



Advantageous Semi-Collusion Revisited: A Note

Kai Zhao

► To cite this version:

| Kai Zhao. Advantageous Semi-Collusion Revisited: A Note. 2011. halshs-00856136

HAL Id: halshs-00856136

<https://shs.hal.science/halshs-00856136>

Preprint submitted on 30 Aug 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Advantageous Semi-Collusion
Revisited: A Note

KAI ZHAO

www.tepp.eu

TEPP - Institute for Labor Studies and Public Policies
TEPP - Travail, Emploi et Politiques Publiques - FR CNRS 3435

Advantageous Semi-Collusion Revisited: A Note

Kai ZHAO*

February 2011

Abstract

In this note we reconsider the paper of Brod and Shivakumar (1999), published in the Journal of Industrial Economics, who analyze a two-stage model in which the firms compete in two dimensions and examine the effect of semi-collusion when the non-production activity is R&D. They shed light on the fact that in the presence of spillovers, firms and consumers could be both better off, peradventure both worse off, by a semi-collusive production cartel. We are motivated to study their setup by this fascinating outcome. Trying to approach the in-depth analysis and to understand the driving forces of this result, we find however that the findings of Brod and Shivakumar (1999) are disputable. By focusing upon their calculative errors, we show how the correct solution can be obtained, furthermore, we explain why the main propositions in their paper don't hold.

Keywords: R&D, semi-collusion, spillover, product differentiation

JEL classification: D43, L13, O31

Introduction

In a one stage game, cartels increase industry profits and exacerbate the consumer surplus. In a model where firms collude in production but compete in R&D, the cartel members may be worse off and consumers better off due to over-investment by firms eager to improve their position in the cartel. Brod and Shivakumar(1999) analyze a two-stage model and examine the effect of semi-collusion when the non-production activity is R&D. Firms choose their R&D

*GAINS-TEPP, Université du Maine;
Address mail: kai.zhao@univ-lemans.fr

effort in a first stage and output in a second stage. They shed light on the fact that in the presence of spillovers, firms and consumers could be both better off, peradventure both worse off, by a semi-collusive production cartel. We are attracted by this fascinating outcome. Thereupon, we try to approach the in-depth analysis and to understand the driving forces of this result. We find however that the findings of Brod and Shivakumar (1999) are disputable. The incorrect Sub Game Perfect Equilibrium values of per-firm R&D effort, output and profit due to improper handling result in the inaccuracy of their main propositions. When the goods are sufficiently substitutable, the proposition 1 doesn't hold. In other words, there is no absolute predominance of production cartel in terms of R&D effort. Since the optimum equilibrium of cartel at the production stage could be negative in certain combination parameters (the degree of product differentiation and the level of spillovers), we find the region D depicted as "*Consumers prefer Production Cartel; firms prefer Competition*" could not always satisfy the conditions mentioned in proposition 2. In what follows, we firstly resume Brod and Shivakumar(1999)'s set-up; secondly, by focusing upon their calculative errors, we show what the correct solution can be and we explain why the main propositions in their paper doesn't hold.

Model

Retrospect to the model of Brod and Shivakumar (1999), two firms with differentiated products engage in upstream R&D and downstream production. Each firm has the same production unit cost of c . This unit cost can be reduced by investing in R&D, where x_i denotes the investment of firm i . Due to spillovers, the R&D expenditure x_i not only benefits the investing firm i but also gives rise to lower unit costs for the rival firm j . Then the effective production unit cost function is given by: $C_i = c - x_i - \beta x_j$ (for $i, j = 1, 2$ and $j \neq i$). The parameter $\beta \in [0, 1]$ measures the size of the spillover effect. R&D spending directly cut down the basic production unit costs c . Nevertheless, investing in R&D is costly, and the R&D cost function is represented by $\frac{1}{2}\delta x_i^2$, where $\delta > 0$. Let $u(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}b(q_1^2 + q_2^2 + 2\gamma q_1 q_2)$, for $(a, b > 0)$, where $\gamma \in [0, 1]$ is a product differentiation parameter whose inverse indicates the strength of production differentiation. The utility function generates the following inverse market demand curve: $p_i = a - b(q_i + \gamma q_j)$

There are two regimes: the one is Competition where firms compete in both the R&D and the output markets; the other one is Production Cartel where the firms compete in the R&D market but collude in output market. The superscript "C" stands for Competition and "P" signifies Production Cartel.

The game is solved by backward induction and we characterize the equilibrium outcomes of this game.

Competition:

The SGPE values of per-firm R&D effort, output and profit are given by:

$$\begin{aligned}x^C &= \frac{2A}{\theta}(2 - \beta\gamma) \\ q^C &= \frac{\delta A}{\theta}(2 - \gamma)(2 + \gamma) \\ \pi^C &= \frac{\delta A^2 \Delta}{\theta^2}\end{aligned}$$

where $A = a - c$, $\theta = (2 - \gamma)(2 + \gamma)^2 b\delta - 2(1 + \beta)(2 - \beta\gamma) > 0$ and $\Delta = (2 - \gamma)^2(2 + \gamma)^2 b\delta - 2(2 - \beta\gamma)^2 > 0$

In the paper of Brod and Shivakumar(1999) (Henceforth "BS"), the expression of Δ displayed in page 225 is however $\Delta_{BS} = (2 - \gamma)^2(2 + \gamma)^2 b\delta - 2(1 + \beta)(2 - \beta\gamma)^2 > 0$. We have $\Delta - \Delta_{BS} = 2\beta(2 - \beta\gamma)^2 > 0$ that generates the underestimate of the real profit.

Production Cartel:

The symmetric equilibrium of R&D effort, output and profit correspond to the following solutions:

$$\begin{aligned}x^P &= \frac{A}{\Phi}(2 - (1 + \beta)\gamma) \\ q^P &= \frac{2\delta A}{\Phi}(1 - \gamma) \\ \pi^P &= \frac{\delta A^2 \Gamma}{2\Phi^2}\end{aligned}$$

where $\Phi = \gamma + \beta^2\gamma + 4b\delta(1 - \gamma^2) - 2\beta(1 - \gamma) - 2$ and $\Gamma = -4 + 8b\delta + 8b\delta\gamma^3 + 4\gamma(1 + \beta - 2b\delta) - \gamma^2(1 + 2\beta + \beta^2 + 8b\delta)$. As mentioned in BS, the product $b\delta$ can be expressed in the same units as output, they assume $b\delta = 1$ to simplify expressions. And we find whether these two expressions(Φ, Γ) are positive or not depends on the combination of parameters γ and β .

Whereas, BS consider that $\Phi_{BS} = 4(1 - \gamma)(1 + \gamma)^2 b\delta - (1 + \beta)(2 - (1 + \beta)\gamma) > 0$ and $\Gamma_{BS} = 8(1 - \gamma)^2 b\delta - (2 - (1 + \beta)\gamma) > 0$. Compared to our results, we have $\Phi - \Phi_{BS} = -4b\delta(1 - \gamma^2)\gamma < 0$. It is clear that there is the underestimate on R&D effort and output. These errors due to improper handling generate the

distinctive change in the following analysis. Furthermore, BS regard mistakenly Φ_{BS} and Γ_{BS} as the positive terms. Taking Φ_{BS} as an example, we illustrate here Φ_{BS} is negative when

- $\gamma \in (0.927441, 0.927886]$ and $\beta \in (\tilde{\beta}_1, \tilde{\beta}_2)$
- $\gamma \in [0.927886, 1]$ and $\beta \in (0, \tilde{\beta}_2)$

with $\tilde{\beta}_1 = \frac{1-\gamma}{\gamma} - \sqrt{\frac{1-4\gamma-4\gamma^2+4\gamma^3+4\gamma^4}{\gamma^2}}$ and $\tilde{\beta}_2 = \frac{1-\gamma}{\gamma} + \sqrt{\frac{1-4\gamma-4\gamma^2+4\gamma^3+4\gamma^4}{\gamma^2}}$

A reappraisal of the main propositions in BS(1999)

Proposition 1

Since $\Phi_{BS} > 0$, BS claimed the R&D effort in regime Production Cartel is always significant, the firms colluding in output spared no effort to invest in R&D for $0 \leq \beta \leq 1$ and for all $0 \leq \gamma < 1$. In fact, their finding is not true, the crux of the matter is that the Φ could be negative¹ in certain circumscription where the optimum equilibrium R&D effort is meaningless. We find that the member firm of cartel could have no interest in R&D processus when the goods are sufficiently homogenous, precisely $\gamma \in (\hat{\gamma}, 1]$ with $\hat{\gamma} = \frac{(1+\beta)^2 + \sqrt{33-28\beta+6\beta^2+4\beta^3+\beta^4}}{8}$. In this instance, the x^P will be inferior to x^C , then the proposition 1 is not always true.

In addition, BS(1999) claimed that "it is easy to show that as β rises, the difference $x^P - x^C$ declines" in page 226. As a matter of fact, the $\frac{\partial(x^P - x^C)}{\partial\beta}$ could be positive. Whether this gap enlarges or shrinks depends upon the combination of two parameters β and γ . In order to be more legible and intuitionistic, we illustrate this outcome with the following graphic.

¹ $\Phi - \Phi_{BS} = -4b\delta(1 - \gamma^2)\gamma < 0$.

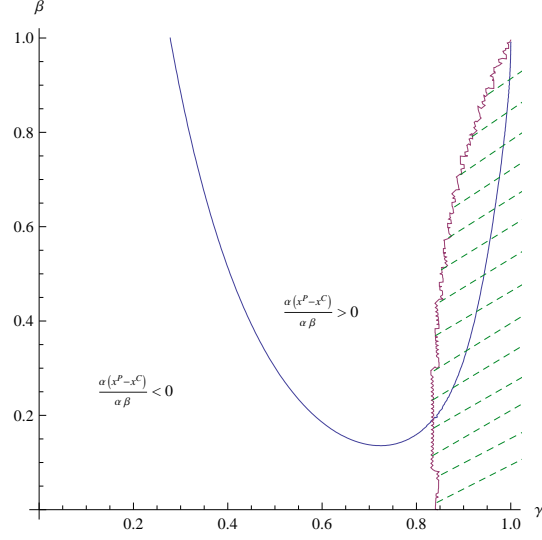


Figure 1: The effect of β on the difference $x^P - x^C$

Apart from the dashed zone which represents the flaw of their proposition 1, we have not only the region, corresponding to the finding of BS, in which the relative valuation of R&D is reduced as spillovers increase, but also the region where the gap enlarges following the rise of spillovers. The primary reason of omitting this positive aspect of β stems from the underestimate of R&D effort in regime P.

Proposition 2

BS(1999) try to compare two mentioned regimes in terms of both individual and collective incentive. They consider output as an index of consumer surplus.

$$\begin{aligned} q^P - q^C &= \frac{2\delta A}{\Phi}(1 - \gamma) - \frac{\delta A}{\theta}(2 - \gamma)(2 + \gamma) \\ &= \frac{A\delta(2(1 - \gamma)\theta - (2 - \gamma)(2 + \gamma)\Phi)}{\Phi\theta} \end{aligned}$$

It is straightforward $q^P - q^C$ has the same sign as the following expression:

$$f(\gamma, \beta) = \frac{2(1 - \gamma)\theta - (2 - \gamma)(2 + \gamma)\Phi}{\Phi\theta} = \frac{f_{BS}(\gamma, \beta)}{\Phi\theta}$$

Due to improper handling and error of judgement about Φ , it is mistakenly deemed that the difference $q^P - q^C$ has the same sign as the expression $f_{BS}(\gamma, \beta) = 2(1 - \gamma)\theta - (2 - \gamma)(2 + \gamma)\Phi = -2\gamma^4 + (\beta^2 + 2\beta + 3)\gamma^3 - 2\gamma^2(2\beta^2 + 3\beta - 3) - 4\gamma(1 - \beta)$ displayed in page 227. As the case stands, the difference $q^P - q^C$ is also influenced by the denominator $\Phi\theta$.

Concerning the difference of profit $\pi^P - \pi^C$,

$$\begin{aligned}\pi^P - \pi^C &= \frac{\delta A^2 \Gamma}{2\Phi^2} - \frac{\delta A^2 \Delta}{\theta^2} \\ &= \frac{A^2 \delta (\Gamma \theta^2 - 2\Delta \Phi^2)}{2\Phi^2 \theta^2} \\ &\neq \frac{A^2 \delta (\Gamma_{BS} \theta^2 - 2\Delta_{BS} \Phi_{BS}^2)}{2\Phi_{BS}^2 \theta^2}\end{aligned}$$

it is straightforward that $\pi^P - \pi^C$ has the same sign as

$$g(\gamma, \beta) = \Gamma \theta^2 - 2\Delta \Phi^2 \neq \Gamma_{BS} \theta^2 - 2\Delta_{BS} \Phi_{BS}^2$$

According to Figure 2 in page 228, there are always $q_{BS}^P > q_{BS}^C$ and $\pi_{BS}^P < \pi_{BS}^C$ in region D. Practically, we can find the inverse outcome $q^P < q^C$ even $\pi^P > \pi^C$ in this region.

REFERENCES

- [1] Brod and Shivakumar, 1999, Advantageous Semi-Collusion, The Journal of Industrial Economics, Vol.47, No.2, 221-230
- [2] Fershtman, C. and N. Gandal, 1994, Disadvantageous semicollusion, International Journal of Industrial Organization, Vol.12, 141-154

TEPP Working Papers 2012

12-1. What drives Health Care Expenditure in France since 1950? A time-series study with structural breaks and non-linearity approaches

Thomas Barnay, Olivier Damette

12-2. How to account for changes in the size of Sports Leagues: The Iso Competitive Balance Curves

Jean-Pascal Gayant, Nicolas Le Pape

12-3. Hedonic model of segmentation with horizontal differentiated housing

Masha Maslianskaia-Pautrel

12-4. Stricter employment protection and firms' incentives to train: The case of French older workers

Pierre-Jean Messe, Bénédicte Rouland

TEPP Working Papers 2011

11-1. The French "Earned Income Supplement" (RSA) and back-to-work incentives

Denis Anne, Yannick L'Horty

11-2. The effect of place of residence on access to employment: a field experiment on qualified young job applicants in Ile-de-France

Yannick L'Horty, Emmanuel Duguet, Loïc du Parquet, Pascale Petit, Florent Sari

11-3. Why is there a faster return to work near the border?

Jonathan Bougard

11-4. Residential Discrimination and Ethnic Origin: An experimental assessment in the Paris suburbs

Emmanuel Duguet, Yannick L'Horty, Pascale Petit

11-5. The Fateful Triangle : Complementarities between product, process and organisational innovation in the UK and France

Gérard Ballot, Fathi Fakhfakh, Fabrice Galia, and Ammon Salter

11-6. How important is innovation? A Bayesian factor-augmented productivity model on panel data

Georges Bressona, Jean-Michel Etienne, Pierre Mohnen

11-7. Fiscal Shocks in a Two Sector Open Economy

Olivier Cardi, Romain Restout

11-8. Productivity, Capital and Labor in Labor-Managed and Conventional Firms

Fathi Fakhfakh, Virginie Pérotin, Mónica Gago

11-9. What is the Natural Weight of the Current Old ?

Damien Gaumont, Daniel Leonard

11-10. Routinization-Biased Technical Change, Globalization and Labor Market Polarization: Does Theory Fit the Facts?

Jaewon Jung, Jean Mercenier

The TEPP Institute

The CNRS **Institute for Labor Studies and Public Policies** (the TEPP Institute, FR n°3435 CNRS) gathers together research centres specializing in economics and sociology:

- the **Centre d'Etudes de l'Emploi** (Centre for Employment Studies), **CEE**, Public Administrative Organization, under the leadership of the Ministers of Work and Research
- **l'Equipe de Recherche sur les Marchés, l'Emploi et la Simulation** (Research Team on Markets, Employment and Simulation), **ERMES**, University of Paris II Panthéon-Assas
- the **Centre d'Etudes des Politiques Economiques de l'université d'Evry** (Research Centre focused on the analysis of economic policy and its foundations and implications), **EPEE**, University of Evry Val d'Essonne
- the **Centre Pierre Naville** (Research on Work and Urban Policies), **CPN**, University of Evry Val d'Essonne
- **l'Equipe de Recherche sur l'Utilisation des Données Temporelles en Economie** (Research Team on Use of Time Data in Economics), **ERUDITE**, University of Paris-Est Créteil and University of Paris-Est Marne-la-Vallée
- the **Groupe d'Analyse des Itinéraires et des Niveaux Salariaux** (The Group on Analysis of Wage Levels and Trajectories), **GAINS**, University of the Maine

The TEPP Institute brings together 147 researchers and research professors and 100 PhD students who study changes in work and employment in relation to the choices made by firms and analyse public policies using new evaluation methods.