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Policy experimentation, political competition, and heterogeneous beliefs

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Policy experimentation, political competition, and heterogeneous beliefs

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Abstract

We consider a two period model in which an incumbent political party chooses the level of a current policy variable unilaterally, but faces competition from a political opponent in the future. Both parties care about voters’ payoffs, but they have different beliefs about how policy choices will map into future economic outcomes. We show that when the incumbent party can endogenously influence whether learning occurs through its policy choices (policy experimentation), future political competition gives it a new incentive to distort its policies – it manipulates them so as to reduce uncertainty and disagreement in the future, thus avoiding the costs of competitive elections with an opponent very different from itself. The model thus demonstrates that all incumbents can find it optimal to ‘over experiment’, relative to a counterfactual in which they are sure to be in power in both periods. We thus identify an incentive for strategic policy manipulation that does not depend on self-serving behavior by political parties, but rather stems from their differing beliefs about the consequences of their actions.

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1 Introduction

Many of the most important public policy problems democratic countries face require cumulative efforts by successive governments to be successfully managed. Consider environmental policy (in particular regulation of stock pollutants such as greenhouse gases), social security reform, sovereign debt management, and public infrastructure development. None of these issues can be tackled in a single legislative term, and the total quantity of resources devoted to them will likely be the result of decisions taken by several governments. As such, the policies incumbent political parties choose to address these issues are heavily influenced by the incentives the political system provides for them to make sound ‘long-run’ policy decisions, even if the effects of those decisions may only be realized once they have left office.

The lack of future political control that is characteristic of democratic systems means that, for the purposes of setting ‘long-run’ policies, incumbents have incentives to manipulate their current policy choices so as to influence both who gets elected in the future and the policy choices future governments will make (Persson & Svensson, 1989; Aghion & Bolton, 1990; Tabellini & Alesina, 1990; Milesi-Ferretti & Spolaore, 1994; Besley & Coate, 1998; Persson & Tabellini, 2000; Azzimonti, 2011). These strategic incentives exist even if parties are not purely office seeking, but have interests that coincide with those of a group of voters, e.g. in models of partisan politics. These effects have traditionally been studied in models with heterogeneous preferences: parties are assumed to have intrinsically different preference parameters, which induce heterogeneous preferences over policies, and hence a strategic incentive for an incumbent party to manipulate present policy choices given that its reelection is uncertain.

While heterogeneity in preference parameters undoubtedly accounts for some of the divergences between political parties’ preferred policies, heterogeneity in beliefs is likely to be an equally important factor. Milton Friedman famously argued that “differences about economic policy among disinterested citizens derive predominantly from different predictions about the economic consequences of taking action...rather than from fundamental differences in basic values” (Friedman, 1966). More recently, public surveys in the US demonstrate a strong polarization in the beliefs of Democrats and Republicans about
a variety of policy issues, including, for example, the likely causes and severity of climate change (Leiserowitz et al., 2012; Borick & Rabe, 2012). Despite the empirical plausibility of belief heterogeneity the consequences of relaxing the common prior assumption have been largely unexplored in the political economy literature on strategic policy choice\(^1\).

The crucial new feature of political competition induced by heterogeneous beliefs is that beliefs are dynamic, and potentially endogenous. Parties’ policy preferences may change over time as their beliefs evolve in response to new information. Moreover this learning process may, at least to some extent, be under the control of the incumbent, who may choose policies with the express purpose of revealing information about their consequences in the future; learning may be ‘active’. Active learning – the idea that current policy choices influence how much is learned in the future – is an old concept in economics (e.g. Prescott, 1972; Grossman et al., 1977), which has been applied to problems in monetary policy (Bertocchi & Spagat, 1993), environmental regulation (Kelly & Kolstad, 1999), and firm behavior (Keller & Rady, 1999). It can be seen as a form of experimentation – we choose an action, observe its consequences, and so learn something new about the relationship between choices and outcomes. In addition, it is often the case that the more intensely we pursue a policy, the more we can separate the ‘signal’ from the ‘noise’, and the more we learn about its effects.\(^2\) Thus when learning is active, and parties have divergent beliefs that they update rationally, the incumbent party has a measure of control over its own, and its opponent’s, future policy preferences. This gives rise to strategic incentives for policy manipulation that are entirely absent when parties merely have different preference parameters.

Our core contribution is to elucidate the interaction between belief heterogeneity, active learning (or experimentation), and political competition, and how this affects the size of

---

\(^1\)Morris (1995) reviews the theoretical arguments for and against the common prior assumption. Acemoglu et al. (2008) demonstrate that Bayesian updating does not generically lead to agreement on posteriors when agents are uncertain about the distribution of possible signals. Glaeser & Sunstein (2013) and Fryer et al. (2013) consider alternative models of belief polarization, and Van den Steen (2004, 2010) consider models of ‘rational’ overoptimism that results from heterogeneous beliefs. We will simply treat belief heterogeneity as an empirical fact, and investigate its consequences for policy choice.

\(^2\)Here are two examples: Consider a policy that decentralizes educational decision making (e.g. management and curriculum decisions) from a central ministry to individual schools. Our ability to discern the causal effect of such a policy on e.g. test scores increases as more schools are included in the program. Next consider a policy that aims to set the allowed level of emissions of a stock pollutant (e.g. greenhouse gases). Suppose that the evolution equation for the stock of pollutant is parametrically uncertain, and contains additive noise. The more of the pollutant we emit, the greater the level of the stock, and the more our observations of the system depend on the underlying dynamics than on stochastic variation. Hence our ability to learn the parameters of the system increases the more we emit (see e.g. Kelly & Kolstad (1999)). Analogous reasoning holds for many public policies.
public programs with uncertain deferred benefits (or costs). Since our concern is specifically to understand how the interaction of these factors determines how incumbents respond to the intertemporal tradeoff inherent in such problems, we abstract from questions of taxation and redistribution, and consider a stylized model in which voters differ only in their beliefs about the benefits of the policy, and parties that represent the beliefs of groups of voters must decide only on the level of some policy variable. We show that the interaction between active learning and political competition gives rise to a new incentive for incumbents to distort their policy choices. This incentive pushes incumbents to choose policies that increase their chances of resolving uncertainty in the future, regardless of their beliefs: they will over experiment. The intuition behind this result is simple – since the preferences of parties with different a priori beliefs converge when learning occurs, incumbents avoid future costly competitive elections with an opponent very different from themselves by choosing policies that reduce disagreement.

We demonstrate this mechanism in a two period model that combines the literature on intertemporal decision making under uncertainty and learning (Arrow & Fisher, 1974; Henry, 1974; Epstein, 1980; Gollier et al., 2000), with a simple but flexible model of political competition (Wittman, 1973, 1983; Roemer, 2001). To demonstrate the effects cleanly, the model assumes that parties care only about voters’ well-being, and disagree only in their beliefs. Thus, in the absence of belief heterogeneity all parties in our model would agree on the correct policy choice, which would also be the optimal policy for the voters. Yet even in the sanguine case where parties are well intentioned and have common objectives, heterogeneous beliefs and political competition will distort their policy choices. We show that when learning is active enough, all incumbents will over-experiment relative to a counterfactual in which they are sure to be in power in the future, regardless of their beliefs and the beliefs of their political opponents.

Section 2 sets out the model structure. Section 3 examines how the interaction between active learning and political competition affects policy choices when beliefs are heterogeneous, without specifying the detailed form of the political competition between parties. To build intuition, a simple model with binary policy choices is discussed first, followed by a more complex model with continuous policy choices. Section 4 specializes to a specific model of political competition: the Wittman model. In our version of this model parties know the distribution of voters’ beliefs, voters vote for their preferred platform, and elections are decided by majority rule. We show that our results hold under plausible primitive conditions on parties’ payoff functions in this case, which apply in both ‘full commitment’
and ‘no commitment’ versions of the model. We reflect on the application of our results to a variety of policy issues in Section 5, before concluding.

Related literature

While the consequences of heterogeneous beliefs and strategic experimentation for the policy choices of incumbents are (to the best of our knowledge) unexplored, several papers investigate some of these factors in other contexts.

Piketty (1995) considers a model of social mobility and redistributive taxation, in which agents hold different beliefs about the relative importance of effort and social class in determining economic outcomes. The beliefs of different agents are updated based on their income mobility experience, and transmitted to their descendants. Piketty shows that belief heterogeneity persists in the steady state, and that experience of income mobility, and not simply income level, contributes to forming political attitudes. While heterogeneous beliefs are at the core of this work, it focusses on voters’ belief formation processes and their interaction with demand for redistribution, and not on strategic policy experimentation by incumbent governments.

Strulovici (2010) is explicitly concerned with strategic experimentation, but focusses on strategic voters, rather than strategic parties. In his model pivotal voters recognize that experimentation reduces their likelihood of being pivotal in the future – this results in under-experimentation in equilibrium. We focus on the behavior of strategic parties, who manipulate their current policies in part to influence the beliefs of future voters. In contrast to Strulovici (2010), we show that parties who have good faith disagreements with their political opponents have an incentive to over experiment.

Callander & Hummel (2013) consider a model that is in some respects close to ours. They examine the efficiency of political turnover, when the only link between successive governments is the information they possess. Incumbents can experiment strategically to influence the information that their successors will use to make their policy choices. They show that, due to the time inconsistency issues that are inherent in political systems with turnover, experimentation can improve the efficiency of policies, as it creates a channel for inter temporal influence. Their work focusses on the efficiency of political turnover as a democratic institution – turnover is imposed exogenously in their model. Our work is concerned with the effect of experimentation and belief heterogeneity on the policy choices of incumbents, and not on turn-taking as an institution. As such, future office holders are
determined endogenously in our model as the equilibrium of a political game, rather than exogenously imposed. Thus incumbents in our model influence both who gets elected, and the policies future governments will implement.

Finally, Hirsch (2013) considers a model of political organization in which a principal and an agent disagree about which policy to implement, but share the same objectives, and can engage in experimentation. Hirsch shows that it may be optimal for the principal to defer to the agent to motivate him to act, or to demonstrate to the agent that his beliefs are incorrect. While the fact that agents in his model differ only in their beliefs is common to our analysis, the roles of the players are exogenously assigned in his work. His model focuses on strategic delegation in a hierarchical organization, rather than strategic interaction between political parties.

Despite the differences in context between our work and that of the last three papers mentioned above, a common overarching theme unites them. In all these cases learning provides a channel for influence, which is used to the advantage of a ‘first-mover’. In Strulovici (2010) this is the pivotal voter, in Hirsch (2013) it is the principle, and in Callander & Hummel (2013) and our own work, it is an incumbent government. Thus our work contributes to a wider recent research program which sees information as a source of strategic control in a variety of contexts.

2 The Model

We consider a two period model, and assume two political parties, indexed by $i \in \{G, B\}$. The parties are well-intentioned: they care only about voters’ well-being, and don’t seek office for their own ends. Our choice of labels for the parties is inspired by an environmental interpretation of the model (‘G’ = Green, ‘B’ = Brown) which we will use to provide intuition at several points in the exposition, but the model is applicable much more widely.

In the first period the incumbent party sets some policy variable $e_1$, which gives rise to certain first period payoffs $U(e_1)$. Second period payoffs $W(e_2|e_1, \lambda)$ depend on the policy $e_2$ that is implemented in the second period, on the legacy of first period policy choices $e_1$, and on an $a$ priori uncertain parameter $\lambda$, which affects the optimal second period policy. The conditions we impose on $U$ and $W$ will be discussed below, for now we focus on the model structure.

It may be helpful to keep in mind an example of a long run policy. Consider a case in which $e_1$ and $e_2$ correspond to the levels of a binding cap on a stock pollutant, such as
greenhouse gases, ozone, or sulphur dioxide. Then we might have

\[ U(e_1) = B(e_1) \]  
\[ W(e_2|e_1, \lambda) = B(e_2) - \lambda C(e_1 + e_2), \]  
where

\[ B' > 0, B'' < 0, C' > 0, C'' > 0. \]

The function \( B(e) \) denotes known short-run benefits from industrial processes that emit the pollutant, and \( C(e_1 + e_2) \) denotes long-run costs (e.g. health impacts or productivity losses) incurred from the accumulation of the pollutant in the atmosphere. The magnitude of these future costs is uncertain, and depends on the realization of \( \lambda \).

Returning to our general model exposition, we assume that \( \lambda \in \{\lambda_L, \lambda_H\} \), where \( \lambda_L < \lambda_H \). The crucial feature of our model is that parties have heterogeneous beliefs about the consequences of policy choices. In the first period, party \( i \) believes that \( \lambda = \lambda_L \) with prior probability \( q_i \). We assume without loss of generality that \( q_G < q_B \). In our environmental example, this implies that the Green party puts more subjective weight on the ‘high damages’ state \( \lambda = \lambda_H \) than the Brown party – hence their labels. Each party’s beliefs are assumed to be representative of the beliefs of some exogenously given set of voters, i.e. the parties position themselves relative to some subset of the distribution of beliefs in the voting population. The heterogeneity in the parties’ beliefs is the only difference between them.

Party \( i \)’s expected utility in the second period, given beliefs \( q_i \), is:

\[ A(e_2|e_1, q_i) := q_i W(e_2|e_1, \lambda_L) + (1 - q_i) W(e_2|e_1, \lambda_H), \]  
and we define

\[ A^*(e_1, q) := \max_{e_2 \geq 0} [q W(e_2|e_1, \lambda_L) + (1 - q) W(e_2|e_1, \lambda_H)] \]  
\[ e^*_2(e_1, q) := \arg\max [q W(e_2|e_1, \lambda_L) + (1 - q) W(e_2|e_1, \lambda_H)]. \]  
\( A^*(e_1, q) \) is thus the payoff a party with beliefs \( q \) expects to receive in the second period if the value of \( \lambda \) remains unknown in the future, and it has exclusive control over which second period policy is implemented. \( e^*_2(e_1, q) \) is the policy this party would choose in this situation.
Similarly, if the value of $\lambda$ is known for sure, both parties will agree that second period payoffs are given by
\[
W^*(e_1, \lambda) := \max_{e_2 \geq 0} W(e_2 | e_1, \lambda).
\] (6)

The parties are dogmatic, in that they do what they think is best for the voters given their beliefs $q_i$, and don’t account for the beliefs of those who disagree with them when making their policy choices. They are however rational, and realize that in the future new observations may be realized that provide information about the value of $\lambda$. They will interpret this new evidence in a rational Bayesian fashion, and update their priors. Moreover, each party knows that the other party will do the same. We compress this incremental learning process into a single period.

To keep the learning process simple we assume that in the second period either the true value of $\lambda$ is revealed (with probability $f(e_1)$), or nothing is learned about the value of $\lambda$ (with probability $1 - f(e_1)$). Crucially, we allow the probability of learning to depend on first period policies. If $f'(e_1) > 0$ then learning is active – the more intensive are first period policies, the greater the chance of learning the value of $\lambda$ in the second period. In this case, policy experimentation carries an informational payoff. Alternatively, if $f'(e_1) = 0$, we say that learning is passive: policy choices have no informational consequences.

Figure 1 illustrates the timing of events in the model. At the beginning of the first period an incumbent chooses a policy $e_1$. At the end of the first period either the true value of $\lambda$ is revealed (with probability $f(e_1)$), or nothing is learned (with probability $1 - f(e_1)$). If $\lambda$ is revealed, parties’ policy preferences are identical in the second period – there is no difference between them as they hold the same beliefs. In this branch of the decision tree there is a ‘trivial’ election in the second period – it doesn’t matter who gets elected, as both parties will choose the same policy. If however $\lambda$ is not revealed, parties’ beliefs remain divergent in the second period. In this case even though the parties have common objectives, they offer different platforms, reflecting their different priors. Thus each party announces a policy platform $e_{2i}$ at the beginning of the second period, and voters decide between them in competitive elections.

We assume for the moment that parties commit to their announced platforms in the

\footnote{Note that this assumption does not violate Bayesian rationality. It is a simplification of the information revelation process (akin to that in e.g. Arrow & Fisher (1974)), and not of agents’ responses to new information. A model with partial learning would be significantly more complex (see e.g. Epstein, 1980), and bring no new qualitative messages about the core interaction we wish to examine. All that we require is that beliefs are ‘closer together’ after observation of a common signal, and that the incumbent can influence the likelihood (or strength) of the signal being received endogenously.}
second period. The case of no commitment can be treated as a special case of this general setup – we pursue this in Section 4.2 below. Since voters believe that parties commit, parties will announce platforms that balance the (subjective) expected benefits of the policy with its ‘electability’. Thus political competition induces parties to offer compromise platforms. We model this electoral game using the Wittman model of political competition (Wittman, 1973, 1983; Roemer, 2001). Under this model, party $i$’s problem is to maximize its payoff $P^i(e_{2i}, e_{2j}|e_1)$ with respect to $e_{2i}$, taking $e_{2j}$ as given, where:

$$P^i(e_{2i}, e_{2j}|e_1) = \pi^i(e_{2i}, e_{2j})A(e_{2i}|e_1, q_i) + (1 - \pi^i(e_{2i}, e_{2j}))A(e_{2j}|e_1, q_i), \quad (7)$$

and $i \neq j \in \{G, B\}$. The function $\pi^i(e_{2i}, e_{2j}) = 1 - \pi^j(e_{2j}, e_{2i})$ is the probability of party $i$ being elected when platforms $e_{2i}, e_{2j}$ are announced. This function will be determined by the distribution of beliefs about $\lambda$ in the voting population, and a model of voter behavior which maps each voter’s beliefs into a vote choice, given a pair of announced platforms. In the interests of generality we leave the details of this function unspecified at this stage, but consider a specific model in Section 4. It is clear from (7) that parties face a tradeoff between increasing their chance of being elected ($\pi$) and having their policy enacted, and choosing a policy that maximizes their expected payoff ($A$).
Roemer (2001) finds conditions that ensure the existence of a pure strategy Nash equilibrium to the political game with payoffs (7). These conditions will always be satisfied for the specifications of $\pi(e_{2i}, e_{2j})$ we consider in Section 4 below, and uniqueness is guaranteed as well for these models. We denote the value of the second period electoral game to party $i \in \{G, B\}$ by

$$\hat{P}_i(e_1) := P^i(\hat{e}_{2i}, \hat{e}_{2j}|e_1)$$

(8)

where we use the $\hat{\cdot}$ symbol to denote optimized quantities that depend on the political equilibrium. The equilibrium policies $\hat{e}_{2i}, \hat{e}_{2j}$ will also depend on $e_1$ in general. This reflects the linkage between the two time periods due to the long run consequences of first period decisions. Note that

$$\hat{P}_i(e_1) \leq A^*(e_1, q_i).$$

(9)

This follows from (7), which shows that $\hat{P}_i(e_1)$ is a convex combination of two terms, each of which is less than or equal to $A^*(e_1, q_i)$. Thus any incumbent party’s payoff is lower when it faces political competition than when it is certain to be in power in the second period. This is a consequence of the loss of control induced by competitive elections.

Summing up all the learning and political components of the model, the optimal first period policy of an incumbent party $i$ is

$$\hat{e}_{1i} := \arg\max_{e_1 \geq 0} \left[ U(e_1) + f(e_1) [q_i W^*(e_1, \lambda_L) + (1 - q_i) W^*(e_1, \lambda_H)] + (1 - f(e_1)) \hat{P}_i(e_1) \right],$$

(10)

where as before the $^*$ symbol denotes an optimized quantity that is independent of political competition.

### 3 Effect of political competition and active learning on policy choice

Our main hypothesis is that the interaction between active learning and political competition gives rise to incentives for incumbents to ‘over experiment’ with their first period policies. This reduces uncertainty and disagreement in the future, and hence avoids costly political competition. In order to demonstrate this in our model, we need to examine the additional effects of active learning, political competition, and their interaction, on policy choice. Thus, we need to define baseline learning and political scenarios which we will compare to the active learning/political competition scenarios.
Table 1: Notation for our four policy scenarios

<table>
<thead>
<tr>
<th></th>
<th>Passive learning</th>
<th>Active learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Optimum</td>
<td>$e_1^0$</td>
<td>$e_1^a$</td>
</tr>
<tr>
<td>Political Competition</td>
<td>$e_1^0$</td>
<td>$e_1^a$</td>
</tr>
</tbody>
</table>

To this end, we define a passive learning scenario, in which first period policies have no effect on the probability of learning the value of $\lambda$ (i.e. $f'(e_1) = 0$), and an active learning scenario, in which increasing $e_1$ increases the chance of learning the value of $\lambda$ (i.e. $f'(e_1) > 0$):

Passive learning: $f(e_1) = f_0$, a constant. \hspace{1cm} (11)

Active learning: $f(e_1) = f_0 + f_a(e_1)$ \hspace{1cm} (12)

where $f_a(0) = 0$, $f_a'(e_1) > 0$, $\lim_{e_1 \to \infty} f_a(e_1) \leq 1 - f_0$. In the active learning case $f_a(e_1)$ represents the additional information that is revealed by enacting policy of intensity $e_1$, over and above the exogenous chance of resolving uncertainty $f_0$. By comparing optimal policies under active learning to optimal policies under passive learning, we will capture the additional effect of the active component of learning $f_a(e_1)$ on policy choice.

In order to isolate the effects of political competition on first period choices in these two learning scenarios, we will contrast the optimal first period decision under political competition with a baseline case in which the incumbent is guaranteed to be in power in both periods – we refer to this as the individual optimum case. In this case, the optimal first period policy of the incumbent $i \in \{G, B\}$ is given by

$$e_{1i}^*: \arg \max_{e_1 \geq 0} \left[ U(e_1) + f(e_1) \left[ q_i W^*(e_1, \lambda_L) + (1 - q_i) W^*(e_1, \lambda_H) \right] + (1 - f(e_1)) A^*(e_1, q_i) \right].$$

(13)

The difference between (13) and (10) is that the value of the ‘no learning’ branch of the decision tree is now given by $A^*(e_1, q_i)$, rather than $\hat{P}^i(e_1)$.

We have thus set up two dimensions of variation in our model – passive vs. active learning, and political competition vs. the individual optimum. Evaluating the optimal policies in (10) and (13) under the two learning scenarios (11) and (12) leads to four policy scenarios. Table 1 summarizes our notation for the optimal first period policies in these four cases.

The passive learning/individual optimum cases allow us to determine the additional effects of active learning/political competition, relative to these baselines. The interaction
between active learning and political competition is captured by looking for differences between the effect of active learning (relative to passive learning) in the two different political scenarios.

Comparing policies in the same column in Table 1 gives us the effect of political competition on the optimal policy choices of an incumbent. When learning is passive this effect is captured by $e_1^* - \hat{e}_1^0$. When learning is active the effect is captured by $e_1^* - \hat{e}_1^a$. Similarly, comparing policies in the same row in Table 1 gives us the effect of active learning. This effect is captured by $e_1^0 - e_1^*$ in the individual optimum, and by $\hat{e}_1^0 - \hat{e}_1^a$ under political competition.

### 3.1 A simple model with binary policy options

In order to build intuition, we consider a simple version of the above model in which first period policies $e_1$ can take only two values $4$: $e_1 \in \{0, 1\}$. The incumbent must either implement a policy ($e_1 = 1$), or do nothing ($e_1 = 0$) in the first period. Second period policies $e_2$ may be discrete or continuous – all that we require is that optimal second period policies $e_2^*(e_1, q)$ depend on the value of $q$.

Our main result in this case is as follows:

**Proposition 1.** Active learning gives any incumbent party an additional incentive to experiment (i.e. choose $e_1 = 1$) relative to the passive learning case, in both the individual optimum and political competition scenarios. However, this additional incentive is greater under political competition than in the individual optimum.

**Proof.** Let $\hat{Y}_i(e_1)$ be the value of policy $e_1$ under political competition, and $Y_i^*(e_1)$ the value of policy $e_1$ in the individual optimum, for party $i$. Let $f_1 = f(1)$, and $f(0) = f_0$, where $f_1 > f_0$ when learning is active, and $f_1 = f_0$ when learning is passive.

From (10) we have that in general

$$\hat{Y}_i(1) - \hat{Y}_i(0) = U(1) - U(0) + f_1 E^i W^*(1, \lambda) - f_0 E^i W^*(0, \lambda) + (1 - f_1) \hat{P}_i(1) - (1 - f_0) \hat{P}_i(0)$$

where $E^i$ denotes an expectation over $\lambda \in \{\lambda_L, \lambda_H\}$ with probability distribution $(q_i, 1 - q_i)$. The incumbent chooses $e_1 = 1$ if and only if $\hat{Y}_i(1) - \hat{Y}_i(0) > 0$. We will refer to the quantity

---

4This simple case can be thought of as an adaptation of the classic model of Arrow & Fisher (1974), which analyses the effect of learning and irreversibility on intertemporal choice, to our political context. Note however that irreversibility plays no role in our analysis.
\[ \hat{Y}_i(1) - \hat{Y}_i(0) \] as the ‘relative benefits’ of \( e_1 = 1 \). Define the difference between the relative benefits of \( e_1 = 1 \) under active and passive learning as:

\[
\Delta_i := [\hat{Y}_i(1) - \hat{Y}_i(0)]_{f_1 > f_0} - [\hat{Y}_i(1) - \hat{Y}_i(0)]_{f_1 = f_0} \\
= (f_1 - f_0) E^i W^*(1, \lambda) - (f_1 - f_0) \hat{P}^i(1) \\
\geq 0 \tag{15}
\]

\( \Delta_i \) measures the additional incentive to choose \( e_1 = 1 \) (rather than \( e_1 = 0 \)) when learning is active, over and above the incentive to choose \( e_1 = 1 \) when learning is passive. The fact that \( \Delta_i \geq 0 \) follows from the convexity of the ‘max’ function (information has positive value), and the second from (9). Thus there is a greater incentive to choose \( e_1 = 1 \) when learning is active than when it is passive when the incumbent faces political competition in the future.

Repeating this calculation in the individual optimum case, one finds that:

\[
\Delta^*_i := [Y^*_i(1) - Y^*_i(0)]_{f_1 > f_0} - [Y^*_i(1) - Y^*_i(0)]_{f_1 = f_0} \\
= (f_1 - f_0) E^i W^*(1, \lambda) - (f_1 - f_0) A^*(1, q_i) \\
\geq 0 \tag{16}
\]

Similarly, there is a greater incentive to choose \( e_1 = 1 \) under active learning (relative to passive learning) when the incumbent is certain to be in office in both periods.

Finally, we show that the interaction between active learning and political competition gives rise to an additional incentive for the incumbent to choose \( e_1 = 1 \), relative to the individual optimum. This is demonstrated by the fact that the ‘difference in differences’ between the effects of active learning in the two political scenarios is:

\[
\Delta_i - \Delta^*_i = (f_1 - f_0) [A^*(1, q_i) - \hat{P}^i(1)] \\
\geq 0 \tag{17}
\]

where again the inequality follows from (9). This inequality holds for any incumbent party, regardless of its beliefs and the beliefs of its opponent. \( \square \)
This result says that the difference between the incumbent’s incentive to choose $e_1 = 1$ under active vs. passive learning is larger when it faces political competition than in its individual optimum. There will thus be cases in which switching from passive to active learning induces the incumbent to switch from $e_1 = 0$ to $e_1 = 1$ under political competition, but not in the individual optimum. The converse, however, can never happen. If a switch from passive to active learning causes the incumbent party to change from $e_1 = 0$ to $e_1 = 1$ in the individual optimum, it must also do so under political competition.

This simple result illustrates the incentive for the incumbent party to over experiment when it faces political competition from an opponent who share its goals, but has differing beliefs. While active learning provides an additional benefit (relative to passive learning) to the $e_1 = 1$ policy under both political scenarios, the difference between the relative benefits of $e_1 = 1$ under active and passive learning is greater under political competition than in the individual optimum. This is so since, under active learning, the incumbent party increases its chance of avoiding a costly election with an opponent different from itself by choosing $e_1 = 1$. It uses its first period policy choice strategically to reduce disagreement in the second period. This result relies critically on the fact that the parties have heterogeneous beliefs. Beliefs are endogenous and amenable to manipulation, whereas preference parameters are not.

### 3.2 Continuous first period policies

The positive interaction between active learning and political competition is easily demonstrated in the binary case examined in Section 3.1. We now extend these results to a continuous model of policy choice. This turns out to be a more complex problem. When first period policies $e_1$ are continuous (and payoffs $W$ are non-linear in $e_1$), one needs to rank optimal policies under different learning and political scenarios; we cannot simply rank differences in payoffs themselves as in the binary policy case. This additional complexity has long been recognized in the literature on the effect of learning on dynamic choice (e.g. Epstein, 1980; Ulph & Ulph, 1997; Gollier et al., 2000; Mäler & Fisher, 2005), which has focussed on conditions that are sufficient to determine the direction of the change in the optimal choice variable under different learning scenarios.

Following in this tradition, we will state sufficient conditions for an analogue of the intuitive results obtained in the binary case to hold in a continuous model. There will clearly be cases in which these conditions are not satisfied. In these cases the effects of active learning and/or political competition on policy choice are ambiguous. The possibility
of an ambiguous effect in the continuous model arises since comparing optimal first period policies under different learning and political scenarios requires us to compare both the levels of second period payoffs under different scenarios (as in the binary case), and the derivatives of these payoffs with respect to $e_1$. The results we obtained in the binary model required us only to rank payoff levels under the different scenarios; in the continuous model there are new effects to consider, as we explain below.

Reconsidering the models of the incumbent’s first period choices in (10) and (13), we now assume that $e_1$ is a continuous choice variable. Our sufficiency result is as follows:

**Proposition 2.** Suppose that first period payoffs $U(e_1)$ are concave in $e_1$, and that unique interior solutions to the first order conditions exist under all the scenarios in Table 1. If for all $e_1$ and $q_i$

\[
\frac{dA^*(e_1, q_i)}{de_1} \geq \frac{d\hat{P}^i(e_1)}{de_1}, \\
\frac{dA^*(e_1, q_i)}{de_1} \leq q_i \frac{dW^*(e_1, \lambda_L)}{de_1} + (1 - q_i) \frac{dW^*(e_1, \lambda_H)}{de_1},
\]

then for any incumbent $i$,

(a) Active learning increases $e_1$ (relative to passive learning) in the individual optimum, and under political competition: $e_1^a > e_1^0$, $\hat{e}_1^a > \hat{e}_1^0$.

(b) Political competition either decreases $e_1$ (relative to the individual optimum), or has no effect on $e_1$, under passive learning: $e_1^0 \leq e_1^*$. 

(c) For any $f(e_1)$ that satisfies

\[
-\frac{d}{de_1} \log(1 - f(e_1)) \geq \frac{d}{de_1} \log(A^*(e_1, q_i) - \hat{P}^i(e_1)) \text{ for all } e_1, q_i,
\]

political competition increases $e_1$ (relative to the individual optimum) under active learning: $\hat{e}_1^a > e_1^a$.

**Proof.** See Appendix A.

When the conditions of the proposition hold, active learning increases $e_1$ (relative to passive learning) in both political scenarios, but increases it more when the incumbent
party faces political competition than when it is certain to remain in power\textsuperscript{5}. The incentive to experiment is thus stronger when the incumbent faces political competition.

To understand the conditions in the proposition, it is helpful to begin by examining a special case. Suppose that $W(e_2|e_1, \lambda)$ is independent of $e_1$. In this case the only way $e_1$ influences second period payoffs is through the effect it has on the probability of learning $f(e_1)$; it does not directly affect parties’ payoffs in the second period. Thus the only linkage between the periods is informational. In this case the conditions (18–19) are satisfied as equalities as their constituent terms are all identically zero, and condition (20) is satisfied for any strictly increasing $f(e_1)$, as its right hand side is zero. Thus the conclusions of the proposition hold identically in this case (with political competition having no effect on $e_1$ under passive learning in conclusion (b)).

The only information that is necessary to deduce the conclusions of the proposition in this special case is the following set of inequalities, which always hold:

\[ \hat{P}^i(e_1) < A^*(e_1, q_i) < q_i W^*(e_1, \lambda_L) + (1 - q_i) W^*(e_1, \lambda_H) \]  \hspace{1cm} (21)

In words, second period payoffs under political competition when $\lambda$ is unknown are always less than payoffs in the individual optimum when $\lambda$ is unknown, which are in turn always less than payoffs when $\lambda$ is known in the second period. These relationships imply the pattern of effects we observed in the binary policy case (we used them in (15–17)), and these effects carry over to the continuous policy case when information is the only linkage between the two periods. The core insight is that in this special case our intuition for how the interaction between active learning and political competition, which was based on comparisons of the \textit{levels} of payoffs under different scenarios, is undisturbed by the \textit{derivatives} of payoffs.

Now consider the more general empirically relevant case in which first period policies affect second period payoffs directly. This introduces new terms into the first order conditions, all of which depend on the \textit{derivatives} of second period payoffs with respect to $e_1$. If we are to obtain similar results in this case we need these additional derivative terms not to disturb the ranking of policies based on comparisons of the \textit{levels} of the payoffs under different learning/political scenarios, as in (21). Notice that we can combine the conditions

\textsuperscript{5}This is a trivial consequence of the conclusions of Proposition 2. Compare $\hat{e}_1^a - \hat{e}_1^0$ to $e_1^a - e_1^0$. Both quantities are positive by conclusion (a). $\hat{e}_1^a > e_1^a$ by conclusion (c), and $\hat{e}_1^0 \leq e_1^0$ by conclusion (b). Hence $\hat{e}_1^a - \hat{e}_1^0 > e_1^a - e_1^0$.\textsuperscript{16}
(18–19) and write them as:

\[ \frac{d}{de_1} \hat{P}_i(e_1) \leq \frac{d}{de_1} A^*(e_1, q_i) \leq \frac{d}{de_1} [q_i W^*(e_1, \lambda_L) + (1 - q_i) W^*(e_1, \lambda_H)] \] (22)

Comparing these inequalities to those in (21), we see that the derivatives of second period payoffs with respect to \( e_1 \) must be ranked in the same way as the levels of the payoffs.

The conditions (18) and (19) are the crucial conditions of Proposition 2. When they are satisfied, there always exist active learning functions \( f_a(e_1) \) such that (20) holds. In the general case however, we need learning to be ‘active enough’ to offset the other derivative terms that appear when comparing the individual optimum and political competition cases under active learning. This is the origin of the condition (20), which requires the rate of decrease of \( 1 - f(e_1) \) to be larger than the rate of increase of \( A(e_1, q_i) - \hat{P}_i(e_1) \) as a function of \( e_1 \). Put another way, it requires \( f(e_1) \) to increase fast enough to offset the difference in the marginal effect of a change in \( e_1 \) between the two political scenarios. Formal details of these arguments can be found in Appendix A.

While the message of the proposition is clear, the conditions (18–19) depend on endogenous quantities, and it is thus not possible to know when they are satisfied without putting more structure on the problem. This is a common feature of learning models (see e.g. Epstein, 1980). In the next section we find primitive conditions on the payoff function \( W(e_1|e_2, \lambda) \) that ensure that the crucial conditions (18–19) hold in two common models of political competition.

4 Application to simple models of political competition

In this section we specialize to a specific model of voter behavior, which allows us to determine the probability of election in (7). This framework leads to simple expressions for equilibrium election platforms, which in turn allow us to write down an analytic expression for \( \hat{P}_i(e_1) \), the equilibrium value of the ‘no learning’ sub-game.

In the model we consider voters’ choices depend only on the platforms parties announce (i.e. they don’t have a party affiliation), and the distribution of their beliefs is known to both parties. We consider two variants of the model – a full commitment case, and a no commitment case – and show that the same primitive conditions on the payoff function \( W(e_2|e_1, \lambda) \) imply that the conditions of Proposition 2 are satisfied in both cases.
4.1 A median voter model (full commitment)

In order to pin down the nature of the political competition parties face we have to specify the probability of election function $\pi^i(e_{2i}, e_{2j})$ in (7). In this variant of the model we assume as before that voters believe that parties commit to any announced policy platform. Recall that $q$ denotes a subjective belief that the realized value of $\lambda$ will be $\lambda_L$. We assume that there is a distribution of voters with different values of $q$ in the population, and denote the cumulative distribution function for $q$ by $F(q)$. As before, parties’ beliefs are exogenously given, and are assumed to be representative of the beliefs of different groups of voters. $\pi^i(e_{2i}, e_{2j})$, the probability of party $i$ winning the election when the announced platforms are $(e_{2i}, e_{2j})$, is modeled through:

$$
\pi^i(e_{2i}, e_{2j}) = \begin{cases} 
1 & \text{if } \Gamma(e_{2i}, e_{2j}) > 0.5 \\
0.5 & \text{if } \Gamma(e_{2i}, e_{2j}) = 0.5 \\
0 & \text{if } \Gamma(e_{2i}, e_{2j}) < 0.5
\end{cases}
$$

(23)

where

$$
\Gamma(e_{2i}, e_{2j}) := F(\{ q : A(e_{2i} | e_1, q) > A(e_{2j} | e_1, q) \})
$$

(24)

is the measure of the set of voters who prefer policy $e_{2i}$ to policy $e_{2j}$. Thus, each voter simply chooses the party whose platform gives it a higher expected utility, and the election is decided by majority rule.

With this specification for $\pi^i(e_{2i}, e_{2j})$, the following lemma holds:

**Lemma 1.** Assume that $W(e_2 | e_1, \lambda)$ is a single-peaked function of $e_2$. Let the median voter’s beliefs be $q_m = F^{-1}(1/2)$, and assume that $q_G < q_m < q_B$. Then the equilibrium outcome of the political game in which parties’ payoffs are given by (7), and the probability of election is given by (23), is that both parties propose the optimal policy of the median voter, $e^*_2(e_1, q_m)$. Thus the value of the electoral game to party $i$ is given by

$$
\hat{P}_i(e_1) = A(e^*_2(e_1, q_m) | e_1, q_i).
$$

(25)

**Proof.** See Appendix B.

Thus when voters’ beliefs are known and they have single peaked preferences, parties’

---

$\text{Note:}$ This does not imply that all voters a party aims to represent will vote for that party. Parties can announce only one platform, and thus cannot ensure that all the voters it aims to represent will prefer that platform to the other platform on offer.
platforms converge completely in the second period – they both offer the median voter’s optimal policy\(^7\).

With this expression for the equilibrium value of the ‘no learning’ sub-game, we can seek conditions on the payoff function \(W\) which ensure that (18–19) of Proposition 2 are satisfied. The next result provides such conditions, without specifying a parametric form for \(W\).

Let subscripts on functions denote partial derivatives, e.g. \(W_2 = \frac{\partial W}{\partial e_2}, W_{2\lambda} = \frac{\partial^2 W}{\partial e_2 \partial \lambda}\). Assume the following conditions on the second period welfare function \(W(e_2|e_1, \lambda)\):

\[
\begin{align*}
W_{22} &< 0 \quad (26) \\
W_{21} &< 0 \quad (27) \\
W_{2\lambda} &< 0 \quad (28)
\end{align*}
\]

(26) is the standard concavity condition, and in addition we assume that solutions to the second period optimization problem are interior, so that the constraint \(e_2 \geq 0\) is not binding. This assumption simplifies our analysis, but our results are not crucially dependent on it. It is readily shown (see Appendix C) that the conditions (27) and (28) imply respectively that the optimal second period policy \(e_2^*(e_1, q)\), is decreasing in \(e_1\), and increasing in \(q\). In our environmental example this implies that the greater is the level of first period emissions, the less parties want to emit in the second period, and similarly, the greater the weight they put on the low damages state \(\lambda_L\), the more they want to emit in the second period.

Now define

\[
\epsilon_{x|y} := \text{Elasticity of } W_{2x} \text{ with respect to } y. \quad (29)
\]

**Proposition 3.** If \(U\) is concave, (26–28) hold, the probability of election \(\pi(e_2^i, e_2^j)\) is given by (23), and

\[
\begin{align*}
\epsilon_{2|2} &\geq \epsilon_{1|2} \quad (30) \\
\epsilon_{1|\lambda} &> \epsilon_{2|\lambda} \quad (31)
\end{align*}
\]

\(^7\)The median voter equilibrium is also the equilibrium that would result if parties maximized their probability of election, and not their idiosyncratic expected payoffs as in (7). This was demonstrated in the classic work of Downs (1957). However, although the equilibrium in the Wittman model with probability of election given by (23) coincides with the Downsian equilibrium, parties’ valuations of the equilibrium differ in our model, as shown in (25), whereas they coincide in the Downsian model. We consider a model of political competition that also permits divergence between parties’ equilibrium platforms in Section 4.2.
then both the conditions (18) and (19) of Proposition 2 hold.

Proof. See Appendix C.

The conditions on the elasticities $\epsilon_{x,ly}$ in this proposition clearly require investigation. To fix ideas however, it may be useful to see what they imply for the simple functional form for $W$ in (2). Substitution of (2) into (30) and (31) shows that these two conditions reduce to

$$\frac{-B'''}{B''} \leq -\frac{C'''}{C''},$$

$$B'' < 0$$

respectively. The first condition requires the marginal costs of emissions $C'$ to be more concave than their marginal benefits $B'$. The second condition is satisfied by assumption. Notice that both conditions are always satisfied in the textbook case of linear marginal benefit and cost functions.

A core consequence of the elasticity conditions (30–31), which aids in their interpretation, is that they imply that second period optimal policies $e^*_2(e_1, q)$ have an ‘increasing differences’ property:

$$\frac{d^2 e^*_2}{de_1 dq} > 0.$$ 

(34)

This is a Spence-Mirrlees sorting condition, which allows us to use beliefs $q$ as an index that tells us how much a change in $e_1$ affects second period optimal policies $e^*_2(e_1, q)$. Recall that (27) implies that $\frac{de^*_2}{de_1} < 0$. Thus (34) says that the higher is $q$ (i.e. the more weight on $\lambda_L$), the less $e^*_2$ is reduced when $e_1$ is increased.

This property has important consequences for the condition (18), which gives rise to conclusion (b) of Proposition 1: under passive learning any incumbent reduces $e_1$ under political competition relative to its individual optimum. This is really the novel condition of the proposition, as the other condition (19), which guarantees that active learning causes incumbents to increase $e_1$ relative to passive learning, is well known; it can be seen as a special case of the sufficient conditions for signing the effect of learning on policy choice derived in Epstein (1980). Conclusion (b) is novel, so it is important to understand how the properties of the payoff function in Proposition 3 give rise to it. This can be seen by thinking about the strategic consequences of (34) in the political competition scenario, as we now explain.
Since \( q_G < q_m < q_B \), we know from (34) that,

\[
\frac{d}{de_1}[e_2^*(e_1, q_m) - e_2^*(e_1, q_G)] > 0 \tag{35}
\]

\[
\frac{d}{de_1}[e_2^*(e_1, q_B) - e_2^*(e_1, q_m)] > 0. \tag{36}
\]

Now by the monotonicity of \( e_2^*(e_1, q) \) in \( q \), we also know that

\[
e_2^*(e_1, q_m) - e_2^*(e_1, q_G) > 0 \tag{37}
\]

\[
e_2^*(e_1, q_B) - e_2^*(e_1, q_m) > 0 \tag{38}
\]

Thus from the inequalities (35–36) and (37–38), we see that if we increase \( e_1 \), the distance between the median voter’s optimum and either parties’ optimum increases. However, reducing \( e_1 \) brings the median voters’ optimum closer to both of the parties’ individual optima. Since it is the median voters’ optimum that is implemented under political competition, and all parties have single peaked preferences over second period policies, all parties want this policy to be as close to their individual optima as possible. Figure 2 illustrates this intuition graphically. The condition (34) thus ensures that regardless of whether the incumbent parties’ beliefs \( q_i \) are greater or less than \( q_m \), it always has a strategic incentive to reduce \( e_1 \) relative to its individual optimum.

### 4.2 Exogenous election probabilities (no commitment)

In this section we replace the model of political competition in Section 4.1, in which the probability of election \( \pi^i(e_{2i}, e_{2j}) \) is determined endogenously, with a model in which \( \pi^i \) is an exogenous parameter that is independent of parties’ platforms. It is readily seen that in this case parties’ equilibrium platforms will coincide with their individually optimal policies. Such a model arises naturally if parties cannot commit to their election platforms. This is the case examined in models of strategic policy manipulation with heterogenous preference parameters by Persson & Svensson (1989) and Aghion & Bolton (1990). In this case all voters know that the parties will implement their individual optima after the election has occurred, and thus the electoral outcome is determined by which individual optimum is
Figure 2: Strategic interactions between the first period choices of an incumbent and the second period choices of the median voter. The condition $\frac{d e_2^*}{de_1dq} > 0$ implies that for any $\delta > 0$, the curves $e_2^*(e_1 + \delta, q)$, $e_2^*(e_1, q)$ and $e_2^*(e_1 - \delta, q)$ as functions of $q$ are ordered as in the figure above. Increasing $e_1$ relative to the individual optimum of the incumbent party increases the difference between the second period optima of the median voter and the incumbent, regardless of whether the incumbent’s $q$ is above or below the median value $q_m$. However, decreasing $e_1$ relative to the incumbent’s individual optimum brings the median voter’s second period optimum closer to the incumbent’s, regardless of the incumbent’s beliefs.

more appealing to the median voter:

$$\pi^i = \begin{cases} 
1 & \text{if } A(e_2^*(e_1, q_i)|e_1, q_m) > A(e_2^*(e_1, q_j)|e_1, q_m) \\
0.5 & \text{if } A(e_2^*(e_1, q_i)|e_1, q_m) = A(e_2^*(e_1, q_j)|e_1, q_m) \\
0 & \text{if } A(e_2^*(e_1, q_i)|e_1, q_m) < A(e_2^*(e_1, q_j)|e_1, q_m)
\end{cases}$$

(39)

We can treat all these cases at once by allowing the probability of election to be an arbitrary constant. We have the following proposition:

**Proposition 4.** Suppose that the conditions on $U$ and $W$ in Proposition 3 are satisfied. Assume that the outcome of the political process is exogenously determined, so that $\pi^i(e_{2i}, e_{2j})$ is an arbitrary constant in $[0,1]$. Then the conclusions of Proposition 3 continue to hold.
Proof. See Appendix D.

Thus the results in Section 4.1, in which the election outcome was endogenously determined by parties’ platforms and parties were assumed to commit, carry over to the case in which election outcomes are exogenous (independent of parties’ platforms) and parties cannot commit.

5 Real world applications

The mechanism we have identified can be applied in many policy contexts. Before we discuss some examples however, we emphasize that we see the effect we have highlighted as only a partial contributing factor to actual policy outcomes. Our purpose has been to highlight an informational channel of inter temporal influence and its effects on the policy choices of incumbents. The model is intentionally idealized, in order to crisply demonstrate the new incentives for policy manipulation that belief heterogeneity gives rise to. Previous literature (e.g. Persson & Svensson, 1989; Aghion & Bolton, 1990) examines strategic policy choice when parties’ objectives are heterogeneous but they have common beliefs; our work assumes the opposite. The real world, of course, falls between these two stark cases.

As a first example of the application of our mechanism, consider the case of public smoking bans. The benefits of reducing second-hand smoke include lowering the risk of lung cancer, reducing health care costs, and improving worker productivity. The costs of bans are born by restaurants and bars who may see a decline in profits. These costs are uncertain before the policy has been implemented. Indeed, the impacts of smoking bans on the restaurant business are still debated (Hyland et al., 1999; Adams & Cotti, 2007). Given this uncertainty, and conflicting sources of information, different public representatives are likely to hold different beliefs about the consequences of these laws. Despite conflicting views, public officials have proven willing to experiment with smoking bans, as their diffusion from a few cities and states (San Luis Obispo, California in 1990, California in 1998, New York City in 2002) to many states and countries illustrates (Adams & Cotti, 2007; Eriksen & Chaloupka, 2007).

Congestion charges provide another application. These taxes provide an immediate social benefit by reducing traffic congestion and improving air quality in city centers. However, they also impose a priori uncertain costs on residents who commute to the city center by car, and hence on centrally located retail businesses. Despite disagreements about the projected impacts of these policies, city administrations have rolled them out.
The charges are usually introduced in an initial experimental phase, which enables affected parties to learn about their consequences. This occurred in Stockholm in 2007, when the charge was implemented for an initial seven month trial period, before being expanded and made permanent (Winslott-Hiselius et al., 2009). Similarly, the London congestion charge in 2003 was adopted after an 18-month public consultation period, which enabled affected parties to learn about its consequences (Leape, 2006).

A further example is provided by policies that aim to regulate national emissions of atmospheric pollutants. Policies that place an annual cap on emissions, such as the cap-and-trade mechanisms of the US Acid Rain Program (established in 1990 to curtail sulphur dioxide and nitrogen oxide emissions), or the the European Emissions Trading Scheme (established in 2005 to reduce greenhouse gas emissions), may reduce the future damages that arise from the accumulation of pollutants. However, different actors hold different beliefs about the magnitude of these damages, which are highly uncertain. Since forcing the atmospheric system with emissions allows us to observe how it responds, emitting more today helps to reduce uncertainty in the future (Kelly et al., 2005). Thus our model suggests that even well-intentioned incumbents have an incentive to set less stringent caps than they would prefer to, so as reduce uncertainty and disagreement in the future. Unlike more conventional explanations of the difficulty of passing stringent abatement policies, which appeal to partisan motives, collective action problems, and the influence of special interest groups, this explanation does not require political parties to act solely in their self-interest; it is simply a consequence of disagreements about matters of fact.

6 Conclusions

Our analysis has shown that when beliefs are the primary source of disagreement between political parties, incumbents have a novel incentive to manipulate their policy choices. This stems from the interaction between active learning – the ability to endogenously influence future information revelation through current policy choices – and political competition. When learning is active incumbents can control the degree of disagreement in the future. Since the incumbent party avoids a costly election with an opponent very different from itself if information is revealed and beliefs converge, it has an incentive to increase the chances of resolving uncertainty in the future, regardless of its initial beliefs. This effect relies crucially on the fact that, unlike intrinsic preference parameters, beliefs are endogenous, and thus subject to manipulation.
The mechanism we identify is widely applicable, and may have explanatory power for any policy issue about which voters and parties have diverse beliefs, and where learning occurs through policy implementation. Unlike conventional explanations of distortions to public good provision due to the influence of politics, our result does not rely on rent-seeking and the influence of special interest groups (Aidt & Dutta, 2007; Bohn, 2007; Battaglini & Coate, 2008), or the institutional structure of government (Persson & Tabellini, 1999; Lizzeri & Persico, 2001; Acemoglu & Robinson, 2001). It can be thought of as a parsimonious causal mechanism that assumes the best of political actors, yet predicts that they will still do more to reduce uncertainty than they would like to.
A Proof of Proposition 2

To prove this result it will be useful to define two operators. For any functional $Y(f)$ of the probability of learning $f(e_1)$, define

$$\Gamma_L Y(f) := Y(f_0) - Y(f_0 + f_a(e_1)).$$

(40)

This operator captures the change in $Y$ when we move from passive learning ($f(e_1) = f_0$) to active learning ($f(e_1) = f_0 + f_a(e_1)$) – the subscript $L$ stands for ‘Learning’.

Similarly, for any functional $Y(\hat{P}^i)$ that depends on the political equilibrium value function $\hat{P}^i(e_1)$, define

$$\Gamma_P Y(\hat{P}^i) := Y(A^*(e_1, q_i)) - Y(\hat{P}^i(e_1)).$$

(41)

This quantity represents the change in $Y$ when we move from the individual optimum, in which second period payoffs are given by $A^*(e_1, q_i)$ when learning does not occur, to political competition, where ‘no learning’ second period payoffs are given by $\hat{P}^i(e_1)$. The subscript $P$ stands for ‘Politics’.

The first order condition associated with the definition of $\hat{e}_{1i}$ in (10), which determines the optimal policies under political competition, can be written as

$$U'(e_1) = \hat{\Phi}_i(e_1)$$

(42)

where

$$\hat{\Phi}_i(e_1) := f'(e_1) [\hat{P}^i(e_1) - q_i W^*(e_1, \lambda_L) - (1 - q_i) W^*(e_1, \lambda_H)] - \frac{d\hat{P}^i(e_1)}{de_1}$$

$$+ f(e_1) \left[ \frac{d\hat{P}^i(e_1)}{de_1} - q_i \frac{dW^*(e_1, \lambda_L)}{de_1} - (1 - q_i) \frac{dW^*(e_1, \lambda_H)}{de_1} \right].$$

(43)

Similarly, the first order condition associated with the definition of $e^*_{1i}$ in (13), which determines the optimal policies in the individual optimum, can be written as

$$U'(e_1) = \Phi^*_i(e_1)$$

(44)
where

\[
\Phi^*_i(e_1) := f'(e_1)[A^*(e_1, q_i) - q_i W^*(e_1, \lambda_L) - (1 - q_i) W^*(e_1, \lambda_H)] - \frac{dA^*(e_1, q_i)}{de_1} \\
+ f(e_1) \left[ \frac{dA^*(e_1, q_i)}{de_1} - q_i \frac{dW^*(e_1, \lambda_L)}{de_1} - (1 - q_i) \frac{dW^*(e_1, \lambda_H)}{de_1} \right].
\] (45)

Now assume that solutions to the first order conditions for \(e_1\) in Table 1 exist and are unique in all cases. Then for any function \(\Phi\),

\[
\Gamma_L(\Phi) > (\prec) 0 \ \forall e_1 \Rightarrow \Gamma_L(e_1) < (\succ) 0
\] (46)

Similarly, for any \(f\),

\[
\Gamma_P(\Phi) > (\prec) 0 \ \forall e_1 \Rightarrow \Gamma_P(e_1) < (\succ) 0
\] (47)

To see these relationships, note that when comparing any two solutions to the first order conditions (42) or (44), the left hand side is always given by the same decreasing function \(U'(e_1)\). Thus, assuming that unique solutions exist, if the sign of the difference in the right hand sides, \(\Gamma_L(\Phi)\) or \(\Gamma_P(\Phi)\) as appropriate, is independent of \(e_1\), we are able to infer the sign of the difference in optimal policies, as illustrated by Figure 3 in the case of \(\Gamma_P(\Phi)\).

From (42) and (44), it follows that

\[
\Gamma_L(\Phi(e_1)) = - \tilde{f}'(e_1) \left[ \tilde{A}^*(e_1) - q_i W^*(e_1, \lambda_L) - (1 - q_i) W^*(e_1, \lambda_H) \right] \\
- \tilde{f}(e_1) \left[ \frac{d\tilde{A}^*(e_1)}{de_1} - q_i \frac{dW^*(e_1, \lambda_L)}{de_1} - (1 - q_i) \frac{dW^*(e_1, \lambda_H)}{de_1} \right]
\] (48)

and

\[
\Gamma_L(\Phi^*(e_1)) = - \tilde{f}'(e_1) \left[ A^*(e_1, q) - q W^*(e_1, \lambda_L) - (1 - q) W^*(e_1, \lambda_H) \right] \\
- \tilde{f}(e_1) \left[ \frac{dA^*(e_1, q)}{de_1} - q \frac{dW^*(e_1, \lambda_L)}{de_1} - (1 - q) \frac{dW^*(e_1, \lambda_H)}{de_1} \right]
\] (49)
Similarly, substitution from \((45)\) and \((43)\) shows that:

\[
\Gamma_P(\Phi) = f'(e_1)(A^*(e_1, q_i) - \hat{P}^i(e_1)) - (1 - f(e_1))(dA^*(e_1, q_i)/de_1 - d\hat{P}^i(e_1)/de_1). \tag{50}
\]

From this expression, we see that:

Passive learning: \(\text{sgn} \left[\Gamma_P(\Phi)\right] = -\text{sgn} \left[\frac{dA^*(e_1, q_i)}{de_1} - \frac{d\hat{P}^i(e_1)}{de_1}\right]\) \tag{51}

Active learning: \(\text{sgn} \left[\Gamma_P(\Phi)\right] = \text{sgn} \left[\frac{f'(e_1)}{1 - f(e_1)} - \frac{dA^*(e_1, q_i)/de_1 - d\hat{P}^i(e_1)/de_1}{A^*(e_1, q_i) - \hat{P}^i(e_1)}\right]\), \tag{52}

where in the second line we have used the fact that \(A^*(e_1, q_i) - \hat{P}^i(e_1)\) is always positive.

The three conclusions of the lemma are now arrived at as follows:

(a) Consider \(\Gamma_L(e_1^*)\), the effect of active learning on \(e_1\) in the individual optimum, which is controlled by \((49)\). By assumption, \(\frac{dA^*(e_1, q_i)}{de_1} \leq q \frac{dW^*(e_1, \lambda_L)}{de_1} + (1 - q) \frac{dW^*(e_1, \lambda_H)}{de_1}\), which means that the second term in \((49)\) is positive. It is also always true that

\[
A^*(e_1, q) < qW^*(e_1, \lambda_L) + (1 - q)W^*(e_1, \lambda_H) \tag{53}
\]

This follows from the fact that \(A^*(e_1, q)\) is the upper envelope of a set of functions.

Figure 3: Effect of a change in the right hand side of the first order conditions induced by political competition on optimal first period policies.
that is linear in \((q, 1 - q)\), and is thus convex in this vector of probabilities (see e.g. Gollier, 2001, p. 359). It follows that \(\Gamma_L(\Phi^*(e_1)) > 0\), and hence \(\Gamma_L(e_1^*) < 0\). Now consider \(\Gamma_L(\hat{e}_1)\), the effect of active learning on \(e_1\) under political competition. This is controlled by \(\Gamma_L(\hat{\Phi}(e_1))\) in equation (48). We know that the first term in this expression is positive since \(\hat{P}_i(e_1) < A^*(e_1, q_i) < q_i W^*(e_1, \lambda_L) + (1 - q_i) W^*(e_1, \lambda_H)\). By assumption, \(d\hat{P}_i(e_1)/de_1 < dA^*(e_1, q_i)/de_1 \leq q \frac{dW^*(e_1, \lambda_L)}{de_1} + (1 - q) \frac{dW^*(e_1, \lambda_H)}{de_1}\), so the second term in (48) is also negative. Hence \(\Gamma_L(\hat{\Phi}(e_1)) > 0\), and \(\Gamma_L(\hat{e}_1) < 0\).

(b) Consider \(\Gamma_P(e_1^0)\), the effect of political competition on \(e_1\) under passive learning, which is controlled by (51). The assumption \(d\hat{P}_i(e_1)/de_1 < dA^*(e_1, q_i)/de_1\) clearly implies that \(\Gamma_P(\Phi) < 0\) in this case, and hence \(\Gamma_P(e_1^0) > 0\).

(c) Consider \(\Gamma_P(e_1^a)\), the effect of political competition on \(e_1\) under active learning, which is controlled by (52). From (52), we see that \(\Gamma_P(\Phi) > 0\), and hence \(\Gamma_P(e_1^a) < 0\), provided that the right hand side of (52) is positive. This condition can be rearranged to yield (20).

B Proof of Lemma 1

This result is an application of a theorem due to Roemer (2001). Recall that in the second period, when no learning occurs, voters’ preferences are given by \(A(e_2|e_1, q)\), where \(e_2\) is the policy variable, and \(q\) is the voters’ type, which is distributed according to \(F(q)\) in the population. We assume that parties’ payoffs are given by (7), with the probability of election \(\pi\) given by (23), and the parties’ values of \(q\) are in \(\{q_G, q_B\}\).

Theorem (Roemer, 2001) Assume that:

1. Voter preferences are continuous in \(q\) and \(e_2\)
2. Voter preferences are single peaked in \(e_2\) for all \(q\).
3. The set of voters who are indifferent between any two policies \(e_2\) and \(e'_2\) has measure zero.
4. (Monotonicity) For every pair of policies \(e_2\) and \(e'_2\) where \(e_2 < e'_2\), there exists a policy \(e''_2\) such that the set of voters who prefer \(e_2\) to \(e'_2\) is equivalent to the set of voters whose optimal policies are less than \(e''_2\).
5. \( \Psi(x) \), the set of voters with optimal policies less than \( x \), is continuous and strictly increasing.

6. The median voter’s optimal policy falls between the optimal policies of the parties.

When conditions 1–6 are satisfied, the unique equilibrium of the game in which parties payoffs are given by (7) and the probability of election \( \pi \) is given by (23) consists of both parties playing \( e_{2m} := e^*_2(e_1, q_m) \), where \( q_m \) satisfies \( F(q_m) = 1/2 \).

We need to check that the conditions of this theorem are satisfied for our model. Condition 1 is satisfied by assumption for our function \( A \), and condition 2 follows from the assumption (26), i.e. \( W_{22} < 0 \). This implies that \( A \) is concave in \( e_2 \), and we have assumed that an interior optimum exists, hence \( A \) is single peaked in \( e_2 \). Condition 3 means that the distribution function \( F(q) \) is continuous on \([0, 1]\), the space of types, and is a mild technical restriction. Conditions 4-5 both rely on the following fact in our model: The optimal policy \( e^*_2(e_1, q) \) is a monotonic (in fact increasing) function of \( q \). We have guaranteed this by assumption (i.e. the assumption \( W_{2\lambda} < 0 \) ensures it). To show monotonicity (condition 4), consider two policies \( e_2 \) and \( e'_2 \), with \( e_2 < e'_2 \). A voter of type \( q \) prefers the former to the latter iff:

\[
A(e'_2|e_1, q) > A(e_2|e_1, q) \\
\Rightarrow qW(e'_2|e_1, \lambda_L) + (1 - q)W(e'_2|e_1, \lambda_H) > qW(e_2|e_1, \lambda_L) + (1 - q)W(e_2|e_1, \lambda_H) \\
\Rightarrow q < N(e_2, e'_2) \tag{54}
\]

where \( N \) is a number which depends on the two policies. Now since the optimal policy function \( e^*_2(q) \) is increasing in \( q \), we can write the condition \( q < N \) equivalently as \( e^*_2(q) < e^*_2(N) \). But this condition is exactly of the form required for monotonicity, i.e. we have identified a policy \( e^*_2(N) \) such that the set of voters who prefer \( e_2 \) to \( e'_2 \) is equivalent to all those voters with optimal policies less than \( e^*_2(N) \). Condition 5 follows from assuming \( F(q) \) is continuous and that the optimal policies are monotonic in \( q \). Finally, Condition 6 requires \( q_G < q_m < q_B \), surely a reasonable assumption. Thus our model fits the conditions of the theorem, and the result is established.
C  Proof of Proposition 3

To prove this proposition begin by considering the condition (18)

\[ \frac{d}{de_1} \tilde{P}^i(e_1) < \frac{d}{de_1} A^*(e_1, q_i) \text{ for all } e_1, q_i \]  \hspace{1cm} (55)

By Lemma 1, we have that

\[ \tilde{P}^i(e_1) = A(e_2^*(e_1, q_m)|e_1, q_i), \]  \hspace{1cm} (56)

hence this condition becomes

\[ \frac{d}{de_1} A(e_2^*(e_1, q_m)|e_1, q_i) - \frac{d}{de_1} A^*(e_1, q_i) < 0 \text{ for all } e_1, q_i. \]  \hspace{1cm} (57)

Now consider the condition (19):

\[ \frac{dA^*(e_1, q_i)}{de_1} - \left( q_i \frac{dW^*(e_1, \lambda_L)}{de_1} + (1 - q_i) \frac{dW^*(e_1, \lambda_H)}{de_1} \right) < 0. \]  \hspace{1cm} (58)

for all \( e_1, q_i \). Since

\[ \frac{dA^*(e_1, q_i)}{de_1} = q_i \frac{dW(e_2^*(e_1, q_i)|e_1, \lambda_L)}{de_1} + (1 - q_i) \frac{dW(e_2^*(e_1, q_i)|e_1, \lambda_H)}{de_1} \]  \hspace{1cm} (59)

we can write the expression on the left hand side of (58) as

\[ q_i \left[ \frac{dW(e_2^*(e_1, q_i)|e_1, \lambda_L)}{de_1} - \frac{dW^*(e_1, \lambda_L)}{de_1} \right] + (1 - q_i) \left[ \frac{dW(e_2^*(e_1, q_i)|e_1, \lambda_H)}{de_1} - \frac{dW^*(e_1, \lambda_H)}{de_1} \right] \]  \hspace{1cm} (60)

\[ = q_i \left[ \frac{dA(e_2^*(e_1, q_i)|e_1, 1)}{de_1} - \frac{dA^*(e_1, 1)}{de_1} \right] + (1 - q_i) \left[ \frac{dA(e_2^*(e_1, q_i)|e_1, 0)}{de_1} - \frac{dA^*(e_1, 0)}{de_1} \right] \]  \hspace{1cm} (61)

Notice that both the square brackets in this expression are of a similar form to (57). Thus both conditions (57) and (58) will be satisfied if

\[ \Omega(q, q') - \Omega(q', q') < 0 \text{ for all } q \neq q'. \]  \hspace{1cm} (62)
where
\[ \Omega(q, q') := \frac{d}{de_1} A(e_2^*(e_1, q)|e_1, q'). \]

This is equivalent to saying that \( \Omega(q, q') \), treated as a function of \( q \), should have a global maximum at \( q = q' \), for all \( q' \). For this to be so it is sufficient for
\[ \text{sgn} \left( \frac{d\Omega(x, q')}{dx} \bigg|_{x=q} \right) = \text{sgn}(q' - q). \]

This sign condition ensures that \( \Omega(q, q') \) is a single peaked function of \( q \), and that its maximum is attained at \( q = q' \).

We are thus interested in computing
\[ \frac{d}{dq} \Omega(q, q') = \frac{d^2}{dqd e_1} A(e_2^*(e_1, q)|e_1, q'). \]

As a preliminary step, note that by definition,
\[ A_2(e_2^*(e_1, q)|e_1, q) = 0 \]

Implicitly differentiating this identity with respect to \( e_1 \) and \( q \) respectively, we have,
\[ \frac{de_2^*}{de_1} = -\frac{A_{21}}{A_{22}} \quad (67) \]
\[ \frac{de_2^*}{dq} = -\frac{A_{2q}}{A_{22}} \quad (68) \]

By assumption (26), \( A_{22} < 0 \), and it is easily shown that assumptions (27) and (28) ensure that
\[ A_{21} < 0 \quad (69) \]
\[ A_{2q} > 0 \quad (70) \]
respectively. Hence the optimal policy \( e_2^*(e_1, q) \) is decreasing in \( e_1 \) and increasing in \( q \).

As a short hand, we will use the symbol \( A \) to refer to the function \( A(e_2^*(e_1, q)|e_1, q) \),
and $A'$ will refer to the function $A(e^*_2(e_1, q)|e_1, q')$. Now,

$$\frac{d^2}{dqde_1} A(e^*_2(e_1, q)|q', e_1)$$

$$= \frac{d}{dq} \left( A'_2 \frac{de^*_2}{de_1} + A'_1 \right)$$

$$= \left( A'_{22} \frac{de^*_2}{dq} + A'_{2q} \right) \frac{de_2}{de_1} + A'_2 \frac{d^2e^*_2}{de_1 dq} + A'_{21} \frac{de^*_2}{dq} + A'_1q$$

$$= \left( A'_{22} \frac{de^*_2}{A_{22}} + 0 \right) -\frac{A_{21}}{A_{22}} + A'_2 \frac{d^2e^*_2}{de_1 dq} + A'_{21} -\frac{A_{2q}}{A_{22}} + 0$$

$$= \frac{A_{2q}}{(A_{22})^2} \left( A'_{22} - A_{2q} \right) + A'_2 \frac{d^2e^*_2}{de_1 dq}$$

The factor in the round brackets in the first term is antisymmetric under the change of variables $q \leftrightarrow q'$, and thus changes sign at $q = q'$. The second term is proportional to $A'_2$ which also changes sign at $q = q'$. Thus the whole expression changes sign at $q = q'$ provided the coefficients that multiply the factors that are switching sign are of definite (and the same) sign, and provided the two factors that switch sign have the same (and not opposite) signs.

From (64), we want it to be the case that when $q < q'$, the whole expression in (75) is positive. Consider the first term in (75) – we want this to be positive when $q < q'$. The fact $A_{2q} > 0$, so we need the factor inside the brackets to be positive for $q < q'$, i.e.

$$q < q' \Rightarrow \frac{A'_{22}}{A'_{21}} > \frac{A_{22}}{A_{21}}.$$  

A sufficient condition that ensures this is:

$$\frac{\partial}{\partial q} \left( \frac{A_{22}}{A_{21}} \right) > 0$$

$$\Rightarrow \frac{A_{22}}{A_{22}} > \frac{A_{21q}}{A_{21}}.$$  

Now consider the second term in (75). $A'_2 > 0$ when $q < q'$ (this follows from $A_{22} < 0$, and from the monotonicity of $e^*_2$ in $q$), so the second term will be positive if $\frac{d^2e^*_2}{de_1 dq}$ is positive. We now look for conditions that ensure this is the case.

Recall that the optimal policy $e^*_2(e_1, q)$ satisfies (66). Implicitly differentiating (66) with
respect to $e_1$ leads to an equation that defines $\frac{d e_2^*}{d e_1}$ in terms of $e_2^*$, $e_1$, and $q$. We then have:

\[
\frac{d^2 e_2^*}{dqde_1} = \left( \frac{\partial}{\partial e_2^*} \frac{d e_2^*}{d e_1} \right) \frac{d e_2^*}{dq} + \frac{\partial}{\partial q} \frac{d e_2^*}{d e_1}
\] (79)

Since $\frac{d e_2^*}{dq} > 0$ by assumption, the following conditions ensure that $\frac{d^2 e_2^*}{dqde_1} > 0$:

\[
\frac{\partial}{\partial e_2^*} \frac{d e_2^*}{d e_1} > 0 \quad (80)
\]

\[
\frac{\partial}{\partial q} \frac{d e_2^*}{d e_1} > 0 \quad (81)
\]

Using the fact that $\frac{d e_2^*}{d e_1} = -\frac{A_{21}}{A_{22}}$, these inequalities become,

\[
\frac{A_{221}}{A_{21}} < \frac{A_{222}}{A_{22}} \quad (82)
\]

\[
\frac{A_{21q}}{A_{21}} < \frac{A_{22q}}{A_{22}} \quad (83)
\]

The second of these inequalities is identical to (78), while the first inequality is a new condition.

For convenience, denote $W^L := W(e_2|e_1, \lambda_L), W^H := W(e_2|e_1, \lambda_H)$. Consider the inequality

\[
\frac{A_{222}}{A_{22}} > \frac{A_{21q}}{A_{21}} \quad (84)
\]

\[
\Rightarrow \frac{W^L_{22} - W^H_{22}}{q W^L_{22} + (1-q) W^H_{22}} > \frac{W^L_{21} - W^H_{21}}{q W^L_{21} + (1-q) W^H_{21}} \quad (85)
\]

\[
\Rightarrow \frac{1}{W^L_{21} W^H_{21}} \left( \frac{W^L_{22}}{W^L_{21}} - \frac{W^H_{22}}{W^H_{21}} \right) > 0, \quad (86)
\]

where the last line follows after two or three lines of algebra. For this inequality to be satisfied, it is sufficient for

\[
\frac{\partial}{\partial \lambda} \frac{W^L_{22}}{W^L_{21}} < 0 \quad (87)
\]

\[
\Rightarrow \frac{W^L_{22\lambda}}{W^L_{22}} < \frac{W^L_{21\lambda}}{W^L_{21}} \quad (88)
\]
Similarly, one can show that
\[
\frac{W_{222}}{W_{22}} > \frac{W_{221}}{W_{21}} \implies \frac{A_{222}}{A_{22}} > \frac{A_{221}}{A_{21}},
\]  
(89)

Thus, we have shown that (88) and (89) are sufficient to ensure \( \Omega(q, q') - \Omega(q', q') < 0 \) for all \( q, q' \). Simple manipulation of these two inequalities shows that they can be written in terms of the elasticities \( \epsilon_{x|y} \) in Proposition 3.

\section*{D Proof of Proposition 4}

When political competition is exogenous, each parties’ equilibrium policy is to offer its own individual optimum, so the value of the political game to party \( i \) is given by
\[
\hat{P}^i(e_1) = k_i A(e_2^*(e_1, q_i)|e_1, q_i) + (1 - k_i) A(e_2^*(e_1, q_j)|e_1, q_i),
\]  
(90)

where \( k_i \) is some constant, and \( k_i = 1 - k_j \). Thus,
\[
\frac{d}{de_1} \left( A^*(e_1, q_i) - \hat{P}^i(e_1) \right) = (1 - k_i) \frac{d}{de_1} \left( A(e_2^*(e_1, q_i)|e_1, q_i) - A(e_2^*(e_1, q_j)|e_1, q_i) \right)
\]
\[
= (1 - k_i)(\Omega(q_i, q_i) - \Omega(q_j, q_i))
\geq 0
\]  
(91)

where the inequality follows from the proof of Proposition 3. Thus the condition (18) in Lemma 2 continues to hold in this case, and we know from Proposition 3 that the conditions (30–31) on \( W \) ensure that the condition (19) in Lemma 2 also holds. Thus the result is established.

\section*{References}


