# Full price elasticities and the value of time: A Tribute to the Beckerian model of the allocation of time

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# Full price elasticities and the value of time:
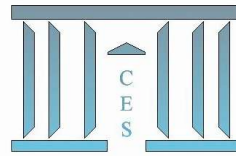## A Tribute to the Beckerian model of the allocation of time

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Abstract

This article adopts Becker's allocation of time framework to describe households' choices concerning both their monetary and time use expenditures in order to propose a new method to derive price elasticity at a micro level. Price and full income elasticities are estimated on a matching of a French Family Budget and a Time Use survey. The utility and home production functions are specified in order to allow the computation of the household's opportunity cost for time, which is shown to be smaller in average than the household's wage net of taxes. This estimate serves to value time dedicated to domestic activities and are used in the definition of full prices. The estimated price elasticities compare well with the estimates by other methods, such as Frisch's model based on independence of preferences assumptions or Hicks-Lewbel's method based on the aggregation of commodities. Finally, the model is applied to the computation of a welfare index, to the estimation of the household's labour supply and to a tentative explanation of the classic difference between cross-section and time-series estimates of income elasticities.

## Introduction

Curiously, it seems that Becker-Muth home production theory has not yet been applied to the practical estimation of full income and full price elasticities. The estimation of price elasticities for large micro-data is an important matter since it may result into more robust parameters than the estimation on macro time-series. It also allows to estimate for different sub-populations (young vs old people, rich vs poor...), which is important for the micro-simulation of public policy, for the measurement of the welfare change associated to price variations and for the test of theoretical assumptions (such as the integrability of demand functions). An important difficulty for such an application of the domestic production model lies in the valuation of time for which a method is proposed in this article.

Four methods have been used to estimate price effects on households' consumption. First, almost all price-elasticities which are presented in the literature are estimated on macro time-series data by means of demand systems under Slutsky constraints. They are generally considered as being not robust to the specification of the demand system and they suffer from aggregation biases and lack of information on price variations. Moreover the stationarity conditions are generally rejected for long-term time-series.

Second, estimation technics on cross-section individual data (such as surveys on consumer expenditures) are either based on separability assumptions over the utility function (Frisch, 1959, Deaton, 1974) which allows to calculate price elasticities in terms of income elasticities and the Frisch's income flexibility (see Theil, 1986, Selvanathan, 1993), or on unit values (ratio of the value of expenditures over the corresponding quantities whenever both are recorded in the survey, generally for food expenditures, see Deaton, 1988). The first method is based on the assumption of strong separability which generally is not supported by the data. The second concerns only those rare datasets which contain both values and quantities of consumption, generally only for food commodities.

Third, arc-elasticity can be computed between two periods (see Gardes, Merrigan, 2011 for such an estimation for tobacco). In contrast to estimation on macro data, this method allows to compute price effects for different types of households, but it necessitates comparable repeated cross-sections or a panel over a period characterized by large price changes. Moreover, it concerns only the direct price effect for the commodity having experienced this price change.

A fourth method consists in computing semi-aggregate price indexes for a set of products using the individual budget shares as a weight of the individual product prices: these aggregate prices are thereby individualized and can be used to estimate price elasticities on a cross-section survey. This method initiates in remarks by Hicks and Stone and was fully discussed by Lewbel (1989). It has been recently applied by Ruiz and Trannoy (2007) on the French Household Expenditures surveys and proved efficient, but perhaps not very robust (considering the volatility of estimates in Ruiz's thesis, 2006). Those techniques imply the endogeneity of semi-aggregate prices (since they are defined by means of current budget shares) which can be corrected by instrumentation. More critical is the assumption that the set of detailed commodities (such as various durables for home cleaning) are sufficiently different to experience different price changes, but pertain to a homogenous group as concerns households' preferences: whenever they differ sufficiently, the difference in the composition of the corresponding aggregate for two households implies that the semi-aggregate does not play the same role in these households' consumption, and thus the price change of the semi-aggregate is largely due to a change in its composition or quality. The hidden assumption of that method (commodities within an aggregate consumption play the same role in the household's consumption) is thus intrinsically illogical: either the commodities are similar, and their prices must follow the same trend; or they differ, and the role of the aggregate in household's consumption depends on the proportion of the detailed commodities in this aggregate.

The method we propose uses full prices for aggregate activities such as food consumption or transport expenditures in order to obtain variations of prices across households, since these full prices depend on the household's opportunity cost of time and its time technology to produce each activity. The model also allows the computation of the opportunity cost of time for each household which is used to calculate full prices. This method has been applied in the literature on transportation costs but in those the opportunity cost of time is calibrated (for instance at the household's net wage rate) or calculated under special hypotheses (see for instance de Vany, 1973, who supposes that income and price elasticities of air travel depend linearly on the trip distance). However, it was recently applied to the purchase of theatre tickets in Germany (Zieba, 2009). The time used for transportation and attendance (leisure price) is valuated at the regional average market wage of the German population multiplied

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by the proportion of households working on the labour market (an empirical proxy of the household's expected wage). The full-income is the sum of the household's monetary disposable income and leisure time income. The specification is double logarithmic and two equations relate the theatre attendance per capita to the ticket price and the price of leisure and, either the household's monetary disposable income, or its full income. The elasticity over the ticket price is estimated at -0.28, while the full price elasticity culminates at -4.16 and the monetary and full income elasticities are respectively 1.16 and 5.65. The full income and full price elasticities thus appear extremely high. In the second specification, with full income and full price, the effect of a change in the opportunity cost of time is twice as big: through the full price elasticity and the time component of the full income. Multiplying the full income elasticity by the ratio of the time component over full income[2] (0.64 as measured on our French statistics, see Table A1 in Appendix A), the sum of the income and price effect of a change in the opportunity cost of time is -0.53, which is close to the ticket price elasticity. Therefore, it seems that the direct specification used in this article is highly disputable, since it mixes the effects of changes in the monetary and time components of the full price with the effect of the opportunity cost of time through the full income.

In section 1 we present Becker's allocation of time model and show that it allows to define an indicator of commodity prices. Section 2 introduces the datasets and presents the statistical matching of the two surveys. Section 3 presents the specification of the utility and domestic production functions which serve to calculate the opportunity cost of time which is used to value the time used in domestic activities. Section 4 applies the model to the computation of the income flexibility and the Arrow-Pratt relative risk aversion index while section 5 presents the econometric specification of the demand system. Section 6 contains the results of the estimation on the French INSEE micro data. Section 7 shows how these estimates can be used to compute elasticities for disaggregate commodities. The last section presents a tentative explanation of the endogeneity biases observed for cross-section income elasticities.
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## 1. Full price elasticity

The new method consists of computing *full prices* for individual agents based on Becker's model of the allocation of time. Full prices incorporate either shadow prices linked to constraints faced by the agent, or shadow prices corresponding to non-monetary resources such as time (see Gardes et al., 2005). The idea in this model is to represent them by the ratio of full expenditures over the monetary, thus suppressing the quantity consumed of the activity.

*Becker's model of the allocation of time with an endogenous opportunity cost of time for home production*

Becker (1965) considers a set of *final goods* the quantities of which $Z_i$, i=1 to m, enter the direct utility of the consumer $u(Z_1, Z_2, \ldots Z_m)$. In order to simplify the analysis, Becker states that a separate activity i produces the final good i in quantity $Z_i$ using a unique market good in quantity $x_i$ and unit time $t_i$ per unit of activity i. finally, time to produce one unit of activity i is supposed to be proportional the quantity of the market factor: $t_i = \tau_i x_i$. Thus the

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[2] A correct specification would use instead the marginal effect of the time component of the full income.

final goods are produced by a set of domestic production functions $f_i$: $Z_i = f_i(x_i, \tau_i; W)$ with all other (socio-economic) characteristics of the household in vector W. This assumption allows him to write the consumer program:

Max $u(Z_1, Z_2, \dots Z_m)$
such that $Z_i = f_i(x_i, \tau_i; W), \sum_i p_i x_i = y$ and $\sum_i \tau_i x_i + t_w = T$

with $y = wt_w + V$ the monetary income which sums labour and other incomes, $t_w$ the labour time on the market and T total disposable time for one period. The market good used in quantity $x_i$ to produce $Z_i$ is supposed to be unique. In case multiple market goods are used in activity i, a generalization to a bundle of market goods used to produce the activity can be performed by defining aggregate commodities of these market goods for i: the monetary price $p_i$ will be defined in this case as a price index for the bundle of corresponding goods coherent with the monetary budget constraint.

These three constraints give together the full budget constraint which depends on the full income $y^f$ defined as the maximum monetary income which could be earned working all disposable time T at the market wage rate net of taxes w:

$$y^f = wT + V = y + w(T - t_w) = y + w \sum_i \tau_i x_i$$

The full price $\pi_i$ for one unit of the final good (activity) i is written: $p_i x_i + \omega t_i$ with an opportunity cost of time $\omega$ which is generally taken as the agent's market wage rate net of taxes. The unit full price of the market good $x_i$ is therefore: $p_i + \omega \tau_i$. We suppose that the agent's opportunity cost $\omega$ differs from her net wage, so that the full budget constraint writes:

$$\sum_i (p_i x_i + \omega t_i) = y^f + (\omega - w)(T - t_w) = y^f + (\omega - w) \sum_i \tau_i x_i \qquad (1)$$

In this formula, the full income is corrected by means of a function of the domestic production time which represents the difference between the market (w) and the personal valuation ($\omega$) of that time: the agent substracts the transaction cost between her leisure and market labour opportunity cost of time from her full income (this correction applies whence the market labour supply $t_w$ is predetermined, which defines the monetary income). Note that this full budget constraint differs from the usual one where leisure or consumption time is valued by the agent's net market wage rate. This particularity allows to propose a new method to estimate the agent's opportunity cost for time.

*Empirical definition of full prices*

The full expenditure for one unit of activity i is: $(\mathbf{p_i + \omega_h \tau_{ih}})\mathbf{x_{ih}}$. It depends on the household characteristics by means of its time participation to activity i: $\boldsymbol{t_{ih} = \tau_{ih} x_{ih}}$ and its opportunity cost of time $\omega_h$. We are now able to indicate the full price for activity i by the ratio of full expenditures over their monetary component:

$$\pi_i = \frac{(p_i + \omega_h \tau_{ih}) x_{ih}}{p_i x_{ih}} = \frac{p_i + \omega_h \tau_{ih}}{p_i} = 1 + \frac{\omega_h \tau_{ih}}{p_i} \qquad (2)$$

Note that under the assumption of a common monetary price $p_i$ for all households, this ratio contains all the information on the differences of full prices through $\boldsymbol{\omega_h}$ and $\boldsymbol{\tau_{ih}}$ (for

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instance its logarithm is approximatively equal to $\frac{\omega_h \tau_{ih}}{p_i}$ for small values of this product and thus the relative change of the full price is approximatively proportional to the absolute change of this ratio). By these definitions, we are able to proxy the changes in the full prices observing only monetary and full expenditures.

Two hypotheses were necessary to derive full prices from monetary and time expenditures: first, domestic production functions are supposed to be Leontief functions with constant production coefficients; second no joint production exist using a common monetary or time expenditure, which may be more easily verified for broad categories of activities such as housing and food.

## 2. Dataset

We use a French dataset from INSEE which combines at the individual level the monetary and time expenditures into a common, unique goods and services consumption structure by a statistical match of the information contained in two surveys: the Family Expenditure Survey (FES, INSEE BDF 2001) and the Family Time Budget (FTB, INSEE BDT 1999). We define 8 types of activities or time use types compatible with the available data both from FES and BDT: *Eating and cooking time* (FTB) and food consumption (FES), *cleaning and home maintenance* and dwelling expenditures (including imputed rent), *clothing maintenance* and clothing expenditures, *education time* and education expenditures, *health care time* and health expenditures, *leisure time* and leisure expenditures, *transport time* and *transport expenditures, miscellaneous time use* and miscellaneous goods and services.

Two matching methods have been used: first, by clustering (into 40 cells) the whole population in terms of age, education, location as key variables, one obtains a good treatment of measurement errors and zero expenditure problems. For each activity and for each household we compute individually the corresponding time use and then the cell weighted average[3]. This way we have for the comparable cells of both surveys the base information for the full time and money expenditure nomenclature. The addition of both will be possible once the individual time value is estimated. This method has been used in Gardes et al. ( 2013) and proved to allow for robust estimation of the full cost of a child (which is estimated to be greater than the monetary). However, it seems better to calculate the time component of each activity at the household level rather than for a cell grouping a lot of different households. The second matching method, used in this article, is based on an individual matching by regression: time use equations for all selected activities are estimated on 30 households' characteristics (income and age class, level of education, location, family type, number of children, socio-professional category and full time or partial labour supply) for all observation units in FTB survey and these estimations serve to predict the time spent on these activities in the corresponding units in the FES survey.

## 3. Estimation of the opportunity cost of time

In the empirical application, three methods are used to value the time spent on domestic activities. First, this value is simply defined as the official minimum wage rate for this period in France: that method supposes that this minimum wage is close to the market wage for domestic activities. In the second method, time is supposed to be perfectly

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[3] Weighting was necessary to take into account the survey interview day (week-end or a work day). The weights are the proportions of the persons interviewed in the week (0.74) or during the week-end (0.26).

exchangeable between the market and non market household's activities, so that the opportunity cost of non-market (domestic) work is computed as the average net wage rate for all working individuals in the family, or by their expected hourly wage rate on the labour market for non working individuals (estimated separately for man and woman using the two-steps Heckman method). Both evaluation methods are adjusted for income taxes and the estimated numbers of working days and hours. The third method is based on the estimation for each household of its opportunity cost of time by means of the first order conditions in the domestic production framework developed later in this section.

In the Becker's original home production theory, the same opportunity cost of time applies for the time factor of the home production and on the labour market. We consider now a model where the agent maximizes a direct utility function depending on the quantities of a set of activities given by the domestic production functions.

First, the classic household model supposes that the household maximizes a direct utility depending on quantities consumed x of an aggregate market good with price $p_x$ and leisure time $t_d$, while adults in the household work on the labour market and at home, with a budget constraint depending on labour time and a time constraint summing market labour, domestic labour and leisure. In this type of model, the opportunity cost of time intervenes in the agent's decision for home production (where it corresponds to the ratio of the marginal utilities of money and time) as well as for its labour supply on the market (by means of the wage rate). Suppose that the utility depends also on the agent's characteristics Z: $u(x, t_d, Z)$ with a domestic production function $Q(t_d, I)$ depending of the number of hours of domestic labour (identified with leisure time) $t_d$ and other inputs I. The income constraints include market labour income, the value of the domestic production and other incomes V: $p_x x + \omega t_d = V + w t_w + p_Q Q$. This model gives rise to a shadow wage $\omega$ defined by the substitution between consumption and leisure time and equal to the ratio of the marginal utilities of the aggregate good and leisure time: $\frac{u'_x}{u'_l} = \frac{\omega}{p_x}$ (3). In the case of a perfect substitution between market and domestic labour, this ratio is equal to the marginal domestic production: $\omega = w = p_q \frac{\partial Q}{\partial h}$. Therefore, in that classic model, a difference between the agent's market wage and the opportunity cost of time is caused by the disutility of market labour due to transportation costs, loss of liberty to organize one's time etc… In fact, the opportunity cost of time may be lower than the market wage not only because of the disutility attached to market labour compare to home production, but also because of the possibility of joint production which characterizes the latter.

This model does not fully correspond to the Becker-Muth domestic production scheme since no utility is defined over the services produced by domestic production, and moreover the agent is not able to substitute income and time through a full income constraint based on an opportunity cost different from the net market wage. In order to analyze that more complex decision model, we suppose, in order to simplify the derivations, a Cobb-Douglas structure both for the utility and the domestic production functions of the final goods $Q_i$ which depend on the monetary and time inputs[4]. The optimization program is, according to the assumptions of section 1 (all variables correspond to a household h which index is omitted in the equations):

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[4] Their parameters will be estimated on each point (for each household in the dataset), so that this specification just supposes the constancy of each household's elasticities of the domestic productions in the utility, and the two factors in the production functions.

$$\max_{m_i,t_i} u(Q) = \prod_i a_i Q_i^{\gamma_i} \ with \ Q_i = m_i^{\alpha_i} t_i^{\beta_i} \qquad (3)$$

under the full income constraint (1):

$$\sum_i (m_i + \omega t_i) = w t_w + \omega(T - t_w) + V.$$

Note that $T - t_w = \sum_i t_i$ and that both the market wage and the shadow wage (i.e. the opportunity cost of time $\omega$) appear in the budget equation: the shadow wage corresponds to the valuation of time in domestic production, and differs from the market wage w whenever there exist some imperfection on the labour market or if the disutility of labour is smaller for domestic production.

In order to estimate the opportunity cost for time, the utility function is re-written:

$$u(Q_i) = \prod_i a_i Q_i^{\gamma_i} = \prod_i a_i \left[ \prod_i m_i^{\alpha_i \gamma_i / \sum \alpha_i \gamma_i} \right]^{\sum \alpha_i \gamma_i} \left[ \prod_i t_i^{\beta_i \gamma_i / \sum \beta_i \gamma_i} \right]^{\sum \beta_i \gamma_i}$$
$$= a m'^{\sum \alpha_i \gamma_i} t'^{\sum \beta_i \gamma_i} \quad (4)$$

with $m'$ and $t'$ the geometric weighted means of the monetary and time inputs with weights $\alpha_i \gamma_i / \sum \alpha_i \gamma_i$ and $\beta_i \gamma_i / \sum \beta_i \gamma_i$. Deriving the utility over income Y and total leisure and domestic

production time T gives the opportunity cost of time :

$$\omega = \frac{\frac{\partial u}{\partial T}}{\frac{\partial u}{\partial Y}} = \frac{\frac{\partial u}{\partial t'}\frac{\partial t'}{\partial T}}{\frac{\partial u}{\partial m'}\frac{\partial m'}{\partial Y}} = \frac{m' \sum \beta_i \gamma_i}{t' \sum \alpha_i \gamma_i} \frac{\frac{\partial t'}{\partial T}}{\frac{\partial m'}{\partial Y}} \quad (5)$$

The ratio $\dfrac{\frac{\partial t'}{\partial T}}{\frac{\partial m'}{\partial Y}}$ is estimated on our data as smaller than one[5].

All parameters of the utility function will be estimated locally (for each household) so that the household's welfare will depend both on the set of parameters $(\alpha, \beta, \gamma)$ and on its monetary and time expenditures $m_i$ and $t_i$. Note that the utility function is defined by formula (5) up to an increasing function. Restricting the transform function so that the formula defining the opportunity cost does not change, all power of $u$ can also act as a utility function:

$$v = u^\delta = a m'^{\delta \sum \alpha_i \gamma_i} t'^{\delta \sum \beta_i \gamma_i} \quad (6)$$

The calculation of all parameters, $\alpha, \beta, \gamma$ are not changed by this transformation, so that the estimated opportunity cost of time $\omega$ also does not depend on this normalization. The Relative Risk Aversion specification of the utility function (6) permits the calculation of the Arrow-Pratt index of relative risk aversion. This Arrow-Pratt index is also equal to the income elasticity of the marginal utility for monetary expenditures or the inverse of Frisch's income

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[5] 0.056753/0.093766=0.61.

flexibility $\breve{\omega}$ (Theil et al., 1981): $-\frac{U''_{Y2}}{U'_Y} = \breve{\omega} = (1 - \delta \sum \alpha_i \gamma_i)$. A method to estimate the Arrow-Pratt index will be presented in section 4 and Appendix C, which allows to compute $\delta = \frac{1-\breve{\omega}}{\sum \alpha_i \gamma_i)}$.

In order to calculate the parameters of the utility and domestic production functions, we consider the substitutions which are possible, first between time and money resources for the production of some activity, second between money expenditures (or equivalently time expenditures) concerning two different activities. It is sufficient to examine the implication of two over these three types of substitution. First, the substitution between time and money in the domestic production function of activity i generates the first order condition:

$$\frac{\frac{\partial u}{\partial t_i}}{\frac{\partial u}{\partial m_i}} = \omega \rightarrow \frac{\alpha_i}{\beta_i} = \frac{m_i}{\omega t_i} \text{ which implies: } \alpha_i = \frac{m_i}{\omega t_i + m_i} \text{ and } \beta_i = \frac{\omega t_{ii}}{\omega t_i + m_i} \quad (7)$$

under the constraint of a constant economy of scale for each production function: $\alpha_i + \beta_i = 1$.

Second, the substitution between times $t_i$ and $t_j$ in the domestic production of two different final goods i and j implies another condition between the parameters of the domestic production functions and the utility function:

$$\frac{\beta_i \gamma_i}{\beta_j \gamma_j} = \frac{t_i^2 m_j}{t_j^2 m_i} \text{ so that: } \frac{t_i}{t_j} = \frac{\gamma_i \alpha_i}{\gamma_j \alpha_j} \text{ or } \gamma_i = \gamma_1 \frac{t_i}{t_1} \frac{\alpha_1}{\alpha_i} \quad (8)$$

All other substitutions between monetary and time resources devoted to different final goods can be derived from (7) and (8). Finally, we suppose that all marginal productivities are positive: $\alpha_i, \beta_i, \gamma_i \geq 0$; that there are no economies of scale in the domestic production functions: $\alpha_i + \beta_i = 1$; and we normalize the utility: $\sum \gamma_i = 1$.

*Estimation of the parameters of the utility function*

In order to estimate these parameters, we calibrate the opportunity cost of time in a first stage, for instance at the constant level of the minimum wage rate. Equations (7) thus gives an estimate of $\alpha_i$ and $\beta_i$ for each household, which gives $\gamma_i$ by equation (8) in term of $\gamma_1$ (supposing for instance $\gamma_1 = 1$). Dividing the $\gamma_i$ by their sum, we obtain values of these parameters summing to one. In the second step, an estimate of the opportunity cost of time $\omega$ is given by equation (5) which allows the computation the individual values of the parameters $\alpha, \beta$ and $\gamma$ for each household using equations (7) and (8). These values enter equation (5) to give for each household the second step estimate of $\omega$.

The estimation is made on six activities, excluding expenditures on health and education, which contains many zeros. The estimation of equations (7) and (8) is made for data grouped into cells (defined by the household size, the education level and age of the head) in order to obtain robust estimates of the parameters $\alpha, \beta$ and $\gamma$ of the utility and domestic production functions. Table B1 in Appendix B contains the average estimates of these parameters. Using equation (5), the estimate of the opportunity cost of time is obtained for each household. It averages 6.72, with a range between 5.3 and 10.3. It is thus

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significantly smaller than the average net wage rate (9.64), with 73% of households having an opportunity cost lower by more than one quarter than its net wage rate.

That estimation, close in average to the minimum wage rate (6.92), corresponds qualitatively to the answer of individuals in direct surveys on their substitution between time and money, which usually give an opportunity cost of time lower than the agent's wage rate net of taxes. Moreover, $\omega$ is positively indexed on the household's net wage (with an elasticity of 0.85) and on income (conditional to net wage: elasticity of 0.19). It is also positively indexed on the household *relative* income, defined by the ratio of the household's average income and the average income of a reference population defined by the aggregation used in the estimation of the parameters of the utility and domestic production functions. It increases till the household's head is 45 years old, then decreases 15 years later. It also increases with family size, especially with respect to the number of adults, which may show that home production is more valued in large families because of economies of scale (i.e. production of public goods).

*A special case*

Another set of first order conditions can be obtained supposing that all substitution have been made between money and time expenditures and that the opportunity cost of time is the same for all activities. In this case, equation (5) rewrites:

$$\omega = \frac{u'_{t_i}}{u'_{m_i}} = \frac{\gamma_i \beta_{ih} m_{ih}}{\gamma_i \alpha_{ih} t_{ih}} \frac{\frac{\partial t'}{\partial T}}{\frac{\partial m'}{\partial Y}} = \frac{\beta_{ih} m_{ih}}{\alpha_{ih} t_{ih}} \frac{\frac{\partial t'}{\partial T}}{\frac{\partial m'}{\partial Y}} \text{ so that } \omega = \frac{\frac{1}{n}\sum_i \beta_{ih} m_{ih}}{\frac{1}{n}\sum_i \alpha_{ih} t_{ih}} \frac{\frac{\partial t'}{\partial T}}{\frac{\partial m'}{\partial Y}} \quad (9)$$

In this case, the knowledge of the utility function (i.e. parameters $\gamma_i$) is no more necessary for the computation of $\omega$ which depends only on the parameters $\alpha_i, \beta_i$ of the domestic production functions (which are the elasticities of domestic production over the monetary and time factors) They are supposed to lie between 0 and 1, so that the opportunity cost can be considered as the ratio of the weighted means of the monetary and time inputs.

Consider first the special case where all domestic production functions have proportional factor elasticities: $\alpha_i = \alpha \lambda_i$ and $\beta_i = \beta \lambda_i$. The opportunity cost of time in that case is:

$$\omega = \frac{\beta m_i}{\alpha t_i} = \frac{\beta}{\alpha} \frac{\sum_i m_i}{\sum_i t_i} = \frac{\beta}{\alpha} \frac{V + w t_w}{w(T - t_w)} w \quad (10)$$

with $V + w t_w$ the monetary component of full income and $w(T - t_w)$ the market value of leisure time used in the domestic production of final goods. Note that the ratio of the monetary component over the time component of full income differs between households. Also, for usual situations, this ratio is most probably smaller than one[6], so that $\omega < \frac{\beta}{\alpha} w$. If we suppose moreover that the domestic production functions have equal money- and time-elasticities of production ($\alpha = \beta = 0.5$) and if we suppose that market labour time is around

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[6] Suppose a bachelor spends half of his disposable time (12 hours per day) working on the market for five days a week and the residual for consumption and leisure. The average market labour time over the year will be around 35 hours per week, while the second component of disposable time should be around 68 hours.

one third of total disposable time in the family, the opportunity cost of time for this model is equal to 3.85, which is no more than the half of market wage net of taxes.

This hypothesis of constant money and time elasticities $\alpha$ and $\beta$ in home production implies the constancy of the monetary over the time components (in monetary value) for all full expenditures. Table A2 in Appendix A shows that this is not at all verified.

Another rough calibration of the opportunity cost of time could also be obtained by considering the average values of the coefficients and variables in this ratio and taking the average monetary budget shares $\overline{w_i}$ as coefficients $\gamma_i$ in the utility function. We replace m' and t' by the empirical means of total expenditures and time for home production (table A1). $\alpha_i$ and $\beta_i$ are defined by their ratio in the first order condition 7 and 8 (using the wage rate as the opportunity cost for time) and by supposing the absence of economy of scale in the domestic productions ($\alpha_i + \beta_i = 1$). Using the results of table 1, we obtain $\sum \alpha_i \overline{w_i} = 0.4534$, $\sum \beta_i \overline{w_i} = 0.5476$ and finally $\omega = 3.19$, which is close to the previous rough estimate.

Therefore, these two calibrations give rise to estimations of the opportunity cost of time which are also smaller than the household average net wage. This result seems more realistic than the equality with the household's net market wage, which is the usual assumption in the literature, since it corresponds to the typical answers in subjective evaluation of their opportunity cost by individuals.

## 4. Income flexibility, risk aversion and welfare index

Suppose we can estimate the exponent $\delta \sum \alpha_i \gamma_i$ of the total monetary expenditure in our utility (equation 4). We remarked that for our constant relative risk aversion specification of the utility function, the Frisch income flexibility $\breve{\omega}$ is $(1 - \delta \sum \alpha_i \gamma_i)$. Frisch (1959) supposed that the income flexibility is equal to -2 for the better part of the population, -0.5 for the middle class. In order to calibrate $\delta$ in the utility function, we propose to estimate a demand system under the assumption of strong separability of the utility (for instance a Linear Expenditure System or a Rotterdam model under additive separability). Indeed, in the case of strong separability of the utility, $\breve{\omega}$ appears in the price coefficient and can be directly estimated within the demand system. Thus, $\delta$ can be recovered for each observation whence $\alpha_i$ and $\gamma_i$ have been estimated.

Theil has shown in various studies based on the estimation of the Rotterdam model *on macro time-series* (see Theil, 1980, Theil-Clements, 1987, Selvanathan, 1993) that the income flexibility is quite stable across time and countries, and averages −0.5 (see Selvanathan, p. 308, for the discussion of 322 estimates over 18 countries and 29 years). This parameter is usually estimated on time-series through a Rotterdam model. The Rotterdam system of demand proceeds from a Taylor expansion of logarithmic demand functions, which ends in differential demand functions which are generally estimated in discrete form between two periods. In Appendix C, we reformulate this Taylor expansion for household h in terms of the difference within sub-populations characterized by similar preferences and budget constraints (cells H defined by crossing various households' socio-economic characteristics):

$$w_{ih}(Dq_{ih} - DQ_h) = \beta_i DQ_h + \breve{\omega}^{-1}(w_{ih} + \beta_i)Dp'_{ih}$$

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with $x_H$ the average of $x_H$ in cell H containing household h, $Dx_h = \ln(x_h/x_H)$, $q_{ih}$ real expenditures by unit of consumption for good i for household h, $DQ_h = \sum_i mw_{ih} . Dq_{ih}$, $w_{iH}$ the average budget share for cell H and $Dp'_{ih} = Dp_{ih} - \sum_j (w_{iH} + \beta_i) Dp_{jh}$. This allows to estimate the Rotterdam demand system on cross-sectional data.

The estimation of the Rotterdam model under the assumption of strong separability of the direct utility gives: $\breve{\omega}^{-1} = -1.18$ ($\sigma = 0.042$) (see detailed results in Appendix C). This value is used to calibrate the exponent of the utility function (4). The negative of the Frisch's flexibility of the marginal utility of money (which is also the Arrow-Pratt index of relative risk aversion) is estimated as: $-\breve{\omega} = 0.845 = (1 - \delta \sum \alpha_i \gamma_i)$. This value is used to calculate price elasticities under strong separability in tables 1 and 2. The multiplicative exponent $\delta$ of the utility function is therefore 0.33 for an estimated $\sum \alpha_i \gamma_i = 0.4695$. Thus, the direct utility function is completely known and can be used to calculate the change in welfare linked to a price change or the change of any other determinant of the demand. Such an application have been performed in Canelas-Gardes-Salazar (2013) for a change in the indirect tax on food consumption in Ecuador and Nicaragua.

## 5. Econometric methodology

The Almost Ideal Demand System is the most commonly used model to estimate demand elasticities. One of the main advantages of the model is that even if the model is nonlinear, one can use a Stone price index to approximate the AI model to its linear version LAIDS, so as to facilitate estimation. Three main problems are derived from this approximation. First of all, as pointed out by Pashardes (1993), the errors coming from that approximation can result in biased parameter estimates, as it can be seen as an omitted variable. The bias is bigger when the AI model is applied to micro-data, because in this case the expenditure effects are highly correlated with the demographic characteristics of the household and thus very heterogeneous between households. In order to correct this bias, Pashardes proposes a simple re-parameterization of the price parameter that circumvents the problem created by the stone price index[7].

Four specific problems appear in the estimation on matched data: *first*, the utility model presented in section 3 indicates a relation between the monetary and time component of the full expenditure (equation 7) which allows to consider an optimization based on the monetary component (for instance a dual model based on a Piglog cost function which leads to a Working specification of the demand system) with full prices being proxies to the scarcity which gouverns the monetary choices: under this postulate, the Almost Ideal demand system writes for the monetary expenditures:

$$w_{im} = \alpha_i + \beta_i \log\left(\frac{y_{mh}}{\pi_h}\right) + Z_{ih}\gamma_i + \varepsilon_{ih} \quad (11)$$

with $w_{im}$ the monetary budget share, $y_{mh}$ the household monetary income, $\pi_h$ the price index and $Z_{ih}$ other explanatory variables (including full prices and socio-economic characteristics).

It is also possible to consider that the optimization applies independently to monetary and for time allocations, but in this case the demand system for full expenditure cannot be

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[7] Two other problems concerning the price index have been recently discussed in the literature: first, the approximation made to linearize the model generates error-in-variable, which yields to inconsistent SUR and IV estimates. Second, the Stone price index is not invariant to changes in units of measurement (see Buse, and for a discussion).

similar to the equations for monetary and time expenditures. Supposing that full expenditures follow an independent optimization scheme, based either on a utility function or a cost function, implies a total substitution between time and monetary household's expenditures. It is more plausible to suppose that two independent optimization exist for monetary and for time allocations. If for instance the cost functions for the monetary and the time expenditures are supposed to be Piglog, both demands are specified as an Almost Ideal demand system (with different parameters): for the monetary expenditures, the demand function writes:

$$w_{im} = \alpha_i + \beta_i \log\left(\frac{y_{mh}}{\pi_h}\right) + Z_{ih}\gamma_i + \varepsilon_{ih} \quad (11)$$

with $w_{im}$ the monetary budget share, $y_{mh}$ the household monetary income, $\pi_h$ the price index and $Z_{ih}$ other explanatory variables (prices, socio-economic characteristics). The same specification can be written for the time expenditures. But in that case, the budget share for full expenditures $w_{if}$ depends on the monetary and time budget shares: $w_{if} = \frac{y_m w_{im} + y_t w_{it}}{y_m + y_t}$ and the resulting demand equation for full expenditure cannot be written under as Almost Ideal specification because of the non-linearity in the income variable.

Following this hypothesis of a separate optimization for the two components of the full expenditure, we can calculate the full income elasticity $E_{if}$ in terms of the separate monetary and time elasticities $E_{im}$ and $E_{it}$ estimated separetly:

$$E_{if} = E_{im}.\frac{w_{im}}{w_{if}}.\frac{1}{1+k} + E_{it}.\frac{w_{it}}{w_{if}}.\frac{k}{1+k} \quad (12)$$

with k the derivative of the temporal income over the monetary income. The income coefficient is fixed in the estimation of the full expenditures demand system.

Both full prices, full total expenditures and full budget shares depend on the opportunity cost of time and the unit times for each activity $\tau_i$. Calibrating income effects in the estimation of the demand system on full expenditures and full prices is a way to suppress the possible endogeneity of the full total expenditure. The endogeneity of full prices can be taken into account by instrumentation or by defining the full price by means of an opportunity cost of time different from what is used to calculate full budget shares (for instance full prices are defined using the household's net wage rate as the opportunity cost for time, while full expenditures are calculated using the minimum wage rate). In our application, the estimation is performed on monetary expenditures over the full prices indicators. In this case, no endogeneity can be provoked by the full prices as budgets shares and total expenditure do not depend on the time component of full expenditures.

*Second*, quality effects are likely to exist in full price and expenditure data. Indeed, an increase (in the cross-section dimension i.e. between two households) of the full price for commodity (activity) i may result either from the difference (between the two agents) of the opportunity cost ω or from the difference of their time allocated to activity i. Both causes may increase the quality of this activity, by means of an increased productivity (which can be supposed to be positively related to ω) or of the time devoted to i. This endogenous quality appears in the same form as in Deaton's technique to estimate price-elasticities on local prices after removing the quality incorporated in unit values (which is the ratio of expenditures over quantities consumed). In our matched dataset, local prices are replaced by the individual full prices for each household.

Deaton (1988) shows that the elasticity of expenditures Q$_i$ over its unit value V$_i$ writes

$$\varepsilon_{ip} = \frac{\partial lnQ_i}{\partial lnV_i} = \frac{E_{ip}}{1+\eta_i\frac{E_{ip}}{E_{iy}}}$$

with $E_{ip}$ the true price-elasticity, $E_{iy}$ the income-elasticity and $\eta_i$ the income-elasticity of the unit value. This formula allows to calculate the true price-elasticity in terms of the other parameters. In order to estimate $\boldsymbol{\eta_i} = \frac{\partial lnV_i}{\partial lny}$, the two equations model *(Deaton's equations 14 and 15)* is written for household h in cluster *C*:

$$w_{hC} = \alpha_1 + \beta_1 lny_{hC} + Z_{hC}\gamma_1 + u_{1hC} \quad (13)$$
$$lnV_{hC} = \alpha_2 + \beta_2 lny_{hC} + Z_{hC}\gamma_2 + u_{2hC} \quad (14)$$

We define clusters as households which are supposed to have the same opportunity cost of time and the same domestic production (thus the same $t_i$) for instance by means of a common age class, location and education of the head. We thus estimate η= $\beta_2$ by (14) within clusters and $E_{ip}=\gamma_1$ and $\beta_1=E_y$ by (13) with full individual prices included in $Z_{hc}$.

The third problem concerns the correction of variances necessitated by the fact that budget times are generated regressors. Budget times are predicted for each household of the Family Expenditures survey from the time budgets recorded in the Time Use survey. These estimated times are added to the household's monetary expenditures to form the household's full expenditures. These expenditures serve to calculate indices of scarcity which are used as full prices $\pi$ in the estimation of the demand system $h(x,\beta,w,\pi)$ where w is the set of variables used in the first step to predict $\pi$ and w, $\beta$ the variables and parameters of the demand functions. Thus, the full prices are generated in a first step before the estimation of the demand system, which necessitate to correct the estimated variances. Murphy and Topel (1985) proposed a method adapted to this case. Their theorem (Greene, 2000, Chapter 10) states that the second step estimator $\beta$ is consistent and asymptotically normal with an asymptotic covariance matrix:

$$V_{\beta^*} = \sigma^2 V_b + V_b[CV_cC' - CV_cR' - RV_cC']V_b$$

where $V_b$ is the covariance matrix given by the second step of the estimation,
$C = n\,plim\frac{1}{n}\sum_{i=0}^{1} x_i^0\hat{\epsilon}_i^2(\frac{\partial h(x,\beta,w,\pi)}{\partial \pi})$ and $R = n\,plim\frac{1}{n}\sum_{i=0}^{1} x_i^0\hat{\epsilon}_i(\frac{\partial g(w,\pi)}{\partial \pi})$ with g depending on the log-likelihood function. In the case where $\pi$ is predicted by a linear regression, C writes:

$$C = d\sum_{i=1}^{n} e_i^2 x_i w_i$$

with $e_i$ the error term of the demand function in the second step and d the coefficients in the estimation of $\pi$ in the first step. R is null if the regression disturbances of the regression in the first and second steps are uncorrelated, which is the case of our model since full prices are predicted from the Time Use surveys and used as a regressor on the family Expenditures survey. Finally, we obtain $C = \sum_{i,j} d_i d_j d_{ij}$ depending on the coefficients and covariance terms of the first step regression.

A simple bootstrap procedure is an alternative to the Murphy-Topel method. Consider that the logarithmic full prices are estimated using the activity times predicted by the actual times observed in the Time use survey and can be considered as instrumented values of the

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actual full prices. The usual method to correct the variances in case of instrumentation compares the residuals $\hat{\varepsilon}_1 = y - X\hat{\beta} - ln\pi^{IV}\hat{\gamma}^{IV}$ estimated in the second step regression using the logarithmic prices defined by the predicted times with the residuals computed with the parameters issued from instrumentation and the actual value for full prices $ln\pi$: $\hat{\varepsilon}_2 = y - X\hat{\beta} - ln\pi\hat{\gamma}^{IV}$. In this case, actual full prices are not observed in the monetary statistics. We can however simulate them knowing the distribution for the set of prices for household h issued from the prediction of prices: $ln\pi_h \sim N(\log[\pi^{IV}(h), V_i]$ supposed to be normal (as indicated by the distribution of the logarithmic full prices) with the estimated price as the mean of the distribution. The variance $V_i$ of the logarithmic price for activity i is estimated by the empirical mean of $ln\pi^{IV}$ computed on the monetary survey. This residual writes for household h:

$$\hat{\varepsilon}_2(h) = \varepsilon_1(h) + \sum \hat{\gamma}_i[ln\pi_i^{IV}(h) - ln\pi_i(h)]$$

so that: $V(\hat{\varepsilon}_2) = V(\hat{\varepsilon}_1) + \sum \hat{\gamma}_i^2 V_i$.

This procedure will be preferred because of its simplicity and the fact that it takes fully into account the non linear nature of predicted logarithmic full prices. Both procedures give similar inflation of standard errors (for instance 156% and 165% for housing expenditures, 119% and 152% for transportation).

Finally, endogeneity may appear in the full demand equations because the opportunity cost of time (and the unit time for activity $\tau_i$) appear both in the full expenditure for i, in the full total expenditure and in the vector of full prices for all commodities. This problem exists because full prices are endogenous, depending on the household type and characteristics (in classic demand systems, prices are on the contrary pre-determined and generally supposed to be constant across the population). This possible endogeneity bias can be taken into account by instrumentation of full prices and full total expenditure or GMM. Also, it is possible to calibrate the full income elasticity by formula (12), using monetary and time elasticities estimated by two demand systems written respectively on monetary and time expenditures. In our estimations, we check that defining prices by an alternative valuation of time than the opportunity cost of time used to define the household's expenditures and full income (for instance, full prices are defined using the minimum wage rate as the opportunity cost of time while full expenditures are computed with an econometric estimate of the household's opportunity cost) gives similar estimates than the model using the opportunity cost to valuate all variables. Another way to estimate full price elasticities consists in estimating the demand system on monetary expenditures and full prices. It gives price elasticities similar to those obtained by the previous methods on full expenditures, so that it is the method which we use in the empirical application.

The resulting price elasticities are computed for semi-aggregate expenditures such as food at home, food away, housing expenditures..., corresponding to the activities of domestic production previously defined. The computation of price elasticities for sub-categories such as bread, wine, tea... is presented in section 7.

## 6. Results

We present the results for the price and income elasticity for the French survey in the tables underneath. Table 3 includes the price elasticities estimated by LAIDS on monetary budget shares with quality correction and under separability restrictions. The price elasticity estimates under strong separability are estimated for two calibration of the inverse of the income flexibility at -0.5 and -1.18 which is the estimated value for this dataset on an independent Rotterdam model.

Table 1 shows that full price elasticities estimated on full budget share by a LAIDS system (calibrating the full income elasticity by formula 12) are very similar to those obtained on the equation using monetary expenditures.

**Table 1**
**Income and full price Elasticities**

|  | Income and time elasticities | | | Full Own-price elasticities | | |
|---|---|---|---|---|---|---|
|  | Monetary income | time | Full income* | a | b | c |
| Food | 0.774 (0.0154) | 0.993 | 0.875 (0.0170) | -0.973 (0.0049) | -0.749 (0.0032) | -0.741 (0.0034) |
| Housing | 1 .023 (0.0129) | 0.698 | 0.899 (0.0204) | -1.350 (.0121) | -1.059 (0.0130) | -1.082 (0.0115) |
| Clothing | 1 .131 (0.0242) | 0.926 | 0.876 (0.0327) | -0.905 (.0040) | -0.827 (0.0042) | -0.819 (0.0041) |
| Transport | 1.021 (0.0174) | 1.342 | 0.913 (0.0222) | -0.873 (.0041) | -0.717 (0.0046) | -0.716 (0.0043) |
| Leisure | 0.920 (0.0190) | 1.045 | 1.213 (0.0144) | -1.126 (.0062) | -0.655 (0.0039) | -0.666 (0.0034) |
| Other | 1.081 (0.0082) | 1.087 | 0.958 (0.0182) | -1.164 (.0089) | -1.006 (0.0040) | -0.994 (0.0039) |

*Equation (12)
Full Own-price elasticities: (a) monetary budget share and income; full price calculated with the estimated opportunity cost of time; (b,c,d) full budget shares; (b) all variables defined by $\hat{\omega}$, full income elasticity calibrated by means of the monetary income and time elasticities (equation 12); (c) full budget share defined by the minimum wage rate, income by the household's net wage rate, full prices by $\hat{\omega}$.

Using full price elasticities $E_\pi$, the elasticities over the own-monetary price $E_{p_i}$, the time used for the consumption activity $E_{t_i}$ and the opportunity cost of time $E_\omega$ for activity i are easily recovered (see for instance De Vany, 1974[8]) and can be calculated by mean of the full expenditures and its monetary and time components:

$$E_{p_i} = E_{\pi_i} \frac{p_i}{\pi_i} = E_{\pi_i} \frac{p_i x_i}{\pi_i x_i} = E_{\pi_i} \frac{m_i}{m_i + \omega t_i}$$

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[8] Note an error in De Vany's formula (11) for the opportunity cost elasticity.

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$$E_{t_i} = E_{\pi_i} \frac{\omega \tau_i}{\pi_i} = E_{\pi_i} \frac{\omega t_i}{m_i + \omega t_i}$$

$$E_\omega = \sum_i E_{\pi_i} \frac{\omega \tau_i}{\pi_i} = \sum_i E_{\pi_i} \frac{\omega t_i}{m_i + \omega t_i}$$

**Table 2**
**Income, price and opportunity cost Elasticities**

| | Monetary income | Full income* | Full price | Monetary price | Time | Opportunity Cost | Grouping method ** | Strong separability*** Φ −0.5      −1.18 |
|---|---|---|---|---|---|---|---|---|
| Food | 0.774 (0.0154) | 0.875 (0.0170) | -0.973 (0.0049) | -0.349 (0.0018) | -0.624 (.0031) | 0.210 (.0062) | -0.810 (0.169) | -0.459   -0.924 (.0111) (.0218) |
| Housing | 1 .023 (0.0129) | 0.899 (0.0204) | -1.350 (.0121) | -0.930 (0.0083) | -0.420 (.0038) | 0.550 (.0104) | -0.383 (0.150) | -0.680   -1.136 (.0082)   (.0118) |
| Clothing | 1 .131 (0.0242) | 0.876 (0.0327) | -0.905 (.0040) | -0.581 (0.0026) | -0.324 (.0014) | 0.532 (.0110) | -0.527 (0.066) | -0.635   -1.281 (.0170)   (.0369) |
| Transport | 1.021 (0.0174) | 0.913 (0.0222) | -0.873 (.0041) | -0.468 (0.0022) | -0.404 (.0019) | 0.469 (.0109) | -0.549 (0.010) | -0.543   -1.191 (.0234)   (.0234) |
| Leisure | 0.920 (0.0190) | 1.213 (0.0144) | -1.126 (.0062) | -0.308 (0.0017) | -0.818 (.0045) | -0.463 (.0062) | -1.306 (0.032) | -0.532   -1.074 (.0144) (.0278) |
| Other | 1.081 (0.0082) | 0.958 (0.0182) | -1.164 (.0089) | -0.661 (0.0051) | -0.503 (.0038) | 0.531 (.0206) | -0.953 (0.142) | -0.592  − 1.245 (.0178) (.0366) |

*Quality effect corrected by Deaton's method

**Hicks-Lewbel method

*** Frisch formulas for the own-price and cross-price elasticities $E_{ii}$ and $E_{ij}$(Φ=-0.5 or -1.18; $E_i$ = income elasticity for expenditure i; $\delta_{ij} = 0, i \neq j; \delta_{ii} = 1$): $E_{ij} = \Phi E_i \delta_{ij} - w_j E_i (1 + \Phi E_j)$; See a discussion of the calibration of the income flexibility in Appendix C.

Some important results come out from the estimations: first of all, we observe that all the (compensated) monetary own-price elasticities are significantly negative. The estimates range from -1 and 0. If we compare to the macroeconomics estimations that oscillate often between -0.1 and -0.3 for semi-aggregate commodities, our estimates are much higher. As we already pointed out elasticities derived from macroeconomic data face measurement errors and aggregation bias.

Second, we observe that the correction of quality decreases by 20% in average the magnitudes of the elasticities estimates, for both full price elasticities and monetary goods

elasticities. This is consistent with the theory as the quality is included in the price, so once the quality effect is corrected the elasticity is smaller.

Third, regarding the estimation under the strong separability assumption of utility, we observe a significant distance with the LAIDS estimations. By the way, the estimation under separability depends heavily over the calibration of the income flexibility. The price elasticity parameters under strong separability (for $\Phi = -0.5$) have a smaller magnitude than those estimated without the latter restriction. In spite of this large difference, we observe that there are at least three commodity groups (food at home, clothing and transportation) for which those estimations are slightly similar, but these parameters differ for the other calibration of the income flexibility.[9] We can also compare the formula used by Pigou which relates the direct price elasticity with the income elasticity coefficient $\frac{|E_{ii}|}{E_i} = 0.5$ to our rather close estimate of this ratio: 0.6. Nevertheless, this ratio is close to one for two commodity groups (Housing and Other expenditures). We can therefore strongly suspect this hypothesis of strong separability.

Fourth, all time elasticities are negative, and their magnitudes are not related to the income elasticity. Their definition shows that this is caused by the absence of systematic relationship between the monetary and time component of the full expenditures. The elasticity of expenditures as regard the opportunity cost of time are positive for all items except leisure expenditures: this is probably explained by the fact that time plays a prominent role for these expenditures (the proportion of time in the full expenditures is equal to 81% compared to 57% for all other expenditures).

Fifth, price and time elasticities change significantly between different types of households, for instance for bachelors, couples without children and families with children (see the results in Appendix IV): all price effect (as well as the elasticities against time or against the opportunity cost for time) increase with the family size. The larger sensibility for prices for large families can be related to the notion that of household's needs increase (conditional to income) with their size (see Gardes and Merrigan, 2007, Gardes and Loisy, 1997).

### 7. Price-elasticities for disaggregated expenditures

Micro-simulation exercices often need to calibrate income and price effect at a more precise level of the expenditures, for instance for alcoholic beverages when a specific tax is applied to them. Suppose this specific item i is included in the broad consumption activity I (food). The total expenditure for this semi-aggregate I writes: $P_I Q_I = \sum_{i \in I} p_i q_i$.

Income elasticities could be estimated, as concerns monetary income as well as full income, by means of the estimation of demand equations for the disaggregate item of expenditures (using for all demand functions the full price index of the broad category to which this item pertains, as no full price can be defined for such a precise consumption). For such a demand function, biases may occur in the estimation of the income coefficient only in the unlikely case where the full price of the broad category is correlated to the full income, while the specific price for the disaggregate item is not.

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[9] Ayanian (1969) compares Barten's estimates of price-elasticities to those obtained through the Frisch formula: the average difference is only 19.4%, and this difference is always smaller than two squared error (and smaller than one squared error for 7 groups over 14).

As far as price effects are concerned, price elasticities for individual items within the semi-aggregate consumption $I$[10] may be estimated using the full prices of the broad activities $P_I$. But this would suppose that all the variations in the specific price of the specific expenditure are reflected in the variations of $P_I$ (which, for instance, is not the case for changes in special taxations on goods or services such as tobacco or restaurants). Moreover, the substitution or complementarity between disaggregate consumptions can only be estimated by means of individual prices for these disaggregate items.

First, in order to calculate price elasticities for disaggregate items, we define the price index $P_I$ for the broad consumption activity I as a true cost-of-living index proportional to the cost function $C(p,u)$[11]: $P_I = kC(p,u)$. This cost corresponds to the expenditure for prices p at the constant utility level u[12]: $C(p,u) = \sum_{i \in I} p_i q_i$ for the hicksian demand $x_i = h_i(p,u)$. Using the Shephard lemma to calculate the hicksian demand we obtain the elasticity of the aggregate price $P_I$ over the individual price $p_i$ as: $E_{P_I/p_i} = \frac{\partial lnP_I}{\partial lnp_i} = \frac{k\partial C}{\partial p_i}\frac{p_i}{kC} = \frac{p_i x}{P_I X_i} = w_i$, so that the logarithmic price index can be written as an Aftalion-Stone index: $lnP_I = a_I + \sum_{i \in I} w_i lnp_i$. The budget share $w_i$ is fixed at its average over the whole population.

Second, in order to establish a relationship between the own-price elasticity of the aggregate expenditure and elasticities of individual expenditure, we calculate the derivative of the aggregate expenditure over its aggregate price:

$$\frac{\partial P_I X_I}{\partial P_I} = \sum_{i \in I} \frac{\partial p_i x_i}{\partial P_I} = \sum_{i,j \in I} \frac{\partial p_i x_i}{\partial p_j}\frac{\partial p_j}{\partial P_I}$$

Conditional to all other prices, the elasticity of the aggregate price index $P_I$ over the individual price $p_i$ is equal to $w_i$, the budget share of the individual item i. This would imply the following relation between aggregate and individual elasticities:

$$E_{P_I X_I/p_I} = \frac{\partial P_I X_I}{\partial P_I}/X_I = \sum_{i,j} \frac{w_i}{w_j} E_{p_i x_i/p_j}$$

In that case, increasing the number of individual consumption in the broad activity would increase the aggregate elasticity if these individual consumption have similar price elasticities. That is highly improbable. The reason is that the general case when considering a change in the price of the aggregate is that all prices of the individual items change too, and the normal hypothesis is that they change approximately at the same rate. The price elasticity for the aggregate can therefore be written:

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[10] Cross-price elasticities of an individual item k (which pertains to another aggregate K) with respect to $p_i$ is supposed to be equal to the elasticity with respect to the aggregate price index $P_I$.

[11] Following the formulas in Fan et al., 1995, appendix. We suppose here a strong separability between expenditures on semi-aggregate commodities I, so that the utility level in this cost function depends only on consumptions in the group I.

[12] That utility level applies to consumptions in group I, so that we have to suppose that preferences are separable between broad consumption bundles. Another way is to consider that this utility is conditional to all other expenditures in other broad groups.

$$E_{P_I X_I / p_I} = \sum_{i,j} E_{p_i x_i / p_j} \frac{p_i x_i}{p_j} \frac{p_j}{p_I X_I} = \sum_{i,j} w_i E_{p_i x_i / p_j} \text{ as } \frac{\partial p_i}{\partial P_I} = \frac{p_i}{P_I} \qquad (15)$$

Third, we assume that individual expenditures are strongly separable within the aggregate. This hypothesis allows to compute direct and cross-price price elasticities by means of the Frisch formula:

$$E_{ij} = \Phi E_i \delta_{ij} - E_i w_j (1 + \Phi E_j)$$

Here $\Phi$ is the Frisch income flexibility (equal to the inverse of the Frisch's flexibility of the marginal utility of money $\breve{\omega}$)[13]. Finally, we normalize these disaggregated price elasticities by multiplying them by the ratio of the sum on the left term of equation (15) over the aggregate elasticity on the right, in order to define price elasticities verifying this additivity constraint.

As an application, we use estimates of the income elasticities given by Darmon (1983) for French time-series of households' expenditures. Two calibration of the income flexibility $\Phi = \breve{\omega}^{-1}$ are used: the value -0.5 proposed by Frisch for the middle income bracket and our estimate -1.18.

**Table 3**
**Price-Elasticity under strong separability**

| Item | Purchase of vehicule | | Expenditure for private transport | | Public transport | | All transport expenditures | Own-price elasticity (Darmon) |
|---|---|---|---|---|---|---|---|---|
| Income flexibility $\Phi$ | -1.18 (0.0042) | -0.5 | -1.18 | -0.5 | -1.18 | -0.5 | - | - |
| Purchase of vehicule | -0.825 (0.061) | -0.323 | 0.339 (0.070) | -0.048 | 0.077 (0.014) | -0.049 | - | -0.21 |
| Private transport expenditures | -0.068 (0.024) | -0.050 | -0.669 (0.012) | -0.436 | 0.284 (0.017) | -0.058 | - | -1.20 |
| Public transport expenditures | 0.057 (0.016) | -0.034 | -0.271 (0.056) | -0.065 | -0.699 (0.054) | -0.264 | - | -1.59 |
| All transport expenditures* | 0.141 (.014) | 0.164 - | 0.600 (.017) | 0.852 - | 0.353 (.010) | 0.130 - | -0.483 | -0.42 |

The income flexibility is calibrated, first at the value estimated on the dataset by a Rotterdam model ($\Phi$ =-1.18), second at the average value proposed by Frisch(-0.5).
Elasticities normalized so that $\sum_{i,j} w_i E_{p_i x_i / p_j} = -0.4683$ (equation 15). Standard errors into parentheses.

$*E_{p_I x_I / p_k} = w_i + \sum_{j \neq k} w_j E_{p_j x_j / p_k}$

Our estimate of the own price elasticity of the total transport expenditure (-0.47) is close to Darmon's time-series estimate (-0.42). On the contrary, time-series own-price elasticities for partial expenditures are very different from our cross-section estimates. Moreover, the weighted sum of Darmon's partial elasticities for the three items (according to

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[13] This parameter would correspond, not to total expenditures, but to the partial expenditure $P_I Q_I$ for the broad consumption activity I. It is calibrated here at the value estimated in section 7.

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formula (15), taking into account only the time-series own-price elasticities given by Darmon, which are the main component of this sum) is much smaller than the own-price elasticity for all transport expenditures, which indicates that these partial elasticities are probably disputable. Thus, the cross-section estimates based on separability and on the price elasticities of the total transport expenditure seem more plausible, although they differ a lot between the two calibrations of the income flexibility.

The substituability between private and public transport expenditures (implying a positive cross-price elasticity), and the complementarity between the two private transport expenditures do not appear clearly. The calibration of the income flexibility Φ changes a lot these estimates of the cross-price elasticities which even change of sign between the two values of Φ. Note that this discussion may be based on the significativity of all these estimations, which depends (for Φ=1.18) on the variance of the income flexibility (since the variance of the income elasticities is not indicated by Darmon). Moreover, the cross-price elasticities are not symmetric, even when both elasticities are significant (as in the case of Private and public transport expenditures)

The calibration of the income flexibility is thus crucial to calculate these price-elasticities. Our preferred set of estimate is obtained for the estimated income flexibility (-1.18), since it affords significant own-price elasticities with a reasonable value compared to the own-price elasticity of total transport expenditures and it is based on the estimate of Φ obtained on the same dataset.

## 8. A tentative explanation of the endogeneity bias on cross-section income elasticities for food

Gardes et al. (2005) explain the difference between cross-section and time-series estimates of income elasticities by an endogeneity effect affecting the cross-section estimation of income effects: the income changes in the cross-section dimension (i.e. the comparison between similar households as far as education, location, family structure… are concerned across the income distribution) does change the household's cost function – in the full dimension (for instance the increase of the opportunity cost of time through the income distribution) or by means of new constraints or a change in some non-monetary resource. These differences between the estimates in cross-section and time-series datasets are due to the presence of latent variables in the cross-section dimension which, being permanent disappear when variables are measured through time. Such are all cohort effects affecting consumption (linked for instance to previous consumption experiments), education level, some permanent disease affecting a member of the family or, to a lesser extent, the household's location. The differences between these local costs can be measured by shadow prices which affect each commodity, so that the structure of expenditures resulting from the household's choices under a local cost function is the same as the expenditures made with a common cost function for the whole population conditional to local shadow prices. These shadow prices give rise to consumption changes which are the same as those produced by the latent variables which operate in the cross-section estimation, but which disappear in the time-series dimension (because they are permanent).

As far as the income effect is concerned, the vector of shadow prices can be revealed comparing the marginal propensities to consume between two households under the local cost functions for household h in period t (demand functions $x_{iht}$) to the marginal propensities defined by a common cost function with local total prices $\pi_{jht}$ aggregating monetary and shadow prices (demand functions $g_i$):

$$\frac{\partial x_{iht}}{\partial z_{ht}^k} = \frac{\partial g_i}{\partial z_{ht}^k} + \sum_j \left(\frac{\partial g_i}{\partial \pi_{jht}}\right) \cdot \left(\frac{\partial \pi_{jht}}{\partial z_{ht}^k}\right)$$

which gives the formula B2 in Gardes et al. (2005):

$$\frac{\partial \pi_i}{\partial z_{ht}^k} = \frac{[\beta_i^{cs} - \beta_i^{ts}]}{\gamma_{ii}} \qquad (16)$$

in terms of parameters $\beta_i$ estimated on cross-section or time-series data. In equation (16), the time-series estimates are supposed to suppress the effect of all permanent variables (such as location or education level of the head), so that the cross-section estimates incorporate these influences while the time-series are independent from them. The marginal change of total prices $\pi_{jht}$ over variables Z thus depends on the difference between these two estimates.

In order to obtain the time-series income elasticity on individual data, the estimation must be performed on panel data. In Cardoso-Gardes (1997), a pseudo-panel of three French Households Expenditures surveys was built in order to obviate the absence of panel data. The cross-section and time-series s of the income elasticity of food at home expenditures are respectively 0.331 and 0.578 in these French surveys (see estimations indicating the same order between cross-section and time-series estimates for Canadian, American and Polish data in Gardes et al., 2005 and 1997). The estimations give a income elasticity of the total food price (for changes over the income distribution in cross-section data) positive and equal to 0.35: a household A having double the income than household B experience, ceteris paribus, food prices, monetary and shadow, greater by 35%. This implies that the estimated income elasticity in the cross-section dimension (between estimate of the income elasticity equal to 0.331), which incorporates the effect of this total price difference, is smaller than the time-series estimate of the income elasticity (within estimate equal to 0.578) for which the influence of all permanent latent determinants of the consumption is cancelled.

The change in the time component of full price parallels the evolution of food total price over the income distribution, as the opportunity cost of time increases with income in the cross-section dimension (with an income elasticity estimated as 0.19) and the time used in food consumption $\tau_i$ can be supposed to be stable across the income distribution. The composition of income and full price change between households A and B can be recovered by the equation, considering only the own price effect of food full price:

$$\frac{\partial x}{\partial Y^{CS}} = \frac{\partial x}{\partial Y}\big|_{\pi^{full}} + \frac{\partial x}{\partial \pi^{full}} \frac{\partial \pi^{full}}{\partial \omega} \frac{\partial \omega}{\partial Y} \qquad (17)$$

which implies a similar equation on elasticities. The elasticity of food expenditures with respect to food full price has been estimated -0.97; the elasticity of full price with respect to

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the opportunity cost of time is the ratio of the time component of expenditures over the corresponding full expenditure. The income elasticity of the opportunity cost is estimated to be 0.19 (conditional to the household's wage rate). This implies that the second term of the left hand side of equation (16) written for elasticities equals -0.14, which is 60% of the difference between the cross-section and the time-series estimated on the French pseudo-panel. Thus, the increase of food full price allows to correct estimates obtained through the Engel curve to obtain estimates incorporating a part of the shadow price influence on expenditures.

## 9. Another estimation of the labour supply function

An alternative approach[14] to the classic estimation of the agent's labour supply involves recognizing that market labor supply is, in our model, the complement of domestic production, their sum being equal to total time available (after deducting time devoted to sleep and other activities necessary to survive). This suggests that it should be possible to estimate labor supply elasticities indirectly, through the estimation of time use functions for home production and consumption. The relation between the allocation of time for market work, home production, investment activities (education, health care…) and leisure can be written as:

$$\sum_i t_i = \sum_i \tau_i x_i = T - t_w \tag{18}$$

Taking the derivative of (10) with respect to the opportunity cost of time gives in terms of elasticities:

$$-E_{t_w/\omega} = \sum_i \frac{\tau_i x_i}{t_w}\left(Ex_{i/\omega} + E\tau_{i/\omega}\right) \tag{19}$$

The elasticity of the time used per unit of market good $\tau_i$ with respect to the opportunity cost of time can be calculated in terms of the elasticity of the market good and the elasticity of substitution between the monetary (market good) and time factors used to home produce the final good (activity) $i$:

$$\sigma_i = \frac{\partial \ln\left(x_i/t_i\right)}{\partial \ln(\omega)} = Ex_{i/\omega} - Et_{i/\omega}$$

so that $E\tau_{i/\omega} = Et_{i/\omega} - Ex_{i/\omega} = -\sigma_i$. We obtain finally the formula for the elasticity of labor supply with respect to the opportunity cost of time:

$$-E_{t_w/\omega} = \sum_i \frac{\tau_i x_i}{t_w}\left(Ex_{i/\omega} - \sigma_i\right) \tag{20}$$

This elasticity of substitution $\sigma_i$ has been estimated by Canelas et al. (2014) as 0.276 ($\sigma = 0.031$) for Food and 0.584 ($\sigma = 0.023$) for all other expenditures. On average, the estimated elasticity of hours of labor supplied with respect to the opportunity cost of time is

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[14] Suggested by David Margolis. See Gardes-Margolis, 2014 for details.

0.861, while the elasticity with respect to wages is 0.732. This result derives mechanically from the relation between wages and the opportunity cost of non-market time (the latter elasticity being multiplied by a factor of 0.85), a result consistent with the literature (see Bargain and Piehl, 2003). Another result worth noting is that labor supply elasticities are estimated to be systematically lower when children are present, regardless of whether the household is a single or a couple. If there is an incompressible minimum amount of time that must be spent on child care in households with children, this will necessarily leave less time available for market work, and this less possible variation in the amount of time spent working in response to any given variation in wages or the opportunity cost of time.

**Conclusion**

Becker's model of domestic production and allocation for time proves convenient to define a new method to estimate price parameter using full prices. This method furnishes precise and quite realistic monetary price elasticities, which compare well with other estimation technics applied to cross-section data. Moreover, it allows to compute elasticities over time use and the opportunity cost of time and offers a new method to estimate the household's labour supply function. Thus, full prices seem to afford good proxies to measure commodities scarcity. Besides, that method allows to estimate price effects for sub-populations (which may be useful for micro-simulation of the effects of tax or price changes) as well as to perform a test of theoretical restrictions such as homogeneity and integrability of the demand functions or separability of preferences. This model allows also, under special conditions, to estimate the household's opportunity cost of time which is used to transform time expenditures into monetary values. This parameter is proved to differ largely between households and to depend on the household's income and wage rate.

The application of the simple scheme proposed by Becker thus allows to estimate: full income and full price elasticities, the monetary price elasticities, time elasticities, the opportunity cost of time and the elasticity of households' expenditures relative to this cost, and finally the household's welfare depending on its monetary expenditures and time uses for domestic production at the individual level. Moreover, an original estimation of models in first difference on cross-sections gives an estimation of the Arrow-Pratt relative risk aversion index for each household.

Another application of the definition of full expenditures and full prices concerns the computation of the full costs of adults and children in the family: an estimation on the same dataset (Gardes-Sayadi-Starzec, 2013, Gardes-Starzec, 2014) proves that the full cost of a child compared to a supplementary adult is greater than the monetary cost, which may have important consequences in terms of public transfers. The definition of individual prices on the cross-sectional survey also permits to prove the existence of substitution through prices (Barten's model, Lewbel's Independent of a Base model) in the estimation of equivalence scales.

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# Appendix A
## Description of the French Households expenditures survey: INSEE Budgets des Familles (2001) and Time-Use survey: Budgets-Temps (1998)

In the *opportunity cost* approach each not working individual is given her expected hourly wage rate on the labour market for one hour of non market domestic activity. Working individuals are given either their actual hourly rate in the market job or a uniform hourly wage rate equal to the legal minimum if their wage appears to be smaller than this minimum. We thus assume that the time use is *perfectly exchangeable* between market and non market activities. Moreover we suppose that the opportunity cost does not depend on the nature of domestic activity nor on the day or period when it is done. We treat the household as a unity where the distribution of monetary and non market budgets is a joint decision of the couple. The potential market wage for the not working was estimated separately for man and woman using the two step Heckman method. The participation equation contains variables describing education, age household type, number of children aged less than 5. In the wage equation (estimated for individuals working full time only) we used education, age, and individual's socio-economic category. For each household we computed the average of the individually estimated expected wages adjusting for income tax (the income tax is computed for every cell using cell averages for taxable income and number of family shares) and obtained hourly wage rate for men and women in the household dividing monthly rate by number of days and hours per day (supposing 22 working days in the month and taking the average of 7h40, 6h labour hours per day respectively for man and woman.). We also compute for each household the average volume of non market activities (working men's non market activities time volume is about 2h36 per day for 3h53 on average in the case of working women) separately for men and women (this data set have prepared in collaboration with C. Starzec, see Gardes-Starzec-Sayadi, 2013, for details).

## Table A1
### Correspondence between expenditures and time use category

| Expenditure | Time use Category |
| --- | --- |
| Food consumption | Eating, cooking, washing up, purchasing food |
| Dwelling expenditures (Including imputed rent) | Cleaning and home maintenance, purchasing goods and services for home, repairing… |
| Clothing | Clothing maintenance (repairing, ironing, washing) |
| Education | Education time (schooling, training, child caring-home , games, readings…) |
| Health care | Health care time (at home and outside) |
| Leisure | Leisure time (cultural, sport, social events, associations, eating away…) |
| Transport | Transport (work, family, friends and associations) |
| Miscellaneous goods and services | Miscellaneous time use |

*Source: Gardes, Starzec, 2014.*

**Table A2**
**Money, time and full expenditure patterns (singles and couples)**

| | Monetary budget shares | Time budget shares | Full expenditure budget shares | yearly monetary expenditure (euros) | full expenditure (money+ time value*) (euros) | Ratio of monetary exp. over time value | Ratio of monetary exp. over time value | Ratio of monetary exp. over time value | Ratio of monetary exp. over time value |
|---|---|---|---|---|---|---|---|---|---|
| | All Population | | | | | | Singles | 2 Adults, no child | 2 Adults with children |
| Eating | 0,180 | 0,278 | 0,232 | 5102 | 14235 | 0.56 | 0.48 | 0.47 | 0.62 |
| Dwelling | 0,331 | 0,11 | 0,223 | 9399 | 13642 | 2.22 | 2.88 | 1.77 | 2.51 |
| Clothing | 0,067 | 0,032 | 0,049 | 1910 | 2977 | 1.79 | 1.60 | 1.28 | 2.24 |
| Leisure | 0,166 | 0,382 | 0,281 | 4706 | 17203 | 0.38 | 0.28 | 0.27 | 0.50 |
| Transport | 0,148 | 0,107 | 0,128 | 4197 | 7821 | 1.16 | 0.72 | 1.03 | 1.25 |
| Other | 0,107 | 0,070 | 0,088 | 3044 | 5362 | 1.31 | 1.08 | 1.13 | 1.53 |
| TOTAL | 1 | 1 | 1 | 2838 | 61240 | 0.86 | 0.78 | 0.69 | 1.01 |

*valuation of time expenditures by the estimated opportunity cost of time.

Under the assumption of strong separability of the utility function, Frisch (1959) showed that price elasticities can be computed by means of the income elasticities and a parameter $\Phi$, named by him the income flexibility, which is the inverse of the income elasticity of the indirect utility (also equal to the Arrow-Prat index of relative risk aversion, see Deaton, 1974). The income flexibility can be recovered by means of an estimation of a demand system under the assumption of separability. Theil has shown in various studies based on the estimation of the Rotterdam model *on macro time-series* (see Theil, 1980, Theil-Clements, 1987, Selvanathan, 1993) that the income flexibility is quite stable across time and countries, between -0.1 and -10, and averages –0.5 for the middle income bracket (see Frisch, 1959, p. 189, and Selvanathan, 1993, p. 308, for the discussion of 322 estimates over 18 countries and 29 years).

This parameter has been estimated through a Rotterdam model. The Rotterdam system of demand proceeds from a Taylor expansion of logarithmic demand functions, which ends in differential demand functions which are generally estimated in discrete form between two periods. We write a similar Taylor expansion in terms of the difference within sub-populations characterized by similar preferences and budget constraints:

$$w_{iH}(Dq_{ih} - Dh) = \beta_i DQ_h + \Phi(w_{ih} + \beta_i)Dp'_{ih}$$

with $Dx_h = \ln(x_h/x_H)$, $q_{it}$ real expenditures by unit of consumption for good i in t, $DQ_h = \sum_i mw_{ih} . Dq_{ih}$, $w_{iH}$ the average budget share for the sub-population H and $Dp'_{ih} = Dp_{ih} - \sum_j (w_{ih} + \beta_i) Dp_{jh}$.

Theil also shows that the inverse of the income flexibility can be derived, for each observation, by considering the covariance between quantities and prices, as:

$\breve{\omega}^{-1} = \frac{\Gamma_h + DQ_h(DP_h - DP'_h)}{\pi'_h}$  with  $\Gamma_h = \sum_i \overline{w}_{iH}(DP_{ih} - DP_h)(DQ_{ih} - DQ_H)$  and  $\pi'_h = \sum_i \theta_i (DP_{ih} - DP_h)^2$, with $x_H$ the average of $x_H$ in cell H containing household h (we reformulate Theil's formula with household h and its reference group H instead of differences between two periods in a time-series).

**Table B1**
**Estimation of the Rotterdam model under strong separability**
**26 mars 2013**

|  | Food | Dwelling | Clothing | Transportation | Other expenditures |
|---|---|---|---|---|---|
| $\boldsymbol{\beta_i}$ | 0.174 (.00062) | 0.371 (.0017) | 0.121 (.00093) | 0.065 (.00044) | - |
| $\Phi$ | -1.184 (.00420) | -1.184 (.00420) | -1.184 (.00420) | -1.184 (.00420) | -1.184 (.00420) |
| $R^2$ | 0.947 | 0.818 | 0.675 | 0.601 | - |

The equation for Other Expenditures is deduced from the additivity constraint.
Number of observations = 8653  Trace of Matrix = 34081.1

## Appendix C
## Price and time Elasticities by sub-population

### Table C1
### Elasticities according to the family size

| Activity | 1 adult, no child | | | 2 adults, no child | | | 2 adults with children | | |
|---|---|---|---|---|---|---|---|---|---|
| | Monetary price | Time | Opportunity Cost of time | Monetary price | Time | Opportunity Cost of time | Monetary price | Time | Opportunity Cost of time |
| Food | -0.327 (.219) | -0.585 (.392) | 0.109 (.839) | -0.359 (.175) | -0.643 (.313) | 0.324 (.675) | -0.360 (.156) | -0.644 (.279) | 0.228 (.594) |
| Housing | -0.863 (1.017) | -0.389 (.459) | 0.658 (1.375) | -0.951 (.682) | -0.429 (.308) | 0.568 (.939) | -1.002 (.631) | -0.452 (.285) | 0.603 (.862) |
| Clothing | -0.557 (.477) | -0.311 (.266) | 0.505 (2.127) | -0.562 (.294) | -0.314 (.164) | 0.494 (1.337) | -0.624 (.315) | -0.348 (.176) | 0.613 (1.466) |
| Transport | -0.435 (.298) | -0.376 (.257) | 0.243 (1.299) | -0.491 (.207) | -0.424 (.179) | 0.625 (1.332) | -0.519 (.195) | -0.448 (.168) | 0.628 (1.212) |
| Leisure | -0.303 (.181) | -0.805 (.481) | -0.638 (.688) | -0.304 (.140) | -0.807 (.373) | -0.473 (.576) | -0.323 (.148) | -0.857 (.394) | -0.412 (.517) |
| Other | -0.587 (.75) | -0.193 (.288) | 0.353 (2.144) | -0.690 (.738) | -0.227 (.243) | 0.707 (2.537) | -0.713 (.608) | -0.235 (.200) | 0.596 (1.969) |
| Population size | 1607 | | | 2688 | | | 3488 | | |

All standard errors multiplied by $10^2$