



Fair management of social risk

Marc Fleurbaey, Stéphane Zuber

► To cite this version:

| Marc Fleurbaey, Stéphane Zuber. Fair management of social risk. 2015. halshs-00973480v2

HAL Id: halshs-00973480

<https://shs.hal.science/halshs-00973480v2>

Submitted on 30 Oct 2015 (v2), last revised 18 Jan 2017 (v3)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Fair management of social risk

Marc FLEURBAEY, Stéphane ZUBER

2014.16R

Version révisée



Fair management of social risk*

Marc Fleurbaey^a

Stéphane Zuber^b

September 2015

Abstract

We provide a general method for extending social preferences defined for riskless economic environments to the context of risk and uncertainty. We apply the method to the problems of managing unemployment allowances (in the context of macroeconomic fluctuations) and catastrophic risks (in the context of climate change). The method guarantees ex post fairness and pays attention to individuals' risk attitudes, while ensuring rationality properties for social preferences, revisiting basic ideas from Harsanyi's celebrated aggregation theorem (Harsanyi, 1955). The social preferences that we obtain do not always take the form of an expected utility criterion, but they always satisfy statewise dominance. We obtain a new characterization of the maximin criterion when expected utility and minimal equity are combined. We also show how non-expected utility individual preferences can be accommodated in the approach, although it raises issues regarding belief aggregation.

Keywords: Social choice, fairness, uncertainty, social risk.

JEL Classification numbers: D63.

* This research is part of the EquiRisk project supported by the Agence Nationale de la Recherche (ANR-12-INEG-0006-01). This research has also been supported by the Chair on Welfare Economics and Social Justice at the Institute for Global Studies (FMSH – Paris) and Franco-Swedish Program on Economics and Philosophy (FMSH & Riksbankens Jubileumsfond). Zuber's research has been supported by the Princeton Institute for International and Regional Studies Research Community "Communicating Uncertainty: Sciences, Institutions and Ethics in the Politics of Global Climate Change".

^aWoodrow Wilson School and University Center for Human Values, Princeton University, USA. E-mail: mflourba@princeton.edu.

^bParis School of Economics – CNRS, France. E-mail: Stephane.Zuber@univ-paris1.fr.

1 Introduction

Public policies are always fraught with uncertainty about their consequences. For the evaluation of uncertain consequences, the literature either adopts the expected utilitarian criterion or incorporates risk and uncertainty as a simple extension of the commodity space – commodities being relabeled as contingent on the realization of a state of nature, as in general equilibrium theory (Debreu, 1959, chap. 7). But either approach is questionable.

The expected utilitarian criterion was famously defended by Harsanyi (1955), who proved the following theorem: if (i) all individuals and the social observer are expected utility maximizers on the set of simple lotteries over a set X ; (ii) whenever all individuals prefer lottery p to lottery q , the social observer also prefers p to q (the Pareto principle); then the social observer's utility function must be an affine combination of individuals' von Neumann-Morgenstern (vNM) utility functions. This result is known as Harsanyi's aggregation theorem.¹ Although Harsanyi viewed this theorem as a positive result justifying utilitarianism, it is more accurate to understand it as pointing to the tension between rationality (expected utility), *ex ante* Pareto, and equity.

Regarding equity in resource allocation, Harsanyi's theorem is clearly problematic. Consider a one-commodity economy with two individuals 1 and 2 who face a risk on their incomes described by random variables \tilde{x}_1 and \tilde{x}_2 . Assume furthermore that the individuals are expected utility maximizers so that their ordinal preferences are represented by $Eu_1(\tilde{x}_1)$ and $Eu_2(\tilde{x}_2)$. Harsanyi's theorem tells us the the society should aim at maximizing $E(u_1(\tilde{x}_1) + u_2(\tilde{x}_2))$ (after an appropriate normalization of the vNM utility functions as in Dhillon and Mertens,

¹Several versions of the theorem have been established for different expected utility models in frameworks involving risk or uncertainty (important references include De Meyer and Mongin, 1995; Mongin, 1995, 1998; Blackorby et al., 1999).

1999, for instance). Thus, on sure prospects, the social welfare criterion should be $u_1(x_1) + u_2(x_2)$. If the individuals have different risk preferences, this social criterion does not satisfy the Pigou-Dalton transfer principle in general.² If u_1 and u_2 are the same, then inequality aversion is constrained by risk aversion, which does not seem particularly appealing either.

Starting from a criterion of equity in resource allocation, one could think that it is natural to adopt the other approach invoked in the first paragraph, dealing with contingent commodities. The problem is that such an approach is likely to entail a violation of rationality at the social level. Indeed, a lottery that increases inequality ex post, thereby making sure that the final consequence is worse in every state of the world, may appear attractive ex ante if everyone has a reasonable chance of being on the winning side. A typical fair criterion extended to lotteries in this way will therefore often violate the basic rationality condition of statewise dominance. This violation appears to be a direct consequence of the combination of the ex ante Pareto principle with equity (inequality aversion). The tension between rationality, ex ante Pareto and equity is therefore serious, and one must seek compromises that trade off these important values.

In this paper, we examine how to evaluate risky prospects while preserving basic rationality conditions (either statewise dominance or expected utility) as well as basic fairness principles. Our approach is compatible with a wide range of social criteria for the evaluation of non-risky allocations, involving various types of interpersonal comparisons and degrees of inequality aversion. In particular, we are not committed to the assumption that individuals' expected utilities provide the correct measure for assessing and comparing individual welfare. This assump-

²At any value x such that $u'_1(x) \neq u'_2(x)$, a small inequality in favor of the individual with greater marginal utility is better than equality. For another illustration of the conflict between the Pareto principle and equity in risky situations, see Gajdos and Tallon (2002).

tion was made in the initial paper by Harsanyi (1955), and in most of the subsequent literature (e.g., Diamond, 1967; Broome, 1991; Epstein and Segal, 1992; Ben Porath et al., 1997; Gajdos and Maurin, 2004; Grant et al., 2010; Fleurbaey, 2010). More recently, Adler (2013) revived the idea of using vNM utilities for the measurement of well-being in the context of evaluations with social welfare functions. But this remains a controversial assumption (see e.g. Chambers and Echenique, 2012) and many alternative welfare measures have been proposed in the literature, including happiness measures, capabilities and money-metric utilities (Decancq et al., 2015). The approach we propose in this paper is compatible with many of these different views.

We first show that statewise dominance and ex ante Pareto restricted to the union of riskless prospects and perfectly egalitarian prospects already characterize a particular way of extending a given social ordering from non-risky allocations to risky prospects. It consists in first constructing a hypothetical prospect where in each state of the world the allocation is replaced by an equally good egalitarian allocation. Then, the social ordering for non-risky allocations is applied to the distribution of certainty-equivalent bundles of this hypothetical prospect.

This extended social ordering always satisfies statewise dominance but may violate stronger rationality conditions, therefore we examine the implications of imposing the expected utility condition on this ordering. It turns out that, if one does not want the social ordering on riskless prospects to depend on individual risk attitudes, then under certain assumptions this social ordering must have the maximin form. We therefore obtain a new characterization of the maximin criterion.

With this new class of criteria, we revisit classical issues of risk management. We first show that the optimal management of unemployment allowance proposed by Gollier (1991) and Drèze and Gollier (1993) is by and large vindicated

but needs adjustment in order to accommodate a special concern for income inequalities that the policy-maker may have. This may attenuate or even reverse Gollier's result that allowances should be countercyclical with wages, for some distributions of wages and some values. The reversal may occur because procyclical allowances induce less inequalities in good states of the world. With a strong degree of priority to the worst-off, the allowances become rigid in order to provide full insurance to the unemployed.

We then revisit Weitzman's dismal theorem (Weitzman, 2009). While he studied transfers of resources to the future, we focus on the prevention of catastrophes (the policy affects probabilities rather than the consumption stream), for which a variant of the dismal theorem obtains with the utilitarian criterion – but it takes the form of sensitiveness to utility parameters rather than a divergence to infinity (not unlike Weitzman's original result, actually). We show that our criterion makes it possible to have an even greater social willingness to pay for the prevention of catastrophes if the catastrophic state is worse than the situation of the current generation, but also a lower willingness to pay in the opposite case.

We finally examine the extension of our analysis to the case in which probabilities are not defined (a case usually labelled as involving uncertainty rather than risk). The bulk of our analysis remains valid in this case, but new questions arise about the aggregation of beliefs. The distributive preferences of the ordering on riskless prospects have an impact on the management of ambiguity at the social level which may appear problematic. Our conclusion is that the aggregation of beliefs should be separated from the aggregation of preferences and should be performed at an earlier stage.

The paper is organized as follows. In Section 2 we introduce the framework, which is Savage's state-contingent model of choice under uncertainty (Savage, 1972), applied to economic environments consisting of allocations of private goods.

Section 3 introduces a basic result showing the general implications of combining a special version of the Pareto principle with the statewise dominance principle for the social evaluation of uncertain allocations. Section 4 specializes the result to the case in which the expected utility condition holds at the social level. In Section 5, with a stronger Pareto condition we provide a complete characterization of a social ordering extended to risky prospects, and discuss its rationality properties. While this result enables us to extend any member of a wide class of social orderings over riskless prospects, we also provide a new characterization of the maximin criterion when the expected utility condition is imposed and the social ordering on riskless prospects is required to be independent of individual risk attitudes. In Section 6, we apply the new criterion to the optimal management of unemployment allowances and of catastrophic risks. Last, in section 7 we explore how to extend our analysis to the context of uncertainty.

2 The model

We consider an economy composed of n individuals and ℓ goods. An allocation x is a collection

$$x = (x_{ik})_{i \in \{1, \dots, n\}, k \in \{1, \dots, \ell\}},$$

where x_{ik} is the quantity of good k consumed by individual i . Let $X = \mathbb{R}_+^{n\ell}$ denote the set of allocations.³ To shorten notations, we define $N = \{1, \dots, n\}$ and $L = \{1, \dots, \ell\}$.

³The symbol \mathbb{R}_+ denotes the set of non-negative real numbers, \mathbb{R}_{++} the set of positive real numbers, and, for any positive integer n , \mathbb{R}_+^n is the n -fold Cartesian product of \mathbb{R}_+ . We use the whole positive orthant for the sake of simplicity. The analysis could be generalized to connected subsets of \mathbb{R}_+^ℓ with appropriate properties guaranteeing that Assumption 1 below can be satisfied.

We consider economies where the final allocation may be uncertain. Hence we let Ω be the set of states of the world, and \mathcal{A} a σ -algebra over Ω . A simple act f is a finite-valued \mathcal{A} -measurable function $f : \Omega \rightarrow X$, and we denote \mathcal{F} the set of simple acts. For any act f , it will be convenient to write $f = (f_1, \dots, f_n)$ where, for each $i \in N$ and each $\omega \in \Omega$, $f_i(\omega) = (x_{ik})_{k \in L}$ whenever $f(\omega) = (x_{ik})_{i \in N, k \in L}$.

Until section 7, we assume that a probability measure p on the measurable space (Ω, \mathcal{A}) appropriately represents the society's beliefs concerning the likelihood of the different events in \mathcal{A} . For a continuous function $u_i : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$, $i \in N$, and an act $f \in \mathcal{F}$, we define the expected utility of f_i as

$$E(u_i \circ f_i) = \int_{\Omega} u_i(f_i(\omega)) dp(\omega).$$

Given any $x \in X$, we abuse notation and let $x \in \mathcal{F}$ also denote the constant act such that $x(\omega) = x$ for all $\omega \in \Omega$, thus identifying X with the subset of the constant acts in \mathcal{F} . In the case of constant acts, we denote by x_i the constant bundle $x_i(\omega)$, $i \in N$, and use the same notation for allocations. For $\eta \in X$, we denote by $(f + \eta)$ (resp., $(f - \eta)$) the act such that, for all $\omega \in \Omega$, $(f + \eta)(\omega) = f(\omega) + \eta$ (resp., $(f - \eta)(\omega) = f(\omega) - \eta$). Similarly, for $\varepsilon \in \mathbb{R}_+^\ell$, we denote by $(f_i + \varepsilon)$ (resp., $(f_i - \varepsilon)$) the act such that, for all $\omega \in \Omega$, $(f_i + \varepsilon)(\omega) = f_i(\omega) + \varepsilon$ (resp., $(f_i - \varepsilon)(\omega) = f_i(\omega) - \varepsilon$).

The problem studied in this paper is the selection of a reasonable social ordering R on \mathcal{F} . More specifically, we want to define such an ordering so as to extend a given social ordering R^0 defined over sure allocations X . The relations R and R^0 are therefore complete, reflexive and transitive over \mathcal{F} and X , respectively. We denote by P (resp., I) the asymmetric (resp., symmetric) part of R , and P_0 (resp., I_0) the asymmetric (resp., symmetric) part of R_0 .

The ordering R^0 is assumed to be based on an ordering R^* on \mathbb{R}^n and on

functions $v_i : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ in the following way: $\forall x, y \in X$,

$$xR^0y \Leftrightarrow (v_i(x_i))_{i \in N} R^* (v_i(y_i))_{i \in N}. \quad (1)$$

The functions v_i are therefore suitable individual well-being functions, and the ordering R^* (whose strict preference and indifference counterparts are denoted P^* and I^* , respectively) aggregates these welfare indices. We make the following two assumptions.⁴

Assumption 1 *Every v_i is continuous increasing and the range of v_i is the same for all i .*

Assumption 2 *The ordering R^* is weakly monotonic, i.e. for any $z, z' \in \mathbb{R}^n$, (a) if $z \geq z'$ then zR^*z' ; (b) if $z \gg z'$ then zP^*z' .*

Assumption 2 is akin to a Pareto principle defined *ex post* on sure allocations, provided the v_i functions are representations of individual preferences. We do not assume the full “Strong Pareto” principle because we want our approach to be general enough to cover social preferences such as the maximin criterion.

Welfare economics has provided various methods for the construction of orderings R^0 on riskless allocations. Most of them take the form exhibited in Eq. (1),⁵ and many of them use v_i functions satisfying Assumption 1. For instance, Assumption 1 is satisfied by the Pazner-Schmeidler social ordering (Pazner, 1979), for which $v_i(x_i) = \min \{ \lambda \in \mathbb{R}_+ \mid (\lambda z) R_i x_i \}$, where R_i represents i 's preferences over bundles and z is a reference bundle. Another example is provided by the

⁴For two vectors $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}^n$, $x \geq y$ if $x_i \geq y_i$ for all $i \in \{1, \dots, n\}$ and $x \gg y$ if $x_i > y_i$ for all $i \in \{1, \dots, n\}$.

⁵An exception is the Egalitarian Walrasian social ordering function defined for instance in Fleurbaey and Maniquet (2011), because the v_i functions depend on relative scarcities, and therefore on the whole allocation (not only on individual bundles).

sum of normalized VNM utilities, axiomatized by Dhillon and Mertens (1999), for which $v_i(x_i) = \frac{u_i(x_i) - \underline{u}_i}{\bar{u}_i - \underline{u}_i}$, where $\bar{u}_i := \sup_{z \in \mathbb{R}_+^\ell} u_i(z)$ and $\underline{u}_i := \min_{z \in \mathbb{R}_+^\ell} u_i(z)$. A last example involves the health utility indices discussed in Drummond et al. (2005).

In the paper, we let R^0 remain unspecified, and focus on its extension to uncertain allocations. In this way our theory is quite general.

Let X^e denote the set of “egalitarian” allocations i.e., $x \in X$ such that

$$\forall i, j \in N, v_i(x_i) = v_j(x_j).$$

Let \mathcal{F}^e denote the set of egalitarian acts, i.e., $\mathcal{F}^e = \{f \in \mathcal{F} \mid f(\omega) \in X^e \text{ for all } \omega \in \Omega\}$.

We now define an equally-distributed quasi-equivalent (EDQE) of an allocation $x \in X$:⁶

Definition 1 *For any given $x \in X$, $x^e \in X^e$ is an equally-distributed quasi-equivalent (EDQE) of x if for all $z \in \mathbb{R}_{++}^{n\ell}$,*

$$(x + z) P^0 x^e \text{ and } (x^e + z) P^0 x.$$

Obviously, for an egalitarian allocation $x \in X^e$, one can take $x^e = x$. Note also that the EDQE allocation is not unique: any egalitarian allocation which is deemed indifferent by all individuals are also EDQE allocations. The definition of the EDQE is straightforwardly extended from allocations to acts by letting, for every $f \in \mathcal{F}$, f^e be defined by $f^e(\omega) = (f(\omega))^e$ for all $\omega \in \Omega$.

With the assumptions made earlier, we have the following lemma:

⁶As explained later, this definition simplifies when continuity is assumed, but at this stage we want our analysis to be valid also for R^0 orderings of the leximin sort, which are frequent in social choice theory and are not continuous.

Lemma 1 *Under Assumptions 1 and 2:*

- (i) *For all $x \in X$, there exists $x^e \in X^e$ such that x^e is an EDQE of x . Furthermore if \tilde{x}^e is another EDQE of x , then $x^e I^0 \tilde{x}^e$.*
- (ii) *$(0, \dots, 0) \in X^e$, and for all $x \in X$ such that $(v_i(x_i))_{i \in N} \gg (v_i(0))_{i \in N}$, one can take $x^e \gg (0, \dots, 0)$.*
- (iii) *For all $x, y \in X$, if $x^e P^0 y^e$, then $x P^0 y$.*

Proof. See Appendix 1. ■

For the moment, we have mostly focused on constant acts with no uncertainty about the final allocation. When there is uncertainty, we will assume that individual i 's interests are appropriately represented by the expected utility $E(u_i \circ f_i)$, where the utility function u_i is continuous and increasing. To ensure coherence between the judgements in situations involving uncertainty and situations comparing sure allocations, we assume that the functions u_i and v_i are ordinally equivalent.

Assumption 3 *For all $i \in N$, for all $x_i, y_i \in \mathbb{R}_+^\ell$,*

$$u_i(x_i) \geq u_i(y_i) \iff v_i(x_i) \geq v_i(y_i).$$

Assumption 3 implies that there exist increasing functions ϕ_i such that $u_i = \phi_i \circ v_i$. We will call ϕ_i the “risk attitude” of individual i , because they embody the aversion to risk on welfare measured by the v_i function.

We now introduce two principles that we will use to derive R from the social ordering on allocations R^0 . The first principle is a restricted version of the Pareto principle, introduced by Fleurbaey (2010) and adapted to the present framework. In contrast to R^0 , we do not require R to be fully Paretian, but only (for the moment) to respect unanimous preferences in the context of equality.

Pareto for Equal Risk For all $f, g \in \mathcal{F}^e$, if $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for all $i \in N$, then pRq ; if $E(u_i \circ f_i) > E(u_i \circ g_i)$ for all $i \in N$, then pPg .

The second principle is a rather natural principle of social rationality, connecting the social orderings R and R^0 .

Dominance For all $f, g \in \mathcal{F}$, if for all $\omega \in \Omega$, $f(\omega)R^0g(\omega)$ then fRg ; if for all $\omega \in \Omega$, $f(\omega)P^0g(\omega)$ then fPg .

This axiom implies in particular that for all $x, y \in X$, xR^0y if and only if xRy (where, in the latter expression, the allocations are identified with the corresponding constant act). So Dominance defines a notion of coherence between the ordering R^0 over allocations, and the ordering R over uncertain allocations.

3 The basic result

Our first result describes the structure of social orderings satisfying Pareto for Equal Risk and Dominance. We show that they consist in an aggregation of the individuals' expected utilities of the EDQE allocations.

Proposition 1 *If the social ordering R satisfies Pareto for Equal Risk and Dominance, then there exists a weakly monotonic ordering R^{**} on \mathbb{R}^n such that for all $f, g \in \mathcal{F}$, fPg whenever there exists $\varepsilon \in \mathbb{R}_{++}^\ell$ such that*

$$\left(E(u_i \circ (f_i^e - \varepsilon))\right)_{i \in N} R^{**} \left(E(u_i \circ (g_i^e + \varepsilon))\right)_{i \in N}.$$

Proof. See Appendix 1. ■

Loosely speaking, Proposition 1 describes a three-step construction of the ordering for uncertain allocations. First, in each state of the world, we compute an EDQE allocation. Then, we compute the expected utilities for the individuals at these EDQE allocations. Last, we assess these expected utilities using

the ordering R^{**} . This presentation is not completely correct, because we do not compare directly the expected utilities of the EDQE allocations, but the expected utilities of the EDQE allocations plus or minus some resources. In particular $(E(u_i \circ f_i^e))_{i \in N} P^{**} (E(u_i \circ g_i^e))_{i \in N}$ would not suffice to guarantee that $f R g$, because the ordering R^{**} need not be continuous.

Observe, however, that if R^0 is continuous, then the EDQE x^e is a true “equally-distributed equivalent” (EDE), i.e., $x^e I^0 x$. In that case Proposition 1 can be formulated in a much simpler way.

Corollary 1 *If the social ordering on allocations R^0 is continuous and the social ordering R satisfies Pareto for Equal Risk and Dominance, then there exists a weakly monotonic ordering R^{**} on \mathbb{R}^n such that for all $f, g \in \mathcal{F}$, $f R g$ if and only if*

$$\left(E(u_i \circ f_i^e) \right)_{i \in N} R^{**} \left(E(u_i \circ g_i^e) \right)_{i \in N}.$$

Proof. See Appendix 1 ■

Proposition 1 does not put any constraint on R^{**} , except weak monotonicity. Therefore, not much can be said on the exact form of R on the sole basis of the two axioms. In the next sections, we study two ways in which a more precise specification can be obtained. But we first describe two cases in which specifying R^{**} is not required in order to obtain precise social welfare criteria.

In the first case, one adopts the Bernoulli assumption that individuals’ expected utilities provide the correct measure for assessing and comparing individual welfare. In our framework, the Bernoulli assumption means that $u_i = v_i$ for all $i \in N$. Then, by definition of \mathcal{F}^e , for all $f \in \mathcal{F}^e$ one always has $E(v_i \circ f_i) = E(v_j \circ f_j)$ for all $i, j \in N$. This makes it unnecessary to seek an exact specification of R^{**} . Indeed, let $f, g \in \mathcal{F}$ be such that $E(u_i \circ f_i^e) > E(u_i \circ g_i^e)$ for some $i \in N$. Because $u_i = v_i$ for all $i \in N$ and $v_i(f_i^e(\omega)) = v_j(f_j^e(\omega))$ for all

$\omega \in \Omega$ and $i, j \in N$, this implies that $E(u_i \circ f_i^e) > E(u_i \circ g_i^e)$ for all $i \in N$. By Pareto for Equal Risk, we thus have $f^e P g^e$, and hence (by transitivity) $f P g$.⁷

Although it has been adopted by most of the literature, the Bernoulli assumption is not uncontroversial. It implies relying on risk aversion to assess the allocation of resources, even when there is no risk. It is a strange idea to rely on vNM functions to make interpersonal comparisons in allocations where risk is absent. Our introductory example also points out that the assumption may imply recommending transfers that increase resource inequality.

There is another case where we do not need to have a precise ordering R^{**} , namely when people have the same risk attitudes as described by functions ϕ_i . Recall that, by Assumption 3, we have $u_i = \phi_i \circ v_i$. If for all $i \in N$ $\phi_i = \phi$, we obtain that for all $f \in \mathcal{F}$, $\omega \in \Omega$ and $i, j \in N$, $u_i(f_i^e(\omega)) = \phi_i \circ v_i(f_i^e(\omega)) = \phi_i \circ v_j(f_j^e(\omega)) = u_j(f_j^e(\omega))$. Hence, the same reasoning as above applies. Note that we do not need to endorse the Bernoulli assumption in this case: vNM utility functions are just acceptable measures of the *level* of welfare but transfers of utility using these functions may still not be meaningful.

In the two cases we have just discussed, Proposition 1 can be seen as adapting Theorem 1 in Fleurbaey (2010) to a framework which explicitly describes the economic environment. The proposition highlights the importance of the Bernoulli assumption made by Fleurbaey (2010). In the next sections we investigate approaches allowing for the possibility that u_i and v_i may be quite different.

⁷The same reasoning implies that when $E(u_i \circ f_i^e) = E(u_i \circ g_i^e)$ we must have $f I g$.

4 Expected utility at the social level

The social ordering described in Proposition 1 satisfies Dominance, but one may argue that this is too weak a condition of social rationality.⁸ To remain close to Harsanyi's initial framework, one may require social criteria on risky allocations to take the form of expected values.

Expected Utility There is a continuous function W such that for all $f, g \in \mathcal{F}$, $f R g$ if and only if $E(W \circ f) \geq E(W \circ g)$.

This principle is logically stronger than Dominance. Note moreover that the continuity of W over allocations is assumed, so that both R^0 and R must be continuous. Such an assumption of continuity has been avoided in Proposition 1 to cover social preferences of the leximin type.

To obtain sharp results, we will introduce a condition on the probability measure p and the set Ω . To do so, additional notation is needed. For any $m \in \mathbb{N} \setminus \{1, 2\}$, we denote by \mathcal{A}^m the set of partitions of Ω in m elements of \mathcal{A} . Let $P^m = \{(p(A_1), \dots, p(A_m)) \in [0, 1]^m \mid (A_1, \dots, A_m) \in \mathcal{A}^m\}$. We make the following assumption:

Assumption 4 *There exists $m \in \mathbb{N} \setminus \{1, 2\}$ such that*

$$P^m = \left\{ \pi = (\pi_1, \dots, \pi_m) \in [0, 1]^m \mid \sum_1^m \pi_m = 1 \right\}.$$

Assumption 4 is rather natural in the context of subjective expected utility *à la* Savage. It is indeed closely related to Axiom P6 in Savage (1972), which guarantees that Ω can be partitioned in a finite number of subsets to obtain subjective probabilities that are as low as one wishes. The assumption is satisfied

⁸Rationality is discussed in more detail in the next section.

whenever Ω is an infinite set and p is a non-atomic probability measure. In the rest of the section we assume that Assumption 4 is satisfied.

Imposing only Pareto for Equal Risk, one first gets the following result.

Proposition 2 *The social ordering R satisfies Pareto for Equal Risk and Expected Utility if and only if there exists a vector $\alpha \in \mathbb{R}_+^n$ such that for all $f, g \in \mathcal{F}$, $f R g$ if and only*

$$\sum_{i \in N} \alpha_i E(u_i \circ f_i^e) \geq \sum_{i \in N} \alpha_i E(u_i \circ g_i^e).$$

Proof. See Appendix 1. ■

Proposition 2 suggests to derive the social utility function used in the expected utility formula from a linear aggregation of individuals' utility indices. The result is similar to Harsanyi's (1955) result, except that the formula is applied only to egalitarian acts (the EDE acts).

The above proposition fails to fully specify the ordering, because the weights α_i remain undetermined. Note again that if one endorses the Bernoulli assumption, or assumes that people have the same risk attitudes, the choice of the weights plays no role. But this is not true in general. The next section discusses a case in which a very specific weighting scheme can be obtained, provided the Pareto condition is strengthened.

5 Strengthening Pareto

As discussed before, Assumption 2 implies that the Pareto principle applies *ex post* to sure allocations, provided the v_i functions are representations of individuals' preferences. We have also applied the Pareto principle *ex ante* to egalitarian acts. In this subsection, we propose to strengthen the Pareto condition to cover comparisons between egalitarian and constant acts.

Pareto for Equal or No Risk For all $f, g \in (\mathcal{F}^e \cup X)$, if $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for all $i \in N$, then fRg .

5.1 A characterization

The main argument of Fleurbaey (2010) to restrict the use of the Pareto principle *ex ante* to egalitarian lotteries is that the social observer may be in a situation that is equivalent to having more information than the individuals in lotteries involving inequality, and may want to ignore the individuals' *ex ante* preferences when she knows the final distribution of their situations, including their *ex post* preferences. Typically, if the uncertainty bears only on who will be the rich and who will be the poor, there is no uncertainty at the social level (the final distribution of welfare is known) even if individuals do face uncertain prospects. In such a case, the observer may judge the situation equivalent to a sure allocation involving the same distribution. The argument therefore does not preclude respecting individual preferences when only sure prospects and egalitarian uncertain prospects are at stake. Pareto for Equal and No Risk therefore seems an acceptable strengthening of Pareto for Equal Risk, in view of this line of argument.

One may also notice that when $u_i = v_i$ for all $i \in N$, Pareto for Equal and No Risk is already almost implied by Pareto for Equal Risk. Indeed, if $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for all $i \in N$ for $f \in \mathcal{F}^e$ and $g \in X$, one has $E(u_i \circ f_i) = E(u_j \circ f_j)$ for all $i, j \in N$, and $E(u_i \circ g_i) \leq \max_j E(u_j \circ g_j) \leq E(u_i \circ f_i)$ for all $i \in N$, so that $E(u_i \circ f_i) \geq E(u_i \circ g_i^e)$ for all $i \in N$. By Pareto for Equal Risk, this implies fRg^e . Therefore, if gIg^e (which occurs when R^0 is continuous), then fRg , so that Pareto for Equal and No Risk holds. A similar reasoning applies when $E(u_i \circ f_i) \leq E(u_i \circ g_i)$ for all $i \in N$. In the framework of Fleurbaey (2010), strengthening Pareto for Equal Risk to Pareto for Equal and No Risk would make no difference.

In the present context, this extension of Pareto for Equal Risk allows us to obtain a quite precise specification of the social ordering R , independently of the definition of the v_i functions, and without invoking the Expected Utility condition of the previous section. To present this result we introduce further notation. For any $f \in \mathcal{F}$, we let $ce(f) \in X$ denote a certainty-equivalent allocation, i.e., an allocation such that for all $i \in N$, $u_i(ce_i(f_i)) = E(u_i \circ f_i)$. When $f_i(\omega) \in \mathbb{R}_{++}^\ell$ for all $i \in N$ and $\omega \in \Omega$, we assume that for convenience one takes $ce_i(f_i) \in \mathbb{R}_{++}^\ell$.

Proposition 3 *If the social ordering R satisfies Pareto for Equal or No Risk and Dominance, then for all $f, g \in \mathcal{F}$, fPg whenever there exists $\eta \in \mathbb{R}_{++}^{n\ell}$ such that*

$$(ce(f^e) - \eta) R^0 (ce(g^e) + \eta).$$

Proof. See Appendix 1. ■

Compared to Proposition 1, Proposition 3 gives an exact description of the order R^{**} , which corresponds to R^0 once expected utilities have been transformed into certainty equivalent bundles.

Like Proposition 1, Proposition 3 describes a three-step construction for ordering uncertain allocations. First, in each state of the world, we compute an EDQE allocation. Then, we compute the certainty equivalent of this EDQE allocation for all individuals. Last, we assess the distribution of the certainty equivalent using the social ordering for riskfree allocations. This summary is actually fully correct only when R^0 is continuous, so that the EDQE is actually an EDE. In that case, we have the following Corollary (whose proof is similar to the proof of Corollary 1).

Corollary 2 *If R^0 is continuous and the social ordering R satisfies Pareto for Equal or No Risk and Dominance, then for all $f, g \in \mathcal{F}$, fRg if and only if*

$$ce(f^e) R^0 ce(g^e). \tag{2}$$

5.2 Rationality issues

The criterion defined in (2) satisfies not just Dominance but also stochastic dominance. Suppose that the probability of getting $f(\omega) R^0 z$ is greater than the probability of $g(\omega) R^0 z$ for all $z \in X$. Then the probability of getting $f^e(\omega) R^0 z$ is greater than the probability of $g^e(\omega) R^0 z$ for all $z \in X$. For every i , $ce_i(f^e)$ increases when the value of $f^e(\omega)$ increases for a positive mass of ω and decreases for at most a null mass. Therefore $ce_i(f^e)$ increases when the distribution of $f^e(\omega)$ improves in the first order stochastic sense.

However, this criterion does not satisfy eventwise dominance, and this is a direct consequence of Pareto for Equal or No Risk, in the presence of inequality aversion. This is shown by the following example. Assume that $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and $p(\omega_1) = p(\omega_2) = p(\omega_3) = 1/3$. There are only two individuals: individual 1 is extremely risk averse whereas individual 2 is risk-neutral. We assume that there is only one commodity, used as the welfare metric v_i . Consider the acts depicted in Table 1.

	Act f			Act g		
States	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
Individual 1	1	3	$3a - 4$	0	b	$3a - 3$
Individual 2	1	3	$3a - 4$	0	b	$3a - 3$
EDE	1	3	$3a - 4$	0	b	$3a - 3$
$ce_1(EDE)$		1			0	
$ce_2(EDE)$		a			$a - 1 + b/3$	
Conditional on $\{\omega_1, \omega_2\}$						
$ce_1(EDE)$		1			0	
$ce_2(EDE)$		2			$b/2$	

Table 1: Example of a violation of eventwise dominance

Conditionally on event $\{\omega_3\}$, act f is obviously worse than act g . Conditionally on event $\{\omega_1, \omega_2\}$, act f is also worse than act g , when inequality aversion is not too high and b is large enough. But, for a given level of inequality aversion and a given b , act f is unconditionally better than act g for a large enough, because the gain $b/3 - 1$ for individual 2 becomes negligible when a is great. One therefore obtains the paradox that f is better than g although it is worse conditionally on each event of a partition of Ω . Such a situation is problematic in itself, and generates additional problems in sequential decisions.

Indeed, let $f^- = f - \varepsilon$ (i.e., ε is deducted from every consumption of every agent in every state), for ε small enough so that the preference for $f - \varepsilon$ over g remains. Consider the following sequential decision problem. At stage 1 the decision maker can opt for f^- or delay the choice until stage 3. At stage 2, the decision maker learns if $\omega \in \{\omega_1, \omega_2\}$ or $\omega = \omega_3$. At stage 3, the decision maker chooses between f and g , if f^- was not chosen at stage 1.

Note that in this sequence, it is possible to delay and secure the outcomes of f whatever happens. As f statewise dominates f^- , it seems irrational to opt for f^- in stage 1. Nevertheless, a decision-maker using the criterion discussed here will face the following dilemma.

The best plan *ex ante* is to wait until stage 3 and choose f . But at stage 3, we know that g will appear better than f conditionally on any piece of information that may be obtained in stage 2. Therefore, time inconsistency is likely to occur here. A resolute decision-maker would moreover appear irrational because, for the criterion under consideration, there is every reason to use the information acquired in stage 2 when one is at stage 3.

Foreseeing that at stage 3 the choice will be for g , at stage 1 it therefore appears better to choose f^- , which entails a quasi-violation of statewise dominance because it is possible to obtain f , which is better in every state of the world.

There is one way to avoid these difficulties. If R^0 is a maximin criterion, in the above example f is better than g conditionally on event $\{1, 2\}$, no matter how great b is. It is clear from the example that only an ordering giving absolute priority to the worst-off escapes the problem. In the next subsection, we show that maximin criteria can be characterized in this way.

5.3 Characterizing the maximin

To avoid the rationality issues arising from eventwise dominance, one possibility is to go back to the expected utility model, and thus requiring that the social ordering is an expected utility, as we did in Section 4. We now examine under what circumstances this condition is compatible with our stronger Pareto requirement.

We first need to introduce the possibility to compare situations where the risk attitudes ϕ_i of the individuals vary (recall that $u_i = \phi_i \circ v_i$). The social ordering R is now a function of the profile $\Phi = (\phi_i)_{i \in N}$. We write R_Φ the social ordering given the profile Φ . Throughout, we assume that individual ordinal preferences and the welfare indices v_i are fixed.

Let \mathcal{I} denote the range of v_i (the same for all individuals by Assumption 1). Let $C(\mathcal{I})$ be the set of continuous increasing functions from \mathcal{I} to \mathbb{R} . Any profile Φ must then belong to the set $C(\mathcal{I})^N$.

We will now focus our attention to the domain \mathcal{C} of profiles such that one individual i has a weakly greater risk aversion than everyone else, in the sense that for all $j \in N$, ϕ_i is at least as concave as ϕ_j , i.e., there is a concave function φ_j such that $\phi_i = \varphi_j \circ \phi_j$.

Let us now introduce the following property, according to which risk attitudes should not matter when risk is absent.

Invariance to risk attitudes for constant acts For all $\Phi \in \mathcal{C}$, for all $x, y \in$

X ,

$$xR_{\Phi}y \iff xR^0y.$$

This property is similar to a property with the same name in Chambers and Echenique (2012) (only the domain is different). As they explain, the property requires that in ranking profiles of risk-free prospects, the social ordering should ignore risk attitudes. This property is therefore a formal rejection of the Bernoulli assumption.

To obtain our next result, we require the social ordering for allocations to satisfy a basic transfer property. We assume that the social ordering R^0 is equitable in the sense that a more equal distribution of the welfare indices v_i is weakly preferred.

Equity For any $\tilde{x}, \tilde{y} \in X$, if there exist $\varepsilon \geq 0$, $i, j \in N$ such that:

1. $v_k(x_k) = v_k(y_k)$ for all $k \in N \setminus \{i, j\}$;
2. $v_i(y_i) + \varepsilon = v_i(x_i) \leq v_j(x_j) = v_j(y_j) - \varepsilon$;

then xR^0y .

In the next proposition, Equity permits to avoid for instance the ‘maximax’ criterion focusing on the best-off individual.⁹

Proposition 4 *If the social ordering R_{Φ} satisfies Pareto for Equal or No Risk, Expected Utility and Invariance of risk attitudes for constant acts for all profiles $\Phi \in \mathcal{C}$, and if the social ordering R^0 satisfies Equity, then, for all $x, y \in X$,*

$$xR^0y \iff \min_i v_i(x_i) \geq \min_i v_i(y_i).$$

⁹The maximax would actually be obtained on a different domain $\bar{\mathcal{C}}$ consisting of profiles where there is always a least risk-averse person.

Proof. See Appendix 1. ■

The spirit of the criterion suggested by Proposition 4 is exactly the opposite of one proposed in Chambers and Echenique (2012, Example 10): they contend that society should use the risk preferences of the least risk-averse individual (which is consistent with their axiom of reduction of risk aversion). Here we argue that fairness and social rationality, on the contrary, require using the risk preferences of the most risk-averse individual.

Indeed, consider a simple case where $\phi_i(v) = v^{1-\gamma_i}/(1-\gamma_i)$, where γ_i measures relative risk aversion with respect to welfare measured by v_i . Assume that for any $x, y \in X$, $x R^0 y \iff \min_{i \in N} v_i(x_i) \geq \min_{i \in N} v_i(y_i)$. And denote $e(x)$ the real number (EDE welfare level) such that $e(x) = v_i(x^e)$ for every $i \in N$. Using Corollary 2, we have:

$$\begin{aligned}
 f R g &\iff \min_{i \in N} \left(\int_{\Omega} (e(f(\omega)))^{1-\gamma_i} dp(\omega) \right)^{\frac{1}{1-\gamma_i}} \geq \min_{i \in N} \left(\int_{\Omega} (e(g(\omega)))^{1-\gamma_i} dp(\omega) \right)^{\frac{1}{1-\gamma_i}} \\
 &\iff \min_{i \in N} \left(\int_{\Omega} \left(\min_{j \in N} v_j(f_j(\omega)) \right)^{1-\gamma_i} dp(\omega) \right)^{\frac{1}{1-\gamma_i}} \geq \\
 &\quad \min_{i \in N} \left(\int_{\Omega} \left(\min_{j \in N} v_j(g_j(\omega)) \right)^{1-\gamma_i} dp(\omega) \right)^{\frac{1}{1-\gamma_i}} \\
 &\iff \int_{\Omega} \left(\min_{j \in N} v_j(f_j(\omega)) \right)^{1-\gamma_{\max}} dp(\omega) \geq \int_{\Omega} \left(\min_{j \in N} v_j(g_j(\omega)) \right)^{1-\gamma_{\max}} dp(\omega).
 \end{aligned}$$

where $\gamma_{\max} = \max_{i \in N} \gamma_i$.

Hence the criterion suggested in Proposition 4 computes the expected utility of the worst welfare level using the most risk averse preferences. This also shows that this result is consistent with Proposition 2 obtaining a weighted sum of expected utilities of the EDE. It amounts to picking a specific weighting scheme, where a positive weight is assigned to the most risk averse individual, while all other agents have a zero weight. This remark should also make clear that the existence of a most risk averse agent is necessary to get the result: if the individual

with the lower certainty-equivalent could change for different egalitarian acts, the weights could not be independent of the act, because the maximin would put all weight on a different individual depending on the act.

One drawback of the maximin criterion is that it does not satisfy the Strong Pareto principle according to which a situation is strictly preferred to another whenever at least one individual strictly prefers the situation while no other individual weakly prefers the alternative. This may result in inefficient choices by a maximin criterion in some economic environments. As a consequence, the social choice literature typically suggests to use a leximin criterion rather than a maximin criterion.

In the context of the present article, the leximin criterion however raises two challenges. First, because it is not continuous, it does not provide an EDE, but only an EDQE which is the same as the EDE for the maximin. Hence Proposition 3 does not completely specify the social ordering: cases with the same EDQE are not specified (while they are fully determined, in the continuous case, thanks to Corollary 2). Second, because the leximin criterion is not continuous, we cannot take its expectation to get an expected utility criterion at the social level.

If we are willing to use a more general setting than expected utility, we can devise a version of leximin in the present setting. To describe this criterion let us introduce some more notation. For any $n \in \mathbb{N}$, and any $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}^n$, we write $x >_{lex} y$ if, after reordering the components of x and y in increasing order, there exists $k \in \{1, \dots, n\}$ such that $x_j = y_j$ for all $j < k$ and $x_k > y_k$; we write $x \geq_{lex} y$ if either $x >_{lex} y$ or x is a permutation of y .

For every $f \in \mathcal{F}$, let $f_{(\cdot)} \in \mathcal{F}$ denote the act that reorders the $v_i(f_i(\omega))$ by increasing order for each $\omega \in \Omega$: for all $i, j \in N$,

$$i < j \implies v_i(f_{(i)}(\omega)) \leq v_j(f_{(j)}(\omega)).$$

Hence act $f_{(\cdot)}$ generates the same distribution of welfare as f in every state of

the world. But it creates fictional prospects for individuals, where individual 1 always has the worst outcome in all states of the world, individual 2 has the second worst outcome, and so forth.

For an individual act $f_k \in (\mathbb{R}_+^\ell)^\Omega$, let $ce_{(\cdot)}(f_k)$ denote the vector of certainty-equivalent levels of welfare (measured by the v functions) ranked in increasing order: for all $i, j \in N$,

$$i < j \implies ce_{(i)}(f_k) \leq ce_{(j)}(f_k).$$

Definition 2 *The fair (ex post) leximin ordering R_{lex} is defined for any $f, g \in \mathcal{A}$, as,*

$$f R_{lex} g \iff \left((ce_{(i)}(f_{(1)}))_{i \in N}, \dots, (ce_{(i)}(f_{(n)}))_{i \in N} \right) \geq_{lex} \left((ce_{(i)}(g_{(1)}))_{i \in N}, \dots, (ce_{(i)}(g_{(n)}))_{i \in N} \right).$$

Note that when individuals have different risk attitudes, in general one does not have

$$\max (ce_{(i)}(f_{(j)}))_{i \in N} \leq \min (ce_{(i)}(f_{(j+1)}))_{i \in N}.$$

So, the leximin criterion proposed here may sometimes give the priority to a more risk-averse individual in a better rank in $f_{(\cdot)}$ over a less risk-averse individual in a worse rank.

We do not seek to provide a complete characterization of this criterion here. But there are two properties satisfied by the criterion that could be used to do so, and we will just describe them informally. A first property is replication invariance, which says that replicas of allocations are ranked the same as the original allocations. The second property is an extension of Pareto for Equal or No Risk to situations which are not equal but in which there is no reranking of individuals across states of nature¹⁰ and in which every bundle is consumed by the

¹⁰An axiom of Pareto for No Reranking is used in Fleurbaey (2010) to axiomatize a leximin criterion. When $u_i = v_i$ for all i , the axiom introduced in the text is identical to it.

same distribution of types of agents (where types are defined as risk preferences). Intuitively, this property extends Pareto to allocations which are not equal, but in which every type of agent is given all bundles in the allocation.

6 Examples

In this section we illustrate how our social evaluation criteria can be applied in two contexts: a macroeconomic risk affecting the marginal productivity of labor, and a catastrophic risk on future generations' consumption.

6.1 Macroeconomic risk

Drèze and Gollier (1993) and Gollier (1991) have studied the design of an unemployment insurance scheme in the presence of a macroeconomic risk.

Drèze and Gollier (1993) have argued in favor of indexing wages on GDP, so as to pool risk efficiently. Their argument focuses on efficiency. As they assume identical agents and as the consumption of all workers, whether employed or not, is equalized in every state, their argument would still be valid when using the certainty-equivalent of the EDE as a criterion.¹¹ In a similar model, Gollier (1991) introduces heterogeneity of reservation wages among workers, and studies the optimal variation of unemployment allowances in the presence of shocks on productivity. Heterogeneity between agents makes the comparison with our approach more interesting and we therefore revisit his analysis.

Consider an economy composed of a unit mass of workers, each having a vNM utility $\phi(c - m_i\ell)$, where $c \in \mathbb{R}_+$ is consumption, m_i is individual i 's reservation wage, and $\ell = 1$ when employed, 0 when unemployed. In any given state s , consumption is equal to w_s for the (employed) workers, with w_s the random wage

¹¹Details are available from the authors.

rate; consumption is equal to an allowance b_s for the unemployed. Note that we assume that all individuals have the same risk preferences described by ϕ . In this case, as discussed after Proposition 1, we only need to consider the expected utility of the equally-distributed equivalent to assess different courses of action. The issue addressed in Section 4 and 5, namely the fact that the social criterion may not be an expected utility, does not arise.

The economy operates under the constraint that the expected profits must attain a threshold: $\sum_s p_s (f_s(L_s) - w_s L_s - (1 - L_s) b_s) \geq \bar{v}$, where p_s is the probability of state s , f_s is the random production function, L_s is total employment, and the term $(1 - L_s) b_s$ in the profit formula corresponds to perfect experience rating (the productive sector funds unemployment allowances).¹² We assume that investors are abstract foreign markets and we focus on the social welfare of the population of workers.

In a given state s , the spot-market equilibrium condition for wages is $f'_s(L_s) = w_s - b_s$. A worker is employed if $w_s - m_i \geq b_s$, so that total employment is determined by

$$L_s = F(w_s - b_s),$$

where F is the cumulative distribution function of the parameter m_i .

The expected utility of an individual with reservation wage m is equal to

$$\sum_s p_s [\phi(\max\{w_s - m, b_s\})].$$

Gollier observes that the spot labor market achieves productive efficiency in every state, but full ex ante Pareto efficiency in risk sharing would require making $\max\{w_s - m_i, b_s\}$ a constant for every worker i , which is impossible with a uniform allowance b_s . Gollier shows that a constrained efficient allowance program

¹²Gollier (1991) also considers the case of risk-averse investors, but we focus here on the simpler case in which they are risk-neutral.

$\{b_s\}$ must make the following expression a constant across states of nature:

$$\int_0^{w_s-b_s} \lambda(m) \phi'(w_s - m) dF(m) + \int_{w_s-b_s}^{+\infty} \lambda(m) \phi'(b_s) dF(m), \quad (3)$$

where $\lambda(m)$ is some weight function reflecting an ethical prioritization of individuals with different values of the m parameter. This is not surprising as ex ante Pareto efficiency corresponds to the maximization of some weighted utilitarian criterion. The consequence is that b_s will be countercyclical with respect to w_s .¹³

In this model, our approach would also look at the distribution of “net-of-labor-disutility” consumptions, $c_i - m_i \ell_i$, but instead of applying the vNM utility function ϕ , we first compute an EDE in every state s . For instance, suppose that the EDE is a generalized mean

$$EDE_s = \varphi^{-1} \left(\int_0^{w_s-b_s} \varphi(w_s - m) dF(m) + \int_{w_s-b_s}^{+\infty} \varphi(b_s) dF(m) \right).$$

As all individuals have the risk attitudes embodied in ϕ , they have the same certainty-equivalent of this EDE, and therefore the optimal policy solves the following program:

$$\max_s \sum p_s \phi \circ \varphi^{-1} \left(\int_0^{w_s-b_s} \varphi(w_s - m) dF(m) + \int_{w_s-b_s}^{+\infty} \varphi(b_s) dF(m) \right)$$

under the same profit constraint and the spot market equilibrium conditions as in the problem considered by Gollier (1991). An analysis similar to Gollier’s one yields that

$$\frac{\phi'(EDE_s)}{\varphi'(EDE_s)} \left(\int_0^{w_s-b_s} \varphi'(w_s - m) dF(m) + \int_{w_s-b_s}^{+\infty} \varphi'(b_s) dF(m) \right)$$

must be a constant across states of nature (see Appendix 1 for details).

When $\varphi = \phi$, one obtains a special case of (3), which is not surprising as the utility of the EDE is then equal to the utilitarian sum of utilities. When φ is

¹³The expression (3) is decreasing in w_s as well as in b_s .

more concave than ϕ , so that $\varphi = g \circ \phi$ for a concave transform g , the expression becomes

$$\frac{1}{g'(EDE_s)} \left(\int_0^{w_s - b_s} \varphi'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi'(b_s) dF(m) \right).$$

In this formula both the numerator and the denominator decrease in w_s and b_s , so that the ratio may depend on these two parameters in a different way and the countercyclical result need not always hold.¹⁴

Interestingly, when the degree of priority of the worst-off goes to infinity, the EDE tends to b_s (assuming that unemployment is positive in every s), the optimal policy is to have a constant b_s (see Appendix 2). The permanently unemployed, who are the worst-off in every state and get full priority, are fully insured, whereas those who work (at least in some states) are only partly insured. Note that this case is also a limit case of (3), when $\lambda(m)$ is positive only for the greatest values of m .

6.2 Prevention of catastrophes

Weitzman (2009) argued that it was hard to bound the amount of effort one should make for the future in the case of a possible future catastrophic climate change. Weitzman and the following literature (for a comprehensive discussion, see Millner, 2013) have focused on a riskless investment that transfers consumption to the future. Here we will examine the different problem of investing for the reduction of the probability of a catastrophe.¹⁵ Consider an economy with two

¹⁴For instance, for $\varphi(x) = g(x) = -x^{-1}$, and F the CDF of the lognormal distribution with mean 1 and standard deviation 2, the ratio increases with w_s and decreases with b_s around $(w_s, b_s) = (1, .2)$.

¹⁵Pindyck and Wang (2009) estimate what a representative-agent economy calibrated on the US would be willing to pay for such a policy.

generations, which is examining whether it is worth taxing the first generation in order to reduce the risk of a catastrophe for the second generation. One can think of the climate change threat, or the meteor threat, as concrete motivations for this example.

The population is $N = N_1 \cup N_2$ (the two generations), and we assume that there is a single consumption good for simplicity. The first generation's consumption is $(x_i)_{i \in N_1}$ and the second generation will have either $(c_i)_{i \in N_2}$ with probability p , which is a catastrophe, or $(y_i)_{i \in N_2}$ with probability $1 - p$. A proportional tax on the first generation would reduce its consumption to $((1 - \tau) x_i)_{i \in N_1}$, and the outcome will be a reduction in the probability p . How much of τ should one accept for a given reduction in p ?

The Utilitarian approach relies on the social welfare function

$$\sum_{i \in N_1} \phi_i((1 - \tau) x_i) + \sum_{i \in N_2} [p \phi_i(c_i) + (1 - p) \phi_i(y_i)]$$

and computes the social willingness to tax as

$$\frac{d\tau}{-dp} = \frac{\sum_{i \in N_2} (\phi_i(y_i) - \phi_i(c_i))}{\sum_{i \in N_1} x_i \phi'_i((1 - \tau) x_i)}.$$

Although Weitzman (2009) looks at investments that raise future consumption rather than reduce future risks, one sees that this expression is very sensitive to $\phi_i(c_i)$ going to large negative values.

For the criterion proposed in this paper, let us assume that R^0 is an inequality averse social ordering represented by the function $\sum_{i \in N} \varphi(x_i)$, with φ a concave function (there is only one good, therefore no issue of heterogeneous preferences).

Let

$$\begin{aligned} EDE^c &= \varphi^{-1} \left(\frac{1}{n} \left(\sum_{i \in N_1} \varphi((1 - \tau) x_i) + \sum_{i \in N_2} \varphi(c_i) \right) \right), \\ EDE^y &= \varphi^{-1} \left(\frac{1}{n} \left(\sum_{i \in N_1} \varphi((1 - \tau) x_i) + \sum_{i \in N_2} \varphi(y_i) \right) \right). \end{aligned}$$

In the economy of this example, the criterion takes the form

$$\sum_{i \in N} \varphi(ce_i(EDE)) = \sum_{i \in N} \varphi(\phi_i^{-1}(p\phi_i(EDE^c) + (1-p)\phi_i(EDE^y))).$$

One then computes

$$\begin{aligned} \frac{d\tau}{-dp} = & \frac{\sum_{i \in N} [\phi_i(EDE^y) - \phi_i(EDE^c)] \frac{\varphi'(ce_i(EDE))}{\phi'_i(ce_i(EDE))}}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i)} \times \\ & \left(\frac{1}{n} \sum_{i \in N} \frac{\varphi'(ce_i(EDE))}{\phi'_i(ce_i(EDE))} \left[\frac{p\phi'_i(EDE^c)}{\varphi'(EDE^c)} + \frac{(1-p)\phi'_i(EDE^y)}{\varphi'(EDE^y)} \right] \right)^{-1}. \end{aligned}$$

In this formula, the first ratio on the right-hand side is similar to the Utilitarian formula, with two key differences. First, the priority of individuals is determined by the same φ function and is not dependent on the scale of utility functions for the members of the two generations. This can be defended as much preferable in terms of equity between individuals (the Pigou-Dalton principle is satisfied in every final consequence). In the case of the second generation, the difference in utility is recalibrated in terms of φ' thanks to the ratio $\varphi'(ce_i(EDE))/\phi'_i(ce_i(EDE))$.

The second difference is that at the numerator all individuals are counted, not just the members of the second generation, because the benefit of reducing the risk of a catastrophe is shared by the whole society as a reduction in the probability of having the low EDE^c . In the social evaluation, the EDE approach implies that one must look at a situation in which all individuals suffer from risk, because if equality was achieved in all states, the risk borne by the second generation would carry over to the first one. Again, this is preferable to the Utilitarian approach because it takes account of the degree of correlation of risk instead of adding up $\sum_{i \in N_2} (\phi_i(y_i) - \phi_i(c_i))$ without checking whether the risk reduction is independent for every individual or concerns a macroeconomic risk.

The final term on the right-hand side of the formula is a calibration term that takes account of how the ratio ϕ'_i/φ' varies across states of the world, and comes

from the fact that the cost of the tax on the first generation is analyzed in terms of a cost on the EDE and is therefore shared by all individuals as well.

To compare the two approaches, let us focus on a simple case in which all individuals in a generation have identical consumption, and for all $i \in N$, $\phi_i = \varphi$. Then the second formula simplifies into

$$\begin{aligned} \frac{d\tau}{-dp} &= \frac{\sum_{i \in N} [\phi_i(EDE^y) - \phi_i(EDE^c)]}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i)} \\ &= \frac{\sum_{i \in N_2} (\varphi(y_i) - \varphi(c_i))}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i)}, \end{aligned}$$

and therefore the two approaches are then identical.

Keeping the assumption that all individuals have the same utility function, and that there is no intra-generational inequality, but assuming that $\phi_i = g \circ \varphi$, for some transform g , one has

$$\frac{d\tau}{-dp} = \frac{n \left[\begin{array}{l} g\left(\frac{1}{n} (\sum_{i \in N_1} \varphi((1-\tau)x_i) + \sum_{i \in N_2} \varphi(y_i))\right) \\ -g\left(\frac{1}{n} (\sum_{i \in N_1} \varphi((1-\tau)x_i) + \sum_{i \in N_2} \varphi(c_i))\right) \end{array} \right]}{\sum_{i \in N_1} x_i \varphi'((1-\tau)x_i) (pg' \circ \varphi(EDE^c) + (1-p)g' \circ \varphi(EDE^y))}.$$

If g is convex (i.e., φ is more concave than ϕ_i), the numerator is less than

$$\sum_{i \in N_2} (\phi_i(y_i) - \phi_i(c_i)),$$

and the denominator is greater or less than $\sum_{i \in N_1} x_i \phi'_i((1-\tau)x_i)$ depending on

$$pg' \circ \varphi(EDE^c) + (1-p)g' \circ \varphi(EDE^y) \gtrless g' \circ \varphi((1-\tau)x_i).$$

Therefore, if c_i and y_i are great enough, the denominator is greater and $d\tau/(-dp)$ is smaller with our criterion than with utilitarianism. If c_i and y_i are small enough, the opposite holds. The conclusions are reversed if g is concave.

When g is extremely convex, the criterion tends to the maximin. Let us assume that $c_i < (1-\tau)x_i < y_i$ (still assuming no intragenerational inequality),

so that, under the maximin:

$$\begin{aligned} EDE^c &= c_i, \\ EDE^y &= (1 - \tau) x_i. \end{aligned}$$

One then obtains

$$\frac{d\tau}{-dp} = \frac{(1 - \tau) x_i - c_i}{(1 - p) x_i} = \frac{(1 - \tau) - \frac{c_i}{x_i}}{1 - p}.$$

In this formula, the social willingness to pay, compared to the utilitarian formula, is diminished by the fact that the good state does not look so bright due to the lower welfare of the present generation, which reduces the comparative loss when a catastrophe occurs. The denominator now features the probability, and the willingness to pay now increases with the value of p , due to the fact that the cost of the tax on present generations is only counted in the good state.

In conclusion, our criterion induces a social willingness to pay for prevention that is not radically different from the utilitarian formula, and, assuming a greater inequality aversion in φ than risk aversion in u_i , is lower when the future is much richer than the present and greater when the future prospects are not so bright.

7 Uncertainty

In this section we examine how to extend the analysis to the case of uncertainty, where individuals may have different beliefs, and perhaps even face ambiguity (represented by a set of probabilities). We first show that our main results extend to that case, so that dealing with uncertainty is possible within the general methodologies we have developed.

We then discuss the consequences of our aggregation method for the model of choice under uncertainty adopted at the social level. We show that, even when people are probabilistically sophisticated, our aggregation method does not

guarantee that the social decision model is also probabilistically sophisticated. Hence, there is no direct link between the individuals' decision model and the social one. We also study the case in which the canonical decision criterion at the individual level is no longer expected utility but the maxmin expected utility. In a specific case, we show that social preferences may be maxmin expected utility, using a specific set of beliefs.

7.1 The general result with Pareto for Equal or no Risk

In the statement of Pareto for Equal or No Risk, we implicitly assume that individual interests under uncertainty are appropriately represented by expected utilities, and that all individuals share the same beliefs described by p . In this section, we show that relaxing the expected utility assumption does not substantially alter Prop. 3. We also discuss the possibility of allowing for different beliefs.

Assume that acts can be ranked from individual i 's point of view via an ordering R_i over the set \mathcal{F}_i of finite-valued \mathcal{A} -measurable functions $f_i : \Omega \rightarrow \mathbb{R}_+^\ell$. We make three assumptions on R_i :

1. *Dominance*: if $f_i(\omega)R_ig_i(\omega)$ for all $\omega \in \Omega$, then $f_iR_ig_i$.
2. *Continuity*: R_i is continuous with respect to the product topology.
3. *Ordinal Equivalence*: for all $x_i, y_i \in \mathbb{R}_+^\ell$, $x_iR_iy_i \iff v_i(x_i) \geq v_i(y_i)$.

Note that the first assumption embodies both the idea of dominance and that of state independence. The third assumption is actually similar to Assumption 3. The first two assumptions imply the existence of a certainty equivalent bundle for any f_i , i.e., a bundle $x_i \in \mathbb{R}_+^\ell$ such that $x_iI_If_i$. As before, we denote such a bundle $ce_i(f_i)$ (there are many such bundles, the choice does not matter).

We modify Pareto for Equal or No Risk in the following way.

Generalized Pareto for Equal or No Risk For all $f, g \in \mathcal{F}^e \cup X$, if $f_i R_i g_i$ for all $i \in N$, then $f R g$; if $f_i P_i g_i$ for all $i \in N$, then $f P g$.

Under our three assumptions, we have the following result.

Proposition 5 *If the social ordering R satisfies Generalized Pareto for Equal or No Risk and Dominance, then, for all $f, g \in \mathcal{F}$, $f P g$ whenever there exists $\eta \in \mathbb{R}_{++}^{n\ell}$ such that*

$$(ce(f^e) - \eta) R^0 (ce(g^e) + \eta).$$

Proof. See Appendix 1. ■

Proposition 5 is very general for it allows individuals to adopt many different ways of making decisions under uncertainty. It also allows them to have different beliefs. From the normative point of view, this raises several issues.

One of these issues is the problem of spurious unanimities (Mongin, 2005). If individuals have different beliefs, they may agree to deem an option better than another one simply because they disagree on the prospects of the different outcomes realizing. For instance a very risk averse individual and a risk neutral individual may agree that an uncertain level of a public good is better than a sure level of public good only because the former believes that there is a very small probability that the level of public good is very low, while the second believes that the expected level of public good is higher than the sure level of public good, but only slightly so. If the first was to realize that he was underestimating the probability of the bad outcome, he would not want to follow the uncertain course of action. This issue has been investigated within the framework of the expected utility model by Gilboa et al. (2004). They propose to weaken the Pareto principle to cases where individuals do share the same beliefs on events (and obtain a Utilitarian aggregation of beliefs and tastes).

We will discuss how beliefs and attitudes towards uncertainty can be aggregated in our model.

7.2 Probabilistic sophistication

Let us first consider the simple case where agents are subjective expected utility maximizers but may have different beliefs about probabilities of different states of the world. This is the case studied by Mongin (2005) and Gilboa et al. (2004).

First, note that the aggregation of preferences with the procedures we propose takes place when individuals face the same egalitarian prospects (which is the meaning of Pareto for Equal Risk). As a consequence, this avoids the strongest form of spurious unanimities in which people have different beliefs *and* different rankings of final consequences in states of the world. The last feature of spurious unanimities is removed by focusing on egalitarian acts.¹⁶

But even without spurious unanimity, it may not be the case that our social aggregation procedure delivers a well-behaved model of social decision under uncertainty. In particular, it may not permit to identify well-defined beliefs. Given that all individuals are expected-utility maximizers, one could argue that the model of social choice should at least be probabilistic sophisticated, so that it aggregates individuals beliefs in a specific way. Probabilistically sophisticated non expected-utility models in the Savage framework have been introduced by Machina and Schmeidler (1992), and further studied by Sarin and Wakker (2000). The following example shows that our social orderings may not be probabilistic sophisticated when subjective expected utility agents have different beliefs.

¹⁶In an Anscombe-Aumann framework, Danan et al. (2015) consider a weakening of Pareto, named Common-Taste Unambiguous Pareto dominance, that also focuses on acts where individuals have the same assessment of the final consequences. They justify this axiom by the absence of strongly spurious unanimities in that case.

First let us recall the definition of probabilistic sophistication by Machina and Schmeidler (1992). A decision maker is probabilistically sophisticated if there exists a probability measure p over Ω such that:

- (i) the agent chooses between acts based on the probability distributions generated over the consequences;
- (ii) first-order stochastic dominance is satisfied.

Sarin and Wakker (2000) showed that – under some mild assumptions about the decision maker’s preferences under uncertainty – probabilistic sophistication implied the following weakened version of the sure-thing principle. For two sure allocations $x, y \in X$ and an event A , denote $x_A y$ the act f such that $f(\omega) = x$ for all $\omega \in A$ and $f(\omega) = y$ for all $\omega \in \Omega \setminus A$.

Sure-thing principle for two-consequence acts For all consequences $x, y \in X$, such that $x \succ y$, and all events A, B, H with $A \cap H = B \cap H = \emptyset$, then

$$x_A y \succeq x_B y \iff x_{A \cup H} y \succ x_{B \cup H} y.$$

Now consider the following simple case. There are two agents $i = 1, 2$ and three states of the world, $\Omega = \{\omega_1, \omega_2, \omega_3\}$. Consider two egalitarian allocations $x^e, y^e \in X^e$, such that $x^e \succ_i y^e$ for the two individuals (and hence for society, by Assumption 2). The two agents are expected utility maximizers and, assuming they have identical risk attitudes, evaluate acts with the utility function:

$$\sum_{k=1}^3 p_i(\omega_k) \phi\left(v_i(f(\omega_k))\right).$$

The two individuals have the same ϕ function by assumption, and therefore the same utility (as measured by the VNM metric) for egalitarian acts. We can define a certainty equivalent of the act, which is such that, for each individual:

$$v_i(ce_i(f)) = \phi^{-1}\left(\sum_{k=1}^3 p_i(\omega_k) \phi\left(v_i(f(\omega_k))\right)\right).$$

Without loss of generality we normalize ϕ so that $\phi(\bar{v}) = 1$ and $\phi(\underline{v}) = 0$ for some values \bar{v}, \underline{v} of the individual welfare metric.

Agent 1 has a subjective probability function such that $p_1(\omega_1) = 1/2$, $p_1(\omega_2) = 1/2$ and $p_1(\omega_3) = 0$. Agent 2 has a subjective probability function such that $p_2(\omega_1) = 1/4$, $p_2(\omega_2) = 0$ and $p_2(\omega_3) = 3/4$. Assume that $x, y \in X^e$ are such that $v_i(x_i) = \bar{v}$ and $v_i(y_i) = \underline{v}$, and consider the four following acts $f = x_{\{\omega_1\}} y$, $g = x_{\{\omega_3\}} y$, $\hat{f} = x_{\{\omega_1, \omega_2\}} y$, and $\hat{g} = x_{\{\omega_2, \omega_3\}} y$. We obtain the following distributions of welfare:

- $v_1(ce_1(f)) = \phi^{-1}(1/2)$ and $v_2(ce_2(f)) = \phi^{-1}(1/4)$;
- $v_1(ce_1(g)) = \phi^{-1}(0)$ and $v_2(ce_2(g)) = \phi^{-1}(3/4)$;
- $v_1(ce_1(\hat{f})) = \phi^{-1}(1)$ and $v_2(ce_2(\hat{f})) = \phi^{-1}(1/4)$;
- $v_1(ce_1(\hat{g})) = \phi^{-1}(1/2)$ and $v_2(ce_2(\hat{g})) = \phi^{-1}(3/4)$.

Assume also that R^0 is such that:

$$xR^0y \iff \psi(v_1(x_1)) + \psi(v_2(x_2)) \geq \psi(v_1(y_1)) + \psi(v_2(y_2)).$$

If ψ is more concave than ϕ (so that $\psi \circ \phi^{-1}$ is concave), then the social criterion defined in Proposition 5 implies fRg but $\hat{g}R\hat{f}$. This contradicts Sure-thing principle for two-consequence acts, and thus probabilistic sophistication for the social criterion.

This example points out that our social criterion does not necessarily inherit the properties of the individuals' preferences. If one insists on the rationality of social choice criteria in situations of uncertainty, the example may also provide an argument for a separate aggregation of tastes and beliefs at the social level. Some authors argue that the ex post approach is precisely one where beliefs and preferences are aggregated separately (Risse, 2003). Most of the literature

in economics about social choice under uncertainty have explored the joint aggregation of tastes and beliefs, following the seminal contribution by Gilboa et al. (2004). Recent contributions include but are not limited to Chambers and Hayashi (2006), Nascimento (2012), and Danan et al. (2015).¹⁷ It seems to us that it is difficult to perform such a joint aggregation, in particular because it is not simple to identify the source of belief divergence from individual choices (for a similar view, see for instance Mongin and Pivato, 2015).

A better procedure would be the following. In a first step, society would try to form the ‘best’ beliefs by aggregating people’s information. Then, it would assess individuals’ welfare on the basis of these best beliefs. In that case, Pareto for equal risk avoids any kind of disagreement between people hinging on different assessments of final situations or different beliefs. The property is only used to aggregate people’s risk attitudes.

7.3 Maxmin expected utility at the social level

In the above example, it seemed appealing to require strong rationality properties for the social ordering, because all individuals satisfied them. But when individuals may not be able to form specific beliefs (in the sense of having a single probability measure), and on the contrary perceive ambiguity, then one may ask how a fair procedure should aggregate the perceived ambiguity.

To investigate this issue, we will assume that individuals are maxmin expected utility maximizers (Gilboa and Schmeidler, 1989). We will assume that the set of states of the world can be written $\Omega = \Omega_1 \times \Omega_2$, where Ω_2 is a set whose σ -algebra

¹⁷There are also papers attempting to only aggregate beliefs by assuming that people have the same tastes: see Cres et al. (2011). Another strands of the literature does not seek to obtain a complete social aggregation, but only to identify Pareto improvements: for instance Gilboa et al. (2014) and Brunnermeier et al. (2014).

\mathcal{A}_2 defines non-ambiguous events for which all individuals can form the same beliefs represented by a common probability measure p over the measurable space $(\Omega_2, \mathcal{A}_2)$. The non ambiguous events will play a role similar to that of roulette lotteries in an Anscombe-Aumann framework; they correspond to situations of “objective” risk where individuals may be expected utility maximizers (but it is objective only in the sense that all agents have the same beliefs represented by a common probability measure).

Individuals perceive ambiguity with respects to events in the set Ω_1 represented by elements of \mathcal{A}_1 . This is formalized in the following way: agent i holds beliefs represented by a (closed and convex) set \mathcal{P}_i of probabilities over the measurable space $(\Omega_1, \mathcal{A}_1)$.

For any $f_i \in (\mathbb{R}^\ell)^\Omega$ and $q \in \mathcal{P}_i$, let us define

$$E_q(u_i \circ f_i) = \int_{\Omega_1} \left(\int_{\Omega_2} u_i(f(\omega_1, \omega_2)) dp(\omega_2) \right) dq(\omega_1).$$

The fact that individual i is a maxmin expected utility maximizer means that for all individual acts $f_i, g_i \in (\mathbb{R}^\ell)^\Omega$,

$$f_i R_i g_i \iff \min_{q \in \mathcal{P}_i} E_q(u_i \circ f_i) \geq \min_{q \in \mathcal{P}_i} E_q(u_i \circ g_i).$$

The issue is to describe how our social orderings would aggregate perceived uncertainty, i.e., the sets \mathcal{P}_i . This can be answered in a very clear way on the basis of Proposition 4, provided it is adapted to the current setting. First, we need Pareto for Equal and No Risk:

Pareto for Equal or No Risk For all $f, g \in \mathcal{F}^e \cup X$, if $\min_{q \in \mathcal{P}_i} E_q(u_i \circ f_i) \geq \min_{q \in \mathcal{P}_i} E_q(u_i \circ g_i)$ for all $i \in N$, then $f R g$.

Second, we need to assume that the social criterion is a maxmin expected utility criterion.

Maxmin Expected Utility There is a set \mathcal{P}_0 of probability measures over the measurable space $(\Omega_1, \mathcal{A}_1)$, and a continuous function W such that for all $f, g \in \mathcal{F}$, fRg if and only if $\min_{q \in \mathcal{P}_0} E_q(W \circ f) \geq \min_{q \in \mathcal{P}_0} E_q(W \circ g)$.

Last, we need to make some assumptions about the domain of profiles of risk attitudes and perceived ambiguity in the society. We thus assume that there exists a collection of functions (ψ_1, \dots, ψ_L) and a collection $(\mathcal{Q}_1, \dots, \mathcal{Q}_L)$ of sets of probability measures over the measurable space $(\Omega_1, \mathcal{A}_1)$ such that:

- For every $i \in N$, there exists $l \in \{1, \dots, L\}$ such that $u_i = \psi_l \circ v_i$.
- For every $i \in N$, there exists $k \in \{1, \dots, K\}$ such that $\mathcal{P}_i = \mathcal{Q}_k$.
- For all $(l, k) \in \{1, \dots, L\} \times \{1, \dots, K\}$, there exists $j \in N$ such that $\mathcal{P}_j = \mathcal{Q}_k$ and $u_j = \psi_l \circ v_j$.
- There exists a most risk averse function, denoted $\bar{\psi}$, in (ψ_1, \dots, ψ_L) .

Adding the conditions of Equity and Invariance to risk attitudes for constant acts of Section 5.3, and a condition of richness similar to Assumption 4 (but applied only to p and the measurable space $(\Omega_2, \mathcal{A}_2)$), we can obtain Proposition 4 by focusing on non-ambiguous acts (i.e., acts such that $f(\omega_1, \omega_2) = f(\omega'_1, \omega_2)$ for all $\omega_1, \omega'_1 \in \Omega_1$ and all $\omega_2 \in \Omega_2$). Next, using Proposition 5, we have that for all $f, g \in \mathcal{F}$:

$$\begin{aligned} fRg &\iff (ce(f^e)) R^0 (ce(g^e)) \\ &\iff \min_i \left[\phi_i^{-1} \left\{ \min_{q \in \mathcal{P}_i} E_q(\phi_i \circ f_i^e) \right\} \right] \geq \min_i \left[\phi_i^{-1} \left\{ \min_{q \in \mathcal{P}_i} E_q(\phi_i \circ g_i^e) \right\} \right] \\ &\iff \min_{k \in \{1, \dots, K\}} \min_{q \in \mathcal{Q}_k} E_q(\bar{\psi} \circ f_i^e) \geq \min_{k \in \{1, \dots, K\}} \min_{q \in \mathcal{Q}_k} E_q(\bar{\psi} \circ g_i^e). \end{aligned}$$

Indeed, only the agents with the most risk averse preferences can have the lowest welfare. The last inequality completely describes how ambiguity is aggregated at the social level. Indeed, if $\mathcal{P}_0 = \cup_{k \in \{1, \dots, K\}} \mathcal{Q}_k$, we evidently have:

$$\min_{q \in \mathcal{P}_0} E_q(\bar{\psi} \circ g_i^e) \geq \min_{q \in \mathcal{P}_0} E_q(\bar{\psi} \circ g_i^e) \iff \min_{k \in \{1, \dots, K\}} \min_{q \in \mathcal{Q}_k} E_q(\bar{\psi} \circ g_i^e) \geq \min_{k \in \{1, \dots, K\}} \min_{q \in \mathcal{Q}_k} E_q(\bar{\psi} \circ g_i^e).$$

Hence, we obtain that social ambiguity is strictly larger than individual ambiguity, because we have to take into account all the possible beliefs people in the society may hold. The social preferences for redistribution, embodied in the R^0 ordering, determine how ambiguity is aggregated. This explains why a preference for the worst-off may imply a focus on the worst possible probability held by people in the society, which may seem extreme. Again, if one wants to avoid this conclusion, a possible path is to completely separate the aggregation of tastes and the aggregation of beliefs.

References

- Ben Porath, E., Gilboa, I., Schmeidler, D. (1997). "On the measurement of inequality under uncertainty," *Journal of Economic Theory*, 75, 194–204.
- Blackorby, C., Donaldson, D., Weymark, J.A. (1999). "Harsanyi's social aggregation theorem for state-contingent alternatives," *Journal of Mathematical Economics*, 32, 365–387.
- Broome, J. (1991). *Weighing Goods. Equality, Uncertainty and Time*, Oxford: Blackwell.
- Brunnermeier, M. K., Simsek, A., Xiong, W. (2014). "A Welfare Criterion for Models with Distorted Beliefs," *Quarterly Journal of Economics*, 129, 1753–1797.
- Chambers, C.P., Echenique, E. (2012). "When does aggregation reduce risk aversion?" *Games and Economic Behavior*, 76, 582–595.
- Chambers, C.P., Hayashi, T. (2006). "Preference aggregation under uncertainty: Savage vs. Pareto," *Games and Economic Behavior*, 54, 430–440.
- Coulhon, T., Mongin, P. (1989). "Social choice theory in the case of Von Neumann-Morgenstern utilities," *Social Choice and Welfare*, 6, 175–187.
- Cres, H., Gilboa, I., Vieille, N. (2011). "Aggregation of multiple prior opinions," *Journal of Economic Theory*, 146(6), 2563–2582.
- Danan, E., Gajdos, T., Hill, B., Tallon, J.-M. (2015). "Aggregating tastes, beliefs, and attitudes under uncertainty," mimeo.

- Debreu, G. (1959). *Theory of Value. An axiomatic analysis of economic equilibrium*, New Haven: Yale University Press.
- Decancq, K., Fleurbaey, M., Schokkaert, E. (2015). "Inequality, income and well-being," in Atkinson, A.B., Bourguignon, F. (Eds.), *Handbook of Income Distribution*, Amsterdam: North Holland.
- De Meyer, B., Mongin, P. (1995). "A note on affine aggregation," *Economics Letters*, 47, 177–183.
- Dhillon, A., Mertens, J.-F. (1999). "Relative utilitarianism," *Econometrica*, 67, 471–498.
- Diamond, P.A. (1967). "Cardinal welfare, individualistic ethics, and interpersonal comparison of utility: comment," *Journal of Political Economy*, 75, 765–766.
- Drèze, J., Gollier, C. (1993). "Risk sharing on the labour market and second-best wage rigidities," *European Economic Review*, 37, 1457–1482.
- Drummond, M.F., Sculpher, M.J., Torrance, G.W., O'Brien, B.L., Stoddart, G.L. (2005). *Methods for the economic evaluation of health care programmes*, Oxford: Oxford University Press.
- Epstein, L.G., Segal, U. (1992). "Quadratic social welfare functions," *Journal of Political Economy*, 100, 691–712.
- Fleurbaey, M. (2010). "Assessing risky social situations," *Journal of Political Economy*, 118, 649–680.
- Fleurbaey, M., Maniquet, F. (2011). *A Theory of Fairness and Social Welfare*, Cambridge: Cambridge University Press.
- Gajdos, T., Maurin, E. (2004). "Unequal Uncertainties and Uncertain Inequalities: An Axiomatic Approach," *Journal of Economic Theory*, 116, 93–118.
- Gajdos, T., Tallon, J.-M. (2002). "Fairness under uncertainty," *Economics Bulletin*, 18, 1–7.
- Gilboa, I., Samet, D., Schmeidler, D. (2004). "Utilitarian aggregation of beliefs and tastes," *Journal of Political Economics*, 112, 932–938.
- Gilboa, I., Samuelson, L., Schmeidler, D. (2014). "No betting Pareto dominance," *Econometrica*, 82, 1405–1442.
- Gilboa, I., Schmeidler, D. (1997). "Maxmin expected utility with non-unique prior," *Journal of Mathematical Economics*, 18, 141–153.

- Gollier, C. (1991). "Unemployment Insurance: Risk Sharing Versus Efficiency," *The Geneva Risk and Insurance Review*, 16, 59–74.
- Grant, S., Kajii, A., Polak, B., Safra, Z. (2010). "Generalized Utilitarianism and Harsanyi's Impartial Observer Theorem," *Econometrica*, 78, 1939–1971.
- Harsanyi, J. (1955). "Cardinal welfare, individualistic ethics and interpersonal comparisons of utility," *Journal of Political Economy*, 63, 309–321.
- Machina, M., Schmeidler, D. (1992). "A more robust definition of subjective probability," *Econometrica*, 60, 745–780.
- Mongin, P. (1995). "Consistent Bayesian aggregation," *Journal of Economic Theory*, 66, 313–351.
- Mongin, P. (1998). "The paradox of the Bayesian expert and state-dependent utility theory," *Journal of Mathematical Economics*, 29, 331–361.
- Mongin, P. (2005). "Spurious unanimity and the Pareto principle", *CPNSS working paper*, vol. 1, no. 5. The Centre for Philosophy of Natural and Social Science (CPNSS), London School of Economics, London.
- Mongin, P., Pivato, M. (2015). "Social choice under uncertainty: beyond ex ante and ex post," mimeo.
- Millner, A. (2013). "On welfare frameworks and catastrophic climate risks," *Journal of Environmental Economics and Management*, 65, 310–325.
- Nascimento, L. (2012). "The ex ante aggregation of opinions under uncertainty," *Theoretical Economics*, 7, 535–570.
- Pazner, E. (1979). "Equity, nonfeasible alternatives and social choice: A reconsideration of the concept of social welfare," in Laffond, J.-J. (Ed.), *Aggregation and Revelation of Preferences*, Amsterdam: North Holland.
- Pindyck R.S., Wang, N. (2009). "The economic and policy consequences of catastrophes," *American Economic Journal: Economic Policy*, 5, 306–339.
- Risse, M. (2003). "Bayesian agents and two modes of aggregation," *Synthese*, 135, 347–377.
- Sarin, R., Wakker, P.P. (2000). "Cumulative dominance and probabilistic sophistication," *Mathematical Social Sciences*, 40, 191–196.
- Savage, L.J. (1972). *The Foundations of Statistics*, 2nd revised ed., New York: Dover Publications.

Szpilrajn, E. (1930), “Sur l’extension de l’ordre partiel,” *Fundamenta Mathematicae*, 16, 386–389.

Weitzman, M.L. (2009). “On modeling and interpreting the economics of catastrophic climate change,” *Review of Economics and Statistics*, 91, 1–19.

Appendix 1: Proofs of the main Propositions

Proof of Lemma 1

Proof of (i). By Assumption 1, $V = \{v = (v_i(x_i))_{i \in N}, x \in X\} = Q^n$ where Q is a convex subset of \mathbb{R} . By Assumption 2, we can therefore apply Lemma 1 in Fleurbaey (2010) to the ordering R^* restricted to V to show that for any $v \in V$ there exist a unique $v^e \in Q$ such that for any $\varepsilon \in \mathbb{R}^n$, $(v + \varepsilon)P^*(v^e, \dots, v^e)$ and $(v^e + \varepsilon_1, \dots, v^e + \varepsilon_n)P^*v$.

For any $x \in X$ such that $(v_i(x_i))_{i \in N} = v$, let x^e be the allocation such that $(v_i(x_i^e))_{i \in N} = v^e$ (it necessarily exists because $v^e \in V$). The allocation is an EDQE of x . Furthermore, by unicity of v^e for each $v \in V$, if \tilde{x}^e is another EDQE of x it must be the case that $(v_i(\tilde{x}_i^e))_{i \in N} = v^e(v_i(x_i^e))_{i \in N}$. By Assumption 2 and the definition of R^0 , we therefore have $x^e I^0 \tilde{x}^e$.

Proof of (ii). By Assumption 1, we necessarily have $v_i(0) = \inf_{x \in X} v_i(x_i)$, and therefore $v_i(0) = v_j(0)$ for all $i, j \in N$ (utility functions v_i have the same range). For any $x \in X$, x^e is such that $v_i(x_i^e) \geq \inf_{i \in N} v_i(x_i)$. Indeed, otherwise, by Assumption 1 (continuity), there exists z such that $v_i(x_i^e + z_i) < v_i(x_i)$ for all $i \in N$, and therefore (by the monotonicity conditions in Assumptions 1 and 2) $xP^0(x^e + z)$ violating the assumption that x^e is an EDQE of x . If $x \in X$ is such that $(v_i(x_i))_{i \in N} \gg (v_i(0))_{i \in N}$, $\inf_{i \in N} v_i(x_i) > v_i(0)$ and hence (by Assumption 1), one can find x^e such that $x_i^e \gg 0$ for all $i \in N$.

Proof of (iii). Assume that for some $x, y \in X$, $x^e P^0 y^e$ and $y R^0 x$. Then, because

y^e is an EDQE of y , for any $z \in R_{++}^{n\ell}$, $(y^e + z)P^0yR^0x$ so that $(y^e + z)P^0x$. Furthermore, because x^e is an EDQE of x , for any $z \in R_{++}^{n\ell}$, $(x + z)P^0x^eP^0y^e$ hence $(x + z)P^0y^e$. Therefore, y^e is an EDQE of x . But this is impossible by statement (i) of the current Lemma, because x^e is an EDQE of x and $x^eP^0y^e$.

Proof of Proposition 1

Let R satisfy Pareto for Equal Risk and Dominance.

Step 1. Let $U = \{z \in \mathbb{R}^n \mid \exists f \in \mathcal{F}^e, z = (E(u_i \circ f_i))_{i \in N}\}$. Define the binary relation \tilde{R}^{**} on \mathbb{R}^n by the following condition: $z\tilde{R}^{**}z'$ if either (a) $z, z' \in U$ and there exists $f, g \in \mathcal{F}^e$ such that fRg and $z = (E(u_i \circ f_i))_{i \in N}, z' = (E(u_i \circ g_i))_{i \in N}$, or (b) $z \geq z'$.

Remark that by Pareto for Equal Risk, if $f, f' \in \mathcal{F}^e$ are such that $(E(u_i \circ f_i))_{i \in N} = (E(u_i \circ f'_i))_{i \in N} = z$, then fIf' . So, by definition of \tilde{R}^{**} , for all $f, g \in \mathcal{F}^e$,

$$(E(u_i \circ f_i))_{i \in N} \tilde{R}^{**} (E(u_i \circ g_i))_{i \in N} \implies fRg.$$

By (a), \tilde{R}^{**} is complete on U , because R is complete on \mathcal{F}^e . Transitivity of R^{**} on U is also easily derived from the transitivity of R . The ordering \tilde{R}^{**} can be extended into an ordering R^{**} on \mathbb{R}^n by Szpilrajn's Lemma (Szpilrajn, 1930).

The relation R^{**} is weakly monotonic by part (b) of the definition of \tilde{R}^{**} . Notice that we do not have a conflict between (a) and (b) of the definition of \tilde{R}^{**} , because, for $z, z' \in U$ such that $z \geq z'$, there exists $f, g \in \mathcal{F}^e$ such that $E(u_i \circ f_i) \geq E(u_i \circ g_i)$ for all $i \in N$, so that, by Pareto for Equal Risk, fRg . Note also that, as before, we have for all $f, g \in \mathcal{F}^e$:

$$(E(u_i \circ f_i))_{i \in N} R^{**} (E(u_i \circ g_i))_{i \in N} \implies fRg.$$

Step 2. Let $f, g \in \mathcal{F}$ be such that for all $\omega \in \Omega$ and for all $i \in N$, $u_i(f_i^e(\omega)) > u_i(g_i^e(\omega))$. By Assumption 3, $v_i(f_i^e(\omega)) > v_i(g_i^e(\omega))$. By Assumption 2, this

implies $f^e(\omega)P^0g^e(\omega)$. By Lemma 1 (iii), this implies $f(\omega)P^0g(\omega)$. Since this is true for all $\omega \in \Omega$, Dominance implies that fPg .

Step 3. Let $f, g \in \mathcal{F}$, be such that there exists $\varepsilon \in \mathbb{R}_{++}^\ell$ such that

$$(E(u_i \circ (f_i^e - \varepsilon)))_{i \in N} R^{**} (E(u_i \circ (g_i^e + \varepsilon)))_{i \in N}.$$

By Assumption 1, there exists $f' \in \mathcal{F}^e$ such that for all $\omega \in \Omega$ and all $i \in N$, $v_i(f_i^e(\omega)) > v_i(f'_i(\omega)) > v_i(f_i^e(\omega) - \varepsilon)$, so that by Assumption 3, $u_i(f_i^e(\omega)) > u_i(f'_i(\omega)) > u_i(f_i^e(\omega) - \varepsilon)$. Similarly, there exists $g' \in \mathcal{F}^e$ such that for all $\omega \in \Omega$ and all $i \in N$, $u_i(g_i^e(\omega)) < u_i(g'_i(\omega)) < u_i(g_i^e(\omega) + \varepsilon)$.

Hence $(E(u_i \circ f'_i))_{i \in N} \gg (E(u_i \circ (f_i - \varepsilon)))_{i \in N}$ and $(E(u_i \circ (g_i + \varepsilon)))_{i \in N} \gg (E(u_i \circ g'_i))_{i \in N}$, so that, by the monotonicity and transitivity of R^{**} ,

$$(E(u_i \circ f'_i))_{i \in N} P^{**} (E(u_i \circ g'_i))_{i \in N}.$$

By Step 1, since $f', g' \in \mathcal{F}^e$, $f'Rg'$.

But considering $f, f' \in \mathcal{F}$ on the one hand, and $g, g' \in \mathcal{F}$ on the other hand, and applying the result of Step 2, we obtain fPf' and $g'Pg$, so that, by transitivity of R , fPg .

Proof of Corollary 1

Step 1 is as in the proof of Proposition 1. Step 2 shows that for all $f \in \mathcal{F}$, fIf^e by Dominance because $f(\omega)I^0f^e(\omega)$ for all $\omega \in \Omega$. Thus

$$\begin{aligned} (E(u_i \circ f_i^e))_{i \in N} R^{**} (E(u_i \circ g_i^e))_{i \in N} &\iff f^e R g^e && \text{(Step 1)} \\ &\iff f R g && \text{(Step 2)} \end{aligned}$$

Proof of Proposition 2

By Expected Utility, R and therefore R^0 are continuous. Because Expected Utility implies Dominance, Corollary 1 applies: there is a weakly monotonic ordering R^{**}

on \mathbb{R} such that for all $f, g \in \mathcal{F}$, fPg if and only if $(E(u_i \circ f_i^e))_{i \in N} R^{**} (E(u_i \circ g_i^e))_{i \in N}$. Moreover, by continuity and monotonicity, R^{**} can be represented by a continuous non-decreasing function $H : \mathbb{R}^n \rightarrow \mathbb{R}$, and therefore R can be represented by $f \rightarrow H((E(u_i \circ f_i^e))_{i \in N})$. By Expected Utility, there is also a function W such that, for some normalisation of H ,

$$H((E(u_i \circ f_i^e))_{i \in N}) = E(W \circ f^e)$$

for all $f \in \mathcal{F}$.

Let $Z = (x_1, \dots, x_m) \in (X^e)^m$ be a collection of m egalitarian allocations (with at least three distinct allocations). Denote $\mathcal{F}_Z = \{f \in \mathcal{F} \mid \exists (A_1, \dots, A_m) \in \mathcal{A}^m, f(\omega) = x_k, \forall \omega \in A_k, \forall k = 1, \dots, m\}$. For each $f \in \mathcal{F}_Z$, define $p_f = (p(\{\omega \mid f(\omega) = x_k\}))_{k=1, \dots, m}$. Also define $V_i(p_f) = E(u_i \circ f_i^e)$, for all $i \in N$, and $V_0(p_f) = E(W \circ f)$.

For all $f \in \mathcal{F}_Z$, we therefore have $H((V_i(p_f))_{i \in N}) = V_0(p_f)$. By Assumption 4, $\{p_f \mid f \in \mathcal{F}_Z\} = \{\pi = (\pi_1, \dots, \pi_m) \in [0, 1]^m \mid \sum_1^m \pi_m = 1\}$, which is a mixture space. By definition, the functions V_i , $i = \{0, 1, \dots, n\}$ are mixture preserving. Hence, by Proposition 2 in Coulhon and Mongin (1989), there exist real numbers $(\alpha_i)_{i \in N}$ and β such that

$$V_0(p) = \sum_{i \in N} \alpha_i V_i(p) + \beta.$$

Since H is non-decreasing, we must have $(\alpha_i)_{i \in N} \in \mathbb{R}_+^n$. We can also change the allocations in Z so as to cover the whole domain of H , and therefore obtain, for all $f \in \mathcal{F}$,

$$H((E(u_i \circ f_i^e))_{i \in N}) = \sum_{i \in N} \alpha_i E(u_i \circ f_i^e) + \beta.$$

Proof of Proposition 3

Let $f, g \in \mathcal{F}$ and assume that there exists $\eta \in \mathbb{R}_{++}^{n\ell}$ such that

$$(ce(f^e) - \eta) R^0 (ce(g^e) + \eta).$$

By Assumptions 1 and 3, the v_i and therefore the u_i functions are continuous and increasing. Therefore, there exist $f', g' \in \mathcal{F}^e$ such that for every $i \in N$,

$$E(u_i \circ (f_i^e)) > E(u_i \circ (f_i')) > u_i(ce_i(f_i^e) - \varepsilon_i)$$

and

$$E(u_i \circ (g_i^e)) < E(u_i \circ (g_i')) < u_i(ce_i(g_i^e) + \varepsilon_i)$$

By Pareto for Equal or No Risk, $f^e P f' P (ce(f^e) - \varepsilon)$ and $(ce(g^e) + \varepsilon) P g' P g^e$. By Dominance $(ce(f^e) - \eta) R^0 (ce(g^e) + \eta)$ implies that $(ce(f^e) - \eta) R (ce(g^e) + \eta)$, so that $f' P g'$. By Step 2 of the proof of Proposition 1, $f^e P f'$ implies $f P f'$ and $g' P g^e$ implies $g' P g$. By transitivity of R , $f P g$.

Proof of Proposition 4

Note that Expected utility implies Dominance. For every $\Phi \in \mathcal{C}$, we know that, by Proposition 3 and Invariance to risk attitudes for constant acts (which posits that every R_Φ extends the same R^0), for all $f, g \in \mathcal{F}$:

$$f R_\Phi g \iff ce(f^e) R^0 ce(g^e). \quad (4)$$

We also know, by Proposition 2, that there exists a vector $\alpha_\Phi \in \mathbb{R}_+^n$ such that, for all $f, g \in \mathcal{F}$,

$$f R_\Phi g \iff \sum_{i \in N} \alpha_\Phi^i E(u_i \circ f_i^e) \geq \sum_{i \in N} \alpha_\Phi^i E(u_i \circ g_i^e). \quad (5)$$

Define the set of allocations which are certainty-equivalent allocations of EDE acts:

$$X_\Phi = \{x \in X \mid \exists f \in \mathcal{F}, x = ce(f^e)\}.$$

Equations (4) and (5) imply that, for all $x, y \in X_\Phi$:

$$xR^0y \iff \sum_{i \in N} \alpha_\Phi^i u_i(x_i) \geq \sum_{i \in N} \alpha_\Phi^i u_i(y_i). \quad (6)$$

The rest of the proof mainly relies on the following Lemma:

Lemma 2 *For all $x, y \in X \cap \mathbb{R}_{++}^{n\ell}$ such that, for some $i \in N$, the following two conditions hold true:*

1. $v_j(x_j) = \underline{v}$ and $v_j(y_j) = \bar{v}$ for all $j \in N \setminus i$;
2. $v_i(y_i) < v_i(x_i) \leq \underline{v} \leq \bar{v}$;

there exists $\Phi \in \mathcal{C}$ such that $x, y \in X_\Phi$ and

$$\sum_{i \in N} \alpha_\Phi^i u_i(x_i) > \sum_{i \in N} \alpha_\Phi^i u_i(y_i).$$

Proof. Let $x, y \in X$ be defined as in the Lemma and denote $\bar{u} = v_i(x_i)$ and $\underline{u} = v_i(y_i)$. Consider the following profile $\Phi \in \mathcal{C}$:

1. For all $j \in N \setminus \{i\}$, $\phi_j(v) = v$ for all $v \in \mathcal{I}$.
2. $\phi_i(v) = \begin{cases} \theta \kappa(v - \underline{u}) & \text{for all } v \leq \underline{u}, \\ \theta(v - \underline{u}) & \text{for all } \underline{u} < v \leq \bar{u}, \\ \theta(\bar{u} - \underline{u}) + v - \bar{u} & \text{for all } v > \bar{u}, \end{cases}$

where the values of κ and θ will be set later.

Step 1: Proof that $y \in X_\Phi$.

By Assumption 4, for any $\pi \in [0, 1]$, there exists a subset \mathcal{A} of Ω such $p(\mathcal{A}) = \pi$. By assumption, \underline{u} and \bar{v} are in the interior of \mathcal{I} . Thus there exists a small

$\varepsilon > 0$ such that $(\underline{u} - \varepsilon) \in \mathcal{I}$ and $(\bar{v} + \varepsilon) \in \mathcal{I}$. And it is always possible to find π such that:

$$\pi(\underline{u} - \varepsilon) + (1 - \pi)(\bar{v} + \varepsilon) = \bar{v}.$$

Indeed, $\pi = (\varepsilon)/(\bar{v} - \underline{u} + 2\varepsilon)$ satisfies this equation.

Let \mathcal{A} be a set such that $p(\mathcal{A}) = \pi$. Then consider the following egalitarian act f . For all $\omega \in \mathcal{A}$, $v_i(f_i(\omega)) = \underline{u} - \varepsilon$ for all $i \in N$; and for all $\omega \in \Omega \setminus \mathcal{A}$, $v_i(f_i(\omega)) = \bar{v} + \varepsilon$ for all $i \in N$. To prove that $y \in X_\Phi$, we need to show that $v_j(ce_j(f)) = \bar{v}$ for all $j \neq i$, and $v_i(ce_i(f)) = \underline{u}$.

For any $j \neq i$, we have:

$$E\phi_j(v_j(f_j^e)) = \pi(\underline{u} - \varepsilon) + (1 - \pi)(\bar{v} + \varepsilon) = \phi_j(\bar{v}),$$

because $\phi_j(v) = v$ for all v , and by definition of π . Hence, $v_j(ce_j(f_j)) = \bar{v}$.¹⁸

For i , we have, by definition of ϕ_i :

$$E\phi_i(v_i(f_i^e)) = -\pi\theta\kappa\varepsilon + (1 - \pi)(\theta(\bar{u} - \underline{u}) + v - \bar{u} + \varepsilon).$$

Letting

$$\kappa = \frac{(1 - \pi)(\theta(\bar{u} - \underline{u}) + v - \bar{u} + \varepsilon)}{\pi\theta\varepsilon},$$

we obtain $E\phi_i(v_i(f_i^e)) = 0 = \phi_i(\underline{u})$, so that $v_i(ce_i(f_i)) = \underline{u}$.

Step 2: Proof that $x \in X_\Phi$.

First note that, for every $0 \leq \tilde{\varepsilon} \leq \varepsilon$, there exists $\pi(\tilde{\varepsilon})$ such that:

$$\pi(\tilde{\varepsilon})(\underline{u} - \tilde{\varepsilon}) + (1 - \pi(\tilde{\varepsilon}))(\bar{v} + \tilde{\varepsilon}) = \underline{v}.$$

Indeed, $\pi(\tilde{\varepsilon}) = (\bar{v} - \underline{v} + \tilde{\varepsilon})/(\bar{v} - \underline{u} + 2\tilde{\varepsilon})$ satisfies this equation.

Let us now consider the function $f(\tilde{\varepsilon}) = \pi(\tilde{\varepsilon})\phi_i(\underline{u} - \tilde{\varepsilon}) + (1 - \pi(\tilde{\varepsilon}))\phi_i(\bar{v} - \tilde{\varepsilon})$. This function is continuous. One has $\pi(0) = 0$, so that $f(0) = \phi_i(\bar{v})$. On the

¹⁸Indeed, if $Eu_i(f_i) = \phi_i(v)$, then by definition of the certainty equivalent, $v_i(ce_i(f_i)) = v$.

other hand,

$$\pi(\varepsilon) = \frac{\bar{v} - \underline{v} + \varepsilon}{\bar{v} - \underline{u} + 2\varepsilon} > \frac{\varepsilon}{\bar{v} - \underline{u} + 2\varepsilon} = \pi,$$

and $\underline{u} - \varepsilon < \bar{v} + \varepsilon$, so that $f(\varepsilon) < \pi\phi_i(\underline{u} - \varepsilon) + (1 - \pi)\phi_i(\bar{v} + \varepsilon) = \phi_i(\underline{u})$, where the last equality comes from Step 1.

Hence $f(\varepsilon) < \phi_i(\bar{u}) < f(0)$. By the intermediate value theorem, and because ϕ_i is increasing, there exists $0 < \bar{\varepsilon} < \varepsilon$ such that $f(\bar{\varepsilon}) = \phi_i(\bar{u})$.

Let \mathcal{A}' be a set such that $p(\mathcal{A}') = \pi(\bar{\varepsilon})$. Then consider the following egalitarian act g . For all $\omega \in \mathcal{A}'$, $v_i(g_i(\omega)) = \underline{u} - \bar{\varepsilon}$ for all $i \in N$; and for all $\omega \in \Omega \setminus \mathcal{A}'$, $v_i(g_i(\omega)) = \bar{v} + \bar{\varepsilon}$ for all $i \in N$. To prove that $x \in X_\Phi$, we need to show that $v_j(ce_j(g_j)) = \underline{v}$ for all $j \neq i$, and $v_i(ce_i(g_i)) = \bar{u}$.

For all $j \neq i$, we have:

$$E\phi_j(v_j(g_j)) = \pi(\bar{\varepsilon})(\underline{u} - \bar{\varepsilon}) + (1 - \pi(\bar{\varepsilon}))(\bar{v} + \bar{\varepsilon}) = \phi_j(\underline{v}),$$

because $\phi_j(v) = v$ for all v , and by definition of $\pi(\bar{\varepsilon})$. Hence, $v_j(ce_j(g)) = \underline{v}$. For i , we have $E\phi_i(v_i(g_i)) = f(\bar{\varepsilon}) = \phi_i(\bar{u})$, therefore $v_i(ce_i(g_i)) = \bar{u}$.

Step 3: Proof that $(n - 1)\alpha_\Phi^i > \sum_{j \neq i} \alpha_\Phi^j$.

Let $\tilde{x}, \tilde{y} \in X$ be such that for some $0 < \varepsilon < (\bar{v} - \underline{v})/n$,

1. $v_j(\tilde{y}_j) = \bar{v}$ and $v_j(\tilde{x}_j) = \bar{v} - \varepsilon$ for all $j \in N \setminus i$;
2. $v_i(\tilde{y}_i) = \underline{v}$ and $v_i(\tilde{x}_i) = \underline{v} + (n - 1)\varepsilon$.

By a reasoning similar to that in Step 2, it is possible to show that $\tilde{y}, \tilde{x} \in X_\Phi$. By $(n - 1)$ successive applications of the Equity principle, we also know that $\tilde{x}R_\Phi^0\tilde{y}$. By Equation (6), this implies that:

$$\sum_{i \in N} \alpha_\Phi^i \phi_i(v_i(\tilde{x}_i)) \geq \sum_{i \in N} \alpha_\Phi^i \phi_i(v_i(\tilde{y}_i)).$$

By definition of Φ , \tilde{x} and \tilde{y} , this yield $\alpha_\Phi^i(n - 1)\varepsilon \geq \sum_{j \neq i} \alpha_\Phi^j \varepsilon$.

Step 4: Conclusion.

We have:

$$\begin{aligned}
\sum_{i \in N} \alpha_{\Phi}^i \phi_i(v_i(x_i)) &= \alpha_{\Phi}^i \theta(\bar{u} - \underline{u}) + \left(\sum_{j \neq i} \alpha_{\Phi}^j \right) \underline{v} \\
&= \left(\sum_{j \neq i} \alpha_{\Phi}^j \right) \bar{v} + \left(\alpha_{\Phi}^i \theta(\bar{u} - \underline{u}) - \left(\sum_{j \neq i} \alpha_{\Phi}^j \right) (\bar{v} - \underline{v}) \right) \\
&\geq \sum_{i \in N} \alpha_{\Phi}^i \phi_i(v_i(y_i)) + \left(\sum_{j \neq i} \alpha_{\Phi}^j \right) \left(\frac{\theta(\bar{u} - \underline{u})}{n-1} - (\bar{v} - \underline{v}) \right).
\end{aligned}$$

Let us set $\theta > (n-1) \frac{\bar{v} - \underline{v}}{\bar{u} - \underline{u}}$. Hence, we obtain:

$$\sum_{i \in N} \alpha_{\Phi}^i \phi_i(v_i(x_i)) > \sum_{i \in N} \alpha_{\Phi}^i \phi_i(v_i(y_i)),$$

and therefore

$$\sum_{i \in N} \alpha_{\Phi}^i u_i(x_i) > \sum_{i \in N} \alpha_{\Phi}^i u_i(y_i).$$

■

We can now complete the proof of Proposition 4. Consider any $x, y \in X$ such that $\min_{i \in N} v_i(x_i) > \min_{i \in N} v_i(y_i)$. Denote $i^* = \arg \min_{i \in N} v_i(x_i)$, $\underline{u} = \min_{i \in N} v_i(y_i)$, $\bar{u} = \min_{i \in N} v_i(x_i)$, $\underline{v} = \min_{i \in N \setminus i^*} v_i(x_i)$, $\bar{v} = \max_{i \in N} v_i(y_i)$. Hence, \underline{u} is the wellbeing of the worst-off individual in y , \bar{u} is the wellbeing of the worst-off individual in x , \underline{v} is the wellbeing of the second worst-off individual in x , \bar{v} is the wellbeing of the best-off individual in y . Define $\bar{x}, \bar{y} \in X$ in the following way. There exists $i \in N$ such that

1. $v_j(\bar{x}_j) = \underline{v}$ and $v_j(\bar{y}_j) = \bar{v}$ for all $j \in N \setminus i$;
2. $v_i(\bar{y}_i) = \underline{u}$, $v_i(\bar{x}_i) = \bar{u}$.

By definition of R^0 and Assumption 2, we necessarily have $x R^0 \bar{x}$ and $\bar{y} R^0 y$. By the same token, if $\underline{v} > \bar{v}$, we have that $\bar{x} P^0 \bar{y}$, which by transitivity implies

xP^0y . Let us thus consider the case $\underline{v} \leq \bar{v}$, and prove that $\bar{x}P^0\bar{y}$, which will conclude the argument by transitivity.

The allocations \bar{x} and \bar{y} satisfy the conditions of Lemma 2. Thus there exists $\bar{\Phi} \in \mathcal{C}$ such that $x, y \in X_{\bar{\Phi}}$ and

$$\sum_{i \in N} \alpha_{\bar{\Phi}}^i u_i(x_i) > \sum_{i \in N} \alpha_{\bar{\Phi}}^i u_i(y_i).$$

But, by Equation (6), since $\bar{x}, \bar{y} \in X_{\bar{\Phi}}$, this implies that $\bar{x}P^0\bar{y}$.

The only remaining case to consider is $\min_{i \in N} v_i(x_i) = \min_{i \in N} v_i(y_i)$. This case can be handled by continuity, because Invariance to risk attitudes for constant acts and Expected utility imply that R^0 should be continuous. Standard arguments imply that if $\min_{i \in N} v_i(x_i) > \min_{i \in N} v_i(y_i) \Rightarrow xR^0y$ then $\min_{i \in N} v_i(x_i) = \min_{i \in N} v_i(y_i) \Rightarrow xI^0y$.

Proof of Proposition 5

If R_i is a continuous ordering satisfying Dominance, Continuity and Ordinal Equivalence, then there exists a continuous monotonic functional $V_i : \mathcal{F}_i \rightarrow \mathbb{R}$, such that $V_i(x_i) = v_i(x_i)$ for all $x_i \in \mathbb{R}^\ell$, and, for all $f_i, g_i \in \mathcal{F}_i$,

$$f_i R_i g_i \iff V_i(f_i) \geq V_i(g_i).$$

Using functional V_i , instead of the expected utility operator $E(u_i \circ f_i)$ in the proof of Proposition 3, delivers the results.¹⁹

¹⁹Note that in the expected utility case we should take $V_i(f_i) = \phi_i(E(u_i \circ f_i))$, where ϕ_i is such that $v_i = \phi_i \circ u_i$.

Appendix 2: Optimal unemployment allowance policy under macroeconomic risk

We first present the original problem studied by Gollier (1991):

$$\begin{aligned} \max \quad & \int_0^{+\infty} \lambda(m) \sum_s p_s [\phi(\max\{w_s - m, b_s\})] dF(m) \\ \text{s.t.} \quad & \sum_s p_s (f_s(L_s) - w_s L_s - (1 - L_s) b_s) \geq \bar{v} \end{aligned} \quad (7)$$

$$L_s = F(w_s - b_s) \quad (8)$$

$$f'_s(L_s) = w_s - b_s \quad (9)$$

The Lagrangian of this problem can be written as

$$\begin{aligned} \mathcal{L} = & \sum_s p_s \left[\int_0^{w_s - b_s} \lambda(m) \phi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \lambda(m) \phi(b_s) dF(m) \right] \\ & + \gamma \sum_s p_s [(f_s(F(w_s - b_s)) - w_s F(w_s - b_s) - (1 - F(w_s - b_s)) b_s) - \bar{v}] \\ & + \eta [f'_s(L_s) - w_s + b_s] \end{aligned}$$

And the first order conditions for an interior solution are:

$$\frac{\partial \mathcal{L}}{\partial w_s} = \int_0^{w_s - b_s} \lambda(m) \phi'(w_s - m) dF(m) - \gamma F(w_s - b_s) - \eta = 0 \quad (10)$$

$$\frac{\partial \mathcal{L}}{\partial b_s} = \int_{w_s - b_s}^{+\infty} \lambda(m) \phi'(b_s) dF(m) - \gamma (1 - F(w_s - b_s)) + \eta = 0 \quad (11)$$

Equations (10) and (11) imply

$$\begin{aligned} & \int_0^{w_s - b_s} \lambda(m) \phi'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \lambda(m) \phi'(b_s) dF(m) \\ & = \gamma F(w_s - b_s) + \gamma (1 - F(w_s - b_s)) = \gamma. \end{aligned}$$

Now consider the alternative problem that we suggest:

$$\begin{aligned} \max \quad & \sum_s p_s \phi \circ \varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right) \\ \text{s.t.} \quad & (7), (8), (9). \end{aligned}$$

The Lagrangian of this problem can be written as

$$\begin{aligned}\mathcal{L} = & \sum_s p_s \phi \circ \varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right) \\ & + \gamma \sum_s p_s [(f_s(F(w_s - b_s)) - w_s F(w_s - b_s) - (1 - F(w_s - b_s)) b_s) - \bar{v}] \\ & + \eta [f'_s(L_s) - w_s + b_s]\end{aligned}$$

And the first order conditions for an interior solution are:

$$\frac{\partial \mathcal{L}}{\partial w_s} = \frac{\phi'(EDE_s)}{\varphi'(EDE_s)} \left(\int_0^{w_s - b_s} \varphi'(w_s - m) dF(m) \right) - \gamma F(w_s - b_s) - \eta = 0 \quad (12)$$

$$\frac{\partial \mathcal{L}}{\partial b_s} = \frac{\phi'(EDE_s)}{\varphi'(EDE_s)} \left(\int_{w_s - b_s}^{+\infty} \varphi'(b_s) dF(m) \right) - \gamma (1 - F(w_s - b_s)) + \eta = 0 \quad (13)$$

Equations (12) and (13) imply

$$\begin{aligned}& \frac{\phi'(EDE_s)}{\varphi'(EDE_s)} \left(\int_0^{w_s - b_s} \varphi'(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi'(b_s) dF(m) \right) \\ & = \gamma F(w_s - b_s) + \gamma (1 - F(w_s - b_s)) = \gamma.\end{aligned}$$

When φ becomes infinitely concave (when an infinite priority is given to the worst-off,

$$\varphi^{-1} \left(\int_0^{w_s - b_s} \varphi(w_s - m) dF(m) + \int_{w_s - b_s}^{+\infty} \varphi(b_s) dF(m) \right) = \min_i \max(w_s - m_i, b_s) = b_s$$

The program hence becomes:

$$\begin{aligned}& \max \sum_s p_s \phi(b_s) \\ & \text{s.t.} \quad (7), (8), (9),\end{aligned}$$

so that the first order conditions are

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial w_s} &= -\gamma F(w_s - b_s) - \eta = 0, \\ \frac{\partial \mathcal{L}}{\partial b_s} &= \phi'(b_s) - \gamma (1 - F(w_s - b_s)) + \eta = 0,\end{aligned}$$

implying $\phi'(b_s) = \gamma$.