the next proposition we provide sufficient conditions of a general nature for Condition (9) to hold under Cournot competition with homogeneous products and Bertrand competition with (symmetrically) differentiated products.

Proposition 13 *(i) Consider the Cournot setting with homogeneous products described in Section 6.1 and assume that assumptions* **C1-C3** *hold. Denote* $Q^e(n, c)$ *the industry equilibrium output when all firms produce at marginal cost* c *and* $Q^m(c)$ *the monopoly output with marginal cost* c. *Then Condition (9) holds whenever* $Q^m(\bar{c} - \epsilon) \leq Q^e(n, \bar{c})$.

(ii) Consider the symmetric version of the Bertrand setting described in Section 6.2 and assume that assumptions **B1-B4** *hold. Denote* $p^e(n, c)$ *the equilibrium price when all firms produce at marginal cost* c *and* $p^m(c)$ *the (multi-product) monopoly price with marginal cost* c *for all products. Then Condition (9) holds whenever* $p^e(n, \bar{c}) \leq p^m(\bar{c} - \epsilon)$.

Proof. See Appendix. ■

In the Cournot setting, it can be easily shown that $Q^m(\bar{c}) < Q^e(n, \bar{c})$ and that the monopoly output $Q^m(c)$ is decreasing in c, which allows to interpret the condition $Q^m(\bar{c} - \epsilon) \leq Q^e(n, \bar{c})$ as ϵ being not too large. To get a sense of how restrictive this condition is, consider the special case of a linear inverse demand $p = a - Q$. Then $Q^m(\bar{c} - \epsilon) \leq Q^e(n, \bar{c})$ if and only if $\epsilon \leq \frac{n-1}{n+1} (a - \bar{c})$. To see why this condition is not very restrictive, note that in this particular setting an innovation is *drastic* if $\epsilon \geq (a - \bar{c})$. Considering now the Bertrand setting, it is straighforward that the condition $p^e(n, \bar{c}) \leq p^m(\bar{c} - \epsilon)$ may also be interpreted as ϵ being not too large (for a given level of product differentiation). Alternatively, it could be interpreted as product differentiation being sufficiently low (for a given innovation size ϵ).

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[25] Note that Condition (3) could also be interpreted in this way as the strategic effect of a *local* decrease in marginal costs $-\frac{\partial \pi^e}{\partial c} (n, c) + q^e (n, c)$ is exactly the opposite of the strategic effect of an increase in marginal cost. However, this does not hold when we consider global conditions such that (9): the strategic effect of an increase in marginal costs (from c to c') is $\pi^e(n, c') + (c' - c)q^e(n, c) - \pi^e(n, c)$ while the strategic effect of a decrease in marginal costs (from c' to c) is $\pi^e(n, c) - (c' - c)q^e(n, c) - \pi^e(n, c')$.

7.3 Licensing uncertain patents to Cournot competitors

In this subsection, we consider the special case of Cournot competition with linear demand $p = a - Q$ and offer a complete characterization of the comparison between per-unit royalties and fixed fees for probabilistic patents of any strength covering innovations such that $\epsilon \leq \frac{a - \bar{c}}{2(n-1)}$. The latter condition implies in particular that $\pi^i(n - 1, \bar{c} - \epsilon) > 0$, i.e. that a challenger would always remain active should the patent be upheld[26] (in constrast to the scenario we focused on in Subsection 7.2). We first establish that there is a unique threshold patent strength below which per-unit royalties dominate and above which fixed fees are preferred, and then show that this threshold patent strength is significantly high for a large set of parameters. This will complement our finding in Subsection 7.2. by proving that even in settings where competition is sufficiently moderate to allow an unsucessful challenger to remain active, the range of patent strength values for which the per-unit royalty mechanism is preferred by the licensor can be substantially wide.

Standard computations show that the relevant equilibrium profits for the case we consider here are given by

$$\pi^e(n, \bar{c} - \epsilon + r) = \frac{(a - \bar{c} + \epsilon - r)^2}{(n+1)^2}, \pi^e(n, \bar{c} - \epsilon) = \frac{(a - \bar{c} + \epsilon)^2}{(n+1)^2}, \pi^i(n-1, \bar{c} - \epsilon + r) = \frac{[a - \bar{c} - (n-1)(\epsilon - r)]^2}{(n+1)^2}.$$

The optimal per-unit royalty $r(\theta)$ is the unique positive solution in r to the equation

$$(a - \bar{c} + \epsilon - r)^2 = \theta[a - \bar{c} - (n-1)(\epsilon - r)]^2 + (1 - \theta)(a - \bar{c} + \epsilon)^2. \tag{10}$$

Solving (10) yields

$$r(\theta) = \epsilon - \frac{\sqrt{(a - \bar{c})^2 [1 + \theta(n-1)]^2 + \epsilon(1 - \theta)[2(a - \bar{c}) + \epsilon]\left[1 - \theta(n-1)^2\right]} - (a - \bar{c})[1 + \theta(n-1)]}{1 - \theta(n-1)^2}.$$

The licensing revenue generated by the optimal per unit royalty contract is given by:

$$P_r(\theta) = \frac{n}{n+1} r(\theta)[a - \bar{c} + \epsilon - r(\theta)].$$

The optimal fixed fee $F(\theta)$ is:

$$F(\theta) = \theta[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)] = \theta \frac{n\epsilon[2(a - \bar{c}) + \epsilon(2 - n)]}{(n+1)^2}.$$

---

[26] A weaker condition, i.e. $\epsilon < \frac{a - \bar{c}}{n-1}$, is actually sufficient to ensure that $\pi^i(n - 1, \bar{c} - \epsilon) > 0$.

and, therefore, the licensing revenue generated by the optimal fixed fee contract is :

$$P_F(\theta) = nF(\theta) = \theta n^2 \epsilon \frac{[2(a - \bar{c}) + \epsilon(2 - n)]}{(n + 1)^2}.$$

The following proposition fully characterizes the patent holder's optimal licensing mechanism (under the constraint of litigation deterrence) for all patent strength values $\theta \in]0, 1[$.

Proposition 14 *There exists a unique threshold $\hat{\theta} \in]0, 1[$ such that $P_r(\theta) \gtreqless P_F(\theta)$ if and only if $\theta \lesseqgtr \hat{\theta}$.*

Proof. See Appendix. ■

It remains only to complement this result with some insight on the possible values of the threshold patent strength $\hat{\theta}$ that can arise in plausible specifications of the linear Cournot model and the innovation parameters. In this setting it is easy to see that the parameters a, $\bar{c}$ and ϵ affect the comparison of $P_r(\theta)$ and $P_F(\theta)$ only through the ratio $\frac{a - \bar{c}}{\epsilon}$. Taking account of the condition $\epsilon \leq \frac{a - \bar{c}}{2(n-1)}$, we compute $\hat{\theta}$ for the set of the 1000 pairs $\left(n, \frac{a - \bar{c}}{\epsilon}\right)$ such that $n = 2, 3, ..., 11$ and $\frac{a - \bar{c}}{\epsilon} = 2(n - 1) + i$ where $i = 1, 2, ..., 100$ and find values that are all greater than 0.5. More precisely, all the values of $\hat{\theta}$ are found to be within the interval $[0.501, 0.678]$. This shows that, even when competition is sufficiently moderate to allow an unsuccessful challenger to remain active, the superiority of the per-unit royalty mechanism is not a phenomenon that arises only for very small values of θ.

8 Welfare implications

Denote $W(k, c)$ the social welfare, defined as the sum of aggregate profits (including the patent holder's) and consumers' surplus, when $k \leq n$ out of the n downstream firms produce at marginal cost $c \leq \bar{c}$, and the remaining $n - k$ firms produce at marginal cost $\bar{c}$. Assume that $W(k, c)$ is increasing in the number k of efficient firms and decreasing in those firms' marginal cost c. This is a very natural assumption, which holds for instance in the usual settings of Cournot oligopoly and symmetrically differentiated Bertrand oligopoly, both with linear demands.

The literature on *ironclad* patents shows that the holders of such patents may have incentives to restrict the number of licensees when using fixed fee contracts while they find it optimal to license all firms if they use per-unit royalty contracts (see e.g. Kamien, 1992): it may hold that $k_r^* = n$ and $k_F^* < n$ where k_r^* (resp. k_F^*) denotes the number of licensees under the patent holder's optimal per-unit royalty (resp. fixed fee) contract. In such a context, a trade-off between the number of licensees and their efficiency exists: it is *a priori* unclear whether welfare when licensing is made through a per-unit royalty contract, i.e. $W(n, \bar{c} - \epsilon + r^*)$ where $r^* > 0$ is the optimal per-unit royalty from the patent holder's perspective, is higher or lower than social welfare when a fixed fee contract is used, i.e. $W(k_F^*, \bar{c} - \epsilon)$. Such ambiguity does not exist in our setting: whenever the patent holder finds it optimal to deter litigation, *all* firms have to be licensed. Therefore, the use of a contract involving the

payment of per-unit royalty $r > 0$ results in a social welfare $W(n, \bar{c}-\epsilon+r)$, while the use of a fixed fee contract yields a social welfare $W(n, \bar{c}-\epsilon)$. It then follows from $r > 0$ that $W(n, \bar{c}-\epsilon+r) < W(n, \bar{c}-\epsilon)$, which means that the fixed fee scheme is socially superior to the per-unit royalty scheme (and to any two-part tariff mechanism involving the payment of a positive per-unit royalty). Thus, whenever the patent holder prefers to offer a per-unit royalty contract, which is the case in particular for weak patents, the privately optimal licensing scheme will be socially suboptimal. This highlights a particular channel through which uncertainty over patent validity can be detrimental to society.

9 Conclusion

The issue of patent quality is one of the most serious problems facing the patent system. Mark Lemley who was arguing some time ago against increasing the resources allocated to the Patent Office in order to improve the examination of patent applications (Lemley, 2001) is now much more concerned about this issue: "*Some bad quality patents award legal rights that are far broader than what their relevant inventors actually invented, and they do so with respect to technologies that turn out to be economically significant. Many Internet patents fall into this category. Rarely a month goes by that some unknown patent holder does not surface and claim to be the true inventor of eBay or the first to come up with now-familiar concepts like hyperlinking and e-commerce. While some such Internet patents may be valid—someone did invent those things, after all—more often the people asserting the patents actually invented something much more modest. But they persuaded the Patent Office to give them rights that are broader than what they actually invented, imposing an implicit tax on consumers and thwarting truly innovative companies who do or would pioneer those fields*" (Lemley, 2012).

One negative consequence of the issuance of weak patents on social welfare stems from the legal costs of patent dispute resolution in court. The use of licensing agreements to deter or settle patent litigation allows to avoid those *direct* costs but induces another *indirect* loss in welfare, in particular due to the use of a licensing mechanism which is potentially suboptimal from a social standpoint. We therefore argue that it is important to get a better understanding of the contracts used for the licensing of technologies covered by dubious patents. In this paper we provide a sufficient condition under which the holder of a weak patent prefers to license its technology through a per-unit royalty contract rather than a fixed fee contract and show that this condition is very mild: it holds under general conditions for a Cournot oligopoly with homogeneous goods and a Bertrand oligopoly with heterogeneous goods, regardless of whether the patent holder is an outsider or an insider to the industry. A significant difference with respect to the literature on the licensing of ironclad patents is that we get a clear-cut result on the comparison of a patent holder's profits under the two schemes, independently of the type of downstream competition, the degree of differentiation between products and whether the patent holder is active or not in the downstream market, while varying any of these three features can overturn the outcome of the comparison when ironclad patents are considered. Furthermore, it is shown that society is always harmed by the patent holder's choice of a per-unit

royalty contract in our setting. Our analysis therefore suggests that constraining or encouraging the licensors of questionable patents to use fixed-fee contracts may yield welfare gains.

Finally, our model generates some testable predictions that might be worth investigating: First, our results suggest that per-unit royalty licenses should be more prevalent in industries with a significant proportion of firms holding questionable patents, e.g., industries relying on some new patentable subject matter (biotechnology, software, business methods,...). Second, if the predictions of our model are correct then under the presumption that the EPO is more stringent in checking the patentability standards than the USPTO, the use of per-unit royalties should be less prevalent in the EU than in the US.

10 References

Allison, J. and M. Lemley, 1998. Empirical evidence on the validity of litigated patents, *AIPLA Quarterly Journal* 26, 185-277.

Amir, R.,1996. Cournot oligopoly and the theory of supermodular games, *Games and Economic Behavior*, 15, 132-148.

Amir, R. and V.E. Lambson, 2000. On the effects of entry in Cournot markets, *Review of Economic Studies* 67, 235-54.

Amir, R. and J. Wooders, 2000. One-way spillovers, endogenous innovator/imitator roles, and Research Joint Ventures, *Games and Economic Behavior*, 31, 1-25.

Anderson, S., A. de Palma and B. Kreider, 2001. Tax incidence in differentiated product oligopoly, *Journal of Public Economics*, 81, 173-192.

Anand, B., and T. Khanna. The structure of licensing contracts, *Journal of Industrial Economics* 48, 103-135.

Arrow, K.J., 1962. Economic welfare and the allocation of resources for invention. In: Nelson, R.R. (Ed.), *The Rate and Direction of Inventive Activity,* Princeton University Press.

Ayres, I. and P. Klemperer, 1999. Limiting patentees' market power without reducing innovation incentives: the perverse benefits of uncertainty and non-injunctive remedies, *Michigan Law Review* 97, 985-999.

Bhattacharya, S., C. d'Aspremont, S. Guriev, D. Sen, and Y. Tauman, 2012. Cooperating in R&D: Patenting, Licensing and Contracting, CORE Discussion Paper 2012/55.

Beggs, A.W., 1992. The licensing of patents under asymmetric information, *International Journal of Industrial Organization* 10, 171–191.

Bessen, J. and M.J. Meurer, *Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk*, Princeton University Press 2008.

Boone, J., 2001. Intensity of competition and the incentive to innovate, *International Journal of Industrial Organization*, 19, 705-726.

Bousquet, A., H. Cremer, M. Ivaldi, and M. Wolkowicz, 1998. Risk sharing in licensing, *International Journal of Industrial Organization* 16, 535–554.

Caballero-Sanz, F., R. Moner-Coloques and J.J. Sempere-Monerris, 2002. Optimal licensing in a spatial model. *Annales d'Economie et de Statistique* 66, 257-279.

Choi, J.P., 2001. Technology transfer with moral hazard, *International Journal of Industrial Organization* 19, 249–266.

Choi, J.P., 2010. Patent pools and cross-licensing in the shadow of patent litigation. *International Economic Review* 51, 441-460.

Encaoua, D. and Y. Lefouili, 2005. Choosing intellectual protection: Imitation, patent strength and licensing, *Annales d'Economie et de Statistique*, 79-80, 241-271.

Encaoua, D. and Y. Lefouili, 2009. Licensing "weak" patents, *The Journal of Industrial Economics* 57, 492-525.

Farrell, J. and R.P. Merges, 2004. Incentives to challenge and defend patents: why litigation won't reliably fix patent office errors and why administrative patent review might help, *Berkeley Technology Law Journal, Annual Review of Law and Technology* 19.

Farrell, J. and C. Shapiro, 2008. How strong are weak patents? *American Economic Review* 98, 1347-1369.

Février, P and L. Linnemer, 2004, Idiosyncratic shocks in a asymmetric Cournot oligopoly, *International Journal of Industrial Organization*, 22, 6, 835-848.

Friebel, G., A. K. Koch, D. Prady and P. Seabright, 2006, Objectives and Incentives at the European Patent Office, Institut d'Economie Industrielle, Toulouse.

Gallini, N.T. and B.D. Wright, 1990. Technology transfer under asymmetric information, *RAND Journal of Economics*, 21, 147–160.

Ginarte , J. and W. Park, 1997. Determinants of patent rights: A cross-national study, *Research Policy* 26, 283-301.

Guellec, D. and B. van Pottelsberghe de la Potterie, 2007. *The Economics of the European Patent System*, Oxford Press.

Kamien, M.I., 1992. Patent licensing. In: Aumann, R.J., Hart, S. (Eds.), *Handbook of Game Theory with Economic Applications*, Elsevier Science, North Holland, vol. 1, pp. 331–354.

Kamien, M.I., S.S. Oren and Y. Tauman, 1992. Optimal licensing of cost-reducing innovation, *Journal of Mathematical Economics* 21, 483–508.

Kamien, M.I. and Y. Tauman, 1984. The private value of a patent: a game theoretic analysis, *Z. Nationalökon* 4 (Supplement), 93–118.

Kamien, M.I. and Y. Tauman, 1986. Fees versus royalties and the private value of a patent, *Quarterly Journal of Economics* 101, 471–491.

Kamien, M.I. and Y. Tauman, 2002. Patent licensing: the inside story, *Manchester School* 70, 7–15.

Katz, M.L. and C. Shapiro, 1985. On the licensing of innovations, *RAND Journal of Economics* 16, 504–520.

Katz, M.L. and C. Shapiro, 1986. How to license intangible property, *Quarterly Journal of Economics* 101, 567–589.

Kimmel, S., 1992. Effects of cost changes on oligopolists' profits, *Journal of Industrial Economics*, 40, 4, 441-449.

Langinier, C. and P. Marcoul, 2009. Monetary and Implicit Incentives of the Patent Examiners, Working Paper, University of Alberta.

Lanjouw, J.O., A. Pakes, and J. Putnam, 1996. How To Count Patents and Value Intellectual Property: Uses of Patent Renewal and Application Data, NBER working paper no.5741, September 1996.

Lei, Z. and B. Wright, 2010. Why Weak Patents? Rational Ignorance or Pro-"Customer" Tilt?, Working Paper, Department of Agricultural and Resource Economics, UC Berkeley.

Lemley, M., 2001, Rational Ignorance at the Patent Office, *Northwestern University Law Review* 95, 4.

Lemley, M., 2012, Fixing the Patent Office, W.P. 18081, NBER, available at http://nber.org/papers/w18081

Lemley, M. and C. Shapiro, 2005, Probabilistic patents, *Journal of Economic Perspectives* 19, 2, 75-98.

Macho-Stadler, I. and J.D. Pérez-Castrillo, 1991. Contrats de licence et asymétrie d'information, *Annales d'Economie et de Statistique* 24, 189–208.

Macho-Stadler, I., X. Martinez-Giralt, X. and J.D. Pérez-Castrillo, 1996. The role of information in licensing contract design, *Research Policy* 25, 25–41.

Milgrom, P. and J. Roberts, 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities, *Econometrica* 58(6), 1255-77.

Muto, S., 1993. On licensing policies in Bertrand competition, *Games and Economic Behavior* 5, 257–267.

Novshek, W., 1985. On the existence of Cournot equilibrium, *Review of Economic Studies* L (II), 85–98.

Poddar, S. and U.B. Sinha, 2004. On patent licensing and spatial competition, *Economic Record* 80, 208–218.

Reinganum, J. F., 1982. Strategic Search Theory, *International Economic Review*, 23, 1-17.

Rockett, K., 1990. The quality of licensed technology, *International Journal of Industrial Organization* 8, 559–574.

Rostoker, M., 1984. A survey of corporate licensing, *IDEA: Journal of Law and Technology* 24, 59–92.

Saracho, A.I., 2002. Patent licensing under strategic delegation. *Journal of Economics and Management Strategy* 11, 225–251.

Seade, J., 1985. Profitable cost increases and the shifting of taxation, The Warwick Economics Research Paper Series.

Sen, D., 2002. Monopoly profit in a Cournot oligopoly, *Economics Bulletin* 4, 1–6.

Sen, D., 2005a. On the coexistence of different licensing schemes, *International Review of Economics and Finance* 14, 393-413.

Sen, D., 2005b. Fee versus royalty reconsidered, *Games and Economic Behavior* 53, 141-147.

Sen D. and Y. Tauman, 2007. General licensing schemes for a cost-reducing innovation, *Games and Economic Behavior* 59, 163-186.

Sen D. and Y. Tauman, 2012. Licensing of cost-reducing innovation in a Cournot oligopoly, Working Paper, Stony Brook University.

Shapiro, C., 1985. Patent licensing and R&D rivalry, *American Economic Review, Papers and Proceedings* 75, 25–30.

Shapiro, C., 2003. Antitrust limits to patent settlements, *RAND Journal of Economics* 34, 391-411.

Stamatopoulos, G., Tauman, Y., 2007. Licensing of a quality-improving innovation, *Mathematical Social Sciences*, 56, 2008, 410-438.

Topkis, D., 1998. *Supermodularity and Complementarity*, Princeton University Press.

Taylor, C. and Z. Silberstone, 1973. *The Economic Impact of the Patent System*, Cambridge Univ. Press, Cambridge.

van Pottelsberghe de la Potterie B., 2011. The quality factor in patent systems, *Industrial and Corporate Change*, 20, 1755-1793

Vives, X., 1999, *Oligopoly pricing: old ideas and new tools*, MIT Press, Cambridge, MA.

Vonortas, N.S. and Y.J. Kim, 2004. Technology licensing, in: *OECD Conference Proceedings: Patents, Innovation and Economic Performance*, 181-199

Wang, X.H., 1998. Fee versus royalty licensing in a Cournot duopoly model, *Economic Letters* 60, 55–62.

Wang, X. H., 2002. Fee versus royalty licensing in a differentiated Cournot duopoly, Journal of Economics and Business, 54, 253-266.

Wang, X.H. and B. Yang, 1999. On licensing under Bertrand competition, *Australian Economic Papers* 38, 106–119.

Zuniga, P.M. and D. Guellec, 2009, Who licenses out patents and why?, *OECD Science, Technology and Industry Working Papers*, 2009/5.

11 Appendix

Proof of Proposition 1

All firms accepting the license offer is an equilibrium if and only if:

$$\pi^e(n, \bar{c} - \epsilon + r) \geq \theta \pi^i(n - 1, \bar{c} - \epsilon + r) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon)$$

which can be rewritten as:

$$g(r, \theta) \equiv \pi^e(n, \bar{c} - \epsilon + r) - \theta \pi^i(n - 1, \bar{c} - \epsilon + r) - (1 - \theta) \pi^e(n, \bar{c} - \epsilon) \geq 0.$$

We have $g(0, \theta) = \theta \left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)\right] \geq \theta \left[\pi^e(n, \bar{c}) - \pi^i(n - 1, \bar{c} - \epsilon)\right]$
$\geq \theta \left[\pi^i(n - 1, \bar{c}) - \pi^i(n - 1, \bar{c} - \epsilon)\right] > 0$ (by **A2** and **A5**) and $g(\epsilon, \theta) = \pi^e(n, \bar{c}) - \theta\pi^i(n - 1, \bar{c}) - (1 - \theta)\pi^e(n, \bar{c} - \epsilon) = (1 - \theta)(\pi^e(n, \bar{c}) - \pi^e(n, \bar{c} - \epsilon)) < 0$ (by **A2**). Combining this with g being continuous (by **A1**) and strictly decreasing in r (by **A3**) yields: i/ the existence and uniqueness of a solution in r to the equation $g(r, \theta) = 0$ (within the interval $[0, \epsilon[$), which we denote by $r(\theta)$; ii/ the equivalence between te inequalities $g(r, \theta) \geq 0$ and $r \leq r(\theta)$.

Proof of Proposition 2

We have already shown (in the main text) that all firms deciding to purchase a license at a fixed fee F is an equilibrium of the second stage subgame if and only if:

$$F \leq F(\theta) \triangleq \theta \left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)\right].$$

All firms but one deciding to purchase a license is an equilibrium of the second stage subgame if the following two conditions hold:

$$\theta\pi^i(n - 1, \bar{c} - \epsilon) + (1 - \theta)\pi^e(n, \bar{c} - \epsilon) \geq \pi^e(n, \bar{c} - \epsilon) - F$$

and

$$\theta\left[\pi^e(n - 1, \bar{c} - \epsilon) - F\right] + (1 - \theta)\pi^e(n, \bar{c} - \epsilon) \geq \theta\pi^i(n - 2, \bar{c} - \epsilon) + (1 - \theta)\pi^e(n, \bar{c} - \epsilon).$$

Thus, all firms but one deciding to purchase a license is an equilibrium if and only if:

$$F(\theta) \leq F \leq F_{n-1} \triangleq \pi^e(n - 1, \bar{c} - \epsilon) - \pi^i(n - 2, \bar{c} - \epsilon).$$

Denoting $F_k \triangleq \pi^e(k, \bar{c} - \epsilon) - \pi^i(k - 1, \bar{c} - \epsilon)$ for each $k = 1, 2, .., n$, we can further show that for any $k = 1, 2, ..., n - 2$, a number k of firms accepting the license offer and the other $n - k$ firms not doing so is an equilibrium if and only if:

$$F_{k+1} \leq F \leq F_k.$$

Moreover, all firms deciding not to buy a license is an equilibrium if and only if:

$$F \geq F_1.$$

If we assume that the sequence $(F_k)_{1 \leq k \leq n}$ is decreasing, i.e. a firm's willingness to pay for a license (under ironclad patent protection) decreases with the number of licensees, and that a firm which is indifferent between accepting and refusing the license offer buys a license, then for any $F \geq 0$, there is a unique equilibrium to the second stage subgame up to a permutation of firms: all the equilibria of the second stage subgame involve the same number of licensees (which allows to define a "demand function" for licenses which is decreasing in the fixed fee F). However, if $(F_k)_{1 \leq k \leq n}$ is not decreasing

then there might exist some values of F for which there is either no (pure-strategy) equilibrium or multiple equilibria with different number of licensees.

However, if we focus on small values of θ and do not care about whether pure-strategy equilibria exist - and which one arises in case they do - if all firms accepting the license offer is not an equilibrium (as in the present paper), then the problem of multiplicity or inexistence of equilibria depicted above does not affect our analysis. The reason is that, to be sure that all firms accepting a license is the unique equilibrium whenever it is an equilibrium, i.e. whenever $F \leq F(\theta)$, we only need the inequality $F(\theta) \leq F_k$ to hold for any $k = 1, 2, ..., n-1$, which, given that $F(\theta) = \theta F_n$, is true if θ is small enough, and more specifically if

$$\theta \leq \bar{\theta} = \frac{\min_{1 \leq k \leq n-1} F_k}{F_n}.$$

Proof of Proposition 3

All firms accepting the payment of a per-unit royalty r is an equilibrium if and only if $r \leq r(\theta)$. Furthermore, **A1** ensures that the licensing revenue function $r \to nrq^e(n, \bar{c} - \epsilon + r)$ is strictly increasing in the neighborhood of 0 (its derivative at $r = 0$ being $q^e(n, \bar{c} - \epsilon) > 0$). Since $r(\theta)$ is continuous (by the Implicit Function Theorem) and increasing and $r(0) = 0$, we can conclude that, for θ sufficiently small, the function $nrq^e(n, \bar{c} - \epsilon + r)$ is increasing over $[0, r(\theta)]$ and, therefore, the optimal per-unit royalty license accepted by all firms involves the payment of the royalty rate $r(\theta)$, that is, the litigation deterrence constraint is binding.

The optimal fixed fee license deterring litigation maximizes the patent holder's revenues nF under the constraint $F \leq F(\theta)$. It is straightforward that solution to this constrained maximization program is $F = F(\theta)$.

Proof of Proposition 4

Since $\tilde{P}_r(0) = \tilde{P}_F(0)$ then $\tilde{P}_r(\theta) > \tilde{P}_F(\theta)$ for θ sufficiently small if:

$$\frac{d\tilde{P}_r(\theta)}{d\theta}\Big|_{\theta=0} - \frac{d\tilde{P}_F(\theta)}{d\theta}\Big|_{\theta=0} > 0 \tag{11}$$

which can be rewritten as:

$$nr'(0)q^e(n, \bar{c} - \epsilon) > n\left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)\right]$$

because $r(0) = 0$. Moreover differentiating at $\theta = 0$ the equation defining $r(\theta)$, that is,

$$\pi^e(n, \bar{c} - \epsilon + r(\theta)) = \theta\pi^i(n - 1, \bar{c} - \epsilon + r(\theta)) + (1 - \theta)\pi^e(n, \bar{c} - \epsilon)$$

we get:

$$r'(0)\frac{\partial\pi^e}{\partial c}(n, \bar{c} - \epsilon) = \pi^i(n - 1, \bar{c} - \epsilon) - \pi^e(n, \bar{c} - \epsilon)$$

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which yields:
$$r'(0) = \frac{\pi^i(n-1,\bar{c}-\epsilon) - \pi^e(n,\bar{c}-\epsilon)}{\frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon)}.$$

Therefore, (11) is equivalent to:

$$n\frac{\pi^i(n-1,\bar{c}-\epsilon) - \pi^e(n,\bar{c}-\epsilon)}{\frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon)}q^e(n,\bar{c}-\epsilon) > n\left[\pi^e(n,\bar{c}-\epsilon) - \pi^i(n-1,\bar{c}-\epsilon)\right]$$

which can be rewritten as:
$$\frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon) > -q^e(n,\bar{c}-\epsilon)$$

because $\pi^i(n-1,\bar{c}-\epsilon) - \pi^e(n,\bar{c}-\epsilon) < 0$.

Proof of Proposition 5

To prove that $\tilde{F}(\theta) = 0$ for θ sufficiently small, it is sufficient to show that the function $h(r,\theta) = rq^e(n,\bar{c}-\epsilon+r)+\pi^e(n,\bar{c}-\epsilon+r)-\theta\pi^i(n-1,\bar{c}-\epsilon+r)-(1-\theta)\pi^e(n,\bar{c}-\epsilon)$ is increasing for sufficiently small values of r and θ (this follows immediately from $r(0) = 0$ and $r(\theta)$ being continuous in θ). We have:
$$\frac{\partial h}{\partial r}(0,\theta) = q^e(n,\bar{c}-\epsilon) + \frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon) - \theta\frac{\partial \pi^i}{\partial c}(n-1,\bar{c}-\epsilon).$$

Let us now assume that $\frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon) > -q^e(n,\bar{c}-\epsilon)$. We need to distinguish between two cases:

- Case 1: $\frac{\partial \pi^i}{\partial c}(n-1,\bar{c}-\epsilon) = 0$: in this $\frac{\partial h}{\partial r}(0,\theta) = \frac{\partial h}{\partial r}(0,0) > 0$ for any $\theta \in [0,1]$
- Case 2: $\frac{\partial \pi^i}{\partial c}(n-1,\bar{c}-\epsilon) \neq 0$: in this case $\frac{\partial h}{\partial r}(0,\theta) > 0$ for any $\theta \in \left[0, \min\left(1, \frac{q^e(n,\bar{c}-\epsilon)+\frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon)}{\frac{\partial \pi^i}{\partial c}(n-1,\bar{c}-\epsilon)}\right)\right)$.

Therefore, in both cases, $\frac{\partial h}{\partial r}(0,\theta) > 0$ for sufficiently small values of θ. Using the continuity of $r \to \frac{\partial h}{\partial r}(r,\theta)$, we can then state that $\frac{\partial h}{\partial r}(r,\theta) > 0$ for sufficiently small values of r and θ, which, as claimed before, is sufficient to complete the proof.

Proof of Proposition 6

Under the per-unit royalty mechanism, the optimal royalty $r_I(\theta)$ for sufficiently weak patents is the solution in r to the following equation:

$$\pi^l(n-1,\bar{c}-\epsilon+r) = \theta\pi^n(n-2,\bar{c}-\epsilon+r) + (1-\theta)\pi^l(n-1,\bar{c}-\epsilon)$$

and the patent holder's overall profit is

$$\tilde{\Pi}_r(\theta) = \pi^p(n-1,\bar{c}-\epsilon+r_I(\theta)) + (n-1)r_I(\theta)q^l(n-1,\bar{c}-\epsilon+r_I(\theta)).$$

Under the fixed fee mechanism, the optimal fee is given by

$$F_I(\theta) = \theta\left[\pi^l(n-1,\bar{c}-\epsilon) - \pi^n(n-2,\bar{c}-\epsilon)\right]$$

and the patent holder's overall profit is then

$$\tilde{\Pi}_F(\theta) = \pi^p(n-1, \bar{c} - \epsilon) + (n-1)\,\theta\left[\pi^l(n-1, \bar{c} - \epsilon) - \pi^n(n-2, \bar{c} - \epsilon)\right].$$

Since $\tilde{\Pi}_r(0) = \tilde{\Pi}_F(0)$ then $\tilde{\Pi}_r(\theta) > \tilde{\Pi}_F(\theta)$ for θ sufficiently small if

$$\frac{d\tilde{\Pi}_r(\theta)}{d\theta}\Big|_{\theta=0} > \frac{d\tilde{\Pi}_F(\theta)}{d\theta}\Big|_{\theta=0} \tag{12}$$

which can be rewritten as

$$r_I'(0)\left[\frac{\partial\pi^p}{\partial c}(n-1, \bar{c} - \epsilon) + (n-1)\,q^l(n, \bar{c} - \epsilon)\right] > (n-1)\left[\pi^l(n-1, \bar{c} - \epsilon) - \pi^n(n-2, \bar{c} - \epsilon)\right].$$

because $r_I(0) = 0$. Moreover, differentiating at $\theta = 0$ the equation defining $r_I(\theta)$, we get

$$r_I'(0).\frac{\partial\pi^l}{\partial c}(n-1, \bar{c} - \epsilon) = \pi^n(n-2, \bar{c} - \epsilon) - \pi^l(n-1, \bar{c} - \epsilon)$$

which yields

$$r_I'(0) = \frac{\pi^n(n-2, \bar{c} - \epsilon) - \pi^l(n-1, \bar{c} - \epsilon)}{\frac{\partial\pi^l}{\partial c}(n-1, \bar{c} - \epsilon)}.$$

Hence, inequality (12) is equivalent to

$$\frac{\pi^n(n-2, \bar{c} - \epsilon) - \pi^l(n-1, \bar{c} - \epsilon)}{\frac{\partial\pi^l}{\partial c}(n-1, \bar{c} - \epsilon)}\left[\frac{\partial\pi^p}{\partial c}(n-1, \bar{c} - \epsilon) + (n-1)\,q^l(n, \bar{c} - \epsilon)\right] > (n-1)\left[\pi^e(n-1, \bar{c} - \epsilon) - \pi^i(n-2, \bar{c} - \epsilon)\right]$$

which can be rewritten as

$$\frac{\partial\pi^p}{\partial c}(n-1, \bar{c} - \epsilon) + (n-1)\,q^l(n, \bar{c} - \epsilon) > -(n-1)\frac{\partial\pi^l}{\partial c}(n-1, \bar{c} - \epsilon)$$

or, equivalently, as

$$\frac{\partial\pi^p}{\partial c}(n-1, \bar{c} - \epsilon) + (n-1)\frac{\partial\pi^l}{\partial c}(n-1, \bar{c} - \epsilon) > -(n-1)\,q^l(n, \bar{c} - \epsilon).$$

Proof of Proposition 7

(a) This follows from the key slope property that every selection of r_i satisfies (see Amir, 1996, and Amir and Lambson, 2000 for details)

$$-1 < \frac{r_i(Q'_{-i}) - r_i(Q_{-i})}{Q'_{-i} - Q_{-i}} < 0 \quad \text{for all} \quad Q'_{-i} > Q_{-i}. \tag{13}$$

(b) We first show that q_i^* is continuously differentiable in c_i. Viewed as a correspondence in the parameter c_i, q_i^* is upper hemi-continuous (or u.h.c.), as a direct consequence of the well-known property of u.h.c. of the equilibrium correspondence for games with continuous payoff functions

(jointly in own and rivals' actions), see e.g., Fudenberg and Tirole, 1990. Since q_i^* is also single-valued in c (from part (b)), q_i^* must be a continuous function. Then the fact that q_i^* is continuously differentiable in c_i follows from the Implicit Function Theorem applied to the first order conditions, and the smoothness of $P(\cdot)$.

The fact that π_i^* is also continuously differentiable in c_i follows directly from the fact that q_i^* has that same property for all i.

The proof for the parameter $c_j, j \neq i$, follows along the same lines.

(c) Throughout part (c), fix i and denote firm i's output, profit and its rivals' total outputs at equilibrium by q_i^*, π_i^* and Q_{-i}^* respectively when the cost vector is $(c_1, c_2, ..., c_n)$. Denote the same three variables by $\widehat{q}_i, \widehat{\pi}_i$ and $\widehat{Q}_{-i}$ after firm i's cost alone changes to $\widehat{c}_i > c_i$, all other firms' unit costs remaining the same.

Adding the n first order conditions at the Cournot equilibrium yields

$$nP(Q^*) + Q^* P'(Q^*) = \sum_{k=1}^{n} c_k. \tag{14}$$

Since the LHS of (14) is strictly decreasing in Q^*, the increase in firm i's cost from c_i to $\widehat{c}_i$ increases the RHS of (14), which causes the solution to (14) to decrease. In other words, $\widehat{Q} < Q^*$.

We now show that for any firm $j \neq i$, we must have $\widehat{Q}_{-j} < Q_{-j}^*$. To this end, first observe that $\widehat{Q}_{-j} + r_j(\widehat{Q}_{-j}) = \widehat{Q} < Q^* = Q_{-j}^* + r_j(Q_{-j}^*)$. Since (13) holds that $Q_{-j} + r_j(Q_{-j})$ is increasing in Q_{-j}, we must have $\widehat{Q}_{-j} < Q_{-j}^*$.

For firm j,

$$\begin{aligned}
\widehat{\pi}_j &= \widehat{q}_j \left[P(\widehat{q}_j + \widehat{Q}_{-j}) - c_j \right] \\
&\geq q_j^* \left[P(q_j^* + \widehat{Q}_{-j}) - c_j \right] \text{ by the Cournot property} \\
&> q_j^* \left[P(q_j^* + Q_{-j}^*) - c_j \right] \text{ since } \widehat{Q}_{-j} < Q_{-j}^* \\
&= \pi_j^*.
\end{aligned}$$

We now show that for firm i, we must have $\widehat{Q}_{-i} > Q_{-i}^*$. To this end, first observe that since for any $j \neq i$, r_j is strictly decreasing (cf. (13)) and $\widehat{Q}_{-j} < Q_{-j}^*$, we have $\widehat{q}_j = r_j(\widehat{Q}_{-j}) > r_j(Q_{-j}^*) = q_j^*$, for every firm $j \neq i$. Then since $\widehat{Q}_{-i} = \sum_{j \neq i} \widehat{q}_j$ and $Q_{-i}^* = \sum_{j \neq i} q_j^*$, we have $\widehat{Q}_{-i} > Q_{-i}^*$.

To show that $\pi_i^* > \widehat{\pi}_i$, consider

$$\begin{aligned}
\pi_i^* &= q_i^* \left[P(q_i^* + Q_{-i}^*) - c_i \right] \\
&\geq \widehat{q}_i \left[P(\widehat{q}_i + Q_{-i}^*) - c_i \right] \text{ by the Cournot property} \\
&> \widehat{q}_i [P(\widehat{q}_i + \widehat{Q}_{-i}) - c_i] \text{ since } \widehat{Q}_{-i} > Q_{-i}^*. \\
&> \widehat{q}_i [P(\widehat{q}_i + \widehat{Q}_{-i}) - \widehat{c}_i] \text{ since } \widehat{c}_i > c_i \\
&= \widehat{\pi}_i.
\end{aligned}$$

For the remaining parts, we consider the case of a symmetric Cournot oligopoly ($c_i = c$ for all i).

(d) Due to the symmetry of the game, asymmetric equilibria, if any, would come in n-tuples. Hence, the conclusion follows from part (a) directly.

(e) Let q^* denote each firm's equilibrium output. Differentiating the first order condition with respect to c, we get:

$$\frac{\partial q^*}{\partial c} \left[(n+1) P' (nq^*) + nq^* P'' (nq^*) \right] = 1. \tag{15}$$

Using the first order condition and **C3**, it is easy to see that the term in brackets is strictly negative, it follows that $\frac{\partial q^*}{\partial c} < 0$.

We now show that per-firm profit decreases in c. Denote the equilibrium variables by q_i^*, π_i^* and Q_{-i}^* when the unit cost is c, and by q_i', π_i' and Q_{-i}' the same variables when the unit cost is $c' > c$.

Differentiating $\pi_i^* = q^* [P(nq^*) - c]$ with respect to c yields

$$
\begin{aligned}
\frac{\partial \pi_i^*}{\partial c} &= \frac{\partial q^*}{\partial c} [P(nq^*) - c] + q^* \left[P'(nq^*) n \frac{\partial q^*}{\partial c} - 1 \right] \\
&= \frac{\partial q^*}{\partial c} (n+1) q^* P'(nq^*) - q^* \quad \text{by (14)} \\
&= -q^* \frac{2P'(Q^*) + Q^* P'' (Q^*)}{(n+1) P' (Q^*) + Q^* P'' (Q^*)} \quad \text{by (15).}
\end{aligned}
\tag{16}
$$

Clearly, **C3** implies that $2P'(Q) + QP''(Q) < 0$ for all Q, so the numerator in the above fraction is < 0. It is then easy to see that the denominator is also < 0. Hence $\frac{\partial \pi_i^*}{\partial c} < 0$.

Proof of Proposition 8

Let us show that Condition (7) holds (which will imply that both Condition (5) and Condition (6) are satisfied).

Total differentiation w.r.t. c_i in

$$\Pi^* = (P(Q^*) - c_i) q_i^* + \sum_{j \neq i} (P(Q^*) - c_j) q_j^*$$

yields

$$\frac{\partial \Pi^*}{\partial c_i} = [P'(Q^*) \frac{\partial Q^*}{\partial c_i} - 1] q_i^* + (P(Q^*) - c_i) \frac{\partial q_i^*}{\partial c_i} + \sum_{j \neq i} [P'(Q^*) \frac{\partial Q^*}{\partial c_i} q_j^* + (P(Q^*) - c_j) \frac{\partial q_j^*}{\partial c_i}].$$

which can be rewritten as:

$$\frac{\partial \Pi^*}{\partial c_i} = -q_i^* + \sum_j \left[P'(Q^*) \frac{\partial Q^*}{\partial c_i} q_j^* + (P(Q^*) - c_j) \frac{\partial q_j^*}{\partial c_i} \right].$$

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When $c_i = c_j = c$, the latter becomes:

$$
\begin{aligned}
\frac{\partial \Pi^*}{\partial c_i} &= -q_i^* + \sum_j \left[P'(Q^*)\frac{\partial Q^*}{\partial c_i}q_j^* + (P(Q^*) - c)\frac{\partial q_j^*}{\partial c_i} \right] \\
&= -q_i^* + P'(Q^*)\frac{\partial Q^*}{\partial c_i}\sum_j q_j^* + (P(Q^*) - c)\sum_j \frac{\partial q_j^*}{\partial c_i} \\
&= -q_i^* + P'(Q^*)\frac{\partial Q^*}{\partial c_i}.Q^* + (P(Q^*) - c)\frac{\partial Q^*}{\partial c_i} \\
&= \underbrace{-q_i^*}_{\text{direct effect}} + \underbrace{\frac{\partial Q^*}{\partial c_i}\left[P'(Q^*).Q^* + (P(Q^*) - c) \right]}_{\text{strategic effect}}.
\end{aligned}
$$

Adding the n first order conditions at the Cournot equilibrium yields

$$
nP(Q^*) + Q^*P'(Q^*) = \sum_{k=1}^{n} c_k = nc.
$$

Thus,

$$
Q^*P'(Q^*) + (P(Q^*) - c) = \frac{n-1}{n}Q^*P'(Q^*) < 0.
$$

Moreover, since we have already shown (in the proof of Proposition 6) that $\frac{\partial Q^*}{\partial c_i} < 0$,

$$
\frac{\partial Q^*}{\partial c_i}\left[P'(Q^*).Q^* + (P(Q^*) - c) \right] > 0
$$

which yields:

$$
\frac{\partial \Pi^*}{\partial c_i} > -q_i^*.
$$

Proof of Proposition 9

First note that for firm i, charging a price of c_i strictly dominates charging any price below c_i. Hence, we restrict attention to the price space $[c_i, \infty)$ as the action set for firm $i, i = 1, 2, ..., n$. Then the transfomed profit function $\log \Pi_i(p_i, p_{-i})$ is well defined.

(a) For the proof that the game with log profits as payoffs is of strict strategic complements, observe that, due to **B3**, each payoff $\Pi_i(p_i, p_{-i})$ satisfies $\partial^2 \log \Pi_i(p_i, p_{-i})/\partial p_i \partial p_{-i} > 0$. Hence, by the strong version of Topkis's Theorem (see Amir, 1996 or Topkis, 1998 p. 79), every selection of $r_i(p_{-i})$ is strictly increasing in p_{-i}. It follows directly from the property of strategic complements, via Tarski's fixed point theorem, that the Bertrand equilibrium set is nonempty. Uniqueness then follows from a well known argument from **B4** (for details, see Milgrom and Roberts 1990, pp. 1271-1272, or Vives, 1999 pp. 149-150).

(b) To show that the equilibrium price p_i^* is increasing in c_i, note that the price game is log-

supermodular (from part (a)), $\log \Pi_i(p_i, p_{-i}) = \log(p_i - c_i) + \log D_i(p_i, p_{-i})$ has the increasing differences property in (p_i, c_i) since $\partial^2 \log \Pi_i(p_i, p_{-i})/\partial p_i \partial c_i = (p_i - c_i)^{-2} > 0$, and the constraint set $[c_i, \infty)$ is clearly ascending in c_i. So the conclusion follows from Theorem 7 in Milgrom and Roberts (1990).

The fact that the equilibrium price p_i^* is also increasing in c_j for any $j \neq i$, follows from a similar argument since $\partial^2 \log \Pi_i(p_i, p_{-i})/\partial p_i \partial c_j = 0$.

(c) We first show that every equilibrium price p_i^* is continuously differentiable in c_j, for all i and j. Viewed as a correspondence in the parameter c_j, p_i^* is u.h.c., by the u.h.c. property of the equilibrium correspondence for games with continuous payoff functions (jointly in own and rivals' actions), see e.g., Fudenberg and Tirole, 1990. Since p_i^* is also single-valued in c_j (from part (i)), p_i^* is a continuous function. Then the fact that p_i^* is continuously differentiable in c_j follows from the Implicit Function Theorem. Finally, continuous differentiability of π_i^* follows from that of all the $p_i^{*\prime}s$.

(d) Differentiating $\pi_i^* = (p_i^* - c_i)D_i(p_i^*, p_{-i}^*)$ with respect to c_j, for $i \neq j$, yields

$$\frac{\partial \pi_i^*}{\partial c_j} = \frac{\partial p_i^*}{\partial c_j} D_i(p_i^*, p_{-i}^*) + (p_i^* - c_i) \sum_k \frac{\partial D_i}{\partial p_k} \frac{\partial p_k^*}{\partial c_j}. \tag{17}$$

Using the first order condition $D_i(p_i^*, p_{-i}^*) + (p_i^* - c_i)\frac{\partial D_i}{\partial p_i} = 0$, (17) reduces to

$$\frac{\partial \pi_i^*}{\partial c_j} = (p_i^* - c_i) \sum_{k \neq i} \frac{\partial D_i}{\partial p_k} \frac{\partial p_k^*}{\partial c_j} \geq 0$$

since $\frac{\partial D_i}{\partial p_k} > 0$ (goods are substitutes) and $\frac{\partial p_k^*}{\partial c_j} \geq 0$ from part (b).

(e) When the Bertrand game is symmetric, the unique Bertrand equilibrium must be symmetric, for otherwise equilibria would come in pairs.

(f) The conclusion follows from the same argument as for part (b) in view of the fact that $\partial^2 \log \Pi_i(p_i, p_{-i})/\partial p_i \partial c = (p_i - c)^{-2} > 0$.

(g) From an argument similar to the proof of part (c), p^* and thus $\pi_i^* = (p^* - c)D_i(p^*, p^*, ..., p^*)$ are differentiable with respect to c. We now derive an expression for $\frac{\partial p^*}{\partial c}$. The FOC at a Bertrand equilibrium is

$$D_i(p^*, ..., p^*) + (p^* - c)\partial D_i(p^*, ..., p^*)/\partial p_i = 0. \tag{18}$$

Using the Implicit Function Theorem and differentiating the FOC with respect to c yields

$$\left(\frac{\partial D_i}{\partial p_i} + \sum_{k \neq i} \frac{\partial D_i}{\partial p_k} \right) \frac{\partial p^*}{\partial c} + (p_i^* - c_i)\frac{\partial p^*}{\partial c} \sum_k \frac{\partial D_i}{\partial p_k \partial p_i} + (\frac{\partial p^*}{\partial c} - 1)\frac{\partial D_i}{\partial p_i} = 0.$$

Hence, using **B2** and **B4**,

$$\frac{\partial p^*}{\partial c} = \frac{\partial D_i/\partial p_i}{2\frac{\partial D_i}{\partial p_i} + \sum_{k\neq i}\frac{\partial D_i}{\partial p_k} + (p_i^* - c_i)\sum_k \frac{\partial D_i}{\partial p_k \partial p_i}} > 0. \tag{19}$$

We can differentiate $\pi^* = (p^* - c)D_i(p^*, p^*, ..., p^*)$ with respect to c to obtain

$$
\begin{aligned}
\frac{\partial \pi^*}{\partial c} &= (\frac{\partial p^*}{\partial c} - 1)D_i(p^*, ..., p^*) + (p^* - c)\frac{\partial p^*}{\partial c}\sum_k \frac{\partial D_i}{\partial p_k} \\
&= D_i(p^*, ..., p^*)\left[-1 - \frac{\sum_{k\neq i}\frac{\partial D_i}{\partial p_k}}{\partial D_i/\partial p_i}\frac{\partial p^*}{\partial c}\right] \text{ from (18)}
\end{aligned}
$$

which yields:

$$
\begin{aligned}
\frac{\partial \pi^*}{\partial c} &= D_i(p^*, ..., p^*)\left[-1 - \frac{\sum_{k\neq i}\frac{\partial D_i}{\partial p_k}}{2\frac{\partial D_i}{\partial p_i} + \sum_{k\neq i}\frac{\partial D_i}{\partial p_k} + (p_i^* - c_i)\sum_k \frac{\partial D_i}{\partial p_k \partial p_i}}\right] \text{ using (19)} \\
&= D_i(p^*, ..., p^*)\left[-\frac{2\sum_k \frac{\partial D_i}{\partial p_k} + (p_i^* - c_i)\sum_k \frac{\partial D_i}{\partial p_k \partial p_i}}{2\frac{\partial D_i}{\partial p_i} + \sum_{k\neq i}\frac{\partial D_i}{\partial p_k} + (p_i^* - c_i)\sum_k \frac{\partial D_i}{\partial p_k \partial p_i}}\right] < 0 \text{ by \textbf{B2} and \textbf{B4}.}
\end{aligned}
$$

Proof of Proposition 10

Let us show that Condition (7) holds (which will imply that both Condition (5) and Condition (6) are satisfied).

We have:

$$\Pi^* = (p_i^* - c_i)D_i^* + \sum_{j\neq i}(p_j^* - c_j)D_j^*$$

then:

$$
\begin{aligned}
\frac{\partial \Pi^*}{\partial c_i} &= \left(\frac{\partial p_i^*}{\partial c_i} - 1\right)D_i^* + (p_i^* - c_i)\frac{\partial D_i^*}{\partial c_i} + \\
&\quad \sum_{j\neq i}\left[\frac{\partial p_j^*}{\partial c_i}D_j^* + (p_i^* - c_i)\frac{\partial D_j^*}{\partial c_i}\right]
\end{aligned}
$$

which can be rewritten as:

$$
\begin{aligned}
\frac{\partial \Pi^*}{\partial c_i} &= -D_i^* + \sum_j \left[\frac{\partial p_j^*}{\partial c_i} D_j^* + \left(p_j^* - c_j \right) \frac{\partial D_j^*}{\partial c_i} \right] \\
&= -D_i^* + \sum_j \left[\frac{\partial p_j^*}{\partial c_i} D_j^* + \left(p_j^* - c_j \right) \sum_k \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right] \\
&= -D_i^* + \sum_j \left[\frac{\partial p_j^*}{\partial c_i} D_j^* + \left(p_j^* - c_j \right) \frac{\partial D_j}{\partial p_j} \cdot \frac{\partial p_j^*}{\partial c_i} + \left(p_j^* - c_j \right) \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right] \\
&= -D_i^* + \sum_j \left[\frac{\partial p_j^*}{\partial c_i} \underbrace{\left(D_j^* + \left(p_j^* - c_j \right) \frac{\partial D_j}{\partial p_j} \right)}_{=0} + \left(p_j^* - c_j \right) \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right] \\
&= -D_i^* + \sum_j \left[\left(p_j^* - c_j \right) \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right]
\end{aligned}
$$

We have already shown that $\frac{\partial p_k^*}{\partial c_i} > 0$ for any k, i (see the proof for part (b) of Proposition 8). Moreover, we have $\frac{\partial D_j}{\partial p_k} > 0$ for any $j \neq k$ (from **B2(ii)**). It then follows that:

$$
\frac{\partial \Pi^*}{\partial c_i} > -D_i^* = -q_i^*
$$

This proof establishes a result which is more general than Condition (7). We have actually shown that $\frac{\partial \Pi^*}{\partial c_i} (c_1, c_2, ..., c_n) > -q_i^* (c_1, c_2, ..., c_n)$ *for any* $(c_1, c_2, ..., c_n)$.

Proof of Proposition 11

In this proof we will explicit in our notations the dependence on ϵ. Differentiating with respect to ϵ the equation $\pi^e(n, \bar{c} - \epsilon + r(\theta, \epsilon)) = \theta \pi^i(n - 1, \bar{c} - \epsilon + r(\theta, \epsilon)) + (1 - \theta) \pi^e(n, \bar{c} - \epsilon)$ defining the optimal royalty rate $r(\theta, \epsilon)$ deterring litigation, and taking the limit $\epsilon \longrightarrow 0$ we obtain:

$$
\frac{\partial \pi^e}{\partial c}(n, \bar{c}) \left[-1 + \frac{\partial r}{\partial \epsilon}(\theta, 0) \right] = \theta \frac{\partial \pi^i}{\partial c}(n - 1, \bar{c}) \left[-1 + \frac{\partial r}{\partial \epsilon}(\theta, 0) \right] - (1 - \theta) \frac{\partial \pi^e}{\partial c}(n, \bar{c})
$$

which yields:

$$
\frac{\partial r}{\partial \epsilon}(\theta, 0) = \theta \frac{\frac{\partial \pi^i}{\partial c}(n - 1, \bar{c}) - \frac{\partial \pi^e}{\partial c}(, \bar{c})}{\frac{\partial \pi^i}{\partial c}(n - 1, \bar{c}) - \theta \frac{\partial \pi^e}{\partial c}(n, \bar{c})}.
$$

Differentiating the patent holder's licensing revenues $P_r(\theta, \epsilon) = nr(\theta, \epsilon) q^e(n, \bar{c} - \epsilon + r(\theta, \epsilon))$ with

respect to ϵ and again taking the limit $\epsilon \longrightarrow 0$, we get:

$$\frac{\partial P_r}{\partial \epsilon}(\theta, 0) = n\frac{\partial r}{\partial \epsilon}(\theta, 0) q^e(n, \bar{c})$$

$$= n\theta q^e(n, \bar{c}).\frac{\frac{\partial \pi^i}{\partial c}(n-1, \bar{c}) - \frac{\partial \pi^e}{\partial c}(n, \bar{c})}{\frac{\partial \pi^i}{\partial c}(n-1, \bar{c}) - \theta\frac{\partial \pi^e}{\partial c}(n, \bar{c})}.$$

Furthermore,

$$\frac{\partial P_F}{\partial \epsilon}(\theta, 0) = n\theta\left[\frac{\partial \pi^i}{\partial c}(n-1, \bar{c}) - \frac{\partial \pi^e}{\partial c}(n, \bar{c})\right].$$

Comparing $\frac{\partial P_r}{\partial \epsilon}(\theta, 0)$ and $\frac{\partial P_F}{\partial \epsilon}(\theta, 0)$ yields the following result:

$$\frac{\partial P_r}{\partial \epsilon}(\theta, 0) > \frac{\partial P_F}{\partial \epsilon}(\theta, 0) \Longleftrightarrow \frac{\partial \pi^e}{\partial c}(n, \bar{c}) > -q^e(n, \bar{c}) + \theta\frac{\partial \pi^i}{\partial c}(n-1, \bar{c}) \Longleftrightarrow \theta > \tilde{\theta} \triangleq \frac{\frac{\partial \pi^e}{\partial c}(n, \bar{c}) + q^e(n, \bar{c})}{\frac{\partial \pi^i}{\partial c}(n-1, \bar{c})}.$$

Combining this with $P_r(\theta, 0) = 0 = P_F(\theta, 0)$ and using the continuity of $P_r(\theta, \epsilon)$ and $P_F(\theta, \epsilon)$, we can state that for any $\theta < \tilde{\theta}$ there exists $\tilde{\epsilon}_\theta > 0$ such that for any $\epsilon \in (0, \tilde{\epsilon}_\theta)$ the patent holder prefers to offer a per-unit royalty contract and for any $\theta > \tilde{\theta}$ there exists $\hat{\epsilon}_\theta > 0$ such that for any $\epsilon \in (0, \hat{\epsilon}_\theta)$ the patent holder prefers to offer a fixed fee contract. Note that the latter scenario (i.e. $\theta > \tilde{\theta}$) can arise for a non-empty set of values of $\theta < 1$ if and only if $\tilde{\theta} < 1$.

Proof of Proposition 13

(i) *Cournot competition with homogeneous products* : In that environment, the strategic effect of a decrease in the common marginal cost from c' to c on individual profits is:

$$\pi^e(n, c) - \pi^e(n, c') - (c' - c)q^e(n, c') = (P(nq^e(n, c)) - c)q^e(n, c) - (P(nq^e(n, c') - c)q^e(n, c')$$

$$= \frac{1}{n}\left[(P(Q^e(n, c)) - c)Q^e(n, c) - (P(Q^e(n, c') - c)Q^e(n, c'))\right].$$

We can easily derive from **C3**, combined with **C1**, that a monopolist's profit $(P(Q) - c)Q$ is concave. Denoting $Q^m(c)$ the monopoly output with marginal cost c[27], it follows that $(P(Q) - c)Q$ is decreasing over the interval $[Q^m(c), Q^e(n, c)]$.[28] We know from part (e) of Proposition 7 that $c < c'$ implies $Q^e(n, c') < Q^e(n, c)$. Therefore, the strategic effect $\pi^e(n, c) - \pi^e(n, c') - (c' - c)q^e(n, c')$ is negative if $Q^m(c) \le Q^e(n, c')$. Thus, for Condition (9) to hold it is sufficient to suppose, beside assumptions **C1-C3**, that $Q^m(\bar{c} - \epsilon) \le Q^e(n, \bar{c})$.[29]

---

[27]It is straigtforward to establish the existence and uniqueness of the solution to the monopolist's maximization program under **C1-C3**.

[28]The inequality $Q^m(c) < Q^e(n, c)$ follows from the fact that $P'(Q)Q + P(Q)$ is decreasing in Q, combined with $P'(Q^m(c))Q^m(c) + P(Q^m(c)) = c$ and $P'(Q^e(n, c))Q^e(n, c) + P(Q^e(n, c)) = c + \frac{n-1}{n}P'(Q^e(n, c))Q^e(n, c) < c$.

[29]Under Cournot competition with a linear inverse demand $P(Q) = a - Q$, this condition holds whenever $\epsilon \le \frac{n-1}{n+1}(a - \bar{c})$. To see why this condition is not very restrictive, recall that in this setting an innovation is *drastic* if $\epsilon \ge (a - \bar{c})$. Moreover, the condition $Q^m(\bar{c} - \epsilon) \le Q^e(n, \bar{c})$ can be weakened to $\bar{Q}^e(n, \bar{c} - \epsilon) \le Q^e(n, \bar{c} - \epsilon)$ where $\bar{Q}^e(n, \bar{c} - \epsilon)$ is the unique value of $Q < Q^m(\bar{c} - \epsilon)$ such that $(P(Q) - c + \epsilon)Q = (P(Q^e(n, \bar{c} - \epsilon)) - c + \epsilon)Q^e(n, \bar{c} - \epsilon)$.

(ii) *Bertrand competition with differentiated products*: Denoting $D_i(p, p, ..., p) \triangleq D(p)$ and $p^e(n, c)$ the (common) equilibrium price when the (common) marginal cost is c, we can rewrite the strategic effect of a decrease in the common marginal cost from c' to c on individual profits as follows:

$$\pi^e(n, c) - \pi^e(n, c') - (c' - c)q^e(n, c') = (p^e(n, c) - c)\, D(p^e(n, c)) - \left(p^e(n, c') - c\right) D(p^e(n, c')).$$

Using **B2** and **B4** we can easily show that the function $(p - c)D(p)$, which can be interpreted as the profit that a multi-product monopolist producing the n varieties and selling them at the same price p derives from each variety, is concave. Denoting $p^m(c) \triangleq \arg\max(p - c)D(p)$, it follows that $(p - c)D(p)$ is increasing over $[p^e(n, c), p^m(c)]$.[30] We know from part (f) of Proposition 9 that $c < c'$ implies $p^e(n, c) < p^e(n, c')$. Therefore, we can state that the strategic effect $\pi^e(n, c) - \pi^e(n, c') - (c' - c)q^e(n, c')$ is negative if $p^e(n, c') \le p^m(c)$. Thus, for Condition (9) to hold it is sufficient to suppose, beside assumptions **B1-B4**, that $p^e(n, \bar{c}) \le p^m(\bar{c} - \epsilon)$.

Proof of Proposition 14

Step 1: Existence of $\hat{\theta}$

We have $P_r(1) = \frac{n}{n+1}\epsilon(a - \bar{c})$ and $P_F(1) = n^2\epsilon\frac{[2(a - \bar{c}) + \epsilon(2 - n)]}{(n+1)^2}$. It is easily verified that $P_r(1) < P_F(1) \iff (n - 2)\epsilon < \frac{n-1}{n}(a - \bar{c})$. Moreover, $\frac{n-1}{n(n-2)} - \frac{1}{2n-1} = \frac{n^2 - n + 1}{n(n-2)(2n-1)} > 0$. Therefore, $P_r(1) < P_F(1)$ for any $\epsilon \le \frac{a - \bar{c}}{2n-1}$. This, combined with the continuity of the function $\theta \to P_r(\theta) - P_F(\theta)$ and the fact that $P_r(\theta) > P_F(\theta)$ for $\theta > 0$ sufficiently small (which holds because the linear demand satisfies **C1-C3** and, therefore, Condition (7) is satisfied), ensures the existence of a value $\hat{\theta}$ such that $P_r(\theta) = P_F(\theta)$.

Step 2: Sufficient condition for the uniqueness of $\hat{\theta}$.

Remark: This part of the proof is not specific to the linear Cournot model considered in this subsection. It holds in any competitive environment such that **A1-A5** hold and $\frac{\partial q^e}{\partial c} < 0$.

To show that there is a unique $\hat{\theta}$, it is sufficient to show that

$$\frac{d\tilde{P}_r(\theta)}{d\theta} < \frac{d\tilde{P}_F(\theta)}{d\theta} \text{ for all } \theta \in\,]0, 1[\text{ such that } \tilde{P}_r(\theta) = \tilde{P}_F(\theta). \tag{20}$$

Now $\frac{d\tilde{P}_r(\theta)}{d\theta} < \frac{d\tilde{P}_F(\theta)}{d\theta}$ iff

$$r'(\theta)\, q^e(n, \bar{c} - \epsilon + r(\theta)) + r(\theta)\frac{\partial q^e(n, \bar{c} - \epsilon + r(\theta))}{\partial c} < \pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon).$$

Evaluating along $\tilde{P}_r(\theta) = \tilde{P}_F(\theta)$, i.e. $nr(\theta)\, q^e(n, \bar{c} - \epsilon + r(\theta)) = n\left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)\right]$, (and dropping arguments) yields

$$r'(\theta)\, q^e + r(\theta)\frac{\partial q^e}{\partial c}r'(\theta) < \frac{r(\theta)\, q^e}{\theta}$$

---

[30] The inequality $p^e(n, c) < p^m(c)$ follows from the fact that $(p - c)\, D'(p) + D(p)$ is decreasing, combined with $(p^m(c) - c)\, D'(p^m(c)) + D(p^m(c)) = 0$ and $(p^e(n, c) - c)\, D'(p^e(n, c)) + D(p^e(n, c)) = (p^e(n, c) - c)\sum_{j \ne i}\frac{\partial D_i}{\partial p_j} > 0$.

or

$$\left[r'\left(\theta\right) - \frac{r\left(\theta\right)}{\theta}\right] q^e + r\left(\theta\right) \frac{\partial q^e}{\partial c} r'\left(\theta\right) < 0.$$

To have this, it is sufficient to have (since $r\left(\theta\right) \frac{\partial q^e}{\partial c} r'\left(\theta\right) < 0$) that along $\tilde{P}_r\left(\theta\right) = \tilde{P}_F\left(\theta\right)$

$$r'\left(\theta\right) - \frac{r\left(\theta\right)}{\theta} \leq 0$$

which can be rewritten as

$$r\left(\theta\right) - \theta r'\left(\theta\right) \geq 0.$$

The function $s(\theta) = r\left(\theta\right) - \theta r'\left(\theta\right)$ is such that $s(0) = 0$ and $s'(\theta) = -\theta r''\left(\theta\right)$ so a sufficient condition for $s(\theta)$ to be non-negative over $[0, 1]$ is that $r\left(\theta\right)$ is weakly concave over $[0, 1]$.

Step 3: Proof for the (weak) concavity of $r\left(\theta\right)$

We have

$$(a - \bar{c} + \epsilon - r(\theta))^2 = \theta[a - \bar{c} - (n-1)(\epsilon - r(\theta))]^2 + (1 - \theta)(a - \bar{c} + \epsilon)^2.$$

Differentiating the latter w.r.t. θ (and dropping the argument of $r(\theta)$) yields

$$-2(a - \bar{c} + \epsilon - r)r'(\theta) = [a - \bar{c} - (n-1)(\epsilon - r)]^2 + 2\theta(n-1)r'(\theta)[a - \bar{c} - (n-1)(\epsilon - r)] - (a - \bar{c} + \epsilon)^2.$$

Solving for $r'(\theta)$

$$r'(\theta) = -\frac{1}{2} \frac{[a - \bar{c} - (n-1)(\epsilon - r)]^2 - (a - \bar{c} + \epsilon)^2}{(a - \bar{c} + \epsilon - r) + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]}. \tag{21}$$

Upon factorization, we have

$$r'(\theta) = \frac{1}{2} \frac{[n\epsilon - (n-1)r][2(a - \bar{c}) - (n-2)\epsilon + (n-1)r]}{a - \bar{c} + \epsilon - r + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]} > 0. \tag{22}$$

Differentiating w.r.t. θ once more yields

$$r''(\theta) = \frac{-(n-1)r'(\theta)[2(a - \bar{c}) - 2(n-1)\epsilon + 2(n-1)r]\{a - \bar{c} + \epsilon - r + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}}{2\{(a - \bar{c} + \epsilon - r) + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}^2}$$
$$-\frac{[n\epsilon - (n-1)r][2(a - \bar{c}) - (n-2)\epsilon + (n-1)r]\{[\theta(n-1)^2 - 1]r'(\theta) + (n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}}{2\{(a - \bar{c} + \epsilon - r) + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}^2}.$$

So $r''(\theta)$ has the sign of

$$-(n-1)r'(\theta)[2(a-\bar{c}) - 2(n-1)\epsilon + 2(n-1)r]\{a - \bar{c} + \epsilon - r + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}$$
$$- [n\epsilon - (n-1)r][2(a-\bar{c}) - (n-2)\epsilon + (n-1)r]\{[\theta(n-1)^2 - 1]r'(\theta) + (n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}.$$

From $r'(\theta) > 0$ it follows that $r''(\theta)$ has the sign of

$$-(n-1)[2(a-\bar{c}) - 2(n-1)\epsilon + 2(n-1)r]\{a - \bar{c} + \epsilon - r + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}$$
$$- [n\epsilon - (n-1)r][2(a-\bar{c}) - (n-2)\epsilon + (n-1)r][\theta(n-1)^2 - 1]$$
$$- \frac{1}{r'(\theta)}[n\epsilon - (n-1)r][2(a-\bar{c}) - (n-2)\epsilon + (n-1)r](n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}.$$

Using (22), we get that $r''(\theta)$ has the sign of

$$-(n-1)[2(a-\bar{c}) - 2(n-1)\epsilon + 2(n-1)r]\{a - \bar{c} + \epsilon - r + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}$$
$$- [n\epsilon - (n-1)r][2(a-\bar{c}) - (n-2)\epsilon + (n-1)r][\theta(n-1)^2 - 1]$$
$$-2\{a - \bar{c} + \epsilon - r + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}.$$

which is the same as

$$-(n-1)[4(a-\bar{c}) - 4(n-1)(\epsilon - r)]\{a - \bar{c} + \epsilon - r + \theta(n-1)[a - \bar{c} - (n-1)(\epsilon - r)]\}$$
$$- [n\epsilon - (n-1)r][2(a-\bar{c}) - (n-2)\epsilon + (n-1)r][\theta(n-1)^2 - 1].$$

Note first that the latter expression is clearly non-positive for any $\theta \geq \frac{1}{(n-1)^2}$. Moreover, a sufficient condition for this expression to be non-positive for any $\theta < \frac{1}{(n-1)^2}$ is that

$$4(n-1)[(a-\bar{c}) - (n-1)(\epsilon - r)][a - \bar{c} + \epsilon - r] - [n\epsilon - (n-1)r][2(a-\bar{c}) - (n-2)\epsilon + (n-1)r] \geq 0$$

for any $r \in [0, \epsilon]$. From $r \geq 0$ and $\epsilon \geq 0$ it follows that a sufficient condition for the latter to hold is

$$4(n-1)[(a-\bar{c}) - (n-1)(\epsilon - r)][a - \bar{c} + \epsilon - r] - (n-1)(\epsilon - r)[2(a-\bar{c}) - (n-2)(\epsilon - r)] \geq 0 \quad (23)$$

for any $r \in [0, \epsilon]$. Denoting $y = \frac{\epsilon - r}{a - \bar{c}}$, it is straightforward to show that (23) can be rewritten as

$$-(n-1)(3n-2)y^2 - (n-1)[4n-6]y + 4(n-1) \geq 0$$

or, equivalently, as

$$(3n-2)y^2 + (4n-6)y \leq 4.$$

Let us show that this inequality holds, which will complete the proof. Since $y \leq \frac{1}{2(n-1)} \leq 1$ for any

$\epsilon \leq \frac{a-\bar{c}}{2(n-1)}$ and $r \in [0, \epsilon]$, it holds that

$$(3n-2)\, y^2 + (4n-6)y \leq (7n-8)\, y \leq \frac{7n-8}{2\,(n-1)} \leq \frac{8(n-1)}{2(n-1)} = 4.$$