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SARAH LE DUIGOU

[www.tepp.eu](http://www.tepp.eu)

TEPP - Institute for Labor Studies and Public Policies  
TEPP - Travail, Emploi et Politiques Publiques - FR CNRS 3435



# Effect of Age on the Wage Distribution: A Quantitative Evaluation Using US Data

SARAH LE DUIGOU \*  
GAINS-TEPP (Le Mans University)  
sarah.le\_duigou.etu@univ-lemans.fr

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## Abstract

The distribution of wages varies with workers'age. In this article we build a model able to explain this evolution by taking into account two channels: the evolution of the wage game described in Burdett and Mortensen (1998), and the evolution of the workers'productivity within a match. The model considers three age groups, the juniors, the adults and the seniors. Using US data, we show that these two channels allow to reproduce quite well the aggregated wage distribution as well as the evolution of it over the workers life-cycle. The channel of the evolution of the wage game accounts for the lower density of low wages in the seniors'wage distribution. Yet firms are naturally induced to create lower quality jobs to seniors because of their short working horizon. In order to fit the data, we show that it is necessary to assume learning by doing of workers. The first aspect of learning by doing is to increase the workers'specific productivity, the second is to improve the adaptation to a new jobs of older workers. These two aspects explain different features of the wage distribution. The first one explains why the mode of the wage distribution is translated over the workers'life-cycle. The second accounts for the existence of high wages within the distribution of older workers.

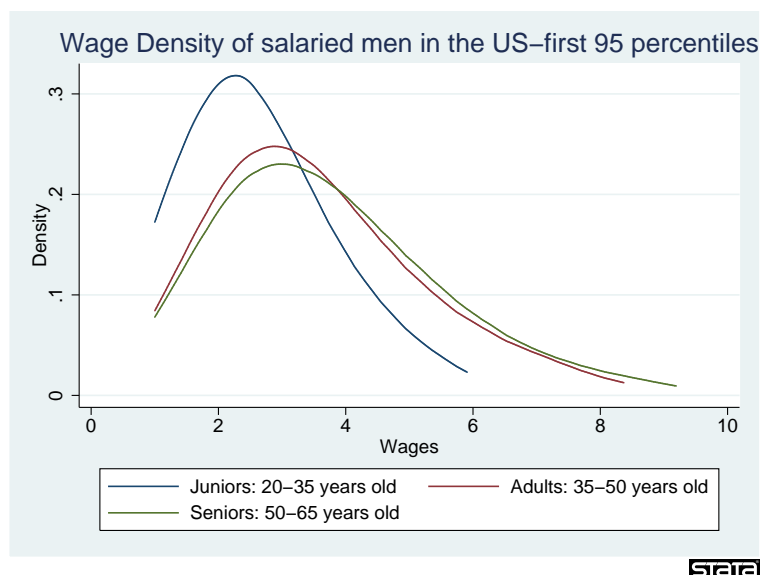
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\*Address: GAINS-TEPP (FR CNRS: 3126), Université du Maine - Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France.

## 1 INTRODUCTION

Empirically, it is well known that the distribution of wages varies significantly with workers' age. Figure 1 displays the wage distribution in the US of men depending on their age. The mean wage of adults is 37% higher than the mean wage of juniors<sup>1</sup>, and the dispersion coefficient<sup>2</sup> 4,4% higher. The difference between adults and seniors is far smaller: the mean wage of seniors is only 8% higher than the mean wage of adults and the dispersion coefficient is almost stationary. The main objective of this article is to construct a structural model able to explain the evolution of the observed wage distribution over workers' life-cycle.

Figure 1: US wage distribution of salaried men (95 percentiles)



Age affects the wage distribution via two channels. First, it affects the wage game. This accounts for the fact that workers of different age even if they have the same productivity have a different wage distribution. Then, it naturally affects the workers' productivity on a job.

Since the article of Burdett and Mortensen (1998), it has been usual to use the search model with frictions to explain the wage game. In this framework, workers search for a job whether they are unemployed or employed. Firms Bertrand compete to recruit and keep their employees. Their wage offers take into account the reservation wage distribution of workers and the wages offered

<sup>1</sup>on normalized wage distribution

<sup>2</sup>The dispersion divided by the mean

by the other firms. Frictions on the labor market give a certain monopsony power to firms via two effects: first, the workers do not find immediately a job when they are unemployed, this generates a reserve army of unemployed workers ready to accept the minimum wage, then workers do not find immediately a better job when employed, this weakens the competition between firms. Frictions rationalize the existence of minimum wage offers. The result of this non cooperative wage game leads to a mixed strategy equilibrium in which firms offer wages along an interval. For the same expected profit, several wage offer strategies are possible: a wage at the bottom of the distribution generates a high instantaneous profit yet a long hiring process and a weak retention, when a wage rather at the top of the distribution leads to a low instantaneous profit yet a fast hiring process and a strong retention.

In this article, to explain how the wage game changes over the workers' life-cycle we augment this model with a finite horizon and three periods of life for the workers: the "junior" period, the "adult" period and the "senior" period. We choose three periods in order to build a model close to the main characteristics of the life-cycle data. Workers start their working life as unemployed juniors. Thanks to unemployment-to-employment mobilities, the unemployment of these juniors entering the labor market is progressively reduced over the three periods. In the model of Burdett and Mortensen (1998), the assumption of job-to-job mobility already allows to insure an increasing trajectory of wage with the time spent on the labor market, since each worker do not leave his current job unless he has a better offer. Thanks to job-to-job mobilities, workers see their wage increasing over the three periods.

Firms direct their search on one of the three workers' market; a job offer often stipulates whether the firms expect a low experience worker (a junior), an experienced worker (an adult) and a very experienced worker (a senior). Given the increase of wage between the three periods induced by the mobilities, and the decrease of the new comers unemployment, firms have on the adults' and on the seniors' market, weaker a monopsony power and the stronger competition between firms leads firms to raise wages by more. The assumption of directed search from the firms allows to highlight the effect of the evolution of the balance of power between firms and the workers over the life-cycle.

The effect of the wage game on the wage trajectory over the life-cycle goes therefore through the ascendant mobility and the evolution of the workers' market power. In the wage game, both aspects depend on the workers' mobility rate. It is therefore also necessary to quantify how the mobility rate is affected by the workers' age in order to assess correctly the contribution of the wage game to the evolution of the wage distribution with age. To do so, we make the number of opportunities that a worker receives at each age, and therefore the workers' mobility rate, endogenous. Firms can create freely a vacancy on each market by paying the same vacancy cost. The number of vacancies on each market becomes therefore endogenous and depends on the profit firms can expect from this vacant job. Quantitative results of our model show that due to their shorter horizon, fewer vacancies are created on the seniors' market. Therefore, seniors face fewer opportunities on their labor market and the unemployment

and job duration increases when workers get closer to their horizon which is consistent with the data. This model does not take into account the search effort of workers. Considering the search effort would go in the same way as the free entry condition and would only reinforce these results (see Hairault, Langot and Sopraseuth (2010) and Hairault, Langot and Chéron (2011)).

The study of the wage game and its evolution over the workers' life cycle shows that at equal productivity, workers of different age have a different wage distribution. Yet, as it is showed in Burdett and Mortensen (1998), taking into account the only wage game do not allow to reproduce the data of the observed wage distribution. The wage game only generates the pure wage dispersion, that is the wage dispersion when workers and firms are homogenous. Empirical studies like Postel-Vinay and Robin (2002) show that the observed wage dispersion can be globally decomposed in three main components: the heterogeneity of the firms, the heterogeneity of the workers and a pure wage dispersion. The criterion representing the heterogeneity of the workers and the firms are often proxy for the match productivity. We therefore take into account the distribution of match productivity, and assess the effect of age on wage distribution also via its effect on the distribution of match productivity.

Age affects the both side of the match productivity: the workers' side through learning by doing, and the firms' side through the incentive to create high or lower quality jobs. Bontemps et al.(1999) and Mortensen (1998) shows that the main problems that the Burdett and Mortensen (1998) model has in fitting the real data could be solved by adding the heterogeneity of firms' productivity. Bontemps et al. show indeed that an exogenous distribution of firm's productivity allows to generate a single mode wage density consistent with the facts, when the model of Burdett and Mortensen (1998) generates a strictly increasing density and too low a dispersion. Yet, quoting Mortensen (1998) : "Existing theory neither explains nor restricts the assumed exogenous distribution of employer productivity". In his paper of (1998), Mortensen shows that introducing the endogenous productivity of firms allows to achieve the same objective, with only a very mild restriction on the shape of the production function<sup>3</sup>. Besides, Hairault, Langot and Chéron (2008) show that the search model with endogenous productivity is not rejected by the data. We choose in this article the Mortensen approach of endogenous productivity. In more concrete terms, this assumption supposes that firms can now decide each time they hire a worker, of an amount to invest in specific human capital. This investment can be associated to a vocational training of the employee at the beginning of the match. This capital is costly and increases the match's productivity during all its tenure. As this investment is costly, the longer the match is expected to be, the more firms invest on it. Ex ante the firms are homogenous, yet at the equilibrium, the same way firms offer different wages, they also have a different productivity. The productivity of a match also depends on the heterogeneity of worker's productivity.

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<sup>3</sup>Only a decreasing return of human capital allows to reproduce the hump shape of the wage distribution

Following the literature on human capital accumulation, (Becker (1964)), we assume aging also allows learning by doing. Over their lives, workers acquire exogenously a corporate knowledge which affects the match productivity in two different ways: first it increases their worker specific productivity (transferable from one job to an other), and second it helps them to adapt in a new job and to learn faster. This latter consequence is taken into account by firms in their training decision and is represented in the model by a least cost of the specific human capital investment. The amount of capital invested therefore depend also on workers'age. Age affects this decision in an ambiguous way. On one hand, firms are induced to create higher quality jobs to young workers who have a long working horizon, on the other hand, older workers are more experienced and are less costly to train.

The complete model with the wage game and the productivity channels is calibrated on US data. The model fits rather well the data of the aggregated wage distribution and its evolution with workers'age. This article shows that the framework developed by Burdett and Mortensen in (1998) augmented with age is able to reproduce the evolution of the wage distribution with age when we take into account the distribution of the match productivity.

The calibrated model allows to explain how each channel contributes to the evolution of the wage distribution over the life-cycle. According to our findings, the evolution of the wage game explains why adult and senior workers are less affected by low wages. The intuition is that those wages being more dedicated to unemployed workers or employed workers with a low working experience, they are simply more scarcely proposed to adult and senior workers. The observed wage dispersion and shape can only be obtained by assuming that firms have different productivity. Yet firms are naturally induced to create lower quality jobs to seniors because of their short working horizon. In order to fit the data, we show that it is necessary to assume learning by doing of workers. The first aspect of learning by doing is to increase the workers'specific productivity, the second is to improve the adaptation to a new jobs of older workers. These two aspects explain different features of the wage distribution. The first one explains why the mode of the wage distribution is translated over the workers'lifecycle. The second accounts for the existence of high wages within the distribution of older workers.

There is a small yet very recent literature studying the effect of age on the wage distribution. The working paper of Bagger et al. (2007) explains the wage progression via job-to-job mobilities and human capital accumulation in the theoretical framework developed by Postel-Vinay, Robin (2002), yet in a finite horizon. Menzio et al. (2010) deal with this link in finite horizon and distinguish the same two channels of wage evolution. They study their effects on the job to unemployment, unemployment to job and job to job transitions as well as on the mean wage trajectory over the life-cycle. The authors use a framework very different from the Burdett and Mortensen (1998). In their model, all the agents behave efficiently and information is perfect. The labor market is divided in small submarkets in which workers who search for only one wage meet employers who only propose one wage. The job research is perfectly



directed on both sides, firms have no monopsony power and the effect of frictions is far weaker than in our framework.

The second section of this article is dedicated to the presentation of the model assumptions. The third section presents the equilibrium of the model and the fourth section the calibration of it on US data. The last section concludes.

## 2 MODEL ASSUMPTIONS

### 2.1 ASSUMPTIONS AND NOTATIONS

The life-cycle is cut in three working life periods. Every worker starts as a junior, indexed in the model by  $i = y$ , then becomes an adult, indexed in the model by  $i = a$ , and eventually a senior, indexed in the model by  $i = s$ . The workers change life period with the probability  $p$ . As this probability is the same at each life period, in steady state the mass of workers noted  $m$  of each age is the same. Between the three periods workers acquire exogenously a corporate knowledge which increase their productivity and their adaptation abilities to a new job.

Workers search for a job while unemployed and employed. The arrival rates of wage offers, which are endogenous, are  $\lambda_i^0$  for the unemployed and  $\lambda_i$  for the employed. The reservation wage of unemployed worker is the labor opportunity cost that we assume to be below the minimum wage. The reservation wage of an employed worker is his current wage. Workers are all homogenous and unemployed when they enter the labor market, yet because of unemployment-to-employment and job-to-job mobility, in the steady state they have different status and reservation wages. The cumulative distribution of wage earned by employed workers is noted  $G_i(w)$  and the density  $g_i(w)$ . The mass  $u_i$  represents the total number of unemployed workers of a period.

We assume directed search from the firms. The employer can observe whether the worker is a junior, an adult or a senior and may target only one out of the three markets. We assume the workers of the three ages are not substitutable: the workers of each age have different and specific abilities, for example juniors have more physical skills when seniors have more intellectual skills. A firm which targets the junior market cannot product anything by employing an adult or a senior. That is naturally the same situation for firms which target the adult or the senior market. The arrival rates of contact with a worker are  $q_i^0$  for the unemployed workers and  $q_i$  for the employed workers. The employer cannot observe the workers' status or the workers' reservation wage: information is not perfect. The firms post wages on the juniors, the adults or the seniors' market and wait to meet a worker with a reservation wage below its proposal. The cumulative distribution of wage offered by firms is noted  $F_i(w)$ , and the density  $f_i(w)$ . Firms can create jobs with different levels of productivity depending on an initial investment which can be seen as a specific training on the job. The match productivity therefore depends on this initial investment but remains constant during the match. The cost of this investment can vary with

the workers' age.

When workers change age periods, the contract is not broken unless the worker's value of keeping the contract obtained in the previous age period becomes lower than the value of being unemployed in his current age period. Firms which target juniors can therefore be exposed to employ adult or even senior workers eventually. Each job can be destroyed due to an exogenous event which occurs with the probability  $s$ .

## 2.2 WORKERS' MOBILITY

*The number of job opportunities in each market*

As the number of job opportunities or vacancies on each of the three markets can be different, mobility of workers may differ in function of their age.

The free entry conditions ruling the number of vacancies on each market are given by:

$$c = \Pi_i(\underline{w}, \theta_i) \text{ for } i = 1, 2 \quad (2-1)$$

with  $c$  the search cost for the firms when the post is vacant,  $\Pi_i$  the expected value of posting a vacancy for the firms in each market, and  $\theta_i$  the labor market tightness on each market. As workers can search on the job,  $\theta_i = \frac{v_i}{u_i + R_\phi(m - u_i)}$ , with  $v_i$  the number of vacancies,  $u_i$  the number of unemployed workers,  $(m - u_i)$  the number of employed workers and  $R_\phi$  the ratio of the effectiveness of the job search of employed workers  $\phi$  and of the unemployed workers  $\phi^0$ . If we consider that the unemployed workers search more intensively than the employed workers, this ratio is for instance inferior to 1.

The mobility rate of workers of each age depends on the probability for a worker to find a job whether he is employed or unemployed. This probability is a function of the labor market tightness.

The number of matches between workers and firms for each age is given by:

$$M_i = \phi v^\eta (u_i + R_\phi(m - u_i))^{1-\eta} \quad (2-2)$$

with  $\eta$  the elasticity of this matching function.

The probabilities of contact between a worker and a firm on the workers' and the firms' side are therefore given by:

$$\begin{aligned} \lambda_i &= \phi \theta_i^{1-\eta} \\ \lambda_i^0 &= \phi^0 \theta_i^{1-\eta} \\ q_i &= \phi \theta_i^{-\eta} \\ q_i^0 &= \phi^0 \theta_i^{-\eta} \end{aligned} \quad (2-3)$$

The mobility of workers depends therefore on both their age and their status.

### *Unemployment by age*

When the unemployed workers receive no benefits, all the workers have the same reservation wage induced by the labor opportunity cost. With no other institution such as the minimum wage, the lowest wage offered by the firms is necessarily equal or above <sup>4</sup> this reservation wage, since no firm have interest to offer a wage that nobody can accept. If there is an institutional minimum wage higher than this reservation wage, firms' lowest wage offer is similarly equal or above this minimum wage. In either case there is no job rejection from unemployed workers.

The total level of unemployment for each age solves the following flows equations:

$$u_y \lambda_y^0 + p u_y = (m - u_y) s + p m \Rightarrow u_y = \frac{sm + pm}{p + s + \lambda_y^0} \quad (2-4)$$

$$u_a \lambda_a^0 + p u_a = (m - u_a) s + p u_y \Rightarrow u_a = \frac{sm + p u_y}{p + s + \lambda_a^0} \quad (2-5)$$

$$u_s \lambda_s^0 + p u_s = (m - u_s) s + p u_a \Rightarrow u_s = \frac{sm + p u_a}{p + s + \lambda_s^0} \quad (2-6)$$

$$(2-7)$$

At the beginning of the first period, all the workers who enter the labor market are unemployed and remain unemployed a certain time before finding his or her first job since labor market is frictional. This mass of unemployed workers is represented in (2-4) by  $pm$  and rises juniors' unemployment. The other sources of unemployment are the exogenous destructions. Yet as they occur at the same rate whatever the workers' age, they affect evenly the unemployment of the juniors, the adults and the seniors. The workers leave the mass of unemployed workers when they change life period or become employed, i.e. when they have a contact with a firm.

Structurally, juniors' unemployment is higher because of their initial condition. When the young, the adults and the seniors have the same mobility rate, as all new entrants become young unemployed workers the unemployment rate decrease with age.

**Property 1.** *When the youth, the adults and the seniors have the same search intensity:  $u_y > u_a > u_s$*

*Proof.* As  $u_y < m$ , necessarily  $u_y > u_a$ . Then as  $u_y > u_a$ , necessarily,  $u_a > u_s$ .  $\square$

Yet, the mobility rate of unemployed workers can differ with age<sup>5</sup>.

**Property 2.** *An increase in the mobility rate of workers of any age decreases the unemployment rate of the age period.*

<sup>4</sup>it can be above on the seniors' or adults' segment, cf section 3

<sup>5</sup>When  $\lambda_y^0$ ,  $\lambda_a^0$ , and  $\lambda_s^0$  are different

Besides, as the unemployment rates are interdependent:

**Property 3.** *An increase in the mobility rate of young workers decreases the unemployment rate of the young, the adults and the seniors. An increase in the mobility rate of adult workers decreases the unemployment rate of the adults and the seniors.*

In this article, the mobility rates are likely to be different yet strictly positive for each age, the evolution of the the unemployment rate with age remains therefore ambiguous.

*Wage distribution by age*

To deduce the masses of workers earning a wage equal or below  $w$  in steady state, we equalize the flows in and out of this category as it follows:

$$\begin{aligned}
(m - u_y)G_y(w) &= \frac{\lambda_y^0}{s + p + \lambda_y(1 - F_y(w))} F_y(w)u_y \\
(m - u_a)G_a(w) &= \frac{\lambda_a^0}{s + p + \lambda_a(1 - F_a(w))} F_a(w)u_a + \frac{p}{s + p + \lambda_a(1 - F_a(w))} (m - u_y)G_y(w) \\
(m - u_s)G_s(w) &= \frac{\lambda_s^0}{s + p + \lambda_s(1 - F_s(w))} F_s(w)u_s + \frac{p}{s + p + \lambda_s(1 - F_s(w))} (m - u_a)G_a(w)
\end{aligned} \tag{2-8}$$

The denominator of each equation represents the expected duration of jobs depending on  $w$ : the higher  $w$ , the lower the poaching risk, the longer the job duration. These equations show that the initial condition affects the wage distribution of the workers.

The densities of wages for each age are given by :

$$\begin{aligned}
g_y(w) &= \frac{u_y}{m - u_y} \frac{\lambda_y^0 f_y(w)(p + s + \lambda_y)}{(s + p + \lambda_y(1 - F_y(w)))^2} \\
g_a(w) &= \frac{u_a}{m - u_a} \frac{\lambda_a^0 f_a(w)(p + s + \lambda_a)}{(s + p + \lambda_a(1 - F_a(w)))^2} + \frac{p}{s + p + \lambda_a(1 - F_a(w))} \frac{m - u_y}{m - u_a} g_y(w) + \frac{p\lambda_a f_a(w)}{(s + p + \lambda_a(1 - F_a(w)))^2} \frac{m - u_y}{m - u_a} G_y(w) \\
g_s(w) &= \frac{u_s}{m - u_s} \frac{\lambda_s^0 f_s(w)(p + s + \lambda_s)}{(s + p + \lambda_s(1 - F_s(w)))^2} + \frac{p}{s + p + \lambda_s(1 - F_s(w))} \frac{m - u_a}{m - u_s} g_a(w) + \frac{p\lambda_s f_s(w)}{(s + p + \lambda_s(1 - F_s(w)))^2} \frac{m - u_a}{m - u_s} G_a(w)
\end{aligned} \tag{2-9}$$

To assess the only selection effect induced by job to job mobility we assume workers face the same lottery of wages and the same job arrival rate for the moment, i.e. they face the same  $F_i(w)$ ,  $\lambda_i^0$  and  $\lambda_i$

**Property 4.** *When workers face an identical lottery of wage offers , the growth rate of the wage density of juniors (according to wage) is always lower than the growth rate of the wage density of adults and the growth rate of the wage density of adults is always lower than the growth rate of the wage density of seniors.*

*Proof.* Comparing the values:  $\frac{g'_y(w)}{g_y(w)}$ ,  $\frac{g'_a(w)}{g_a(w)}$ , and  $\frac{g'_s(w)}{g_s(w)}$ , when  $g_i$  are the derivative of  $G_i$ .  $\square$

This property highlights the fact that over the life-cycle workers benefit from job opportunities which allow them to raise their wage.

What happens when the job to job mobility rate differs with age? An increase in the job to job mobility rate allows workers to climb quicker the wage ladder.

**Property 5.** *An increase in the job to job mobility rate of workers reinforces the selection effect and the increasing trajectory of wage.*

*Proof.* The last term of the denominators of 2-8 represents the resignation rate of workers due to a better offer in function of their wage. An increase in the job to job mobility rate  $\lambda_i$  raises this resignation rate.  $\square$

### 2.3 FIRMS' BEHAVIOR IN COMPETITION

In this subsection, we study the most profitable strategies for firms on each market since wages generating the higher profit are those offered by a larger number of firms, and this directly affects the shape of the wage density.

*The case of the monopsony: From Diamond (1971) to Burdett and Mortensen (1998)*

To understand exactly the role of each of these elements on the wage setting decision, we present the situation of firms in these two seminal articles in which horizon is infinite. In these articles, the expected profit depends of three elements: the hiring probability of a worker, the instantaneous profit yielded by the worker and the job duration. The way wage affects these four elements rules the balance of the power between workers and firms. First, we assume workers cannot search on the job. This economy is the economy of the monopsony described by Diamond (1971). In this economy, the wage does not affect the hiring probability since all the workers who search for a job are unemployed without benefit, it does not affect the job duration, since workers have no interest to resign. The only effect of the wage is to reduce the instantaneous profit of firms. All the firms therefore decide in such economy to offer a wage equal to the labor opportunity cost. The monopsony power of firms is total and the workers have no market power. When the workers can search on the job, the monopsony power of firms decreases and the firms have to compete to recruit and keep their worker. That is the economy described by Burdett and Mortensen (1998). In this economy, the higher the wage compared to the other firm, the higher the hiring probability of a worker and the job duration of an employee. This effect of wage is reinforced by the introduction of the matches heterogeneity which strengthen the competition to increase the job duration. Now, in this game, the only power for the firms is given by the labor market frictions which decrease the intensity of competition between firms. Frictions

generate a mass of unemployed workers for whom there is no need to compete (they accept any wage) and a certain time before the employed workers find a better wage in a competitor's firm.

*The hiring probabilities over the life-cycle*

Given mobilities, offering a wage at the bottom of the distribution allows to hire only a small part of workers, the unemployed workers, while offering a wage at the top of the distribution allows to hire a large part of the workers, the unemployed and a part of the employed workers. There exists a large difference between the beginning of the life-cycle of an agent and its end. At the beginning of the life cycle, the agent enters in the labor market as an unemployed worker. In addition, the youngest agents have not had the time to largely improve their careers. At the opposite, at the end of the life cycle, a large majority of workers are integrated in the firms. Their experiences have given them the opportunities to find the better wage offers. This clearly suggests that if we allow firms to direct their search on each market, the juniors, the adults, and the seniors, the hiring probabilities in function of a wage offer  $w$  will be different. The hiring probability that the firms face on each market depends on the repartition of workers according to their reservation wage on each market and is given by:

$$h_y(w) = q_y^0 u_y + q_y(m - u_y)G_y(w) \quad (2-10)$$

$$h_a(w) = q_a^0 u_a + q_a(m - u_a)G_a(w) \quad (2-11)$$

$$h_s(w) = q_s^0 u_s + q_s(m - u_s)G_s(w) \quad (2-12)$$

The first term of the right hand side of these three expressions represents the probability the firm has to contact an unemployed worker, who necessarily accept his offer, and the second term, the probability the firm has to contact an employed worker who accepts his offer. The hiring probabilities are always increasing with wage. Yet according to age they can be more or less strongly increasing. This depends mostly on the size of the reserve army on each market that we note  $h_i(\underline{w})$ . For instance, when the unemployment rate is low and when the employed workers are concentrated on low wages, the probability increases less strongly with wage. In that case the wage competition between firms which target the population is low and leads to rather low wage offers from firms. In other words, firms have higher a monopsony power on this market.

**Property 6.** *When workers face an identical lottery of wage offers<sup>6</sup>, the reserve army of the juniors  $h_y(\underline{w})$  is larger than the reserve army of adults and seniors  $h_a(\underline{w})$  and  $h_s(\underline{w})$ .*

*Proof.* We assume workers face an identical lottery of wage offers and compare

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<sup>6</sup>Same  $F_i(w)$ , same  $\lambda_i^0$ ,  $\lambda_i$  and same  $q_i^0$

the size of the reserve armies:

$$h_y(\underline{w}) = q^0 u_y + q(m - u_y)g_y(\underline{w}) \quad (2-13)$$

$$h_a(\underline{w}) = q^0 u_a + q(m - u_a)g_a(\underline{w}) \quad (2-14)$$

$$h_s(\underline{w}) = q^0 u_s + q(m - u_s)g_s(\underline{w}) \quad (2-15)$$

After substitution of the values of the unemployment and of the values of the density evaluated in the minimum wage, we get:  $h_y(\underline{w}) > h_a(\underline{w})$ . The comparison between  $h_a(\underline{w})$  and  $h_s(\underline{w})$  remain ambiguous.  $\square$

This indicates that the monopsony power of firms tends to decrease between the first and the second period. Yet, as the monopsony power of firms depends on the unemployment rate and the repartition of the employed workers according to wage, it therefore also depends on the mobility the workers have experienced, from unemployment to employment and the selection made thanks to the job to job mobility.

**Property 7.** *An increase in the mobility rate of workers reduces the firms' monopsony power.*

If it is clear that the monopsony power of firms depends on the age of workers, the algebra does not allow us to affirm on which market the firms have the higher monopsony power.

#### *Heterogenous productivity*

Mortensen (1998) shows that even if workers are similar in terms of productivity, the Bertrand competition between firms induces firms to train them differently. At the equilibrium, similar workers have different level of productivity. This assumptions allows to reproduce more realistic a wage distribution. In this part we show that this assumption is not neutral in a finite horizon. When we consider life-cycle, even when workers are similar in terms of productivity ex ante, at the equilibrium the level of productivity of workers depends on their age. Then we show how the difference of productivity of workers affects the wage offers.

Firms choose the wage  $w$  and the amount of specific human capital  $k$  which maximizes their profit on each market

$$\begin{aligned} \Pi_y(w, k) &= \max_{w > \underline{w}, k > 0} \{h_y(w)(J_y(w, k) - \beta_y k)\} \\ \Pi_a(w, k) &= \max_{w > \underline{w}, k > 0} \{h_a(w)(J_a(w, k) - \beta_a k)\} \\ \Pi_s(w, k) &= \max_{w > \underline{w}, k > 0} \{h_s(w)(J_s(w, k) - \beta_s k)\} \end{aligned} \quad (2-16)$$

The  $J_i(w, k)$  are the value function of a filled vacancy and the  $\beta_i$  are parameters which represent the cost of training for each age. This cost depends on the general human capital workers have accumulated over his life. The value

functions of a filled vacancy depend on the specific human capital invested in the match and are given by:

$$\begin{aligned}
J_y(w, k) &= \frac{y_y(k) - w + pJ_a(w, k)}{r + p + s + \lambda_y(1 - F_y(w))} \\
J_a(w, k) &= \frac{y_a(k) - w + pJ_s(w, k)}{r + p + s + \lambda_a(1 - F_a(w))} \\
J_s(w, k) &= \frac{y_s(k) - w}{r + p + s + \lambda_s(1 - F_s(w))}
\end{aligned} \tag{2-17}$$

The intertemporal profit induced by a worker once hired is composed of the margin depending on the productivity of the match and the wage, and the job duration. The terms  $pJ_a(w, k)$  and  $pJ_s(w, k)$  respectively in value function of the young and of the adults show that if at the rate  $p$  an employer employing a senior lose his worker, on the opposite, an employer employing a junior or an adult can keep him. These value functions show therefore how the the difference of horizon affects the profit of firms

The production function depending on the level of specific human capital is given by:

$$\begin{aligned}
y_y(k) &= y_y + \left(\frac{q}{\alpha}\right) k^\alpha \\
y_a(k) &= y_a + \left(\frac{q}{\alpha}\right) k^\alpha \\
y_s(k) &= y_s + \left(\frac{q}{\alpha}\right) k^\alpha
\end{aligned} \tag{2-18}$$

where  $q$  and  $\alpha$  are strictly positive exogenous parameters and  $y_i$ , the workers' specific productivity which depends on age.

The amount of human capital invested in a job depends on the age of the worker. For each age of the worker, the capital invested in function of wage is the result of the maximization of the profit according to  $k$  and is given by:

$$\begin{aligned}
k_s(w) &= \left( \frac{q}{\beta_s(r + p + s + \lambda_s(1 - F_s(w)))} \right)^{\frac{1}{1-\alpha}} \\
k_a(w) &= \left( \frac{q \left( 1 + \frac{p}{r+p+s+\lambda_s(1-F_s(w))} \right)}{\beta_a(r + p + s + \lambda_a(1 - F_a(w)))} \right)^{\frac{1}{1-\alpha}} \\
k_y(w) &= \left( \frac{q \left( 1 + \frac{p}{r+p+s+\lambda_a(1-F_a(w))} \right) \left( 1 + \frac{p}{r+p+s+\lambda_s(1-F_s(w))} \right)}{\beta_y(r + p + s + \lambda_y(1 - F_y(w)))} \right)^{\frac{1}{1-\alpha}}
\end{aligned} \tag{2-19}$$



*Proof.* The derivative of the profit of firms targeting each age according to the capital is given by:

$$\begin{aligned}
\frac{\partial \Pi_s(w, k)}{\partial k} &= qk^{\alpha-1} - \beta_s(r + p + s + \lambda_s(1 - F_s(w))) \\
\frac{\partial \Pi_a(w, k)}{\partial k} &= qk^{\alpha-1} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) - \beta_a(r + p + s + \lambda_a(1 - F_a(w))) \\
\frac{\partial \Pi_y(w, k)}{\partial k} &= qk^{\alpha-1} \left( 1 + \frac{p}{r + p + s + \lambda_a(1 - F_a(w))} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) \right) \\
&\quad - \beta_y(r + p + s + \lambda_y(1 - F_y(w)))
\end{aligned} \tag{2-20}$$

The second derivative is given by:

$$\begin{aligned}
\frac{\partial^2 \Pi_s(w, k)}{\partial k^2} &= (\alpha - 1)qk^{\alpha-2} \\
\frac{\partial^2 \Pi_a(w, k)}{\partial k^2} &= (\alpha - 1)qk^{\alpha-2} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) \\
\frac{\partial^2 \Pi_y(w, k)}{\partial k^2} &= (\alpha - 1)qk^{\alpha-2} \left( 1 + \frac{p}{r + p + s + \lambda_a(1 - F_a(w))} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) \right)
\end{aligned} \tag{2-21}$$

When the production function has a decreasing return to capital, the second derivatives are negative. The first order condition  $\frac{\partial \Pi_i(w, k)}{\partial k} = 0$ , for  $i = y; a; s$  gives the result of equations 2-19.  $\square$

Note that these investments do not depend on the workers'specific productivity.

**Property 8.** *The amount of specific human capital unambiguously increases with the wage offered to the workers. We have for each age,  $\frac{\partial k_i}{\partial w} > 0$ , for  $i = y; a; s$ .*

Reversionally, the choice of a high match productivity allows therefore the firms to set high wages.

**Property 9.** *When the workers are homogenous in terms of productivity and mobility, ie.  $\beta_y = \beta_a = \beta_s$  and  $\lambda_y = \lambda_a = \lambda_s$ , the investment in specific human capital of firms offering the minimum wage decreases with age:  $k_s(\underline{w}) < k_a(\underline{w}) < k_y(\underline{w})$*

*Proof.*

$$\begin{aligned}
k_s(\underline{w}) &= \left( \frac{q}{\beta_s(r + p + s + \lambda_s)} \right)^{\frac{1}{1-\alpha}} \\
k_a(\underline{w}) &= \left( \frac{q}{\beta_a(r + p + s + \lambda_a)} + \frac{p}{\beta_a(r + p + s + \lambda_s)(r + p + s + \lambda_a)} \right)^{\frac{1}{1-\alpha}} \\
k_y(\underline{w}) &= \left( \frac{q}{\beta_y(r + p + s + \lambda_y)} + \frac{p}{\beta_y(r + p + s + \lambda_y)(r + p + s + \lambda_a)} + \frac{p}{\beta_y(r + p + s + \lambda_y)(r + p + s + \lambda_a)(r + p + s + \lambda_s)} \right)^{\frac{1}{1-\alpha}}
\end{aligned} \tag{2-22}$$

$\square$

This property shows that at equal productivity and mobility when we do not consider the effect of wage on the investment, so when firms offer the minimum wage, the firms choose to train more the younger workers. Indeed, these workers, at equal mobility, have a longer discounting horizon.

**Property 10.** *For each age, an increase in the mobility rates (here job to job) of workers decrease the amount of specific human capital that firms invest in the match. Yet, this effect decreases when the wage increases, and at the maximal wage in the economy ( $\bar{w}_i$  such that  $F_i(\bar{w}_i) = 1$  for each  $i$ ), the mobility rate has no effect any more.*

When firms anticipate that the the worker they hire have a lot of other opportunities, they do not train him much unless they choose a high wage strategy which protects the firms from the future workers' resignation. Note that when firms decide to train a young or an adult worker, they take into account the mobility rate of workers of next periods, since the worker can still be employed in the firm during these periods.

**Property 11.** *For each age, an increase of the training cost  $\beta_i$  decreases the amount of specific human capital invested by the firms.*

This parameter allows to take into account the heterogeneity of workers according to age in terms of productivity. Notably, if senior workers are less costly to train because they have already worked in several other companies, this may compensate the fact that they have a short horizon, and firms can choose to create high quality jobs for these workers. Firms can indeed create higher quality jobs (higher productivity) with lower training costs. In this context, it is more accurate to consider the capital invested in terms of quality of the job than in terms of training investment.

The workers' specific productivity, the other parameter representing the learning by doing, naturally affects the productivity of the match. Yet it does not affect the wage distribution directly via the firms' behavior. This parameter increases the profit of the firms, therefore it increases the number of vacancies and eventually the workers' mobility.

### 3 EQUILIBRIUM

We search the equilibrium wage distribution. The firms' behavior is the center of this model and the distribution of workers' wages depends on the number of opportunities that firms create and the level of wage they offer.

Firms spread their wage offers along a wage interval. The firms' maximum instantaneous profit is obtained when the firms post the minimum wage, when firms increase their offer, their instantaneous profit decreases, yet as  $F(w)$  increases, the hiring probability, the retention, and the productivity increase. As  $F(w)$  cannot be superior to 1, there exists in each market a  $\bar{w}$  above which firms have no interest to post wages. We assume there exists a minimum wage  $\underline{w}$ .

This value is computed for each age by:

$$\begin{aligned}\Pi_y(\underline{w}) &= \Pi_y(\overline{w}_y) \\ \Pi_y(\underline{w}_a) &= \Pi_y(\overline{w}_a) \\ \Pi_y(\underline{w}_s) &= \Pi_y(\overline{w}_s)\end{aligned}\tag{3-23}$$

Note that the level of capital invested in each match can be expressed only in function of wage, we therefore integrate it in the expression of the profit. As the profit is different and can evolve differently from one market to another with wage, it is likely that the maximum wage would be different in each market. In 3-23, note that we specify  $\underline{w}_a$  and  $\underline{w}_s$ . Indeed, for the first period the maximum profit is necessarily reached for the lowest wage above the unemployed worker reservation wage. If there exists a minimum wage as in this economy, this profit is reached as in the Burdett and Mortensen 1998 model in  $\underline{w}$ , the minimum wage. For the other periods, firms face a labor supply which depends on the heritage of the wage distribution of the previous period. The lowest wage that the firms have interest to offer is therefore not necessarily the institutional minimum wage. These two wages can be computed as it follows:

$$\begin{aligned}\underline{w}_a &= \underset{w}{\operatorname{argmax}} \Pi_a(w) \\ \underline{w}_s &= \underset{w}{\operatorname{argmax}} \Pi_s(w)\end{aligned}\tag{3-24}$$

with  $\Pi_i$  the profit of firms offering the lowest wage of the market, ie. the expression of the profit when  $F_i(w) = 0$ .

Eventually the distribution of the wages offered by the firms solves: From  $\underline{w}$  to  $\overline{w}_y$

$$\Pi_y(\underline{w}) = \Pi_y(w)\tag{3-25}$$

From  $\underline{w}_a$  to  $\overline{w}_a$

$$\Pi_a(\underline{w}_a) = \Pi_y(w)\tag{3-26}$$

From  $\underline{w}_s$  to  $\overline{w}_s$

$$\Pi_s(\underline{w}_s) = \Pi_y(w)\tag{3-27}$$

Firms can move freely from one market to another. At the equilibrium, the profit in each market must be therefore equal. For that we compute the value of  $\theta_y, \theta_a, \theta_s$ , such that:

$$\Pi_y(\underline{w}) = \Pi_a(\underline{w}_a) = \Pi_s(\underline{w}_s) = c\tag{3-28}$$

The number of vacancies and the distribution of offered wages allows easily to compute the distribution of workers' wage  $G_y, G_a$ , and  $G_s$ , thanks to the equations 2-8.

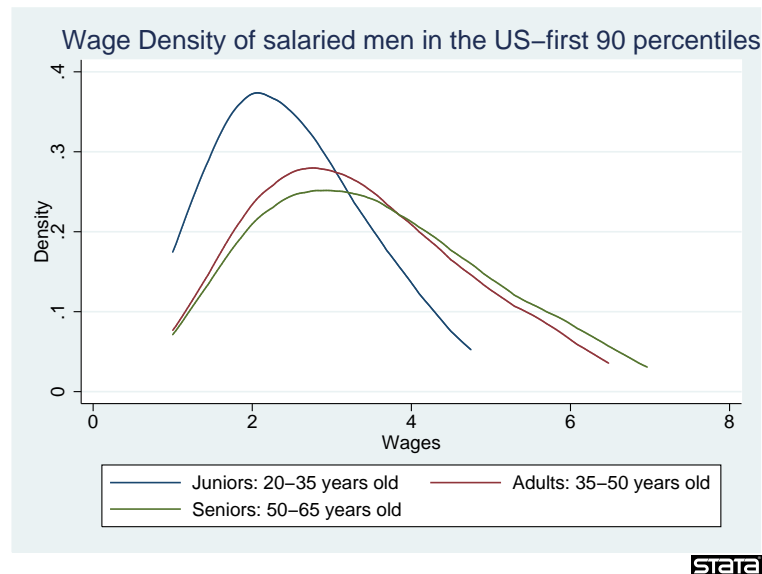
As the algebra does not allow us to solve this model analytically, we carry on a simulation of the model.

## 4 CALIBRATION EXERCISE ON US DATA

### 4.1 THE DATA

We calibrate this model on US data. The data comes from the Current Population Survey (CPS) of 2002. All the wages are expressed in minimum wages. We only keep male wage-earner workers between 20 and 65 years old. It is difficult to pretend that the model developed in the first section can reproduce the extreme wages existing in a wage distribution since in this model workers are ex ante homogenous (when they arrive on the labor market). We therefore calibrate this model on a wage distribution corresponding to the first nine deciles of the wage distribution of each age. This distribution is presented in figure 2. The figure 1 in the introduction displays the first ninety five percentiles of the wage distribution of each age.

Figure 2: US wage distribution of salaried men (9 deciles)



### 4.2 THE CALIBRATION

The moments used for the calibration are mainly aggregated moments: the unemployment rate, the unemployment duration, the mean wage, and four ratio of moments of the wage distribution in order to capture the shape of it. Yet these aggregated moments are not sufficient to calibrate the two age-specific parameter, therefore, we also use the three modes of the distribution by age. The table 1 sums up the annual value of the parameters and the targets used to calibrate them.

Table 1: Calibration parameters

Fixed and Institutional Parameters		
$r$	0.04	Discounting rate
$p$	1/15	Age period duration
$\beta_y$	1	Normalized
$w$	1	Minimum wage
Calibrated on cps 2002		
$s$	0.1	Unemployment rate
$\phi_0$	10	Unemployment duration
$\phi_1$	1	C25/C10
$\eta$	0.8	C50/C25
$q$	0.34	Mean Wage
$\alpha$	0.81	Median
$y_y$	2	Mode of young
$y_a$	2.8	Mode of adults
$y_s$	2.7	Mode of seniors
$\beta_a$	0.89	C75/C50
$\beta_s$	0.67	C90/C75

The discounting rate is set at 4% per year as it is usual in the literature. We assume three life periods of 15 years, the probability to change period is therefore  $\frac{1}{15}$ . The minimum wage and the cost of training a junior are normalized to one. The costs of training workers by age are only important in difference between them.

The job destruction rate and the efficiency of the matching function between firms and unemployed are naturally set to match the unemployment rate and the unemployment duration. The efficiency of the matching function between firms and unemployed and the elasticity of the matching function affect the mobility mainly at the beginning of the distribution, we set these parameters in order to match moments of the first half of the distribution. The two parameters of the production function  $q$  and  $\alpha$  are set to reproduce the mean wage and the median wage. The workers'specific productivities are set to match the mode of the distribution by age. Eventually, the cost of training for adults and seniors allows to explain a great part of the second half of the distribution, we calibrate these parameters in order to match moments of this part.

Note that we also tried to make a calibration of the model on the same US data with only the endogenous productivity of firms. It is indeed necessary to introduce this heterogeneity in the firms' productivity to obtain the hump shape of the wage distribution according to age. Yet, we omit the effect of learning by doing ( $y_i$  and  $\beta_i$  are in this case equal for each age). No calibration is possible without this last assumption. Indeed in this specification, there is a conflict between an accurate wage distribution shape and the increasing path of wage with age.

### 4.3 SIMULATION RESULTS

The table 2 displays the result of the simulation. The first part presents the values of the targets reached, the second part presents extra moments that we did not search to reproduce in order to test the reliability of the model.

Table 2: Results

Moments to match	Data	Model
Mean Wage	3.08	3.09
Median Wage	2.9	2.89
Q1/D1	1.36	1.33
D5/Q1	1.39	1.41
Q3/D5	1.34	1.39
D9/Q3	1.27	1.25
Unemployment rate	0.05	0.05
Unemployment duration	0.3	0.42
Mode young	2.05	2
Mode adults	2.75	2.7
Mode seniors	2.85	2.8
Extra moments and results on life-cycle	Data	Model
Standard disp	1.29	1.30
Mean $Wage_s$ / Mean $Wage_a$	1.06	1.088
Mean $Wage_a$ / Mean $Wage_y$	1.35	1.34
Median $Wage_s$ / Median $Wage_a$	1.07	1.12
Median $Wage_a$ / Median $Wage_y$	1.35	1.37
$\sigma_s / \sigma_a$	1.08	1.06
$\sigma_a / \sigma_y$	1.41	1.37
Junior Employment	74%	93.4%
Adult Employment	80%	95.91%
Senior Employment	77%	95.68%

This model allows to reproduce quite well the moments targeted as well as some extra moments on the aggregated distribution and the wage distribution according to life-cycle<sup>7</sup>.

The wage distribution of employed workers is presented in figure 3. The step that we observe on the simulated distribution of adults when the wage is close to 4.7, corresponds to the fact that all adult workers earning a wage above 4.7 can only have been hired as an adult since junior workers cannot earn such wages. There is no report from junior employed workers to adult employed workers above the wage 4.7. The step that we observe on the simulated distribution of seniors after this same wage is the report of the step on the adults' distribution.

The figure 4 displays the distribution of wages offered by the firms. The distributions  $g_i$  of figure 3 are more on the right than the distributions of  $f_i$ . This can be explained by the fact that high paid jobs last longer than low paid

<sup>7</sup>The difference observed between the employment rate between the model and the data mainly comes from the extend of inactivity which does not exist in the model. Yet the model reproduces correctly the hump trend over the life cycle.

Figure 3: Simulation: Complete model

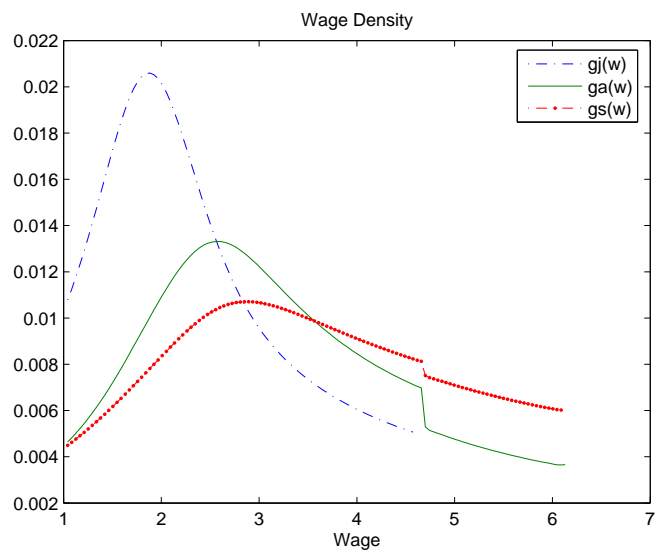
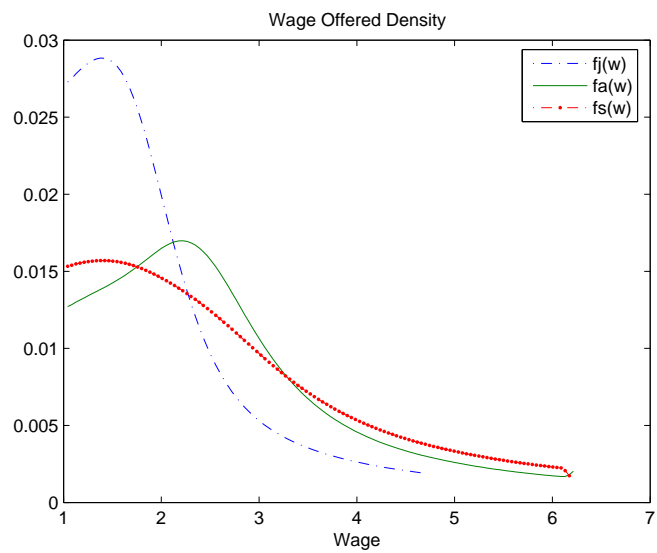


Figure 4: Simulation: Complete model

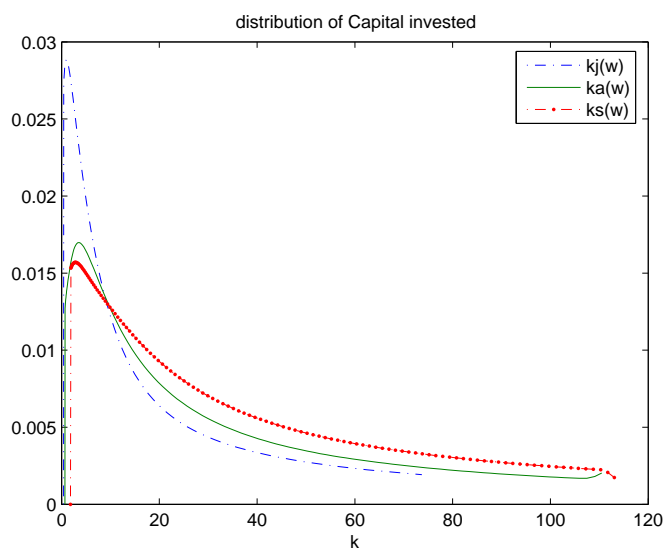


jobs. Yet, we notice the translation rightwards of the older workers' distribution is stronger than for the the younger. This difference highlights the selection effect of jobs and the fact that this selection accounts for a part of the increasing wage trajectory with age.

The figure 5 displays the distribution of capital invested by the firms. As firms make the decision of the wage and of the amount of invested capital in the same time, the distribution of wage offered on figure 4 and of capital on figure5 are interdependent. Therefore an impact on the choice of the wage offered for the firms affects the amount of capital invested and the other way around.

Eventually the figure 6 displays the distribution of the productivity of the workers newly hired. This distribution shows the translation of the productivity compared to the amount of capital invested due to the effect of learning by doing on the workers'specific productivity. The distribution of figure 6 represent the production function 2-18 in function of the distribution of the invested capital of figure 5.

Figure 5: Simulation: Complete model

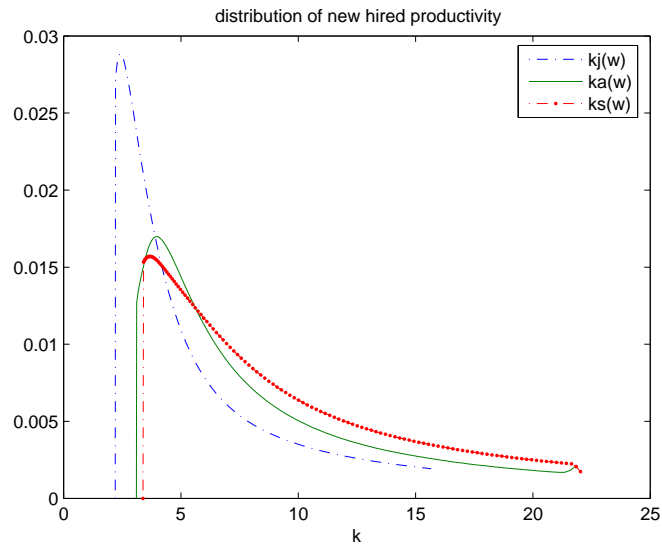


#### 4.4 THE EFFECT OF PRODUCTIVITY ON WAGE DISTRIBUTIONS BY AGE

In this article, we introduce the channel of match productivity to account for the evolution of wage distribution over the workers'life-cycle. Two components linked to the match productivity have been introduced: the firms'component



Figure 6: Simulation: Complete model



(the capital invested and therefore the quality of the match created) and the workers' component (the workers' specific productivity). To assess the effect of these two components, we take off these effects of the calibrated model and compare the situation with and without the component.

*The workers' specific productivity*

Let's assume, the workers' specific productivity is similar for all the workers. In the calibration, we set  $y_y = y_a = y_s$ . The results of these new simulations are presented in figures 7, 8, and 9 and in the table ???. Notice that the modes of the wage distribution are close to be similar for each age. What is the mechanism behind this difference? The fixed parameter of the production function only affect the profit of the firms and therefore the number of vacancies and the mobility of workers. In this simulation, the older workers' mobility is therefore lower. This has two main consequences. First the selection effect is weaker between the distributions of figure 7 and the distribution of figure 8. Then the monopsony power of firms is higher for the adults and the seniors since the unemployment rate of these older workers are higher, and the risk of poaching and the selection effect weaker. Firms compete less to employ the older workers and offer them wages more concentrated on the bottom of the distribution (figure 8). Firms create in the same time a greater number of low quality jobs for these workers (figure 9) since these low paid jobs are jobs with a high turnover (high concentration of offers around the same wages).

Table 3: Results for  $y$  constant

Moments to match	Data	Model
Mean Wage	3.08	2.65
Median Wage	2.9	2.51
Q1/D1	1.36	1.3
D5/Q1	1.39	1.38
Q3/D5	1.34	1.36
D9/Q3	1.27	1.2
Unemployment rate	0.05	0.05
Unemployment duration	0.3	0.44
Mode young	2.05	1.7
Mode adults	2.75	1.9
Mode seniors	2.85	2.1
Extra moments and results on life-cycle	Data	Model
Standard disp	1.29	1
Mean $Wage_s$ / Mean $Wage_a$	1.06	1.08
Mean $Wage_a$ / Mean $Wage_y$	1.35	1.13
Median $Wage_s$ / Median $Wage_a$	1.07	1.11
Median $Wage_a$ / Median $Wage_y$	1.35	1.15
$\sigma_s/\sigma_a$	1.08	1.05
$\sigma_a/\sigma_y$	1.41	1.17
Junior Employment	74% <sup>8</sup>	93.9%
Adult Employment	80%	95.42%
Senior Employment	77%	95.30%

Figure 7: Simulation: constant  $y$

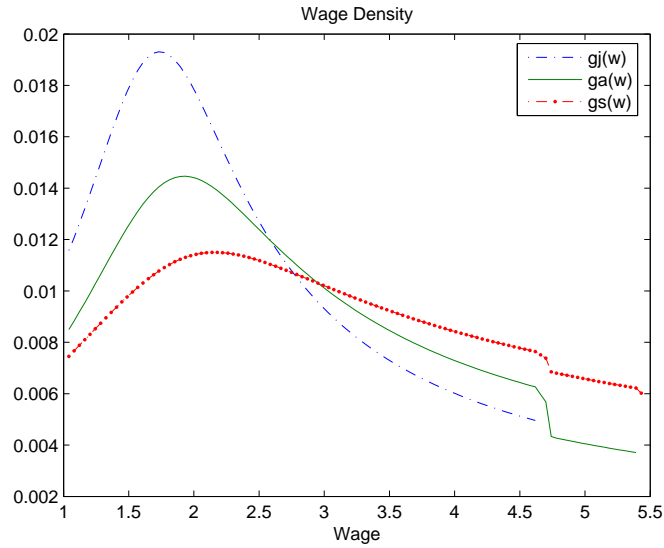


Figure 8: Simulation: constant  $y$

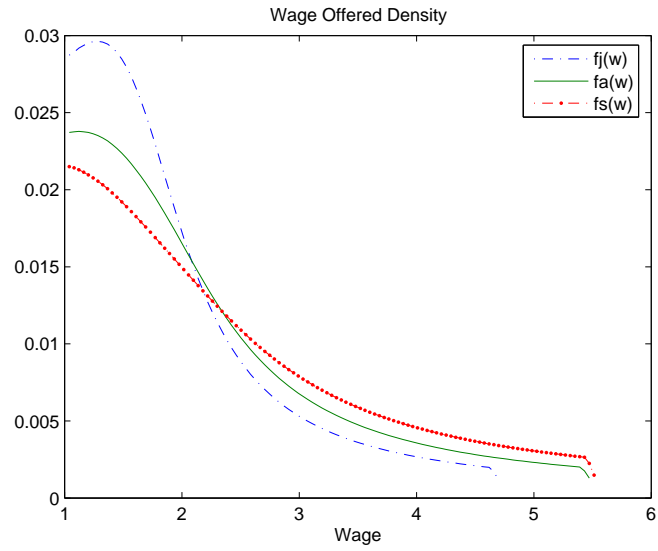
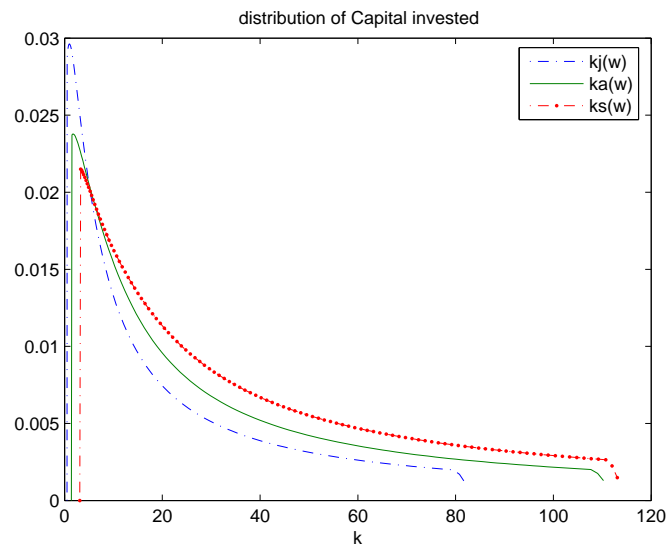


Figure 9: Simulation: constant  $y$



*The cost of training*

Let's assume, the cost of training is similar for all the workers. In the calibration, we set  $y_y = y_a = y_s = 2$  and  $\beta_y = \beta_a = \beta_s = 1$ <sup>9</sup>. The results of these new simulations are presented in figures 12, 11, and 10 and in the table ???. This new assumption mainly affects in terms of behavior the training decision. The figure 10 shows clearly that without the learning by doing allowing a better adaptation of older workers, the firms would be reluctant to create high quality jobs to seniors, and in a least amplitude to adults. The horizon effect affects very negatively the senior workers in terms of age. This training decision has for consequence that firms are reluctant to offer high wages to senior workers.

Table 4: Results for  $y$  and  $\beta$  constant

Moments to match	Data	Model
Mean Wage	3.08	2.35
Median Wage	2.9	2.19
Q1/D1	1.36	1.27
D5/Q1	1.39	1.29
Q3/D5	1.34	1.34
D9/Q3	1.27	1.24
Unemployment rate	0.05	0.052
Unemployment duration	0.3	0.46
Mode young	2.05	1.7
Mode adults	2.75	1.8
Mode seniors	2.85	2
Extra moments and results on life-cycle	Data	Model
Standard disp	1.29	0.84
Mean $Wage_s$ / Mean $Wage_a$	1.06	0.84
Mean $Wage_a$ / Mean $Wage_y$	1.35	1.02
Median $Wage_s$ / Median $Wage_a$	1.07	0.8
Median $Wage_a$ / Median $Wage_y$	1.35	1.08
$\sigma_s/\sigma_a$	1.08	0.82
$\sigma_a/\sigma_y$	1.41	0.88
Junior Employment	74% <sup>10</sup>	93.9%
Adult Employment	80%	95.46%
Senior Employment	77%	95%

*The monopsony power*

This case where the workers'specific productivity and the training cost are similar can also be used to observe the effect of the monopsony power of firms on the wage distribution of workers of different age. Indeed, in spite of the training decision not in the favor of senior workers, we can notice on figure 11 that the mode of the wage offered to seniors remains a bit more on the right. This suggests that senior workers still have higher a market power than the other workers and that the capital investment only affects the high wages not the medium wages.

<sup>9</sup>We set the values of  $y_i$  and  $\beta_i$  at the level of the value of  $y_y$  and  $\beta_y$

Figure 10: Simulation: constant  $\gamma$  and constant  $\beta$

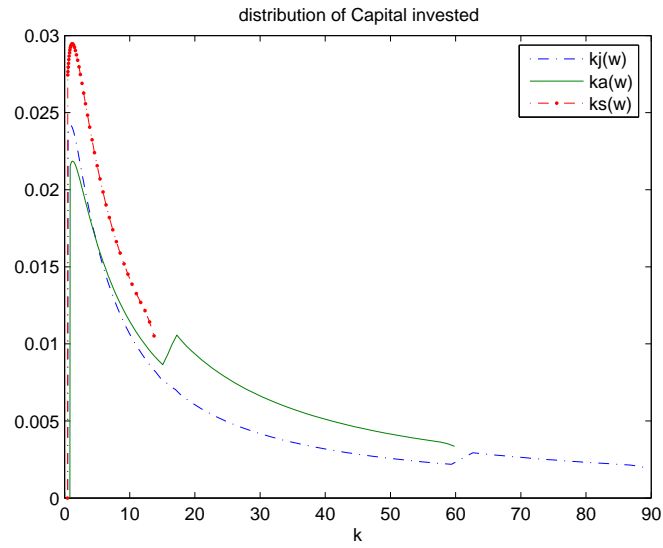


Figure 11: Simulation: constant  $\gamma$  and constant  $\beta$

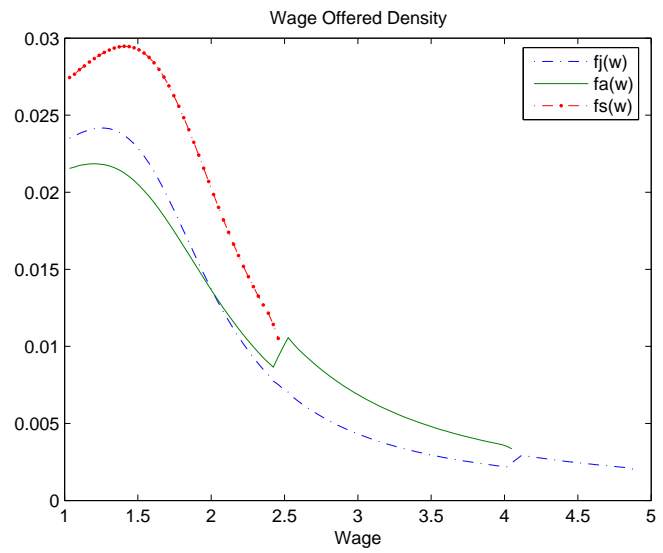
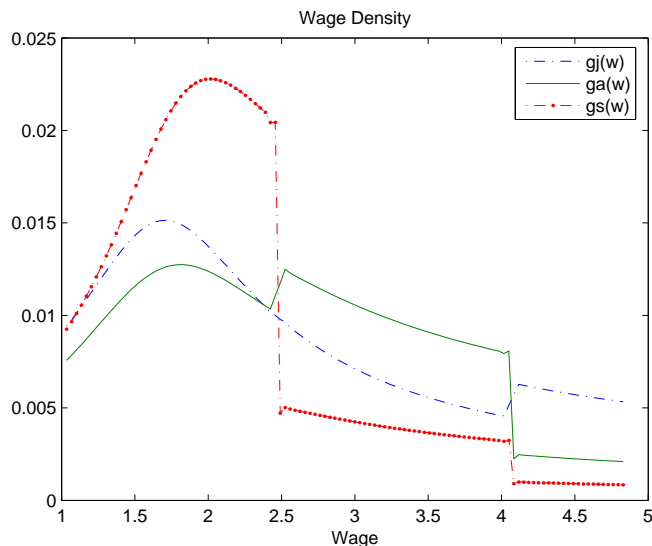


Figure 12: Simulation: constant  $y$  and constant  $\beta$



## 5 CONCLUSION

This article shows that the evolution of the wage game described in Burdett and Mortensen (1998) and the evolution of the match productivity account for the main features of the evolution of the wage distribution over the life-cycle. The decrease in the monopsony power of firms over the life-cycle accounts for the lower density of low wages in the seniors' wage distribution. The observed wage dispersion and shape can only be obtained by assuming that firms have different productivity. Yet firms are naturally induced to create lower quality jobs to seniors because of their short working horizon. Therefore the evolution of wage distribution with age does not fit the data. It is only possible to reproduce the correct evolution of the wage distribution by assuming learning by doing of workers. In this model the learning by doing has two consequences. First it increases the workers' specific productivity. This strengthens the monopsony power of older workers and allows to account for the translation of the mode of the wage distribution. Second it improves the adaptation to a new jobs. This induces firms to create high quality jobs to the older workers in spite of their short horizon and accounts for the existence of high wages within the distribution of seniors.

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