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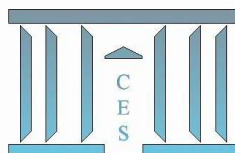
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Fair management of social risk

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Fair management of social risk*

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Abstract – We provide a general method for extending fair social preferences defined for riskless economic environments to the context of risk and uncertainty. We apply the method to the problems of managing unemployment allowances (in the context of macroeconomic fluctuations) and catastrophic risks (in the context of climate change). It requires paying attention to individuals' risk attitudes and rationality properties of social preferences, revisiting basic ideas from Harsanyi's seminal work (Harsanyi, 1955). The social preferences that we obtain do not in general take the form of an expected utility criterion, but they always satisfy statewise dominance. We also show how non-expected utility individual preferences can be accommodated in the approach.

Résumé – Nous présentons une méthode générale pour étendre les préférences sociales équitables définies dans des environnements économiques non risqués aux situations mettant en jeu du risque et de l'incertitude. Nous appliquons cette méthode aux problèmes de l'indexation des allocations chômage (en cas de fluctuations macroéconomiques) et des risques catastrophiques (dans le contexte du changement climatique). Cela implique de s'intéresser aux comportements des individus face au risque et d'étudier l'exigence de rationalité des préférences sociales, revisitant ainsi le travail fondateur de Harsanyi (Harsanyi, 1955). Les préférences sociales que nous obtenons ne sont en général pas des espérances d'utilité mais elles satisfont toujours un principe de dominance état du monde par état du monde. Nous montrons aussi comment notre modèle peut s'adapter au cas où les préférences individuelles ne sont pas des espérances d'utilité.

Keywords: Social choice; Uncertainty; Economic environment; Social risk.

Mots clés : Choix social; Incertitude; Environnement économique; Risque social.

JEL Classification numbers: D63.

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1 Introduction

Public policies are always fraught with uncertainty about their consequences. Large parts of the public economics literature incorporate risk and uncertainty as a simple extension of the commodity space. Indeed, the extension to risk and uncertainty appears immediate when commodities can be relabelled as contingent on the realization of a state of nature, as in general equilibrium theory (Debreu, 1959, chap. 7). In this extended commodity space, the standard social welfare analysis can be applied, it is thought, with little change.¹ In particular, the Pareto principle should be applied *ex ante*.

Most of the literature on risk and insurance policy, for instance in health, unemployment and environmental hazards,² adopts this *ex ante* methodology without acknowledging the difficulties it raises for key principles of social rationality and *ex post* equity. Harsanyi (1955), though, had early on warned about the specific constraints that the context of risk imposes on social welfare evaluation. He proved the following theorem: if (i) all individuals and the social observer are expected utility maximizers on the set of simple lotteries over a set X ; (ii) whenever all individuals prefer lottery p to lottery q , the social observer must also prefer p to q (the Pareto principle); then the social observer's utility function must be an affine combination of individuals' von Neumann-Morgenstern (vNM) utility functions. This result is known as Harsanyi's aggregation theorem.³ Although Harsanyi viewed this theorem as a positive result justifying utilitarianism, it is more accurate to understand it as pointing to the tension between rationality (expected utility), *ex ante* Pareto, and equity.

Regarding equity, the affine social welfare function obtained in the theorem can indeed be criticized for implying insufficient attention to the distribution of welfare, both *ex ante* and *ex post*. Diamond (1967) provided the following example. Consider two prospects, described by matrices in which a cell gives the utility of an individual in a particular state

¹There are of course technical differences, as the domain of individual preferences is more specific if one assumes that they satisfy the von Neuman-Morgenstern axioms.

²For health, see the overviews in Pauly et al. (2011, in particular chapters 3, 5, 7, 11); for unemployment, see Nicholson and Needels (2006); for environmental hazards, see Mäler and Vincent (2005, in particular chapters 12, 13, 17, 20).

³Several versions of the theorem have been established for different expected utility models in frameworks involving uncertainty (important references include De Meyer and Mongin, 1995; Mongin, 1995, 1998; Blackorby et al., 1999).

of the world (rows are for two individuals, columns for two equiprobable states):

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

By Harsanyi's theorem, the society is indifferent between those two prospects, which yields the same social utility in the two states of the world. Diamond (1967) argued that this should not be so, because individuals' expected utilities are unequal in the second prospect.

In addition, Broome (1991, p. 185) and Ben Porath et al. (1997) noted that Harsanyi's result may be questioned for not paying enough attention to *ex post* equity. Using the following examples:

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

one may argue that an egalitarian should prefer the first lottery which does not yield *ex post* inequality, while both individuals are *ex ante* indifferent, and so are the social preferences induced by Harsanyi's theorem as well as any social preferences adopting the *ex ante* approach.

Diamond's example can also be used to show that the popular *ex ante* approach is problematic at the bar of social rationality. A strictly quasi-concave symmetrical social welfare function applied to individual expected utilities is not an expected utility and violates not just the vNM independence axiom or Savage's sure-thing principle, but even the most basic rationality principle of statewise dominance. In Diamond's example, such a criterion prefers the first prospect, but also says that whatever the state of nature, the outcomes of the two prospects are equivalent. And it is easy to generate examples in which the preferred prospect induces *worse* consequences in every state. The *ex ante* approach, unless it embraces utilitarian-like indifference to inequalities, is bound to generate such blatant violations of rationality — which seems even more troublesome than the neglect of *ex post* equity.

A serious limitation of the above discussion of equity is that it implicitly assumes that vNM utilities provide the right well-being measure to discuss equity issues under uncertainty. It is actually an assumption made by most of the literature on social choice under uncertainty (recent references include Epstein and Segal, 1992; Ben Porath et al., 1997;

Gajdos and Maurin, 2004; Grant et al., 2010). The question of whether the vNM utilities are the adequate metrics for wellbeing has appeared in the literature on Harsanyi's theorem only in relation to whether it vindicates utilitarianism (vNM utilities do not necessarily represent welfare in a traditional utilitarian sense; on this issue, see Sen, 1986; Weymark, 1991; Fleurbaey and Mongin, 2012).

The question of the measurement and comparison of wellbeing has been a longstanding issue in social choice theory. In this paper we follow the approach recently developed in the theory of fair social choice (Fleurbaey and Maniquet, 2011). It consists in combining the theory of social choice (using ordinal non-comparable preferences) and the idea of resource equality in the spirit of the theory of fair allocation to jointly derive individual welfare indices and a social choice rule. The welfare indices are grounded in the description of the economic environment, and based on the idea that resources available to individuals are comparable.

From the viewpoint of resource equality, Harsanyi's theorem is clearly problematic. Consider a one-commodity economy with two individuals 1 and 2 who face a risk on their incomes described by random variables \tilde{x}_1 and \tilde{x}_2 . Assume furthermore that the individuals are expected utility maximizers so that their ordinal preferences are represented by $Eu_1(\tilde{x}_1)$ and $Eu_2(\tilde{x}_2)$. Harsanyi's theorem tells us the the society should aim at maximizing $E(u_1(\tilde{x}_1) + u_2(\tilde{x}_2))$ (after an appropriate normalization of the vNM utility functions as in Dhillon and Mertens, 1999, for instance). Thus, on sure prospects, the social welfare criterion should be $u_1(x_1) + u_2(x_2)$. If the individuals have different risk preferences, this social criterion does not satisfy the Pigou-Dalton transfer principle in general.⁴ If u_1 and u_2 are the same, then inequality aversion is constrained by risk aversion, which does not seem particularly appealing either.

In this paper, we show how a reasonable combination of rationality, equity, and efficiency induces a three-step construction for the social assessment of uncertain allocations: first, an equally-distributed equivalent (EDE) allocation is computed in each state of the world (embodying equity and efficiency); then, a certainty-equivalent bundle of the EDE allocations is computed for each individual (embodying attention to individual risk attitudes);

⁴At any value x such that $u'_1(x) \neq u'_2(x)$, a small inequality in favor of the individual with greater marginal utility is better than equality.

last, the society evaluates the distribution of these certainty-equivalent bundles (reflecting equity and efficiency concerns again). The social evaluation criterion that we obtain is *not*, in general, an expected utility, although we exhibit cases where it is closely related to an expected utility (or even precisely an expected utility). The social criterion however always satisfies a dominance requirement, which is arguably a sufficient rationality requirement for choice under uncertainty. We also examine what happens if the social criterion is forced to be an expected value of social welfare.

With this new criterion, we revisit classical issues of risk management. We first show that the optimal management of unemployment allowance proposed by Gollier (1991) and Drèze and Gollier (1993) is by and large vindicated but needs adjustment in order to accommodate a special concern for income inequalities that the policy-maker may have. This may attenuate or even reverse Gollier's result that allowances should be countercyclical with wages, for some distributions of wages and some values. The reversal may occur because pro-cyclical allowances induce less inequalities in good states of the world. With a strong degree of priority to the worst-off, the allowances become rigid in order to provide full insurance to the unemployed.

We then revisit Weitzman's dismal theorem (Weitzman, 2009). While he studied transfers of resources to the future, we focus on the prevention of catastrophes (the policy affects probabilities rather than the consumption stream), for which a variant of the dismal theorem obtains with the utilitarian criterion—but it takes the form of sensitiveness to utility parameters rather than a divergence to infinity (not unlike Weitzman's original result, actually). We show that our criterion makes it possible to have an even greater social willingness to pay for the prevention of catastrophes if the catastrophic state is worse than the situation of the current generation, but also a lower willingness to pay in the opposite case.

The paper is organized as follows. In Section 2 we introduce the framework, which is Savage's state-contingent model of choice under uncertainty (Savage, 1972), applied to economic environments consisting of allocations of private goods. Section 3 introduces a basic result showing the general implications of combining a special version of the Pareto principle with the statewise dominance principle for the social evaluation of uncertain allocations. Section 4 considers several ways of refining our basic result in order to obtain precise social preferences. In subsection 4.1, we briefly review the standard approach assuming that vNM

utility functions can be used to compare individuals' wellbeing, and obtain an expected EDE criterion as in Fleurbaey (2010). In subsection 4.2, we introduce a stronger Pareto principle and show how it yields a precise recommendation for social choice under uncertainty, based on certainty-equivalent bundles. This is the main result of our paper. In subsection 4.3, we discuss whether this social criterion can be an expected utility, and argue that this may imply the consequence that the social ordering on sure allocations is partly determined by individuals' risk attitudes rather than fairness principles. In Section 5, we apply the new criterion to the optimal management of unemployment allowances and of catastrophic risks. In Section 6, we explore the induced deviations from the expected utility approach. In a specific framework involving two goods (health and standard consumption goods), we show in subsection 6.1 that the deviation from the expected utility model may actually be small if the fair ordering for sure allocations is of the maximin or leximin type. This may however not be true for other social orderings. Last, in subsection 6.2 we show that our main result extends to the case where individuals are non-expected utility maximizers, although this opens new and interesting questions about how to aggregate beliefs and how to assess individuals' *ex ante* interests.

2 The model

Let \mathbb{R}_+ denote the set of non-negative real numbers, \mathbb{R}_{++} the set of positive real numbers, and, for any positive integer n , let \mathbb{R}_+^n be the n -fold cartesian product of \mathbb{R}_+ . For two vectors $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}^n$, $x \geq y$ if $x_i \geq y_i$ for all $i \in \{1, \dots, n\}$ and $x \gg y$ if $x_i > y_i$ for all $i \in \{1, \dots, n\}$.

We consider an economy composed of n individuals and ℓ goods. An allocation x is a collection

$$x = (x_{ik})_{i \in \{1, \dots, n\}, k \in \{1, \dots, \ell\}},$$

where x_{ik} is the quantity of good k consumed by individual i . Let $X = \mathbb{R}_+^{n\ell}$ denote the set of allocations. To shorten notations, we define $N = \{1, \dots, n\}$ and $L = \{1, \dots, \ell\}$.

We consider economies where the final allocation may be uncertain. Hence we let Ω be the set of states of the world, and \mathcal{A} a σ -algebra over Ω . A simple act f is a finite-valued \mathcal{A} -measurable function $f : \Omega \rightarrow X$, and we denote \mathcal{F} the set of simple acts. For any act

f , it will be convenient to write $f = (f_1, \dots, f_n)$ where, for each $i \in N$ and each $\omega \in \Omega$, $f_i(\omega) = (x_{ik})_{k \in L}$ whenever $f(\omega) = (x_{ik})_{i \in N, k \in L}$.

Until subsection 6.2, we assume that a probability measure p on the measurable space (Ω, \mathcal{A}) appropriately represents the society's beliefs concerning the likelihood of the different events in \mathcal{A} . For a continuous function $u_i : \mathbb{R}^\ell \rightarrow \mathbb{R}$, $i \in N$, and an act $f \in \mathcal{F}$, we denote $E(u_i \circ f_i) = \int_\Omega u_i(f_i(\omega)) dp(\omega)$. Given any $x \in X$, by abuse notation we let $x \in \mathcal{F}$ also denote the constant act such that $x(\omega) = x$ for all $\omega \in \Omega$, thus identifying X with the subset of the constant acts in \mathcal{F} . In the case of constant acts, we denote by x_i the constant bundle $x_i(\omega)$, $i \in N$, and use the same notation for allocations. For $\eta \in X$, we denote by $(f + \eta)$ (resp., $(f - \eta)$) the act such that, for all $\omega \in \Omega$, $(f + \eta)(\omega) = f(\omega) + \eta$ (resp., $(f - \eta)(\omega) = f(\omega) - \eta$). Similarly, for $\varepsilon \in \mathbb{R}_+^\ell$, we denote by $(f_i + \varepsilon)$ (resp., $(f_i - \varepsilon)$) the act such that, for all $\omega \in \Omega$, $(f_i + \varepsilon)(\omega) = f_i(\omega) + \varepsilon$ (resp., $(f_i - \varepsilon)(\omega) = f_i(\omega) - \varepsilon$).

The problem studied in this paper is the selection of a reasonable social ordering R on \mathcal{F} . More specifically, we want to define such an ordering so as to extend a given social ordering R^0 defined over sure allocations X . The relations R and R^0 are therefore complete, reflexive and transitive over \mathcal{F} and X , respectively. We denote by P (resp., I) the asymmetric (resp., symmetric) part of R , and P_0 (resp., I_0) the asymmetric (resp., symmetric) part of R_0 .

The ordering R^0 is assumed to be based on an ordering R^* on \mathbb{R}^n and on functions $v_i : \mathbb{R}^\ell \rightarrow \mathbb{R}$ in the following way: $\forall x, y \in X$,

$$xR^0y \Leftrightarrow (v_i(x_i))_{i \in N} R^* (v_i(y_i))_{i \in N}. \quad (1)$$

The functions v_i are therefore suitable individual wellbeing functions, and the ordering R^* (whose strict preference and indifference are denoted P^* and I^* , respectively) aggregates these welfare indices. We make the following two assumptions.

Assumption 1 *Every v_i is continuous increasing and the range of v_i is the same for all i .*

Assumption 2 *The ordering R^* is weakly monotonic, i.e. for any $z, z' \in \mathbb{R}^n$, (a) if $z \geq z'$ then zR^*z' ; (b) if $z \gg z'$ then zP^*z' .*

Assumption 1 is satisfied by the egalitarian-equivalent orderings when individuals have continuous monotonic preferences, for instance. Assumption 2 is akin to a Pareto principle

defined *ex post* on sure allocations, provided the v_i functions are representations of individual preferences. We do not assume the full “Strong Pareto” principle because we want our approach to be general enough to cover social preferences such as the maximin criterion.

The theory of fair social orderings has been focussing on the construction of orderings R^0 . Not all of them take the form exhibited in Eq. (1), because for some strongly non-separable orderings the v_i functions depend on the whole allocation (not only on individual bundles). But all *egalitarian-equivalent orderings* can be written in this way (Fleurbaey and Maniquet, 2011). For instance, the Pazner-Schmeidler ordering defines $v_i(x_i) = \min \{ \lambda \in \mathbb{R}_+ \mid (\lambda \Omega) R_i x_i \}$, where R_i represents i 's preferences over bundles and Ω is a reference bundle. Besides, egalitarian-equivalent orderings usually take R^* to be the maximin or leximin criterion. In this paper, we let R^0 remain unspecified, and focus on the extension of R^0 to uncertain allocations. In this way our theory is quite general.

Let X^e denote the set of “egalitarian” allocations i.e., $x \in X$ such that

$$\forall i, j \in N, v_i(x_i) = v_j(x_j).$$

Let \mathcal{F}^e denote the set of egalitarian acts, i.e., $\mathcal{F}^e = \{f \in \mathcal{F} \mid f(\Omega) \subset X^e\}$.

We now define an equally-distributed quasi-equivalent (EDQE) allocation of an allocation $x \in X$:

Definition 1 *For any $x \in X$, $x^e \in X^e$ is a corresponding equally-distributed quasi-equivalent (EDQE) if for all $z \in \mathbb{R}_{++}^{n\ell}$,*

$$(x + z) P^0 x^e \text{ and } (x^e + z) P^0 x.$$

Obviously, for an egalitarian allocation $x \in X^e$, one can take $x^e = x$. Note also that the EDQE allocation is not unique: any allocation which is deemed indifferent by all individuals are also EDQE allocations. The definition of the EDQE is straightforwardly extended from allocations to acts by letting, for any $f \in \mathcal{F}$, f^e be defined by $f^e(\omega) = (f(\omega))^e$.

With the assumptions made earlier, we have the following lemma:

Lemma 1 *Under Assumptions 1 and 2:*

- (i) For all $x \in X$, there exists $x^e \in X^e$ such that x^e is an EDQE of x . Furthermore if \tilde{x}^e is another EDQE of x , then $x^e I^0 \tilde{x}^e$.
- (ii) $(0, \dots, 0) \in X^e$, and for all $x \in X$ such that $(v_i(x_i))_{i \in N} \gg (v_i(0))_{i \in N}$, one can take $x^e \gg (0, \dots, 0)$.
- (iii) For all $x, y \in X$, if $x^e P^0 y^e$, then $x P^0 y$.

Proof.

Proof of (i). By Assumption 1, $V = \{v = (v_i(x_i))_{i \in N}, x \in X\} = Q^n$ where Q is a convex subset of \mathbb{R} . By Assumption 2, we can therefore apply Lemma 1 in Fleurbaey (2010) to the ordering R^* restricted to V to show that for any $v \in V$ there exist a unique $v^e \in Q$ such that for any $\varepsilon \in \mathbb{R}^n$, $(v + \varepsilon)P^*(v^e, \dots, v^e)$ and $(v^e + \varepsilon_1, \dots, v^e + \varepsilon_n)P^*v$.

For any $x \in X$ such that $(v_i(x_i))_{i \in N} = v$, let x^e be the allocation such that $(v_i(x_i^e))_{i \in N} = v^e$ (it necessarily exists because $v^e \in V$). The allocation is an EDQE of x . Furthermore, by unicity of v^e for each $v \in V$, if \tilde{x}^e is another EDQE of x it must be the case that $(v_i(\tilde{x}_i^e))_{i \in N} = v^e(v_i(x_i^e))_{i \in N}$. By Assumption 2 and the definition of R^0 , we therefore have $x^e I^0 \tilde{x}^e$.

Proof of (ii). By Assumption 1, we necessarily have $v_i(0) = \inf_{x \in X} v_i(x_i)$, and therefore $v_i(0) = v_j(0)$ for all $i, j \in N$ (utility functions v_i have the same range). For any $x \in X$, x^e is such that $v_i(x_i^e) \geq \inf_{i \in N} v_i(x_i)$. Indeed, otherwise, by Assumption 1 (continuity), there exists z such that $v_i(x_i^e + z_i) < v_i(x_i)$ for all $i \in N$, and therefore (by the monotonicity conditions in Assumptions 1 and 2) $x P^0(x^e + z)$ violating the assumption that x^e is an EDQE of x . If $x \in X$ is such that $(v_i(x_i))_{i \in N} \gg (v_i(0))_{i \in N}$, $\inf_{i \in N} v_i(x_i) > v_i(0)$ and hence (by Assumption 1), one can find x^e such that $x_i^e \gg 0$ for all $i \in N$.

Proof of (iii). Assume that for some $x, y \in X$, $x^e P^0 y^e$ and $y R^0 x$. Then, because y^e is an EDQE of y , for any $z \in R_{++}^{n\ell}$, $(y^e + z)P^0 y R^0 x$ so that $(y^e + z)P^0 x$. Furthermore, because x^e is an EDQE of x , for any $z \in R_{++}^{n\ell}$, $(x + z)P^0 x^e P^0 y^e$ hence $(x + z)P^0 y^e$. Therefore, y^e is an EDQE of x . But this is impossible by statement (i) of the current Lemma, because x^e is an EDQE of x and $x^e P^0 y^e$. ■

For the moment, we have mostly focused on constant acts with no uncertainty about the final allocation. When there is uncertainty, we will assume that individual i 's interests are appropriately represented by the expected utility $E(u_i \circ f_i)$, where the utility function u_i

is continuous and increasing. To ensure coherence between the judgements in situations involving uncertainty and situations comparing sure allocations, we assume that the functions u_i and v_i are ordinally equivalent.

Assumption 3 For all $i \in N$, for all $x_i, y_i \in \mathbb{R}_+^\ell$,

$$u_i(x_i) \geq u_i(y_i) \iff v_i(x_i) \geq v_i(y_i).$$

We now introduce two principles that we will use to extend the social ordering on allocations R^0 . The first principle is a restricted version of the Pareto principle, introduced by Fleurbaey (2010) and adapted to the present framework. In contrast to R^0 , we do not require R to be fully Paretian, but only (for the moment) to respect unanimous preferences in the context of equality.

Pareto for Equal Risk For all $f, g \in \mathcal{F}^e$, if $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for all $i \in N$, then pRq ; if $E(u_i \circ f_i) > E(u_i \circ g_i)$ for all $i \in N$, then pPq .

The second principle is a rather natural principle of social rationality, allowing to connect the social orderings R and R^0 .

Dominance For all $f, g \in \mathcal{F}$, if for all $\omega \in \Omega$, $f(\omega)R^0g(\omega)$ then fRg ; if for all $\omega \in \Omega$, $f(\omega)P^0g(\omega)$ then fPg .

This axiom implies in particular that for all $x, y \in X$, xR^0y if and only if xRy (where, in the latter expression, the allocations are identified with the corresponding constant act). So Dominance defines a notion of coherence between the ordering R^0 over allocations, and the ordering R over uncertain allocations.

3 The basic result

Our first result describes the structure of social orderings satisfying Pareto for Equal Risk and Dominance. We show that they consist in an aggregation of the individuals' expected utilities of the EDQE allocations.

Proposition 1 *If the social ordering R satisfies Pareto for Equal Risk and Dominance, then there exists a weakly monotonic ordering R^{**} on \mathbb{R}^n such that for all $f, g \in \mathcal{F}$, fPg if there exists $\varepsilon \in \mathbb{R}_{++}^\ell$ such that*

$$(E(u_i \circ (f_i^e - \varepsilon)))_{i \in N} R^{**} (E(u_i \circ (g_i^e + \varepsilon)))_{i \in N}.$$

Proof. Let R satisfy Pareto for Equal Risk and Dominance.

Step 1. Let $U = \{z \in \mathbb{R}^n \mid \exists f \in \mathcal{F}^e, z = (E(u_i \circ f_i))_{i \in N}\}$. Define the binary relation \tilde{R}^{**} on \mathbb{R}^n by the following condition: $z \tilde{R}^{**} z'$ if either (a) $z, z' \in U$ and there exists $f, g \in \mathcal{F}^e$ such that fRg and $z = (E(u_i \circ f_i))_{i \in N}, z' = (E(u_i \circ g_i))_{i \in N}$, or (b) $z \geq z'$.

Remark that by Pareto for Equal Risk, if $f, f' \in \mathcal{F}^e$ are such that $(E(u_i \circ f_i))_{i \in N} = (E(u_i \circ f'_i))_{i \in N} = z$, then fIf' . So, by definition of \tilde{R}^{**} , for all $f, g \in \mathcal{F}^e$,

$$(E(u_i \circ f_i))_{i \in N} \tilde{R}^{**} (E(u_i \circ g_i))_{i \in N} \implies fRg.$$

By (a), \tilde{R}^{**} is complete on U , because R is complete on \mathcal{F}^e . Transitivity of R^{**} on U is also easily derived from the transitivity of R . The ordering \tilde{R}^{**} can be extended into an ordering R^{**} on \mathbb{R}^n by Szpilrajn's Lemma (Szpilrajn, 1930).

The relation R^{**} is weakly monotonic by part (b) of the definition of \tilde{R}^{**} . Notice that we do not have a conflict between (a) and (b) of the definition of \tilde{R}^{**} , because, for $z, z' \in U$ such that $z \geq z'$, there exists $f, g \in \mathcal{F}^e$ such that $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for all $i \in N$, so that, by Pareto for Equal Risk, fRg . Note also that, as before, we have for all $f, g \in \mathcal{F}^e$:

$$(E(u_i \circ f_i))_{i \in N} R^{**} (E(u_i \circ g_i))_{i \in N} \implies fRg.$$

Step 2. Let $f, g \in \mathcal{F}$ be such that for all $\omega \in \Omega$ and for all $i \in N$, $u_i(f_i^e(\omega)) > u_i(g_i^e(\omega))$. By Assumption 3, $v_i(f_i^e(\omega)) > v_i(g_i^e(\omega))$. By Assumption 2, this implies $f^e(\omega)P^0g^e(\omega)$. By Lemma 1 (iii), this implies $f(\omega)P^0g(\omega)$. Since this is true for all $\omega \in \Omega$, Dominance implies that fPg .

Step 3. Let $f, g \in \mathcal{F}$, be such that there exists $\varepsilon \in \mathbb{R}_{++}^\ell$ such that

$$(E(u_i \circ (f_i^e - \varepsilon)))_{i \in N} R^{**} (E(u_i \circ (g_i^e + \varepsilon)))_{i \in N}.$$

By Assumption 1, there exists $f' \in \mathcal{F}^e$ such that for all $\omega \in \Omega$ and all $i \in N$, $v_i(f'_i(\omega)) > v_i(f_i^e(\omega) - \varepsilon)$, so that by Assumption 3, $u_i(f'_i(\omega)) > u_i(f_i^e(\omega) - \varepsilon)$. Similarly, there exists $g' \in \mathcal{F}^e$ such that for all $\omega \in \Omega$ and all $i \in N$, $u_i(g'_i(\omega)) < u_i(g_i^e(\omega) + \varepsilon)$.

Hence $(E(u_i \circ f'_i))_{i \in N} \gg (E(u_i \circ (f_i - \varepsilon)))_{i \in N}$ and $(E(u_i \circ (g_i + \varepsilon)))_{i \in N} \gg (E(u_i \circ g'_i))_{i \in N}$, so that, by the monotonicity and transitivity of R^{**} ,

$$(E(u_i \circ f'_i))_{i \in N} P^{**} (E(u_i \circ g'_i))_{i \in N}.$$

By Step 1, since $f', g' \in \mathcal{F}^e$, $f' R g'$.

But considering $f, f' \in \mathcal{F}$ on the one hand, and $g, g' \in \mathcal{F}$ on the other hand, and applying the result of Step 2, we obtain $f P f'$ and $g' P g$, so that, by transitivity of R , $f P g$. ■

Loosely speaking, Proposition 1 describes a three-step construction of the ordering for uncertain allocations. First, in each state of the world, we compute an EDQE allocation. Then, we compute the expected utilities for the individuals at these EDQE allocations. Last, we assess these expected utilities using the ordering R^{**} . This presentation is not completely correct, because we do not compare directly the expected utilities of the EDQE allocations, but the expected utilities of the EDQE allocations plus or minus some resources. In particular $(E(u_i \circ f_i^e))_{i \in N} P^{**} (E(u_i \circ g_i^e))_{i \in N}$ would not suffice to guarantee that $f R g$, because the ordering R^{**} need not be continuous.

Observe, however, that if R^0 is continuous, then the EDQE is a true “equally-distributed equivalent” (EDE), i.e., $x^e I^0 x$. In that case Proposition 1 can be formulated in a much simpler way.

Corollary 1 *If the social ordering on allocations R^0 is continuous and the social ordering R satisfies Pareto for Equal Risk and Dominance, then there exists a weakly monotonic ordering R^{**} on \mathbb{R}^n such that for all $f, g \in \mathcal{F}$, $f R g$ if and only if*

$$(E(u_i \circ f_i^e))_{i \in N} R^{**} (E(u_i \circ g_i^e))_{i \in N}.$$

Proof. Step 1 is as in the proof of Proposition 1. Step 2 shows that for all $f \in \mathcal{F}$, fIf^e by Dominance because $f(\omega)I^0f^e(\omega)$ for all $\omega \in \Omega$. Thus

$$\begin{aligned} (E(u_i \circ f_i^e))_{i \in N} R^{**} (E(u_i \circ g_i^e))_{i \in N} &\iff f^e R g^e && \text{(Step 1)} \\ &\iff f R g && \text{(Step 2)} \end{aligned}$$

■

Proposition 1 does not put any constraint on R^{**} , except weak monotonicity. Therefore, not much can be said on the exact form of R on the basis of these only two axioms. In the next section we study three ways in which more precise specifications can be obtained.

4 Characterizations

In order to provide a more precise specification of the social ordering R , we can introduce several additional assumptions or requirements. In subsection 4.1, we consider the case prominently studied in the literature on social choice under uncertainty. It assumes that the utility functions used to compute individuals' expected utilities provide the right metrics to assess individual's welfare. The social criterion then relies on the expected value of the EDQE utility. In subsection 4.2, we show that a mild and quite compelling strengthening of the Pareto principle delivers an ordering that coincides with the expected EDQE when vNM utilities are used for interpersonal comparisons but can be quite different when other welfare metrics are used, as in the theory of fair social choice. In subsection 4.3, we investigate the consequences of requiring the social criterion to take the form of an expected utility (as requested by Harsanyi in his aggregation theorem).

4.1 The expected utility measure of wellbeing

Most of the literature on social choice under uncertainty, stemming from Harsanyi (1955), has considered that the utility functions used to compute individual's expected utilities provided the correct measure for assessing and comparing individual welfare. As recalled in the introduction, the main criticisms raised against Harsanyi's theorem are formulated on the basis of a concern for the distribution of vNM utilities (e.g., Diamond, 1967; Broome, 1991; Epstein and Segal, 1992; Ben Porath et al., 1997; Gajdos and Maurin, 2004; Grant

et al., 2010; Fleurbaey, 2010). Although the fair allocation literature suggests that other wellbeing indexes may be preferable, it is interesting to briefly examine in our model what happens under this classical approach.

Let us therefore assume that $u_i = v_i$ for all $i \in N$. Then, by definition of \mathcal{F}^e , for all $f \in \mathcal{F}^e$ one always has $E(v_i \circ f_i) = E(v_j \circ f_j)$ for all $i, j \in N$. Therefore, for all $f \in \mathcal{F}$, $E(v_i \circ f_i^e) = E(v_j \circ f_j^e)$ for all $i, j \in N$. This makes it unnecessary to seek an exact specification of R^{**} , as stated in the following Proposition.

Proposition 2 *If $u_i = v_i$ for all i , and the social ordering R satisfies Pareto for Equal Risk and Dominance, then for all $f, g \in \mathcal{F}$, fPg if $E(u_i \circ f_i^e) > E(u_i \circ g_i^e)$ for some (and therefore all) $i \in N$.*

Proof. Because the social ordering R satisfies Pareto for Equal Risk and Dominance, we know by Proposition 1 that there exists a weakly monotonic ordering R^{**} on \mathbb{R}^n such that for all $f, g \in \mathcal{F}$, if there exists $\varepsilon \in \mathbb{R}_{++}^\ell$ such that

$$(E(u_i \circ (f_i^e - \varepsilon)))_{i \in N} R^{**} (E(u_i \circ (g_i^e + \varepsilon)))_{i \in N}$$

then fPg .

Let $f, g \in \mathcal{F}$ be such that $E(u_i \circ f_i^e) > E(u_i \circ g_i^e)$ for some $i \in N$. Because $u_i = v_i$ for all $i \in N$ and $v_i(f_i^e(\omega)) = v_j(f_j^e(\omega))$ for all $\omega \in \Omega$ and $i, j \in N$, this implies that $E(u_i \circ f_i^e) > E(u_i \circ g_i^e)$ for all $i \in N$.

By Assumptions 1 and $u_i = v_i$ for all $i \in N$, the u_i functions are continuous, so that there must exist $\varepsilon \in \mathbb{R}_{++}^\ell$ such that $(E(u_i \circ (f_i^e - \varepsilon)))_{i \in N} \geq (E(u_i \circ (g_i^e + \varepsilon)))_{i \in N}$. Because the order R^{**} is weakly monotonic, this implies that there exists $\varepsilon \in \mathbb{R}_{++}^\ell$ such that

$$(E(u_i \circ (f_i^e - \varepsilon)))_{i \in N} R^{**} (E(u_i \circ (g_i^e + \varepsilon)))_{i \in N}.$$

And by Proposition 1, we obtain that fPg . ■

Proposition 2 adapts Theorem 1 in Fleurbaey (2010) to the present framework which explicitly describes the economic environment. This proposition highlights the importance of the assumption that $u_i = v_i$ for all $i \in N$. In an economic environment, however, this assumption may be rather implausible as suggested earlier. If interpersonal comparisons

are made for instance in the typical money-metric or resource-metric utilities that one finds in the theory of fair social orderings, then it is unlikely that individual preferences over lotteries would consist in maximizing the expected value of such utilities. Economic theory typically assumes some form of risk aversion with respect to money or resources.

It is therefore interesting to investigate approaches allowing for the possibility that u_i and v_i may be quite different. To do so, we will propose an extension of the Pareto principle used in Proposition 1.

4.2 *Strengthening Pareto for Equal Risk*

As discussed before, Assumption 2 implies that the Pareto principle applies *ex post* to sure allocations, provided the v_i functions are representations of individuals' preferences. We have also applied the Pareto principle *ex ante* to egalitarian acts. In this subsection, we propose to strengthen the Pareto condition to cover comparisons between egalitarian and constant acts.

Pareto for Equal or No Risk For all $f, g \in (\mathcal{F}^e \cup X)$, if $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for all $i \in N$, then fRg .

The main argument of Fleurbaey (2010) to restrict the use of the Pareto principle *ex ante* to egalitarian lotteries is that the social observer may be in a situation that is equivalent to having more information than individuals in lotteries involving inequality, and may want to ignore the individuals' *ex ante* preferences when he knows the final distribution of their situations, including their *ex post* preferences. Typically, if the uncertainty bears on who will be the rich and who will be the poor, there is no uncertainty at the social level (the final distribution of welfare is known) while individuals do face uncertain prospects. In that case, the observer may judge the situation equivalent to a sure allocation involving the same distribution. The argument therefore does not preclude respecting individual preferences when only sure prospects and egalitarian uncertain prospects are at stake. Pareto for Equal and No Risk therefore seems an acceptable strengthening of Pareto for Equal Risk, in view of this line of argument.

One may also notice that when $u_i = v_i$ for all $i \in N$, Pareto for Equal and No Risk is already almost implied by Pareto for Equal Risk. Indeed, if $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for

all $i \in N$ for $f \in \mathcal{F}^e$ and $g \in X$, one has $E(u_i \circ f_i) = E(u_j \circ f_j)$ for all $i, j \in N$, and $E(u_i \circ g_i) \leq \max_j E(u_j \circ g_j) \leq E(u_i \circ f_i)$ for all $i \in N$, so that $E(u_i \circ f_i) \geq E(u_i \circ g_i^e)$ for all $i \in N$. By Pareto for Equal Risk, this implies fRg^e . Therefore, if gIg^e (which occurs when R^0 is continuous), then fRg , so that Pareto for Equal and No Risk holds. A similar reasoning applies when $E(u_i \circ f_i) \leq E(u_i \circ g_i)$ for all $i \in N$. In the context of the previous subsection, strengthening Pareto for Equal Risk to Pareto for Equal and No Risk would make no difference.

In the present context, this extension of Pareto for Equal Risk allows us to obtain a quite precise specification of the social ordering R , independently of the definition of the v_i functions. To present this result we introduce further notation. For any $f \in \mathcal{F}$, we let $ce(f) \in X$ denote a certainty-equivalent allocation, i.e., an allocation such that for all $i \in N$, $u_i(ce_i(f_i)) = E(u_i \circ f_i)$. When $f_i(\omega) \in \mathbb{R}_{++}^\ell$ for all $i \in N$ and $\omega \in \Omega$, we assume that for convenience one takes $ce_i(f_i) \in \mathbb{R}_{++}^\ell$.

Proposition 3 *If the social ordering R satisfies Pareto for Equal or No Risk and Dominance, then for all $f, g \in \mathcal{F}$, fPg if there exists $\eta \in \mathbb{R}_{++}^{n\ell}$ such that*

$$(ce(f^e) - \eta) R^0 (ce(g^e) + \eta).$$

Proof. Let $f, g \in \mathcal{F}$ and assume that there exists $\eta \in \mathbb{R}_{++}^{n\ell}$ such that

$$(ce(f^e) - \eta) R^0 (ce(g^e) + \eta).$$

By Assumptions 1 and 3, the v_i and therefore the u_i functions are continuous and increasing. Therefore, there exist $f', g' \in \mathcal{F}^e$ such that for every $i \in N$,

$$E(u_i \circ (f_i^e)) > E(u_i \circ (f_i')) > u_i(ce_i(f_i^e) - \varepsilon_i)$$

and

$$E(u_i \circ (g_i^e)) < E(u_i \circ (g_i')) < u_i(ce_i(g_i^e) + \varepsilon_i)$$

By Pareto for Equal or No Risk, $f^e P f' P (ce(f^e) - \varepsilon)$ and $(ce(g^e) + \varepsilon) P g' P g^e$. By Dominance $(ce(f^e) - \eta) R^0 (ce(g^e) + \eta)$ implies that $(ce(f^e) - \eta) R (ce(g^e) + \eta)$, so that

$f'Pg'$. By Step 2 of the proof of Proposition 1, f^ePf' implies fPf' and $g'Pg^e$ implies $g'Pg$. By transitivity of R , fPg . ■

Compared to Proposition 1, Proposition 3 gives an exact description of the order R^{**} , which corresponds to R^0 once expected utilities have been transformed into certainty equivalent bundles.

Like Proposition 1, Proposition 3 describes a three-step construction for ordering uncertain allocations. First, in each state of the world, we compute an EDQE allocation. Then, we compute the certainty equivalent of this EDQE allocation for all individuals. Last, we assess the distribution of the certainty equivalent using the social ordering for riskfree allocations. This summary is actually fully correct only when R^0 is continuous, so that the EDQE is actually an EDE. In that case, we have the following Corollary (whose proof is similar to the proof of Corollary 1).

Corollary 2 *If R^0 is continuous and the social ordering R satisfies Pareto for Equal or No Risk and Dominance, then for all $f, g \in \mathcal{F}$, fRg if and only if*

$$ce(f^e)R^0ce(g^e). \quad (2)$$

It is worth checking that the criterion defined in (2) satisfies not just Dominance but also stochastic dominance. Suppose that the probability of getting $f(\omega)R^0z$ is greater than the probability of $g(\omega)R^0z$ for all $z \in X$. Then the probability of getting $f^e(\omega)R^0z$ is greater than the probability of $g^e(\omega)R^0z$ for all $z \in X$. For every i , $ce_i(f^e)$ increases when the value of $f^e(\omega)$ increases for a positive mass of ω and decreases for at most a null mass. Therefore $ce_i(f^e)$ increases when the distribution of $f^e(\omega)$ increases in the first order stochastic sense.

4.3 Expected utility at the social level

One important feature of Proposition 2 is that the social ordering takes the form of an expected utility. The previous section provides a quite precise specification of R , but, while it satisfies Dominance, it does not necessarily take the form of an expected utility. It is therefore interesting to examine whether R can be an expected utility while satisfying Pareto for Equal and No Risk. To investigate this issue, we introduce the following principle.

Expected Utility There is a continuous function W such that for all $f, g \in \mathcal{F}$, fRg if and only if $E(W \circ f) \geq E(W \circ g)$.

This principle is logically stronger than Dominance. Note moreover that the continuity of W over allocations is assumed, so that both R^0 and R must then be continuous. Such an assumption of continuity has been avoided in most results of the previous sections in order to cover social preferences of the leximin type.

To obtain sharp results, we will introduce a condition on the probability measure p and the set Ω . To do so, additional notation is needed. For any $m \in \mathbb{N} \setminus \{1, 2\}$, we denote by \mathcal{A}^m the set of partitions of Ω in m elements of \mathcal{A} . Let $P^m = \{(p(A_1), \dots, p(A_m)) \in [0, 1]^m \mid (A_1, \dots, A_m) \in \mathcal{A}^m\}$. We make the following assumption:

Assumption 4 *There exists $m \in \mathbb{N} \setminus \{1, 2\}$ such that*

$$P^m = \left\{ \pi = (\pi_1, \dots, \pi_m) \in [0, 1]^m \mid \sum_{i=1}^m \pi_i = 1 \right\}.$$

Assumption 4 is rather natural in the context of subjective expected utility *à la* Savage. It is indeed closely related to Axiom P6 in Savage (1972), which guarantees that Ω can be partitioned in a finite number of subsets to obtain subjective probabilities that are as low as one wishes. The assumption is satisfied whenever Ω is an infinite set and p is a non-atomic probability measure. In the rest of the section we assume that Assumption 4 is satisfied.

Imposing only Pareto for Equal Risk, one first gets the following result.

Proposition 4 *The social ordering R satisfies Pareto for Equal Risk and Expected Utility if and only if there exists a vector $\alpha \in \mathbb{R}_+^n$ such that for all $f, g \in \mathcal{F}$, fRg if and only*

$$\sum_{i \in N} \alpha_i E(u_i \circ f_i^e) \geq \sum_{i \in N} \alpha_i E(u_i \circ g_i^e).$$

Proof. By Expected Utility, R and therefore R^0 are continuous. Because Expected Utility implies Dominance, Corollary 1 applies: there is a weakly monotonic ordering R^{**} on \mathbb{R} such that for all $f, g \in \mathcal{F}$, fPg if and only if $(E(u_i \circ f_i^e))_{i \in N} R^{**} (E(u_i \circ g_i^e))_{i \in N}$. Moreover, by continuity and monotonicity, R^{**} can be represented by a continuous non-decreasing function $H : \mathbb{R}^n \rightarrow \mathbb{R}$, and therefore R can be represented by $f \rightarrow H((E(u_i \circ f_i^e))_{i \in N})$. By

Expected Utility, there is also a function W such that, for some normalisation of H ,

$$H((E(u_i \circ f_i^e))_{i \in N}) = E(W \circ f^e)$$

for all $f \in \mathcal{F}$.

Let $Z = (x_1, \dots, x_m) \in (X^e)^m$ be a collection of m egalitarian allocations (with at least three distinct allocations). Denote $\mathcal{F}_Z = \{f \in \mathcal{F} \mid \exists (A_1, \dots, A_m) \in \mathcal{A}^m, f(\omega) = x_k, \forall \omega \in A_k, \forall k = 1, \dots, m\}$. For each $f \in \mathcal{F}_Z$, define $p_f = (p(\{\omega \mid f(\omega) = x_k\}))_{k=1, \dots, m}$. Also define $V_i(p_f) = E(u_i \circ f_i^e)$, for all $i \in N$, and $V_0(p_f) = E(W \circ f)$.

For all $f \in \mathcal{F}_Z$, we therefore have $H((V_i(p_f))_{i \in N}) = V_0(p_f)$. By Assumption 4, $\{p_f \mid f \in \mathcal{F}_Z\} = \{\pi = (\pi_1, \dots, \pi_m) \in [0, 1]^m \mid \sum_1^m \pi_m = 1\}$, which is a mixture space. By definition, the functions V_i , $i = \{0, 1, \dots, m\}$ are mixture preserving. Hence, by Proposition 2 in Coulhon and Mongin (1989), there exist real numbers $(\alpha_i)_{i \in N}$ and β such that

$$V_0(p) = \sum_{i \in N} \alpha_i V_i(p) + \beta.$$

Since H is non-decreasing, we must have $(\alpha_i)_{i \in N} \in \mathbb{R}_+^n$. We can also change the allocations in Z so as to cover the whole domain of H , and therefore obtain, for all $f \in \mathcal{F}$,

$$H((E(u_i \circ f_i^e))_{i \in N}) = \sum_{i \in N} \alpha_i E(u_i \circ f_i^e) + \beta.$$

■

Proposition 4 suggests to derive the social utility function used in the expected utility formula from a linear aggregation of individuals' utility indices. The result is similar to Harsanyi's (1955) result, except that the formula is applied only to egalitarian acts (the EDE acts).

If one further strengthens Pareto for Equal Risk into Pareto for Equal or No Risk in Proposition 4, one however obtains a substantial constraint on R^0 . Let $C^e = \{x \in X \mid \exists f \in \mathcal{F}^e, x = ce(f)\}$ denote the subset of allocations which contain the certainty-equivalent bundles of an egalitarian act, so that $X^e \subset C^e \subset X$. The social ordering R^0 on C^e is severely constrained under Pareto for Equal or No Risk

Proposition 5 *If the social ordering R satisfies Pareto for Equal or No Risk and Expected Utility, then there exists a vector $\alpha \in \mathbb{R}_+^n$ such that for all $x, y \in C^e$, xR^0y if and only*

$$\sum_{i \in N} \alpha_i u_i(x_i) \geq \sum_{i \in N} \alpha_i u_i(y_i).$$

Proof. Pick any $x, y \in C^e$. By definition of C^e , there exists $f, g \in \mathcal{F}^e$ such that $x = ce(f)$ and $y = ce(g)$. By Pareto for Equal or No Risk, xIf and yIg so that $xRy \iff fRg$. By Proposition 4, there exists a vector $\alpha \in \mathbb{R}_+^n$ such that fRg if and only if $\sum_{i \in N} \alpha_i E(u_i \circ f_i) \geq \sum_{i \in N} \alpha_i E(u_i \circ g_i)$. The fact that $x = ce(f)$ and $y = ce(g)$ implies $u_i(x_i) = E(u_i \circ f_i)$ and $u_i(y_i) = E(u_i \circ g_i)$ for all $i \in N$, so that

$$xRy \iff \sum_{i \in N} \alpha_i u_i(x_i) \geq \sum_{i \in N} \alpha_i u_i(y_i).$$

As for all $x, y \in X$, xRy if and only if xR^0y , this proves the result. ■

This proposition means that on the domain C^e , R^0 must be a weighted utilitarian criterion, using the utility functions embodying risk preferences as the correct measure of utility. This can be viewed as a negative result if one believes that there is no reason to let R^0 be influenced by the risk attitudes of the individuals.

How substantial the domain C^e is, and therefore how substantial the problem is, depends on the diversity of risk preferences over equal allocations in the population. More precisely, we know by Assumption 3 that for each $i \in N$ there exists a continuous and increasing function ϕ_i such that $u_i(x_i) = \phi_i \circ v_i(x_i)$ for all $x_i \in \mathbb{R}_{++}^\ell$. The ϕ_i function represents a form of risk attitude with respect to risks in the welfare index v_i .⁵ If $\phi_i = \phi_j = \phi$ for all $i, j \in N$, then for all $x \in C^e$ there exists $f \in \mathcal{F}^e$ such that $\phi \circ v_i(ce_i(f_i)) = u_i(ce_i(f_i)) = E(u_i \circ f_i^e) = E(\phi \circ v_i \circ f_i^e)$. Hence $v_i(ce_i(f_i)) = \phi^{-1}(E(\phi \circ v_i \circ f_i^e))$, and because, by definition, $v_i(f_i^e(\omega)) = v_j(f_j^e(\omega))$ for all $i, j \in N$ and $\omega \in \Omega$, we necessarily have $v_i(ce_i(f_i)) = v_j(ce_j(f_j))$ for all $i, j \in N$. So, in the case $\phi_i = \phi_j = \phi$ for all $i, j \in N$, $C^e = X^e$. In that case Proposition 5 has little bite. If however people have sufficiently diverse preferences, in the sense of the ϕ functions being sufficiently different, one can expect that the result applies

⁵The functions ϕ_i do not embody risk preference strictly speaking, because we know from Kihlstrom and Mirman (1974, Proposition 5) that individuals are comparable in terms of risk aversion in a multi-commodity framework only if they have the same ordinal preferences on sure prospects, which is not generally the case in our framework.

to a large set of allocations.

In view of Proposition 5, it appears that the expected utility property may not be desirable when considering economic environments, precisely because such environments offer reasons to ignore risk attitudes in the evaluation of riskless allocations. Such reasons take the form of axioms of fairness that impose certain orderings of allocation based only on individual preferences over sure bundles. In the remainder of the paper, we therefore stick to the social ordering characterized in Proposition 3, and no longer assume that the social criterion is an expected utility.

5 Examples

In this section we illustrate the criterion obtained in Corollary 2 in two classical contexts.

5.1 Macroeconomic risk

Drèze and Gollier (1993) and Gollier (1991) have studied the design of an unemployment insurance scheme in the presence of a macroeconomic risk.

Drèze and Gollier (1993) have argued in favor of indexing wages on GDP, so as to pool risk efficiently. Their argument focuses on efficiency. As they assume identical agents and as the consumption of all workers, whether employed or not, is equalized in every state, their argument would still be valid when using the certainty-equivalent of the EDE as a criterion.⁶ In a similar model, Gollier (1991) introduces heterogeneity of reservation wages among workers, and studies the optimal variation of unemployment allowances in the presence of shocks on productivity. Heterogeneity between agents makes the comparison with our approach more interesting and we therefore revisit his analysis.

Consider an economy composed of a unit mass of workers, each having a VNM utility $u(c - m_i \ell)$, where $c \in \mathbb{R}_+$ is consumption, m_i is individual i 's reservation wage, and $\ell = 1$ when employed, 0 when unemployed. In any given state s , consumption is equal to w_s for the (employed) workers, with w_s the random wage rate; consumption is equal to an allowance b_s for the unemployed.

The economy operates under the constraint that the expected profits must attain a

⁶Details are available from the authors.

threshold: $\sum_s p_s (f_s(L_s) - w_s L_s - (1 - L_s) b_s) \geq \bar{v}$, where p_s is the probability of state s , f_s is the random production function, L_s is total employment, and the term $(1 - L_s) b_s$ in the profit formula corresponds to perfect experience rating (the productive sector funds unemployment allowances).⁷ We assume that investors are abstract foreign markets and we focus on the social welfare of the population of workers.

In a given state s , the spot-market equilibrium condition for wages is $f'_s(L_s) = w_s - b_s$. A worker is employed if $w_s - m_i \geq b_s$, so that total employment is determined by

$$L_s = F(w_s - b_s),$$

where F is the cumulative distribution function of the parameter m_i .

The expected utility of an individual with reservation wage m is equal to

$$\sum_s p_s [u(\max\{w_s - m, b_s\})].$$

Gollier observes that the spot labor market achieves productive efficiency in every state, but full ex ante Pareto efficiency in risk sharing would require making $\max\{w_s - m_i, b_s\}$ a constant for every worker i , which is impossible with a uniform allowance b_s . Gollier shows that a constrained efficient allowance program $\{b_s\}$ must make the following expression a constant across states of nature:

$$\int_0^{w_s - b_s} \lambda(m) u'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \lambda(m) u'(b_s) dF(m), \quad (3)$$

where $\lambda(m)$ is some weight function reflecting an ethical prioritization of individuals with different values of the m parameter. This is not surprising as ex ante Pareto efficiency corresponds to the maximization of some weighted utilitarian criterion. The consequence is that b_s will be countercyclical with respect to w_s .⁸

In this model, our approach would look at the distribution of “net-of-labor-disutility” consumption (which is unidimensional) and compute an EDE in every state s . For instance,

⁷Gollier (1991) also considers the case of risk-averse investors, but we focus here on the simpler case in which they are risk-neutral.

⁸The expression (3) is decreasing in w_s as well as in b_s .

suppose that the EDE is a generalized mean

$$\varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right).$$

As all individuals have the same utility u , they have the same certainty-equivalent of this EDE, and therefore the optimal policy solves the following program:

$$\max_s \sum p_s u \circ \varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right)$$

under the same profit constraint and the spot market equilibrium conditions as in the problem considered by Gollier (1991). An analysis similar to Gollier's one yields that

$$\frac{u'(EDE_s)}{\varphi'(EDE_s)} \left(\int_0^{w_s - b_s} \varphi'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi'(b_s) dF(m) \right)$$

must be a constant across states of nature (see Appendix 1 for details).

When $\varphi = u$, one obtains a special case of (3), which is not surprising as the utility of the EDE is then equal to the utilitarian sum of utilities. When φ is more concave than u , so that $\varphi = g \circ u$ for a concave transform g , the expression becomes

$$\frac{1}{g'(EDE_s)} \left(\int_0^{w_s - b_s} \varphi'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi'(b_s) dF(m) \right).$$

In this formula both the numerator and the denominator decrease in w_s and b_s , so that the ratio may depend on these two parameters in a different way and the countercyclicality result need not always hold.⁹

Interestingly, when the degree of priority of the worst-off goes to infinity, the EDE tends to b_s (assuming that unemployment is positive in every s), the optimal policy is to have a constant b_s (see Appendix 1). The permanently unemployed, who are the worst-off in every state and get full priority, are fully insured, whereas those who work (at least in some states) are only partly insured. Note that this case is also a limit case of (3), when $\lambda(m)$ is positive only for the greatest values of m .

⁹For instance, for $\varphi(x) = g(x) = -x^{-1}$, and F the CDF of the lognormal distribution with mean 1 and standard deviation 2, the ratio increases with w_s and decreases with b_s around $(w_s, b_s) = (1, .2)$.

5.2 Prevention of catastrophes

Weitzman (2009) argued that it was hard to bound the amount of effort one should make for the future in the case of a possible future catastrophic climate change. Weitzman and the following literature (for a comprehensive discussion, see Millner, 2013) have focused on a riskless investment that transfers consumption to the future. Here we will examine the different problem of investing for the reduction of the probability of a catastrophe.¹⁰ Consider an economy with two generations, which is examining whether it is worth taxing the first generation in order to reduce the risk of a catastrophe for the second generation. One can think of the climate change threat, or the meteor threat, as concrete motivations for this example.

The population is $N = N_1 \cup N_2$ (the two generations), and we assume that there is a single consumption good for simplicity. The first generation's consumption is $(x_i)_{i \in N_1}$ and the second generation will have either $(c_i)_{i \in N_2}$ with probability p , which is a catastrophe, or $(y_i)_{i \in N_2}$ with probability $1 - p$. A proportional tax on the first generation would reduce its consumption to $((1 - \tau)x_i)_{i \in N_1}$, and the outcome will be a reduction in the probability p . How much of τ should one accept for a given reduction in p ?

The Utilitarian approach relies on the social welfare function

$$\sum_{i \in N_1} u_i((1 - \tau)x_i) + \sum_{i \in N_2} [pu_i(c_i) + (1 - p)u_i(y_i)]$$

and computes the social willingness to tax as

$$\frac{d\tau}{-dp} = \frac{\sum_{i \in N_2} (u_i(y_i) - u_i(c_i))}{\sum_{i \in N_1} x_i u'_i((1 - \tau)x_i)}.$$

Although Weitzman (2009) looks at investments that raise future consumption rather than reduce future risks, one sees that this expression is very sensitive to $u_i(c_i)$ going to large negative values.

For the criterion proposed in this paper, let us assume that R^0 is an inequality averse social ordering represented by the function $\sum_{i \in N} \varphi(x_i)$, with φ a concave function (there

¹⁰Pindyck and Wang (2009) estimate of what a representative-agent economy calibrated on the US would be willing to pay for such a policy.

is only one good, therefore no issue of heterogeneous preferences). Let

$$\begin{aligned} EDE^c &= \varphi^{-1} \left(\frac{1}{n} \left(\sum_{i \in N_1} \varphi((1-\tau)x_i) + \sum_{i \in N_2} \varphi(c_i) \right) \right), \\ EDE^y &= \varphi^{-1} \left(\frac{1}{n} \left(\sum_{i \in N_1} \varphi((1-\tau)x_i) + \sum_{i \in N_2} \varphi(y_i) \right) \right). \end{aligned}$$

In the economy of this example, the criterion takes the form

$$\sum_{i \in N} \varphi(ce_i(EDE)) = \sum_{i \in N} \varphi(u_i^{-1}(pu_i(EDE^c) + (1-p)u_i(EDE^y))).$$

One then computes

$$\begin{aligned} \frac{d\tau}{-dp} &= \frac{\sum_{i \in N} [u_i(EDE^y) - u_i(EDE^c)] \frac{\varphi'(ce_i(EDE))}{u'_i(ce_i(EDE))}}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i)} \times \\ &\quad \left(\frac{1}{n} \sum_{i \in N} \frac{\varphi'(ce_i(EDE))}{u'_i(ce_i(EDE))} \left[\frac{pu'_i(EDE^c)}{\varphi'(EDE^c)} + \frac{(1-p)u'_i(EDE^y)}{\varphi'(EDE^y)} \right] \right)^{-1}. \end{aligned}$$

In this formula, the first ratio on the right-hand side is similar to the Utilitarian formula, with two key differences. First, the priority of individuals is determined by the same φ function and is not dependent on the scale of utility functions for the members of the two generations. This can be defended as much preferable in terms of equity between individuals (the Pigou-Dalton principle is satisfied in every final consequence). In the case of the second generation, the difference in utility is recalibrated in terms of φ' thanks to the ratio $\varphi'(ce_i(EDE)) / u'_i(ce_i(EDE))$.

The second difference is that at the numerator all individuals are counted, not just the members of the second generation, because the benefit of reducing the risk of a catastrophe is shared by the whole society as a reduction in the probability of having the low EDE^c . In the social evaluation, the EDE approach implies that one must look at a situation in which all individuals suffer from risk, because if equality was achieved in all states, the risk borne by the second generation would carry over to the first one. Again, this is preferable to the Utilitarian approach because it takes account of the degree of correlation of risk instead of adding up $\sum_{i \in N_2} (u_i(y_i) - u_i(c_i))$ without checking whether the risk reduction

is independent for every individual or concerns a macroeconomic risk.

The final term on the right-hand side of the formula is a calibration term that takes account of how the ratio u'_i/φ' varies across states of the world, and comes from the fact that the cost of the tax on the first generation is analyzed in terms of a cost on the EDE and is therefore shared by all individuals as well.

To compare the two approaches, let us focus on a simple case in which all individuals in a generation have identical consumption, and for all $i \in N$, $u_i = \varphi$. Then the second formula simplifies into

$$\begin{aligned} \frac{d\tau}{-dp} &= \frac{\sum_{i \in N} [u_i(EDE^y) - u_i(EDE^c)]}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i)} \\ &= \frac{\sum_{i \in N_2} (\varphi(y_i) - \varphi(c_i))}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i)}, \end{aligned}$$

and therefore the two approaches are then identical.

Keeping the assumption that all individual have the same utility function, and that there is no intra-generational inequality, but assuming that $u_i = g \circ \varphi$, for some transform g , one has

$$\frac{d\tau}{-dp} = \frac{n \begin{bmatrix} g\left(\frac{1}{n}(\sum_{i \in N_1} \varphi((1-\tau)x_i) + \sum_{i \in N_2} \varphi(y_i))\right) \\ -g\left(\frac{1}{n}(\sum_{i \in N_1} \varphi((1-\tau)x_i) + \sum_{i \in N_2} \varphi(c_i))\right) \end{bmatrix}}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i) (pg' \circ \varphi(EDE^c) + (1-p)g' \circ \varphi(EDE^y))}.$$

If g is convex (i.e., φ is more concave than u_i), the numerator is less than

$$\sum_{i \in N_2} (u_i(y_i) - u_i(c_i)),$$

and the denominator is greater or less than $\sum_{i \in N_1} x_i u'_i((1-\tau)x_i)$ depending on

$$pg' \circ \varphi(EDE^c) + (1-p)g' \circ \varphi(EDE^y) \gtrless g' \circ \varphi((1-\tau)x_i).$$

Therefore, if c_i and y_i are great enough, the denominator is greater and $d\tau/(-dp)$ is smaller with our criterion than with utilitarianism. If c_i and y_i are small enough, the opposite holds. The conclusions are reversed if g is concave.

In conclusion, our criterion induces a social willingness to pay for prevention that is not

radically different from the utilitarian formula, and, assuming a greater inequality aversion in φ than risk aversion in u_i , is lower when the future is much richer than the present and greater when the future prospects are not so bright.

6 Beyond expected utility

6.1 How far from expected utility at the social level?

To explore the consequence of adopting a criterion such as the one highlighted in Proposition 3, we study a simple model with two goods, a standard private consumption good $c \in \mathbb{R}_+$ and health $h \in [0, 1]$, where $h = 1$ denotes full health. For the sake of simplicity, we assume that individual ordinal preferences are represented by the function $(c, h) \mapsto ck_i(h)$ with k_i an increasing function such that $k_i(h) \geq 0$ and $k_i(1) = 1$.

We assume that the welfare index used to compare individual's welfare is the “healthy-equivalent consumption”, i.e. c_i^* such that $c_i^*k_i(1) = c_i k_i(h_i)$; therefore, $v_i(c_i, h_i) = c_i^* = c_i k_i(h_i)$. For $f_i : \Omega \rightarrow \mathbb{R}_+ \times [0, 1]$, and $\omega \in \Omega$, we use the following piece of notation: $f_i(\omega) = (c_{f_i}(\omega), h_{f_i}(\omega))$. The prospect for individual i is evaluated by an expected utility

$$E(\phi_i \circ v_i \circ f_i) = \int_{\omega \in \Omega} \phi_i(c_{f_i}(\omega)k_i(h_{f_i}(\omega)))dp(\omega)$$

or by a certainty-equivalent healthy-equivalent consumption:

$$ce_i(f_i) = \phi_i^{-1} \left(\int_{\omega \in \Omega} \phi_i(c_{f_i}(\omega)k_i(h_{f_i}(\omega)))dp(\omega) \right)$$

We discuss two different social orderings R , defined as in Proposition 3, based on the form taken by the ordering R^0 (or, more precisely, its associated R^*).

R^* is maximin or leximin. Fleurbaey (2005) provides an axiomatization suggesting that an allocation $((c_i, h_i))_{i \in N}$ is strictly better than an allocation $((\tilde{c}_i, \tilde{h}_i))$ whenever $\min_{i \in N} v_i(c_i, h_i) > \min_{i \in N} v_i(\tilde{c}_i, \tilde{h}_i)$.

Assume that R^* is a maximin. Since the maximin criterion is continuous, we can compute the EDE for an allocation $f(\omega) = ((c_{f_i}(\omega), h_{f_i}(\omega)))_{i \in N}$, which is $f^e(\omega) = ((\min_{j \in N} v_j(c_{f_j}(\omega), h_{f_j}(\omega)), 1))_{i \in N}$: the allocation where all individuals are in full health and receive the lowest level of healthy-

equivalent consumption.¹¹ Therefore, for any $i \in N$, and any simple act $f : \Omega \rightarrow (\mathbb{R}_+ \times [0, 1])^n$,

$$ce_i(f_i^e) = \phi_i^{-1} \left(\int_{\Omega} \phi_i \left(\min_{j \in N} c_{f_j}(\omega) k_j(h_{f_j}(\omega)) \right) dp(\omega) \right)$$

Using Proposition 3, or more precisely its Corollary 2, we obtain that:

$$fRg \iff \min_{i \in N} \phi_i^{-1} \left(\int_{\Omega} \phi_i \left(\min_{j \in N} c_{f_j}(\omega) k_j(h_{f_j}(\omega)) \right) dp(\omega) \right) \geq \min_{i \in N} \phi_i^{-1} \left(\int_{\Omega} \phi_i \left(\min_{j \in N} c_{g_j}(\omega) k_j(h_{g_j}(\omega)) \right) dp(\omega) \right).$$

The criterion therefore consists in maximizing the lowest certainty-equivalent of the lowest healthy-equivalent consumption.

One easily realizes that the social ordering R can be an expected utility criterion provided the individual with the lowest certainty-equivalent is always the same. Within the framework of this section this happens whenever there exists $i \in N$ such that for each $j \in N$ there exists a continuous increasing and weakly concave function φ_j such that $\phi_i = \varphi_j \circ \phi_j$. When this is the case, the social criterion consists in maximizing the expected utility of the equally-distributed equivalent using the utility function of the most risk averse individual (where risk aversion is defined with respect to healthy-equivalent consumption only). For instance, if $\phi_i(c) = c^{1-\gamma_i}/(1-\gamma_i)$, letting $\gamma_{\max} = \max_{i \in N} \gamma_i$, the social ordering is:

$$fRg \iff \frac{1}{1-\gamma_{\max}} \int_{\Omega} \left(\min_{j \in N} c_{f_j}(\omega) k_j(h_{f_j}(\omega)) \right)^{1-\gamma_{\max}} dp(\omega) \geq \frac{1}{1-\gamma_{\max}} \int_{\Omega} \left(\min_{j \in N} c_{g_j}(\omega) k_j(h_{g_j}(\omega)) \right)^{1-\gamma_{\max}} dp(\omega).$$

Note that the spirit of this criterion is exactly the opposite of one proposed in Chambers and Echenique (2012, Example 10): they contend that the society should use the risk preferences of the least risk-averse individual (which is consistent with their axiom of reduction of risk aversion). We argue that fairness on the contrary requires to use the risk preferences of the most risk-averse individual.

Notice also that, when individuals are well-ordered in terms of risk preference with

¹¹Recall that the EDE is not uniquely defined, but it is convenient to express it in terms of the minimal healthy-equivalent consumption.

respect to healthy-equivalent consumption, this result does not contradict Proposition 4. Indeed, in the social objective highlighted in Proposition 4, we can set $\alpha_i = 1$ for i who is the most risk averse and $\alpha_j = 0$ for all other $j \neq i$, and maximize the certainty-equivalent of the most risk-averse individual.

One difficulty with the maximin criterion is that it does not satisfy the Strong Pareto principle according to which a situation is strictly preferred to another whenever at least one individual strictly prefers the situation while no other individual weakly prefers the alternative. This may result in inefficient choices by a maximin criterion in some economic environments. As a consequence, the literature on fair social choice typically suggests to use a leximin criterion rather than a maximin criterion, in order to obtain a strongly Paretian social ordering.

In the context of the present article, the leximin criterion however raises several challenges. First, because it is not continuous, it does not provide an EDE, but only an EDQE which is the same as the maximin EDE. Hence Proposition 3 cannot be used in situations where the minimal healthy-equivalent consumptions might be the same in the two allocations, but some other individuals are better-off in one of the allocation, in some states of the world. These are precisely the situations where it is interesting to use the leximin rather than the maximin criterion *ex post*. Proposition 3 cannot be used either to distinguish situations where the certainty equivalent of the EDQE is the same in the two allocations for the person with the lowest certainty equivalent, but not for other individuals.

Hence Proposition 3 alone cannot describe how to use the leximin criterion in many cases of interest. We propose a particular way to define an extension of a leximin R^0 to uncertain acts. For any $f \in \mathcal{F}$, define $f_{(\cdot)} \in \mathcal{F}$ as the act such that for all $\omega \in \Omega$, $f_{(\cdot)}(\omega) = (c_{f_{(1)}}^*(\omega), \dots, c_{f_{(n)}}^*(\omega))$. In each state of the world, $f_{(\cdot)}$ gives the vector of healthy-equivalent consumptions reordered in an increasing sequence. With the maximin criterion, only $f_{(1)}$ matters. But with the leximin, all ranks become relevant. For each $j \in N$ and each f , let $ce_{(\cdot)}(f_{(j)}) = (ce_{(1)}(f_{(j)}), \dots, ce_{(n)}(f_{(j)}))$ be the vector of certainty-equivalent healthy-equivalent consumptions at rank j reordered in increasing order. We propose the following social ordering R . For all $f, g \in \mathcal{F}$, fRg if and only if

$$\left((ce_{(i)}(f_{(1)}))_{i \in N}, \dots, (ce_{(i)}(f_{(n)}))_{i \in N} \right) \geq_{lex} \left((ce_{(i)}(g_{(1)}))_{i \in N}, \dots, (ce_{(i)}(g_{(n)}))_{i \in N} \right),$$

where \geq_{lex} is the leximin ordering that lexicographically checks the lowest value, then the second lowest, and so on. Note that when individuals have different risk attitudes, in general one does not have $(ce_{(i)}(f_{(j)}))_{i \in N} \leq (ce_{(i)}(f_{(j+1)}))_{i \in N}$. So, the leximin criterion proposed here may give the priority to a risk averse individual in a better situation.

In order to justify this criterion, two axioms could be introduced and will only be informally described here. The first axiom is replication invariance, which says that replicas of allocations are ranked the same as the original allocations. The second is an extension of Pareto for Equal or No Risk to situations which are not equal but in which there is no reranking of individuals across states of nature¹² and in which every bundle is consumed by the same distribution of types of agents (where types are defined as risk preferences with respect to healthy consumption). Hence, $c_{f_{(1)}}^*(\omega) = \dots = c_{f_{(m)}}^*(\omega)$ is consumed by a subgroup of m individuals, $c_{f_{(m+1)}}^*(\omega) = \dots = c_{f_{(2m)}}^*(\omega)$ is consumed by another subgroup of m individuals, which has the same distribution of risk preferences as the first subgroup, and so on. Intuitively, this axiom extends Pareto to allocations which are not equal, but in which every type of agent is given all bundles in the allocation. It is easy to check that if there is no reranking of the bundles $(c_{f_1}^*(\omega), \dots, c_{f_n}^*(\omega))$ across states, and if the population N is replicated n times, such that the n members of each of the n types receive the whole list $(c_{f_1}^*(\omega), \dots, c_{f_n}^*(\omega))$, then the list certainty-equivalent bundles is $\left((ce_{(i)}(f_{(1)}))_{i \in N}, \dots, (ce_{(i)}(f_{(n)}))_{i \in N}\right)$.

R^* is additive separable. Another very common assumption is that R^* is additive separable in the sense that there exists a continuous increasing function $G : \mathbb{R} \rightarrow \mathbb{R}$ such that, for any $z, z' \in \mathbb{R}^n$

$$z R^* z' \iff \sum_{i \in N} G(z_i) \geq \sum_{i \in N} G(z'_i).$$

In this case, the ordering R^0 is continuous and the EDE healthy-equivalent consumption for an allocation $((c_i, h_i))_{i \in N}$ is:

$$G^{-1} \left(\frac{1}{n} \sum_{i \in N} G(c_i k_i(h_i)) \right).$$

¹²An axiom of Pareto for No Reranking is used in Fleurbaey (2010) to axiomatize a leximin criterion. When $u_i = v_i$ for all i , the axiom introduced in the text is identical to it.

By Corollary 2, if R satisfies Pareto for Equal or No Risk and Dominance, we have that, for all $f, g \in \mathcal{F}$, fRg if and only if

$$\begin{aligned} & \sum_{i \in N} G \circ \phi_i^{-1} \left(\int_{\Omega} \phi_i \circ G^{-1} \left(\frac{1}{n} \sum_{i \in N} G(c_{f_i}(\omega) k_i(h_{f_i}(\omega))) \right) dp(\omega) \right) \\ & \geq \sum_{i \in N} G \circ \phi_i^{-1} \left(\int_{\Omega} \phi_i \circ G^{-1} \left(\frac{1}{n} \sum_{i \in N} G(c_{g_i}(\omega) k_i(h_{g_i}(\omega))) \right) dp(\omega) \right) \end{aligned} \quad (4)$$

When G is affine, the above criterion resembles the one proposed by Chambers and Echenique (2012, Corollary 7), except that the sum of the certainty-equivalent bundles is taken with respect to the common EDE.

In general, the criterion represented in Equation (4) is not an expected utility criterion, even when individuals are ordered in terms of risk preferences with respect with healthy-equivalent consumption. The criterion is ordinally equivalent to an expected utility only when $\phi_i = \alpha_i \phi + \beta_i$ for all $i \in N$, that is, when all individuals have the same risk preferences. Indeed, the criterion (4) does not generally satisfy Savage's sure-thing principle¹³ when individual functions ϕ_i embody different risk attitudes.

This can be seen using the following simple example. Assume that there are only two individuals 1 and 2. Individual 1 is risk-averse with respect to healthy consumption $\phi_1(c) = 2c^{1/2}$. Individual 2 is risk-neutral with respect to healthy consumption, $\phi_2(c) = c$. Society is inequality-averse with respect to healthy consumption: $G(z) = 2z^{1/2}$ (this choice is made for the sake of convenience, but the reasoning does not depend on the assumption that society's relative inequality-aversion is equal to individual 1's relative risk-aversion).

Consider the following four simple acts f, f', g and g' . There exists $A \in \mathcal{A}$ such that, for all $i \in N$, $f_i(\omega) = f'_i(\omega) = (c_f(\omega), 1)$ and $g_i(\omega) = g'_i(\omega) = (c_g(\omega), 1)$ for all $\omega \in A$. And there exist $c \in \mathbb{R}_+$, $\varepsilon \in \mathbb{R}_{++}$ such that for all $\omega \in (\Omega \setminus A)$, for all $i \in N$, $f_i(\omega) = g_i(\omega) = (c, 1)$, $f'_i(\omega) = g'_i(\omega) = (c + \varepsilon, 1)$.

¹³There is often a confusion between the "true" sure-thing principle, which is just an extension of the dominance principle to preferences conditional on events (see Savage 1972, p. 21), and Savage's axiom P2, which includes a separability condition that is not part of the sure-thing principle itself (but is convenient to define conditional preferences). In the text we follow the usual, abusive convention of calling P2 "Savage's sure-thing principle".

Assume that fIg . Given our assumptions, and applying Equation (4), this implies that

$$\begin{aligned} \int_A (c_f(\omega))^{1/2} dp(\omega) + \left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))c \right)^{1/2} \\ = \int_A (c_g(\omega))^{1/2} dp(\omega) + \left(\int_A c_g(\omega) dp(\omega) + (1 - p(A))c \right)^{1/2}. \end{aligned}$$

One can assume, without loss of generality, that $\int_A c_f(\omega) dp(\omega) < \int_A c_g(\omega) dp(\omega)$ and $\int_A (c_f(\omega))^{1/2} dp(\omega) > \int_A (c_g(\omega))^{1/2} dp(\omega)$. This is an egalitarian situation where the risk-averse individual prefers act f but the risk-neutral one prefers g .

Savage's sure-thing principle implies that $fIg \iff f'Ig'$. But actually, for ε small, one has:

$$\begin{aligned} \left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))(c + \varepsilon) \right)^{1/2} \approx \left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))c \right)^{1/2} \\ + \frac{1-p(A)}{2} \varepsilon \left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))c \right)^{-1/2}. \end{aligned}$$

Since $\int_A c_f(\omega) dp(\omega) < \int_A c_g(\omega) dp(\omega)$,

$$\left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))c \right)^{-1/2} > \left(\int_A c_g(\omega) dp(\omega) + (1 - p(A))c \right)^{-1/2},$$

so that:

$$\begin{aligned} \int_A (c_f(\omega))^{1/2} dp(\omega) + \left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))(c + \varepsilon) \right)^{1/2} \\ \approx \int_A (c_f(\omega))^{1/2} dp(\omega) + \left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))c \right)^{1/2} \\ + \frac{1-p(A)}{2} \varepsilon \left(\int_A c_f(\omega) dp(\omega) + (1 - p(A))c \right)^{-1/2} \\ > \int_A (c_g(\omega))^{1/2} dp(\omega) + \left(\int_A c_g(\omega) dp(\omega) + (1 - p(A))c \right)^{1/2} \\ + \frac{1-p(A)}{2} \varepsilon \left(\int_A c_g(\omega) dp(\omega) + (1 - p(A))c \right)^{-1/2} \\ \approx \int_A (c_g(\omega))^{1/2} dp(\omega) + \left(\int_A c_g(\omega) dp(\omega) + (1 - p(A))(c + \varepsilon) \right)^{1/2}. \end{aligned}$$

Hence $f'Pg'$.

The violation of Savage's sure-thing principle exhibited in this example seems rather general. When R^* is additive separable the society's evaluation is likely to significantly differ from the expected utility model.

We do not consider that violating Savage's sure-thing principle, which is just a separability axiom, is a serious rationality failure. The presence of bad social outcomes in some states of nature may alter the *ex ante* evaluation of what happens in other states of nature because individuals' certainty-equivalent bundles are lower as a consequence, and lower certainty bundles may be evaluated with a greater focus on the worse-off, for instance. This argument is really specific to social evaluation and does not seem to carry to individual decision-making. Perhaps the closest story for individual decision-making, in superficial appearance, would see the individual becoming more nervous about taking risk in a subset of states of nature when things are worse in other states, for instance because he does not want the probability of ending up below a certain utility level to go beyond a certain threshold — this is the essence of Allais' paradox. But this is really different because there is nothing in our approach that deviates from expected utility if individuals in the population have the same risk attitudes as measured by the ϕ_i functions.

6.2 Non expected utility at the individual level

In the statement of Pareto for Equal or No Risk, we implicitly assume that individual interests under uncertainty are appropriately represented by expected utilities, and that all individuals share the same beliefs described by p . In this section, we show that relaxing the expected utility assumption would not significantly alter our main results. We also discuss the possibility of allowing for different beliefs.

Assume that acts can be ranked from individual's i point of view via an ordering R_i over the set \mathcal{F}_i of finite-valued \mathcal{A} -measurable functions $f_i : \Omega \rightarrow \mathbb{R}^\ell$. We make three assumptions on R_i :

1. *Dominance*: if $f_i(\omega) R_i g_i(\omega)$ for all $\omega \in \Omega$, then $f_i R_i g_i$.
2. *Continuity*: R_i is continuous with respect to the product topology.
3. *Ordinal Equivalence*: for all $x_i, y_i \in \mathbb{R}_+^\ell$, $x_i R_i y_i \iff v_i(x_i) \geq v_i(y_i)$.

Note that the first assumption embodies both the idea of dominance and that of state independence. The third assumption is actually similar to Assumption 3. The first two assumptions imply the existence of a certainty equivalent bundle for any f_i , that is a bundle $x_i \in \mathbb{R}_+^\ell$ such that $x_i I_i f_i$. As before, we denote this bundle $ce_i(f_i)$.

We modify Pareto for Equal or No Risk in the following way.

Generalized Pareto for Equal or No Risk For all $f, g \in (\mathcal{F}^e \cup X)$, if $f_i R_i g_i$ for all $i \in N$, then $f R g$; if $f_i P_i g_i$ for all $i \in N$, then $f P g$.

Under our three assumptions, we have the following result.

Proposition 6 *If the social ordering R satisfies Generalized Pareto for Equal or No Risk and Dominance, then, for all $f, g \in \mathcal{F}$, $f P g$ if there exists $\eta \in \mathbb{R}_{++}^{n\ell}$ such that*

$$(ce(f^e) - \eta) R^0 (ce(g^e) + \eta).$$

Proof. If R_i is a continuous ordering satisfying Dominance, Continuity and Ordinal Equivalence, then there exists a continuous monotonic functional $V_i : \mathcal{F}_i \rightarrow \mathbb{R}$, such that $V_i(x_i) = v_i(x_i)$ for all $x_i \in \mathbb{R}^\ell$, and, for all $f_i, g_i \in \mathcal{F}_i$,

$$f_i R_i g_i \iff V_i(f_i) \geq V_i(g_i).$$

Using functional V_i , instead of the expected utility operator $E(u_i \circ f_i)$ in the proof of Proposition 3, delivers the results.¹⁴ ■

Proposition 6 is very general for it allows individuals to adopt many different ways of making decisions under uncertainty. It also allows them to have different beliefs about the probabilities of events. From the normative point of view, this raises several issues.

One of these issues is the problem of spurious unanimities (Mongin 2005). If individuals have different beliefs, they may agree to deem an option better than another one simply because they disagree on the prospects of the different outcomes realizing. For instance a very risk averse individual and a risk neutral individual may agree that an uncertain level of a public good is better than a sure level of public good only because the former believes that

¹⁴Note that in the expected utility case we should take $V_i(f_i) = \phi_i(E(u_i \circ f_i))$, where ϕ_i is such that $v_i = \phi_i \circ u_i$.

there is a very small probability that the level of public good is very low, while the second believes that the expected level of public good is higher than the sure level of public good, but only slightly so. If the first was to realize that he was underestimating the probability of the bad outcome, he would not want to follow the uncertain course of action. This issue has been investigated within the framework of the expected utility model by Gilboa et al. (2004). They propose to weaken the Pareto principle to cases where individuals do share the same beliefs on events (and obtain a Utilitarian aggregation of beliefs and tastes).

For the above problem to arise, it must of course be the case that the notion of beliefs is itself well-defined. This raises a second issue with the very general result in Proposition 6: for the model to be relevant from the normative point of view, it needs to be the case that the model used to represent individuals *ex ante* interests is itself normatively appealing. In particular, one could argue that a minimal condition would be that this model is probabilistic sophisticated and therefore provides a well-defined notion of beliefs. Probabilistically sophisticated non expected-utility models in the Savage framework have been introduced by Machina and Schmeidler (1992), and further studied by Grant et al. (2008). All these models rely on the existence of a probability measure p on the sets of events \mathcal{A} .

We admit that a complete account of social choice under uncertainty would need to develop normative principles guiding the model of choice under uncertainty used to evaluate the *ex ante* interests of individuals and the definition of social beliefs. The aim of the present paper is not to study this question, but here is a brief sketch of how this issue could be tackled. It might be useful to distinguish two tasks.

The first task is the construction of a consistent evaluation of social situations by an ethical observer. Such an observer should rely on the best information available and the best normative model for the evaluation of individual and social interests. Our paper has been dealing with this task, focusing on the social side and assuming up to this section that expected utility for well-defined probabilities was the best approach for the assessment of individual interests. This paper has not explored the normative measurement of individual well-being, neither under certainty (which is the object of the theory of fair social orderings) nor under uncertainty.

The second task is the collective decision-making process, which has to aggregate individual beliefs and individual views about the assessment of individual well-being and social

welfare. For this second task, we suggest that seeking a mechanical aggregation formula is actually completely inappropriate — and the theory of social choice has often misleadingly suggested it was seeking such a formula. Democratic deliberation should take a leading role in this process, only assisted by the normative theories provided for the first task and which can be taken as advice or platforms for discussion, not more.

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Appendix 1: Optimal unemployment allowance policy under macroeconomic risk

We first present the original problem studied by Gollier (1991):

$$\begin{aligned} \max \quad & \int_0^{+\infty} \lambda(m) \sum_s p_s [u(\max\{w_s - m, b_s\})] dF(m) \\ \text{s.t.} \quad & \sum_s p_s (f_s(L_s) - w_s L_s - (1 - L_s) b_s) \geq \bar{v} \end{aligned} \quad (5)$$

$$L_s = F(w_s - b_s) \quad (6)$$

$$f'_s(L_s) = w_s - b_s \quad (7)$$

The Lagrangian of this problem can be written as

$$\begin{aligned} \mathcal{L} = & \sum_s p_s \left[\int_0^{w_s - b_s} \lambda(m) u(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \lambda(m) u(b_s) dF(m) \right] \\ & + \gamma \sum_s p_s [(f_s(F(w_s - b_s)) - w_s F(w_s - b_s) - (1 - F(w_s - b_s)) b_s) - \bar{v}] \\ & + \eta [f'_s(L_s) - w_s + b_s] \end{aligned}$$

And the first order conditions for an interior solution are:

$$\frac{\partial \mathcal{L}}{\partial w_s} = \int_0^{w_s - b_s} \lambda(m) u'(w_s - m) dF(m) - \gamma F(w_s - b_s) - \eta = 0 \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial b_s} = \int_{w_s - b_s}^{+\infty} \lambda(m) u'(b_s) dF(m) - \gamma (1 - F(w_s - b_s)) + \eta = 0 \quad (9)$$

Equations (8) and (9) imply

$$\begin{aligned} & \int_0^{w_s - b_s} \lambda(m) u'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \lambda(m) u'(b_s) dF(m) \\ &= \gamma F(w_s - b_s) + \gamma (1 - F(w_s - b_s)) = \gamma. \end{aligned}$$

Now consider the alternative problem that we suggest:

$$\begin{aligned} \max \quad & \sum_s p_s u \circ \varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right) \\ \text{s.t.} \quad & (5), (6), (7). \end{aligned}$$

The Lagrangian of this problem can be written as

$$\begin{aligned} \mathcal{L} = & \sum_s p_s u \circ \varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right) \\ & + \gamma \sum_s p_s [(f_s(F(w_s - b_s)) - w_s F(w_s - b_s) - (1 - F(w_s - b_s)) b_s) - \bar{v}] \\ & + \eta [f'_s(L_s) - w_s + b_s] \end{aligned}$$

And the first order conditions for an interior solution are:

$$\frac{\partial \mathcal{L}}{\partial w_s} = \frac{u'(EDE_s)}{\varphi'(EDE_s)} \left(\int_0^{w_s - b_s} \varphi'(w_s - m) dF(m) \right) - \gamma F(w_s - b_s) - \eta = 0 \quad (10)$$

$$\frac{\partial \mathcal{L}}{\partial b_s} = \frac{u'(EDE_s)}{\varphi'(EDE_s)} \left(\int_{w_s - b_s}^{+\infty} \varphi'(b_s) dF(m) \right) - \gamma (1 - F(w_s - b_s)) + \eta = 0 \quad (11)$$

Equations (10) and (11) imply

$$\begin{aligned} & \frac{u'(EDE_s)}{\varphi'(EDE_s)} \left(\int_0^{w_s - b_s} \varphi'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi'(b_s) dF(m) \right) \\ &= \gamma F(w_s - b_s) + \gamma (1 - F(w_s - b_s)) = \gamma. \end{aligned}$$

When φ becomes infinitely concave (when an infinite priority is given to the worst-off,

$$\varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right) = \min_i \max(w_s - m_i, b_s) = b_s$$

The program hence becomes:

$$\begin{aligned} \max \quad & \sum_s p_s u(b_s) \\ \text{s.t.} \quad & (5), (6), (7), \end{aligned}$$

so that the first order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w_s} &= -\gamma F(w_s - b_s) - \eta = 0, \\ \frac{\partial \mathcal{L}}{\partial b_s} &= u'(b_s) - \gamma(1 - F(w_s - b_s)) + \eta = 0, \end{aligned}$$

implying $u'(b_s) = \gamma$.