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Marc Fleurbaey, Marie-Louise Leroux, Pierre Pestieau, Grégory Ponthière. Fair Retirement Under Risky Lifetime. 2013. halshs-00857945

**HAL Id: halshs-00857945**

**<https://shs.hal.science/halshs-00857945>**

Preprint submitted on 4 Sep 2013

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**PARIS SCHOOL OF ECONOMICS**  
ÉCOLE D'ÉCONOMIE DE PARIS

**WORKING PAPER N° 2013 – 31**

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**PARIS-JOURDAN SCIENCES ÉCONOMIQUES**

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# Fair Retirement under Risky Lifetime\*

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September 4, 2013

## Abstract

A premature death unexpectedly brings a life and a career to their end, leading to substantial welfare losses. We study the retirement decision in an economy with risky lifetime, and compare the laissez-faire with egalitarian social optima. We consider two social objectives: (1) the maximin on expected lifetime welfare (*ex ante*), allowing for a compensation for unequal life expectancies; (2) the maximin on realized lifetime welfare (*ex post*), allowing for a compensation for unequal lifetimes. The latter optimum involves, in general, decreasing lifetime consumption profiles, as well as raising the retirement age, unlike the *ex ante* egalitarian optimum. This result is robust to the introduction of unequal life expectancies and unequal productivities. Hence, the postponement of the retirement age can, quite surprisingly, be defended on egalitarian grounds –although the conclusion is reversed when mortality strikes only after retirement.

*Keywords:* risky lifetime, mortality, labour supply, retirement, compensation.

*JEL codes:* I14, I18, J10, J22.

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\*The authors are grateful to Benoit Decerf for his comments on this paper.

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<sup>||</sup>Marc Fleurbaey and Gregory Ponthiere acknowledge the financial support of the ANR Equirisk (Equity in Risky Intertemporal Economic Environments) (ANR-12-INEG-0006-01).

# 1 Introduction

The continuous rise in life expectancy observed in industrialized economies in the last two centuries hides a fundamental feature of human life. Despite major advances in medical sciences, a human life remains a lottery, with a significant variance in longevity outcomes. This point is well illustrated by survival curves, which give us the probabilities to reach all ages of life (based on the age-specific mortality rates prevailing at the year under study). As shown by Figure 1 for French males, provided mortality rates remain constant, more than 13 percent of the cohort born in 2010 will die before having reached the official retirement age of 62 years.<sup>1</sup> Moreover, about 20 percent will die before age 67.

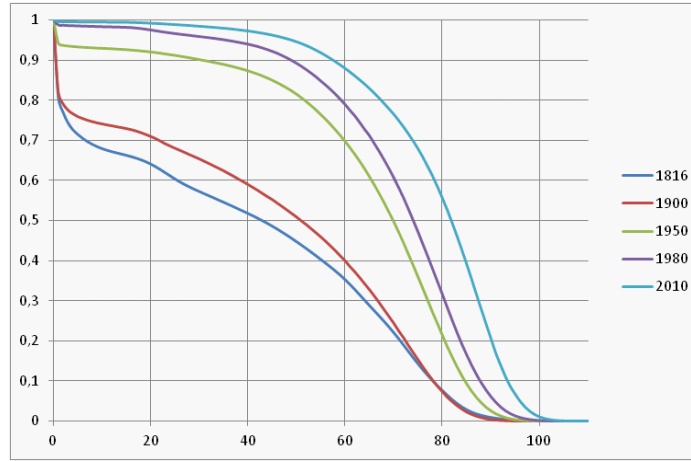


Figure 1: Period survival curves: French males (1816 to 2010)

Average life expectancy statistics therefore hide this important uncertainty, and the induced inequalities in longevity outcomes. Only 61 percent of French males will reach the average life expectancy, equal to 78.04 years. The remaining 39 percent of the population will have a shorter life.

Those observations are not without consequences for optimal policy-making, in particular when considering the design of optimal pension system. From a policy perspective, the observed annual three-months increase in average life expectancy has often been used to justify postponing the legal retirement age in countries with Pay-as-You-Go pensions systems. Indeed, if individuals tend, "on average", to live longer, and if the retirement age remains the same, the sustainability of the pension system requires, under a constant fertility, either to increase the pension contributions, or to reduce the replacement ratio.<sup>2</sup>

The problem is that such a reasoning relies on a "on average" way of looking at things. In reality, there remain, as shown above, large longevity inequali-

<sup>1</sup>Sources: the Human Mortality Database (2013).

<sup>2</sup>On the potential gains from postponing the age at retirement, see Cremer and Pestieau (2000, 2003).

ties, and it is not at all clear that policy-makers should concentrate on average outcomes. An obvious reason why more attention should be paid to longevity inequalities is that a significant proportion of those inequalities are due to factors on which individuals have no influence at all, and, thus, are circumstances for which agents can hardly be regarded as responsible.<sup>3</sup> For instance, Christensen *et al* (2006) claim that about one quarter to one third of longevity inequalities within a cohort can be explained by differences in genetic background.

Given that longevity inequalities are, to a significant extent, independent from individual behavior, there is a strong ethical support for a social security system that does not penalize the short-lived. The goal of this paper is to examine the design of the optimal retirement system in an economy with risky lifetime, with a social objective that incorporates a concern for inequalities.

In the context of risk, there are two main ways of taking account of inequalities. One can first focus on inequalities in expected lifetime utility, and in particular give priority to people having lower life expectancy. This connects well to the policy proposal of making retirement age depend on working conditions which affect longevity. But one can also look at the final distribution of longevity and realized utility across individuals, and thereby take account of the residual inequalities due to good or bad luck in the lottery of longevity. This relates to the policy discussions about giving less priority to the (lucky) elderly people who had their "fair innings". The former approach we call *ex ante* egalitarianism, and the latter *ex post* egalitarianism.

Here is a brief summary of our results. We first study the retirement decision in a two-period economy with identical agents and a risky lifetime. In such a simple framework, the laissez-faire allocation coincides with the *ex ante* egalitarian social optimum, but not with the *ex post* egalitarian optimum because of the remaining longevity inequalities. The latter optimum involves a declining consumption profile over the life cycle and a later retirement than in the laissez-faire. We then introduce inequalities in life expectancy which make the *ex ante* egalitarian approach depart from the laissez-faire, and show how the *ex ante* and the *ex post* approaches still differ significantly. We finally focus on the result that the *ex post* approach implies a later retirement, and we show that the opposite result can be obtained when mortality occurs mostly after retirement. The reason is that when mortality occurs early, a late retirement helps providing high consumption to the young, in order to help those among them who will have a short life. In contrast, when death strikes after retirement, it helps the short-lived to give them an early retirement. Finally, we study a more general model with wage heterogeneity and tax redistribution, and confirm our results, qualitatively, in this setting.

In sum, the present paper suggests that adopting an *ex post* egalitarian social objective would lead to a strong reorganization of the lifecycle in terms of consumption and labour. As such, the present study complements the existing literature in several ways. First, it complements the existing positive literature

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<sup>3</sup>See Fleurbaey (2008) on the distinction between circumstances and responsibility characteristics.

on optimal labour and retirement age (see Sheshinski 1978, Crawford and Lilien 1981, Kahn 1988), by applying a model of labour and retirement decisions in a context of risky lifetime. Second, it also complements the normative literature on (socially) optimal labour and retirement age (see Cremer *et al* 2004), which relies on classical (Benthamite) utilitarianism, unlike the present, egalitarian ethical framework.<sup>4</sup> Third, it complements recent attempts to apply *ex post* egalitarian social criteria to economies with risky lifetimes (see Fleurbaey *et al* 2011), but which did not consider labor and retirement decisions.

The rest of the paper is organized as follows. Section 2 presents the basic framework where individuals who are identical *ex ante* turn out to have unequal longevity, and compares the laissez-faire with the social optimum under *ex ante* and *ex post* egalitarianism. Section 3 introduces unequal life expectancies and discusses several difficulties with the *ex ante* egalitarian approach. Section 4 studies the *ex post* approach under different assumptions about when mortality strikes in the lifecycle. Section 5 introduces a more general model with double heterogeneity: life expectancy and labour earnings. Section 6 concludes.

## 2 Identical individuals facing mortality risk

We consider a two-period model with risky lifetime. The population is a continuum of agents, with a measure normalized to 1. Agents live either one period or two periods. The old age (second period) is reached with a probability  $\pi$ . During the young age (first period), agents supply their labour inelastically, consume and save resources for the old age. At the old age, agents can still work some subperiod, of length  $z$ , and are retired over the remaining subperiod of length  $1 - z$ .

We assume that lifetime welfare takes a time-additive form, and that old-age temporal welfare is additively separable in the utility of consumption and the disutility of labour. Assuming that agents have preferences satisfying the expected utility hypothesis, their preferences can be represented by:<sup>5</sup>

$$u(c) + \pi [u(d) - v(z)], \quad (1)$$

where  $c$  and  $d$  denote first- and second-period consumptions,  $z$  denotes the retirement age, while  $u(\cdot)$  is the temporal utility from consumption, whereas  $v(\cdot)$  denotes the disutility of old-age labour. As usual,  $u(\cdot)$  satisfies  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ , and  $v(\cdot)$  satisfies  $v(0) = 0$ ,  $v'(\cdot) > 0$  and  $v''(\cdot) > 0$ . Let  $u(c_0) = 0$  for a given  $c_0 > 0$ .

For analytical convenience, we assume that the labour market is perfectly competitive. Workers are paid at a wage rate  $w$ , which is taken as given by

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<sup>4</sup>Utilitarianism also prevails in optimal retirement age studies in a dynamic setting (see Michel and Pestieau 2002, Crettez and Le Maitre 2002, Lacombe and Lagos 2005).

<sup>5</sup>As usual, the utility of death is normalized to 0. Moreover, we assume away pure time preferences for the simplicity of presentation. Note that the survival probability acts as a kind of "natural" discount factor.

them. There also exists a perfect annuity market, with actuarially fair returns. This amounts to assuming that consumption at the old age is:

$$d = \frac{Rs}{\pi} + wz \quad (2)$$

where  $R$  equals one *plus* the interest rate, while  $s$  denotes savings. For the sake of presentation, we will, throughout this paper, assume that  $R$  equals 1.

## 2.1 Laissez-faire and ex ante egalitarianism

In this simple setting, the laissez-faire allocation involves no *ex ante* inequality, and is therefore also the *ex ante* egalitarian optimum.

Agents choose their consumptions, savings and retirement age so as to maximize their expected lifetime welfare subject to their budget constraint:

$$\begin{aligned} \max_{c,d,z} & u(c) + \pi [u(d) - v(z)] \\ \text{s.t. } & c + \pi d = w(1 + \pi z) \end{aligned}$$

From the first-order conditions, we obtain:

$$u'(c) = u'(d) \quad (3)$$

$$c = w(1 + \pi z) - \pi d \quad (4)$$

$$v'(z) = u'(d)w \quad (5)$$

The laissez-faire involves a perfect smoothing of consumption across periods, i.e.  $c = d$ , as well as a retirement age such that the marginal disutility of old-age labour (LHS) equals the marginal welfare gain from working (RHS).

Let us provide the intuition for comparative statics with a few figures. To study the impact of a change in  $\pi$ , consider the following two equations, which derive from (4) and (5) after substituting  $c$  for  $d$

$$c = w \frac{1 + \pi z}{1 + \pi}, \quad (6)$$

$$v'(z) = u'(c)w. \quad (7)$$

From equation (7) one can write consumption as  $c = f(z)$ , with  $f'(z) < 0$  and  $f''(z) > 0$ . This gives a decreasing convex curve (see Figure 2) that is intersected by the line of equation (6) with intercept  $w/(1 + \pi)$  and slope  $w\pi/(1 + \pi)$ . When  $\pi$  increases (raising life expectancy), only the budget equation (6) is affected<sup>6</sup> and this line goes down (its value at  $z = 1$  remains unchanged, at  $w$ ), which implies that  $z$  goes up and  $c = d$  go down.

To study the impact of a change in  $w$ , consider the different system formed by (6) and the following equation, which derives from (7) after substituting  $w$  for its value in (6):

$$u'(c)c = v'(z) \frac{1 + \pi z}{1 + \pi}. \quad (8)$$

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<sup>6</sup>This, however, depends on our assumption of perfect annuities. Otherwise life expectancy would affect the choice of labour directly.

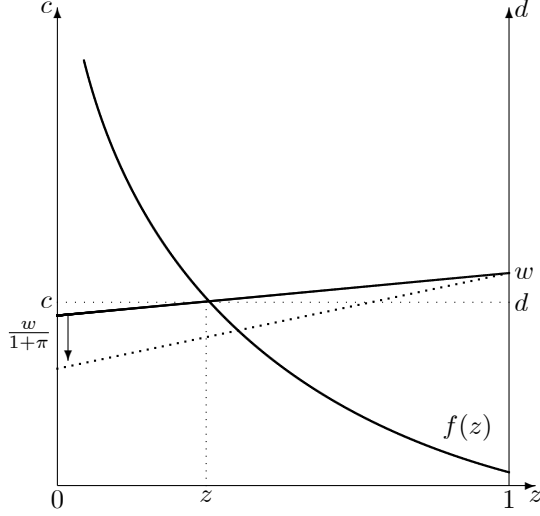


Figure 2: Impact of  $\pi$  on laissez-faire

This equation provides a vertical line when  $u(c) = \ln c$ , a decreasing curve if  $u$  is more concave than the logarithm, and an increasing curve when it is less concave than the logarithm (see Figure 3). In the former case, an increase in  $w$ , which raises the line of equation (6), triggers a decrease in  $z$  and an increase in  $c = d$ ; in the latter case, provided that the curve of equation (8) is steeper than the line (6) at the intersection — this turns out to be always true — an increase in  $w$  triggers an increase in both  $z$  and  $c = d$ . When  $u(c) = \ln c$ ,  $z$  is unaffected by a change in  $w$ .

As explained in the introduction, this allocation may not be satisfactory when it exhibits large welfare inequalities between the long-lived and the short-lived individuals. The latter's lifetime welfare  $U^{sl}$  equals  $u(c)$ , whereas the former's lifetime welfare  $U^{ll}$  equals  $2u(c) - v(z)$ . Long-lived agents are better off than short-lived agents if and only if:

$$u(d) - v(z) = u\left(\frac{w(1 + \pi z)}{1 + \pi}\right) - v(z) > 0 \quad (9)$$

that is, if the second period is worth being lived. We focus on this case here, and ignore the opposite case of poor economies in which it is a calamity to live in old age.

Let us briefly examine how this inequality depends on the parameters. As far as life expectancy is concerned, given that a rise in  $\pi$  reduces  $d$  and raises  $z$  and, thus,  $v(z)$ , it follows that lifetime welfare inequalities due to unequal lifetimes are smaller in economies with a higher expectancy.<sup>7</sup>

<sup>7</sup>The intuition is that economies with higher life expectancy are characterized by more



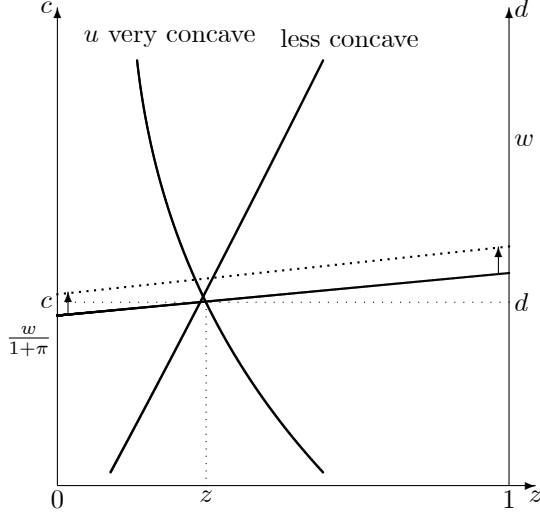


Figure 3: Impact of  $w$  on laissez-faire

When the effect of productivity on retirement age is negative, or positive but small enough, a rise in  $w$  increases lifetime welfare inequalities between long-lived and short-lived.

The following proposition summarizes our results.

**Proposition 1** • *At the laissez-faire, we have:*

$$\begin{aligned}
 c &= d = \frac{w(1 + \pi z)}{1 + \pi} \\
 v'(z) &= u'(d)w \\
 \frac{dz}{dw} &\geq 0 \iff R_R(c) \leq 1 \\
 \frac{dc}{dw} &= \frac{dd}{dw} > 0, \\
 \frac{dz}{d\pi} &> 0, \frac{dc}{d\pi} = \frac{dd}{d\pi} < 0,
 \end{aligned}$$

where  $R_R(c) = -u''(c)c/u'(c)$  is the degree of relative risk aversion.

- *Long-lived agents enjoy a higher lifetime welfare than short-lived agents if and only if*

$$u\left(\frac{w(1 + \pi z)}{1 + \pi}\right) - v(z) > 0.$$

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labour and less consumption at the old age, so that a premature death is necessarily less damaging there, in comparison with economies with worse survival conditions. This is, again, influenced by the perfect annuity assumption.

*Lifetime welfare inequalities between long-lived and short-lived are decreasing in life expectancy; they increase with productivity if*

$$\frac{1}{1+\pi} \left( 1 - (1+\pi z) \frac{v''(z)}{v'(z)} \right) < R_R(c).$$

**Proof.** See the Appendix. ■

Finally, note that, if the retirement age were fixed, i.e.  $z = \bar{z}$ , then a rise in productivity would raise lifetime welfare inequalities between short-lived and long-lived, as this would increase consumption at the old age, and, hence, make premature death more damageable *ceteris paribus*. The endogeneity of the retirement decision prevents us from drawing such conclusions: agents may, under a higher productivity, work more or less, depending on whether the substitution effect dominates the income effect or not.

## 2.2 Ex post egalitarian first-best optimum

Let us now characterize the ex post egalitarian social optimum of this economy. Under this social objective, the social planner's problem can be rewritten as:

$$\begin{aligned} \max_{c,d,z} \min \{ & u(c), u(c) + u(d) - v(z) \} \\ \text{s.t. } & c + \pi d = w(1 + \pi z) \end{aligned}$$

The objective function of that planning problem is continuous, but not differentiable. However, we can rewrite that problem in a simpler form assuming that the optimum features  $u(d) = v(z)$ . This condition is necessary and sufficient to insure the equality of lifetime welfare across individuals, whatever their longevity. Therefore, the social planner's problem can be rewritten as maximizing  $c$  under the constraints

$$\begin{aligned} u(d) &= v(z) \\ c + \pi d &= w(1 + \pi z). \end{aligned}$$

This boils down to maximizing the quantity

$$c = w(1 + \pi z) - \pi u^{-1} \circ v(z).$$

This is a concave function because  $u^{-1} \circ v$  is convex. The first order condition is

$$wu'(u^{-1} \circ v(z)) = v'(z),$$

which is just (5) because  $u^{-1} \circ v(z) = d$ .

We focus on the case of an interior solution. As the FOC is just (5), it is easy to graphically compare the laissez-faire and the egalitarian optimum. From  $wu'(d^*) = v'(z^*)$ , one derives  $d^* = f(z^*)$ , where, as in Figure 2,  $f(z^*) = u^{-1}(v'(z^*)/w)$ , with  $f'(z^*) < 0$  and  $f''(z^*) > 0$ . Moreover,  $c^* = w(1 + \pi z^*) - \pi d^*$  can be represented in the  $(z^*, c^*)$  space as a concave function  $c^* = w(1 +$

$\pi z^*) - \pi f(z^*)$  that is below the line  $c^* = w(1 + \pi z^*)$  and has this line as an asymptote (see Figure 4).<sup>8</sup> The socially optimum levels of  $d^*$  and  $z^*$  are obtained at the intersection of  $d^* = f(z^*)$  and the egalitarian constraint  $u(d^*) = v(z^*)$ , rewritten as  $d^* = u^{-1} \circ v(z^*)$ .<sup>9</sup> Given that  $\lim_{z \rightarrow 0} f(z) > \lim_{z \rightarrow 0} u^{-1} \circ v(z)$  and  $\lim_{z \rightarrow 1} f(z) < \lim_{z \rightarrow 1} u^{-1} \circ v(z)$ , such an intersection always exists. Then, for that level of  $z^*$ , we obtain, from  $c^* = w(1 + \pi z^*) - \pi f(z^*)$ , the socially optimal consumption  $c^*$ .

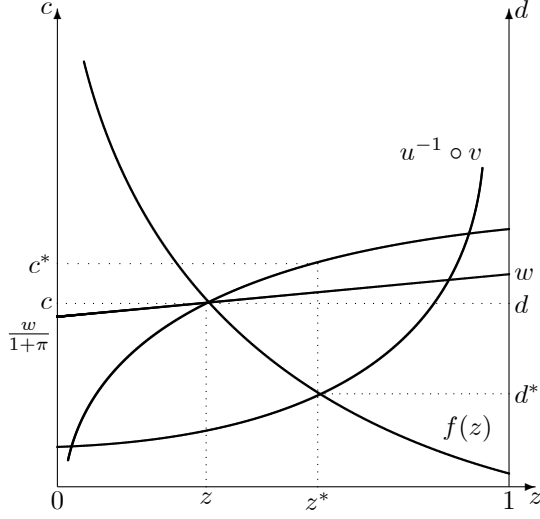


Figure 4: Comparing laissez-faire and social optimum

The *ex post* egalitarian optimum can be compared with the laissez-faire as follows. At the laissez-faire, the optimal level of  $z$  is characterized by the intersection of the decreasing curve  $d = f(z)$  with the increasing concave curve  $c = w(1 + \pi z) - \pi f(z)$ . On the contrary, at the *ex post* egalitarian optimum, the optimal level of  $z$  is determined by the intersection of  $d = f(z)$  with the increasing convex curve  $d = u^{-1} \circ v(z)$ . The intersection between  $d = f(z)$  with  $d = u^{-1} \circ v(z)$  is, in general, on the south east of the intersection between  $d = f(z)$  and  $c = w(1 + \pi z) - \pi f(z)$ .<sup>10</sup> Hence the retirement age at the *ex post* egalitarian optimum exceeds its laissez-faire level, and that old-age consumption is lower than at the laissez-faire, whereas young age consumption is higher than at the laissez-faire. The following proposition specifies our results.

**Proposition 2** *Comparing the ex post egalitarian optimum (\*) with the laissez-faire, if  $u(d) > v(z)$  at the laissez-faire, then:*

$$c^* > c = d > d^*, \quad z^* > z.$$

<sup>8</sup> Observe that the curves of equations  $c = w(1 + \pi z) - \pi f(z)$ ,  $c = f(z)$ , and  $c = w(1 + \pi z)/(1 + \pi)$  intersect at the same point.

<sup>9</sup> Note that, if  $z^* = 0$ ,  $d^* = u^{-1}(0) = c_0 > 0$ .

<sup>10</sup> Exceptions include very poor economies (see *supra*).

**Proof.** Both the laissez-faire and the egalitarian optimum satisfy

$$\begin{aligned} c &= w(1 + \pi z) - \pi f(z) \\ d &= f(z), \end{aligned}$$

where  $f(z) = u'^{-1}(v'(z)/w)$ . The first expression is increasing in  $z$  whereas the second is decreasing. Therefore,  $c^* > c$  and  $d^* < d$  if and only if  $z^* > z$ .

The laissez-faire, in addition, satisfies  $c = d$ ; in contrast, the egalitarian optimum has  $u(d^*) = v(z^*)$ .

At the laissez-faire, one therefore has  $w(1 + \pi z) - \pi f(z) = f(z)$ , i.e.,  $f(z) = w(1 + \pi z)/(1 + \pi)$ . At the egalitarian optimum, one has  $u(f(z^*)) = v(z^*)$ , i.e.,  $f(z^*) = u^{-1} \circ v(z^*)$ .

Under the assumption that  $u(d) > v(z)$  at the laissez-faire, then  $w(1 + \pi z)/(1 + \pi) > u^{-1} \circ v(z)$ , which implies that  $f(z) > u^{-1} \circ v(z)$ . As  $f$  is decreasing and  $u^{-1} \circ v$  is increasing, the equation  $f(z^*) = u^{-1} \circ v(z^*)$  requires  $z^* > z$ . ■

The proof actually shows an "if and only if" fact: if  $u(d) < v(z)$  at the laissez-faire (a poor economy), then the inequalities are reversed, and the egalitarian optimum transfers resources from the young to the old in order to make life bearable in old age:  $c^* < c$ ,  $d^* > d$  and  $z^* < z$ .

Hence, in the case of affluent economies, *ex post* egalitarianism involves longer lifetime working periods. This somewhat counterintuitive result can be explained as follows. From an *ex post* egalitarian perspective, the only thing that matters is to maximize the welfare of the worst-off. In advanced economies with a sufficiently large output, the worst-off is, unambiguously, the short-lived. Therefore, if one wants to minimize *ex post* welfare inequalities, all resources need to be targeted towards the short-lived individuals. Given that those can hardly be identified *ex ante*, nor compensated after their death, the only solution is to allow young age consumption that is as large as possible. This is achieved by making the surviving old work longer than at the laissez-faire.

This has some consequences when one thinks about the direction of intertemporal resource transfers, that is, the issue of positive or negative savings. The laissez-faire is characterized by positive savings, since  $c = \frac{w(1+\pi z)}{1+\pi} < w$  for  $\pi, z < 1$ , leading to a transfer of resources from the young age of life towards the old age. On the contrary, under the *ex post* egalitarian optimum, the direction of intertemporal resources transfers is the opposite: from old age towards the young age. Indeed, from the budget constraint, we have:

$$c^* = w + \pi(wz^* - d^*)$$

The curve of equation  $d = wz$  is an increasing linear curve that lies above the curve  $u(d) = v(z)$  at  $z = z^*$  if  $w$  is high enough, so that the factor  $(wz^* - d^*)$  is then positive, which implies the occurrence of negative savings. Thus, the *ex post* egalitarian optimum involves intertemporal transfers from the old age towards the young age, in sufficiently productive economies.

If, for some reason (e.g., political constraints), intertemporal transfers from the old age towards the young age are not possible, one can seek to obtain

a constrained egalitarian optimum, which has  $c = w$  and  $d = wz$ . One can show that  $z$  is then at an intermediate value between the laissez-faire and the unconstrained optimum. Inequalities are then not fully eliminated and long-lived agents are now better off than the short-lived.

### 3 Unequal life expectancies

Not all human activities are characterized by the same mortality risks. Blanpain (2011) shows that, in France (2000-2008), life expectancy at age 35 is equal to 40.9 years for a blue collar, against 47.2 years for an executive. In this light, it makes sense to explore what our results become under unequal life expectancies.

For this purpose, we introduce two kinds of individuals, with unequal mortality risks, in the two-period model:

- Type- $H$  agents, who have a high life expectancy  $1 + \pi_H$ ;
- Type- $L$  agents, with a lower life expectancy  $1 + \pi_L$ , with  $\pi_L < \pi_H$ .

For simplicity, we assume that there exists an equal proportion of type- $H$  agents and of type- $L$  agents in the population (at the young age). We also assume that there exist type-specific annuity markets for the two types of individuals. Hence type- $i$  agents who survive to the old age benefit from savings returns equal to  $\frac{1}{\pi_i}$ .

#### 3.1 Laissez-faire

In the laissez-faire, individuals of type  $i \in \{H, L\}$  face the following problem:

$$\max_{s_i, z_i} u(w - s_i) + \pi_i \left[ u\left(\frac{s_i}{\pi_i} + z_i w\right) - v(z_i) \right]$$

The FOCs are:

$$u'(c_i) = u'(d_i) \tag{10}$$

$$v'(z_i) = u'(d_i)w \tag{11}$$

From Prop. 1, we directly deduce that:

$$c_L = d_L = \frac{w(1 + z_L \pi_L)}{1 + \pi_L} > c_H = d_H = \frac{w(1 + z_H \pi_H)}{1 + \pi_H}$$

and that  $z_L < z_H$ .

Thus, type- $L$  agents enjoy higher consumption at all periods and retire earlier. Despite this, type- $L$  agents may have a lower expected utility, due to their greater mortality, i.e., it is possible to have

$$u(c_H) + \pi_H [u(c_H) - v(z_H)] > u(c_L) + \pi_L [u(c_L) - v(z_L)]$$

in spite of  $c_H < c_L$  and  $u(c_H) - v(z_H) < u(c_L) - v(z_L)$ .

**Proposition 3** *At the laissez-faire, if  $u(c_H) - v(z_H) > \max_{z \in [z_L, z_H]} v'(z)(1 - z)/(1 + \pi_L)$ , then*

$$u(c_H) + \pi_H [u(c_H) - v(z_H)] > u(c_L) + \pi_L [u(c_L) - v(z_L)].$$

**Proof.** Consider the comparative statics of Prop. 1. The expression  $u(c) - v(z)$  is decreasing in  $\pi$ . Therefore if  $u(c_H) - v(z_H) > \max_{z \in [z_L, z_H]} v'(z)(1 - z)/(1 + \pi_L)$ , one has  $u(c) - v(z) > v'(z)(1 - z)/(1 + \pi)$  for all  $\pi \in [\pi_L, \pi_H]$ .

The expression  $u(c) + \pi [u(c) - v(z)]$  is increasing in  $\pi$  if

$$(1 + \pi) u'(c) \frac{dc}{d\pi} - \pi v'(z) \frac{dz}{d\pi} + u(c) - v(z) > 0.$$

Using  $wu'(c) = v'(z)$ , one obtains

$$u'(c) \left[ (1 + \pi) \frac{dc}{d\pi} - \pi w \frac{dz}{d\pi} \right] + u(c) - v(z) > 0.$$

One has  $c = w(1 + \pi z)/(1 + \pi)$ . The expression in bracket is therefore equal to

$$\begin{aligned} (1 + \pi) \frac{dc}{d\pi} - \pi w \frac{dz}{d\pi} &= (1 + \pi) \left[ -\frac{w(1 + \pi z)}{(1 + \pi)^2} + \frac{wz}{1 + \pi} + \frac{w\pi}{1 + \pi} \frac{dz}{d\pi} \right] - \pi w \frac{dz}{d\pi} \\ &= -[w(1 - z)/(1 + \pi)]. \end{aligned}$$

The result is obtained by using  $wu'(c) = v'(z)$  once again. ■

Observe that it is not sufficient to have  $u(c) - v(z) > 0$  to make it a benefit to have greater life expectancy. The reason is that with perfect annuities, the rate of return on savings decreases, making the initial consumption-labour bundle unaffordable after an increase in  $\pi$ . If we assumed that all agents faced the same rate of return, then it would be easier to obtain inequalities in expected utility to the benefit of the agents with greater life expectancy.

### 3.2 *Ex ante* egalitarian optimum

A social planner whose aim is to maximize the *minimum* expected lifetime welfare faces the following problem:

$$\begin{aligned} &\max_{c_i, d_i, z_i} \min \{u(c_H) + \pi_H [u(d_H) - v(z_H)], u(c_L) + \pi_L [u(d_L) - v(z_L)]\} \\ \text{s.t. } &\sum_{i=H,L} [c_i + \pi_i d_i] = \sum_{i=H,L} [w + wz_i] \end{aligned}$$

We can thus reformulate the social planner's problem as follows:

$$\begin{aligned} &\max_{c_i, d_i, z_i} \mu [u(c_L) + \pi_L [u(d_L) - v(z_L)]] + (1 - \mu) [u(c_H) + \pi_H [u(d_H) - v(z_H)]] \\ \text{s.t. } &\sum_{i=H,L} [w + \pi_i wz_i] - \sum_{i=H,L} [c_i + \pi_i d_i] = 0 \end{aligned}$$

where  $\mu$  is set so as to ensure that we obtain:

$$[u(c_L^*) + \pi_L [u(d_L^*) - v(z_L^*)]] = u(c_H^*) + \pi_H [u(d_H^*) - v(z_H^*)]$$

We obtain the following first-order conditions:

$$u'(c_L^*) = u'(d_L^*) = \frac{\lambda}{\mu} \quad (12)$$

$$u'(c_H^*) = u'(d_H^*) = \frac{\lambda}{1-\mu} \quad (13)$$

$$v'(z_L^*) = u'(d_L^*)w \quad (14)$$

$$v'(z_H^*) = u'(d_H^*)w \quad (15)$$

Under the objective of maximizing the utility of the worst-off agent, and assuming that the type- $L$  agent is the worst off, we have that  $\mu > 1 - \mu$ , so that type- $L$  consumptions are larger than the ones of a type- $H$  agent.

We thus have consumption smoothing, but at different levels for the two types of agents. Type- $L$  agents enjoy higher consumption at the two periods. Hence, from the last two FOCs, it follows also that type- $H$  agents should also work longer, and retire later than type- $L$  agents, on the grounds of the equalization of expected lifetime welfare.

**Proposition 4** *Under  $\pi_H > \pi_L$ , the ex ante egalitarian optimum involves:*

$$\begin{aligned} c_L^* &= d_L^* > c_H^* = d_H^* \\ z_L^* &< z_H^* \end{aligned}$$

**Proof.** The proof follows from the FOCs. ■

Thus, at the *ex ante* egalitarian optimum, individuals with lower survival chances should consume more than individuals with high survival chances, and retire earlier than these. Unequal consumption patterns and retirement ages are indeed the only way to yield an equalization of expected lifetime welfare across groups with unequal survival prospects.

Let us mention that, under asymmetric information, there would be a conflict between the *ex ante* egalitarian constraint and the incentive compatibility constraint. Indeed, if the social planner cannot observe individual survival probabilities, and proposes the first-best contracts, type- $H$  agents have an interest in claiming to be type- $L$  ones. Hence, under asymmetric information, one needs to add an incentive constraint of the form,

$$u(c_H) + \pi_H [u(d_H) - v(z_H)] - [u(c_L) + \pi_H [u(d_L) - v(z_L)]] \geq 0 \quad (16)$$

to the first-best problem so as to prevent mimicking from type- $H$  agents. In equilibrium, this constraint is binding and no contract  $\{(c_L, d_L, z_L), (c_H, d_H, z_H)\}$  can satisfy both the incentive compatibility constraint (16) and the egalitarian constraint. There is thus a conflict between egalitarianism and incentive compatibility.

This analysis reveals two problems with *ex ante* egalitarianism.<sup>11</sup> First, equalizing expected lifetime welfare across groups with different life expectancies may still leave large *ex post* welfare inequalities between long-lived and short-lived individuals. Indeed the above allocation of resources only equalizes the *expected* lifetime utility across groups, but there still remain large inequalities within groups, as a consequence of unequal lifespans.

The second problem is perhaps less apparent, but no less deep. It is assumed here that  $\pi_H, \pi_L$  are simultaneously the average survival rate in the two subgroups and the individual survival probability of each member of the group. In real-life applications, this assumption never holds. The average life expectancy of blue-collar workers is not the individual life expectancy of every blue-collar worker. It is in fact deeply problematic for the *ex ante* egalitarian approach to require defining individual life expectancies. First, individuals may not be trusted to have the best estimate of their own life expectancy, because average subjective life expectancy generally differs from observed average life expectancy (Brouwer and van Exel 2005). Second, the very notion of an individual life expectancy is problematic. In a deterministic world, individual life expectancy and individual final longevity coincide. The *ex post* approach, by focusing on the distribution of actual longevities, may in fact be the best version of the *ex ante* approach, because it relies on the true individual life expectancy, not on subgroup average life expectancy. In a sense, the *ex ante* egalitarian approach remains utilitarian at the subgroup level (individual expected utility being abusively proxied by subgroup average utility). In welfare economics, individualistic approaches are better. If inequality aversion is introduced, it should be at the individual level, not at the subgroup level. Therefore, from now on we focus mainly on the *ex post* approach.

### 3.3 *Ex post* egalitarian optimum

From an *ex post* perspective, the planner's problem is:

$$\max_{c_i, d_i, z_i} \min \{U_H^{ll}, U_H^{sl}, U_L^{ll}, U_L^{sl}\} \quad \text{s.t.} \quad \sum_{i=H,L} [c_i + \pi_i d_i] = \sum_{i=H,L} [w + \pi_i z_i w]$$

We can rewrite the social planning program in a more convenient way, by adding egalitarian constraints. Three egalitarian constraints are needed: two constraints guaranteeing that long-lived and short-lived agents for a given type are equally well:

$$u(d_L) - v(z_L) = u(d_H) - v(z_H) = 0,$$

and one additional constraint guaranteeing that individuals with some equal longevity are equally well-off across *ex ante* types:  $u(c_H) - u(c_L) = 0$ . The latter implies  $c_H^{**} = c_L^{**} = c^{**}$ .

---

<sup>11</sup> Additional rationality arguments against *ex ante* egalitarianism (i.e., it violates statewise dominance) can be found in Fleurbaey (2010).



The budget constraint, supplemented with the egalitarian constraints, implies

$$2c^{**} = w(2 + \pi_H z_H^{**} + \pi_L z_L^{**}) - \pi_H u^{-1} \circ v(z_H^{**}) - \pi_L u^{-1} \circ v(z_L^{**}).$$

Maximizing  $c^{**}$  under this constraint implies  $w = (u^{-1} \circ v)'(z_H^{**}) = (u^{-1} \circ v)'(z_L^{**})$ . Therefore,  $z_H^{**} = z_L^{**}$ , hence  $d_H^{**} = d_L^{**}$ . The two subgroups have the same consumption-labour bundle, independent of their survival rate:

$$\begin{aligned} c^{**} &> d_H^{**} = d_L^{**} \\ z_H^{**} &= z_L^{**} \end{aligned}$$

We thus have an equal optimal age at retirement, as well as equal old-age consumptions, which are inferior to first-period consumption.<sup>12</sup>

To compare this with the laissez-faire, remember first that the *ex post* egalitarian optimum  $z^{**}$  is obtained as the intersection of the  $f$  curve  $d = f(z)$  with the egalitarian constraint  $d = u^{-1} \circ v(z)$ . This intersection determines  $d_i^{**}$  and  $z_i^{**}$  for the two types, which can then be substituted in the resource constraint, to obtain first-period consumptions, i.e.,  $2c^{**} = w(2 + \pi_L z^{**} + \pi_H z^{**}) - (\pi_L + \pi_H)f(z^{**})$ .

The laissez-faire equilibrium is represented as the intersection of the  $f$  curve with the corresponding  $c$  curves, i.e.  $c_i = w(1 + \pi_i z_i) - \pi_i f(z_i)$ . Observe that the previous curve for  $c^{**}$  is the average of these two curves.

From Proposition 2 it is clear that, if  $u(d_H) > v(z_H)$  (which implies  $u(d_L) > v(z_L)$ ),  $c_L^{**} = c_H^{**} > c_H$ ,  $d_L^{**} = d_H^{**} < d_H < d_L$ , and  $z_L^{**} = z_H^{**} > z_H > z_L$ , but the comparison between  $c_L^{**}$  and  $c_L$  (which is greater than  $c_H$ ) is ambiguous. However, if one assumes that at the egalitarian optimum,  $wz^{**} > d^{**}$ , then  $z^{**}$  is greater than the solution  $z^+$  to  $wz^+ = f(z^+)$ . At  $z^+$ , the two curves  $c = w(1 + \pi_i z) - \pi_i f(z)$  cross. When  $z^{**} > z^+ > z$ , therefore, necessarily  $c^{**} > c_L, c_H$ , as can be seen from Figure 5.

The following proposition summarizes our results.

**Proposition 5**     • *The ex post egalitarian optimum involves:*

$$\begin{aligned} c_L^{**} &= c_H^{**} > d_L^{**} = d_H^{**} \\ z_L^{**} &= z_H^{**}. \end{aligned}$$

• *In comparison to the laissez-faire, assuming that  $u(d_H) > v(z_H)$ , we have:*

$$\begin{aligned} c_L^{**} &= c_H^{**} > c_H \\ d_L^{**} &= d_H^{**} < d_H < d_L \\ z_L^{**} &= z_H^{**} > z_H > z_L. \end{aligned}$$

<sup>12</sup> As above (section 2.2), we focus here on the case of sufficiently rich economies. In very poor economies, the optimal lifecycle consumption profile would be increasing rather than decreasing, as the low productivity level would not allow for  $u(d_i) - v(z_i) \geq 0$ .

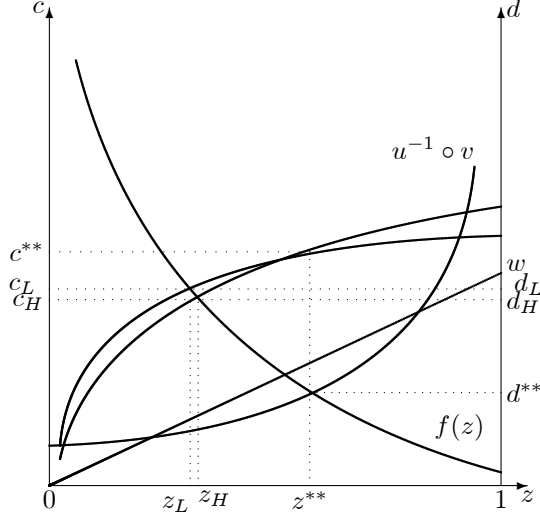


Figure 5: Comparing laissez-faire and social optimum

- If, in addition, one assumes that  $wz^{**} > d^{**}$ , we have:

$$c_L^{**} = c_H^{**} > c_L > c_H.$$

**Proof.** See above. ■

In sum, the introduction of unequal life expectancies does not affect the major features of the *ex post* egalitarian optimum. Note also that this allocation is still implementable under asymmetric information, as the allocation is independent of the survival probabilities. In comparison to the *ex ante* egalitarian optimum, the *ex post* egalitarian optimum recommends more consumption at the young age (except possibly for type *L*), and less consumption at the old age. Moreover, the optimal retirement age is also later under the *ex post* egalitarian optimum than under the *ex ante* egalitarian optimum. Thus, adopting an *ex post* view leads to a deterioration of the living conditions of surviving type-*L* agents, that is, of lucky, long-lived agents who faced initially a lower life expectancy, as well as a widening of consumption possibilities for type-*H* agents at the young age, on the grounds of a possible premature death even within the type-*H* group.

## 4 Egalitarianism and retirement age

The conclusion that egalitarian concerns require postponing retirement is driven by the fact that the worse-off are individuals who die before retiring. The most one can do for them is to increase their consumption, and to this effect it is beneficial if the lucky long-lived work even more to provide for more consumption resources to transfer to the young.

This reasoning may not be very robust. It might be possible to improve the lot of the disadvantaged by granting them early retirement. This section explores variants of the model in which this opposite conclusion may arise.

The idea is to introduce three periods so that mortality may be described as striking primarily before or after retirement. Let us now assume that agents work in the first and second periods, retire during the second period and then, if still alive, enjoy their retirement in the third period. The probability to reach period 2 is now denoted by  $\pi_2$ , whereas the probability to reach the old age, conditionally on survival to period 2, is denoted by  $\pi_3$ .

#### 4.1 Laissez-faire

At the laissez-faire, the individual problem is now:

$$\max_{s_1, s_2, z} u(w - s_1) + \pi_2 \left[ u \left( \frac{s_1}{\pi_2} + zw - s_2 \right) - v(z) \right] + \pi_2 \pi_3 u \left( \frac{s_2}{\pi_3} \right)$$

where  $s_1, s_2$  denotes first- and second-period savings. Assuming an interior solution, the FOCs can be rewritten as:

$$u'(c) = u'(d) = u'(b) \quad (17)$$

$$v'(z) = u'(d)w \quad (18)$$

$$c = w(1 + \pi_2 z) - \pi_2 d - \pi_2 \pi_3 b \quad (19)$$

where  $b$  is third-period consumption. We thus have a perfect consumption smoothing across periods. Substituting in the budget constraint yields:

$$c = d = b = \frac{w(1 + \pi_2 z)}{1 + \pi_2 + \pi_2 \pi_3}$$

One can thus represent the laissez-faire as the intersection, in the  $(z, c)$  space, of the increasing line  $c = \frac{w}{1 + \pi_2 + \pi_2 \pi_3} + \frac{w \pi_2}{1 + \pi_2 + \pi_2 \pi_3} z$  with the decreasing curve  $d = f(z)$  obtained from  $v'(z) = u'(d)w$ . A rise in productivity  $w$  pushes the  $d$  curve up, and raises both the intercept and the slope of the  $c$  line. Hence, it contributes to increase consumption, but has an effect on retirement age that depends the elasticity of  $u'$ , as in Prop. 1. A rise in  $\pi_2$  leaves the function  $d = f(z)$  unchanged, but lowers the  $c$  line, implying a later retirement age, as well as a lower consumption (see Figure 2 —the only difference here is that the fixed point of the  $c$  line when  $\pi_2$  changes occurs for  $z = 1 + \pi_3$  instead of  $z = 1$ ). Similarly, a rise in  $\pi_3$  leaves the function  $d = f(z)$  unchanged, and lowers the  $c$  line, implying a higher  $z$ , as well as a lower consumption. Thus changes in  $\pi_2$  and  $\pi_3$  have qualitatively similar effects on retirement age at the laissez-faire.

Regarding welfare inequalities, individuals who live the maximum length of life are better off than those who die after the second period if and only if:

$$u \left( \frac{w(1 + \pi_2 z)}{1 + \pi_2 + \pi_2 \pi_3} \right) > 0$$

Moreover, individuals who live only two periods are better off than those who live only one period if and only if:

$$u\left(\frac{w(1 + \pi_2 z)}{1 + \pi_2 + \pi_2 \pi_3}\right) - v(z) > 0$$

From this, it follows that, if agents who live only two periods are better-off than those who live only 1 period, agents living the maximum life will also be better off than those living the minimum life. When considering lifetime welfare inequalities between agents with different longevity, it appears that a rise in productivity, by raising consumption at all ages of life, raises the welfare inequalities between those living three periods and those living only two periods. A rise in  $\pi_2$ , by implying a later retirement and a lower consumption, will reduce welfare inequalities between those living only one period and the other individuals, who enjoy longer lives. The same is true for a rise in  $\pi_3$ .

**Proposition 6** • *At the laissez-faire, we have:*

$$\begin{aligned} c &= d = b = \frac{w(1 + \pi_2 z)}{1 + \pi_2 + \pi_2 \pi_3} \\ v'(z) &= u'(d)w \\ \frac{dz}{dw} &\leq 0, \frac{dz}{d\pi_2} > 0, \frac{dz}{d\pi_3} > 0 \end{aligned}$$

- *Agents with the minimum longevity are the worst-off if and only if*

$$u\left(\frac{w(1 + \pi_2 z)}{1 + \pi_2 + \pi_2 \pi_3}\right) - v(z) > 0$$

- *Lifetime welfare inequalities between agents living 3 periods and agents living 1 period are increasing in  $w$ , but decreasing in  $\pi_2$  and  $\pi_3$ .*

**Proof.** See the Appendix. ■

## 4.2 Egalitarian first-best optimum

Here again, we will not focus on the *ex ante* egalitarian optimum, which yields the same allocation as the laissez-faire, and focus instead on the *ex post* welfare of the worst-off individual.

Note that, under  $\pi_2 < 1$ , that egalitarian constraint includes two parts:

$$u(d) - v(z) = 0 \quad \text{and} \quad u(b) = 0$$

The first part states that individuals living one more period are not better off than those who die after period 1, whereas the second part states that individuals living two more periods are not better off either. This implies that  $b^{**} = c_0$ . But we also need to set  $u(d) - v(z) = 0$ , so as to avoid inequalities between

agents who live two periods and those who live only one. As in the previous section with only two periods, the social planning problem can be written in a simple form:

$$\max_z c = w(1 + \pi_2 z) - \pi_2 u^{-1} \circ v(z) - \pi_2 \pi_3 c_0.$$

As far as  $z$  is concerned, this boils down to maximizing  $w(1 + \pi_2 z) - \pi_2 u^{-1} \circ v(z)$ , a problem that has already been studied in Subsection 2.2. If  $\pi_2 < 1$ , we thus obtain a *decreasing* consumption profile over the lifecycle:<sup>13</sup>

$$c^{**} > d^{**} > b^{**} = c_0.$$

But let us now examine the special case in which there is no premature death during the career, so that  $\pi_2 = 1$ . In that case, the egalitarian constraint is only  $u(b^{**}) = 0$ , leading to  $b^{**} = c_0$ . Hence the social planner's problem becomes:

$$\max_{c,d,z} u(c) + u(d) - v(z) + \lambda [w + zw - c - d - \pi_3 c_0]$$

where  $\lambda$  is the Lagrange multiplier associated to the resource constraint of the economy. Hence the FOCs are now:

$$u'(c^{**}) = u'(d^{**}) \tag{20}$$

$$v'(z^{**}) = u'(d^{**})w \tag{21}$$

$$w + z^{**}w = c^{**} + d^{**} + \pi_3 c_0 \tag{22}$$

From which we now have:<sup>14</sup>

$$c^{**} = d^{**} > b^{**} = c_0$$

Hence, in that case, there is perfect consumption smoothing during the active life (1st and 2nd life-periods), which is akin to what happens in the *laissez-faire*.

But  $c^{**} = d^{**}$  are greater than at the *laissez-faire* (in affluent economies), because they determine the well-being of the short-lived. Given the relationship  $v'(z^{**}) = u'(d^{**})w$ , a greater  $d^{**}$  implies a lower  $z^{**}$ . Therefore we now obtain that optimal retirement occurs earlier than at the *laissez-faire*.

This result can be understood as follows. As death strikes only during retirement, the goal is to enhance the well-being of the active and the young pensioners. As efficiency requires retirement to occur when working more has a marginal disutility equal to the marginal utility of the consumption it pays, a better situation for these people means that they have a lower marginal utility of consumption, therefore should stop working before marginal disutility rises

<sup>13</sup>We leave aside the extreme case of very poor economies, where the low productivity level would always make the long-lived worse off than the short-lived, and where the *ex post* egalitarian optimum would involve an increasing optimal consumption profile, contrary to what prevails under non-poor economies.

<sup>14</sup>Once again, we assume here that the productivity level is sufficiently large, so as to allow longer-lived agents to be better off than shorter-lived agents (see above).

too much. Their extra welfare (compared to the laissez-faire) is split between extra consumption and extra leisure.

In sum, the extent to which the egalitarian optimum differs from the laissez-faire depends on the timing of premature deaths. If premature deaths occur during the career, the retirement age should be *postponed* from an egalitarian perspective. However, if death only arises at the *end* of the career, then the retirement age should be *advanced*.

**Proposition 7** • Suppose that  $\pi_2 < 1$ . Comparing the *ex post* egalitarian optimum  $(^{**})$  with the laissez-faire, we have:

$$c^{**} > c; d^{**} < d; b^{**} < b; z^{**} > z$$

• Suppose that  $\pi_2 = 1$ . Then, comparing the *ex post* egalitarian optimum  $(^{**})$  with the laissez-faire, we have:

$$c^{**} > c; d^{**} > d; b^{**} < b; z^{**} < z$$

**Proof.** See above. ■

In any case, consumption at the young age is always too low at the laissez-faire in comparison with the *ex post* egalitarian optimum. But the crucial difference lies in the treatment of individuals in the second part of their career. The *ex post* egalitarian optimum recommends to reduce their consumption and to postpone their retirement with respect to the laissez-faire when some premature deaths occur during the active life, whereas it recommends the opposite when death only occurs once the career is fully completed. Our advanced societies are still very much concerned with the first case, but, over time, may come closer and closer to the second case, as illustrated on Figure 1 for French males.

### 4.3 Moderate egalitarianism

The above analysis reveals a tension between the interests of the very worse-off who die at the end of the first period, and for whom increasing consumption (therefore postponing retirement) is beneficial, and the slightly less disadvantaged who die one period later and would benefit from earlier retirement.

Full egalitarianism focuses only on the very worse-off, but if one considered a moderate form of egalitarianism, it may happen that the fate of the less disadvantaged looms larger, for instance if they are sufficiently many while the worse-off are few. It is therefore worth exploring the possibility to obtain a policy of early retirement even when mortality strikes at all periods, provided that mortality in period 2 is small and that the degree of priority for the worse-off is moderate.

A moderate egalitarian social planner seeks to maximise the following social welfare function, where  $G(\cdot)$  is an increasing concave transform:

$$(1-\pi_2)G(u(c))+\pi_2(1-\pi_3)G(u(c)+u(d)-v(z))+\pi_2\pi_3G(u(c)+u(d)-v(z)+u(b))$$

subject to the following resource constraint:

$$w(1 + \pi_2 z) \geq c + \pi_2 d + \pi_2 \pi_3 b$$

The first order conditions of this problem are written as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c} &= [(1 - \pi_2)G'(U_1) + \pi_2(1 - \pi_3)G'(U_2) + \pi_2\pi_3G'(U_3)]u'(c) - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial d} &= [(1 - \pi_3)G'(U_2) + \pi_3G'(U_3)]u'(d) - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial b} &= G'(U_3)u'(b) - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial z} &= -[(1 - \pi_3)G'(U_2) + \pi_3G'(U_3)]v'(z) + \lambda w = 0 \end{aligned}$$

where  $U_t$  is the *ex post* utility of an agent living  $t = 1, 2, 3$  periods of life:  $U_1 = u(c)$ ;  $U_2 = u(c) + u(d) - v(z)$ ;  $U_3 = u(c) + u(d) - v(z) + u(b)$ .

The above FOCs do not allow us to draw unambiguous results about the form of the moderate egalitarian optimum. In the following, we provide a numerical example aimed at comparing the moderate egalitarian optimum with the full egalitarian optimum. To do so, we use the following functional forms:  $u(c) = \ln(c)$ ,  $v(z) = 2z^2$  and  $G(u(x)) = \frac{u(x)^\varepsilon}{\varepsilon}$ . We also assume that  $w = 10$  and  $\pi_2 = 0.9$  and  $\pi_3 = 0.8$ . We make the parameter of inequality aversion  $\varepsilon$  vary between 1 and  $-30$ .<sup>15</sup> The following table contrast the laissez-faire (Section 4.1) with the full egalitarian optimum (Section 4.2) and the moderate egalitarian optimum, under different degrees of inequality aversion.

	LF		Moderate Egalitarianism								Full Egalit.
$\varepsilon$	1	0.5	0.1	-0.5	-1	-2	-3	-10	-30		
$c$	5.41	5.41	5.67	5.93	6.40	6.85	7.76	8.58	11.30	12.58	13.26
$d$	5.41	5.41	5.37	5.32	5.21	5.09	4.83	4.58	3.78	3.43	3.25
$b$	5.41	5.41	5.12	4.87	4.48	4.15	3.55	3.07	1.74	1.23	1.00
$z$	0.46	0.46	0.47	0.47	0.48	0.49	0.52	0.55	0.66	0.73	0.77
$U_1$	1.69	1.69	1.73	1.78	1.86	1.92	2.05	2.15	2.42	2.53	2.59
$U_2$	2.95	2.95	2.98	3.01	3.05	3.07	3.09	3.08	2.88	2.70	2.59
$U_3$	4.63	4.63	4.62	4.59	4.55	4.49	4.35	4.20	3.43	2.91	2.59

Under full and moderate egalitarianism, first-period consumption should be increased with respect to the laissez-faire, whereas second- and third-period consumptions should be decreased.<sup>16</sup> However, the extent to which optimal and laissez-faire levels differ depends on the degree of aversion toward inequality: the difference between first-period and subsequent-periods consumption levels is maximum under full egalitarianism, but decreases as  $\varepsilon$  increases.

<sup>15</sup> Assuming  $\varepsilon = 1$  under moderate egalitarianism is equivalent to assuming a utilitarian social planner maximising *ex ante* utilities.

<sup>16</sup> That result is in line with a previous study by Bommier *et al* (2011).

Regarding the optimal retirement age, we also obtain that, under moderate egalitarianism, the optimal  $z$  increases with the degree of inequality aversion. To interpret those results, one can assume that each period lasts 25 years, with the active life starting when the individual is aged 25. At the laissez-faire, the age at retirement is equal to about 61.5 years. When  $\varepsilon$  remains high, the optimal retirement age under moderate egalitarianism is close to the one at the laissez-faire. For instance, the optimal age at retirement when  $\varepsilon = -1$  is equal to about 62.5 years. The largest retirement age under moderate egalitarianism, obtained under  $\varepsilon = -30$ , is equal to 68.25 years. This remains below the optimal retirement age under full egalitarianism, which is equal to 69.25 years. The underlying intuition behind those higher retirement ages under a higher inequality aversion is the following: making individuals work longer tends to reduce the welfare differentials between those who live a short life and those who enjoy a longer life. Note, however, that the optimal retirement age remains, even under full egalitarianism, limited, because of the high desutility of old-age labour postulated in this numerical example.

Note also that, while inequalities in utilities between individuals living one, two and three periods are maximum under the laissez-faire and utilitarianism, those inequalities are reduced as inequality aversion increases. Under full egalitarianism, we find, not surprisingly, that *ex post* utilities are equalized across individuals having unequal realized longevity.

How would an improvement of survival conditions affect the social optimum under moderate and full egalitarianism? To answer that question, the following table shows the laissez-faire, the moderate egalitarian optimum and the full egalitarian optimum when  $\pi_2$  is as high as 0.99.

	LF		Moderate Egalitarianism								Full Egalit.
$\varepsilon$	1	0.5	0.1	-0.5	-1	-2	-3	-10	-30		
$c$	5.28	5.28	5.36	5.43	5.56	5.68	6.01	6.43	9.63	12.16	13.59
$d$	5.28	5.28	5.33	5.36	5.41	5.45	5.47	5.43	4.46	3.65	3.25
$b$	5.28	5.28	5.07	4.90	4.62	4.38	3.91	3.47	1.90	1.25	1.00
$z$	0.47	0.47	0.47	0.47	0.46	0.46	0.46	0.46	0.56	0.68	0.77
$U_1$	1.66	1.66	1.68	1.69	1.71	1.74	1.79	1.86	2.26	2.50	2.59
$U_2$	2.88	2.88	2.91	2.94	2.98	3.01	3.08	3.13	3.13	2.86	2.59
$U_3$	4.54	4.54	4.53	4.52	4.51	4.49	4.44	4.37	3.77	3.08	2.59

An interesting result concerns the optimal age at retirement. At the laissez-faire, the age at retirement is now larger than under  $\pi_2 = 0.9$ . However, under full egalitarianism, the optimal age at retirement remains, under  $\pi_2 = 0.99$ , exactly the same as under  $\pi_2 = 0.9$ , and equal to 0.77 (or, in usual units, 69.25 years). On the contrary, under moderate egalitarianism, the optimal retirement age is, under  $\pi_2 = 0.99$ , significantly lower than under  $\pi_2 = 0.9$ . Hence, while an improvement in survival conditions makes individuals work longer at the laissez-faire, the two kinds of egalitarian optima have very different implications in terms of the optimal age at retirement.



In sum, this non-exhaustive numerical illustration shows that moderate egalitarianism involves, depending on the degree of inequality aversion, a large span of intermediate social optima - including different optimal retirement ages - between two extreme situations: on the one hand, the laissez-faire (equivalent here to the utilitarian optimum), and, on the other hand, the full egalitarian optimum. Moreover, an improvement of survival conditions is shown to have quite distinct effects on the optimal age at retirement under the laissez-faire and under the two egalitarian optima considered. While raising  $\pi_2$  leaves the optimal age at retirement unchanged under full egalitarianism, this raises the retirement age at the laissez-faire and reduces the retirement age under moderate egalitarianism.

## 5 Unequal life expectancy and productivity

Whereas the previous sections relied on a unique source of heterogeneity *ex ante* - unequal life expectancy -, let us conclude this study by studying the double heterogeneity case, where both individual survival prospects and individual productivities differ. For that purpose, we turn back to the baseline 2-period model, but introduce four types of agents *ex ante*, who differ in their survival type  $i \in \{H, L\}$  and in their productivity type  $j \in \{h, \ell\}$ . We have:

$$\begin{aligned}\pi_H &> \pi_L \\ w_h &> w_\ell\end{aligned}$$

Hence, there exist four distinct groups of agents *ex ante*: types  $Hh$ ,  $H\ell$ ,  $Lh$  and  $L\ell$ . For simplicity, we assume that there exists an equal proportion of the four types of agents in the population (at the young age).

### 5.1 Laissez-faire

As in the previous sections, there exists a perfect annuity market, which yields a return on annuities,  $1/\pi_i$  where the gross interest rate is  $R = 1$ . Individuals of type  $\{i, j\}$  face, at the laissez-faire, the following problem:

$$\max_{s_{ij}, z_{ij}} u(w_j - s_{ij}) + \pi_i \left[ u\left(\frac{s_{ij}}{\pi_i} + z_{ij}w_j\right) - v(z_{ij}) \right]$$

and the FOCs are:

$$u'(c_{ij}) = u'(d_{ij}) \tag{23}$$

$$v'(z_{ij}) = u'(d_{ij})w_j \tag{24}$$

As in the previous cases, consumption should be equalized across time so that

$$c_{ij} = d_{ij} = \frac{w_j(1 + z_{ij}\pi_i)}{1 + \pi_i}.$$

Using the results of Proposition 1, we obtain the following ranking

$$\begin{aligned} c_{Lh} &= d_{Lh} > c_{Hh} = d_{Hh} \\ c_{L\ell} &= d_{L\ell} > c_{H\ell} = d_{H\ell} \\ c_{Lh} &= d_{Lh} > c_{L\ell} = d_{L\ell} \\ c_{Hh} &= d_{Hh} > c_{H\ell} = d_{H\ell} \end{aligned}$$

or equivalently

$$c_{Lh} > c_{Hh} \geq c_{L\ell} > c_{H\ell}$$

The ranking of retirement ages follows that of Proposition 1 so that  $dz_{ij}/dw_j \leq 0$  (depending on the value of  $R_R(c)$ ) and  $dz_{ij}/d\pi_i > 0$ . Hence, it is not possible to have a complete ranking of the retirement ages as a function of the different sources of heterogeneity. Only it is possible to show that:

$$z_{Hj} > z_{Lj} \quad \forall j \in \{h, \ell\}$$

Long-lived agents are better off than short-lived ones if and only if:

$$u(d_{ij}) - v(z_{ij}) > 0$$

Using results from section 2.1 (and the conditions set in Proposition 1), we obtain that welfare inequalities between long-lived and short-lived agents of the same *ex ante* type  $ij$  are smaller for agents with higher life expectancy and higher for agents with higher productivity under the condition on  $R_R(c)$  set in Proposition 1. Putting together these results, we obtain that welfare inequalities between short- and long-lived agents according to their types are as follows:

$$u(d_{Lh}) - v(z_{Lh}) > u(d_{Hh}) - v(z_{Hh}) \leq u(d_{L\ell}) - v(z_{L\ell}) > u(d_{H\ell}) - v(z_{H\ell})$$

Agents with high productivity and low life expectancy face a higher welfare loss if they die at the end of the first period than, at the other extreme, agents with low productivity and high life expectancy.

## 5.2 Perfect information

### 5.2.1 *Ex ante* egalitarian optimum

A social planner whose aim is to maximize the *minimum* expected lifetime welfare faces the following problem:

$$\begin{aligned} &\max_{c_{ij}, d_{ij}, z_{ij}} \min \{u(c_{ij}) + \pi_i [u(d_{ij}) - v(z_{ij})]\} \\ \text{s.t. } &\sum_{i=H,L} \sum_{j=h,\ell} [c_{ij} + \pi_i d_{ij}] = \sum_{i=H,L} \sum_{j=h,\ell} w_j [1 + \pi_i z_{ij}] \end{aligned}$$

The problem can be formulated as follows:

$$\begin{aligned} &\max_{c_{ij}, d_{ij}, z_{ij}} \sum_{i=H,L} \sum_{j=h,\ell} \{ \mu_{ij} [u(c_{ij}) + \pi_i [u(d_{ij}) - v(z_{ij})]] \\ &\quad + \lambda [c_{ij} + \pi_i d_{ij} - w_j (1 + \pi_i z_{ij})] \} \end{aligned}$$

where the  $\mu_{ij}$  represent the weights which ensure the equality in *ex ante* utilities for all types  $\{i, j\}$ . Assuming that low productivity and low survival probability agents are the worst-off agents (as it would be the case with standard utilitarianism), the ranking of  $\mu_{ij}$  is such that:

$$\mu_{L\ell} > \mu_{H\ell} \lesssim \mu_{Lh} > \mu_{Hh}.$$

We obtain the following first order conditions:

$$u'(c_{ij}^*) = u'(d_{ij}^*) = \frac{\lambda}{\mu_{ij}} \quad (25)$$

$$v'(z_{ij}^*) = \frac{\lambda w_j}{\mu_{ij}} \quad (26)$$

from which we obtain the following ranking:

$$\begin{aligned} c_{L\ell}^* &= d_{L\ell}^* > c_{H\ell}^* = d_{H\ell}^* \lesssim c_{Lh}^* = d_{Lh}^* > c_{Hh}^* = d_{Hh}^* \\ z_{L\ell}^* &< z_{H\ell}^* \lesssim z_{Lh}^* < z_{Hh}^* \end{aligned}$$

This ranking reproduces what we had in the one-source of heterogeneity cases and ensures the equalization of *ex ante* utilities across types.

For each type  $ij$ , consumption is smoothed across the lifecycle. The consumption profile of agents with low life expectancy and low productivity (i.e. type  $L\ell$ ) is higher than the one of agents with high life expectancy and high productivity (i.e. type  $Hh$ ), whereas there is no clear ranking in terms of consumption for the intermediate types (i.e. types  $H\ell$  and  $Lh$ ). In their case, one needs additional assumptions on the relative size of differences in longevity and productivity (i.e. whether inequalities are higher in one or the other dimension of heterogeneity) in order to obtain a clear ranking of their consumptions.

Regarding the optimal retirement ages, we obtain that agents with low productivity (i.e. type  $i\ell$ ) should, *ceteris paribus*, retire earlier than agents with high productivity (i.e. type  $ih$ ). That result is standard in the literature. We obtain also that agents with low life expectancy (i.e. type  $Lj$ ) should, *ceteris paribus*, retire earlier than agents with high life expectancy (i.e. type  $Hj$ ). That earlier retirement would be a kind of compensation for the lowest survival prospects. Here again, there is no clear ranking when considering the optimal retirement ages for the intermediate groups: low productivity / high life expectancy, and high productivity / low life expectancy.

### 5.2.2 *Ex post* egalitarian optimum

In this section, we thus have eight types of agents, depending on whether they are long-lived or short-lived. The planner's problem is thus written as follows:

$$\max_{c_{ij}, d_{ij}, z_{ij}} \min \{U_{ij}^{sl}, U_{ij}^{ll}\} \text{ s. t. } \sum_{i=H,L} \sum_{j=h,\ell} (c_{ij} + \pi_i d_{ij}) = \sum_{i=H,L} \sum_{j=h,\ell} w_j (1 + \pi_i z_{ij})$$

where as before, one has

$$\begin{aligned} U_{ij}^{sl} &= u(c_{ij}) \\ U_{ij}^{ll} &= u(c_{ij}) + u(d_{ij}) - v(z_{ij}) \end{aligned}$$

As in the preceding sections, we can rewrite this problem in a more convenient way by expliciting the egalitarian constraints. First, short-lived and long-lived of given types must be equally well:

$$u(d_{ij}^{**}) - v(z_{ij}^{**}) = 0 \quad \forall i, j \quad (27)$$

Note that this corresponds to a case where one can have  $s_{ij}^{**} \leq 0$ .<sup>17</sup> Second, short-lived agents have to be equally well:

$$u(c_{ij}^{**}) = \bar{u} \quad \forall i, j$$

The latter constraint implies that  $c_{ij}^{**} = c^{**}$  for every agent. Using the same approach as in section 3.3, we have that

$$4c^{**} = \sum_{i=H,L} \sum_{j=h,\ell} w_j (1 + \pi_i z_{ij}^{**}) - \pi_i u^{-1} \circ v(z_{ij}^{**})$$

Maximising  $c^{**}$  with respect to  $z_{ij}^{**}$  yields:

$$w_j = (u^{-1} \circ v)'(z_{ij}^{**}) \quad (28)$$

and thus, that  $z_{Lh}^{**} = z_{Hh}^{**} > z_{L\ell}^{**} = z_{H\ell}^{**}$ . From (27) this implies that  $d_{Lh}^{**} = d_{Hh}^{**} > d_{L\ell}^{**} = d_{H\ell}^{**}$ . That allocation is independent of survival rates. This is a direct consequence of the *ex post* egalitarian approach, as what matters is the *realized* outcome and not the expected one. Regarding the form of the lifecycle consumption profile, and focusing on the case of sufficiently rich economies, we obtain, as in the baseline model, that the optimal consumption profile is, in general, decreasing over time for each type, so that  $c^{**} > d_{ih}^{**}, d_{i\ell}^{**}$ . Finally, note that *ex post* utilities are the same for short and long-lived agents and across types. This is a direct consequence of the first period consumption equalization and of (27).

The following proposition summarizes our results under perfect information.

**Proposition 8** *Consider a two-period economy with risky lifetime, and inequalities in terms of life expectancy ( $\pi_H > \pi_L$ ) and productivity ( $w_h > w_\ell$ ).*

- *The laissez-faire involves:*

$$\begin{aligned} c_{ij} &= d_{ij} \quad \forall i \in \{H, L\}, \forall j \in \{h, \ell\} \\ c_{Lh} &> c_{Hh} \geq c_{L\ell} > c_{H\ell} \\ d_{Lh} &> d_{Hh} \geq d_{L\ell} > d_{H\ell} \\ z_{Hj} &> z_{Lj} \quad \forall j \in \{h, \ell\} \end{aligned}$$

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<sup>17</sup>If  $s_{ij}$  was constrained to be positive, one would have that second period utility would be equal across types but not necessarily null.

- The *ex ante* egalitarian optimum involves:

$$\begin{aligned}
c_{ij}^* &= d_{ij}^* \quad \forall i \in \{H, L\}, \forall j \in \{h, \ell\} \\
c_{L\ell}^* &> c_{H\ell}^* \geq c_{Lh}^* > c_{Hh}^* \\
d_{L\ell}^* &> d_{H\ell}^* \geq d_{Lh}^* > d_{Hh}^* \\
z_{L\ell}^* &< z_{H\ell}^* \leq z_{Lh}^* < z_{Hh}^*
\end{aligned}$$

- The *ex post* egalitarian optimum involves, if  $u(d_{ij}) - v(z_{ij}) > 0$  for all  $i, j$  at the *laissez-faire*:

$$\begin{aligned}
c_{ij}^{**} &= c^{**} > d_{ij}^{**} \quad \forall i \in \{H, L\}, \forall j \in \{h, \ell\} \\
d_{Lh}^{**} &= d_{Hh}^{**} > d_{L\ell}^{**} = d_{H\ell}^{**} \\
z_{Lh}^{**} &= z_{Hh}^{**} > z_{L\ell}^{**} = z_{H\ell}^{**}
\end{aligned}$$

**Proof.** See above. ■

The *laissez-faire* and the *ex ante* egalitarian social optimum exhibit a smoothed consumption profile, unlike under the *ex post* egalitarian social optimum. The main dissonance between the three allocations concerns the retirement age. At the *laissez-faire* and at the *ex ante* egalitarian optimum, agents with a higher life expectancy should retire later on, in comparison with agents with low life expectancy. That result does not prevail under the *ex post* egalitarian optimum, where agents with the same productivity level should retire at the same age, independent of their life expectancy. Hence differences in survival chances are irrelevant under the *ex post* egalitarian optimum. The only relevant piece of information is productivity: high productivity agents should retire later than low productivity agents. That result was also prevailing under the *ex ante* egalitarian optimum. Therefore the main source of tension between *ex ante* and *ex post* egalitarianisms remains the treatment of life expectancy differentials, exactly as in the model without wage heterogeneity (see section 3.3).

### 5.3 Asymmetric information

We now study the planner's problem under asymmetric information. For that purpose, we will proceed in two stages. In a first stage, we will show that, with both *ex post* or *ex ante* egalitarianism, under asymmetric information, the equalization of utilities is *not* incentive compatible (as it was already shown in Section 3.2). Whereas in the first-best, the maximin criterion implies *equal* utilities, in the second-best the self-selection constraint implies utilities that are necessarily *unequal*. Then, in a second stage, we will derive the maximin solution under asymmetric information.

To make our point, we assume a *positive correlation* between life expectancy and productivity.<sup>18</sup> That assumption is in line with the existing literature, which suggests that there exists, in general, a positive correlation between labour

<sup>18</sup>We make this assumption so as to avoid unnecessary complexification of the analysis when studying the second-best optimum. Our argument could be extended to the four-type case.

earnings and life expectancy (see Duleep 1986, Deaton and Paxton 1998). We assume that type- $H$  agents have better survival prospects as well as a higher productivity, while it is the reverse for type- $L$  agents.

### 5.3.1 Incentive constraints and egalitarianism

Under asymmetric information, if agents were proposed the first-best allocations, type- $H$  agents would always have interest in claiming to be  $L$ -type agents. Hence, one needs to add to the previous problem the following incentive constraint:

$$u(c_H) + \pi_H \left[ u(d_H) - v\left(\frac{y_H}{w_H}\right) \right] \geq u(c_L) + \pi_H \left[ u(d_L) - v\left(\frac{y_L}{w_H}\right) \right]$$

This constraint has to be binding in equilibrium. This is clearly not compatible with the equality of *ex ante* utilities, that is

$$u(c_H) + \pi_H \left[ u(d_H) - v\left(\frac{y_H}{w_H}\right) \right] = u(c_L) + \pi_L \left[ u(d_L) - v\left(\frac{y_L}{w_L}\right) \right].$$

The same problem arises with *ex post* egalitarianism. The only difference is that first-period consumptions are the same across types, so as to ensure that there is no inequality among agents who live for only one period. Saying this differently, mimicking is possible only in the second period. The incentive constraint leads to setting:

$$u(c) + u(d_H) - v\left(\frac{y_H}{w_H}\right) \geq u(c) + u(d_L) - v\left(\frac{y_L}{w_H}\right)$$

which is binding in equilibrium. This is obviously not compatible with the egalitarian constraint,

$$u(c) + u(d_H) - v\left(\frac{y_H}{w_H}\right) = u(c) + u(d_L) - v\left(\frac{y_L}{w_L}\right).$$

All in all, under asymmetric information, the egalitarian and the incentive constraints cannot be satisfied at the same time. This result is independent of whether we use an *ex ante* or an *ex post* approach.

In the following, we will thus use an alternative social welfare criterion, i.e. the maximin criterion so as to solve the planner's problem under asymmetric information.

### 5.3.2 The maximin *ex ante* solution

Under asymmetric information, the *ex ante* maximin solution is obtained from solving the following problem:

$$\begin{aligned} & \max_{c_i, d_i, y_i} u(c_L) + \pi_L \left[ u(d_L) - v\left(\frac{y_L}{w_L}\right) \right] \\ \text{s. to } & \begin{cases} \Sigma_i (c_i + \pi_i d_i) \leq \Sigma_i w_i (1 + \pi_i z_i) \\ u(c_H) + \pi_H \left[ u(d_H) - v\left(\frac{y_H}{w_H}\right) \right] \geq u(c_L) + \pi_H \left[ u(d_L) - v\left(\frac{y_L}{w_H}\right) \right] \end{cases} \end{aligned}$$

The first-order conditions for this problem can be rearranged so as to obtain the following marginal rates of substitution:

$$\frac{u'(c_H^*)}{u'(d_H^*)} = 1 \quad (29)$$

$$\frac{v'(y_H^*/w_H)}{u'(d_H^*)} = w_H \quad (30)$$

$$\frac{u'(c_L^*)}{u'(d_L^*)} = \frac{1 - \gamma \frac{\pi_H}{\pi_L}}{1 - \gamma} < 1 \quad (31)$$

$$\frac{v'(y_L^*/w_L)}{u'(d_L^*)} = w_L \left[ \frac{1 - \gamma \frac{\pi_H}{\pi_L}}{1 - \gamma \frac{\pi_H}{\pi_L} \frac{w_L}{w_H} \frac{v'(y_L^*/w_L)}{v'(y_H^*/w_H)}} \right] < w_L \quad (32)$$

where  $\gamma$  is the Lagrange multiplier associated to the incentive constraint. First, note that we obtain the usual result of no distortion at the top for the type- $H$  agent. For this agent, consumptions are equalized across periods and the marginal rate of substitution between consumption and labour is equal to his productivity. In contrast, type- $L$  agent consumptions should be decreasing over time, i.e.  $c_L^* > d_L^*$  and the retirement age should be distorted downwards. Under asymmetric information, it is optimal to put an implicit tax on the retirement age of type- $L$  agents, so as to avoid mimicking from type- $H$  agents.

Finally, note that we have the following ranking of utilities between the short- and the long-lived:

$$\begin{aligned} u(c_H^*) + \pi_H \left[ u(d_H^*) - v\left(\frac{y_H^*}{w_H}\right) \right] &= u(c_L^*) + \pi_H \left[ u(d_L^*) - v\left(\frac{y_L^*}{w_H}\right) \right] \\ &> u(c_L^*) + \pi_L \left[ u(d_L^*) - v\left(\frac{y_L^*}{w_L}\right) \right] \end{aligned}$$

Clearly, at the second-best optimum, the type- $H$  agent needs to have higher utility than type- $L$  agents, so as to avoid mimicking behavior.

### 5.3.3 The maximin *ex post* solution

Let us now turn to the *ex post* approach. As in the perfect information case, we want to avoid inequalities in first-period consumption, i.e.,  $c_H^{**} = c_L^{**} = c^{**}$ , to ensure that short-lived agents are equally well-off. Yet, as we showed before, under asymmetric information, the equalization of utilities is not possible, since incentive constraints lead to:

$$u(d_H) - v\left(\frac{y_H}{w_H}\right) = u(d_L) - v\left(\frac{y_L}{w_H}\right) > u(d_L) - v\left(\frac{y_L}{w_L}\right) = 0$$

where we got rid of the utility of first-period consumption. The first part of the equality ensures that type- $H$  will not be tempted to pretend to be a type- $L$ , while the second inequality ensures that the long-lived and short-lived of type  $L$  are equally well off. Obviously, under asymmetric information, one cannot

avoid that the long-lived  $H$ -type is better-off than the rest of the population. Because of the incentive problem, one has to leave him some rent. This being said, the problem of the planner can be written as follows:

$$\begin{aligned} \max_{d_i, y_i} \quad & 2c = w_H + w_L + \pi_H y_H + \pi_L y_L - \pi_H d_H - \pi_L d_L \\ \text{s.to} \quad & \begin{cases} u(d_H) - v(\frac{y_H}{w_H}) = u(d_L) - v(\frac{y_L}{w_H}) \\ u(d_L) - v(\frac{y_L}{w_L}) = 0 \end{cases} \end{aligned}$$

We assume that the social planner does not use the information revealed in the previous period regarding wage levels, as otherwise the planner could observe who is effectively of type  $L$  or  $H$ .

Denoting by  $\gamma$  and  $\mu$  the Lagrange multipliers associated with respectively the incentive constraint and the egalitarian constraint, we obtain the following marginal rates of substitution between the retirement ages and second-period consumption:

$$\frac{v'(y_H^{**}/w_H)}{u'(d_H^{**})} = w_H \quad (33)$$

$$\frac{v'(y_L^{**}/w_L)}{u'(d_L^{**})} = w_L \left[ \frac{\mu - \gamma}{\mu - \gamma \frac{w_L}{w_H} \frac{v'(y_L^{**}/w_H)}{v'(y_L^{**}/w_L)}} \right] < w_L \quad (34)$$

Hence the trade-off between consumption and retirement is not distorted for type- $H$  agent (this is the usual result of no distortion at the top), but we find that it is distorted downwards for the type- $L$ . Hence taxation of the retirement age would be justified on equity grounds.<sup>19</sup>

Let us look at the ranking of second-period consumptions. Using the fact that:

$$\frac{v'(y_H^{**}/w_H)}{u'(d_H^{**})} > \frac{v'(y_L^{**}/w_L)}{u'(d_L^{**})}$$

and the binding incentive constraint, one can show that the only possible solution is such that  $y_H^{**} > y_L^{**}$  and  $d_H^{**} > d_L^{**}$ .

The following proposition summarizes our results at the second-best.

**Proposition 9** *Consider a two-period economy with risky lifetime, and inequalities in terms of life expectancy ( $\pi_H > \pi_L$ ) and productivity ( $w_h > w_\ell$ ). Assume asymmetric information about agents types  $ij$ , and a positive correlation between life expectancy and productivity, implying two types  $H$  and  $L$ . We obtain:*

- *The equalization of expected or realized lifetime utilities across ex ante types violates incentive compatibility constraints.*
- *Under the ex ante maximin, we have:*

$$\begin{aligned} c_H^* &= d_H^*; c_L^* > d_L^* \\ v'(z_H^*) &= w_H u'(d_H^*); v'(z_L^*) < w_L u'(d_L^*) \end{aligned}$$

<sup>19</sup>See Gruber and Wise (1999) for reviewing evidence of the existence of an implicit tax on retirement.



- Under the *ex post* maximin, we have:

$$\begin{aligned} c_H^{**} &= c_L^{**} = c^{**} \\ d_H^{**} &> d_L^{**}, \quad y_H^{**} > y_L^{**} \\ v'(z_H^{**}) &= w_H u'(d_H^{**}); \quad v'(z_L^{**}) < w_L u'(d_L^{**}) \end{aligned}$$

**Proof.** See above. ■

The introduction of asymmetric information affects the form of the *ex ante* egalitarian optimum at three levels. First, the optimum now involves inequalities in terms of the expected lifetime welfare between types  $H$  and  $L$ , since equalizing expected lifetime welfare is incompatible with incentive constraints. A higher expected lifetime welfare for type  $H$  is necessary to prevent type- $H$  agents from pretending to be of type  $L$ . Second, the optimal consumption profile is no longer flat for type- $L$  agents, but is now decreasing, unlike at the first-best. Moreover, type- $L$  agent's optimal retirement age is here distorted downwards. Making type- $L$  agents consume less and retire earlier at the second-period makes mimicking unappealing for type- $H$  agents. Regarding the *ex post* egalitarian optimum, the same incompatibility with incentive constraints holds, and therefore inequalities in realized welfare remain at the optimum, between type- $H$  and type- $L$  agents. Those inequalities are not related to the young age, since first-period consumptions are equalized across all agents. On the contrary, second-period consumptions are not equal, since type  $H$  enjoy a higher consumption than type  $L$ . Here again, the age at retirement is distorted downwards for the type- $L$  agent, in such a way as to make mimicking by type- $H$  unappealing.

Note, however, that, although asymmetric information makes both the equalization of *expected* lifetime welfares and *realized* lifetime welfares across agents incompatible with incentive constraints, the major difference between the two social optima remains: their different sensitivity to life expectancy differentials. Under the *ex post* planning problem, survival probabilities  $\pi_H$  and  $\pi_L$  matter only through the economy's resource constraint. On the contrary, under the *ex ante* planning problem, survival probabilities affect not only the resource constraint, but, also, how a given  $(c_{ij}, d_{ij}, z_{ij})$  bundle turns out to be valued by agents. As shown above, that different treatment of demographic information is far from neutral for the design of the optimal age at retirement. That general result remains valid under asymmetric information.

## 6 Concluding remarks

Despite major medical advances, human lives remain inherently risky. The corollary of that uncertainty consists, in a laissez-faire world, in large inequalities in lifetime welfare between those who enjoy a long life, and those who suffer from a premature death. Given that a significant part of longevity inequalities are due to factors on which individuals have no influence at all, there is a strong

ethical support for adopting an egalitarian social objective, which tries to reduce welfare inequalities between short-lived and long-lived individuals.

Two egalitarian objectives were explored here. On the one hand, *ex ante* egalitarianism, which recommends to maximize the minimum expected lifetime welfare in the population. On the other hand, *ex post* egalitarianism, which recommends to maximize the minimum realized lifetime welfare. Those two social objectives were shown to provide quite different views of the optimal organization of the lifecycle in terms of labour and retirement age.

Whereas the *ex ante* egalitarian optimum coincides with the laissez-faire allocation in an economy of *ex ante* identical individuals, the *ex post* egalitarian optimum recommends, in an advanced economy, higher consumption at the young age, and lower consumption at the old age, as well as a later retirement. Old-age labour thus appears to be a major ingredient of a fair production under risky lifetime: the longer lives are, the longer the working period should be.

That result also holds in a more general economy with three life-periods, provided some individuals turn out to die during their career. In that extended framework, premature deaths during the career support the postponement of retirement, and not an earlier retirement. On the contrary, the absence of early deaths during the career tends to support the advancement of the retirement age. Once heterogeneous life expectancies and productivities are introduced, the *ex ante* and *ex post* egalitarian optima still differs significantly, whatever we consider full information on agents' characteristics or asymmetric information.

Hence, adopting an *ex post* egalitarian approach has sizeable effects on the design of the optimal lifecycle. Such an approach would have major corollaries when thinking about the real world. To see this, consider, for instance, the hardness of labour, which leads, through occupational diseases, to a lower life expectancy. Such a lower life expectancy is often used to legitimate earlier retirement for high-risk workers. This is in conformity with the *ex ante* egalitarian optimum, but it is compatible with the *ex post* egalitarian optimum only if early deaths occur only after retirement, or if the number of deaths before retirement is small enough and the degree of egalitarianism is moderate.

The choice between *ex ante* and *ex post* egalitarianisms in a risky environment is a complex issue. There is no space here to provide a complete treatment of that problem, which is examined in Fleurbaey (2010). Nonetheless, an egalitarianism that focuses on inequalities in life expectancies rather than in actual longevities may miss its target. At the end of the day, what matters is what people achieve, not what they expected to achieve. A person dying at the age, let us say, of 45 years, suffers from a serious damage, even if he enjoyed a life expectancy of 78 years. That damage would hardly be affected if the life expectancy was, instead, equal to 77 years. Moreover, *ex ante* egalitarianism that is based on average mortality statistics is not really *ex ante* egalitarian as it fails to track individuals' true life expectancy. In a deterministic world, the true *ex ante* perspective coincides with the *ex post* perspective.

In sum, life expectancy statistics, although most useful for various purposes, can be quite misleading. That average figure hides the inequalities that really matter: inequalities in the lifespan actually enjoyed. Policy-makers should def-

initely not fall under the tyranny of life expectancy. But, as we showed in this paper, adopting such an *ex post* egalitarian perspective would have substantial implications for the organization of the lifecycle. Under risky lifetime and substantial mortality at working age, fairness in terms of realized well-being may require the elderly to work more, not less.

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## 8 Appendix

### 8.1 Proof of Proposition 1

The first two equalities in the first point of Proposition 1 follow immediately from the FOCs. Regarding the effects of  $w$  and  $\pi$  on  $z$ ,  $c$  and  $d$ , assuming differentiability to the desired order, one obtains the following results. Fully differentiating

$$c = w \frac{1 + \pi z}{1 + \pi}, \quad (35)$$

$$v'(z) = u'(c)w. \quad (36)$$

with respect to  $w$  and rearranging terms, one obtains:

$$\begin{aligned} \frac{dz}{dw} &= \frac{u''(c)c + u'(c)}{v''(z) - u''(c)\frac{w^2\pi}{1+\pi}} \\ &= \frac{u'(c)(1 - R_R(c))}{v''(z) - u''(c)\frac{w^2\pi}{1+\pi}} \end{aligned}$$

where  $R_R(c) = -u''(c)c/u'(c)$  so that  $dz/dw \geq 0$  if  $R_R(c) \leq 1$  and  $dz/dw < 0$  if  $R_R(c) > 1$ . We also obtain:

$$\frac{dc}{dw} = \frac{(1 + \pi z)v''(z) + w\pi u'(c)}{(1 + \pi) \left[ v''(z) - u''(c)\frac{w^2\pi}{1+\pi} \right]} > 0.$$

Fully differentiating (35) and (2) with respect to  $\pi$  and rearranging terms, we obtain:

$$\frac{dc}{d\pi} < 0 \text{ and } \frac{dz}{d\pi} > 0.$$

whenever  $z < 1$ , which is always the case here.

Inequalities in lifetime welfare increase with  $w$  if

$$u'(c) \frac{dc}{dw} > v'(z) \frac{dz}{dw}.$$

Substituting for the expressions of  $dc/dw$  and  $dz/dw$  above, we obtain that this inequality holds if and only if

$$\frac{(1 + \pi z)v''(z)}{(1 + \pi)v'(z)} + \frac{\pi}{(1 + \pi)} > (1 - R_R(c))$$

from which we deduce the second part of Proposition 1.

## 8.2 Proof of Proposition 6

Fully differentiating  $c$  with respect to  $w$ , one obtains

$$\frac{dc}{dw} = \frac{1 + \pi_2 z}{1 + \pi_2 (1 + \pi_3)} + \frac{w\pi_2}{1 + \pi_2 (1 + \pi_3)} \frac{dz}{dw}$$

and differentiating the FOC with respect to  $z$ , one has:

$$v''(z) \frac{dz}{dw} = wu''(d) \frac{dc}{dw} + u'(c)$$

Substituting one into the other and rearranging terms, we get:

$$\frac{dz}{dw} = \frac{wu''(d) \frac{1+\pi_2 z}{1+\pi_2(1+\pi_3)} + u'(c)}{v''(z) - \frac{w\pi_2}{1+\pi_2(1+\pi_3)} u''(c)} \leq 0$$

Substituting into  $dc/dw$ , we obtain:

$$\frac{dc}{dw} = \frac{1}{1 + \pi_2 (1 + \pi_3)} \left[ \frac{(1 + \pi_2 z) v''(z) + u'(c) w\pi_2}{v''(z) - \frac{w\pi_2}{1 + \pi_2 (1 + \pi_3)} u''(c)} \right] > 0.$$

Using the same reasoning for comparative statics with respect to  $\pi_2$  and  $\pi_3$ , we obtain

$$\begin{aligned} \frac{dz}{d\pi_2} &> 0 \text{ and } \frac{dd}{d\pi_2} < 0 \\ \frac{dz}{d\pi_3} &> 0 \text{ and } \frac{dd}{d\pi_3} < 0 \end{aligned}$$

Differentiating the expressions for welfare inequalities and using the above comparative statics, we obtain the last point of Proposition 4.