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Oskar Lange and how IS-LM came to be interpreted as a Walrasian model

Goulven Rubin¹

Abstract:

A few years after the publication of *The General Theory*, a number of economists began to present Keynes's model, identified with *IS-LM*, as a particular case of the Walrasian model. This view of *IS-LM* has often been rationalized by a basic syllogism: *IS-LM* was invented by John Hicks, Hicks was a Walrasian, hence *IS-LM* was Walrasian. But as some historians of macroeconomics have shown, this syllogism is false. Considering this confusion as an established fact, this article studies how and why *IS-LM* came to be considered as Walrasian. It shows that the standard view took its roots in "The Rate of Interest and the Optimum Propensity to Consume", a paper published by Oskar Lange in 1938, and resulted from a need to clarify the foundations of Keynes's theory.

I. INTRODUCTION

The growing importance attributed to the Walrasian theory is a striking feature of the history of contemporary macroeconomics. This is particularly paradoxical as far as Keynesian theory is concerned. John Maynard Keynes was trained as a Marshallian economist and, except for its general equilibrium perspective, the model of his 1936 *General Theory* had nothing to do with Léon Walras' work. Yet, a few years after the publication of Keynes' book, a number of economists began to present his model, identified with *IS-LM*, as a particular case of the Walrasian model. This idea appeared in Franco Modigliani in 1944, in Lawrence Klein in 1947. A perfect illustration of this position appears in Don Patinkin's entry on Keynes for the *New Palgrave Dictionary of economics*. There he wrote:

Thus a basic contribution of the General Theory is that it is in effect the first practical application of the Walrasian theory of general equilibrium: 'practical', not in the sense of empirical (...), but in the sense of reducing Walras's formal model of n simultaneous equations in n unknowns to a manageable model from which implications for the real world could be drawn (Patinkin 1991, p. 27).

This view of *IS-LM*, endorsed by defenders, critics and historians alike, has often been rationalized by the following syllogism: *IS-LM* was invented by John Hicks, Hicks was a Walrasian, hence *IS-LM* was Walrasian. Alessandro Vercelli (1999) offers a good example of the standard story:

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As is well known, the first prototype of IS-LM model was introduced by Hicks (1937) as an hermeneutic device for clarifying the relationship between Keynes's *General Theory* (from now on *GT*) and General Equilibrium (from now on *GE*) classical theory in a language that could be understood also by the emerging group of econometricists and mathematical economists (Hicks, 1982, p.100). At that time Hicks was busy in writing *Value and Capital* (1939) meant to clarify the foundations of Walrasian *GE* theory in order to build on them more manageable models for economic analysis and policy. Therefore it came natural to him to represent the bulk of *GT* in a small-scale semi-aggregate *GE* model and compare it with an analogous *GE* model of Walrasian inspiration in order to isolate and discuss the differences between them (Vercelli 1999, pp. 201-2).²

In a well known paper published in the *Journal of Post-Keynesian Economics*, Hicks himself seemed to accept this view:

IS-LM was in fact a translation of Keynes's non-flexprice model into my terms. (...) It will readily be understood, in the light of what I have been saying, that the idea of the *IS-LM* diagram came to me as a result of the work I had been doing on three-way exchange, conceived in a Walrasian manner (Hicks 1980, p. 51).

But as some historians of macroeconomics have shown, the basic syllogism is false. The usual story has been disputed by Ingo Barends (1999), Barends and Volker Caspari (1999), Alain Béraud (2012), Michel De Vroey (2004a), Robert Dimand (2007), Warren Young (1987) and William Darity and Young (1995). In a few words, Hicks did not invent *IS-LM* and the original *IS-LM* models were not Walrasian. A confusion has plagued the most common understanding of *IS-LM*.

This confusion has played a decisive role in the history of macroeconomics. For instance, one may argue that it was at the origin of the disequilibrium theories of Jean-Pascal Benassy, Jacques Dreze and Edmond Malinvaud. Or, on the other side of the battlefield, it may also be said that it played a role in the genesis of Robert Lucas's Walrasian macroeconomics³. Whether this confusion was a bad or a good thing is a question that we will leave open. It may have hampered the development of a truly Keynesian macroeconomics but it also stimulated a variety of research programs.

Considering the confusion surrounding *IS-LM* as an established fact, this article offers to probe into its origins: how and why did *IS-LM* come to be considered as Walrasian? We will also take stock on the literature concerning the nature of Keynesian models and add some arguments to dispel the usual confusion.

The standard view took its roots in "The Rate of Interest and the Optimum Propensity to Consume", a paper published by Oskar Lange in 1938. Lange asserted that *IS-LM* was a simplified Walrasian model. In other words, according to him there was a continuity between the two models. With the benefit of hindsight we will show that this presentation was inconsistent with the reality of his own version of *IS-LM*. Actually, the 1938 article illustrated the incompatibility between Keynes' theory and Walras theory of prices. The discovery of this incompatibility led to the mutation that macroeconomics experienced during the 1970's.

Section 1 and 2 explain why it is incorrect to say that Hicks invented *IS-LM* and why nobody considered it to be Walrasian before Lange that is between 1933 and 1938. Section 3 present Lange's 1938 continuity claim and the reason why it is false. This leaves us with a last question. If Lange was the first to say that *IS-LM* was Walrasian, it does not prove that he was the one that was responsible for its establishment. Hence, section 4 discusses the

² This argument is ubiquitous. A more compact formulation can be found in Szenberg and Ramrattan's book on Franco Modigliani: « Hicks was wedded to the Walrasian approach. It first appeared in his 1939 *Value and Capital*, and in his (1937) 'Mr. Keynes and the classics' (2008, p. 27). See also Kriesler and Nevile (2002) or Franco Donzelli (2012, pp. 91-2).

³ See Robert Lucas (2004).

influence of Lange's article on the young American economists who developed the neoclassical synthesis.

II. THE ORIGINS OF IS-LM

The IS-LM model was born out of the Marshallian tradition and its first versions had nothing to do with the Walrasian theory of general equilibrium. To make this point I need to touch upon a number of controversial issues concerning the nature of the Marshallian and the Walrasian traditions or the nature of the *General Theory*. Since these controversies are not the subject of this paper, I will content myself with a few points on which most historians agree.

Two prominent economic traditions

Different economic traditions interacted during the thirties, the Austrian, the institutionalist, the Marshallian, the Swedish and the Walrasian to name only the most prominent (David Laidler 1999). Among them, two are particularly important for our story: the Marshallian and the Walrasian. Contrary to what is often said, these two traditions offered competing, not complementary, representations of the functioning of a market economy. This point has been recognized by a number of economists and historians of economic thought, notably Axel Leijonhufvud (2006) or De Vroey (1999a).

A basic distinction concerns the way both traditions approach the formation of equilibrium. Within the Marshallian tradition, the formation of equilibrium and the nature of the equilibrium that is reached depend on the assumption that production is carried out before market opening and on agents full knowledge of market conditions. Based on this knowledge, agents form expectations of current market prices and take their decisions. If agents' knowledge is perfect, a normal equilibrium is reached. If they make mistakes, an inferior form of equilibrium is reached.

The Walrasian representation of the market is based on the tâtonnement metaphor. Agents are knowledgeable about the states of nature and past prices and quantities but are ignorant of current excess demands. Their price taking behavior calls for the intervention of agents announcing the prices and modifying them on the basis of observed excess demands, Walras' "crieur de prix" and "courtiers" or Arrow and Debreu's "market participant". This approach implies that agents do not anticipate current market conditions, though they may have to form expectations concerning future price.

Another interesting difference is stressed by De Vroey. Whereas, Marshallians tend to see the economy as constituted by a number of autonomous markets, Walrasian would rather see it as one big market. Hence, the Marshallian approach will tend to insist on the specificity of each market whereas the Walrasian approach will treat all "sub-markets" as symmetric. The Walrasian approach is reflected in the discipline imposed on agents by a unique budget constraint. This constraint implies that agents decide simultaneously what they will supply and demand on each market. It also implies that the decision concerning one market is dependent on the decisions taken on all other markets. From this point of view, the Marshallian approach is more flexible. Agents are generally supposed to determine their behavior on one market assuming that the *ceteris paribus* clause applies on other markets. As a consequence, the budget constraint they face changes as they visit the economy one market after the other. A related characteristic is the pragmatic approach of Marshall and many of his followers concerning the representation of agents' behaviors not necessarily tied to strict optimization assumptions.

The contrast between these representation of the market economy means that a model built in one tradition, say the Marshallian, could not be characterized as part of the other tradition, say Walrasian.

Keynes as a Marshallian

As many argued before, Keynes was brought up in the Marshallian tradition.

Peter Groenewegen (1995) has insisted on the depth of the relation between Keynes and Alfred Marshall himself and identified three periods during which Keynes was the “pupil of the master”⁴. The last one was posthumous as Keynes had to immerse himself in the papers of Marshall to write a biographical *Memoir* and edit his *Official Papers* for the Royal Economic Society.

Groenewegen also noted the closeness between Keynes’ methodology and the methodology of Marshall, in stark contrast with Arthur Cecil Pigou who, according to Groenewegen, “never absorbed Marshall’s message on method, conceptualization, the nature of abstraction, style and vision” (1995, p. 140). The relation between Keynes’ method and the method of Marshall has also been explored by Kevin Hoover (2006) who notes that “Keynes spent his intellectual childhood dangled, as it were, on Alfred Marshall’s knee”⁵.

As far as economic theory is concerned, the secondary literature usually presents the young Keynes as a Marshallian economist participating in the elaboration of the Marshallian theories of money and business fluctuations (Bridel 1987, p. 89; Laidler 1999, p. 106). The case becomes more controversial when it comes to the *General Theory*, the result of “a long struggle of escape”, as is well known. Yet, today, Keynes’ scholars seem to admit that it is difficult to understand the content of the *General Theory* if one ignores the Marshallian background of Keynes. Actually, Joan Robinson was probably one of the first to point out the Marshallian roots of the book:

Keynes’ *General Theory of Employment* is an application to output as a whole of the analysis developed by Marshall of the short-period equilibrium of a particular industry (Robinson 1954 [1982], p. 69).

The idea that the principle of effective demand resulted from a series of modifications of Marshall’s theory of the market has been taken up by a long list of commentators since then⁶. The Marshallian ingredients of the theory of the rate of interest in the *General Theory* have been studied by Pascal Bridel (1987) and Michael S. Lawlor (2006). Lawlor also studied the influence of the Marshallian tradition of labor market analysis on Keynes’ magnum opus. In short, if the *General Theory* attempted to escape the limitations of Marshallian theories, this attempt was an internal criticism, it was a struggle fought within the Marshallian tradition⁷. For, one thing that is certain is that Keynes did not try to switch from Marshall to Walras, as this quotation, taken from a letter to Hicks, illustrates: “All the same, I shall hope to convince you some day that Walras’ theory and all the others along those lines are little better than nonsense!” (Letter from Keynes to Hicks, December 9th 1934)⁸

⁴ See also Vaizey (1977).

⁵ Some consequences of this methodological filiation are presented below.

⁶ See for instance, Asimakopulos, 1984 and 1991; Clower, 1989; De Vroey, 1999b; Dos Santos Ferreira and Michel, 1985; Leijonhufvud, 2006; Parrinello, 1985; Schefold, 1997.

⁷ Vaizey makes the same point: “But the fact that even in old age Pigou could still go along with Keynes’ fundamental revision of the theory of employment suggests that the Keynes innovation, though great, was well within the general corpus of Cambridge theory. That is a point of view that the Keynesians would have contested at the time, but it is clear from the French edition of the *General Theory* that Keynes himself was aware of the complexity of his own position” (1977, p. 15). Vaizey goes on quoting the French preface in which Keynes notes that “historian of doctrine will regard this book as in essentially the same tradition” (meaning the orthodoxy which he “learnt, taught and wrote”).

⁸ This quotation was called to my attention by Romain Plassard who found a copy of Keynes’ letter in the Robert Clower Papers at Duke University. This copy was sent to Clower by Hicks himself.

Keynes' invention of IS-LM

Ironically enough, the first IS-LM model was developed by Keynes himself who used it when presenting his new theory to his students in December 1933.

As noted by Young (1987, pp. 12-13) and Dimand (2007), the lecture notes assembled by Thomas K. Rymes (1989) document this fact. In the notes taken during the Final lecture of the Michaelmas term given the 4th of December, we find the following set of equations:

$$M = A(W, \rho)$$

$$Y = C + I$$

$$C = \varphi_1(W, Y)$$

$$I = \varphi_2(W, \rho)$$

Where A was the “state of the liquidity preference”, W “the state of the news” and ρ the rate of interest. Y was the income of the community. Its measure was not indicated in the notes for the 4th of December but the 6th of November, Tarshis noted that output could be valued in terms of money or in terms of employment (1989, pp. 103). These equations were used by Keynes to summarize the “relevant forces” that he had “isolated” and show how they determined the level of employment. But he did not use his model to produce one of the now familiar comparative static exercises. Instead, he warned his students:

The equations are only illustrations and [by themselves] are not good. The real tool is a method of thought and there are many other things [to consider?] (1989, p. 126).

Yet, IS-LM was there for the first time and Keynes was its father.

IS-LM taken up by Keynes' students and two members of the circus

IS-LM was taken up after the publication of *The General Theory*. But the first who brought it to the fore, David G. Champernowne, Brian Reddaway, James Meade and Roy Harrod, were young Marshallians who had little knowledge of Walrasian economics. Actually, their models came from their readings of *The General Theory* or from Keynes' lectures and the ideas that circulated at Cambridge during the elaboration of the book (Dimand 2007, p. 82; Young 1987, p. 33 and p. 60).

The first versions came from Champernowne and Reddaway, two students of Cambridge University that Keynes had supervised. The story of Reddaway's contribution, reported by Young (1987, p. 75), is particularly illustrative of the fact that the equations of the IS-LM model were taken from Keynes' book and had nothing to do with the works of the Lausanne school. In 1936, Reddaway published a review of the *General Theory* in an Australian journal, *The Economic Record*, containing an equational representation of Keynes' central message. This came out of his reading of a copy of the book he had been given by Keynes' himself and that he had brought with him on the boat taking him back to Australia. As he told in his interview with Young: “I wanted to make sure that the thing was the right number of equations for the right number of unknowns and so I scribbled this thing down” (Young 1987, p. 76).

The following quotation from Champernowne illustrates the fact that he read the *General Theory* having in mind the debates between the main figures of the Marshallian school:

I was aiming to elucidate the relation of Keynes's new model with the Marshall-Pigou-Robertson type of model and to provide mnemonics for those wishing to use the Keynes model for investigating the likely effects of particular shifts of the 'state of the news' in the sense of changes which would alter some of the functions underlying some or all of the curves used as mnemonics (quoted by Young 1987, p. 85).

Meade's training located him also in the Marshallian tradition. In 1930, as a fellow of Hertford College in Oxford, Meade was sent to Cambridge to learn economics from Dennis Robertson (David Vines, 1991). There he became an active member of the circus, the small group of young economists discussing Keynes' ideas on the road from the *Treatise on Money* to the *General Theory*. This experience left him with most of the ingredients used in the paper that expounded his version of IS-LM.

When he introduced his version of IS-LM, Harrod remarked that Keynes' theory might not appear revolutionary to "those whose main interest is in the general theory" (1937, p. 75). This "general theory" referred quite clearly (albeit implicitly) to the general equilibrium approach of the Lausanne school. Does this mean that Harrod considered Keynes' system as a version of the Walrasian system? Nowhere did he say something as explicit. Actually, in the same paper, he contrasted the "general theory" to the "departmental studies" aimed at "particular problems" and offering the "main findings of economic theory" for the "ordinary working economists". For these economists, Keynes' theory was revolutionary. The tone of Harrod suggests that he had little interest in the "general theory" in the hands of a minority "standing at the philosophical end of the economic array". Harrod's attitude towards Walras and his school was made clear in his review of the translation of the *Eléments d'économie politique pure* published by William Jaffé in 1954. Two quotations suffice to illustrate the tone of the article: "Almost all those general qualities that made Marshall's *Principles* a Great classic, despite the fact that its original contribution to pure theory are admittedly limited, are lacking in Walras" (1956, pp. 311-2) or "Marshall has far greater scope and depth than Walras" (1956, pp. 315-6). In spite of his ambiguous reference to a "general theory", Harrod was also a Marshallian economist.

For all these economists, it would probably have been strange to see the IS-LM framework presented as belonging to the Walrasian approach.

The disappearance of IS-LM in The General Theory : cause and consequences

Being conceived within the Marshallian tradition, IS-LM could not be a Walrasian model⁹. But if this point is accepted it also becomes difficult to understand how it came to be seen as Walrasian. Of course, one may say that no theory of market interdependencies was developed in the Marshallian camp before the appearance of the *General Theory* (with the exception of the elusive note 21 in the mathematical appendix of the *Principles*). As a consequence, most economists tended to equate general equilibrium systems with Walrasian systems.

This confusion has certainly played an important role. But another reason has to do with Keynes' version of the Marshallian methodology (Hoover 2006). Marshallians consider that theory must be "purpose built". The aim must not be to offer a comprehensive model of the whole economy. Or, in other words, the elaboration of a price theory explaining the functioning of an entire market system cannot be a preliminary step. Such an undertaking would be sterile because the economy is too complex to be captured this way. The economist must elaborate theoretical tools adapted to the particular problems he wishes to study. The partial equilibrium scheme is the most obvious illustration of this approach. Hoover (2006) explains how Keynes developed the concept of "causal nexus" as a particular version of this Marshallian methodology. The model of the *General Theory* aimed to isolate the part of the economic mechanism relevant to explain unemployment. This means that the principle of effective demand and the liquidity preference theory of the rate of interest are only parts of a larger economic system, a system too complex to be captured in its entirety. This methodological trait of the *General Theory* clarifies the nature of IS-LM.

⁹ We will offer analytical arguments in section III on Hicks' version of the model.

Because of his methodological stance, in his 1936 book, Keynes refused to lay down the mathematical model that would have synthesized his theory. Unlike in the lessons given in 1933, he scattered his equations in the different chapters without any attempt to homogenize the mathematical notations. Finally, in a famous paragraph of chapter 21 he warned against the dangers of “pseudo-mathematical methods of formalizing a system of economic analysis” (Keynes 1936, p. 297). As a result, the young readers of the *General Theory* were left on their own to reconstruct Keynes’ model. But in the process, the Marshallian microfoundations present in the *General Theory* got lost.

For the *General Theory* is not devoid of microfoundations (Dos Santos Ferreira 2000; Hoover 2012). The most obvious instance is chapter 3 and the analysis of the goods market, as mentioned above. The liquidity preference function is also clearly defined starting from the choice of individual agents (Cartelier 1995). But in keeping with his conception of a good theory, Keynes did not try to relate his analysis to a systematic theory of prices that is a Keynesian theory of the market system. Such a theory did not exist and he did not try to build it. For him, probably, such an undertaking would have led to lose “sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols” (Keynes 1936, p. 298). As a result, the elements of microfoundations in the *General Theory* remained unsystematic and difficult to grasp for the uninformed reader. This explains what happened to Keynes’ theory in the hands of the young economists looking for the central message of his book. These readers tried to gather the main elements of the theory, what Benetti (1998) called the “logical structure of the *General Theory*”. As a result they produced a model that was quite agnostic with regards to price theory and microfoundations. Though, as we will stress again in the case of Hicks, the structure of the model bore a number of traces of its Marshallian origins.

III. JOHN R. HICKS AND THE NATURE OF IS-LM

What role did Hicks play in our story? During the 1930s, Hicks wrote *Value and Capital* (1939), a book which is said to have introduced the Walrasian theory in the anglo-saxon world. So, the story runs, when he conceived his IS-LM model, Hicks simply translated Keynes’ theory in the language of Walras. But as a number of historians before me have realized, this interpretation of Hicks’s contribution does not stand up scrutiny. The main reason, documented in the previous sections of this paper, and first uncovered by Young (1987) is that Hicks did not invent the model he presented in 1937¹⁰. Young proved, in particular, that before writing “Mr Keynes and the Classics”, Hicks had seen the models of Harrod and Meade¹¹. What Hicks invented was the graphic representation of these equations, the SI-LL diagram. The elaboration of his diagram was probably facilitated by Hicks’s familiarity with general equilibrium models (Hicks, 1980; Young, 1987, p. 101) but this does not mean that his model and his diagram were Walrasian.

Hicks on the Marshallian origins of IS-LM

As a matter of fact, there was no reference to Walras or to the Lausanne school in “Mr Keynes and Classics”. Hicks mentioned Knut Wicksell, a reader of Walras, but only to remark that the *IS-LM* framework could account for cumulative inflation when the IS and LM curves were both horizontal. The paper explicitly located the *IS-LM* apparatus and Keynes’ theory within the Marshallian tradition. Hicks’ paper compared Keynes’ model to a Classical model.

¹⁰ It should be noted that Young (1987) himself did not discuss thoroughly the idea that Hicks SI-LL was a Walrasian product. Yet, his investigations provide ample evidence in favor of our case.

¹¹ Darity and Young (1995, p. 6) make exactly that point.

The latter was said to descend from “Ricardo and Marshall” and was the result of further qualifications by the “successors of Marshall”. But Keynes’ model itself was only a qualified version of the Classical model (1937, p. 150). In Hicks’s terms:

With this revision [the introduction income of in the liquidity preference function], Mr. Keynes takes a big step back to Marshallian orthodoxy, and his theory becomes hard to distinguish from the revised and qualified Marshallian theories, which, as we have seen, are not new (1937, p. 153).

Hicks maintained this position in *A Contribution to the Theory of the Trade Cycle* (1950):

[O]ne must never forget that the General Theory is in essentials a formalization (and sometimes over-formalization) of the great Cambridge tradition in monetary economics, which descends from Marshall to Keynes, not without significant contributions from Pigou and Lavington, Robertson and Kahn (1950, p. 4).

Furthermore, in different writings after 1937 and even in interviews, Hicks explicitly dissociated the work presented in 1937 from the research leading to *Value and Capital*. In “Mr Keynes and the Classics”, Hicks tried to assemble the main ingredients of Keynes’ *General Theory*. His aim was to extract Keynes’ model from the 1936 book. In *Value and Capital*, Hicks attempted to develop his own dynamic theory on the basis of his readings of Walras, Pareto, the Swedes and Marshall. Then, in chapter 20, 21 and 22, he tried to reconstruct and evaluate Keynes’ theory. As a result, he offered a presentation of Keynes quite distinct from the 1937 presentation¹². Here is, for instance, what Hicks said in an interview to the *Eastern Economic Journal* in 1988:

The actual writing of *Value and Capital* must have been largely done at Cambridge; though some of the static part must have been written in 1934 (year II of my six). Quite six months out of year IV, my first at Cambridge, were occupied in writing two papers on Keynes’ theory, one of which is very well known. But, I would say that these papers had little to do with *Value and Capital*. There are indeed some references in it to the *General Theory*, but they come in as extras. The book could have been completed, with its main lines much the same, if I had never been asked to write those papers on Keynes, but had been able to put his book aside until I finished my own job (Hicks 1988, p. 1).

Here, Hicks stresses the fact that there was next to no relation between *Value and Capital* and his IS-LM paper. Maybe more to the point, in *The Crisis of Keynesian Economics*, one can read:

But it [SI-LL] was never intended as more than a representation of what appeared to be a central part of the Keynes theory. As such, I think it is still defensible. But I have never regarded it as complete in itself. In fact, only two years later, in *Value and Capital* (1939), I myself put forward what is surely a very different formulation. This also has had much effect; the version of Keynes that is put forward in many modern writings (especially, perhaps, those descended from Patinkin) looks to me more like the *Value and Capital* formulation than like Keynes’ own (Hicks 1974, pp. 6-7).

Here, Hicks insists on the fact that the aim of his 1937 paper was to present Keynes’ model and not to translate it in Walrasian language. He also stresses the distance between the result of this approach and the result of his more Walrasian analysis of Keynes in *Value and Capital*. In his 1987 book, Young presents other quotations showing that according to Hicks his 1937 models were extracted directly from the *General Theory*:

What is common between my paper and [champernowne’s] seems to be no more than what any intelligent person could have got from a careful reading of the *General Theory*.” (Letter from Hicks to Young, quoted by Young 1987, p. 95)

¹² On Hicks’s presentation of Keynes in *Value and Capital* see Rubin (2011a). The contrast between What Hicks said in different places about the relation between *IS-LM* and *Value and Capital* and the position adopted in “IS-LM and explanation” (1980) was noted by Young (1987, p. 46 and p. 152).

Analytical basis for the distinction between SI-LL and a Walrasian model

To complete the demonstration we need to consider the content of Hicks's SI-LL model and explain why it was not a translation of Keynes in Walrasian terms¹³.

I propose to begin with the following question: if Hicks had used the concepts and the method he used in his Walrasian works during the 1930s, would he have presented the theory of Keynes in the form of his *SI-LL* model? I think the answer is no. During the 1930s, Hicks built two macro-models in line with the general equilibrium approach that he presented in *Value and Capital*. The first one appeared in a paper published in 1935 entitled "Wage and Interest: the Dynamic Problem", the second was presented in chapter 22 of *Value and Capital*. Both models differed markedly from the *SI-LL* model.

Two characteristics of the *SI-LL* models developed by Hicks illustrate their Keynesian origin and locate them in the Marshallian approach. To begin with, these models offer an asymmetric and recursive representation of markets. In a Walrasian perspective each supply and each demand depend on all the prices of the system so that interdependency and symmetry are the rule. In the *SI-LL* models each market has its specificities and the systems are recursive. In all models, the labor market is characterized by rigid wages whereas prices in other markets are flexible. Besides, labor demands depend only on real wages and not on the rate of interest. In Hicks's classical model, for instance, money demand depends on income only, so that causality runs from the money market, on which money income is determined, to the saving-investment equation, that determines the rate of interest, to the supply side equation and the labor demand functions allowing to determine the levels of employment in each sector of the economy. Even in the last model presented by Hicks, in which money demand, savings and investment depend symmetrically on income and the rate of interest, money income and the rate of interest are determined before the level of employment and the price levels. A second characteristic of Hicks's 1937 models that shows their Keynesian origin is the presence of money income instead of real income as an argument of behavior functions. This way of writing behavior functions is obviously at odds with an analysis of choice starting from the budget constraints. But it was used by Keynes in the *General Theory* as a way to dodge aggregation problems¹⁴.

Moreover, turning to the macromodels that Hicks analyzed in 1935 and 1939, one can see that in contrast with the approach of "Mr Keynes and the Classics", he took very seriously the symmetry issue and the issue of the relation between aggregate behavior functions and the choice of rational agents.

In "Wage and Interest: the Dynamic Problem" (1935), Hicks analyzed a temporary equilibrium model comprising three goods: labor, bread and equipment (capital). This economy contained three agents: a representative laborer, a representative entrepreneur and a representative rentier. In order to define the supplies and the demands on each market, Hicks analyzed the intertemporal optimization problems of agents. He obtained demand and supply

¹³ Barrens (1999) meets this challenge by showing very carefully how Hicks's model can be extracted from the text of the *General Theory*. He insists in particular on the fact that Keynes himself made use of the idea of a "simultaneous equations system" in the *General Theory* and in drafts. In particular, if his correspondence with Hicks shows that Keynes was critical about various aspects of Hicks interpretation of his theory in his 1937 article, he did not object to Hicks using a simultaneous equation system. Eventually, Barrens concludes: "Furthermore, the system of simultaneous equations developed in Section 3 [Hicks's model] does not in any way represent an example of Walrasian general equilibrium. By contrast, it represents an example of Marshallian (or Marshall-esque) macroeconomic analysis." (1999, p. 105) De Vroey (2004, pp. 72-3) argues that *IS-LM* describes a monetary economy "composed of markets that function separately" in which there is not auctioneer, all characteristics that would single out its Marshallian belonging.

¹⁴ On the problem faced by Keynes and on the inconsistencies implied by this procedure in Hicks (1937) see Béraud (2012).

functions that all depended symmetrically on the two prices of the system: the real wage and the rate of interest. In this context, behavior functions could hardly depend on money income since money was assumed away. The structure of the model was clearly Walrasian and very different from the structure of the 1937 models.

In chapter 22 of *Value and Capital* (1939), Hicks attempted to summarize the “laws of working” of his temporary equilibrium system. At this stage, he reduced his system to a three goods economy that is a macro-model comprising commodities, securities and money. He analyzed the behavior of the system when faced to shifts in demands from one good to another. In contrast with the IS-LM approach, Hicks did not specify aggregate behavior functions from which all conclusions could be mechanically drawn. Rather, he used the concepts and the methods developed in the microeconomic part of *Value and Capital*. Shifts in demands would lead to price changes that would entail substitution and income effects the nature of which would depend upon the relation of substitutability and complementarity between the three goods of the economy. That being said, Hicks proceeded with a systematic examination of the consequences of various “demand shifts”. He first assumed inelastic expectations after what he superimposed the consequences of elastic expectations. Nowhere did the aggregate money income enter the picture, behavior depended only on prices and the rate of interest. The effects of wage rigidity were discussed but their introduction in the discussion was curiously abrupt since Hicks started without mentioning the existence of a labor market. Again it is quite clear that we are far from the approach of the 1937 paper.

It is impossible to close this discussion without considering the content of the text published by Hicks in 1980 in which he claimed that “*IS-LM* was in fact a translation of Keynes’s non-flexprice model into my terms”. In this paper Hicks offered to show how an IS-LM model could be derived from a Walrasian system. But as noted already by Barrens (1999, pp. 105-106), the model obtained by Hicks was not the model of 1937. Whereas the first contained four goods and three markets, the latter comprised five goods and four markets (two markets for commodities, consumer goods and investment goods). Whereas the first assumed fixed wages and fixed prices, the latter assumed only fixed wages. Whereas adjustment depended on real income in 1980, it depended on money income in 1937. Paradoxically, what Hicks showed in 1980 was precisely that his *SI-LL* model could not be derived from a Walrasian model.¹⁵

IV. OSKAR LANGE AND THE CONTINUITY VIEWPOINT

Oskar Lange was the first to present an *IS-LM* model as the simplified version of a Walrasian general equilibrium model in “The Rate of Interest and the Optimum Propensity to Consume”, a paper published in 1938 in *Economica*. The object of the following section is to explain how and why he introduced this interpretation.

Lange’s model

It should be clear to the reader that the point on which I will insist is only a small part of Lange’s article and not its main argument. Lange began his paper by a presentation of his version of *IS-LM*. First, he said very clearly that he owed his framework to Keynes:

¹⁵ Kaldor’s reaction when Young asked him if Hicks presented him his model as a Walrasian framework is worth mentioning in this respect: “No. I don’t think so. Hicks lectured on Walras long before. But this wasn’t connected with Walras. This was macroeconomics and Walras wasn’t [...]. I am very surprised that Hicks ever says that Walras has anything to do with this.” (Kaldor, quoted by Young, 1987, p. 109)

By introducing liquidity preference into the theory of interest Mr. Keynes has provided us with an analytical apparatus of great power to attack problems which hitherto have successfully resisted the intrusion of the economic theorist (Lange, 1938, p. 12).

Lange's version of *IS-LM* resembles the fixed-price version of textbooks. It differed from Hicks' version on several scores. First, all magnitudes were measured in wage-units, that is in real terms¹⁶. Second, the investment function depended on consumption. Third, the IS condition was considered to be an identity and not an equilibrium relation. Finally, Lange's model possessed no supply side. The equations were the following:

$$M = L(i, Y) \quad (1)$$

$$C = \varphi(Y, i) \quad (2)$$

$$I = F(i, C) \quad (3)$$

$$Y \equiv C + I \quad (4)$$

$$Q = wM \quad (5)$$

The symbols used by Lange, distinct from Hicks's notations, were to become the standard ones except for M which was the real supply of money and Q the nominal quantity of money. Everything being measured in wage-units, M was the quantity of money Q divided by the money wage w .

In order to clarify the functioning of his model, Lange detailed the "process of determination of the rate of interest" that it implied. Given a real income, the liquidity preference equation determined an equilibrium rate of interest. This rate of interest and the initial income then determined consumption hence investment. Equation (4) then defined a new real income. If it was different from the initial income, the "process of mutual adjustment" would go on "until equilibrium is attained" (1938, p. 17). This analysis appeared like the continuation of the experiment sketched in chapter 18 of the *General Theory*. But the ease with which Lange proceeded was probably the consequence of his acquaintance with Walras' tâtonnement¹⁷.

Lange used his model to answer two different questions. First, just like Hicks (1937) he discussed the nature of Keynes' message starting from the analysis of a rise in the marginal efficiency of investment. In the general case, a rise in investment would raise the equilibrium rate of interest and a decline of the propensity to consume (a rise of savings) would lower that rate. Then he introduced Mr. Keynes' theory and the traditional theory by comparing equations (1a) and (1b):

$$M = L(i) \quad (1a)$$

$$M = L(Y) \quad (1b) \text{ (or } M = kY)$$

Equation (1a) captured the Keynesian case in which variations of I and S had no effect on i , hence equilibrium was obtained through variations of Y alone. The reverse was the case with the "traditional theory". Just like Hicks, Lange concluded:

¹⁶ As stressed by Alain Béraud (2012) this device was inspired by Keynes' himself. Moreover it solved a defect of Hicks' model, namely the fact that when wages were supposed to be variables, Hicks model always had a solution, even in the liquidity trap scenario. Béraud also remarks that this issue was pointed out by Keynes himself in his famous letter to Hicks about *IS-LM*. An interesting discussion of Lange's assumptions can also be found in Lampa (2013).

¹⁷ This should be compared to Modigliani's analysis of the dynamics of *IS-LM* in his 1944 article (see Rubin, 2004) or even to Hicks' presentation of the determination of income and employment in "Mr Keynes and the Classics" saying absolutely nothing about the processes at work in his model.

Thus both the Keynesian and the traditional theory of interest are but two limiting cases of what may be regarded to be the general theory of interest (Lange 1938, p. 20).

But “Mr Keynes’ theory” was more general than the “Keynesian case”. Keynes assumed that money demand depended on income hence his theory was well represented by equations (1) to (5). The key assumption then was that “the interest-elasticity of the demand for liquidity is infinite” (1938, p. 19).

The second aim of the paper was to show the existence of an optimum propensity to consume. “Optimum” here only meant “leading to the maximum level of investment”. The differentiation of the equations of the model allowed Lange to define a mathematical condition defining the optimum propensity to consume. This condition was unrelated to the choices of agents. Hence the conclusion of the paper:

In a society where the propensity to save is determined by the individuals there are no forces at work which keep it automatically at its optimum and it is well possible, as the underconsumption theorists maintain, that there is a tendency to exceed it (1938, p. 32).

The existence of an optimum level of consumption implied that consumption could be either too high or too low. The first case corresponded to the traditional theory where more saving led to more investment. The second case showed the validity of the under consumption theories of Malthus or Rosa Luxembourgh and proved to be particularly relevant under the assumption of the Keynesian case. Here, more consumption led to more investment and a higher level of income.

The above account shows that a reference to the Walrasian theory of prices was not necessary to Lange’s presentation. Lange borrowed the apparatus of the *General Theory*, introduced modifications like the accelerator effect, and used the resulting model to show that a capitalist system was prone to under-accumulation, a claim that, as we will show latter, belonged to his own agenda. Yet, the Walrasian benchmark played an important role in Lange’s interpretation of *IS-LM*.

IS-LM as a simplified version of the Walrasian model

According to Lange, the *IS-LM* model and the Walrasian model were in a relation of continuity. I will document this continuity viewpoint before turning to its assessment.

Lange first suggested that *IS-LM* could be obtained by aggregation of a Walrasian system of equations. In other words, there was a relation of correspondence between the *IS-LM* model and an underlying “Walrasian system”. This idea appeared as Lange tried to clarify his notion of real magnitudes. Income and cash balances could be measured in terms of wages units or in terms of any other numéraire. According to Lange:

This presupposes, of course, that the ratio of the price of each commodity or service to the price of the commodity or service which is chosen as the numéraire is given. These ratios may be thought of as determined by the Walrasian or Paretian system of equation of general equilibrium. Thus index numbers are not involved in this procedure (1938, p. 13).

Suppose that the underlying “Walrasian or Paretian system” contains N commodities or services and that the first one is labor, if Y is the real income in terms of wage units, in accordance with the preceding quotation it can be defined by the following equation:

$$Y = \frac{p_2}{w} q_2 + \dots + \frac{p_N}{w} q_N \quad (6)$$

As long as the ratios $p_2/w, \dots, p_N/w$ are given, a variation of Y reflects a variation of the quantities q . Of course it is difficult to see how quantities can vary independently from relative prices in a Walrasian setting. Anyhow, Lange used the assumption of an underlying

Walrasian system to define the aggregate magnitudes of his *IS-LM* model. The equilibrium of this Walrasian system provided a vector of “given” relative prices.

Later in the paper, Lange introduced the claim that the main ingredients of *IS-LM* had been previously discovered by Walras:

It is a feature of great historical interest that the essentials of this general theory are contained already in the work of Walras (Lange 1938, p. 20).

This contention was argued with precise references to the *Etudes d'économie politique appliquée* (1898) and to two editions of the *Eléments d'économie politique pure* (1874, 1900). Walras had already pointed out the relation between the demand for money and the rate of interest. He also had a saving function and an investment function depending on the rate of interest. This left Lange with three issues that needed to be clarified in order to clinch his argument.

The first issue was one that would not prove important today. Until the forties at least, many readers of Keynes considered that the IS relation was an identity and not an equilibrium relation¹⁸. This was the case with Lange. According to him, unlike in the *General Theory*, in Walras, the IS equation appeared as an equilibrium condition. But though this difference was “important”, Keynes’ approach was presented as a betterment of the theory “recognized by many economists before” him (1938, note 3 p. 22) and (seemingly) of little consequence as far as the relation between the Walrasian and the Keynesian frameworks were concerned. Lange went on claiming that the identity between investment and savings could be obtained by aggregation of “the budget equations in the Walrasian system” (1938, p. 23).¹⁹ In other words, equation 4 could be derived from the underlying Walrasian system.

A more central issue was the role that aggregate income played in the Keynesian framework and its apparent absence in the theory of Walras. Lange noticed that Walras omitted income in his money demand function, but this was an error without consequences: “But Whatever the shortcomings of his presentation, the liquidity preference function has been indicated clearly by Walras” (1938, p. 21). Lange’s general answer to the problem raised by the insertion of aggregate income in Walrasian behavior functions was revealed on the following pages. If income did not appear as an argument of these functions in Walras’ presentation it was only because “by introducing the prices of all commodities he [brought] income indirectly into the propensity to save” (1938, p. 22). The same argument was used at a higher level concerning the role of aggregate income as an adjustment variable of the economic system. Lange considered the fact that income appeared as an adjustment variable in the *IS-LM* framework as an advantage over the Walrasian presentation. Indeed, Walras’ equilibrium condition on the capital market (IS relation) “[did] not show how total income changes so as to bring saving actually performed always into equality with investment” (1938, p. 22). But according to Lange: “in the process of *tâtonnements* described by Walras all the prices change and thus total income changes, too” (Lange, 1938, note 4, p. 22). In other words, the variations of aggregate income in the *IS-LM* model reflected the variations of commodity prices in the Walrasian model. Aggregate income was the substitute for the price vector in the aggregate behavioral functions: “by introducing the prices of all commodities [Walras] brings income indirectly into the equation expressing the propensity to save” (1938, pp. 21-22). In the end, this led Lange to consider the *IS-LM* model as a valuable simplification of the Walrasian framework: “Thus Mr Keynes’ apparatus involved a considerable simplification of the theory” (1938, p. 23). Without knowing it, Keynes had devised a

¹⁸ This idea came from Keynes’ discussion of the definition of income, savings and investment in chapter 6 and 7 of the *General Theory* although Keynes himself kept writing about an equilibrium relation (see in particular chapter 3 of the *General Theory*).

¹⁹ More on this puzzling statement below.

Walrasian model. But in offering a theory of income determination, this model was able to deal with issues that remained intractable with the complex version of the theory. It was thus a very important contribution.

A third stumbling block in his attempt to bridge Keynes and Walras was considered, albeit implicitly, by Lange. Keynes's theory aimed to show the possible coexistence of equilibrium and involuntary unemployment. If involuntary unemployment is defined as a situation of excess supply, as it generally is, it is barely compatible with the occurrence of a Walrasian general equilibrium in which all markets must be cleared. But Lange was not inconsistent on this score. On the last page of his article, he offered a definition of involuntary unemployment compatible with labor market clearing:

If involuntary unemployment of a factor is defined by its supply being elastic, it is absent whenever the elasticity of supply is finite (1938, p. 31).

Lange assumed that the labor supply curve was horizontal for a certain level of wages and up to a certain level of employment. As long as the labor demand curve cut the supply curve on its horizontal portion there was involuntary unemployment. This meant that involuntary unemployment coincided with market clearing. Hence it was compatible with a state of general equilibrium in the Walrasian meaning. But this general equilibrium was not optimal in the sense that employment and real income were not at their maxima²⁰. Lange's assumption with respect to the labor supply curve allowed him to consider Keynes' underemployment equilibrium as a particular case of Walrasian equilibrium. This is what I call the continuity viewpoint.

Inconsistency between Lange's model and the way Lange presented it

With the benefit of hindsight, one realizes that what Lange said about his model was not consistent with its actual content.

First of all, the way Lange reconciled the presence of aggregate income among the arguments of Keynesian behavior functions and its absence in Walras' behavior functions was too superficial. It is not possible to say that, in general, there is a mechanism of income adjustment in a Walrasian setting just like in IS-LM. Variations of aggregate income in IS-LM cannot be treated as a simple reflection of variations of relative prices in a Walrasian general equilibrium model. As Clower (1965) would point out in 1962 at the Royaumont Conference, there is a difference of nature between the two frameworks on this score. The role played by income as an adjustment variable in the Keynesian models implies that income is given when households make their decisions. But in a Walrasian setting, income is a dependent and not an independent variable. For instance, household will decide their supply of labor on the basis of the price vector of the economy. As a result, relative prices and not income should appear in the behavior functions of agents unlike in the *IS-LM* framework. One may even say that the concept of income is not necessary to a Walrasian theory. But, indeed, a surprisingly long span of time elapsed before macroeconomists understood this issue²¹.

Lange's assumption concerning the form of the labor supply curve offered a better, yet implicit, rationalization of his contentions concerning the possibility of an aggregate income adjustment mechanism in a Walrasian setting. Lange remained ambiguous with regards to the nature of his horizontal supply curve assumption. Did it imply a fixed money wage or a fixed

²⁰ See Rubin (2002), for Patinkin's accurate critique of the way Lange interpreted Keynes concept of involuntary unemployment. Using De Vroey's classification (De Vroey, 2004b) one may say that Lange dealt with underemployment and not with involuntary unemployment. Employment was not at its maximum level yet unemployment implied no departure from agent's optimizing plans.

²¹ On this subject, see Rubin (2005) and Rubin (2012).

real wage? This was not clarified in 1938. In a latter text, Lange repeated his interpretation of Keynes' involuntary concept and mentioned money wages:

“involuntary unemployment' in the Keynesian sense is not an excess supply of labor but an equilibrium position obtained by intersection of a demand and a supply curve, the supply curve of labor, however, being infinitely elastic over a wide range with respect to money wages (...).” (Lange 1944, p. 6)

On the basis of this quotation, let us say that Lange assumed money wages to be fixed for low levels of employment. In this context, suppose the demand for certain goods increased in the Walrasian framework. This would raise their prices and reduce real wages. The producers would demand more labor in order to increase output. The fact that the labor supply curve was horizontal implied that the level of employment was totally determined by the choice of firms. Employment and aggregate income appeared as independent variables to households just like in a standard *IS-LM* model. Finally, a new equilibrium would be reached with a higher level of employment but the same money wage. In terms of *IS-LM*, the rise of prices increased Y . w remained even but the p_i and the q_i increased in equation (6) above. Consumption increased but not as much as Y . M or Q/w was constant and the rate of interest increased. One sees how a certain level of consistency between an *IS-LM* and a Walrasian model could be achieved along these lines.

The problem with this rationalization is that it referred to ingredients that were absent from Lange's system of equations, namely the supply side of the model. If the model combined “rigid” wages and flexible prices, it should have featured an aggregate supply function depending on the price level until the “full employment” level was reached. Moreover, Lange's arguments concerning the link between *IS-LM* and a Walrasian system and his definition of involuntary unemployment implied the assumption that prices were flexible. But the equations of his model and his contention that “the ratio of the price of each commodity or service to the price of the commodity or service which is chosen as the numéraire is given” seemed to imply that prices were fixed. Since the numéraire was labor and since its price was constant (in the underemployment zone at least), all other prices should also be constant²². Finally, even if we interpret Lange as saying that money and not real wages are fixed we are bound to come up against the gap between his *IS-LM* model and the way he presented it.

In addition, there is the puzzle of Lange's definition of equation (4) as the sum of “budget equations”. This statement was based on the definition of budget constraints as the equality between “individuals' incomes” and “the sum of expenditure on consumption and investment”. But in a Keynesian economy like in a Walrasian economy, the budget constraint should include incomes derived from property shares and bond holdings and expenditures on bonds and cash balances. As a result, the sum of budget constraints should give Walras' law. So what had happened to the excess demands for labor, bonds and money? Finally, in spite of his horizontal supply curve of labor assumption, Lange's overall interpretation of *IS-LM* was marred by a number of inconsistencies.

Why Lange referred to Walras or the foundational issue

We have seen how Lange asserted the existence of a theoretical continuity between the Keynesian and the Walrasian models. But we have also seen that this assertion was unwarranted. Besides, it was introduced quite oddly as an anecdotal aside and it was not necessary to the main line of the article. So, why did Lange insist on the relation between *IS-LM* and the Walrasian framework? Lange himself was not very explicit on this matter and we lack archival material to clarify his motivations. So we shall content ourselves with a few tentative remarks.

²² For this reason, Béraud (2012) concludes that Lange assumed fixed real wages in his 1938 article.

The most obvious reason why Lange could have had the idea of comparing Keynes' 1936 theory and the theory of Walras is the fact that both developed an approach of economics centered on markets interdependencies. As shown by Lendjel (2001), in early works, Lange (1932) opposed the approach of Walras and Pareto, presented as those who had been "the first to show the methodological consequences of the fact of the general interdependency between all economic goods"²³, and the approach of Marshall identified with the "method of isolation". In other words, since the *General Theory* dealt with the interaction between the goods and the money markets and since there was no Marshallian general equilibrium theory, Lange could have thought that it had to be related to the theories of Walras and Pareto. Moreover, like Hicks, Lange believed that the distinct theories of Marshall and Walras were complementary and could be integrated in a grand synthesis (Lendjel, 2001). But these ideas are in the nature of a necessary condition. They are not sufficient for they do not tell us why Lange felt that he needed to point out, and so strongly, the connection between Keynes and Walras in his 1938 article.

A simple way of answering this question is by studying the context of Lange's reference to Walras in his article. This suggests that Lange used the reference to the Walrasian model in order to clarify the foundations of his *IS-LM* model. We have seen in the first section that the lack of explicit foundations is a defining trait of *IS-LM* and a consequence of its origins. For Lange this would have been a shortcoming to which he tried to answer by referring to the Walrasian framework. As we have seen above, he first used the Walrasian reference to clarify the definition of aggregate magnitudes supposedly built upon the relative prices of "a Walrasian or Paretian system". He used it also to clarify the nature of the IS relation oddly considered as the aggregation of budget constraints. Then he used it to clarify the nature of the adjustment mechanism at work in the model. The fact that *Y* could be seen as an adjustment variable (in Keynes' theory) was justified with reference to the adjustment of relative prices and individual incomes in a general equilibrium system. Finally, referring to a general equilibrium system probably helped him interpret the nature of the equilibrium state reached in Keynes' model. This equilibrium was simply a Walrasian equilibrium with underemployment due to the form of the supply curve of labor.

Another motivation behind Lange's parallel may have had to do with his famous fight for socialism. Until the establishment of a communist regime in Poland in 1945, Lange developed a research program dominated by two basic goals. First, he aimed at showing that, contrary to Von Mises claims, a socialist economy was viable. Second, he wanted to prove that capitalist economies were dysfunctional in a way that made a socialist system superior. Lange's vision of capitalism was made clear in a text co-authored by Marek Breit and published in Polish in 1934 and in "Marxian economics and economic theory" (1935). In these texts, Lange endorsed the Marxian analysis of the long run tendencies in the development of the capitalist system or Marx vision of history. Yet, he believed that Marx's theory of value was obsolete. In order to base his demonstrations, Lange preferred to use "bourgeois economics" meaning "the Austrian School, Pareto and Marshall" or "Wieser, Boehm-Bawerk, Pareto or even Marshall" (Lange 1935).²⁴

Lange's main arguments before 1938 appeared in his famous pair of papers "On the Economic Theory of Socialism" (1936-1937). In those papers, Lange played with the opposition between the ideal and the actual capitalist systems. In the first part of his essay, he had shown, using the Walrasian theory of tâtonnement, how "free competition tends to enforce rules of behavior similar to those in an ideal planned economy" (1937, p. 123). In other words, in principle, socialism and capitalism seemed to offer the same results. But then,

²³ Quoted by Lendjel (2001, p. 83).

²⁴ For a more detailed analysis of this aspect of Lange's thinking see Rubin (2011).

asked Lange, “what is the use of bothering about socialism?” (ibidem). The answer, offered in the second part of the essay, was developed in two stages. Lange began by a comparison of the two systems in their pure forms. There were two differences explaining why a pure socialist system was superior to a pure capitalist system. Firstly, in a capitalist system, the distribution of wealth was arbitrary and this prevented the attainment of maximum social welfare. Secondly, a planned economy would cope more efficiently with externalities. But these arguments were not enough to settle the issue. In a second stage, Lange put forward the gap between the actual economic system and the pure capitalist system. The first source of this gap was the imperfection of competition analyzed by Robinson and Chamberlin. And this, according to Lange, added a “much more powerful argument to the economist’s case for socialism” (1938, p. 126). It is also in this context that Lange referred to the works of Robertson (1926) and Keynes (1936) showing that the “actual rate of saving in capitalist society” could prove inadequate. What all this shows is that, in 1937, in order to prove the superiority of socialism, Lange thought that he had to insist on the distance between the actual economy and a perfectly competitive system.

This reading of Lange’s previous works leads me to the following conjecture concerning his 1938 article. One may assume that it would have seemed more convincing from Lange’s own perspective to be able to argue that a perfectly competitive capitalist system could suffer from deep deficiencies beyond the problems of distribution and externalities mentioned in 1937. But as a matter of fact, presenting IS-LM as a kind of Walrasian system meant precisely that it depicted an ideal capitalist system. Now this gave a special meaning to the conclusions derived in the 1938 paper. It implied that Keynes’ apparatus allowed to demonstrate that two flaws of the capitalist system, namely the tendency towards under-accumulation and under-employment, were part and parcel of its ideal version and not only the result of the distance between the actual and a perfectly competitive economy. Whereas in the 1937 article *The General Theory* was presented as a book dealing with an actual system characterized by imperfect competition, in 1938 it was associated with an assumption of perfect competition. And this allowed Lange to go much further than in his 1937 article. In a way, this was again a matter of foundations. To say that a Walrasian general equilibrium model laid at the foundation the IS-LM model clarified the scope and the significance of its properties and gave more weight to the Keynesian’s and to the under-consumptionists’ conclusions.

To conclude, I want to insist on the implications of Lange’s continuity viewpoint. Lange referred to Walras to clarify the foundations of IS-LM. But at the same time, since, for him, IS-LM was a Walrasian model, the bridge with a more complex system was not a problem. The continuity viewpoint amounted to say that there was nothing more to say on what would come to be called “microfoundations of macroeconomics”. One may say that Lange planted the seeds of the microfoundation research program but seeds that would have to wait for a critical reader to pick them up.²⁵

V. IMMEDIATE INFLUENCE OF LANGE’S 1938 PAPER

We still have to show how the belief in an organic link between *IS-LM* and the Walrasian model put forward in the 1938 essay made its way in the writings of the founders of the neoclassical synthesis. These may be classified in three categories. The first category encompasses the Keynesians who remained foreign to the idea of a relation between Walras’s

²⁵ This explains why Clower could not consider Lange as a forerunner, a fact that seems to puzzle Young (2008). Lange considered the Keynesian equilibrium in the market clearing sense. This was polls apart from Clower’s interpretation of Keynes in a disequilibrium perspective.

theory and the IS-LM framework. Unsurprisingly, this category should be large for this idea was not necessary to the elaboration of the hydraulic Keynesianism and the derivation of its various multipliers. The two other categories comprise Keynesian economists who were truly influenced by Lange's 1938 article. The second category encompasses those like Klein and Modigliani who can be characterized as promoters of Lange's viewpoint. The third category emerged out of the contribution of Patinkin who originated a view leading to the disequilibrium theories of the 1970's. We will only consider the immediate impact of Lange's article say in the late thirties and in the forties.²⁶

Hansen and Lerner

Looking at the literature from which the Keynesian mainstream stemmed, one discovers that Lange's 1938 article may have been as influential as Hicks's "Mr Keynes and the Classics" in forging the new consensus. A beta-search on JSTOR for the period going from 1938 to 1950 gave us 16 citations for "Mr Keynes and the Classics a Suggested Interpretation" against 25 for "The Rate of Interest and the Optimum Propensity to consume". But this does not mean that all defenders of IS-LM took up his views.

Alvin Hansen illustrates our first category of economists, those who never related IS-LM and the theory of Walras. His *Guide to Keynes* did not contain a single reference to Walras whereas Marshall, Mitchell or Veblen were quoted. Yet, in *Fiscal Policy and Business Cycles*, a book published in 1941, he quoted Lange (1938) about the relation between investment and consumption.

Abba Lerner (1944) also illustrates the influence of Lange (1938), though his position remained critical. Lerner referred to Lange in his defense of the liquidity preference theory of the rate of interest against the loanable fund theory. A way of approaching the issue was that of Lange (1938):

One solution is to construct simplified Walrasian or general equilibrium schemes in which there are a small number of variable representing composite quantities, such as output of consumption goods in general or output of investment goods in general. Perhaps the most enlightening of these schemes and the easiest for the non-mathematical economist to understand is that constructed by Professor Lange. (Lerner, 1944: 89)

Lerner did not share Lange approach though and explain that Keynes' method was different. Keynes had showed that "partial analysis can be made a little more complicated, bringing in three or four variables instead of the Marshallian two, and yet remain manageable" (1944: 89). Using this method one could show that Keynes had a distinct theory of the rate of interest.

Samuelson

Lange (1938) was certainly an important paper for Paul Anthony Samuelson. He adopted his argument about the tendency towards under-accumulation (Samuelson [1942](1966), p. 1162) in his analysis of "Fiscal Policy and Income Determination" and in a critical review of Pigou's *Employment and Policy* ([1941](1966), p. 1187). In a famous paper, Samuelson (1946, 146) explained that no one in Cambridge, Massachusetts, could understand Keynes' *General Theory* before the appearance of *IS-LM*. At that point, he put on an equal footing the mathematical models of Meade, Harrod, Hicks and Lange. This was already the case in his 1941 paper on "The Stability of Equilibrium: Comparative Statics and Dynamics" in which he

²⁶ Our presentation leaves aside the importance of Hicks' *Value and Capital* (1939) and Lange's *Price Flexibility and Employment* (1944) in the diffusion of the idea that Keynes' and Walras' contribution could and had to be related somewhat. These works certainly played an important role though they were not concerned with IS-LM proper. This is the subject of a work in progress (but see Rubin, 2011b).

analyzed the “simple Keynesian model as outlined in the *General Theory*” (1941, p. 113) explaining that: “various writers, such as Meade, Hicks, and Lange, have developed explicitly in mathematical form the meaning of the Keynesian system” (ibidem). This text appeared without modification in the second part of the *Foundations of Economic Analysis* in 1947. As a matter of fact, the model analyzed by Samuelson was closer to Lange’s version than to any other. This may explain his reference to the “Keynes-Lange system” (1947, p. 354) in the conclusion of the second part of the *Foundations*.

If Samuelson was clearly influenced by Lange’s 1938 article, he never wrote explicitly that *IS-LM* was a version of the general equilibrium model of Walras and Pareto. Actually, as pointed out by Arrow, he never dealt with the microfoundation issue²⁷. This could mean that he accepted implicitly Lange’s continuity viewpoint. As pointed out by John E. King (2011, p. 86), Samuelson probably perceived the difficulty associated with Lange’s position and decided to adopt a more cautious approach. Nevertheless, in the *Foundations of Economic Analysis*, he presented the general equilibrium framework and the Keynesian framework side by side without mentioning any problem of compatibility. According to De Vroey (2009) and De Vroey and Pedro Garcia Duarte (2013), Samuelson would have considered that Walras’s theory described the long run position of the economy. The Keynesian theory in turn would have captured the disequilibrium states of the system moving towards its long run position, a vision a bit different from Lange’s viewpoint.

Modigliani

In 1944, Modigliani published a paper entitled “Liquidity Preference and the Rate of Interest”, originally a PhD thesis submitted at the New School for Social Research. This discussion of the *IS-LM* model became a landmark in the development of old Keynesian macroeconomics (De Vroey 2000; Pentti Kouri 1986; Leijonhufvud 1994). On the first pages of his article, Modigliani indicated that his system of equations was “partly taken from earlier writings on the subject” among which he only referred to Hicks (1937) and Lange (1938). But Lange’s works were particularly important for Modigliani.

Modigliani can be presented as a proponent of the continuity viewpoint. The following quotation illustrates:

As a first step in the analysis, we must set up a system of equations describing the relation between the variables to be analyzed. In doing this we are at once confronted with a difficult choice between rigor and convenience; the only rigorous procedure is to set up a complete ‘Walrasian’ system and to determine the equilibrium prices and quantities of each good: but this system is cumbersome and not well suited to an essentially literary exposition such as we intend to develop here. The alternative is to work with a reduced system: we must then be satisfied with the rather vague notions of ‘physical output’, ‘investment’, ‘price level’, etc. In what follows we have chosen, in principle, the second alternative, but we shall check our conclusions with a more general system whenever necessary (1944, p. 46)

Like Lange, Modigliani presented the *IS-LM* framework as a “reduced” or simplified version of the more complex Walrasian system. The assumed continuity between the two models appeared in the idea that one could check some of his macro-conclusions in the “general system”. Besides, like Lange, Modigliani claimed that Keynes’ assumed a horizontal supply curve of labor²⁸. In accordance with Lange’s optimism, he did not study seriously the relation between his macro-framework and the Walrasian model.

²⁷ “Samuelson has not addressed himself to one of the major scandals of current price theory, the relation between microeconomics and macroeconomics” (Arrow, 1967, p. 734)

²⁸ Although in practice, he based his formalization of wage rigidity on the introduction of an exogenous wage floor.

Klein

In 1947, Lawrence Klein published *The Keynesian Revolution*, a book inspired by discussions with Samuelson (Klein 2006, p. 171). Like Modigliani and Samuelson, Klein read the *General Theory* through the lenses of an *IS-LM* model. But his “Keynes’ model” seemed so obvious to him that he did not admit a debt towards those who had extracted the model from Keynes’ book²⁹. So neither Hicks, nor Lange received credit for their contributions to the development of the neo-Keynesian framework. But Lange’s 1938 article was quoted, among others.

To some extent, Klein can also be presented as a proponent of the continuity viewpoint. In the chapter of *The Keynesian Revolution* presenting the new Keynesian theory, Klein started from the microfoundation problem:

A problem that has never been adequately considered by Keynesians is the derivation of a theory in terms of communities of individuals and groups of commodities from a basic theory in terms of individuals and single commodities. In modern economic terminology this is the problem of passing from micro to macroeconomics, i.e. aggregation (1947, p. 56).

Note that Klein reduced the foundation issue to one of aggregation at this stage. Now, what followed made clear that the microeconomic framework was the Walrasian one:

The theories of individual behavior provide a complete set of inter-relationships within the economy; e.g., they give us the demand-and-supply relationships of every commodity in the system. This is the famous Walrasian system of general equilibrium. A mathematical representation of this system would probably involve several million equations in several million unknowns, and incomprehensible maze. To make any useful economic judgments, one must simplify this system into a manageable number of relationships among aggregates of the fundamental prices and quantities (1947, p. 57).

In the end, the Keynesian macro model was presented as an aggregate version of the Walrasian model just like in Lange. It is particularly striking to see how Klein derived the Keynesian consumption function from the standard microeconomic theory of the consumer starting from a given income in the technical appendix of his book.

The last test, to see if Klein followed Lange concerns his assumption concerning the labor supply curve. In *The Keynesian Revolution*, Klein asserted explicitly that Keynes assumed a horizontal supply curve of labor with respect to money wages. Starting from this point, he wrote that Keynesians had to face the challenge of obtaining Keynesian results in Walrasian framework:

In a perfect Walrasian equilibrium where the classical supply curve is used, can one get the result of under-employment equilibrium with the Keynesian system? (1947, p. 75)

To answer this question, Klein proposed to abandon temporarily Keynes’s special assumption and see what happened if the labor supply function depended on real wages. He then argued that Keynes’ results were based on an interest-inelastic investment function. In a Walrasian context, the result was the possibility that the full employment equilibrium did not exist. Finally, to obtain an (unemployment) equilibrium it was necessary to reintroduce Keynes’s assumption concerning the labor supply curve. Note that this assumption was erroneously conflated with the assumption of money wage rigidity.

A common feature of the works of Klein, Modigliani and Samuelson is that they never pointed out the difference in the nature of the microeconomic foundations of a Walrasian and a Keynesian economy. A relation of continuity was assumed but they did not take the time to examine seriously its reality. My conjecture would be that, under the influence of Lange, they considered this to be a benign neglect.

²⁹ Klein may have been influenced by his reading of lectures notes showing the equations presented by Keynes in 1933: “a copy of some unpublished notes by R. B. Bryce, who was then at Cambridge, has been made available in mimeographed form” (1947: 42).

Patinkin and the birth of a discontinuity viewpoint

Last but not least, and also in 1947, Patinkin submitted a PhD thesis containing a discussion of the integration of money to general equilibrium systems and an aborted theory of involuntary unemployment which he would publish in a deeply rewritten form as *Money, Interest and Prices*, his magnum opus, in 1956. Patinkin's contributions were also very important in the stabilization of the neoclassical synthesis (Antoine d'Autume 2000; De Vroey 2009; Rubin 2004). Patinkin was a student of Lange at Chicago University in 1944 and 1945. In his autobiography, he explained that, in his course on business cycles, Lange offered a "detailed presentation of Keynes' General Theory (based on Lange's own interpretation in his 'Rate of interest and the optimum propensity to consume, 1938)" (1995, p. 371). Though neither Lange (1938) nor Hicks (1937) appeared in the bibliography of Patinkin's PhD thesis, the influence of Lange was most visible in the text as will be explained infra.³⁰

On the surface, Patinkin could appear as a promoter of the continuity viewpoint along with Klein and Modigliani. Our quotation in the introduction illustrates this fact. Patinkin (1991) did write that IS-LM was a "reduced" version of Walras' system of equations. The titles of the chapters of *Money, Interest, and Prices* (1956) in which he presented his macro models, "The Workings of the Model: Full Employment" and the "Workings of the Model: Involuntary Unemployment" suggested that they were subtypes of a broader apparatus. And this apparatus was clearly conceived as Walrasian by Patinkin. But if one looks carefully at Patinkin's 1947 PhD thesis, a different picture emerges. The true "heir" of Lange, as Robert Solow (1982) called him, departed markedly from his teacher, though, like Modigliani and Klein, he was still deeply influenced by Lange's 1938 position. In other words, in 1947, Patinkin inaugurated a "discontinuity viewpoint" with respect to the relation between the theory of Walras and IS-LM. Since I have elaborated elsewhere my views concerning Patinkin's PhD thesis (Rubin 2012) my argumentation will be brief.

From Lange (1938), Patinkin took two fundamental ideas. Firstly, he adopted Lange's interpretation of the *General Theory* in terms of IS-LM. Secondly, he considered, like Lange, that in order to clarify the foundations of the Keynesian macro model, one had to start from the Walrasian model. But Patinkin broke with Lange's definition of involuntary unemployment based on the horizontal supply curve of labor. According to Patinkin, involuntary unemployment implied that some workers (at least) could not satisfy their desire to work. But this was not compatible with a representation in which the labor market cleared. To express the involuntariness of unemployment, worker had to be "off their labor curve". The situation should be one of excess supply. This criticism led Patinkin to realize that a theory of involuntary unemployment presupposed a choice theory allowing for such possibility, a theory incorporating "additional restraints". Moreover, in order to explain how these "additional restraints" appeared, he was led to develop new assumptions regarding market adjustment mechanisms. At this point, a clear discontinuity appeared between the Walrasian model and the Keynesian model. The latter was not a straightforward simplification of the former. The Walrasian model was still the benchmark of the Keynesian model. But what Patinkin proposed to do was to reconstruct the Keynesian model by modifying the microeconomics of the Walrasian framework. As I explained in a previous paper³¹, the result was not totally satisfactory, to say the least. This led Patinkin to abandon his attempt at formalizing mathematically the Keynesian model in chapter 13 and 14 of *Money, Interest and Prices*. As a consequence, the idea of a discontinuity between the Walrasian and the Keynesian theory receded into the background. But it was still there. In a famous footnote, Patinkin stressed in particular the difficulty in formalizing the behavior of firms in a

³⁰ Patinkin referred to other pieces by Lange that triggered his work namely Lange (1942) and Lange (1945).

³¹ See Rubin, 2012.

disequilibrium context and the incompatibility between the assumptions of the standard theory and the behavior implied by his interpretation of the Keynesian theory. Torn between his insights and the requirements of rigorous theorizing, he adopted a conservative and somewhat ambiguous attitude reflected by the quotation offered in the introduction of the present paper. But Patinkin's insights finally opened the way to the contribution of Clower (1965) and to the disequilibrium theories of Drèze and Bénassy in the 1970's.

CONCLUSION

Our study of the secondary literature about the relation between Marshall, Walras, Keynes and the *IS-LM* model shows that on several points there should be a consensus today. The Marshallian and the Walrasian traditions offered competing and not complementary representations of the market economy. The *IS-LM* model was developed by economists trained in the Marshallian tradition. Actually, it was devised by Keynes himself in order to summarize his new theory. As a consequence, this model should not have been considered as a Walrasian framework. As noted by some historians, "Mr Keynes and the Classics" was not the origin of the confusion that developed from the 1940s onward. Hicks wrote explicitly that *IS-LM* was a "formalization of the great tradition of Cambridge monetary economics" and something distinct from the line he was pursuing in *Value and Capital*. If he had attempted to translate Keynes into his own terms, in the language of *Value and Capital*, he would have come up with a different model.

But, as suggested in our first section, the genealogy of *IS-LM* itself explains how the confusion could spread. Because of his commitment to the methodology of Marshall, in 1936, Keynes carefully avoided to reduce his theory to a set of simultaneous equations. The task of gathering the equations scattered in the book was left to his young followers. But in the process, they left aside the sophisticated and complex foundations of the *General Theory*. The result was a mathematical framework whose relation to price theory was left hanging in the air.

This may be the key to understand Lange's 1938 contention about the relation between *IS-LM* and the theory of Walras. Lange was clearly the first among Keynes interpreters to claim that such a relation existed. He did not explain why he needed to put this claim forward but a careful reading of his text led us to the conclusion that he did so in order to clarify the foundations of the model and the scope of its predictions. Lange's position in this respect was particularly radical. According to him, *IS-LM* was the simplified version of a Walrasian model. Interpreting involuntary unemployment as the result of a horizontal labor supply curve, he considered Keynes' underemployment equilibrium a kind of Walrasian equilibrium.

There is a paradox here. If Lange's contention concerning the nature of *IS-LM* was probably motivated by an attempt to clarify its foundations, his continuity viewpoint implied that the relation between Keynes's aggregate model and Walras's model was obvious and did not require further research. Yet, as we have shown above, Lange's viewpoint did not hold water. His equations were deprived of any supply side and required fixed prices, all things incompatible with his main claim.

Finally, we have shown how, in spite of this issue, Lange's *IS-LM* article was very influential during the genesis of the neoclassical synthesis in the 1940's. Four among the main architects of neo-keynesianism were directly inspired by Lange. Samuelson, Klein and Modigliani propagated the idea that general equilibrium theory and Keynesian macroeconomics could live in perfect harmony. Samuelson abstained from raising any question concerning microfoundations while Klein and Modigliani clearly endorsed Lange's continuity viewpoint. In principle, a bridge could be built between the Walrasian and the

Keynesian model, so much so that the exact nature of the bridge need not be clarified. Patinkin's attitude was somewhat different. Starting from an internal criticism of Lange's views, he realized that building a Keynesian model on Walrasian foundations required substantial modifications of Walrasian microeconomics. But this discontinuity viewpoint bore fruit only twenty years later with the works of disequilibrium theorists.

BIBLIOGRAPHY

- Arrow, Kenneth. J. 1967. "Samuelson Collected", *Journal of Political Economy*, 75: 730-37.
- Asimakopulos, Athanasios. 1984. "The General Theory and Its Marshallian Microfoundations", *Metroeconomica*, 36 (2-3): 161-175.
- Asimakopulos, Athanasios. 1991. *Keynes's general theory and accumulation*, Cambridge: Cambridge University Press.
- Barens, Ingo. 1999. "From Keynes to *Money, Markets and Method: Essays in Honour of Robert W. Clower*, Cheltenham: Edward Elgar, pp. 85-120. Barens, Ingo. Caspari, Volker. 1999. "Old View and New Perspectives: on Rereading 'Mr Keynes and the Classics'", *European Journal of the History of Economic Thought*, 6 (2): 216-241.
- Barro, Robert. J. Grossman, Herschel. I. 1971. "A General Disequilibrium Model of Income and Employment". *American Economic Review* 61 : 82-93.
- Benetti, Carlo. 1998. "La structure logique de la Théorie générale", *Cahiers d'économie politique*, 30-31 : 11-52.
- Béraud, Alain. 2012. "La formulation du modèle keynésien et le choix de l'unité de mesure", 14ième colloque Charles Gide, Université de Nice, juin.
- Breit, Marek. Lange, Oskar. [1934] 2003. "The Way to the Socialist Planned Economy", translated by Jan Toporowski, *History of Economics Review*: 51-70.
- Cartelier, Jean. 1995. *L'économie de Keynes*, Bruxelles: De Boeck université.
- Champernowne, David. G. 1936. "Unemployment, Basic and Monetary: The Classical Analysis and the Keynesian", *Review of Economic Studies*, 3 (3): 201-216.
- Clower, Robert. 1965. "The Keynesian Counter-Revolution: a Theoretical Appraisal". In *The Theory of Interest Rate* edited by F. H. Hahn. F. P. R. Bretchling. London: Macmillan, pp. 103-25.
- Darity, William. Young, Warren. 1995. "IS-LM: An Inquest", *History of Political Economy*, 27 (1): 1-41.
- De Vroey, Michel. 1999a. "The Marshallian Market and the Walrasian Economy. Two Incompatible Bedfellows", *Scottish Journal of Political Economy*, 46 (3): 19-38
- De Vroey, Michel. 1999b. "Keynes and the Marshall-Walras Divide", *Journal of the History of Economic Thought*, 21 (2), pp. 117-36
- De Vroey, Michel. 2004a. "The History of Macroeconomics Viewed Against the Background of the Marshall- Walras Divide", in M. De Vroey and K. Hoover (eds.), *The IS-LM Model. Its Rise, Fall and Strange Persistence*, *History of Political Economy*, Annual Supplement to Volume 36: 57-91.

- De Vroey, Michel. 2004b. *Involuntary Unemployment: The elusive quest for a theory*, London, New York: Routledge.
- De Vroey, Michel. .2006. “Getting rid of Keynes? A reflection on the history of macroeconomics”, *Discussion paper, Département des sciences économiques de l’université de Louvain*, 51.
- De Vroey, Michel. 2009. *Keynes, Lu cas d’une macroéconomie à l’autre*, Paris : Dalloz.
- De Vroey, Michel. Garcia Duarte, P. 2013. “In Search of Lost Time: the Neoclassical Synthesis”, *The B.E. Journal of Macroeconomics*, 13 (1): 1-31.
- Dimand, Robert. 2007. “Keynes, IS-LM, and the Marshallian Tradition”, *History of Political Economy*, 39 (1) : 81-95.
- Dos Santos Ferreira, Rodolphe. 2000. « Keynes et le développement la théorie de l’emploi dans une économie monétaire », in *Nouvelle histoire de la pensée économique*, Béraud, A. Faccarello, G. (eds), Paris : La Découverte&Syros.
- Dos Santos Ferreira, Rodolphe. Michel, Phillipe. 1985. « Réflexions sur les fondements microéconomiques de la fonction keynésienne d’offre globale », in *Keynes aujourd’hui: théories et politiques*, Barrère, A. (ed), Economica, Paris, pp. 241-252.
- Friedman, Milton. 1946. “Lange on Price Flexibility and Employment: a Methodological Criticism”. *The American Economic Review*. 36 (4): 613-631.
- Garcia Duarte, Pedro. Tadeu Lima, Gilberto. 2012. *Microfoundations Reconsidered, The Relationship of Micro and Macroeconomics in Historical Perspective*, Cheltenham, Edward Elgar.
- Groenewegen, Peter. 1995. “Keynes and Marshall: Methodology, Society, and Politics”, in *New Perspectives on Keynes*, Cottrell, A. F. Lawlor, M. S. (eds) *History of Political Economy*, Annual Supplement to Volume 27: 29-155.
- Harrod, Roy. 1937. “Mr Keynes and Traditional Theory”, *Econometrica*, 5 (1): 74-86.
- Harrod, Roy. 1946. “Price Flexibility and Employment by Oscar Lange”. *The Economic Journal*. 56 (221): 102-7.
- Harrod, Roy. 1956. “Walras: a Re-Appraisal”. *The Economic Journal*. 66 (262): 307-316.
- Hicks, John. R. 1934. “Leon Walras”, *Econometrica*, 2 (4): 338-48.
- Hicks, John. R. 1935. “Wage and Interest : The Dynamic Problem”, *The Economic Journal*, 45 (179): 456-468.
- Hicks, John. R. 1937. “Mr Keynes and the Classics: a suggested interpretation”. *Econometrica* 5: 147-59.
- Hicks, John. R. [1939] 1946. *Value and Capital. An Inquiry Into Some Fundamental Principles of Economic Theory*. Oxford: Clarendon Press.
- Hicks, John. R. 1950. *A Contribution to the Theory of the Trade Cycle*, Oxford, The Clarendon Press.
- Hicks, John. R. 1974. *The Crisis in Keynesian Economics*, Basil Blackwell: Oxford.
- Hicks, John. R. 1980. “IS-LM: An Explanation”, *Journal of Post Keynesian Economics*, 3: 139-54.

- Hoover, Kevin. D. 2006. "Doctor Keynes: economic theory in a diagnostic science", in the *Cambridge companion to Keynes*, Backhouse, R. and Bateman, B. (editors), Cambridge University Press: Cambridge.
- Hoover, Kevin. D. 2012. "Microfoundational Programs", in Garcia Duarte, P. Tadeu Lima, G. (eds) *Microfoundations Reconsidered, The Relationship of Micro and Macroeconomics in Historical Perspective*, Cheltenham, Edward Elgar.
- Keynes, John Maynard. [1936] 1991. *The General Theory of Employment, Interest and Money*. San Diego, New York, London: Harvest/Harcourt Brace.
- King, James. E. 2011. *The Microfoundation Delusion, Metaphor and Dogma in the History of Macroeconomics*, Edward Elgar, Cheltenham, Northampton.
- Klein, Lawrence. 1946. "Macroeconomic and the theory of rational behavior", *Econometrica*, 14 (2): 93-108.
- Klein, Lawrence. 1947. "Theories of effective demand and employment", *Journal of Political Economy*, 55 (2): 108-131.
- Klein, Lawrence. 1947. *The Keynesian Revolution*. Second edition, The Macmillan Compagny, New York.
- Klein, Lawrence. 2006. "Paul Samuelson as a 'Keynesian' economist", in *Samuelsonian Economics and the Twenty-First Century*, Szenberg, M. Ramrattan, L. Gottesman, A. A. (eds), Oxford: Oxford University Press.
- Lampa, Roberto. 2013. "A 'Walrasian Post-Keynesian' model? Resolving the paradox of Oskar Lange's 1938 theory of interest", *Cambridge Journal of Economics*, first published online July 8, 2013, doi:10.1093/cje/bet023.
- Lange, Oskar. 1932. "Die allgemeine Interdependenz der Wirtschaftsprognosen und die Isolierungsmethode", *Zeitschrift für Nationalökonomie*, 4(1): 52-78.
- Lange, Oskar. 1935. "Marxian Economics and Modern Economic Theory", *The Review of Economic Studies*, 2 (3), pp. 189-201.
- Lange, Oskar. 1936. "On the Economic Theory of Socialism Part I". *Review of Economic Studies*. 4 (1): 53-71.
- Lange, Oskar. 1937. "On the Economic Theory of Socialism Part II". *Review of Economic Studies*. 4 (2): 123-42.
- Lange, Oskar. 1938. "The rate of interest and the optimum propensity to consume", *Economica*, 5: 12-32.
- Lange, Oskar. 1942. "Say's law: a restatement and criticism", in Lange O. et al (eds), *Studies in Mathematical Economics and Econometrics*, Chicago.
- Lange, Oskar. 1945. *Price Flexibility and Employment*, San Antonio: Principia Press of Trinity University.
- Lawlor, Michael. S. 2006. *The Economics of Keynes in Historical Context, An intellectual History of the General Theory*, New York, Palgrave macmillan.
- Lendjel, Emeric. 2001. "Le tâtonnement marshallien dans les premiers écrits d'Oskar Lange". *Cahiers d'économie politique*, 38 : 79-134.
- Lucas, Robert. E. 2004. "Keynote Address to the 2003 HOPE conference: My Keynesian Education", *History of Political Economy*, annual supplement to volume 36: 12-24.

- Meade, James. E. 1937. "A simplified Model of Mr Keynes's System", *Review of Economic Studies*, 4 (2): 98-107.
- Modigliani, Franco. 1944. "Liquidity Preference and the Theory of Interest and Money". *Econometrica* 12: 45-88.
- Parrinello, Sergio. 1985. "Le noyau marshallien de la Théorie générale" in *Keynes aujourd'hui: théories et politiques*, Barrère, A. (ed), Economica, Paris, pp. 421-430.
- Patinkin, Don. 1947. *On the Consistency of Economic Models: A Theory of Involuntary Unemployment*. Doctoral dissertation. University of Chicago, August.
- Patinkin, Don. 1956. *Money, Interest and Prices*. Row, Evanston: Peterson and Compagny.
- Patinkin, Don. 1991. "John Maynard Keynes." In *The New Palgrave: A Dictionary of Economics*, J. Eatwell, M. Milgate, and P. Newman (eds), London: Macmillan. 3: pp. 19-41.
- Patinkin, Don. 1995. "The training of an economist". *Banco Nazionale Del Lavoro Quarterly Review*. 195: 359-95.
- Ramrattan, Lall. Szenberg, Michael. 2008. *Franco Modigliani*, Palgrave macmillan, Basingstoke.
- Reddaway, Brian. W. 1936. "The General Theory of Employment, Interest and Money", *Economic Record*, 12 (june): 44-88.
- Robinson Joan 1982. [1954] "The Generalisation of the General Theory" in *The Rate of Interest and Other Essays*, Hyperion Press : Westport.
- Rubin, Goulven. 2004. "Patinkin on IS-LM: An alternative to Modigliani", *History of Political Economy*, annual supplement to vol. 36: 57-91.
- Rubin, Goulven. 2005. "Patinkin and the Pigou effect: or how a Keynesian came to accept and anti-Keynesian argument", *The European Journal of the History of Economic Thought*, 12(1): 47-72.
- Rubin, Goulven. 2008. "Patinkin's interpretation of Keynesian economics: a genetic approach", in Leeson, R. (ed.) *Archival Insights into the Evolution of economics*. Vol. 1 *The Keynesian tradition*, Routledge: 99-125.
- Rubin, Goulven. 2011a. « Hicks et l'économie de la dépression », *Recherches Economiques de Louvain*, 77 (4) : 57-87.
- Rubin, Goulven. 2011b. « Oskar Lange and the influence of Walrasian theory during the Keynesian revolution », ESHET conference, Bogazici University, HAL working papers, halshs-00817758.
- Rubin, Goulven. 2012. « Don Patinkin's Ph.D dissertation as the prehistory of disequilibrium theories », *History of Political Economy*, 44 (1): 235-76.
- Rymes, Thomas. K. 1989. *Keynes's Lectures, 1932-35, Notes of a Representative Student*, London: Macmillan.
- Samuelson, Paul. A. 1941. "Professor Pigou's Employment and Equilibrium", *American Economic Review*, in *The collected scientific papers of Paul A. Samuelson*, Stiglitz J. E. (ed), The MIT Press, Cambridge and London, pp. 1183-1190.
- Samuelson, Paul. A. 1941. "The Stability of Equilibrium: Comparative Statics and Dynamics". *Econometrica*. 9: 97-120.

- Samuelson, Paul. A. 1942. "Fiscal Policy and Income Determination", *Quarterly Journal of Economics*, in *The collected scientific papers of Paul A. Samuelson*, Stiglitz J. E. (ed), The MIT Press, Cambridge and London, pp. 1140-1170.
- Samuelson, Paul. A. 1947. *Foundations of Economic Analysis*, Harvard University Press, Cambridge.
- Samuelson, Paul. A. 1947. "The General Theory by Paul A. Samuelson" in *The New Economics*, Harris, S. (ed), Dennis Dobson LTD, London, pp. 145-160.
- Schefold, Bertram. 1997. *Normal Prices, Technical Change and Accumulation*, London: Macmillan.
- Solow, Robert. M. 1982. "Cowles and the Tradition of Macroeconomics", Cowles Fiftieth Anniversary Volume.
- Timlin, Mabel. 1946. "Price Flexibility and Employment. Review article". *The Canadian Journal of Economics and Political Science*. 12 (2): 204-213.
- Vaizey, John. 1977. "Keynes and Cambridge", in *The End of the Keynesian Era*, Skidelsky, R. (ed), London: Macmillan.
- Vercelli, Alessandro. 1999. "The evolution of IS-LM models: empirical evidence and theoretical presuppositions", *Journal of Economic Methodology*, 6(2): 199-219.
- Vines, David. 1991. "Meade, James", In *The New Palgrave: A Dictionary of Economics*, J. Eatwell, M. Milgate, and P. Newman (eds), London: Macmillan, 3, pp. 410-416.
- Young, Warren. 1987. *Interpreting Mr Keynes The IS-LM enigma*, Cambridge: Polity Press.
- Young, Warren. 2008. "Is IS-LM a Static or a Dynamic 'Keynesian Model'?" in Leeson, R. (ed) *The Keynesian Tradition, Archival Insights into the Evolution of Economics*, Palgrave macMillan, Basingstoke.