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## **The ontological status of geometrical objects in the commentary on the Elements of Euclid of Jacques Peletier du Mans (1517-1582)**

Angela Axworthy

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In Peletier's commentary on the *Elements* (1557), the ontological status of the objects of geometry is defined, on one hand, through the assertion of their archetypal role in the constitution of the Universe and, on the other hand, through the description of their modes of apprehension in the frame of geometrical research. The first perspective, which is qualified by the statement of the limits of the human mind faced with the true genesis of geometrical objects, tends to give way to a more human representation of geometry, which is mainly fulfilled through the non-passive and rational study of the objects of Euclidean geometry.

**Working Papers Series**

# The ontological status of geometrical objects in the commentary on the Elements of Euclid of Jacques Peletier du Mans (1517-1582)

Angela Axworthy

Août 2013

## L'auteur

Angela Axworthy a obtenu un doctorat en philosophie en 2011 au Centre d'Études Supérieures de la Renaissance de Tours. Elle poursuit ses recherches au Max-Planck-Institut für Wissenschaftsgeschichte de Berlin depuis septembre 2012, tout d'abord en tant que bénéficiaire d'une Bourse Fernand Braudel-IFER de la Fondation Maison des sciences de l'homme (de septembre 2012 à mai 2013), puis en tant que Postdoctoral Fellow du MPIWG (de juin 2013 à juin 2014). Ses recherches portent sur le statut des mathématiques dans le cadre de la tradition euclidienne du XVI<sup>e</sup> siècle et plus particulièrement sur le statut ontologique des objets de la géométrie.

## Le texte

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## Abstract

In Peletier's commentary on the *Elements* (1557), the ontological status of the objects of geometry is defined, on one hand, through the assertion of their archetypal role in the constitution of the Universe and, on the other hand, through the description of their modes of apprehension in the frame of geometrical research. The first perspective, which is qualified by the statement of the limits of the human mind faced with the true genesis of geometrical objects, tends to give way to a more human representation of geometry, which is mainly fulfilled through the non-passive and rational study of the objects of Euclidean geometry.

## Keywords

Peletier du Mans, Euclid, Proclus, geometry, ontology

## Le statut ontologique des objets géométriques dans le commentaire des *Éléments* d'Euclide de Jacques Peletier du Mans (1517-1582)

### Résumé

Dans le commentaire des *Éléments* de Peletier (1557), le statut ontologique des objets de la géométrie est défini, d'une part, à travers l'affirmation de leur rôle archétypal dans la constitution de l'Univers et, d'autre part, à travers la description de leurs modes d'apprehension au sein de la recherche du géomètre. La première perspective, qui est nuancée par l'affirmation des limites de l'esprit humain face à la genèse réelle des objets géométriques, tend à laisser place à une représentation plus humaine de la géométrie, laquelle se réalise avant tout dans l'examen proprement opératoire et rationnel des objets de la géométrie euclidienne.

### Mots-clefs

Peletier du Mans, Euclide, Proclus, géométrie, ontologie

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Jacques Peletier du Mans, French poet and mathematician born in 1517 and deceased in 1582, is mainly known by historians of mathematics for his contributions to the development of sixteenth-century algebra<sup>1</sup> and to the Early modern controversy on the status of the angle of contact<sup>2</sup>, as well as for his rejection of the method of superposition in the context of the *Elements* of Euclid<sup>3</sup> and for his discussions and reinterpretations of the Euclidean definitions and demonstrations. He also contributed to the advancement and promotion of sixteenth-century mathematics through the publication of treatises on practical arithmetic and geometry, as well as through various poetic writings which aimed to display the nature and perfections of mathematics<sup>4</sup>. He has also played an important role in the promotion of French as a scientific language<sup>5</sup>.

This paper will focus on the conceptions Peletier set forth, in his commentary on the six first books of Euclid's *Elements*, published in Lyon in 1557, regarding the ontological and gnoseological status of geometrical objects, issue which was crucial to the development of pre-modern epistemology of mathematics, as it aimed to establish the function of the study of abstract magnitudes in the

1. Giovanna Cifoletti, *Mathematics and rhetoric. Jacques Peletier, Guillaume Gosselin and the French Algebraic Tradition*, Ph.D. Thesis, Princeton University, 1992 and *id.*, "The algebraic art of discourse. Algebraic *dispositio*, invention and imitation in sixteenth-century France", *Boston studies in the philosophy of science* 238 (2004), p. 123-135 (regarding the interactions between algebra and rhetoric).

2. On this controversy, see notably Luigi Maièrù, "John Wallis: lettura della polemica fra Peletier e Clavio circa l'angolo di contatto", in M. Galluzzi (ed.), *Atti del Convegno "Giornate di storia della matematica"*, Editel, Rende, 1991, p. 318-65; François Loget, *La Querelle de l'angle de contact (1554-1685). Constitution et autonomie de la communauté mathématique entre Renaissance et Âge baroque*, Ph.D. Thesis, EHESS (Paris), 2000, p. 165-280 and Sabine Rommevaux, "Un débat dans les mathématiques de la Renaissance: le statut de l'angle de contingence", *Le Journal de la Renaissance* IV/4 (2006), p. 291-302.

3. Although this issue is connected to the ontological status of geometrical objects, we will not discuss it in this paper, as we plan to investigate this issue in a more specific context.

4. On this aspect of Peletier's poetic work, see notably Stephen Bamforth, "Peletier du Mans and 'scientific eloquence'", *Renaissance Studies*, III/2 (1989), p. 202-211 and Albert-Marie Schmidt, *La Poésie scientifique en France au seizième siècle*, Paris, A. Michel, 1938.

5. On this theme, see Giovanna Cifoletti, "Du français au latin: *L'Algèbre* de Jacques Peletier et ses projets pour une nouvelle langue des sciences", in C. Roger & C. Pietro (eds.), *Sciences et langues en Europe*, Paris, Centre Alexandre Koyré, 1994, p. 95-105.

contemplation of the Universe and in the moral and technical aspects of human life.

On this issue, Peletier played an important role in promoting, in sixteenth-century France, the ontological model inherited from Platonism and Neoplatonism, which gave a central place to mathematics in the contemplation of physical and divine substances, faced with the abstractionist conceptions of the Peripatetics, which, on the contrary, considered mathematics as disconnected from the study of the causes of beings.

Our aim, in this paper, will be to give an overview of the way Peletier defined, in his commentary on Euclid's *Elements*, the status of geometrical objects, their nature and origin, as well as the modes through which these are apprehended by the geometer in the frame of his investigation.

## Geometrical forms and the constitution of the universe

In the epistle dedicated to Charles de Lorraine, Peletier defines the status of geometrical objects mainly by setting forth the correspondence between the compositions of geometrical figures and the structure of the universe and of its components. He asserts indeed that "the various forms of the figures", which the geometer studies, "represent the diversity of things in a superior manner"<sup>6</sup>. This, to him, would explain the "incredible pleasure" (*incredibilem voluptatem*) which the geometer receives from the study of his objects. The assertion of the correspondence between geometrical and natural compositions holds a central place in Peletier's promotion of mathematics, as it enabled him to define geometry as a privileged instrument to explore the functions and mutual relations of the components of the universe<sup>7</sup>.

6. Jacques Peletier, *In Euclidis elementa geometrica demonstrationum libri sex*, Lyon, Jean de Tournes, 1557 [EE], sig. 4v: "In diversis autem Figurarum formis, quae rerum varietatem eximiè referunt, incredibilem concipit voluptatem".

7. This conception appears in particular as Peletier asserts the usefulness of geometry in exploring Nature. *EE, ibid.*: "Geometricae positiones, quae operas auxiliarias inter se praesentant, omnia in rerum natura mutuis alternisque subsidijs niti & consistere declarant. Quinetiam amicitiae ipsius iura, in Figurarum similitudine, quarum colligationem Diameter efficit, conspicua sunt. Ad summam, haec imago & facies Geometrica eiusmodi est, ut in ea Mundi quandam theorian possis agnoscere".

By saying that geometrical compositions represent the diversity of things in a superior manner (*eximiè*), Peletier seems to point to the purity and intelligible nature of geometrical objects. This implicit statement, as well as the assertion of the higher esthetic pleasure received by the student of geometry, is not without evoking the Platonic definition of mathematics as the study of an intelligible form of substances, though not the highest of all (*Republic* 510d-511c), and which is able to procure the superior form of pleasure defined in the *Philebus* (51d-52b) as obtained from the contemplation of unchanging entities. The thesis of the correspondence between the intelligible order of geometry and the order of Nature also echoes the Platonic ontological system, such as presented in the *Republic* (510c-511e) and in the *Timaeus* (27d-29b and 36e-37a). Indeed, in these dialogues, Plato defined the compositions of changing realities as the material image or reflection of mathematical substances, themselves being conceived as the divisible and multiple images of the pure and undivided principles of all things, as well as the instrument of the constitution of the Universe<sup>8</sup>.

The fact that Peletier here conceives the objects of geometry as pertaining to an intelligible form of substances is confirmed by his assertion of the eternal nature and divine origin of geometrical knowledge.

Of the beginnings [of geometry], I have decided to present nothing here. I would not search its origin among the Egyptians, the Chaldeans or the Phenicians. Indeed, I have always considered that the sciences are eternal, and that, just as the constitution of the world has been eternally established in the divine mind, so the disciplines are certain celestial seeds which are settled in us and bring us profit in proportion to how much we cultivate them<sup>9</sup>.

This passage sets a parallel between the “celestial seeds” of geometrical knowledge and the divine

8. On Peletier's relation to Platonism, see notably Guy Demerson, “*Dialectique de l'amour et Amour des amours chez Peletier du Mans*”, in *Actes du colloque Renaissance-classicisme du Maine*, Paris, Nizet, 1975, p. 263-282; Isabelle Pantin, “Microcosme et ‘Amour volant’ dans *L'Amour des Amours* de Jacques Peletier du Mans”, *Nouvelle Revue du XVIe Siècle*, 2 (1984), p. 43-54.

9. *EE*, sig. 4v: “De cuius initijs huc nihil afferre constitui. Non ab Aegyptijs, non à Chaldaeis, non à Phoenicibus, illius originem requiram. Scientias quippè aeternas esse semper ex-

constitution of the universe, the former being connaturally settled within us as the latter is eternally established within God's mind<sup>10</sup>. Peletier thus seems to situate the origin of geometrical notions within the human soul, in conformity with the Platonic conception of mathematical knowledge<sup>11</sup>, and not in material substances, as was defended by Aristotle and the Peripatetics<sup>12</sup>.

In view of the eternal character of geometry and of the innate presence of its principles within all men, it would be pointless, for Peletier, to attribute the invention of geometry to any particular individual or nation<sup>13</sup>. Hence, although this eternal knowledge is not immediately accessible to men and is mainly actualised by those who care to cultivate its universal seeds, the unveiling of the geometrical principles should be attributed to all students of geometry throughout the ages. Regarding this issue, Peletier's position clearly differs from the humanistic attitude which was held by many sixteenth century commentators of Ancient mathematicians<sup>14</sup>.

Admitting, in this context, the divine origin of geometrical notions and the ontological correspondence between geometrical and physical compositions, Peletier therefore goes on to present geometry as a means to reconstitute the structure of the Universe and, more fundamentally, to reach towards the infinite wisdom and power of God.

Everywhere hides a certain power of geometry. Whether it may be held more as a power of nature or as a power of the art cannot

istimavi: atque ut in Mente divina, ab aeterno infixam fuisse Mundi constitutionem: sic disciplinas, caelestia quaedam semina esse: quae in nobis insita, & pro rata cuiusque portione exulta, fructum edunt”.

10. Here, the notion of *semina* indirectly evokes the Stoic notion of *logoi spermatikoi* which was connected to the timaeian representation of a “sowing God” and which was interpreted in metaphysical terms by Plotinus, Proclus and later by Augustine, by whom mainly this notion was transmitted to the Medieval thinkers. On the development of this notion from the Antiquity to the Renaissance, see Hiro Harai, *Le concept de semence dans les théories de la matière à la Renaissance*, Turnhout, Brepols, 2005, p. 23-32.

11. Indeed, in the *Meno* (81b-85b) and in the *Phaedo* (75c-77a), Plato describes mathematical notions as acquired by the soul before its conjunction with the body.

12. For Aristotle, see notably *De anima* III.7, 431b12-16.

13. Peletier's position on this question is mainly developed in his letter to Jean Fernel (*EE*, sig. p4v).

14. On this issue, see François Loget, *La Querelle de l'angle de contact...*, p. 135-148.

be perceived well enough, unless insofar as the practice itself reveals it. The more progress we will make in this contemplation, the closer we will seem to get to God. And as the eternal mind seizes the past, distinguishes the present, perceives the future, while embracing and governing all things simultaneously, the excellent geometer turns his unified thoughts towards his object and, through a universal speculation, carefully examines a world of his own<sup>15</sup>.

The geometer, through his study of the order which governs all geometrical figures, would therefore attempt to imitate the process by which God embraces all things and presides over their essence and existence. Yet, what this passage also shows is that the geometer's reconstitution of the divine intuition and conception of the universe is only carried out to the scale of the human mind and that there is, in other words, an ontological disproportion between the geometrical order contemplated by the geometer and the transcending order which it aims to recreate. Indeed, the manner in which Peletier describes geometrical research here, that is to say, as the constitution within the geometer's mind of a world which is the reflection of God's work, clearly evokes the parallel presented in Cusanus' *De coniecturis* between the world made by God and the re-creation of this world by the human mind through the construction and the study of its rational objects<sup>16</sup>. Now, in this context, Cusanus establishes not only the similitude, but also the disproportion, between the infinite and divine intuition that led to the constitution of the Universe

and the finite and conjectural notions by which the human understanding aims to reach it<sup>17</sup>.

## Geometry and the limits of human knowledge

The idea that the study of geometrical compositions only imitates and reveals the constitution of the Universe in a finite and somewhat inadequate manner finds its confirmation in Peletier's commentary on df. I.16. In this context, he presents the various modes of definition of the circle, and notably its cinematic mode of definition, which is, to him, as admissible, from a geometrical point of view, as the non-cinematic definition proposed by Euclid in df. I.15<sup>18</sup>. At this occasion, he dismisses the problem of establishing the most adequate mode of definition by stating that it is useless to try to determine the real order of procession between the circle and the straight line, given that the human mind is too weak to reach the true genesis and essence of geometrical objects and may only, regarding their origin, formulate conjectures.

And nobody should exert himself to find out which is first between the straight and the round. But if someone is forced to give his opinion, he will judge appropriately and as a philosopher, if he declares that both exist simultaneously. For the circle turned about in the plane produces the straight line. Indeed, for the mind, nothing will come before or after. The understanding itself can barely seize points as having existed before lines, or lines before surfaces, or finally surfaces before bodies. Just as, among philosophers, to say that the universe once was not exceeds the capacity of our souls and to say that it has always been is above all admiration. But, as for us, we aim to establish all things in their proper order and to reduce them to art to the measure of our understanding, and attempt to decide,

15. *EE*, sig. 4rv: "Ubique latet vis quaedam Geometriae: quae utrum plus naturae habeat an artificij, non satis perspicere potest: nisi quatenus explicat ipsa exercitatio. In qua meditatione quanto maiores progressus fecerimus, tanto propius ad Deum accedere videmur. Ac quemadmodum Mens illa aeterna, praeteritorum meminit, praesentia cernit, futura perspicit, simul verò omnia amplectitur & moderatur: ita praeclarus Geometriae artifex suas cogitationes in unum collatas, ad rem suam convertit, et suum quendam Mundum universa speculatione intuetur".

16. Nicholas of Cusa, *De coniecturis*, I.1.5 (*Nicolai de Cusa Opera omnia*, vol. III, eds. Joseph Koch and Karl Bormann, Hamburg, Felix Meiner, 1972, p. 7): "Dum enim humana mens, alta dei similitudo, fecunditatem creatricis naturae, ut potest, participat, ex se ipsa, ut imagine omnipotentis formae, in realium entium similitudine rationalia exserit". On Cusanus' influence on Peletier, see Hans Staub, *Le curieux désir...*, p. 19-21 and Sophie Arnaud, *Ratio et Oratio: la voix de la Nature dans l'œuvre de Jacques Peletier du Mans (1517-1582)*, Paris, Honoré Champion, 2005, p. 50, 73 and 77.

17. *Ibid.*: "Coniecturas a mente nostra, uti realis mundus a divina infinita ratione, prodire oportet. [...] Coniecturalis itaque mundi humana mens forma exstitit ut realis divina".

18. *EE*, p. 6: "Circulus, est vestigium lineae rectae in plano circumductae, altero extremorum manente fixo, donec ipsa unde duci coepit, redierit. Ut, si linea *ab* super *a* puncto duci incipiat in orbem à puncto *b*, per *c*, *d*, & *e* puncta, donec ipsa rursus *ab* facta sit: descriptus erit Circulus *bcd*. [...] Tota demum linea *eb* circumducta, Superficiem describit quae Circulus dicitur. Unde manifestum est omnes lineas à centro Circuli exeuntes, aequales esse: quum sint ex unius lineae vestigio".

as much as possible, what is probable and the least false. Indeed, order is the most reliable guide in the disciplines. But this diversity of things exercises us, in which it is enough, for us, to apply the conjecture to a use. For what do we think we can accomplish by art in the things which Nature has so ingeniously made? Or what can we understand regarding the things which have emanated in a divine manner, when we judge them in a human manner. So the circle, while originating from itself, seems to come from the straight; it is infinite, but similar to the finite; it contains all things, as it is the most capable figure, but however appears to receive something from outside<sup>19</sup>.

According to this text, the limits of our intellectual faculties faced with the genesis of the straight line and of the circle would be due to the unsolvable gap which separates the human creature from God and from the things which may be understood as direct emanations of the divine<sup>20</sup>. This ontological and gnoseological disproportion between the human and the divine would be due to the fact that, in God's mind and in the divine constitution of the Universe, all divine things subsist undividedly and simultaneously. Therefore, the temporal modes according to which the geometer aims to reach the constitution of his objects would not enable him reach their true genesis and mode of existence. As a consequence, for Peletier, to conceive the straight line as emanating from the circle would be just as improper as conceiving the circle as emanating from the straight line.

19. *EE*, p. 6: "Neque est quòd quisquam se fatiget inquirendo, utrum sit prius Rectum an Rotundum. Sed si quis sententiam ferre cogatur: ut Philosophus, rectè iudicabit, si utrunque simul esse pronuntiaverit. Nam & Circulus in plano rotatus, Rectum procreat. Menti quippè nihil prius neque posterius. Immò puncta ante lineas: aut lineas ante Superficies: aut denique superficies ante corpora fuisse, vix cogitatio ipsa complecti potest. Sicut apud Philosophos, Universum aliquando non fuisse, captum animorum excedit: semper fuisse, supra omnem admirationem est. Nos autem, quantum cogitatione assequimur, omnia suo ordine statuere, atque ad artem reducere: conamur iudicio quoad eius fieri potest, probabili, minimeque fallaci. Ordo enim in Disciplinis dux certissimus. Sed nos haec rerum varietas exercet: in qua satis nobis est coniecturam ad usum accommodare. Quid enim nos efficere posse putamus arte, in ijs quae Natura tam affabrè fecit? aut quid ingenio consequi, quum de his quae divinitus emanarunt, humanitùs iudicamus. Circulus igitur ex se ipse ortus, ex Recto provenire videtur: infinitus, ac finito similis: omnia continens, ut capacissimus, & tamen aliquid extrà se in speciem admittens".

20. See also Hans Staub, *Le curieux désir: Scève et Peletier du Mans, poètes de la connaissance*, Genève, Droz, 1967, p. 20-21.

The same may be said about the causality and the interrelations we conceive between points, lines, surfaces and bodies.

The difficulties we would have in determining the genesis and mode of being of divinely originating entities is illustrated through the example of the circle, in which seem to coincide the circular and the straight, the finite and the infinite, the self-sufficiency of the principle and the condition of created things, none of which may be fully understood simultaneously through human means. This set of oppositions, as well as the assertion of the limits of our understanding faced with the mode of existence of divine entities, clearly evokes the conception of Cusanus, who asserted that, at the level of the divine, all opposites coincide and that this *coincidentia oppositorum* properly exceeds the capacities of the human intellect<sup>21</sup>.

The limits of the human mind faced with the infinite and the undivided is clearly expressed by Peletier in the commentary on df. I.1, in which he asserts that geometry only considers finite magnitudes<sup>22</sup>, since "there is no science of the infinite"<sup>23</sup>. This assertion, which relates to Aristotle's conception of scientific knowledge<sup>24</sup> (and which was also crucial to Cusanus' epistemology<sup>25</sup>), thus restricts the domain of scientifically knowable objects to those which are finite, comparable and non-contradictory. In this context, the limits of

21. See notably *De docta ignorantia* I.4 (ed. by P. Wilpert, F. Meiner, Hamburg, 1994, p. 16-18). The set of oppositions presented here in relation to the circle also evokes the thought of Plato's commentator Proclus (fifth century A.D.), who affirmed, in his *Commentary on the first book of Euclid's Elements*, the mutual implication, within the circle, of the circular and the straight, of the Limit and the Unlimited (the two fundamental principles of all mathematical beings), of the unconditioned and the conditioned (*Procli Diadochi in primum Euclidis Elementorum librum commentarii*, ed. G. Friedlein, Leipzig, Teubner, 1873 [FR.], p. 153-154). As we will see, Proclus seems to have been an important source of Peletier's conception of geometrical objects. In 1557, Proclus's commentary on Euclid was mainly accessible through a partial latin version in Giorgio Valla's *De expendis et fugiendis rebus opus* (Venice, A. Manutius, 1501) or in Greek, through Simon Grynaeus's *aeditio princeps* of Euclid's *Elements* (Basel, I. Hervagius, 1533). The first official and authoritative latin translation, that of Francesco Barozzi (Padua, G. Percazzino), dates from 1560.

22. *EE*, p. 2: "Geometria magnitudines considerat, easque finitas".

23. *Ibid.*: "Geometria ubique infinitum devitans (infiniti enim nulla est scientia)".

24. See for example *Physics*, I.4, 187b7-10.

25. *De docta ignorantia* I.1, ed. Wilpert, p.6-8.



the human mind faced with the infinite and the uncomparable is also brought forth to explain why, in Euclidean geometry, lines, surfaces and solids are regarded as non-homogeneous. Peletier says indeed that without this separation of lines, surfaces and solids in geometry, “the substance of things would be vague and confused”<sup>26</sup>, here evidently describing the way these objects appear to the human mind. Indeed, the non-homogeneity of lines, surfaces and solids would not representative of the true essence of magnitudes, which are, in God as in the constitution of the Universe, properly undistinguished and homogeneous. As Peletier will put it much later in a set of poem entitled the *Louanges*, the distinctions and the causality the geometer conceives between points, lines, surfaces and solids is just as inadequate as the distinction and the causality we conceive between atoms, matter, form and body in the constitution of physical substances, since all these would subsist undividedly in God’s mind and would have come about simultaneously in the constitution of the Universe<sup>27</sup>. Thus, to consider points as parts of lines, lines as parts of surfaces, and surfaces as parts of bodies, would be attempting to understand these things according to a mode which is only accessible to God<sup>28</sup>.

Nevertheless, this does not mean that geometrical research is, for Peletier, devoid of any epistemic value. As he states it in his commentary on df. I.16, the geometer must try to approach truth

26. *EE*, p. 2: “Sed quia Magnitudinum partes, naturam totius denominationemque retinent, partes enim Linearum, lineae sunt: Superficierum, superficies: & Corporum, corpora: alioqui vaga & confusa esset rerum substantia”. This passage could be a quotation of Proclus’ commentary on the first book of Euclid’s *Elements*, [FR. 91]: “Most people, observing that limits exist imperfectly in limited things, have a confused conception of their being” (transl. by G. Morrow, *Commentary on the first book of Euclid’s Elements*, Princeton, Princeton University Press, 1970, p. 75).

27. *Suite de la Science*, in *Euvres poétiques intitulées Louanges*, Paris, R. Coulombel, 1581, f° 57r: “tout cet Univers, à pris sa Forme ansamble: / Tous Nombres ont etè, e sont, aussi tot qu’Un: / Matiere, e Forme, e Tout, n’uret principe aucun: / A coup, e an l’instant les poins, qui s’estandiret, / Lignes, Eres, e Cors an l’Infini randiret” (p. 283 in the edition of S. Arnaud, S. Bamforth and J. Miernowski, in *Oeuvres complètes X*, Paris, H. Champion, 2005). See also Arnaud, *Ratio et Oratio...*, p. 76 and 81-82 and Staub, *Le curieux désir...*, p. 32-33.

28. *Ibid.*, f° 56v (ed. Arnaud, Bamforth and Miernowski, p. 282): “Mes chercher dans le Cors, les Eres, plus ou moins, / E les Lignes an l’Ere, an la Ligne les Poins, / Ni pourquo il an vient tele, ou tele facture, / C’et vouloer defonser l’armoere de Nature, / Pour comter le trezor de ce grand Immortel, / An soe seul infini, e seul se sachant tel”.

as much as possible, notably by applying, in his conjectures and in his teaching, the rational principle of order, which, in the epistle to Charles de Lorraine, is said to govern both the creative process of Nature and the development of geometrical science<sup>29</sup>.

Moreover, the truthfulness and certainty of geometrical knowledge, which Peletier clearly opposed, in this context, to the uncertainty and refutability of opinion-based disciplines<sup>30</sup> and which also enabled him a bit further to define geometry as the very source from which all disciplines draw the method to distinguish truth from falseness<sup>31</sup>, are only established in relation to our discursive and comparative modes of knowledge.

What we therefore have to consider now is how Peletier defined the ontological status, as well as the mode of apprehension, of the objects which are properly reached and studied by the Euclidean geometer.

29. *Ibid.*, sig. 4r: “Ut enim ab exiguis initijs, Natura ad operis perfectionem & absolutionem sensim pervenit : ita Geometria ab infimis ad altissima, rectè & gradatim sese extollit. Quid Puncto simplicius ? quid Circulo absolutius ? at ex illo omnia emanant, in hoc omnia concluduntur. Ut ne Puncto quidem desit infinitatis admiratio. Quid enim tam mirabile, quàm à medij Circuli puncto, quod Centrum vocamus, tot lineas exire, quot ad Peripheriam desinunt ? Iam verò ipsa & Theoremata, quum alia ex alijs consequantur, rerum agendarum seriem nobis opportunè referunt: monentque nihil praeposterè, nihil sine consilio aggrediendum, sed omnia ad rationis normam esse dirigenda. Ad haec, sicut Natura novi quippiam assiduè molitur, ita Geometria semper aliquid egredium dispicit, exquirat, excogitat”.

30. *EE*, sig. 4r: “Nam quum caeterae artes probabili quadam opinione constent, certè Geometria (inter Mathematicas spectatissima) veritatis confirmatione seipsam tuetur & commendat. Quumque res omnes aut ornatu amplificare, aut testimonio deprimere, aut denique circuitione invertere possimus : haec ipsa suo innixa praesidio, perpetua, simplex & uniusmodi est. Nulla huc controversia, nulla disceptatio incidit, quae non statim ad veritatis fidem probationemque referatur. Eius quippè rationes non persuadent, sed cogunt : etiam ordine ipso, immò adeò Naturae ductu quodam”.

31. *Ibid.*, p. 12: “Demonstrationem verò appellant Dialectici, Syllogismum qui faciat scire: nempè qui ex probatissimis concludat. Atque haec à Geometria ortum habet. Immò omnis quae ad verum perducit probatio, Geometrica est. Ut verissimè dictum sit, neminem scire verum à falso distinguere, cui Euclides non fuerit familiaris”. On the epistemological status of the principles and the demonstrations of geometry according to Peletier’s commentary on the *Elements*, see Giovanna Cifoletti, “From Valla to Viète: The Rhetorical Reform of Logic and Its Use in Early Modern Algebra”, *Early Science and Medicine XI/4* (2006), p. 390-423.

## The modes of apprehension and the ontological status of the objects of geometrical research

### The role of the senses in the apprehension of geometrical objects

Due to the weakness of the human mind faced with the divine essence and genesis of his objects, it seems that, for Peletier, the geometer is forced to access his objects in a manner which is adequate to his bodily condition, that is to say, by starting from the compositions found in the objects of the senses. This mode of apprehension of geometrical objects is first of all presented in the commentary on df. I.1, in which the notion of point is presented as induced within the intellect by the observation of a physical form of *minima*.

For since in external things something minimal could be presented to the senses, it was certainly conform to reason to also give to the intellect something which is the smallest possible thing<sup>32</sup>.

Peletier does not say here which are these very small physical beings which enable the geometrical notion of point to arise in the intellect. These do not seem to be the atoms of Democritus and Epicurus, although Peletier established an essential correspondence between geometrical points and physical atoms<sup>33</sup>, since the latter were traditionally considered as imperceptible to the senses<sup>34</sup>. It seems rather that Peletier has here in mind certain natural substances which the senses perceive as partless. This would be confirmed by a passage of Proclus' *Commentary on the first book of Euclid's Elements*<sup>35</sup> which Peletier seems to be

32. *Ibid.*, p. 2: "Nam quum in rebus externis aliquid sensui obiceretur minimum: sanè rationabile fuit, intellectui quoque aliquid dari quo nihil esset minus".

33. *EE*, df. I.1, p. 2: "[Puncta] sunt Epicuri Atomii, omnium rerum semina" and df. I.5, p. 3: "Puncta igitur, atomi sunt". In both cases, Peletier's aim is to show the identity between the structure of physical substances and the structure of geometrical magnitudes (these being both related to the structure of numbers). On this issue, see also Arnaud, *Ratio et Oratio*, p. 70-71.

34. See, for example, Charles Mugler, "L'invisibilité des atomes. À propos d'un passage d'Aristote", *Revue des Études Grecques*, LXXVI/361 (1963), p. 397-403.

35. On Proclus and on the means through which Peletier

quoting here. Comparing, in his commentary on df. I.1, the way the physicist and the mathematician each apprehend indivisible particules in their own science, Proclus says: "The former defines as element what is simple to sense-perception, the other what is simple in thought; and each of them is right with regard to his own science<sup>36</sup>".

Now, this implicit quotation does not seem insignificant here, since, as we will see further, Peletier seems, in his commentary on the *Elements*, to have shared Proclus' views regarding the mode of apprehension of geometrical objects. In his own commentary on Euclid, Proclus clearly rejected the idea that our notions of geometrical objects find their proper origin within the sensible world and that they are obtained through abstraction<sup>37</sup>, in other words through a process which consists in mentally separating the quantitative properties of physical realities from matter in order to study the intrinsic properties of magnitudes<sup>38</sup>. Proclus rather defended a conception in which the objects of the senses play the role of awakening within us the innate notions of geometrical objects through the structural similarities they present with the latter, these corresponding to the intelligible archetypes of all sensible compositions<sup>39</sup>.

Now, not only Peletier clearly assumes, in his commentary on Euclid, the correspondence between natural and geometrical entities and the intelligibility of the latter as did Proclus, but he also never refers, even indirectly, to the Aristotelian notion of abstraction. This seems rather significant here, as Aristotle's ontological system had a canonical

might have had access to his commentary on the first book of the *Elements*, see note 22.

36. Proclus, *A commentary on the first book of Euclid's Elements*, transl. by G. Morrow, Princeton, Princeton University Press, 1970, p. 76 (FR. 93).

37. Proclus, FR. 49 and FR. 139-140.

38. On the development of this conception, from Aristotle to his commentators from late Antiquity, see Ian Mueller, "Aristotle on Geometrical Objects", *Archiv für Geschichte der Philosophie* XLII/2 (1970), p. 156-171 and *id.* "Aristotle's Doctrine of Abstraction in the commentators", in R. Sorabji (ed.), *Aristotle Transformed: the Ancient Commentators and their Influence*, Ithaca, Cornell University Press, 1990, p. 463-479.

39. Proclus, *ibid.*, p. 112 (FR. 140): "Prior to sense objects, therefore, are the self-moving intelligible and divine ideas of the figures. Although we are stirred to activity by sense objects, we project the ideas within us, which are images of things other than themselves; and by their means we understand sensible things of which they are paradigms and intelligible and divine things of which they are likenesses".

status during the late Middle Ages as well as in the Renaissance, and was still commonly referred to in sixteenth-century commentaries on Euclid, even by authors who did not fully defend the ontological conception which is associated to it. This is the case, for example, of Oronce Fine, who published in France a commentary on the *Elements* prior to Peletier<sup>40</sup>.

Peletier gives more precisions regarding the ontological and gnoseological relation of geometrical and physical objects in his commentary on df. I.5 of the *Elements*, which presents the notion of surface. Peletier then reaffirms the importance of sensation in the apprehension of geometrical notions, but he does not go as far as to imply that geometrical figures hold their origin from sensed objects. He rather asserts the identity or correspondence between their modes of composition and shows that the human mind, which is not able to reach intelligible objects directly, without the assistance of the senses, may find a physical example of the primary notions of geometry in the formal constitution of certain physical realities.

Although it seems that the point, the line and the surface can only be seized by the intellect, that they do not exist in reality and that they cannot be shown, each of these things can however be represented by something in nature. Points are indeed assimilated to the indivisible corpuscles which play together in the rays of the Sun, lines, to those very rays, the surfaces, to shadows, since they can never go underground, or even colours, according to the opinion of the Pythagoreans<sup>41</sup>.

When Peletier says that geometrical points, lines and surfaces do not seem to exist in reality (*non re existere videntur*), he is not discussing their degree of reality in an absolute manner, but is only describing the way they initially appear to us. Indeed, from the point of view of the human mind, points, lines and surfaces do not, at first, seem to present

the same level of concreteness as the things which are more easily and commonly known to us, that is to say, the objects of the senses. The intellect would therefore be too weak to reach suprasensible objects directly and would initially need to start his scientific investigations with the observation of sensible realities. This certainly evokes the Platonic epistemological model, according to which the human soul cannot directly access the intelligible notions it contains because of its conjunction with the imperfections of the body. However, Peletier does not, as Plato, describe geometrical knowledge as resulting from the transcendence of the imperfections of the material world, but is rather suggesting, as Proclus did, that the compositions found in nature suggest the innate notions of geometrical figures, and calls them to our intellect from our souls, through the similarities they present with them.

Moreover, if the intellect seems to depend upon the senses to reach its notions of point, line and surface, it must be noted that it is not any type of sensed objects that seems able to awaken these intelligible notions within us, but mainly phenomena pertaining to light and vision, such as corpuscles which move around in light, rays of light, shadows and colours. These physical representations of points, lines and surfaces, which were partly set forth by Proclus<sup>42</sup> (and before him by Hero of Alexandria<sup>43</sup>), are probably not mentioned here just because light and colour are the only kind of sensible realities which may subsist physically without depth, but also because of the particular status of light in the Neoplatonic tradition and in the medieval cosmological theories which were underpinned by a Neoplatonic understanding of the universe<sup>44</sup>. In this context, light was considered as the direct emanation of God and as the first and the most perfect physical expression of the divine archetypes of the Universe, as well as its material cause. Now, this metaphysical conception of light seems occasionally referred to in Peletier's writings, notably in his scientific poetry<sup>45</sup>.

40. On Oronce Fine's conceptions on the status of geometrical objects, see Angela Axworthy, *Le Statut des mathématiques en France au XVIe siècle: le cas d'Oronce Fine*, Ph.D. Thesis, CESR (Tours), 2011, p. 94-112.

41. *EE*, p. 3: "Ac tametsi, Punctum, Linea, & Superficies intellectu tantum capi videantur, non re existere, neque ostendi posse: habet tamen unumquodque horum in rerum natura quo representetur. Puncta enim corpusculis insecabilibus assimilantur, quae in radijs Solis collidunt: Lineae radijs ipsis: Superficies umbris, ut quae terram nunquam subeant: seu etiam coloribus, ex Pythagoreorum sententia".

42. Proclus, FR. 115.

43. Hero of Alexandria, *Definitiones* 8, in *Heronis Alexandrini opera*, ed. by J.L. Heiberg, IV, Stuttgart, Teubner, 1974, p. 21.

44. James McEvoy, "The Metaphysics of Light in the Middle Ages", *Philosophical Studies*, 26 (1978), p. 126-145.

45. On the treatment of light in Peletier's scientific poetry, see notably Isabelle Pantin, "Microcosme et 'Amour volant'...", (quoted in note 9); *id.*, *La Poésie du ciel en France dans la sec-*

Now, as we have seen, if Peletier attributes to the senses the function of awakening within us the intelligible notions of the primary geometrical objects, it is however the intellect which seems to hold the main role in the study of their properties and in the constitution of geometrical knowledge.

### The role of the intellect in the study of geometrical objects

Regarding the role of the intellect, Peletier presents interesting considerations within the commentary on df. I.12, which concerns the angle. In this context, he discusses the Euclidean definition of the geometrical angle and presents his own definition. Indeed, while Euclid, in df. I.8 of the *Elements*, defined the angle as the mutual inclination of two lines<sup>46</sup>, Peletier chose to define it rather as the intersection of the two lines<sup>47</sup>. This redefinition of the angle played a crucial role in his discussion of the angle of contact, as it enabled him to declare that the angle of contact or of contingence (that is to say, the angle formed by the contact of the circumference of a circle with its tangent) is not an angle, in order to avoid the parallogisms which arised from its admission in Euclidean geometry (notably the problem of its non-comparability with rectilinear angles)<sup>48</sup>.

Peletier's discussion of this matter brings him to define the angle as consisting in a point<sup>49</sup>, rather than in a line or in a surface<sup>50</sup>. Now, in order to demonstrate that his definition of the angle (as

consisting in a point) does not contradict the definition of the angle as a magnitude<sup>51</sup>, given that the geometrical point is by definition not a magnitude<sup>52</sup> and could therefore not be rightfully considered as the cause of the quantity of the angle, Peletier attempts to show that, in the case of the point of intersection of two lines, the intellect seizes the point as configured and extended, although it admits it as indivisible by definition.

The angle surely consists in one point, but the inclination makes it either greater or smaller. [...] On this matter, one can very well see that the point of intersection is pressured and made somewhat more narrow by the cutting line according to its inclination. Will the point then be a quantity? By no means. Indeed, once the intellect has received something as the smallest possible thing, it cannot divide it any further, but nothing stops it however from contracting and compressing it. And so that nobody thinks that what we are saying is contradictory, it should be thought that the point in geometry is not considered as nothing, but rather as something. And as we diminish the unit in arithmetics, so do we diminish the point in its own way in the continuous, so that the source from which all things come from may also offer the representation and image of all those things, that is to say, the straight, the oblique, the long, the large and the deep<sup>53</sup>.

According to this passage, the intellect (*intellectus*) would indeed receive the point as indivisible through its definition, but, within the angle,

*onde moitié du XVI<sup>e</sup> siècle*, Genève, Droz, 1995, p. 214-224 and Kathryn Banks, "Space and light: Ficinian neoplatonism and Jacques Peletier du Mans's *Amour des Amours*", *Bibliothèque d'Humanisme et Renaissance*, 69/1 (2007), p. 83-101.

46. Euclid, *The Thirteen Books of the Elements*, transl. by T. L. Heath, 1956, vol. I, p. 153: "A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line".

47. *EE*, p. 4: "Angulus Planus, est duarum linearum in plano sectio".

48. *Ibid.*, Prop. III, 15, p. 73-78. We will unfortunately not be able to consider this discussion in this paper. For works relating to it, see note 3.

49. Proclus, in his commentary on df. I.8 (FR. 123), attributes to Apollonius a definition of the angle as the "contracting of a surface at a point under a broken line, or of a solid under a broken surface" (transl. Morrow, p. 99), which has in common, with Peletier's definition, the assimilation of the angle to a point.

50. *EE*, p. 5: "Linea quidem lineam secans Angulum constituit: sed non propterea Angulus pars est Lineae: sicut nec Lineae ipsae sunt Superficieis partes, quamvis sine Lineis Superficieis esse non possit quae ipsam terminent. Neque igitur Angulus pars erit Superficieis, quod ipsam occludat".

51. *Ibid.*: "Quaesitum est à nonnullis, esset ne Angulus quantitas an qualitas. [...] Ego verò omnino Angulum ut quantitatem attendi debere puto. Nam etiam quatenus rectus, obtusus, aut acutus est, ipsius sola dimensio consideratur [...]". This discussion is set forth by Proclus in his own commentary on the Euclidean definition of the angle (FR. 121-123).

52. *Ibid.*, p. 2: "Nam sicut Unitas est Numerorum veluti supposita materia & origo, quae Numerus non est: ita Punctum, Magnitudinis, quum magnitudo non sit".

53. *EE*, p. 5: "Angulum quidem in uno puncto consistere: sed inclinationem esse quae majorem ipsum aut minorem efficit. [...] In quo sanè intueri licet, punctum ipsum sectionis, à linea secante premi & quodammodo angustius fieri, pro inclinationis modo. Punctum ergo erit quantitas? minime. Intellectus enim quod semel minutissimum receperit, id amplius non dividit: sed contrahere tamen & constringere nihil vetat. Ac nequis nos repugnancia dicere putet, is attendat in Geometria Punctum non considerari ut nihil: immò ut aliquid. Et ut in Arithmetice Unitatem minuiamus, sic & Punctum suo modo in Continuis: scilicet ut id ex quo omnia emergunt, omnium etiam repraesentationem imaginemque exhibeat: nempe recti, obliqui, longi, lati, & profundi".

would seize it as extended and configured, and as spatially augmentable or diminishable in function of the mutual position of the two intersected lines. The intellect therefore seems to act here as a faculty which represents rational concepts in a figured and spatial manner and which can operate on these figured representations in a near-to-concrete manner (in this case, by augmenting or reducing the opening of the two branches of the angle). Peletier here justifies this state of things by saying that, although the point is by definition indivisible and deprived of quantity, it is truly “something” for the geometer (*Geometria Punctum non considerari ut nihil: immò ut aliquid*) and may therefore be treated in the same manner as any other geometrical object. Hence, it may be imaginarily endowed with a spatialised configuration and operated on in a manner which evokes the condition of material objects (enabling it to be cut, stretched or pressured), only without adding to these any corruptible matter. The same would apply to the arithmetical unit, which may be represented as divisible and operated on in the manner of a number, when studied in relation to fractions, although it is indivisible by definition.

Now, what may be said about this intellectual representation of the point as extended is that it is not fully disconnected from the condition of material objects, insofar as the geometrical point seems endowed with properties which belong to physical beings. It also shows that the study of geometrical properties includes a form of operation, which involves the construction and manipulation of imaginary objects, and may therefore not be considered as a purely passive contemplation of universal concepts.

In the following part of the commentary on df. I.12, Peletier asserts the relation between the intellect’s representation of the point, the line and the angle, and the properties of material realities. He then shows that it is this connection between the structures of geometrical and physical entities which incites and also authorises the intellect to represent the point of intersection of the two branches of the angle as configured and extended.

So since geometry represents Nature everywhere, insofar as it is its mirror, let us think that, as in the physical angle, the two lines (however thin they may be) cannot cut each other mutually unless one falls on the other at a point of intersection, so in the straight section of mathematical lines, the point will be

somewhat square, in the obtuse section, more blunted, and in the acute, more compressed and narrow. And these things are seized by the intellect, which only falls at rest with nature. Hence, it continuously diminishes the point, until the falling line is made one with the stretched-out line. For this reason, when the intellect asserts that the point is deprived of division, it conceives it as being neither a line, nor a surface, nor a body. But when it comes to the angle, which consideration completely differs from that of other quantities, then he will want to diminish what he had already assumed as being partless, so that, as we have said before, that from which quantity emerges also suggests the nature of quantity<sup>54</sup>.

According to this passage, the intellect would not apprehend and conceive the point of intersection of two lines in the same manner as the point which is defined in df. I.1. When conceiving the latter, the intellect would proceed by negation of the three dimensions of the body, since its aim is to reach the representation of something which is by definition deprived of quantity. But when conceiving the point which constitutes the angle, the intellect would seize the indivisible point as divisible, or rather as extended and variable in configuration, so that it may be properly understood as the cause of the quantity of the angle. Peletier justifies this by saying that the angle does not have the same mode of consideration as other quantities, which is probably due to the fact that, although the angle is a quantity, it consists in something that is by definition not a quantity. But he also justifies this by stating the correspondence between the intellect’s representation of the geometrical object and the structure of the physical object, arguing that the intersection of

54. *EE*, p. 5: “Quum igitur Geometria Naturam ubique referat, ut est ipsius speculum: cogitemus, sicut in angulo physico duae lineae quantunlibet tenues se mutuò non possunt scindere, nisi altera alteri cedat in puncto decussationis: ita in linearum Mathematicarum sectione recta, Punctum quodammodo esse quadratum: in obtusa, hebetius; in acuta, pressius & angustius. Atque haec intellectus assequitur: qui nisi cum natura nunquam conquiescit. Unde Puncti imminuendi finem non facit: donec linea cadens cum iacente una facta sit. Quum itaque Punctum divisionis expers esse intellectus ponit: id concipit, ut neque Linea, neque Superficies, neque Corpus sit. Sed quum ad Angulum devenit, qui aliam omnino habet considerationem à caeteris quantitibus: tunc quod sine partibus receperat, id iam minuendum assumit: scilicet, ut, quod paulò antè diximus, id ex quo quantitas nascitur, naturam etiam quantitatis sapiat”.

two material lines (however thin they may be<sup>55</sup>) is that which incites the intellect to represent the point of intersection as extended and configured.

What Peletier's discussion of the angle mainly shows us is that, for him, the object reached by the geometer seems to present two different modes of apprehension and, from this point of view, two different modes of being: one pertaining to its spatial representation, which is related to the structure of physical objects, and one pertaining to its logical definition, which is, on the other hand, properly disconnected from division and multiplicity. In the case of the point defined in df. I.1, the two modes of apprehension of the point (rational and figured) do not contradict each other. But when it comes to the point of intersection of two lines, the rational definition and the figured representation do not seem to coincide. Peletier does not however consider this a contradiction, since he distinguishes the apprehension of the geometrical object by the intellect, which has the ability to seize it as extended and divisible, from its apprehension by a purely rational faculty, which would seize its essential properties only in a conceptual and logical manner<sup>56</sup>. Therefore, when the intellect (which seems here attributed the power of a properly imaginative faculty) seizes the point as pressured, square or blunted, this representation should not affect the logical definition of the point as a partless and indivisible object. As Peletier clearly says it here, this spatial representation of the point does not cause it to be defined as a quantity. Now, what is important for us to determine is whether, for Peletier, the object properly aimed by the geometer is the object of the figured representation or the object of the rational definition and also how these are apprehended and related to each other.

### *Intellectus and cogitatio* and the status of geometrical objects

Such a distinction between the rational and the figured concept in geometry strongly evokes Proclus' conception of geometrical knowledge. Indeed, in order to preserve the intelligible nature and origin (as well as their necessity and universality) of geometrical objects and to authorise, at the same time, the geometer to represent his objects

with certain physical qualities (such as divisibility, multiplicity, and even movement) and to operate on them in near-to-concrete manner, Proclus distinguished the apprehension of the reason or the understanding (*dianoia*), which reaches the unextended and indivisible notions of geometrical objects, from the apprehension of the imagination (*phantasia*), which seizes these notions in a figured and spatialised mode<sup>57</sup>. According to Proclus, the objects of geometrical knowledge belong to the *dianoia*, which apprehends their properties through a deductive process which starts from hypothetical premises (contrary to *noësis*, which is said to grasp the undivided and unfigurable principles of all things, in other words the Platonic Ideas). But the *dianoia*, due to its absolute separation from matter and from empirical perception, would however not be able to seize its objects according to the mode of multiplicity and divisibility, although the essential properties of numbers and magnitudes may only be revealed through the study of their modes of division and composition. The *dianoia* would thus need to project its concepts (the *logoi* of geometrical figures) onto a material, but however intelligible screen, provided by the imagination (*phantasia*), which would enable to represent them according to the mode of multiplicity and divisibility. Geometrical objects, such as those which are constructed and operated on in the frame of the geometer's research, would therefore initially correspond to those imagined entities, which may be divided, multiplied, measured, extended or reduced, and treated in any possible way like material objects, only without being connected to any corruptible matter. For Proclus, this would legitimately allow geometers, against what Plato thought<sup>58</sup>, to talk about their objects as if they were dealing with concrete realities, speaking of squaring, cutting or superposing. However, in conformity with Plato's

55. Peletier is probably here thinking of the intersection of rays of light.

56. The tension between these two modes of presentation of the point may be linked to Proclus, FR. 91-92.

57. Proclus, FR. 54-55, trans. Morrow, p. 43-44: "For the circle [in the understanding] is one, yet geometry speaks of many circles, setting them forth individually and studying the identical features in all of them ; and that circle [in the understanding] is indivisible, yet the circle in geometry is divisible. Nevertheless we must grant the geometer that he is investigating the universal [...]. For the understanding contains the ideas but, being unable to see them when they are wrapped up, unfolds and exposes them and presents them to the imagination sitting in the vestibule ; and in imagination, or with its aid, it explicates its knowledge of them, happy in their separation from sensible things and finding in the matter of imagination a medium apt for receiving its forms".

58. *Republic*, 527a-b.

conception, Proclus clearly stated that the geometer must not remain attached to these divisible and spatially extended objects and should ultimately aim to reach, through these, the unextended and undivided *logoi* settled in the *dianoia*. It would therefore be these *logoi*, more than their figured representation, which would correspond, for Proclus, to the proper objects of the geometer.

Now, did Peletier also follow Proclus on this issue? Although he did not speak of imagination in the commentary to the *Elements*<sup>59</sup> (in latin, such a faculty would be called *imaginatio* or *phantasia*), the notion of *imaginatio* may be indirectly found through the use of the term *imago* in the context of the commentary on df. I.12, as Peletier says that the point, as the principle of magnitudes, displays the image and representation of the various types of magnitudes<sup>60</sup>. It is in any case clear that he attributed a role to a power of figuration in the apprehension and study of geometrical objects, which is attributed, in the dfs. 1, 5 and 12 of Book I, to the *intellectus*. The fact that the *intellectus*, for Peletier, contains a power to represent spatially the conceptual notions of the definitions and to operate on them in an imaginary manner is also suggested by his use of the verb *intelligere* throughout the commentary.

What could enable us to make a parallel between Proclus' and Peletier's descriptions of the apprehension of geometrical objects is the role Peletier gives, in this context, to the *cogitatio*, which seems to be comparable to the role played by the *dianoia* in Proclus' epistemological system. Peletier recurrently uses the term *cogitatio* in his commentary on the *Elements*, either to describe the proper notions of the geometer (*cogitationes*)<sup>61</sup> or to describe the faculty which possesses these notions and which is always described as a discursive and comparative faculty. In the commentary on df. I.16, as in the commentary on prop. I.46, the *cogitatio* is presented as the faculty through which the geometer seeks to understand the genesis and the ontological relation of its objects, but

which is also restricted in its capacity to do so<sup>62</sup>. Indeed, it is because of the limits of his understanding that the geometer would be restricted in his apprehension of the relations between points, lines, surfaces and bodies<sup>63</sup> or of the connection between the circle and the straight line, notably when it comes to conceiving the circle as possessing an infinite number of sides and angles<sup>64</sup>.

The passage in which the function of the *cogitatio* is presented in the most significant way is situated in the commentary on prop. III.15, in which Peletier aims to disprove that the angle of contact is an angle, as well as other correlative propositions, such as "the interior contact of two circles is a quantity"<sup>65</sup>. He then says that "nothing falls in the understanding which geometry could not somewhere represent" (*nihil in cogitationem cadat, quod semel uspiam Geometria non repraesentet*)<sup>66</sup>. By saying this, in this context, Peletier seems to conceive geometry as a means to display and explore the universal notions of the *cogitatio* in a manner which allows to reveal their truth or falseness in regards to the nature of quantity.

This function of geometry would also coincide with the function Peletier attributed to problems and theorems within the *Elements* of Euclid. Indeed, in Peletier's description of problems and theorems (which is again drawn from Proclus<sup>67</sup>), the geometer is said to deal with the constitution and the study of constructed and divisible figures<sup>68</sup>, in other words with objects which are

59. It appears however in his other mathematical texts, for example, in *L'Algèbre* (Lyon, 1554, p. 5) or in the *De contactu linearum* (in *Commentarii tres*, Basel, 1563, p. 46). The function it is given in those contexts would require a comparison which we unfortunately do not have the space to do here.

60. See note 54.

61. This is only in the epistle to Charles de Lorraine. See note 16.

62. In this sense, the notions of *mens*, *captum animae* or *ingenium*, such as presented in the commentary on df. I.16, seem used as synonyms of *cogitatio*, but it is mainly this notion, such as used throughout the commentary, which seems to properly designate the discursive faculty at work in the geometer's investigation.

63. See note 20.

64. *EE*, p. 45: "quum sit ipse [circulus], si huc cogitatio pertingere potest, infinitorum angulorum & infinitorum laterum".

65. *Ibid.*, p. 73-78.

66. *Ibid.*, p. 75.

67. Proclus, FR. 77.

68. *EE*, p. 12: "Problemata, ortus Figurarum comprehendunt, sectiones, additamenta: eaque omnia in arte, quae faciendae proponuntur. Atque, ut in Philosophia, Problemata dicuntur dubia quaedam quae nobis examinanda & solvenda proponimus: sic in Geometricis, Problemata vocamus constructiones ex arte depromptas: à quibus speculationes oriuntur, seu Theoremata: nempè quae factas Figuras comitantur, proprietates & affectiones: quaeque scientiae ipsi inhaerent & ipsam efficiunt. Nam in assertione consistunt praeceptorum sicut Problemata in constitutione Figurarum".

figurable and thereby rationally explorable. This seems to confirm that, for Peletier, the geometer would deal, in his investigation, with constructed and spatialised objects, which would project or display the universal notions of the *cogitatio* in a divisible and multiple manner, just as the representations of the Proclean *phantasia* in relation to the notions of the *dianoia*. Indeed, defined as such, geometry would therefore have the function of “explicating” the concepts which are “complicated” in the *cogitatio* (although this complication does not take place in a suprarational manner)<sup>69</sup>. If Peletier does not explicitly speak of projection (which is termed *probolè* by Proclus), this notion indirectly appears through his description of geometry as the “mirror of Nature”<sup>70</sup>, description which echoes Proclus’ description of geometrical figures as the mirror of the notions of the *dianoia*<sup>71</sup>.

### The role of the *intelligentia* in geometry

As in Proclus’ epistemological account, Peletier seems to admit, above the rational faculty involved in geometrical knowledge, a higher and more intuitive faculty, which he calls intelligence (*intelligentia*). In the presentation of the principles, intelligence is described in a somewhat negative way, Peletier stating that we do not firmly seize the definitions of geometrical objects through the “natural and innate intelligence”<sup>72</sup>.

69. The terms of complication and explication were fundamental in the Neoplatonic ontological tradition, from Plotinus and Proclus to Cusanus, and had a great influence on Renaissance philosophers, such as Charles de Bovelles. The term explication aims to present the material world, and all the things which exist and which are apprehended according to the mode of division and distinction, as the exteriorised deployment and unfoldment of the undivided essence of the divine principle of all thing, state which describes the term of complication.

70. *EE*, sig. A2r: “Rerum naturam in Geometria tanquam in speculo elucere, eamque è Geometricis speculationibus pendam esse docuimus”; df. I.12, p. 5: “Geometria Naturam ubique referat, ut est ipsius speculum”; prop. I.20, p. 26: “Naturae quaedam praestantia, quae in Geometricis passim relucet”.

71. FR. 141, trans. Morrow, p. 113: “Therefore just as nature stands creatively above the visible figures, so the soul, exercising her capacity to know, projects on the imagination, as on a mirror, the ideas of the figures; and the imagination, receiving in pictorial form these impressions of the ideas within his soul, by their means affords the soul an opportunity to turn inward from the pictures and attend to herself”.

72. *EE*, *Principiorum explicatio*, p. 1: “Definitiones quidem naturali insitaque intelligentia non statim concipimus: sed in

What we receive from this faculty are the axioms or the common notions, which Peletier calls “notions of the soul” (*animi notiones*)<sup>73</sup>. Following Aristotle<sup>74</sup> (which Proclus also followed on this issue<sup>75</sup>), Peletier defined these as self-evident principles, which are common to all demonstrative sciences, but which, in geometry, govern the properties and relations of magnitudes<sup>76</sup>. In other passages, such as the commentary on Prop. V.1, Peletier refers to the *communis intelligentia*, which is also called, in the same text, “common judgement” (*communis iudicium*). This faculty, which is explicitly associated to the common notions, is then said to contain the whole matter (or principles) of proportions<sup>77</sup>, these being also defined as the main objects of geometry<sup>78</sup>.

Now, for Peletier, the proportions and the order we seize in geometry, as in all things, would precisely be that by which the geometer aims to reach the archetypal principles which are undivided in God and which govern the constitution of the Universe<sup>79</sup>. Therefore, as the *cogitatio* attempts to discover the universal principles of proportions, even in a discursive manner and through the study of figured and comparable magnitudes, it would attempt, for Peletier, to reach back to the pure essence of the principles of all things. And from this point of view, Peletier’s *cogitatio* would indeed relate to the principal notions of the *intelligentia* as Proclus’ *dianoia* relates to the transcending notions of the *nous*.

Now, what would mainly distinguish Proclus’ and Peletier’s account of the apprehension of geometrical objects is that, for the latter, the human

... eas, quum proponuntur, sponte consentimus: sic dictante rei cuiusque natura”.

73. *Ibid.*: “Sed qui Notiones non recipit, etiam sensu communi caret: unde & animi sensa vocantur”. On the status of those “notions of the soul” in Peletier’s commentary on Euclid, see Cifoletti, “From Valla to Viète”, quoted in note 32.

74. *Posterior analytics* I.10, 76a37-76b2 and 76b23-35.

75. FR. 76.

76. *EE*, p.1: “Quum igitur in Geometricis animi Notiones, Quantitates respiciant”.

77. *EE*, p. 119: “Tota igitur Proportionum materia ferè in communi intelligentia consistit”.

78. *Ibid.*, prop. V.1, p. 120: “Geometria enim quantacunque est, tota in Proportionibus est: neque aliud quicquam spectat, quàm ut Lineas Lineis, Superficies Superficiebus, & Corpora Corporibus componat & comparet”.

79. See note 8. See also, on this theme, the introduction to Arnaud, Bamforth and Miernowski’s edition of Peletier’s *Euvres poetiques intitulez louanges* (quoted in note 28), p. 22-24 and Arnaud, *Ratio et Oratio*..., p. 51-53.



mind never seems able to actually transcend the level of discursive knowledge in order to embrace the true essence and condition of the divine principles. The principles of the *intelligentia*, though present within all men and at the foundation of all sciences, could never be grasped in their purity and undividedness, beyond the study of comparable quantities and the conjectures of the geometer, in other words such as they subsist in God's mind and in his intuition of the *cosmos*.

We can certainly find, notably in the epistle to Charles de Lorraine, the idea that the universal notions reached by the geometer, in other words his *cogitationes*, are ultimately bound to serve a purer and more primordial pursuit, that is the imitation and the contemplation of God's work. Peletier also asserts the proximity, and even in some cases the identity, between geometrical and philosophical speculations<sup>80</sup>. As he presents it throughout his commentary on Euclid, this proximity would be first of all due to the intrinsic connection between Nature and geometry, as well as to its exemplary role in the discovery of the method and the order to apply in all scientific investigations<sup>81</sup>, both aspects being intrinsically related.

However, Peletier's rather skeptical vision of human knowledge clearly qualifies his assertion of the propaedeutical function of geometry (at least as a means to reach the divine). The epistle to Charles de Lorraine, in which the research of the geometer is compared to the divine contemplative and creative process, certainly presents geometry as a means to get closer to God's understanding of the world, but surely not as a means to reach it. Although geometry is said to represent and to reveal the constitution of things, it only seems to reach the deployed and explicated expression accessible to the senses and to the human mind, and not the complicated form it presents within God's mind. Hence, if geometry allows us to get closer to the divine, it is only through a symbolic form of knowledge<sup>82</sup>, which only allows to reach the divine infinity and unity, and its relation to

the finiteness and variety of Nature, in a conjectural manner.

## Conclusion

The various elements pertaining to the ontological status of geometrical objects, in Jacques Peletier's commentary on the six first books of the *Elements*, seem to offer different levels of definition of the nature and mode of apprehension of the objects of study. In the epistle to Charles de Lorraine and in the commentary on df. I.16, Peletier presents geometrical objects as entities originating from a divine source and which preside over the order of Nature, in conformity with Plato's ontological system. However, such as grasped and studied by the geometer, these may only be considered as a conjectural imitation of the divine constitution of the Universe. Following Cusanus, Peletier sets indeed a limit to our ability to reach the true genesis and mode of being of geometrical notions. Under these conditions, the objects which are defined and studied in the geometrical discipline seem to bear, for him, an irreducible distance with the divine mathematical order these ultimately aim to unveil.

In this framework, Peletier presents a theory of geometrical knowledge which, in various aspects, evokes that which is set forth by Proclus in his *Commentary on the first book of the Elements of Euclid*. As Proclus, Peletier did not conceive the objects of the senses as the origin of geometrical objects, but as the means to awaken their notion within the intellect through the similarities of their structures and modes of composition. As Proclus, Peletier admitted that the study of geometrical objects involves two suprasensible faculties, one which seizes these objects as extended and divisible figures and which operates on them in a near-to-concrete manner (the *intellectus*, which then seems closely related to the Proclean *phantasia*), and one which seizes them as unextended rational notions and which reaches the universal properties of magnitudes through discursive means (the *cogitatio*, which corresponds to Proclus' *dianoia*). The intellect would indeed display the rational notions of the *cogitatio* in a spatial and figured manner, in order to reveal their modes of composition and mutual relations. Through the rational exploration of the latter, the geometer would ultimately aim to reach towards the transcending order God settled in the Universe, as in our souls under the form of "celestial

80. *EE*, Prop. III.15, p. 77: "Atque ut rationes quoque philosophicas (immò quae Philosophiae pars in Geometria non latet?) Geometricis speculationibus immisceamus: Circulus ipse omnia in se recipit, ob sui perfectionem".

81. See note 32.

82. Staub, *Le curieux désir...*, p. 19 and Isabelle Pantin, "La représentation des mathématiques chez Jacques Peletier du Mans: Cosmos hiéroglyphique ou ordre rhétorique?", *Rhetorica* XX/4 (2002), p. 375-389.

seeds”, although this ascension may never be fully accomplished due to the finiteness of the human creature and of its modes of knowledge.

In view of this conception, if geometry is presented, in this commentary on the *Elements*, as the privileged means to cultivate of the seeds of knowledge settled by God in all human beings and to unveil the order of the Universe, this process would only occur, for Peletier, according to an infinite progression and at the scale of humanity, since the individual can only contribute to its accomplishment to the measure of his limited abilities. Therefore, this constant research and practice of all the generations of mathematicians, which allows and even urges each accomplished geometer to discuss and amend the conjectures of his predecessors, would above all contribute to the constitution of an art or a discipline, in other words of a properly human form of knowledge. In this perspective, what defines the object of the geometer, for Peletier, is not only its universality and necessity, but also its ability to be constructed and manipulated in an imaginary manner and to thereby provide the human mind with the adequate matter to exercise its innate powers of discernment.

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