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Bridging the Learning Gap in the Market for Higher Education: E-learning and Public Subsidies

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ABSTRACT – This article aims at analyzing the adoption patterns which apply on the market for higher education when two types of learning organizations – namely, traditional learning and e-learning organizations – provide educational programs. We focus on the impact of public subsidies to e-learning providers in order to evaluate the conditions under which the learning gap is bridged. A welfare analysis is introduced to estimate the relevance of such ‘pro e-learning’ public policies. Our first results show that public subsidies enable the e-learning organization to provide quality-based and pricing strategies that tend to be similar to those of the brick’n mortar organization. Besides, we find that such short-term policies positively impact on the global level of quality which is provided by both providers. Nevertheless, our welfare analysis underlines contrasted results about the relevance of such short-term public policies.

KEY WORDS: adoption patterns, costs, e-learning, public subsidies, quality, welfare

Introduction

Information and communication technologies have deeply changed the scope and the nature of the market for higher education (Bates, 2005). Indeed, virtual campuses and e-learning programs have emerged as relevant alternatives to traditional ‘brick’n mortar’ learning organizations, thus providing somehow distant learning features. Following the developing of such new types of learning providers, it has been suggested that it is possible to consider a market for higher education learning services in which academic institutions and virtual campuses are key actors (Allen and Shen, 1999; Belfield and Levin, 2002; Brasington, 2003; Sosin et al., 2004). The advent of virtual campuses and e-learning programs leads to two major changes in the market for higher education. A first significant change affects the scope, the boundaries and the nature of competition in such a market. A second major change is related to the provision of quality for programs in this market inasmuch as e-learning programs tend to stimulate the developing of new – ICT-related – uses.

The quality of the learning services each channel is likely to provide represents one key issue following the appearing of new competition dynamics on the market for higher

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education. ICTs have been shown to enable teachers and learning staff to develop pedagogical innovations (Becker and Watts, 2001; Rivkin et al., 2005; Cukusic et al., 2010). Friedman (1962), Hoxby (1994; 2000), West (1997), Dee (1998) and Jaag (2006) find that educational mobility leads learning organizations to provide higher-leveled quality for their programs whereas lesser optimistic results are obtained by Apple and Romano (1998), Hoyt and Lee (1998), Jepsen (2002) and McMillan (2004). Although the impact of the competition effect on learning quality remains unclear, such contributions have opened alternative paths to study the delivering of educational services. The European Union has taken various initiatives so as to promote e-learning programs and to overcome the so-called ‘learning gaps’. Some countries of its Members States (i.e., France and Spain) have hugely subsidized these programs. Although the impact of government subsidy on performance and social outcomes in the traditional market for higher education has largely been explored by scholars (Schneider, 2010; Yamauchi, 2011), the whole impact of such policies remains unclear in the case of e-learning and deeper analyses have to be carried out.

The aim of this article is to analyze the impact of public subsidies to e-learning providers in order to evaluate the conditions under which the learning gap is bridged. A welfare analysis is introduced to estimate the relevance of such ‘pro e-learning’ public policies in a framework in which traditional learning and e-learning organizations compete by providing educational programs. Our first results show that public subsidies enable the e-learning organization to provide quality-based and pricing strategies that tend to be similar to those of the brick’n mortar organization. Besides, we find that such short-term policies positively impact on the global level of quality which is provided by both providers. Nevertheless, our welfare analysis underlines contrasted results about the relevance of such short-term public policies.

The paper is organized as follows. Section two presents the settings of the model (2). Section three identifies optimal quality-based and pricing strategies when the market for higher education partially and shared (3). Section four analyzes the impact of public subsidies on both competition outcomes and welfare (4). Section five concludes (5).

**The model**

We present a market in which two commercial organizations act as duopolists when providing learning services. We introduce two types of learning providers, namely brick’n mortar campuses and virtual campuses. These traditional organizations are more likely to provide mass-oriented services whereas virtual ones rather provide customized services to better match the needs of the students. They also differ in the nature of the constraints students have to face when adopting one of the two services. We develop a model in a ‘à la’ Hotelling framework in which providers sell differentiated services, the traditional campus (resp. the virtual campus) being located at 0 (resp. 1). Adopters are uniformly distributed on the Hotelling line and their total mass \( N \) is equal to 1. They adopt at most one learning service that is provided by either the traditional campus or the virtual campus. Be \( x \) the location of each product on the line (\( x \in [0;1] \)). Utility functions are defined as follows:
\[ U_q = \begin{cases} 
 r + q_1 - t x - p_1 & \text{if adopts from the traditional campus} \\
 r + q_2 - t (1-x) - p_2 & \text{if adopts from the virtual campus} \\
 0 & \text{if does not adopt} 
\end{cases} \]

\( r \ (r > 0) \) is the gross utility adopters derive from learning services. \( t x \) (resp. \( t (1-x) \)) is the transportation disutility adopters get from adopting the learning program provided by the traditional campus (resp. the virtual campus). Transportation cost \( t \) captures the nature of the constraints which lead students to adopt at most one of the two educational services. Such constraints are related to both technical features (e.g., equipments) and abilities (e.g., competences) to use one of the two types of service. \( x \) (resp. \( 1-x \)) represents the distance between any adopter’s ideal product and that provided by the traditional campus (resp. the virtual campus). \( p_1 \ (p_1 > 0) \) is the price the brick’n mortar provider charges to students when adopting learning programs and \( p_2 \ (p_2 > 0) \) represents the price students have to pay to adopt e-learning programs. \( q_i \ (q_i > 0) \) (resp. \( q_2 \) \( q_2 > 0) \) is the level of quality that is provided by the traditional provider (resp. the virtual provider). We here interpret learning qualities as the efforts carried out by the organizations to hire high-leveled learning staff and to contribute to the provision of high-leveled diploma. We suppose that the content of learning programs provided by the two types of learning organizations is the same. Product differentiation can only be measured by the way educational services are released.

Both learning providers are driven by pure for-profit motives to develop learning programs. We define their objective functions as follows:

\[
\begin{align*}
\pi_1 &= n_1 p_1 - F_1 - \left( q_1^2 / 2 \right) \\
\pi_2 &= n_2 p_2 - n_2 f_2 - \left( q_2^2 / 2 \right)
\end{align*}
\]

\( \pi_1 \) (resp. \( \pi_2 \)) is the profit function of the traditional learning provider (resp. the virtual learning provider) when supplying her programs. We define \( n_1 \ (n_1 \in \mathbb{R}_+; 1] \) as the mass of the students who adopt the programs provided by the brick’n mortar campus whereas \( n_2 \ (n_2 \in \mathbb{R}_+; 1] \) represents the mass of the students who adopt e-learning services. Both organizations face costs whose cost structures are not the same. The traditional learning provider has to support fixed costs \( F_1 \) to carry out the provision of learning services inasmuch as the setting out of brick’n mortar activities requires buildings to be allocated and both teaching and administrative staffs to be paid. Nevertheless, the mass of the students who adopt traditional learning services is not likely to strongly affect the levels of costs of the learning supplier since her activity is based on a mass-consumption scheme. For simplification purposes, we set the level of marginal costs to zero. The provider of e-learning programs has to face marginal costs when carrying out her activity inasmuch as it is based on a customization-based approach. Indeed, since each student is likely to be individually provided a specific learning program and teachers are likely to be more involved in participating to personal trainings, marginal costs \( f_2 \) have to be taking into account when providing learning services. We however suggest that fixed costs are much lower-leveled in the case of commercial e-learning activities. Again, for simplification purposes, we set the
level of fixed costs to zero. As generally assumed, we eventually suppose that both producers face innovation production costs whose shapes are quadratic. Such costs lead both learning providers to design suitable pricing and quality strategies to maximize their profits.

We define the adoption decision process as a four-step game:

- at step $t = 0$, both the traditional organization and the virtual organization decide whether to provide or not to provide learning programs;
- at step $t = 1$, both organizations simultaneously set qualities $q_1$ and $q_2$;
- at step $t = 2$, both organizations set prices $p_1$ and $p_2$;
- at step $t = 3$, students decide to adopt or not to adopt the product released by either the brick’n mortar organization or the virtual campus.

Providers and potential adopters have full and common knowledge of the production outcomes, whether they concern prices and qualities. We suppose that their expectations about the way prices and qualities are defined do apply.

We consider the specific case in which the market is partially served and shared. Indeed, the gross utility adopters derive from getting access to learning services (i.e., $r$) is low so that access to learning services is limited. Therefore, we here focus on the case in which both the cultural and political landscapes of the countries are likely to generate learning gaps.

**Optimal strategies and competition outcomes**

Potential adopters have a low intrinsic valuation for learning services and/or constraints to access educational services that are high so that no-adooption patterns are introduced. The two learning providers consequently act as ‘local’ monopolists.

*Figure 1. Adoption patterns when the market is partially served*

Our analysis holds for specific values for $t$ and $f_2$.

**Assumption 1a.** Transportation costs $t$ are large so that $t > 1$.

Mobility is limited so that there exist adoption constraints which do not allow potential adopters to easily switch from one learning provider to the other one.

**Assumption 1b.** Marginal costs $f_2$ are lower-bounded so that $f_2 < t$.

Assumption 1b is introduced to restrict our analysis to a framework in which marginal costs are low-leveled enough so that the e-learning provider may generate a positive profit from her commercial activity.
In addition, we restrict our analysis to ‘reasonable’ values of $r$, namely $r$.

**Assumption 2.** The intrinsic valuation for learning services $r$ is defined so that

$$0 < f_2 < r < (2t - 1 + f_2)/2.$$  

Assumption 2 presents the conditions for values of $r$ we have to consider for the market for higher education to be partially-served. Let us stress that assumption 2 can be extended and expressed so that $0 < f_2 < r < (2t - 1 + f_2)/2 < 2t - 1 < 2t - 1 + f_2$.

Solving backward, we identify optimal levels for quality, prices and adoption patterns:

$$\begin{align*}
q_1^* &= \frac{r}{2t - 1} \\
q_2^* &= \frac{r - f_2}{2t - 1} \\
p_1^* &= \frac{tr}{2t - 1} \\
p_2^* &= \frac{tr + f_2(t - 1)}{2t - 1} \\
n_1^* &= \frac{r}{2t - 1} \\
n_2^* &= \frac{r - f_2}{2t - 1}
\end{align*}$$

(3a), (3b) and (3c)

Marginal costs $f_2$ only affect the optimal level for quality, price and adoption of the e-learning provider. This results from the partially-served nature of the market. One can easily see from assumptions 1a, 1b and 2 that levels for $p_1^*$, $p_2^*$, $q_1^*$, $q_2^*$, $n_1^*$ and $n_2^*$ are all positive.

**Proposition 1.** Both learning providers reach out positive profits and partially serve the market when potential adopters have a ‘low’ intrinsic valuation for learning services.

**Proof of proposition 1.** One can easily observe from assumption 1a that the traditional learning provider generates a positive profit from her activity provided that fixed costs $F_t$ are sufficiently small. It is straightforward to show that the profit of the e-learning provider is positive at the optimal state for any value of $f_2$ ($0 < f_2 < t$).

Traditional learning and e-learning activities are thus both found to be sustainable inasmuch as their respective optimal levels of profit are always shown to be positive.

**Proposition 2.** When the market for higher education is partially served, the traditional learning organization sets out higher (resp. lower) quality (resp. price) levels for their learning programs than the ones which are provided by the e-learning organization.

**Proof of proposition 2.** From assumptions 1a and 1b, one can easily find that

$$p_1^* - p_2^* = f_2 [(1 - t)/(2t - 2)] < 0 \text{ and } q_1^* - q_2^* = f_2/(2t - 1) > 0.$$ 

The traditional learning provider appears more likely to attract a higher scope of potential adopters. Nevertheless, let us remind that no-adoption patterns apply due to the structure of the market for higher education.

**Learning divides’ and public policies**

No-adoption patterns are represented by the optimal mass of non-adopters, which is

$$n_{\infty}^* = 1 - (n_1^* + n_2^*) = [2(t - r) + f_2 - 1]/(2t - 1)$$

(4)

---

1 If $r$ is shown to be ‘too’ low, global no-adoption patterns are likely to apply on the market for education. We do not analyze such a case in this article.
We obviously find that $n_\varphi^*$ is positive for any value of $t$, $f_2$ and $r$ under assumptions 1a, 1b and 2. Moreover, $n_\varphi^* = n_\varphi^* (f_2, r)$ is unsurprisingly found to be an increasing function for $f_2$ and a decreasing function for $r$. Consequently, one may see appropriate to set up public policies which tend to make levels for marginal costs $f_2$ lower and levels for valuation for educational programs $r$ higher. Although such policies are both shown to reduce the gap for learning access, they differ in the amount of time which is needed for positive outcomes to be reached out. The setting up of information campaigns which are led to develop the intrinsic valuation for educational programs are somehow likely to be efficient in the long run. We therefore restrict our public analysis to the example of public subsidies to the e-learning provider (e.g., public support to purchase IT terminals or computers). The total amount allocated by the public authorities thus depends on the number of adopters of e-learning services. Marginal public subsidy is noted $s$ ($s > 0$) and is defined so that

$$\frac{\partial (f_2 - s)}{\partial s} < 0$$

$s$ refers to the amount which is allocated by public players to the virtual campus to enhance the providing of e-learning programs and to reduce the so-called ‘learning divides’. The impact of such a type of public policies can be measured by analyzing the effect that marginal costs $f_2$ have on optimal price and quality levels, as well as on profits. The level of marginal subsidy $s$ is of course assumed to be lower to that of marginal cost $f_2$ (i.e., $s \leq f_2$).

We first analyze the effect of public subsidies on the optimal levels we previously pointed out (4.1.). A welfare study is then carried out to see to what extent public subsidies may be socially-improving (4.2).

**Public subsidies and competition outcomes**

We define $Q^* = q_1^* + q_2^*$ as the quality effort which is jointly provided by the two learning organizations. As we have supposed that educational programs are homogeneous products, one can interpret $Q^*$ as a – yet preliminary – measure of social outcome. We show that

$$Q^* = (2r - f_2) / (2t - 1)$$

The effect of public subsidies on the optimal quality and pricing strategies of both learning providers, as well as that on their profits, are expressed as follows:

$$\begin{align*}
\frac{\partial q_1^*}{\partial f_2} &= 0 \\
\frac{\partial q_2^*}{\partial f_2} &= - \frac{1}{2t - 1} < 0 \\
\frac{\partial p_1^*}{\partial f_2} &= 0 \\
\frac{\partial p_2^*}{\partial f_2} &= \frac{t - 1}{2t - 1} > 0 \\
\frac{\partial \pi_1^*}{\partial f_2} &= 0 \\
\frac{\partial \pi_2^*}{\partial f_2} &= - \frac{r - f_2}{2t - 1} < 0
\end{align*}$$

(7a), (7b) and (7c)

**Lemma 1.** When potential adopters have a low intrinsic valuation for learning services, public subsidies have a positive (resp. negative) impact on the quality (resp. pricing) strategy of the e-learning provider whereas they have no effect on that of the brick’n mortar learning provider. Public subsidies enable the e-learning provider to increase her profits whereas such subsidies do not affect that of the traditional organization.
The effect of public subsidies on the global quality effort is given by
\[
\frac{\partial Q^*}{\partial f_2} = -\frac{1}{2t-1} < 0
\]  
(8)

**Lemma 2.** Public subsidies improve the level of global quality effort.

Lemma 2 provides first social insights about the providing of public subsidies. It somehow stresses that the providing of marginal subsidies does negatively impact on the joint level of quality delivered by both types of learning organizations. As such, exclusive public support to the e-learning provider is likely to lead both providers to offer higher-leveled services.

**Public subsidies and welfare**

We discuss welfare outcomes to identify the impact of public subsidies on welfare. Aggregate adopters surplus is equal to \( AS^* = AS^*_1 + AS^*_2 \). The surpluses of both learning organizations are equal to the sum of the profit of the brick’n mortar provider \( \pi_1^* \) and that of the e-learning provider \( \pi_2^* \). Welfare is defined by the sum of both adopters and providers’ surpluses, as well as of the level of total public subsidy (i.e., public deficit) \( S = S(n_2^*) = s \times n_2^* \).

Indeed, we here consider that public authorities take rational decisions when designing public policies inasmuch as subsidies are allocated if and only if two conditions are met. These conditions state that (i) the impact of public subsidies on both the producers’ and consumers’ surpluses is shown to be positive, and (ii) public deficit \( S \) is overcome by the increase of the level of total surplus which results from the setting up of such a public policy.

When public subsidies are provided to the e-learning organization, its marginal cost decreases to a level that is equal to \( f_2 - s \). We define the optimal level of welfare \( W^* \) as
\[
W^* = \int_{x=0}^{x^*} (r + q_1^* - tx - p_1^*) \, dx + \int_{x=x^*_2}^{x^*} (r + q_2^* - t(1-x) - p_2^*) \, dx + \pi_1^* + \pi_2^* - s(1-x_2^*)
\]

We find that
\[
W^* = -F_1 + \frac{1}{2} \left( \frac{1}{2t-1} \right)^2 \left[ 3r - 2[(r - (f_2 - s))^2] - (s) \left( \frac{r - (f_2 - s)}{2t-1} \right) \right]
\]  
(9)

One can find that levels for welfare are positive, provided that fixed costs \( F_1 \) are sufficiently small (see proof in appendix A). The impact of public subsidies on welfare is expressed as follows:
\[
\frac{\partial W^*}{\partial s} = \left( \frac{1}{2t-1} \right)^2 \left[ (r - f_2)(t) + (s)(1-t) \right]
\]  
(10)

**Proposition 3.** When the market is partially-served, public subsidies are beneficial to welfare for ‘low’ values for marginal public subsidy \( s \), and detrimental otherwise.
Proof of proposition 3. We find that \( \partial W^*/\partial s > 0 \) for values for \( s \) that are set out by public authorities so that \( s < (t/t-1)(r - f_2) \) and that \( \partial W^*/\partial s < 0 \) for values for \( s \) that are defined so that \( s > (t/t-1)(r - f_2) \).

We find that public subsidies may be relevant to reach out welfare-improving states provided that the level of marginal subsidy does not exceed a maximal value. Upper-bounded public support is likely to be welfare-improving for areas in which access to learning programs is limited.

The optimal decision-making of public authorities can be seen as the result of a welfare-maximizing program of which \( s^* = (t/t-1)(r - f_2) \) is the result.\(^2\) Let us note that such an optimal level for \( s \) is higher (resp. lower) than marginal cost \( f_2 \) if \( f_2 > (t/2t-1)r \) (resp. \( f_2 < (t/2t-1)r \)). When public authorities define levels for public subsidies so that \( 0 \leq s \leq f_2 \), \( s^* \) is set out at a level which is equal to \( (t/t-1)(r - f_2) \) if values for \( f_2 \) and \( r \) are so that \( f_2 \geq (t/(2t-1))r \), and \( s^* = f_2 \) if values for \( f_2 \) and \( r \) are so that \( f_2 \leq (t/(2t-1))r \).

It is needed to see if the setting out of public subsidies whose marginal amount is \( s^* = (t/t-1)(r - f_2) \) or \( s^* = f_2 \) eventually leads to the disappearing of ‘learning divides’ or if they only attenuate them.

Proposition 4. Although public subsidies eventually foster the adoption of educational services, they may not lead to the disappearing of the ‘learning gap’.

Proof of proposition 4. See appendix B.

Proposition 4 suggests that short-term oriented public policies may not be sufficient for ‘learning divides’ to disappear. Indeed, one can see that a – yet narrower – ‘learning gap’ may apply, depending on the values of both \( r \) and \( f_2 \). Short-termed public support to e-learning organizations cannot thus be seen as a great ‘equalizer’ which would lead to the developing of universal access to educational services.

Conclusion

The aim of this article has been to analyze to what extent the setting out of public subsiding may affect the bridging of ‘learning divides’ while stimulating the providing of high levels of quality for educational programs. Following the increasing popularity of e-learning services, we have built a model in which a traditional learning organization competes with a virtual one while having to deal with differing cost structures so as to measure the relevancy of ‘pro’ e-learning support policies from a social point of view. One of our major research directions has been to identify how public policies should be designed to eventually facilitating access to learning services while not levelling down overall educational quality.

\(^2\) As \( \partial W^*/\partial s' = (1/2t-1)(1-t) < 0 \), the value \( s' \) for \( s \) which is defined so that \( \partial W^*/\partial s = 0 \) depicts a maximum state.
The impact of public subsidies has been analyzed, notably to measure to what extent they are likely to reduce the ‘learning divides’ one may observe in areas in which intrinsic valuation for educational services is low. Our welfare analysis has evidenced that the providing of public subsidies that level is too high is detrimental from a social viewpoint. It has also revealed that – under some specific circumstances – public subsidies may not be efficient enough to fully-cover the ‘learning gap’. This last result can lead to several interpretations. A somehow pessimistic interpretation is that it may not be possible to provide universal access to educational services through the designing of short-termed public policies. Although the setting out of public policies is found to ‘equalize’ the competitive game between the two types of learning organizations from both a pricing and quality-based point of view, detrimental effects to welfare levels may appear. We suggest that the mixing of short-term (e.g., public subsidies) and long-term (e.g., educational campaigns) public policies may be appropriate to lead to the disappearing of ‘learning divides’. Yet, such a practice is likely to generate coordination failures and seems to provide efficient results only when political contexts are found to be stable. Nevertheless, we would like to address another – rather optimistic – interpretation. Indeed, our findings pinpoint that it may be possible for public authorities to widen access to educational services while preserving their ‘cultural’ learning-provision model. Put it differently, they may allow a larger scope of people to get access to educational services while avoiding the emerging of a mass-consumption model for such higher services.

The analysis of adoption patterns for educational services raises major concerns from both market-based and public viewpoints. We strongly believe that both empirical and model-based further contributions will lead to the better understanding of adoption dynamics within the market for higher education and to the shaping of suitable public policies.

Appendixes

Appendix A. Positive welfare levels

The level of welfare that applies when the market for higher education is partially served is given by

\[ W^* = -F_1 + \frac{1}{2} \left[ \frac{1}{2t-1} \right]^2 \left[ (2t-1) \left[ 2r(t-f_2) + (f_2 - s)(f_2 + s) \right] + \left[ t(c^2 + (t-f_2 + s)^2) \right] \right] \]

From assumptions 1a and 2, it is straightforward to find that \( W^* > 0 \) for sufficiently small values of \( F_1 \).

Appendix B. Proof of Proposition 2

As we previously pointed out, two cases have to be taken into account: (i) \( (s^* = f_2; f_2 \leq (t-c/2t-1)) \) and (ii) \( (s^* = (t-1)(t-f_2); f_2 \geq (t-c/2t-1)) \).

Let us first focus on the \( (s^* = f_2; f_2 \leq (t-c/2t-1)) \) case. We show that \( n_{\phi} > 0 \) if the following conditions are simultaneously met:
\[
\begin{align*}
& \left\{ r < (1/2)(2t-1) \\
& f_2 \leq \left( t r / 2t - 1 \right) \\
& 0 < f_2 < r < (1/2)(2t-1) < 2t-1 < 2t-1 + f_2 \\
& 0 < f_2 < r < (1/2)(2t-1 + f_2) < 2t-1 < 2t-1 + f_2
\end{align*}
\]

Such conditions can be summarized as follows:

\[
0 < f_2 < \left( (2t-1) f_2 \right) / t < r < (1/2)(2t-1)(2t-1 + f_2) < 2t-1 < 2t-1 + f_2
\]

In a similar fashion, we show that \( n^{**} > 0 \) if the following conditions are simultaneously met:

\[
\begin{align*}
& \left\{ r > (1/2)(2t-1) \\
& f_2 \leq \left( t r / 2t - 1 \right) \\
& 0 < f_2 < r < (1/2)(2t-1 + f_2) < 2t-1 < 2t-1 + f_2 \\
& 0 < \max \left\{ (1/2)(2t-1); f_2 \right\} < r < (1/2)(2t-1 + f_2) < 2t-1 < 2t-1 + f_2
\end{align*}
\]

Such conditions can be summarized as follows:

\[
0 < \max \left\{ (1/2)(2t-1); f_2 \right\} \lessgtr r < (1/2)(2t-1 + f_2) < 2t-1 < 2t-1 + f_2
\]

We thus identify possible conditions under which public subsidies are found to fully bridge the ‘learning gap’ and other conditions under which such policies are not found to be efficient enough to fully cover this gap.

Similar results are found when the \( s^* = (t - t - 1)(r - f_2); f_2 \geq \left( t r / 2t - 1 \right) \) case is dealt.

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