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Multiplicity of Ecolabels, Environment and Competition

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Abstract

The purpose of this article is to develop a vertical differentiation model in order to understand the competition and environmental impacts of the multiplication of ecological labels within a given economic sector. We focus on the studing of the informational contents of these ecological labels and if they reflect or not the true offered environmental qualities. Two situations are considered. The first one, is a benchmark case, where we assume that information is complete (the consumers know the true qualities produced by the firms). The second, common situation, corresponds to a situation of incomplete information where the consumers do not know the true environmental qualities presented but use the price signal in order to approach environmental qualities. Our results show that: in the case of complete information the introduction of a second ecological label in a market improves the environmental qualities offered by the two firms. In the case of incomplete information, the introduction of a second label led to an increase in prices practiced by the two firms and a reduction in environmental qualities of both firms. This situation needs specific regulation and that information must be revealed by a benevolent social planer.

Nomenclature

i = H, L: represents the firm that offers the high environmental (H) or the low (L) environmental quality.

 L_H : high environmental quality label.

 L_L : low environmental quality label.

 π^i : firm's i profit

 q^i : quality of good i.

 p^i : price of good i.

 $F(q^i)$: Fixed cost to obtain the label L_i

 c_i : marginal production cost of the quality of the product i.

 θ : quality taste parameter.

 $U(q^i, p^i; \theta)$: utility of a consumer with type θ .

 $\tilde{\theta}$: marginal consumer who is indifferent between high or low environmental quality.

 D^i : demand function for good i.

 p^{HC} : price of the product H at the equilibrium of the complete information case.

 p^{LC} .: price of the product L at the equilibrium of the complete information case.

 D^{LC} : demand function of L at the equilibrium of the complete information case.

 D^{HC} .: demand function of L at the equilibrium of the complete information case.

 π^{HC} : Firm's H profit at the equilibrium of the complete information case.

 π^{LC} : Firm's L profit at the equilibrium of the complete information case.

 $\mu(p^H, p^L)$: is the posterior believes of the consumers that the firm which plays p^H offers the high environmental good whereas the other which plays p^L is a low environmental quality one.

 π^{HI} : Firm's H profit at the equilibrium of the incomplete information case.

 π^{LI} : Firm's L profit at the equilibrium of incomplete information case.

 $\pi^H(p^H, p^L, \mu(p^H, p^L))$: firm's H profit when H fixes its price at p^H , and L offers a product at price p^L .

 $\rho(\mu)$: quality that consumers expect to obtain.

 $D^{I}(p,p)$: demand function when prices give no information concerning quality.

1 Introduction

Eco-labels are nowadays widespread worldwide and participate in changing the patterns of consumption in most EOCD Countries (EOCD, 1997). They exist for a variety of products such as batteries, textile, paper, wood products, fisheries and tourism... Their primarily goal is to solve the adverse selection problem and to give the consumers the information they need in order to make the right choice in the market. This policy is successful for a large variety of eco-labels (Teisl et al. 2002, Bjorner et al. 2002).

However, for a decade we have observed the multiplicity of eco-labels in a single market and for the same category of products. For example, in the German electricity market three alternative labels co-exist (Grüner Storm Label, Energie Vision, OK Power...) (Truffer et al. 2001). This is also the case for wood products where two eco-labels are competing (FSC and PEFC). Countries like Sweden have three official competing eco-labels: the flower Pan European eco-label, the Swedish falcon and the Nordic Swan. The International Standardization Organisation (ISO) is aware of the multiplicity of eco-labelling and distinguishes between three different types of eco-labels. Eco-labels type I are those based on a voluntary multi-criteria product life cycle assessment of environmental effects with verification by a third party. Producers, importers, and retailers of products and services base eco-labels type II on self-environmental claims. Eco-labels type III provide quantified product information according to pre-set indices similar to consumer information on product packaging. The co-existing of such eco-labels in a single market may lead to a confused situation where asymmetric information remains unsolved. Consumers are not able to distinguish between the different eco-labels and their informational contents.

Agriculture and Food products are also subject to such dynamics and a great variety of eco-labels exist (Nilson, 2004; Van Amstel et al. 2006). For example, five competing eco-labels exist for arable farming in Europe (EurepGAP¹, Demeter², EKO³, MK⁴ and ESP⁵). Farmers and firms are using them differently depending on competition setting and their initial production conditions. Food scarcity and diseases may lead firms to communicate their production methods and contents more through eco-labels (Nilson, 2004).

These dynamics raise several questions both at theoretical and economic policy view.

From a theoretical point of view two questions at least need to be answered. First, what are the environmental values of the additional eco-labels? Do they improve or harm the environment? How do consumers solve their informational dilemma when they take into account the different eco-labels? Second, how the additional labels impacts the competition in the market. From an empirical view, one can ask what could be the optimal policies vis-à-vis these new voluntary approaches if they do not reach their environmental goals? Do we need regulation of voluntary approaches?

Starting from these observations, the aim of our paper is to challenge this point and to develop a theoretical model capturing the environmental and competition effects of the multiplicity of eco-labels in a given industry. Little attention was given to this specific point in the economic literature. Our main result shows that in the perfect information case the additional eco-label benefits environmental qualities and an increase in the environmental performance is observed. However in the asymmetric information

¹EurepGAP: is a label for the certification of agricultural products around the globe. The main goal of this label is to reduce detrimental environmental impacts of farming operations, optimize the use of inputs and to ensure a responsible approach to worker health and safety (www.eurepgap.org).

²Demeter: is an eco-label for the bio dynamic agriculture. It aims to take care to the earth; to regenerate the earth, to better integrate animals' agriculture and the cultivation of the soil in a same area. (www.bio-dynamie.org)

³EKO: The "SKAL International Foundation" (Netherlands) monitors and examines the biologic and organic production of the entire world. This foundation gives the eco-label EKO.

⁴MK: is an eco-label, which is based on life cycle analysis.

⁵ESP: "Recognized Regional Product"

case firms will practice "Greenwashing"⁶. The multiplicity of eco-labels could lead to a price increase in the market and a reduction in environmental qualities.

The present paper is structured as follows. Section two presents the literature review and shows the need for a specific model of the multiplicity of eco-labels. Section three develops a general model. Section four deals with the benchmark case (the model with complete information case). Section five develops the incomplete information model. We compare the case of complete and incomplete information. Section six concludes.

2 Literature review

The existing literature on ecological labelling has never developed a theoretical model of this specific case. However, several papers have considered the borders of this issue.

The first set of articles focused on the information value of the eco-labels.

Ibanez and Stenger (2000) studied the information impacts of the eco-labels through a vertical differentiation model in three different information situations: complete, imperfect and partial. They try to understand if information on food safety is consistent with a higher level of environmental quality of the agriculture. They showed on one hand, that labelling could be efficient from an environmental point of view: it depends on the proportion of high environmental quality products in the market. On the other hand, they showed that the labelling policy can decrease consumer' surplus. However, a possible extension of their work is the informational content of two eco-labels. Consumers cannot always infer correctly high attributes from labelled products. Producers can be willing to cheat and claim false value of their environmental qualities through labels.

Bougherara, Grolleau and Tjiébaut (2005) have studied the general impact of the consumption of two different goods on the environment when labelling schemes are introduced. They have shown that the net effect on the environment is worse than without labelling schemes. Firstly, this article differs from ours in the analytical framework. Indeed, the authors have developed the concept of the environmental elasticity of demand in order to study this specific case. Secondly, the authors have assumed that consumers' preferences are identical. Thirdly, the authors have not studied the problem of the asymmetric information linked to the presence of two eco-labels. So what happens when there are two differentiated products differing by their eco-label's type and two kinds of consumers (concerned and unconcerned)?

Heyes and Maxwell (2004), compare the environmental implications and social welfare of the "World Environmental Organization" (WEO) labelling and the "Non-Governmental" NGO labelling. They analyze the interaction between these two approaches when WEO is subject to pressure from firms. They have shown that if each of these approaches do not depend from the other then the presence of the NGO induces a resistance toward the WEO and this case may reduce social welfare. If these two approaches run in parallel then it may reduce the resistance of the producers towards the WEO labelling which increases welfare. However, they have not studied the impact of the co-existence of both eco-labels on the environment and on the competition.

Mason (2006) studies a market under asymmetric information where technologies are fixed and output are not in which firms decide to choose an eco-label or not. Mason has shown that the social welfare can increase or decrease when eco-labelling option is introduced. This depends upon parameter values and the decision to choose the production technology. However, he has not studied the problem of the asymmetric information due to the introduction of a second eco-label in the market.

Our paper is extending this line of research.

⁶The Green washing means that producers sell a product that seems to be more environmentally friendly than it really. Boulding and Kirmani (1993), have demonstrated that the consumers do not perceive warranties as a credible signal of product quality. Cason and Ganghadaran (2002) have shown that greenwashing is due some passivity of the regulation of the environmental claims. All these authors have not studied the effect of greenwashing due to the presence of two eco-labels in the market.

The second set of articles studied the strategic interaction in the different stages of the eco-labelling program. They study the effect of an eco-label on competition between firms.

Amacher et al. (2004) have studied the conditions of eco-labelling through a vertical differentiation model with two firms using eco-labels. They showed that if firms have the same fixed cost then they will always invest. If one firm is more efficient then the other in investments, then the other firm will invest or not depending on the unit investment cost. They have shown that in this case quality dispersion remains unchanged but generally firms are incited to differentiate more in order to relax price competition. They have shown that eco-labelling increases the environmental qualities that are too low. However, the existence of a second eco-label may impact the price competition and the environmental quality competition. Firms can therefore have an opportunistic behaviour. Our paper contributes to this line of research and shows that firms which compete with two different eco-labels are incited to cheat.

Ben Youssef and Lahmandi-Ayed (2008) show that firms may lobby at the criteria definition stage in order to modify the competition setting. They found that several issues are possible. Firms, which are not satisfied at the criteria stage, may try to develop their own label. The authors mention this issue but they have not give any theoretical model.

Brien and Teisl (2004) study the environmental certification and eco-labelling for forest products. Their results show that changes in eco-labelling policies affect the consumer's willingness to pay. As an extension of their work one may expect that the presence of a second eco-label may impact the consumer's willingness to pay.

Arora and Gangopadhay (1994) has developed a model of overcompliance in a two stage game. In this game, consumers can value the environmental quality. They have shown that under these assumptions, the market is segmented and that the firm with the lower cost always overcomplies. However, these authors do not study the case where the consumers cannot value the environmental quality due to the presence of two eco-labels in the market.

Nilson et al. (2004) analyze the credibility of the increased number of eco-labels in the food industry. They conclude that a great number of consumers fail to find the real information due to the presence of several eco-labels. We can extend this work by studying the willingness to cheat by the producers who are using these eco-labels. Does the presence of several eco-labels in the industry incite the different producers to cheat and to produce a lower environmental quality that it is claimed through the eco-label? Our article partially gives some answers this issue.

More recently and particularly in matter of Food and agricultural products, Van Amstel et al. (2006) compare the informational content of five eco-labels in the food industry. They conclude that the eco-labels fail to survey the message of the environmental impacts to consumers. Three complementary issues are developed in our article. First, Amstel et al. (2006) gives empirical findings without modelling the situation. Our article gives the analytical arguments for their findings. Secondly, our work analyzes theoretically the impacts of the multiplicity of labels on the global environmental quality. Thirdly, when the eco-labels fail to survey the message of the environmental impacts to consumers, are the producers incited to cheat? We consider this issue also.

3 The structure of the model

In this section, we introduce the main assumptions of a vertical differentiation model where the environmental quality is the element of discrimination. We begin by analyzing firms' behaviour after what we present the consumers' behavior.

3.1 Firms' behaviour

We suppose here that there are two ecolabels, each firm adopts one of them.

For simplicity, we suppose that two firms are sharing the market and are producing the same good. Firm H offers the high environmental quality (q^H) good at price p^H with an eco-label (L_H) and a firm L is producing a low environmental quality (q^L) at price p^L good with an eco-label $(L_L)^7$.

We assume that the two labels L_H and L_L are different. For illustration, suppose that L_H is the official label (type I), and L_L is the label of type II e.g. it is developed by a group of firms. We can suppose for example that the two firms adopt the ecolabels EKO and MK.

We assume that $q^H > q^L$ and that $q^i \in [0, \infty]$ where i = H, L.

We assume that firm H investment \cos^{8} is $F(q^{H}) = aq^{H^{2}}$ where a > 0 to obtain the label L_{H} . Firm L has also an investment cost $F(q^{L}) = \gamma aq^{L^{2}}$ where $\gamma > 0$ to obtain the eco-label L_{L} . This fixed costs can be interpreted as the necessary cost to adopt the label and to maintain it.

 γ reflects the efficiency of the firm L in term of investment in quality. If $\gamma \leq 1$ then L is efficient in investment and invests in the impovement of environmental quality whereas if $\gamma > 1$ then L is not efficient in investment and invests weakly in environmental quality. Finally, we assume that L does not invest too much in environmental quality e.g. $\gamma > 1$ since it provides the low environmental quality.

In order to focus on investment decisions in quality, we assume that the marginal cost of quality is constant and for simplicity let this cost normatized to zero: $c_H = c_L = 0$.

3.2 Consumers' behaviour

We assume that consumers have a weak ecological awareness in the sense that they all prefer the most environment friendly product if they have the choice between several "environmental qualities" sold at the same price. Thus we model this situation by using an appropriate vertical differentiation model. In this model, all consumers buy almost one unit of the product. The consumers are identified by a taste parameter θ for the environmental quality. We assume that θ is uniformly distributed on [0,1]. The consumers pay p^i for environmental quality q^i (i=H,L).

We consider, as Mussa and Rosen (1978) that indirect utility of a consumer of type θ buying from firm i (i = H, L) a unit good of environmental quality q^i at price p^i is given by:

$$U(q^i, p^i; \theta) = \theta q^i - p^i$$
 where $i = H, L$.

Let θ the taste parameter which, represents the marginal consumer who is indifferent between high or low environmental quality.

⁷There are two kinds of labels: endogenous and exogenous (Kirchhoff 2000).

Endogenous Labels are created by the firm itself (Label type II or III). In this case one can not monitor firm's claim and moral hazard arises. Firms can claim producing a certain level of quality through its eco-label but offers really another level of environmental quality. In this case, firm can uses 'greenwashing".

Exogenous or third party Labeling is created by a third party and independent labeling authority (label type I). Under these systems the third party authority defines the criteria of the eco-label for different products and the firm can apply for a license to use the eco-label (eco-label Blue Angel for example) if it gives the evidence that its product is conform with the eco-label. In the two types of eco-labels firms can cheat but if its labels is a third party labeling authority than its licence to use the label is withdrawn. A firm can then select a type of eco-label but offers a different environmental quality. The exogenous label is also more credible for consumers than the endogenous label.

⁸We assume also that the two firms have an investment cost $F(q^i)$ where i = H, L.

Like Ronnen (1991), we suppose that the investment costs $F(q^i)$ and that the marginal investment costs $F'(q^i)$ where i = H, L are increasing with quality for all $q^i \in [0, \infty]$ and i = H, L. We assume also that $F'''(q^i) \ge 0$.

The investment costs has a quadratic form. This particular form is necessary for the existence and the uniqueness of the equilibrium (Ronnen (1991).

We have $\tilde{\theta} = \frac{p^H - p^L}{q^H - q^L}$. We suppose that the market is not entirely covered. The consumers whose $\theta \in \left[\frac{p^L}{q^L}, \tilde{\theta}\right]$ buy the good of environmental quality q^L . The consumers whose $\theta \in \left[\tilde{\theta}, 1\right]$ buy the good of environmental quality q^H . The consumers whose $\theta \in [0, p^L]$ don't buy the good.

Thus the demands functions for low and high environmental quality respectively are given by:

$$D^L(p^L, p^H, q^L, q^H) = \tilde{\theta} - \frac{p^L}{q^L} = \frac{p^H - p^L}{q^H - q^L} - \frac{p^L}{q^L} \qquad (2.1)$$

$$D^H(p^H, p^L, q^H, q^L) = 1 - \tilde{\theta} = 1 - \frac{p^H - p^L}{q^H - q^L} \qquad (2.2)$$
 In the next section, we will develop situation where the information is complete.

$$D^{H}(p^{H}, p^{L}, q^{H}, q^{L}) = 1 - \tilde{\theta} = 1 - \frac{p^{H} - p^{L}}{q^{H} - q^{L}}$$
 (2.2)

The complete information case 4

In this section, we develop a model where all consumers are able to know perfectly the environmental qualities q^L and q^H . The game is like the followings: in the first step, firms H and L with eco-labels L_H and L_L compete in environmental qualities. In the second stage, the two firms compete in prices. In the third stage, the consumer choose to buy the good H or L or not to buy. The aim of this section is to understand the effective impacts of two differents eco-labels on the environmental qualities when the consumers know perfectly the quality levels.

4.1 Price choice: the equilibrium prices

The profit of firm H is given by:

$$\pi^{H}(p^{H}, p^{L}, q^{H}, q^{L}) = p^{H}D^{H}(p^{H}, p^{L}, q^{H}, q^{L}) - aq^{H^{2}}$$
The profit of form L is given by:
$$(3.1)$$

The profit of firm L is given by:

$$\pi^{L}(p^{L}, p^{H}, q^{L}, q^{H}) = p^{L}D^{L}(p^{L}, p^{H}, q^{L}, q^{H}) - \gamma aq^{L^{2}}$$
(3.2)

We resolve the game by backward induction. We start by resolving the second stage of the game in which firms choose their prices in function of their preceding choice of qualities; we obtain the following equilibrium prices⁹:

$$p^{HC} = \frac{2q^H(q^H - q^L)}{4q^H - q^L}$$

$$p^{LC} = \frac{q^L(q^H - q^L)}{4q^H - q^L}$$
(3.3)

$$p^{LC} = \frac{q^L(q^H - q^L)}{4q^H - q^L} \tag{3.4}$$

Through the results (3.3) and (3.4), we found that $p^{HC} > p^{LC}$. In fact, $p^{LC} = \frac{1}{2} \frac{q^L}{q^H} p^{HC}$. This implies that the low quality firm fixes a price that is below the half of the price imposed by the firm H. We note at the same time that the more q^L is closer to q^H the more our model tends to be a Bertrand model where prices are equal to marginal costs normalized here to zero (the Bertrand paradox).

The demand functions at the equilibrium of the complete information case are:

$$D^{HC} = \frac{2q^H}{4q^H - q^L} \tag{3.5}$$

$$D^{LC} = \frac{q^H}{4a^H - a^L} \tag{3.6}$$

 $D^{LC} = \frac{q^H}{4q^H - q^L}$ (3.6) The quantity sold by firm L is equal to the half of the quantity sold by the high environmental quality firm.

Finally, the equilibrium profits are:
$$\pi^{HC}(q^{H}, q^{L}) = \frac{4q^{H^{2}}(q^{H} - q^{L})}{(4q^{H} - q^{L})^{2}} - aq^{H^{2}}$$
 (3.7)

⁹The proofs are given in the annex A1.

$$\pi^{LC}(q^L, q^H) = \frac{q^L q^H (q^H - q^L)}{(4q^H - q^L)^2} - \gamma a q^{L^2}$$
 (3.8)

 $\pi^{LC}(q^L,q^H) = \frac{q^L q^H (q^H - q^L)}{(4q^H - q^L)^2} - \gamma a q^{L^2} \qquad (3.8)$ We note $R^{HC} = \frac{4q^{H^2} (q^H - q^L)}{(4q^H - q^L)^2}$ and $R^{LC} = \frac{q^L q^H (q^H - q^L)}{(4q^H - q^L)^2}$ the revenues of the high and the low environmental qualities respectively.

We have
$$\frac{\partial R^{HC}}{\partial q^L} = -4 \frac{q^{H^2 (4q^H - q^L)} (2q^H - q^L)}{(4q^H - q^L)^4} < 0$$
 (3.9)

$$\frac{\partial R^{LC}}{\partial q^H} = \frac{q^L (4q^H - q^L) \left(6q^H - 2q^L + 8q^{H^3} + 8q^{H^2} q^{L^2} \right)}{(4q^H - q^L)^4} > 0 \tag{3.10}$$

- (3.9) implies that a reduction in q^L increases firm's H revenue.
- (3.10) implies that an increase in q^H increases firm's L revenue.

These results mean that the more the products are differentiated in term of environmental qualities the more competition in prices is relaxed and the more the firms increase their revenues as Amacher et al. (2004). We can conclude here that despite the presence of the two eco-labels in the market, firms must maximise the differentiation between their environmental qualities to a maximum. The presence of a second ecolabel improves the hole performance of the sector in matter of environment. From this point, one may conclude that the best solution for firm H is to abandon its ecolabel and to choose the minimum environmental quality. This result is also find in different analysis like Ben Youssef and Lahmandi-Ayed (2008).

In the next section, we will study the equilibrium environmental qualities in the case of complete information.

4.2 The existence and the uniqueness of the equilibrium in quality

In the first stage of the game firms compete simultaneously in quality. Each firm fixes its quality taking the quality of the other firm as given.

The profits of firm H and L are respectively:

$$\pi^{HC}(q^H, q^L) = \frac{4q^{H^2}(q^H - q^L)}{(4q^H - q^L)^2} - aq^{H^2}$$

$$\pi^{LC}(q^L, q^H) = \frac{q^L q^H (q^H - q^L)}{(4q^H - q^L)^2} - \gamma aq^{L^2}$$
(3.8)

Equations (3.7) and (3.8) are equivalent to:

$$\pi^{HC}(q^H, q^L) = R^{HC} - aq^{H^2}$$

$$\pi^{LC}(q^L, q^H) = R^{LC} - \gamma aq^{L^2}$$
(3.11)
(3.12)

$$\pi^{LC}(q^L, q^H) = R^{LC} - \gamma a q^{L^2}$$
 (3.12)

Nash equilibrium in environmental qualities must satisfy the following first order conditions:

$$\frac{\partial \pi^{HC}}{\partial a^H} = \frac{\partial R^{HC}}{\partial q^H} - 2aq^H = 0 \tag{3.13}$$

$$\frac{\partial \pi^{HC}}{\partial q^H} = \frac{\partial R^{HC}}{\partial q^H} - 2aq^H = 0$$

$$\frac{\partial \pi^{LC}}{\partial q^L} = \frac{\partial R^{LC}}{\partial q^L} - 2\gamma aq^L = 0$$
(3.13)

In this case, we notice that optimal qualities are obtained by equalizing marginal cost of investment in environmental quality and marginal revenue.

The second order and stability conditions are globally satisfied 10 .

There is a unique equilibrium since $\frac{\partial R^{HC}}{\partial q^H}$ and $\frac{\partial R^{L\tilde{C}}}{\partial q^L}$ are positives¹¹. The question is now to know if there is a unique equilibrium in quality.

¹⁰the proofs are given in the Annex A2.

¹¹The proofs are given in the Annex A2.

Thus if quality q^H is an equilibrium it must satisfy $\frac{\partial R^{HC}}{\partial q^H} = 2aq^H$.

At the same time, if quality q^L is an equilibrium it must satisfy $\frac{\partial R^{LC}}{\partial q^L} = 2\gamma a q^L$.

The slopes of the reaction functions in quality space are given by:

$$\begin{split} \frac{\partial q^{H}}{\partial q^{L}} &= -\frac{\frac{\partial^{2}R^{HC}}{\partial q^{H}\partial q^{L}}}{\frac{\partial^{2}\pi^{HC}}{\partial q^{H^{2}}}} > 0\\ \frac{\partial q^{L}}{\partial q^{H}} &= -\frac{\frac{\partial^{2}R^{HC}}{\partial q^{L}\partial q^{H}}}{\frac{\partial^{2}\pi^{LC}}{\partial q^{L^{2}}}} > 0 \end{split}$$

The reaction functions have positive slopes, which implies that the products are strategic complements. Indeed, since an increase in q^L reduces the difference $q^H - q^L$ and intensifies price competition, firm H has an incentive to increase its environmental quality in order to relax price competition. Taking into account that we are in the complete information case, firm L must invest in environmental quality. Since $\gamma > 1$, the more γ tends to 1, and the more firm L will be efficient and the more quality q^L will be high and consequently the more firm H will invest in quality in order to increase its quality and then to relax price competition.

PROPOSITION 1: In the complete information case, the introduction of a second ecolabel in the market increases the environmental quality of the two firms and subsequently the global environmental quality.

This result is very important since it shows that when there are several eco-labels in the market, and if the firms offer the product with the real environmental quality proposed by each eco-label than the global environmental quality will be raised. If firms do not pratice opportunistic behavior (if each firm offers its product with the environmental quality announced in the eco-label) then the presence of several eco-labels will increase the global environmental quality.

In the next section, we will develop the incomplete information case. We will assume that consumers know that there are two eco-labels in the market but they don't know which firm produces the high environmental quality and which one offers the low environmental quality.

5 The incomplete information case

5.1 Definition: the model and the assumptions

By incomplete information or asymmetric information, we mean the situation where the consumers are not able to observe the true qualities. When consumers face several eco-labels, the information cost is prohibitive and they have no ability to associate an environmental quality to a special ecolabel. We suppose here that there is no reputation effects. The presence of two or several different eco-labels in the market leads the consumers to ask themselves about the value of the ecological information of these labels. Each producer claims offering a high environmental quality through its own eco-label. But who tells the truth? Consumers are uncertain about the truthfulness of these eco-labels. They will not know which label really increases the environmental quality. The most important question here is what kind

of eco-labels gives the higher value? The cost of verifying the environmental performance of the firm is prohibitive for consumers. For example in the food industry, the eco-labels' criteria are different depending on which label it is. Some stresses on the presence of pesticide residue, others on the additives and preservatives but they all pretend to be environmentally friendly. How will do consumers to know that each firm is not "greenwashing" (Van Amstel et al. (2006), Kirchhoff (2000))

We assume that the consumers have no information on the environmental qualities of the ecolabels L_L and L_H . Indeed, consumers can be confused or will not take into account the eco-label. They don't know which eco-label corresponds to the high environmental quality¹². Their basic belief is that the two eco-labels are different and that one is more performing than the other. The only observable variable is prices. They may have an initial belief about the performance of each firm. Intuitively, we think that the consumers will observe the prices to infer qualities levels. The consumers observe price p^L of firm L which offers the label L_L , and the price p^H of the firm H which offers the label L_H . In this case, consumers have diffused believes on the identity of the high environmental quality firm. Actually, they don't know which firm is the high environmental quality firm and which one is the low environmental quality firm. They will observe the price vector (p^H, p^L) to update their believes. We assume that a priori for the consumers there is a fifty percent chance that firm H or L has a high environmental quality. There is a broad literature on price as a signal of quality. Daughety and Reinganum (1995) give a model in which quality can be interpreted as safety. They have shown that higher prices signal safer products when consumers support a sufficiently high portion of the loss. Hertzendorf and Overgaard (2001) develop a model with two firms where consumers don't know the quality of each firm. They have shown that the model has several equilibrium. Fluet and Garella (2002) have used price and advertising to signal quality. They show that when the price difference is small, advertising is necessary to signal quality. However, all these authors have not studied the case of signaling environmental quality through prices. In the present paper, it is the presence of two eco-labels in the market that makes it unbelievable for the consumer. Hence the consumers use the price as a signal for quality.

Mahenc (2008) has studied prices as signals of environmental performances for polluting products. He has shown that high environmental performance can be signaled either through a high or a low price depending on the link between environment and competitiveness. The author has used the price as a signal for environmental performances in his work. Nevertheless, in his work he does not study the asymmetric problem through a model of price as environmental quality signal as consequences of the presence of two eco-labels in the market.

Let $\pi^H(p^H, p^L, \mu\left(p^H, p^L\right))$ be firm H profit. This profit depends on the term $\mu\left(p^H, p^L\right)$. $\mu\left(p^H, p^L\right)$ which is the consumer's posteriori belief that the firm which offers the price p^H really offers a high environmental quality product while the other firm which offres p^L is a low environmental quality firm.

Consumers know that there is one firm of each type:

$$\mu\left(p^{H}, p^{L}\right) = 1 - \mu\left(p^{L}, p^{H}\right).$$

Consumers observe the vector of firms' strategies (p^H, p^L) and update their believes.

Since for the consumers there is a fifty percent chance that firm H or L has a high environmental quality, we note $\mu^0 = \frac{1}{2}$. μ^0 is the a priori consumers believes.

Firms are symmetric for the consumers in the sense that no firm has a reputation advantage on the other, this implies that $\mu\left(p^{i},p^{i}\right)=\frac{1}{2}$ i=L,H. In fact, if the two firms set the same price then consumers' posterior believes remain diffuse.

Finally, if $\mu\left(p^{H},p^{L}\right)=1$, then consumers are sure that the firm which offers p^{H} is a high environmental quality firm. If $\mu\left(p^{H},p^{L}\right)=\frac{1}{2}$ for the consumers there is a fifty percent chance that firm H or L has a high environmental quality. Finally, if $\mu\left(p^{H},p^{L}\right)=0$; then consumers are sure that the firm which set p^{H} is a low environmental quality firm.

Let $\rho(\mu) = \mu q^H + (1-\mu)q^L$. $\rho(\mu)$ be the quality that consumers expect to obtain. We have $\rho^0 =$

¹²This situation is due to the multiplicity of the eco-labels from the one hand and to the different direction each label is retending to be from the other hand.

$$\frac{q^H + q^L}{2} = \rho \left(\mu^0\right).$$

If the price vector doesn't give any information on the environmental quality demand¹³ will always be the same for each firm. It will be equal to:

$$D^{I}(p,p) = \frac{1}{2}(1 - \frac{p}{\rho^{0}}) = \frac{1}{2}(1 - \frac{2p}{q^{H} + q^{L}})$$
(4.1)

 $D^{I}(p,p) = \frac{1}{2}(1-\frac{p}{\rho^{0}}) = \frac{1}{2}(1-\frac{2p}{q^{H}+q^{L}})$ (4.1) In this section, the game is as follows: in the first step, firms H and L with eco-labels L_{H} and L_{L} compete in environmental qualities. In the second stage, the two firms compete in prices. In the third stage, the consumers observe prices, update their believes and decide to buy the good H or L or not to buy.

The aim of this section is to know the real impact of two different eco-labels on the environmental qualities when the consumers don't know the environmental qualities offered by the two firms. Is the global environmental quality level higher or lower than in the complete information case?

5.2Sequential equilibrium and firms' incentives

5.2.1 Definition of a sequential equilibrium

Definition: a sequential equilibrium is a pair of strategies (\hat{p}^L, \hat{p}^H) and a system of believes $\mu(p^L, p^H)$

- (1) \hat{p}^L maximizes π^L
- (2) \hat{p}^H maximizes π^H

- (3) If $\hat{p}^{L} \neq \hat{p}^{H}$ then $\mu(\hat{p}^{L}, \hat{p}^{H}) = 0$ and $\mu(\hat{p}^{H}, \hat{p}^{L}) = 1$ (4) If $\hat{p}^{L} = \hat{p}^{H}$ then $\mu(\hat{p}^{L}, \hat{p}^{H}) = \mu(\hat{p}^{H}, \hat{p}^{L}) = \frac{1}{2}$ (5) If $(p^{L}; p^{H}) \neq (\hat{p}^{L}, \hat{p}^{H})$ then $\mu(p^{L}, p^{H}) + \mu(p^{H}, p^{L}) = 1$ and if $p^{L} = p^{H}$ then $\mu(p^{L}, p^{H}) = \frac{1}{2}$

(1) and (2) imply that each firm will choose a price that maximizes its profit taking as given the strategy of the other firm and the belief of the consumers (it is the sequential rationality). The points (3), (4) and (5) imply that beliefs have to make sense with the structure of the game and with the strategies of the firms (consistency). If firms choose different strategies then consumers will be able to know exactly which firm is a high or a low environmental quality firm. In constrast if the firms choose the same strategy consumers will then return to their prior beliefs that either firm is the high quality firm.

5.2.2Firms' incentives

If we assume that firms' strategies are to set prices equal to the complete information prices p^{HC} , p^{LC} , then their respective profits will be equal to π^{HC} , π^{LC} . If we assume now that firm's L strategy is to model firm's H strategy and then to set price p^{HC} instead of p^{LC} . We will have: $\pi^{LI}(p^{HC}, p^{HC}, \mu^0) = p^{HC}D^I - \gamma aq^{L^2}$

By replacing D^I and p^{HC} by their respective values defined by (4.1) and (3.3) respectively, we obtain: $\pi^{LI}(p^{HC}, p^{HC}, \mu^0) = \frac{q^H(q^H - q^L)}{4q^H - q^L} \left(1 - 4\frac{q^H(q^H - q^L)}{(4q^H - q^L)(q^H + q^L)}\right) - \gamma a q^{L^2}$ (4.2)

PROPOSITION 2¹⁴: In the incomplete information case, the low environmental quality firm has an incentive to replicate the strategy of the high environmental quality firm.

¹³The proofs are given in the Annex B1.

¹⁴The proofs are given in the annex B2.

This proposition implies that if the low environmental quality firm replicates the high environmental quality firm's strategy then its profits will be higher than in the complete information case.

We assume now that the high environmental quality firm replicates the low environmental quality firm's strategy and sets price p^{LC} instead of p^{HC} .

We have then: $\pi^{HI}(p^{LC}, p^{LC}, \mu^0) = p^{LC}D^I - aq^{H^2}$

By replacing D^{I} and p^{LC} by their respective values defined by (4.1) and (3.4) respectively, we obtain: $\pi^{HI}(p^{LC}, p^{LC}, \mu^{0}) = \frac{1}{2} \frac{q^{L}(q^{H} - q^{L})}{4q^{H} - q^{L}} (1 - \frac{2q^{L}(q^{H} - q^{L})}{(4q^{H} - q^{L})(q^{H} + q^{L})}) - aq^{H^{2}}$ (4.3)

PROPOSITION 3¹⁵: The high environmental quality firm has no incentive to replicate firm's L strategy.

This proposition implies that if the high environmental quality firm replicates the low environmental quality firm's strategy then its profits will be lower than in the complete information case.

Propositions 2 and 3 that the low environmental quality firm is always incited to mimic the full information price of the high environmental quality irrespective with the quality difference of the firms. In contrast, the high environmental quality firm is never incited to mimic the full information price of the low quality firm. This result implies that the full information prices can never be a separating equilibrium in the incomplete information case.

5.3 Characterization of a separating equilibrium

We consider a separating equilibrium. In a separating equilibrium each firm will choose different prices $\hat{p}^L \neq \hat{p}^H$ where $\hat{p}^L < \hat{p}^H$. It is an equilibrium where consumers infer correctly environmental quality levels: $\mu(\hat{p}^{L}, \hat{p}^{H}) = 0 \text{ and } \mu(\hat{p}^{H}, \hat{p}^{L}) = 1.$

5.3.1 Definition of a separating equilibrium

The price vector (\hat{p}^L, \hat{p}^H) where $\hat{p}^L < \hat{p}^H$ is a separating equilibrium if:

- (1) \hat{p}^L maximizes $\pi^L(p^L, \hat{p}^H, 0)$ (2) $\pi^L(\hat{p}^L, \hat{p}^H, 0) \succeq \pi^L(\hat{p}^H, \hat{p}^H, \frac{1}{2})$ (3) $\pi^H(\hat{p}^H, \hat{p}^L, 0) \succeq \pi^H(\hat{p}^L, \hat{p}^L, \frac{1}{2})$

The point (1) implies that given the separation of firms, the low environmental quality firm chooses its price to maximize its profit.

- (2) implies that if L chooses \hat{p}^L its profit will be higher than its profit if it choose the to mimic the high environmental price \hat{p}^H .
 - (3) implies that H prefers choosing \hat{p}^H than mimicking L and choosing \hat{p}^L .
 - (2) and (3) represent incentive constraints.

We can deduce from (2) and (3) that the strategies vector (p^{LC}, p^{HC}) can never be a separating equilibrium.

In the next section, we will define the different prices possible in a separating equilibrium ¹⁶.

¹⁵The proofs are given in the Annex B3.

¹⁶Separating equilibrium is always used to characterize incomplete information case (See Fluet and Garella (2002); Daughety and Reinganum (2007)).

Characterization of a separating equilibrium 5.3.2

PROPOSITION 4¹⁷: If (\hat{p}^L, \hat{p}^H) is a separating equilibrium then:

(a)
$$\hat{p}^L = \frac{1}{2} \frac{q^L}{q^H} \hat{p}^H$$
 (4.4)

(a)
$$\hat{p}^{L} = \frac{1}{2} \frac{q^{L}}{q^{H}} \hat{p}^{H}$$
 (4.4)
(b) $\frac{2q^{H}(q^{H}-q^{L})(q^{H}+q^{L})}{(q^{H}+q^{L})q^{L}+4q^{H}(q^{H}-q^{L})} < \hat{p}^{H} < \frac{q^{H^{2}}(4q^{H}-q^{L})(q^{H}+q^{L})(q^{H}-q^{L})}{2q^{H}(2q^{H}-q^{L})(q^{H}+q^{L})-q^{L^{2}}(q^{H}-q^{L})}$ (4.5)

This proposition implies that if any deviation from the equilibrium prices leads to a confusion of the consumers and therefore consumers will not know which firm is offering which quality. A separating equilibrium is equilibrium where the firm offering the high environmental quality conveys a signal that the low environmental quality firm cannot mimic otherwise the equilibrium is pooling (Spence (1973)). It is equilibrium where consumers can distinguish the high from the low quality firm. Any deviation from this equilibrium implies that consumers cannot distinguish the high from the low environmental quality

From (4.5), we can conclude that there is an interval for the existence of \hat{p}^H .

We can conclude from (4.4) and (4.5), that if we take a \hat{p}^H from (4.5), it will correspond to a unique \hat{p}^L . From (4.5), we can conclude that there exists an interval for the existence of \hat{p}^H . We let $\hat{p}_{inf}^H =$ $\frac{2q^H\left(q^H-q^L\right)\left(q^H+q^L\right)}{\left(q^H+q^L\right)q^L+4q^H\left(q^H-q^L\right)}.\ \hat{p}_{\inf}^H \text{ is the smallest possible value for } \hat{p}^H. \text{ We want to calculate } \hat{p}_{\inf}^H-p^{HC}.$

Recall that
$$p^{HC} = \frac{2q^H(q^H - q^L)}{4q^H - q^L}$$
.

we have:
$$\hat{p}_{\inf}^{H} - p^{HC} = \frac{2q^{H}(q^{H} - q^{L})(q^{H} + q^{L})(4q^{H} - q^{L}) - 2q^{H}(q^{H} - q^{L})((q^{H} + q^{L})q^{L} + 4q^{H}(q^{H} - q^{L}))}{((q^{H} + q^{L})q^{L} + 4q^{H}(q^{H} - q^{L}))(4q^{H} - q^{L})}$$
 If we simplify this equation, we obtain:
$$\hat{p}_{\inf}^{H} - p^{HC} = \frac{qL(4q^{H} - q^{L})}{((q^{H} + q^{L})q^{L} + 4q^{H}(q^{H} - q^{L}))(4q^{H} - q^{L})} > 0 \tag{4.6}$$

$$\hat{p}_{\inf}^{H} - p^{HC} = \frac{qL(4q^H - q^L)}{((q^H + q^L)q^L + 4q^H(q^H - q^L))(4q^H - q^L)} > 0 \tag{4.6}$$

This result leads to our next proposition.

 $\begin{aligned} & \textbf{PROPOSITION 5: for all } \hat{p}^{H} \textbf{ in the interval} \\ & \left[\hat{p}_{\text{inf}}^{H} = \frac{2q^{H} \left(q^{H} - q^{L}\right) \left(q^{H} + q^{L}\right)}{(q^{H} + q^{L})q^{L} + 4q^{H} (q^{H} - q^{L})}, \frac{q^{H^{2}} \left(4q^{H} - q^{L}\right) \left(q^{H} + q^{L}\right) \left(q^{H} - q^{L}\right)}{2q^{H} (2q^{H} - q^{L}) (q^{H} + q^{L}) - q^{L^{2}} (q^{H} - q^{L})} = \hat{p}_{\sup}^{H} \right], \text{ we always have } \hat{p}^{H} > p^{HC}. \end{aligned}$ Since $\hat{p}^L = \frac{1}{2} \frac{q^L}{a^H} \hat{p}^H$ and $\hat{p}^H > p^{HC}$, we have equally $\hat{p}^L > p^{LC}$.

This proposition implies that separating equilibrium prices are always higher than complete information equilibrium prices. This result is due to the competition between the two firms. Indeed, the low environmental quality firm has a best response that is directly linked with the separate price chosen by the high environmental quality firm. This result is interesting and can be explained by the following intuition. Firms manipulate prices because they are in the incomplete information case: they take advantage of the asymmetric information case. In that way, when consumers see these high prices they will automatically think that the environmental quality represented by the eco-label is high.

In the proposition 6¹⁸, we are interested in the existence and conditions of existence of separating equilibrium.

¹⁷The proofs are given in the Annex B4.

¹⁸The proofs are given in the Annex B5.

The necessary condition (b) of proposition 4 implies that if $q^L \to q^H$, (if the quality difference is small) then the intensity of the Bertrand competition leads to nul separating profits. It also implies that the high environmental quality firm is only willing to make marginal distortion of its price above its complete information price p^{HC} to ensure separation. We conclude therefore that there is a limit or value of the environmental quality difference noted Δq defined as the solution of $\hat{p}_{\rm inf}^H = \hat{p}_{\rm sup}^H$. Above Δq there exists a separating equilibrium and under it there isn't any separating equilibrium. It follows the following proposition.

PROPOSITION 6:

- (1)— If the low environmental quality firm offers the same quality as the high environmental quality one, then there isn't any separating equilibrium.
 - (2)- If $q^H q^L < \Delta q$ then there isn't any separating equilibrium.
 - (3)– If $q^H q^L > \Delta q$ then there exists a separating equilibrium.

This proposition establishes that if the quality difference is sufficiently large there is an intervalle of prices $[\hat{p}_{\rm inf}^H, \, \hat{p}_{\rm sup}^H]$ for the high environmental quality firm that separates it from the low environmental quality firm. For any $\hat{p}^H \in [\hat{p}_{inf}^H, \hat{p}_{sup}^H]$ there exists a unique best response for the low environmental quality firm.

Furthermore, this proposition implies that any deviation \hat{p}^L or \hat{p}^H is suboptimal. A deviation such as $(\hat{p}^H; \hat{p}^H)$ implies $\mu(\hat{p}^H; \hat{p}^H) = \frac{1}{2}$. This situation is dominated by $\mu(\hat{p}^L; \hat{p}^H) = 0$. So the low environmental quality firm is never incited to deviate from \hat{p}^L . Similarly, the high environmental quality firm is never incited to deviate of its strategy \hat{p}^H . If H chooses $\tilde{p}^H > \hat{p}^L$, then it will face no demand because consumers will think that it offers a low environmental quality product with a higher price than L. If $\hat{p}^H = \hat{p}^L$ then this situation is dominated by $\hat{p}^H > \hat{p}^L$. If now it chooses \tilde{p}^H such that $\tilde{p}^H < \hat{p}^L$ then we will have $\mu\left(\tilde{p}^{H};\hat{p}^{L}\right)=0$. We show in the annex B.5, that firm H isn't incited to choose \tilde{p}^{H} instead of \hat{p}^{H} .

In the next section, we are interested in environmental quality choice of the third stage of the game. Recall that the third stage of the game concerns a simultaneous quality competition.

5.4 Quality competition and firms' incitations

We are interested in this section in the third stage of the game where firms compete simultaneously in qualities¹⁹. We want here, especially, to compare the qualities of complete and incomplete information cases.

Comparison of the revenues of the complete and incomplete information cases

In the incomplete information case, firms' H and L profits are respectively:

$$\pi^{H}(\hat{p}^{H}, \hat{p}^{L}, 1) = \left(1 - \frac{\hat{p}^{H}(2q^{H} - q^{L})}{2q^{H}(q^{H} - q^{L})}\right)\hat{p}^{H} - aq^{H^{2}}$$

$$\pi^{L}(\hat{p}^{L}, \hat{p}^{H}, 0) = \frac{1}{4} \frac{q^{L}}{q^{H}(q^{H} - q^{L})}\hat{p}^{H^{2}} - \gamma aq^{L^{2}}$$

$$(4.7)$$

In the complete information case, firms' H and L profits are respectively:
$$\pi^{HC}(p^H;q^H,q^L) = (1 - \frac{p^H \left(2q^H - q^L\right)}{2q^H \left(q^H - q^L\right)})p^H - aq^{H^2} \tag{4.9}$$

$$\pi^{LC}(p^H; q^L, q^H) = \frac{1}{4} \frac{q^L}{q^H(q^H - q^L)} p^{HC^2} - \gamma a q^{L^2}$$
(4.10)

¹⁹The existence, concavity and stability conditions have been studied in the complete information case and results are exactly the same.

Firm's H revenues in incomplete and complete information case are respectively:

$$R^{H}(\hat{p}^{H},\hat{p}^{L},1) = (1 - \frac{\hat{p}^{H}(2q^{H} - q^{L})}{2q^{H}(q^{H} - q^{L})})\hat{p}^{H};$$

$$R^{H}(p^{HC}; q^{H}, q^{L}) = (1 - \frac{p^{HC}(2q^{H} - q^{L})}{2q^{H}(q^{H} - q^{L})})p^{HC}$$

Firm's L revenues in incomplete and complete information case are respectively:

$$R^{L}(\hat{p}^{L}, \hat{p}^{H}, 0) = \frac{1}{4} \frac{q^{L}}{q^{H}(q^{H} - q^{L})} \hat{p}^{H^{2}}$$

$$R^L(p^{HC};q^L,q^H) = \frac{1}{4} \frac{q^L}{q^H(q^H-q^L)} p^{HC^2}$$

PROPOSITION 7: we show that 20 :

$$(1) - R^{H}(\hat{p}^{H}, \hat{p}^{L}, 1) > R^{H}(p^{HC}; q^{H}, q^{L})$$

$$(2) - R^{L}(\hat{p}^{L}, \hat{p}^{H}, 0) > R^{L}(p^{HC}; q^{L}, q^{H})$$

$$(2) - R^{L}(\hat{p}^{L}, \hat{p}^{H}, 0) > R^{L}(p^{HC}; q^{L}, q^{H})$$

Proposition 7 implies that in the incomplete information case, firms' revenues are always higher of those obtained in the complete information case. This situation can be explained with proposition 5. Proposition establishes that the asymmetric information prices are always higher than the complete information prices. Therefore the revenues of the two firms in the asymetric information case are higher than in the complete information case. Secondly, we show in proposition 6 that separating equilibrium exist if the quality difference is sufficiently high and in this case the market is well segmented and price competition is relaxed and revenue are higher.

Quality choice and firms' incitations

PROPOSITION 8²¹: We let \hat{q}^L and \hat{q}^H the environmental qualities of firm H and L in the incomplete information case. In the asymmetric information case, each firm is incited to choose an environmental quality level lower or equal to their complete information environmental quality. We will have $q^{HC} \succeq \hat{q}^H$ and $q^{LC} \succeq \hat{q}^L$.

This result is very important in our paper. First, this proposition establishes that even if the products have eco-labels, their environmental qualities can be very low and do not match with the environmental quality expected or announced by the eco-label. One must notice in this case that it is the multiplicity of eco-labels that permits firms to practice the "green washing" phenomenon. Second, this proposition entails that the global level of environmental quality has decreased since the $q^{HC} \succeq \hat{q}^H$ and $q^{LC} \succeq \hat{q}^L$. So the eco-labels don't reflect the real level of the environmental qualities when several eco-labels are present in the market. Therefore, the consumer will buy a labelled product with a higher price but a lower environmental quality. Despite the fact that an eco-label is a means to reduce the informational gap between producers and consumers, consumers will not use the eco-label information to buy the product.

6 Conclusion

The objective of this article was to develop a model which, explains how the presence of several ecological labels in the same market affects both the information value of these labels and the environmental qualities.

²⁰The proofs are given in the annex B6.

²¹The proofs are given in the annex B7.

We have supposed that there are two firms with different eco-labels in the market. One firm offers the high environmental quality product and the other the low environmental quality one. We have developed a complete information model where consumers know perfectly which firm offers the high or the low environmental quality. Secondly, we have developed an incomplete information model where consumers don't know which one offers which environmental quality.

Our paper shows two important results:

In the case of complete information the introduction of a second ecological label in a market improves the environmental qualities offered by the two firms. If firms do not pratice opportunistic behavior (if each firm offer its product with the environmental quality announced in the eco-label) than the presence of several eco-labels will increase the global environmental quality.

In the case of incomplete information, the introduction of a second label led to an increase in prices practiced by the two firms and a reduction in the environmental qualities of both firms.

The improvement of the information related to the environmental quality of the products for the consumers becomes the principal stake of the environmental policies.

The presence of a second eco-label decreases considerably the global environmental quality level of both firms. This result is contradictory since an eco-label is a message of a high environmental quality. When there are multiplicity of ecolabels, the incentives of each firm are to diminish their respective qualities, and the consumer will not take into account the label in his decision to buy the product.

One solution observed in different markets is that some firms are applying for several eco-labels at the same time in order to send a credible signal for consumers.

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ANNEX

Annex A: Complete information case

Annex A.1- Computation of equilibrium prices, stability and concavity

(i)- Equilibrium prices:

Reaction functions of firm H and L are obtained by maximizing (3.1) and (3.2) with respect to p^H and p^L respectively: $\frac{\partial \pi^H}{\partial p^H} = 0 \Leftrightarrow 1 - \frac{p^H - p^L}{q^H - q^L} + p^H \left(-\frac{1}{q^H - q^L} \right) = 0$ We have $\frac{\partial \pi^L}{\partial p^L} = 0 \Leftrightarrow \frac{p^H - p^L}{q^H - q^L} - \frac{p^L}{q^L} + p^L \left(-\frac{1}{q^H - q^L} - \frac{1}{q^L} \right) = 0$ We obtain the following reaction functions: $p^H(p^L) = \frac{1}{2}(q^H - q^L) + \frac{1}{2}p^L$ The second $p^L(p^H)$ are the reaction functions:

$$\frac{\partial \pi^H}{\partial p^H} = 0 \Leftrightarrow 1 - \frac{p^H - p^L}{q^H - q^L} + p^H \left(-\frac{1}{q^H - q^L}\right) = 0$$

We have
$$\frac{\partial \pi^L}{\partial p^L} = 0 \Leftrightarrow \frac{p^H - p^L}{q^H - q^L} - \frac{p^L}{q^L} + p^L \left(-\frac{1}{q^H - q^L} - \frac{1}{q^L} \right) = 0$$

$$p^{H}(p^{L}) = \frac{1}{2}(q^{H} - q^{L}) + \frac{1}{2}p^{L}$$

$$p^L(p^H) = \frac{1}{2} \frac{q^L}{q^H} p^H$$
 where $p^H(p^L)$ and $p^L(p^H)$ are the reaction functions of H and L respectively.

To obtain equilibrium prices of H and L, we must resolve the system formed by the two reaction

functions. We then find:
$$p^{HC} = \frac{2q^H(q^H - q^L)}{4q^H - q^L}$$

$$p^{LC} = \frac{q^L(q^H - q^L)}{4q^H - q^L}$$

$$WRT$$

(ii)- Concavity

We have
$$\frac{\partial^2 \pi^H}{\partial p^{H^2}} = -\frac{2}{q^H - q^L} < 0$$
 and $\frac{\partial^2 \pi^L}{\partial p^{L^2}} = -\frac{2}{q^H - q^L} - \frac{2}{q^L} < 0$: the concavity conditions are verified.

(iii)- Stability conditions

We must have
$$\left(\frac{\partial^2 \pi^L}{(\partial p^L)^2}\right) \left(\frac{\partial^2 \pi^H}{(\partial p^H)^2}\right) - \frac{\partial \pi^{2L}}{\partial p^L \partial p^H} \frac{\partial \pi^{2H}}{\partial p^H \partial p^L} > 0,$$

We have:
$$\left(\frac{\partial^2 \pi^L}{\left(\partial p^L\right)^2}\right) \left(\frac{\partial^2 \pi^H}{\left(\partial p^H\right)^2}\right) - \frac{\partial^2 \pi^L}{\partial p^{H^2}} \frac{\partial^2 \pi^H}{\partial p^{L^2}} = -\frac{2}{q^H - q^L} \left(-\frac{2}{q^H - q^L} - \frac{2}{q^L}\right) - \frac{1}{q^H - q^L} \frac{1}{q^H - q^L} \left(-\frac{1}{q^H - q^L}\right) - \frac{2}{q^H - q^L} \left(-\frac{2}{q^H - q^L} - \frac{2}{q^L}\right) - \frac{1}{q^H - q^L} \frac{1}{q^H - q^L} \left(-\frac{1}{q^H - q^L}\right) > 0$$
 Stability conditions are satisfied. WRT

Annexe A2: Concavity and stability conditions for the quality case

We have
$$\frac{\partial^2 \pi^{HC}}{\partial q^{H^2}} = \frac{\partial^2 R^{HC}}{\partial q^{H^2}} - 2a$$

and
$$\frac{\partial R^{HC}}{\partial q^H} = \frac{4q^H}{(4q^H - q^L)^3} \left(\left(3q^H - 2q^L \right) \left(4q^H - q^L \right) - 8(q^{H^3} - q^L q^{H^2}) \right)$$

Then
$$\frac{\partial R^{HC}}{\partial q^H} = \frac{4q^H}{(4q^H - q^L)^3} \left(4q^{H^2} + 2q^{L^2} - 3q^L q^H \right) > 0$$

$$\frac{\partial^2 R^{HC}}{\partial \alpha^{H^2}}$$
 =

$$= \frac{4}{(4q^H - q^L)^6} \left(\left(12q^{H^2} + 2q^{L^2} - 6q^Lq^H\right) (4q^H - q^L)^3 - 12 \left(4q^H - q^L\right)^2 \left(4q^{H^3} + 2q^Hq^{L^2} - 3q^Lq^{H^2}\right) \right)$$

$$\begin{split} &= -\frac{8}{(4q^H - q^L)^4} (5q^H q^{L^2} + q^{L^3}) < 0 \\ &\text{Then } \frac{\partial^2 \pi^{HC}}{\partial q^{H^2}} < 0 \text{ ; the first concavity condition is satisfied.} \\ &\text{In the same way, we have } \frac{\partial^2 \pi^{LC}}{\partial q^{L^2}} = \frac{\partial^2 R^{LC}}{\partial q^{L^2}} - 2\gamma a \\ &\text{and } \frac{\partial R^{LC}}{\partial q^L} = \frac{q^H}{(4q^H - q^L)^3} \left(\left(q^H - 2q^L \right) (4q^H - q^L) + 2(q^L q^H - q^{L^3}) \right) \\ &\text{Then } \frac{\partial R^{LC}}{\partial q^L} = \frac{q^{H^2}}{(4q^H - q^L)^3} (4q^H - 7q^L) > 0 \text{ if } \frac{q^H}{q^L} > \frac{7}{4}. \\ &\frac{\partial^2 R^{LC}}{\partial q^{L^2}} = -\frac{q^{H^2}}{(4q^H - q^L)^4} (16q^H + 14q^L) < 0 \\ &\text{finally } \frac{\partial^2 \pi^{LC}}{\partial q^{L^2}} < 0 \end{split}$$

Concavity conditions are satisfied.

(ii)- Stability conditions

(ii)- Stability conditions We must verify the following condition:
$$\frac{\partial^2 \pi^{HC}}{\partial q^{H^2}} \frac{\partial^2 \pi^{LC}}{\partial q^{L^2}} - \frac{\partial^2 \pi^{HC}}{\partial q^H \partial q^L} \frac{\partial^2 \pi^{LC}}{\partial q^L \partial q^H} > 0$$
 We have:
$$\frac{\partial^2 \pi^{HC}}{\partial q^H \partial q^L} = \frac{\partial^2 R^{HC}}{\partial q^H \partial q^L} = \frac{8}{(4q^H - q^L)^4} (5q^{H^2}q^L + q^{L^2}q^H)$$
 Then
$$\frac{\partial^2 \pi^{HC}}{\partial q^H \partial q^L} = -\frac{q^H}{q^L} \frac{\partial^2 R^{HC}}{\partial q^{H^2}} > 0$$
 and
$$\frac{\partial^2 \pi^{LC}}{\partial q^L \partial q^H} = \frac{\partial^2 R^{LC}}{\partial q^L \partial q^H} = \frac{2q^H}{(4q^H - q^L)^4} (7q^{L^2} + 8q^Lq^H)$$
 Then
$$\frac{\partial^2 \pi^{LC}}{\partial q^L \partial q^H} = -\frac{q^L}{q^H} \frac{\partial^2 R^{HC}}{\partial q^{H^2}} > 0$$
 The stability condition becomes:
$$\left(\frac{\partial^2 R^{HC}}{\partial q^{H^2}} - 2a\right) \left(\frac{\partial^2 R^{LC}}{\partial q^{L^2}} - 2\gamma a\right) - \left(-\frac{q^H}{q^L} \frac{\partial^2 R^{HC}}{\partial q^{H^2}}\right) \left(-\frac{q^L}{q^H} \frac{\partial^2 R^{HC}}{\partial q^{H^2}}\right) = A$$

$$A = -2\gamma a \left(\frac{\partial^2 R^{HC}}{\partial q^{H^2}}\right) - 2a \frac{\partial^2 R^{LC}}{\partial q^{L^2}} + 4\gamma a^2 > 0$$

The stability condition is verified. WRT.

Annex B: The case of incomplete information

Annex B1: Demand function

 $U = \theta \rho^0 - p$ is the consumers utility function in the incomplete information case. The consumer indifferent has a null utility thus

$$\theta = \frac{p}{\rho^0}.WRT.$$

Annex B2: Proof for proposition 2:

We have:
$$\pi^{LI}(p^{HC},p^{HC},\mu^0) = \frac{q^H(q^H-q^L)}{4q^H-q^L}(1-4\frac{q^H(q^H-q^L)}{(4q^H-q^L)(q^H+q^L)} - \gamma aq^{L^2}$$
 and
$$\pi^{LC}(q^L,q^H) = \frac{q^Lq^H(q^H-q^L)}{(4q^H-q^L)^2} - \gamma aq^{L^2}$$

$$\pi^{LI}(p^{HC},p^{HC},\mu^0) - \pi^{LC}(q^L,q^H) = \frac{q^H(q^H-q^L)}{(4q^H-q^L)^2(q^H+q^L)}(8q^Hq^L) > 0$$
 Finally
$$\pi^{LI}(p^{HC},p^{HC},p^{HC},\mu^0) > \pi^{LC}(q^L,q^H)$$

$$WRT$$

Annex B.3- Proof for proposition 3:

We have
$$\pi^{HI}(p^{LC},p^{LC},\mu^0) = \frac{1}{2} \frac{q^L(q^H-q^L)}{4q^H-q^L} \left(1 - \frac{2q^L(q^H-q^L)}{(4q^H-q^L)(q^H+q^L)}\right) - aq^{H^2}$$
 and
$$\pi^{HC}(q^H,q^L) = \frac{4q^{H^2}(q^H-q^L)}{(4q^H-q^L)^2} - aq^{H^2}$$

$$\pi^{HI}(p^{LC},p^{LC},\mu^0) > \pi^{HC}(q^H,q^L) \text{ if and only if } \frac{\pi^{HI}(p^{LC},p^{LC},\mu^0)}{\pi^{HC}(q^H,q^L)} > 1$$
 However,
$$\frac{\pi^{HI}(p^{LC},p^{LC},\mu^0)}{\pi^{HC}(q^H,q^L)} = 2\frac{q^L}{q^{H^2}} \frac{1}{4q^H-q^L} \left(1 - \frac{2q^L(q^H-q^L)}{(4q^H-q^L)(q^H+q^L)}\right) < 1$$
 Finally
$$\pi^{HI}(p^{LC},p^{LC},\mu^0) < \pi^{HC}(q^H,q^L)$$

$$WRT.$$

Annex B.4: Characterization of the separating equilibrium: proof for proposition

4

To characterize the separating equilibrium (when it exists) we must analyse the definition.

The first point (1) of (4.3.1) implies that $\hat{p}^L = \frac{1}{2} \frac{q^L}{q^H} \hat{p}^H$

Firm's L profit become:
$$\pi^{L}(\hat{p}^{L}, \hat{p}^{H}, 0) = \left(\frac{\hat{p}^{H} - \frac{1}{2} \frac{q^{L}}{q^{H}} \hat{p}^{H}}{q^{H} - q^{L}} - \frac{1}{2} \frac{\hat{p}^{H}}{q^{H}}\right) \left(\frac{1}{2} \frac{q^{L}}{q^{H}} \hat{p}^{H}\right) - \gamma a q^{L^{2}}$$

If we develop this expression we obtain:
$$\pi^{L}(\hat{p}^{L}, \hat{p}^{H}, 0) = \frac{1}{4} \frac{q^{L}}{q^{H}(q^{H} - q^{L})} \hat{p}^{H^{2}} - \gamma a q^{L^{2}}$$

The second point implies that firm L is never incited to mimic the strategy of firm H:

$$\pi^{L}(\hat{p}^{L}, \hat{p}^{H}, \hat{0}) \succeq \pi^{L}(\hat{p}^{H}, \hat{p}^{H}, \frac{1}{2})$$

But
$$\pi^{L}(\hat{p}^{H}, \hat{p}^{H}, \frac{1}{2}) = \frac{1}{2}\hat{p}^{H}(1 - \frac{2\hat{p}^{H}}{q^{H} + q^{L}}) - \gamma aq^{L^{2}}$$

 $\pi^{L}(\hat{p}^{L}, \hat{p}^{H}, 0) \succeq \pi^{L}(\hat{p}^{H}, \hat{p}^{H}, \frac{1}{2})$
 $\Rightarrow \frac{1}{4}\frac{q^{L}}{q^{H}(q^{H} - q^{L})}\hat{p}^{H^{2}} - \gamma aq^{L^{2}} > \frac{1}{2}\hat{p}^{H}(1 - \frac{2\hat{p}^{H}}{q^{H} + q^{L}}) - \gamma aq^{L^{2}}$
 $\iff \frac{1}{4}\frac{q^{L}}{q^{H}(q^{H} - q^{L})}\hat{p}^{H^{2}} > \frac{1}{2}\hat{p}^{H}(1 - \frac{2\hat{p}^{H}}{q^{H} + q^{L}})$

$$\Longleftrightarrow \hat{p}^H > \tfrac{2q^H \left(q^H - q^L\right) \left(q^H + q^L\right)}{\left(q^H + q^L\right)q^L + 4q^H \left(q^H - q^L\right)}$$

 $\iff \hat{p}^H > \frac{2q^H(q^H - q^L)(q^H + q^L)}{(q^H + q^L)q^L + 4q^H(q^H - q^L)}$ In the same manner, the third point implies that firm H is never incited to mimic the strategy of firm

L:

$$\begin{split} \pi^{H}(\hat{p}^{H},\hat{p}^{L},0) &\succeq \pi^{H}(\hat{p}^{L},\hat{p}^{L},\frac{1}{2}) \\ \text{But } \pi^{H}(\hat{p}^{H},\hat{p}^{L},1) = (1 - \frac{\hat{p}^{H} - \frac{1}{2} \frac{q^{L}}{q^{H}} \hat{p}^{H}}{q^{H} - q^{L}}) \hat{p}^{H} - aq^{H^{2}} \\ \text{By developing this expression, we obtain:} \\ \pi^{H}(\hat{p}^{H},\hat{p}^{L},1) &= (1 - \frac{\hat{p}^{H}(2q^{H} - q^{L})}{2q^{H}(q^{H} - q^{L})}) \hat{p}^{H} - aq^{H^{2}} \\ \text{We even have:} \\ \pi^{H}(\hat{p}^{L},\hat{p}^{L},\frac{1}{2}) &= \frac{1}{2} \hat{p}^{L} (1 - \frac{2\hat{p}^{L}}{q^{H} + q^{L}}) - aq^{H^{2}} \\ &= \frac{1}{4} \frac{q^{L}}{q^{H}} \hat{p}^{H} (1 - \frac{q^{L}}{q^{H}(q^{H} + q^{L})} \hat{p}^{H}) - aq^{H^{2}} \\ \text{but } \pi^{H}(\hat{p}^{H},\hat{p}^{L},1) &\succeq \pi^{H}(\hat{p}^{L},\hat{p}^{L},\frac{1}{2}) \\ &\Rightarrow (1 - \frac{\hat{p}^{H} - \frac{1}{2} \frac{q^{L}}{q^{H}} \hat{p}^{H}}{q^{H} - q^{L}}) \hat{p}^{H} - aq^{H^{2}} > \frac{1}{4} \frac{q^{L}}{q^{H}} \hat{p}^{H} (1 - \frac{q^{L}}{q^{H}(q^{H} + q^{L})} \hat{p}^{H}) - aq^{H^{2}} \\ &\iff (1 - \frac{\hat{p}^{H} - \frac{1}{2} \frac{q^{L}}{q^{H}} \hat{p}^{H}}{q^{H} - q^{L}}) \hat{p}^{H} > \frac{1}{4} \frac{q^{L}}{q^{H}} \hat{p}^{H} (1 - \frac{q^{L}}{q^{H}(q^{H} + q^{L})} \hat{p}^{H}) \\ &\iff \hat{p}^{H} < \frac{q^{H^{2}} (4q^{H} - q^{L}) (q^{H} + q^{L}) (q^{H} - q^{L})}{2q^{H} (2q^{H} - q^{L}) (q^{H} + q^{L}) - q^{L^{2}} (q^{H} - q^{L})} \end{split}$$

Annex B.5: Proof for proposition 6

We must show that none of the firms L and H want to deviate from $(\hat{p}^L; \hat{p}^H)$.

Any deviation \hat{p}^L or \hat{p}^H is sub optimal. A deviation such that $(\hat{p}^H; \hat{p}^{\check{H}})$ implies $\mu(\hat{p}^H; \hat{p}^H) = \frac{1}{2}$. This situation is dominated by $\mu\left(\hat{p}^L;\hat{p}^H\right)=0$. The low environmental quality firm is never incited to deviate from \hat{p}^L . In the same manner, the high environmental quality firm is never incited to deviate of its strategy \hat{p}^H . If H chooses $\hat{p}^H > \hat{p}^L$, then it will face no demand because consumers will think that it offers a low environmental quality product with a higher price than L. If $\tilde{p}^H = \hat{p}^L$ then this situation is dominated by $\hat{p}^H > \hat{p}^L$. If now it chooses \tilde{p}^H such that $\tilde{p}^H < \hat{p}^L$ then we will have $\mu\left(\tilde{p}^H; \hat{p}^L\right) = 0$.

We have:
$$D^{H} = \frac{\hat{p}^{L} - \tilde{p}^{H}}{q^{H} - q^{L}} - \frac{\tilde{p}^{H}}{q^{L}}, \text{ and }$$

$$\pi^{H}(\tilde{p}^{H}, \hat{p}^{L}, 0) = \tilde{p}^{H}(\frac{\hat{p}^{L} - \tilde{p}^{H}}{q^{H} - q^{L}} - \frac{\tilde{p}^{H}}{q^{L}}) - aq^{H^{2}}$$

Profit maximisation with respect to \tilde{p}^H gives $\tilde{p}^H = \frac{q^L}{2a^H}\hat{p}^L$. Or $\hat{p}^L = \frac{q^L}{2a^H}\hat{p}^H$. This implies that $\tilde{p}^H = \frac{q^{L^2}}{4a^{H^2}} \hat{p}^H.$

We will have:
$$\pi^H(\tilde{p}^H, \hat{p}^L, 0) = \frac{q^{L^3}(2-q^H)}{16q^{H^4}(q^H-q^L)}\hat{p}^{H^2} - aq^{H^2}$$
.

We will show now that H is not incited to choose \tilde{p}^H more than \hat{p}^H

We compute $\pi^H(\hat{p}^H,\hat{p}^L,1)-\pi^H(\tilde{p}^H,\hat{p}^L,0)$:

$$\pi^{H}(\hat{p}^{H}, \hat{p}^{L}, 1) - \pi^{H}(\tilde{p}^{H}, \hat{p}^{L}, 0) = \left[(1 - \frac{\hat{p}^{H}(2q^{H} - q^{L})}{2q^{H}(q^{H} - q^{L})})\hat{p}^{H} - aq^{H^{2}} \right] - \left[\frac{q^{L^{3}}(2 - q^{H})}{16q^{H^{4}}(q^{H} - q^{L})}\hat{p}^{H^{2}} - aq^{H^{2}} \right]$$

After computation, we obtain

$$\pi^{H}(\hat{p}^{H}, \hat{p}^{L}, 1) - \pi^{H}(\hat{p}^{H}, \hat{p}^{L}, 0) = \hat{p}^{H} \left[1 - \frac{16q^{H^{4}} - 8q^{H^{3}}q^{L} + q^{L^{3}}(2 - q^{H})}{16q^{H^{4}}(q^{H} - q^{L})} \hat{p}^{H} \right]$$

$$\pi^{H}(\hat{p}^{H}, \hat{p}^{L}, 1) - \pi^{H}(\hat{p}^{H}, \hat{p}^{L}, 0) > 0$$
if $1 - \frac{16q^{H^{4}} - 8q^{H^{3}}q^{L} + q^{L^{3}}(2 - q^{H})}{16q^{H^{4}}(q^{H} - q^{L})} \hat{p}^{H} > 0$

$$\implies \text{if } \hat{p}^{H} < \frac{16q^{H^{4}}(q^{H} - q^{L})}{16q^{H^{4}} - 8q^{H^{3}}q^{L} + q^{L^{3}}(2 - q^{H})}$$
However, we have $\hat{p}^{H} < \frac{q^{H^{2}}(4q^{H} - q^{L})(q^{H} + q^{L})(q^{H} - q^{L})}{2q^{H}(2q^{H} - q^{L})(q^{H} + q^{L}) - q^{L^{2}}(q^{H} - q^{L})}.$

We must show that:
$$\frac{q^{H^2}(4q^H-q^L)(q^H+q^L)(q^H-q^L)}{2q^H(2q^H-q^L)(q^H+q^L)-q^{L^2}(q^H-q^L)} < \frac{16q^{H^4}(q^H-q^L)}{16q^{H^4}-8q^{H^3}q^L+q^{L^3}(2-q^H)}$$
 We have :
$$\frac{q^{H^2}(4q^H-q^L)(q^H+q^L)(q^H-q^L)}{2q^H(2q^H-q^L)(q^H+q^L)-q^{L^2}(q^H-q^L)} < \frac{16q^{H^4}(q^H-q^L)}{16q^{H^4}-8q^{H^3}q^L+q^{L^3}(2-q^H)}$$

$$\Leftrightarrow \frac{2q^H(2q^H-q^L)(q^H+q^L)-q^{L^2}(q^H-q^L)}{q^{H^2}(4q^H-q^L)(q^H+q^L)} - \frac{16q^{H^4}-8q^{H^3}q^L+q^{L^3}(2-q^H)}{16q^{H^4}(q^H-q^L)} > 0$$

$$\Leftrightarrow \frac{2}{q^H(4q^H-q^L)} - \frac{q^{L^2}}{q^{H^2}} \frac{1}{(4q^H-q^L)(q^H+q^L)} > \frac{16q^{H^4}-8q^{H^3}q^L+q^{L^3}(2-q^H)}{16q^{H^4}(q^H-q^L)} = \frac{8q^{H^3}(2q^H-q^L)+q^{L^3}(2-q^H)}{16q^{H^4}(q^H-q^L)}$$
 But
$$\frac{16q^{H^4}-8q^{H^3}q^L+q^{L^3}(2-q^H)}{16q^{H^4}(q^H-q^L)} = \frac{8q^{H^3}(2q^H-q^L)+q^{L^3}(2-q^H)}{16q^{H^4}(q^H-q^L)} > 0$$
 Because
$$q^H \in [0,1]$$
. Finally H is not incited to choose
$$\tilde{p}^H$$
 more than
$$\hat{p}^H$$
 WRT

Annex B.6: Proof for proposition 7:

We know that
$$\hat{p}^H > p^{HC}$$
 and $\hat{p}^L > p^{LC}$.
$$R^H(\hat{p}^H, \hat{p}^L, 1) - R^H(p^H; q^H, q^L) = 1 * (\hat{p}^H - p^{HC}) - (\hat{p}^H - p^{HC}) \frac{(2q^H - q^L)}{2q^H(q^H - q^L)}$$

$$Or \frac{(2q^H - q^L)}{2q^H(q^H - q^L)} < 1.$$
 Indeed,
$$\frac{(2q^H - q^L)}{2q^H(q^H - q^L)} = \frac{1}{(q^H - q^L)} (1 - \frac{q^L}{2q^H}).$$
 But $\frac{q^L}{q^H} < 1$; $\frac{1}{(q^H - q^L)} (1 - \frac{q^L}{2q^H}) < 1$ Finally we have:
$$R^H(\hat{p}^H, \hat{p}^L, 1) - R^H(p^H; q^H, q^L) = A$$

$$A = 1 * (\hat{p}^H - p^{HC}) - (\hat{p}^H - p^{HC}) \frac{(2q^H - q^L)}{2q^H(q^H - q^L)} > 0$$
 We also have:
$$R^L(\hat{p}^L, \hat{p}^H, 0) - R^L(p^{HC}; q^L, q^H) = B$$

$$B = \frac{1}{4} \frac{q^L}{q^H(q^H - q^L)} (\hat{p}^{H^2} - p^{HC^2}) > 0$$
 because $\hat{p}^H > p^{HC}$
$$WRT$$

Annex B.7 Proof for proposition 8:

We have
$$\pi^H(\hat{p}^H, \hat{p}^L, 1) = (1 - \frac{\hat{p}^H(2q^H - q^L)}{2q^H(q^H - q^L)})\hat{p}^H - aq^{H^2}$$

$$\pi^L(\hat{p}^L, \hat{p}^H, 0) = \frac{1}{4} \frac{q^L}{q^H(q^H - q^L)}\hat{p}^{H^2} - \gamma aq^{L^2}$$

First, we have $\mu\left(\hat{p}^L;\hat{p}^H\right)=0$ and $\mu\left(\hat{p}^H,\hat{p}^L\right)=1$; then firms are incited to offer higher prices and smallest qualities. They are not incited to set quality levels higher than in the complete information case. Second, we have $R^H(\hat{p}^H,\hat{p}^L,1)>R^H(p^{HC};q^H,q^L)$. Firm H will decrease its quality to a maximum in order to maximize its revenue. It will then choose a quality such that $q^{HC}\succeq\hat{q}^H$. For firm L, we have $R^L(\hat{p}^L,\hat{p}^H,0)>R^L(p^{HC};q^L,q^H)$, L will choose a quality level that minimises its fixed cost γaq^{L^2} . WRT