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Cross-Market Spillovers with ‘Volatility Surprise’*

Sofiane Aboura[†] and Julien Chevallier[‡]

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Abstract

This article adopts the asymmetric DCC with one exogenous variable (ADCCX) model developed by Vargas (2008), by updating the concept of ‘volatility surprise’ to capture cross-market relationships. Current methods for measuring spillovers do not focus on volatility interactions, and neglect cross-effects between the conditional variances. This paper aims to fill this gap. The dataset includes four aggregate indices representing equities, bonds, foreign exchange rates and commodities from 1983 to 2013. The results provide strong evidence of spillover effects coming from the ‘volatility surprise’ component across markets. Against the background of the recent financial crisis, the aim is to contribute to the literature on the interdependencies of financial markets, both in conditional means and (co)variances. In addition, asset management implications are derived.

Keywords: Cross-market relationships, Volatility surprise, Volatility spillover, ADCCX, Asset management.

JEL Codes: C32, C4, G15.

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[†]DRM Finance, Université Paris Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, France. Email: sofiane.aboura@dauphine.fr

[‡]*Corresponding Author.* IPAG Business School (IPAG Lab), 184 Boulevard Saint-Germain, 75006 Paris, France. Tel: +33 (0)1 49 40 73 86. Fax: +33 (0)1 49 40 72 55. Email: julien.chevallier04@univ-paris8.fr

1 Introduction

The study of volatility interaction is of interest to both academics and practitioners. Changes in variance are said to reflect the arrival of information, and the extent to which the market evaluates and assimilates new information.¹ The transmission pattern in variance provides an insight concerning the characteristics and dynamics of economic and financial prices, and such information can be used to construct better econometric models describing the temporal dynamics of the time series.

A rising research interest is directed towards the topic of international transmission mechanisms – attributable to the ever-increasing degree of interdependence among world financial markets – which seem to become more pronounced during financial crises. Regarding returns transmission, the study of *returns* co-movements begins with the investigation of the benefits from international diversification at various frequencies (Schwert (1989), Susmel and Engle (1994), Andersen and Bollerslev (1997)). Returns, volatility and correlation changes are closely related in financial models (Orlowski (2012), Bekiros (2013)).

Regarding volatility transmission, Ross (1989) shows that it is the *volatility* of an asset price, not the asset's price change, that is related to the rate of information flow to the market. This empirical justifies the study of international volatility transmission, in addition to returns contagion. Schwert, French and Stambaugh (1987) and Campbell and Hentschel (1992) introduce the notion of the volatility feedback effect: volatility is typically higher after a stock market decrease than after it increases, which explains the negative correlation between stock returns and future volatility. In the same field, various studies examine the volatility spillover effects with univariate and multivariate GARCH models (Lin, Engle and Ito (1994)). These models typically provide practical applications for optimal portfolio selection or option pricing (Al Janabi (2012), Konermann, Meinerding and Sedova (2013)).

Let us now discuss the concept of volatility surprise. In finance, the attention is usually focused on the *predictable variance*, such as the conditional variance or the implied variance. However, according to Engle (1993), it is the difference that cannot be forecast between the squared residuals

and the conditional variance that is worthy of interest. Such a quantity has been coined a ‘volatility surprise’. Hamao, Masulis and Ng (1990) were the first to interpret this quantity as a volatility surprise since it lags behind the conditional variance. This new concept paved the way for numerous studies (Kim and Rogers (1995), Chan-Lau and Ivaschenko (2003)).

Since the financial crisis of 2008, the topic of international volatility transmission across markets has once more attracted a considerable attention. Many researchers have concentrated on ways of measuring systematic risk, and spillovers have become a central issue. Some studies analyse the extent of cross-market linkages over different asset classes: stocks and bonds (Straetmans and Candelon (2013)), stocks and FX (Wang, Wu and Lai (2013)), stocks-bonds-oil-gold and real estate markets (Chan, Treepongkaruna, Brooks and Gray (2011)), metals and energy (Chng (2009)), gold and stocks (Hood and Malik (2013)), energy-food and gold (Mensi, Beljid, Boubaker and Managi (2013)). Whereas the econometric methodologies sometimes differ from one study to another (e.g. DCC models or copulas), the global conclusion gears towards the frequent identification of cross-market links in recent empirical studies. Previous studies have mostly examined the spillovers in multivariate GARCH-type models (Engle, Ito and Lin (1990), Hassan and Malik (2007), Cai, Howorka and Wongswan (2008)), or with the BEKK VECM-GARCH model (Kavussanos et al. (2014)).

In this paper, we contribute to the literature by proposing an alternative for modelling cross-market relations with multivariate volatility processes, on the basis of the asymmetric dynamic conditional correlation model with one exogenous variable (ADCCX) newly defined by Vargas (2008). This model represents a parsimonious specification for measuring cross-market relations. It is flexible in the sense that each market’s shock may be fitted separately as a spillover on any combination of bivariate volatility models. Computationally, Vargas (2008) has established the consistency of the estimates in presence of high-dimensional optimization problems.

In contrast with previous works, this paper focuses on *volatility* interactions between equities, bonds, foreign exchange rates and commodities, as further evidence is emerging for volatility to be autocorrelated within its own market and also to be cross-correlated with volatility in other asset markets. As an extension to the work of Hamao et al. (1990), its key contribution is to document

the spillover effects coming from each market's 'volatility surprise' component to the remaining pairs of covariance volatilities.² The two-step econometric methodology consists of (1) computing the mean-zero 'volatility surprise' component from univariate GARCH models, and (2) plugging it into the ADCCX model. As sensitivity tests, the performance of this technique is also examined during sub-periods.

To summarize our results, this paper aims to empirically model and measure volatility spillovers between four segments of the US financial markets: stocks, bonds, commodities, and foreign exchange rates. The selected model is that of Vargas (2008), who presents an asymmetric dynamic conditional correlation model with exogenous variables in the covariance matrix's movement equation (ADCCX model). The main contribution of the paper consists in defining and calculating 'volatility surprises' for each market, and in asking whether a volatility surprise in one market affects the volatility of other markets (evaluated for each pair of assets). The paper finds evidence of volatility spillovers with, apparently, the stock markets being identified as the main source of volatility spillovers. By examining time-varying correlations, we are able to identify rising interdependencies between financial and commodity markets – pointing to the 'financialization of commodity markets' phenomenon (Tang and Xiong (2012)) – that are especially visible since 2008. This conclusion holds true for both volatility and return shocks.

The volatility risk transmission channel can well explain the theoretical underpinnings behind spillovers, whereby asset markets are inter-related through their dynamic conditional correlation structure. Our analysis greatly enhances the understanding of volatility cross-market dynamics, both in turbulent and calm times. Besides, we attempt to build implications for asset managers. Finally, one central methodological contribution is brought to the attention of practitioners, related to the use of the 'volatility surprise' component (alongside other traditional measures of volatility) to apprehend fully the sensitivity of financial markets to volatility shocks.

The remainder of this paper is organized as follows. Section 2 contains a detailed description of the new methodology proposed. Section 3 outlines the data set. Section 4 contains the illustration with empirical results, along with a sensitivity analysis. Section 5 reflects on the implications in terms of asset management. Section 6 concludes.

2 The model

Vivid research areas in financial econometrics have attempted to model the time-varying volatility of financial returns. Indeed, capturing the time-varying correlations between different securities appears necessary for portfolio optimization, asset pricing and risk management. In this section, we outline the building blocks of this quest for modelling multivariate processes. The representation of the conditional covariance matrices adopted belongs to the DCC family.

2.1 The DCC family models

Multivariate GARCH (henceforth, MVGARCH) models are useful developments regarding the parameterization of conditional dependence. Different classes of MVGARCH models have been proposed in the literature³. The first-generation models were introduced by Bollerslev, Engle, and Wooldridge (1988), as well as Engle and Kroner (1995). The numerical difficulties encountered with these models are linked to the large number of parameters to be estimated. Overparameterization will lead to a flat likelihood function, making statistical inference intrinsically difficult and computationally troublesome.

To overcome these difficulties, Bollerslev (1990) has proposed a new class of MVGARCH model in which the conditional correlations are constant (CCC). Even with such a simple specification, the estimation typically involves solving a high-dimensional optimization problem as, for example, the Gaussian likelihood function cannot be factorized into several lower dimensional functions.

The CCC assumption is relaxed by Engle (2002) and Tse and Tsui (2002), who generalize Bollerslev's (1990) model by making the conditional correlation matrix time-dependent. The dynamic conditional correlation (DCC) model constrains the time-varying conditional correlation matrix to be positive definite, and the number of parameters to grow linearly by following a two-step procedure. The first step fits each conditional variance with an univariate GARCH(1,1) model. The second step allows the computation of the dynamic conditional correlations given the conditional volatility estimated in the first step. The log-likelihood is therefore written as a sum of a volatility part and a correlation part. This two-step estimation procedure provides adequate fitting when the

bivariate systems exhibit different dynamic correlation structures, and minimizes the biases that are inevitable in such an estimation strategy for the conditional correlation.

Cappiello, Engle and Sheppard (2006) extend the DCC model to account for asymmetries in the correlation dynamics. Their asymmetric DCC (ADCC) model fits the leverage effects observed in equity markets better. Vargas (2008) introduces the ADCCX model, which allows for spillover effects. Therefore, it becomes possible to test for spillover effects in the correlation dynamics.⁴ Compared with the DCC method, the ADCCX approach involves modeling of inter-series dynamics. This paper makes use of the ADCCX model in order to gauge the spillover effects coming from the ‘volatility surprise’ component across equities, bonds, foreign exchange rates and commodities.

Let us recall how to obtain first the volatility surprise component and second, the ADCCX model that nests the ADCC model as a generalization of the DCC model.

2.2 The volatility surprise

Following Engle (1993), we define the ‘volatility surprise’ as the volatility component that cannot be forecast. Consider the mean equation of a standard GARCH (1,1) specification:

$$r_t = \mu + \epsilon_t \tag{1}$$

with r_t the asset price returns, μ the unconditional mean, and $\epsilon_t = z_t\sigma_t$ the innovations. z_t denotes a strong white noise, with $E(z_t) = 0$ and $E(z_t^2) = 1$. σ_t denotes the conditional variance: $V(r_t|r_{t-1}) = \sigma_t^2$, with r_{t-1} the historical information available at time $t - 1$. The purpose of the time-varying conditional σ_t is to capture as much of the conditional variance in the residual ϵ_t as follows:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

More specifically, Engle (1993) defines the so-called ‘volatility surprise’, ς , as the difference between the squared residuals ϵ_t^2 and the conditional variance σ_t^2 . For scaling purposes, we normalize this quantity by the conditional variance σ_t^2 . The ‘normalized volatility surprise’, $\tilde{\varsigma}$, is given by:

$$\tilde{\zeta}_t = \frac{(\epsilon_t^2 - \sigma_t^2)}{\sigma_t^2} \quad (3)$$

The volatility surprise component for each univariate time series can be computed accordingly. It is easy to see that, once we have specified $\tilde{\zeta}_t$, we obtain $E(\tilde{\zeta}_t|r_{t-1}) = \frac{E(\epsilon_t^2|r_{t-1}) - \sigma_t^2}{\sigma_t^2} = 0$, since by construction $E(\epsilon_t^2|r_{t-1}) = \sigma_t^2$. Hence, we are able to verify that – as a pre-requisite for an input to any MVGARCH model – the ‘volatility surprise’ computed in eq.(3) is indeed mean-zero.

2.3 The ADCCX model

Let (r_t) and $(\tilde{\zeta})$ represent, respectively, the price returns and the volatility surprise component. The returns are computed as the logarithmic first difference of the asset price, i.e. $r_t = \log(P_t/P_{t-1})$. Let x_t be a $n \times 1$ vector of spillover variables, ξ be a $n \times 1$ vector of parameters, and I be an $n \times n$ identity matrix. The variables (x_1, x_2, \dots, x_n) correspond to the cross-market spillover effects coming from the volatility surprises computed in eq.(3).

Let us consider the vectorial process χ_t as representing the $n \times 1$ vector of unpredictable observations $(\epsilon_t$ or $\tilde{\zeta}_t)$ at time t , which is assumed to be conditionally normal with mean zero and covariance $n \times n$ matrix H_t :

$$\chi_t|\Omega_{t-1} \sim N(0, H_t) \quad (4)$$

with Ω_{t-1} the information set at time $t-1$. The conditional covariance matrix H_t can be decomposed as follows:

$$H_t = D_t R_t D_t \quad (5)$$

with R_t the $n \times n$ time-varying correlation matrix. $D_t = \text{diag}(\sqrt{h_{1,t}}, \dots, \sqrt{h_{i,t}}, \dots, \sqrt{h_{n,t}})$ is the $n \times n$ diagonal matrix of time-varying standard deviations extracted from univariate GARCH models with $\sqrt{h_{i,t}} = \sigma_{i,t}$ on the i^{th} diagonal.

The dynamic conditional correlation structure in matrix form is given by:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \quad (6)$$

An element of R_t has the following form:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} \quad (7)$$

$Q_t^* = \text{diag}(\sqrt{q_{ii,t}})$ is a diagonal matrix composed of the square root of the diagonal elements of the covariance matrix Q_t . The covariance matrix Q_t of the ADCCX model evolves according to:

$$Q_t = \left(\bar{Q} - A' \bar{Q} A - B' \bar{Q} B - \Gamma' \bar{N} \Gamma - I \xi' \bar{x} \right) + A' \left(e_{t-1} e'_{t-1} \right) A + B' \left(Q_{t-1} \right) B + \Gamma' \left(\eta_{t-1} \eta'_{t-1} \right) \Gamma + I \xi' x_{t-1} \quad (8)$$

where the unconditional covariance matrix \bar{Q} is composed of the $n \times n$ vector of standardized residuals $e_{i,t} = \frac{X_{i,t}}{\sqrt{h_{i,t}}}$ computed from the first stage procedure for which $e_{i,t} \leftrightarrow N(0, R_t)$. $\eta_t = 1_{[e_t < 0]} \circ e_t$ is a $n \times 1$ dummy variable equals to unity when standardized residuals $e_t < 0$ and zero otherwise. The asymmetry caused by η_t happens for any i^{th} element $e_{i,t} < 0$. A , B and Γ are $n \times n$ diagonal matrices where $A = \text{diag}(\sqrt{a})$, $B = \text{diag}(\sqrt{b})$ and $\Gamma = \text{diag}(\sqrt{\gamma})$. The model implies that all correlations are equally influenced by any exogenous variable. ξ is the $n \times 1$ vector of parameters capturing the influence of the x_t spillover variables. I is set as an identity matrix. Capiello, Engle and Sheppard (2006) show that to ensure positive definiteness of the covariance matrix Q_t , a sufficient condition is $a^2 + b^2 + \delta\gamma^2 < 1$ where δ is the maximum eigenvalue of $(\bar{Q}^{-1/2})(\bar{N})(\bar{Q}^{-1/2})$. a , b and γ are scalar parameters. Furthermore, $\bar{Q} = T^{-1} \sum_{t=1}^T e_t e'_t$, $\bar{N} = T^{-1} \sum_{t=1}^T \eta_t \eta'_t$ and $\bar{x} = T^{-1} \sum_{t=1}^T x_t$.

Vargas (2008) extends the scalar ADCC model of Capiello et al. (2006) by allowing exogenous variables to drive correlations. The model implies that all correlations are equally influenced by any

exogenous variable. The scalar version of the ADCCX model can be written as:

$$Q_t = \left(\bar{Q} - a^2 \bar{Q} - b^2 \bar{Q} - \gamma^2 \bar{N} - I \xi' \bar{x} \right) + a^2 \left(e_{t-1} e'_{t-1} \right) + b^2 \left(Q_{t-1} \right) + \gamma^2 \left(\eta_{t-1} \eta'_{t-1} \right) + I \xi' x_{t-1} \quad (9)$$

The ADCCX model reduces to the ADCC version when $\xi = 0$, and to the DCC version when $\gamma = 0$ and $\xi = 0$. In other words, models are nested in the sense that the DCC and ADCC may be viewed as special cases of the more general ADCCX. To ensure the positive definiteness of the conditional correlation matrices, Vargas (2008) bounds the vector that measures the impact of the exogenous variables x_t between 0 and 1. We view the ADCCX as a model that is capable of catching sophisticated dynamical correlation structures, but this poses limitation on the dimensionality of the spillover variable(s) with the available computing capacity. That is why we restrict our analysis by including only one exogenous variable at a time.

The resulting constrained maximum likelihood function is maximized with respect to two sets of parameters in succeeding steps: first variances, second unconditional correlations and other terms in the intercepts of Q_t , third conditional correlation parameters.⁵ The minimization problem is typically solved by iterative algorithms.

3 Descriptive statistics

We apply the ADCCX method to investigate the cross-market relations based on real examples from financial and commodity markets. Our analysis is performed on the long data span of 7,828 daily observations covering the period from 25 January 1983 to 25 January 2013. Four markets are represented by aggregate indices retrieved from Thomson Financial Datastream:

1. *GSCI* is the S&P Goldman Sachs Commodity Spot Price Index (GSCI).
2. *SP500* is the S&P 500 Price Index.
3. *BMUS10Y* is the US Benchmark 10-Year DS Government Index - Clean Price Index.
4. *UKDOLLR* is the UK GBP to USD (WMR) Exchange Rate.

Insert Figure 1

Insert Figure 2

Raw series can be observed in Figure 1. They are characterized by various shocks impacting financial, foreign exchange and commodity markets over the period. Volatility surprise series, computed from eq.(3), are pictured in Figure 2. The ADCCX framework takes as an input these volatility series during the estimation process. What is striking for the financial economist is that we are using quantities that are very different from returns visually.

Insert Table 1

Table 1 provides the descriptive statistics of the four time series used in the paper, under the form of both returns and volatility surprises. In particular, the interested reader can verify (by means of one-sided t -tests) that the volatility surprise components calculated from eq.(3) are mean-zero, as they will be re-used as an input to the ADCCX model.

Besides, we observe that the unconditional cross-correlations are typically very low (0.2 or below) between our variables of interest, which will not undermine our econometric work. We have also applied Chow break tests to the time series under consideration, which indicated the need to resort to sub-sampling estimates.

To evaluate how the markets are inter-related, we run preliminary data analyses by means of OLS regressions. The gist of the results can be seen at the bottom of Table 1. First, in most cases, the intercept is not significant. Second, UKDOLLR returns receive a negative impact from the three other variables, which are all significant. Indeed, the S&P500, bonds and commodity aggregates are accounted for in the construction of the UKDOLLR parity expectation. Third, commodities can be explained only by the S&P500, either for returns or for the volatility surprise component. Finally, the S&P500 lagged volatility surprise is the only variable that influences all three other market volatility surprises. The bottom line is that whatever the specification, no variable seems always to have a statistically significant impact on the other markets. For that reason, we continue to explore the covariance structure of these four aggregate indices with volatility surprise spillover effects in the ADCCX framework.

Insert Table 2

Table 2 lists the univariate GARCH, EGARCH and TGARCH models estimated to fit σ_t^2 for each time series (as the preliminary step before computing the volatility surprise component). The identification of the univariate volatility process that provides the best goodness-of-fit to each time series calls for a broader search than the initial GARCH(p,q) model. That is why we also consider thresholds and leverage effects. Asymmetric effects are present in equities (as documented by Cappiello et al. (2006)) and FX. In contrast, standard GARCH models are preferred (according to the AIC criterion) for sovereign bonds and commodities. To account for the conditional mean of the return series, a vector AR(1) model, which was selected by the AIC, was first fitted to the data. In the remainder of the paper, we shall similarly model the conditional covariance matrix process with DCC(1,1) terms.

4 Empirical findings

In this section, we present the main empirical results obtained by plugging the volatility surprise component into the ADCCX model to evaluate cross-market volatility spillovers. The advantages of this new approach include (i) flexibility in modelling each market spillover effect separately on bivariate volatility models, and (ii) allowing the volatility model for one pair of assets to depend on the (lagged) value of the market at the origin of the spillover effect. We check the accuracy of the estimation by running additional sub-sample tests.

4.1 Volatility surprise effect on pairwise volatility

Let us consider the volatility surprise introduced as a spillover variable⁶ x_{t-1} in the ADCCX model, and examine its impact on the covariance structure of two other volatility surprises. To obtain parsimonious specifications, we select only one exogenous variable at a time. By doing so, we obtain the spillover effects coming from the ‘volatility surprise’ of one time series $\{\tilde{\zeta}(GSCI(-1), SP500(-1), BMUS10Y(-1), UKDOLLR(-1))\}$ on the covariance structure of two other volatility surprises $\{\tilde{\zeta}(GSCI), \tilde{\zeta}(SP500), \tilde{\zeta}(BMUS10Y), \tilde{\zeta}(UKDOLLR)\}$. In other words, we only have $\tilde{\zeta}$ variables⁷ entering the ADCCX model given by eq.(9). We report below the specification

adopted:

$$Q_t = \left(\bar{Q} - a^2 \bar{Q} - b^2 \bar{Q} - \gamma^2 \bar{N} - K \xi' \bar{x} \right) + a^2 \left(e_{t-1} e'_{t-1} \right) + b^2 (Q_{t-1}) + \gamma^2 \left(\eta_{t-1} \eta'_{t-1} \right) + K \xi' x_{t-1} \quad (10)$$

By identifying which market is at the origin of the shock, it will give us an understanding about the other markets' responses. Table 3 presents quasi maximum-likelihood estimates and diagnostic statistics of the twelve possibilities to explore. Standard errors based on the normality assumption are given in parentheses below parameter estimates.

Insert Table 3

Let us now detail the volatility spillover effects at stake between the financial, foreign exchange, and commodity markets under analysis.

The pairwise 'volatility surprises' of GSCI and S&P500 ($\zeta(GSCI) + \zeta(SP500)$) is symmetric (as the γ coefficient is not statistically significant), which implies that the covariance function between volatilities reacts similarly after a positive or a negative volatility variation. Interestingly, both one-day lagged volatility surprises coming from the US 10-Year Sovereign Bond ($\zeta(BMUS10Y(-1))$) and the UK/Dollar parity ($\zeta(UKDOLLR(-1))$) appear positive and significant (at the 1% level) when introduced (separately) as exogenous variables. This means that there is a clear volatility transmission channel from the bonds / foreign exchange markets to the joint volatility of equities and commodities. In addition, the conditional correlation of the GSCI and S&P500 volatility surprises are highly persistent, as shown by the sum of the significant parameter estimates of the ADCC model.

When looking at the pairwise 'volatility surprises' of the GSCI and the US 10-Year Sovereign Bond ($\zeta(GSCI) + \zeta(BMUS10Y)$), we remark that the covariance function is symmetric, and that the lagged volatility spillovers coming from either the S&P500 or the UK/Dollar parity are significant (at 1%) and positive. Hence, we may consider that strong volatility spillovers stem from the UKDOLLR and SP500 (when set as exogenous variables in the ADCCX model) on the pairwise volatility of bonds and commodities.

The third combination of GSCI and UK/Dollar ‘volatility surprises’ ($\tilde{\zeta}(GSCI) + \tilde{\zeta}(UKDOLLR)$) yields to the same line of comments. The covariance structure is symmetric. The added explanatory variable is positive and significant, when considering separately as spillover the S&P500 and the US 10-Year Sovereign Bond.

Globally, we have been able to identify strong volatility spillovers channeling through commodity markets (when paired with each one of the three other aggregate indices). These spillover effects inform us that the volatility of commodity markets is very sensitive to that of the equity, bond and foreign exchange markets. Such volatility spillovers also reflect that the risks attached to traditional financial markets can easily be transmitted to commodity markets.

The linkages between these traditional asset markets are also interesting to comment upon. The joint ‘volatility surprises’ of equities and bonds ($\tilde{\zeta}(SP500) + \tilde{\zeta}(BMUS10Y)$) exhibit an asymmetric covariance function (as the γ coefficient is statistically significant at the 5% level), which implies that the correlation function increases more after negative returns than after positive ones. Moreover, the equity/bond covariance is positively affected by lagged ‘volatility surprises’ spillover effects coming from commodities and the UK/Dollar parity (at the 1% level).

The same comments apply for the pair of $\tilde{\zeta}(SP500) + \tilde{\zeta}(UKDOLLR)$ when setting as exogenous variables either commodities or bonds.⁸ Across our models, we may conclude that the ‘volatility surprise’ component of the S&P500 is highly correlated to that of the UK/Dollar parity.

Last but not least, we examine the case where the pair $\tilde{\zeta}(BMUS10Y) + \tilde{\zeta}(UKDOLLR)$ is impacted by the ‘volatility surprises’ coming from either GSCI or SP500. While the covariance function is found to be symmetric, both lagged volatility spillovers are significant and positive at the 1% level.

Overall, we uncover as the most striking feature that all the volatility surprises introduced (separately) as spillover variables into the combinations of pairwise volatilities are statistically significant (at the 1% level), and positive. The spillover coming from the lagged S&P500 volatility surprise is the most influential, as the coefficient estimates of $\tilde{\zeta}(SP500)$ have the highest magnitude. Therefore, Table 3 shows clear evidence of cross-market volatility spillovers between the equity, bond, foreign exchange and commodity markets during our sample period (1983-2013). In terms of interpretation,

we may conclude that higher risk levels are transmitted across markets through higher volatilities. Conversely, in presence of lagged spillover volatility effects, market risks are more correlated.

In light of these strong volatility surprise spillovers, commodities do not appear as a segmented market, since they affect and are affected by the three other financial markets. There exists a sparse literature concerning the statistical patterns observed between commodities and standard assets.⁹ Kat and Oomen (2007a,b), Gorton and Rouwenhorst (2005) and Erb and Campbell (2006) have previously established that commodities exhibit a long-term correlation with traditional asset classes. These inter-relationships in volatility surprise series between commodities and financial markets might be interpreted as a result of the growing integration among various asset classes (e.g. phenomenon of ‘financialization’ of commodity markets (Tang and Xiong (2012))).

Our MVGARCH tests encompass the usual checks, such as testing the null of no ARCH in standardized errors (LM-test), misspecification tests (functional form, symmetry and parameter constancy, see Lundbergh and Terasvirta (2002)). Following the lead of Reinsel (1997), we employ a multivariate portmanteau statistic to test for the autocorrelation in the vectorized cross-product of residuals. What concerns parameter stability tests, we have conducted LR-type tests on our models with spillovers (ADCCX) against models without spillovers (ADCC). We are able to verify that the effects coming from the spillover variables significantly modify the likelihood. Hence, the twelve ADCCX models carry additional explanatory power regarding the cross-market relationships at stake, that would be missed without plugging in the exogenous variables. Overall, diagnostic statistics indicate that the ADCCX models provide a good fit for this particular data set.¹⁰

Insert Figure 3

Figure 3 displays the time plots of the estimated conditional variance processes for each of our twelve models. It is evident from these plots that the volatility surprise stemming from one market strikes back on the conditional correlation function of two other markets’ volatility surprises. The overall interpretation behind positive trends for all assets is that risks are spreading from one market (e.g., equities, fixed income, FOREX, commodities) to another. Thanks to the spillover variable that is set in each model, we can identify from where the shock is coming, and how it impacts

dynamically the time-varying conditional correlation for pairwise assets. Hence, we are able to show the superiority of the ADCCX model in uncovering market interdependencies, which have come under the scrutiny of financial economists with a greater intensity since the 2008 financial crisis. Moreover, these graphs confirm the time-varying nature of the correlation function, and hence the need to resort to DCC-type models. The most striking feature of the figures is the change in correlation during the simultaneous recession and oil price shock of 1991-1992, the subsequent two recessions in 2001 and 2007-2009 (the latter overlapping the oil shocks of 2005-2008). Note that correlations have become less negative since the onset of the European debt crisis in 2010.

4.2 Sub-periods decomposition

Insert Table 4

Given the rather long data sample (thirty years), structural breaks in the relation across series must be evaluated. To do so, we rely on Chow break tests (Chow (1960)) to determine the sub-periods. According to Table 4, the following break dates are identified:

- 16 January 1995, i.e. with fears that the soaring U.S. trade deficit might destabilize the world economy and rising geopolitical tensions (economic sanctions against Iran).
- 13 November 2000, i.e. in the aftermath of the dot-com bubble burst.
- 06 March 2009, i.e. in the midst of the 2008 sub-primes crisis (and after the crash and rally of commodities due to the July 2008 oil price swing).

Therefore, these results suggest a decomposition of the 1983-2013 full sample into four sub-periods:

- Sub-period #1: 25 January 1983 to 16 January 1995 (3,124 observations)
- Sub-period #2: 17 January 1995 to 13 November 2000 (1,520 observations)
- Sub-period #3: 14 November 2000 to 06 March 2009 (2,169 observations)

- Sub-period #4: 07 March 2009 to 25 January 2013 (1,014 observations)

In Figure 1, structural break dates are represented by vertical dotted lines. Next, we move to the estimation of our ADCCX model composed of volatility surprise series during sub-periods.

Insert Table 5

Insert Table 6

Insert Table 7

Insert Table 8

Results are given in Tables 5 to 8. The main purpose of these additional Tables is to test the stability of the parameter estimates obtained during the full period, and thus they may be seen as robustness checks of the main results. We find significant cross-effects across the volatility models, with asymmetric volatility response to positive and negative shocks, and high levels of time-varying correlations. By and large, sub-period results tend to confirm the remarkable stability of our findings with respect to cross market volatility spillovers. During the sub-period #1 (1983-1995), parameter estimates for exogenous spillover variables are found to be statistically significant for each of the twelve pairs of assets considered (Table 5). The stability of the results is confirmed during the sub-period #3 going from 2000 to 2009 (Table 7), as well as during the sub-period #4 covering the recession period after the 2008 financial crisis and the 2011 European debt crisis (Table 8). Only during the sub-period #2 (1995-2000), we are able to identify seven statistically significant parameters for the exogenous spillover variables (out of twelve). Vanishing ‘volatility surprise’ impacts stem from bonds, FX and commodities. However, the S&P 500 impacts are always statistically significant. We may infer from this last set of comments that the S&P 500 stands out as the most influential market. Besides, the sub-period decomposition allows us to conclude that the cross-market spillover effects are very satisfactory when considering volatility surprise spillovers on pairs of assets in the ADCCX framework.¹¹

5 Asset management implications

Time-varying covariances are crucial inputs for many tasks of financial, portfolio and risk management. In particular, they are worthy of interest not only from an empirical perspective, but also from a practitioner’s point of view.

First, the most striking result concerns Table 3 in which all the volatility surprise spillover parameters are highly significant. Therefore, we uncover that the transmission channels across asset markets are related to the volatility surprise phenomenon, which means that a volatility shock is more likely to be transmitted from one market to another than a return shock. In addition, the most influential variable is the equity market (as judged by the size of the parameter and/or the statistical significance).

From an asset manager’s standpoint, these findings imply that the cross-market unpredicted risks are cumulative. To build a hedging strategy against such risks, investors need to take volatility shocks into account. Variance swaps, computed on unpredicted components of implied volatility and historical volatility, can be seen as an ideal tool in this context, in order to buy and sell the volatility stemming from each market (Carr and Wu (2009)). This constitutes the key asset management implication to take from our paper.¹²

Second, a shared belief in the asset management industry is that including commodities brings diversification to a portfolio. However, this study shows that commodities might be considered as an integrated market as they contribute to global risk. This result contradicts the view that commodities form an alternative asset class for investments (i.e. a segmented market), as suggested by the bulk of the literature (see Buyukshahin, Haigh and Robe (2010)). Our results are more in line with Daskalaki and Skiadopoulos (2011), who challenged the alleged diversification benefits of commodities by resorting to mean-variance spanning tests.

Finally, practitioners will also find the following methodological contribution of the paper to be of interest: we update the concept of ‘volatility surprise’ that matches considerably the needs of any hedging strategy implemented by asset managers. Indeed, what hurts asset managers more is the unpredicted volatility. By construction, the ‘volatility surprise’ component addresses precisely this

issue since it represents the difference between the squared residuals (representing the unpredicted squared returns) and the conditional variance (representing the observable risk). Besides standard volatility series (e.g. GARCH volatility, implied volatility, realized volatility, variance risk premium), we would advise market practitioners also to include the series of volatility surprises in their standard econometric toolbox.

By doing so, their portfolio assets will be better protected against unfavorable volatility shocks. Why? Simply because the volatility surprise represents the unexpected volatility component that is typically ignored, whereas common practice focuses on the predictable variance (e.g. GARCH). With reference to Ben Bernanke's view on the Fed monetary policy, GARCH conditional variances correspond to the 'known unknown'. It is important to keep in mind such a measure, of course. However, according to Engle (1993), the volatility surprise is the non predictable variance from which the unexpected risks arises, and that can adversely affect the P&L of the asset manager. This unexpected risk could be called the 'unknown unknown'.

6 Conclusional remarks

Analysis of conditional covariances is indisputably one of the most important topics in multivariate financial time series modelling. Our basic idea consists in investigating volatility interactions stemming from the 'volatility surprise' component (Hamao et al. (1990)) with cross effects and time-varying correlations in financial and commodity markets. The objective consists in studying the transmission channels of a long forgotten risk measure – e.g. the volatility surprise – that is invisible but hurts the asset manager.

Indeed, there has been recently increasing interest in the transmission of variance across various financial asset price movements. In our setting, volatility spillovers are quantified by resorting to Vargas's (2008) asymmetric DCC model with one exogenous variable (ADCCX). The model entails fitting a univariate model that incorporates changing variances to each time series, and fitting the resulting volatility surprise series in a multivariate GARCH model with an additional spillover. The ADCCX model is both original and flexible in the sense that any variable can be set as the

spillover, which leads to uncovering new results across financial and commodity markets through pairwise conditional correlations.

Most of the literature uses classical pairwise DCC models. We take the analysis one step further, since we investigate how one spillover variable might impact pairs of assets. To this end, the ADCCX model brings forth a novel analysis. The appealing features of this approach make it very useful in practice, by providing information on the interaction between time series data and also by helping construct a more complex multivariate model. Following a two-step procedure, we separate the estimation task first by fitting each variance with an appropriate univariate volatility model (such as GARCH), and second by plugging the resulting volatility surprise quantities into the ADCCX equation.

The dataset includes four aggregates indices representing equities, bonds, foreign exchange rates and commodities from 1983 to 2013. Setting the ‘volatility surprise’ component as a spillover variable yields strong volatility transmission effects in the ADCCX framework. Depending on model specifications, volatility spillovers are shown to exist within financial markets and across commodities. This observation motivates us to conclude in favor of the ‘financialization of commodities’ phenomenon, recently highlighted by Tang and Xiong (2012). We also check the reliability of these results by running sub-period estimates. Besides, we have applied several diagnostic checking statistics to assess the various fitted models.

The further interdependence between asset prices therefore concerns the links between their dynamic conditional correlations. It seems reasonable for volatility to vary across time and space, so that volatility in one asset market moves contemporaneously or leads the volatility of other assets. These effects are most of the time due to the processing and transmission of economic news (e.g. 1987 Black Monday, 1997 Asia Crisis, 2000 Dot.com, 2008 Financial Crisis, 2011 European Debt Crisis) and/or changes in the patterns of the trading volumes.

Although spillovers are frequently studied, little is known about their dynamics. The spillovers of volatility surprises (on pairwise volatilities) can be interpreted as a permanent transmission channel of risk. It is shown indeed that the traditional asset markets (equities, bonds, FX) as well as commodities are mutually sensitive to each other’s volatilities across our model estimates. For

instance, the transmission of risk affects the covariance structure of equities and bonds through the commodity variable. Overall, the econometric methodology based on the ADCCX model provides useful information on the temporal dynamics and the interactions at stake between commodity and financial markets.

The picture would not be complete without deriving some asset management implications, based on the full span of derivatives products. The prevention of unwanted volatility shocks can be achieved by resorting to variance swaps, or by purchasing options. Hedging equity return spillovers can be performed by relying on equity futures. Last but not least, we bring to the fore the need for market practitioners and fund managers to reconsider the use of the ‘volatility surprise’ component – along with more traditional measures of volatility – to capture fully the cumulative risks at stake on financial markets.

Notes

¹Ross (1989) shows that, in a no-arbitrage economy, the variance of price changes is directly related to the rate of information flow to the market. Engle, Ito and Lin (1990) attribute movements in variance to the time required by market participants in processing new information.

²Alternative econometric methodologies, to cite few, include exploiting high-frequency data for improved (realized) covariance matrix measurement (Andersen, Bollerslev, Christoffersen and Diebold (2007)), and the Multivariate Realized GARCH model (Hansen, Lunde and Voev (2014)). The extension of these models to allowing one exogenous spillover variable between pairs of assets is left for future research.

³One of the most general multivariate generalized auto-regressive conditional heteroskedasticity GARCH(p,q) models is the BEKK representation (Engle and Kroner (1995)). Although the form of this model is quite general, it suffers from overparameterization. Hence, we do not detail further BEKK-type models. For a survey, see Bauwens, Laurent and Rombouts (2006).

⁴Sheppard (2008) proposes an alternative approach, whereby the conditional covariance matrix is a linear function of one or more exogenous variables, by using spectral decomposition. However, in this paper, we wish to employ Vargas's (2008) model.

⁵The interested reader may refer to Vargas (2008) for further discussion regarding the applicability of the asymptotic normal distributions in the analysis, and the conditions for the stationarity of the covariance process in eq.(9).

⁶Introduced with a lag to judge its predictive power.

⁷Recall that the notation ζ stands for the volatility surprise component.

⁸Note however that the covariance symmetric function is symmetric.

⁹Tang and Xiong (2012) argue that the increase of investments in commodities via commodity indexes (financialization of commodities) tends to integrate the equity with the commodity markets (see also Singleton, 2014). Hong and Yogo (2012) find that there are common variables which predict commodity futures and equity returns.

¹⁰To conserve space, these results are not reproduced in the paper, and can be accessed upon request.

¹¹Dynamic conditional correlations recorded during each sub-period are not reproduced to conserve space, and can be accessed upon request to the authors.

¹²Moreover, we remark that the bond and currency volatilities contribute the most to explaining equity returns. Thus, another asset management implication would consist of hedging risks in priority against bond and currency volatility spillovers. Equity funds can implement this strategy by purchasing call options on bonds and currencies. Interestingly, the most influential market in terms of return spillovers on pairwise returns is the S&P 500 variable, contrary to the US 10-year Sovereign Bond which is the least influential. To address this issue, one last piece of advice for asset managers would be to construct a hedging strategy relying on futures contracts on the S&P500 index futures.

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7 Appendix

7.1 Figures

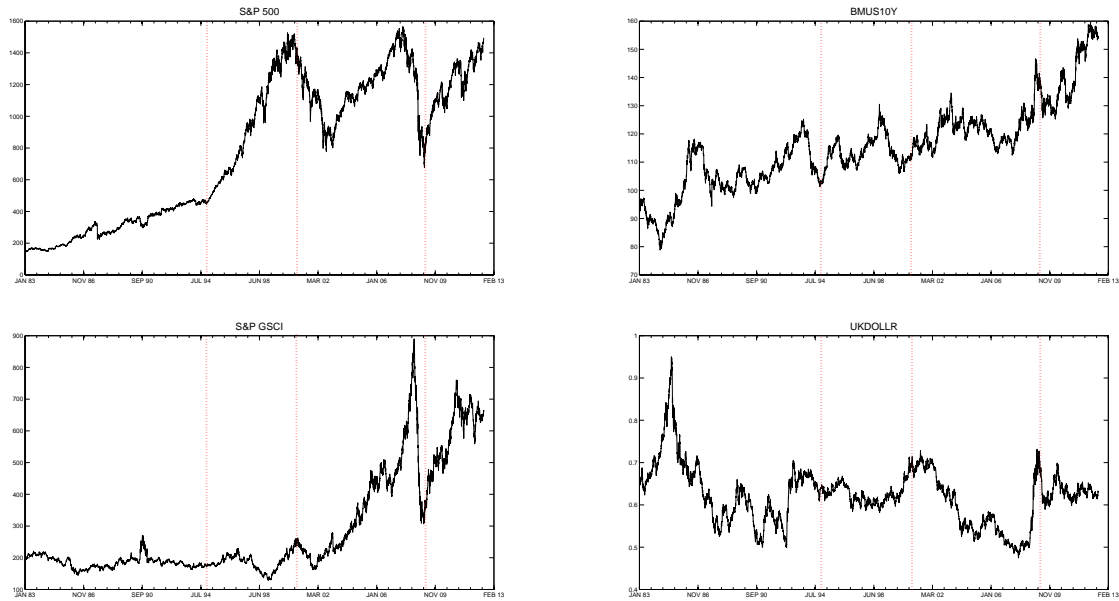


Figure 1: Raw series: S&P 500 (top left), Bonds (top right), S&P GSCI (bottom left), UK/Dollar FX (bottom right)

Note: Structural breaks identified by means of Chow tests are represented by vertical dotted lines.

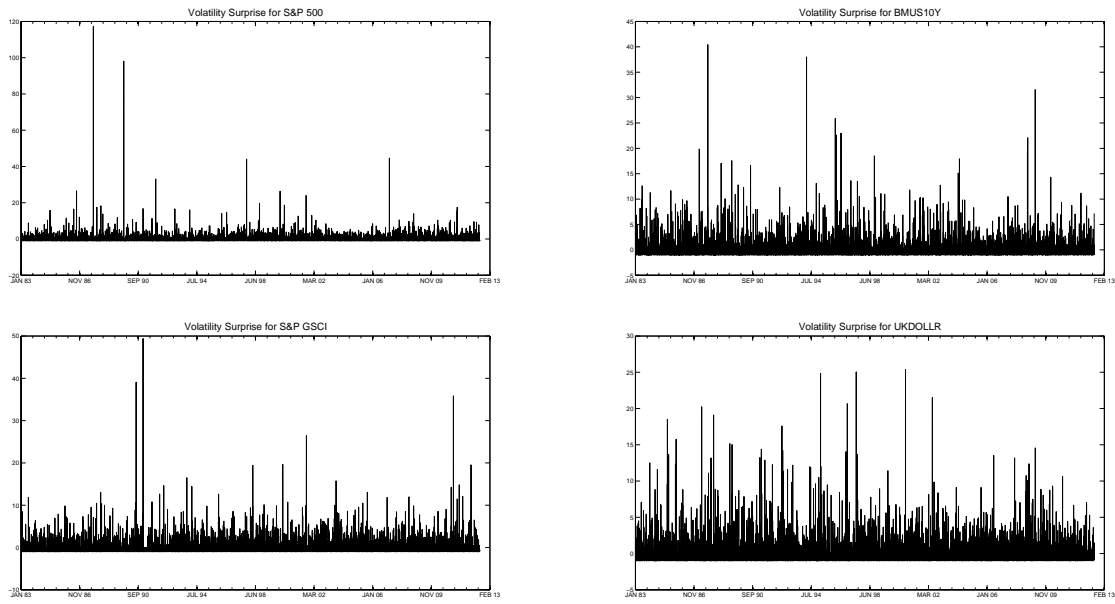


Figure 2: ‘Volatility surprise’ series: S&P 500 (top left), Bonds (top right), S&P GSCI (bottom left), UK/Dollar FX (bottom right)

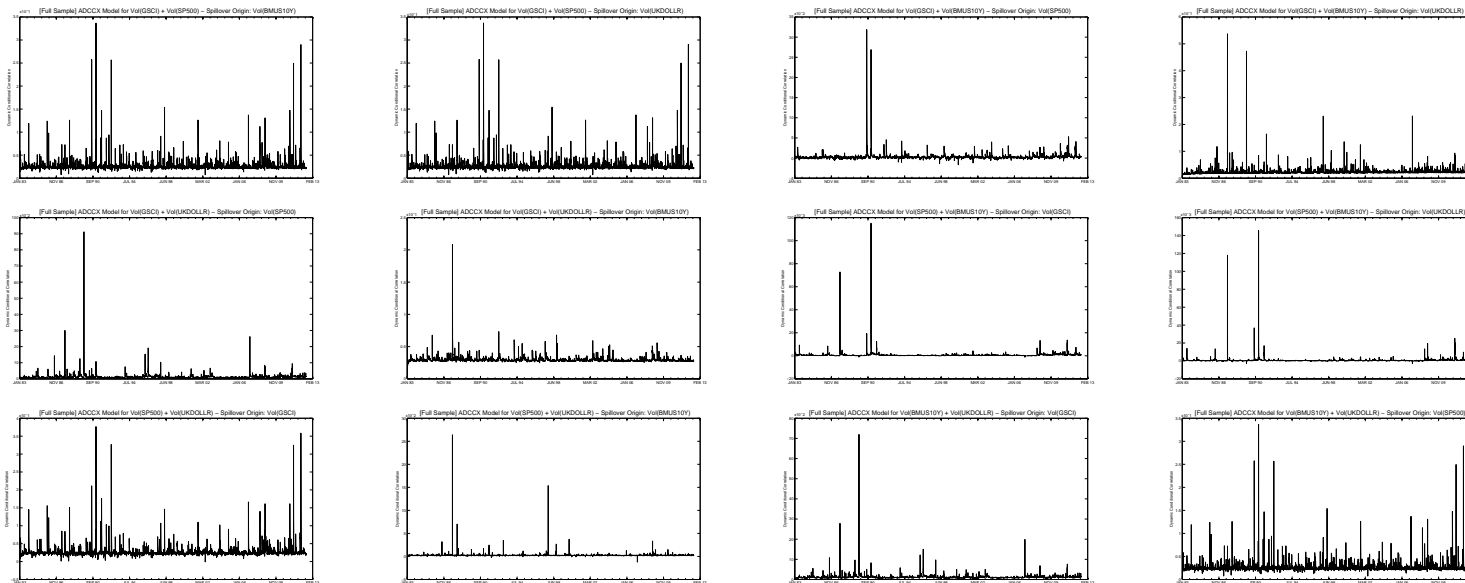


Figure 3: Dynamic Conditional Correlations computed from the ADCCX Model during the Full Sample [1983-2013]

7.2 Tables

Table 1: Descriptive statistics

Returns	Mean	Standard Deviation	Skewness	Kurtosis	Standardized Skewness	Standardized Kurtosis
GSCI	0.02	1.25	-0.60	12.97	-0.48	10.35
SP500	0.03	1.14	-1.27	31.69	-1.11	27.74
BMUS10Y	0.01	1.14	-1.58	43.13	-1.39	37.95
UKDOLLR	0.00	0.62	-0.01	6.68	-0.01	10.83

Volatility Surprise	Mean	Standard Deviation	Skewness	Kurtosis	Standardized Skewness	Standardized Kurtosis
GSCI	0.04	187.59	7.38	123.23	0.03	0.65
SP500	-0.10	258.26	22.01	844.44	0.08	3.26
BMUS10Y	0.22	194.06	6.34	79.25	0.03	0.41
UKDOLLR	0.32	181.24	4.80	40.31	0.02	0.22

Unconditional cross-correlations				
Returns	GSCI	SP500	BMUS10Y	UKDOLLR
GSCI	1	0.12	-0.13	-0.14
SP500		1	-0.06	-0.03
BMUS10Y			1	-0.01
UKDOLLR				1

Unconditional cross-correlations				
Volatility Surprise	GSCI	SP500	BMUS10Y	UKDOLLR
GSCI	1	0.13	0.09	0.07
SP500		1	0.21	0.05
BMUS10Y			1	0.08
UKDOLLR				1

OLS Regressions						
Returns	C	GSCI(-1)	SP500(-1)	BMUS10Y(-1)	UKDOLLR(-1)	R ² ×100
1 GSCI	0.01		0.06***	0.02	0.03	0.32
2 SP500	0.01**	-0.03**		0.11***	0.01	0.32
3 BMUS10Y	0.01	0.01	0.01		-0.01	0.03
4 UKDOLLR	0.01	-0.02***	-0.03***	-0.04***		0.41

OLS Regressions						
Volatility Surprise	C	GSCI(-1)	SP500(-1)	BMUS10Y(-1)	UKDOLLR(-1)	R ² ×100
1 GSCI	0.01		0.02**	0.01	-0.01	0.07
2 SP500	-0.01	0.02		-0.01	-0.01	0.02
3 BMUS10Y	0.01	-0.01	0.11***		-0.02	1.97
4 UKDOLLR	0.01	-0.01	0.03***	0.04***		0.43

Note: The standardized skewness and kurtosis are the skewness and kurtosis of the returns and ‘volatility surprise’ components standardized by their estimated standard deviation. Parameter estimates are given for the OLS Regressions, which have been estimated with Newey-West Standard Errors and Covariance. The statistical significance is indicated at the 1%(***), 5%(**), and 10%(*) levels. *C* stands for the intercept, and (-1) for lag one. Diagnostic tests for OLS Regressions (Ljung-Box-Pierce residuals autocorrelation in particular) can be accessed upon request.

Table 2: Univariate GARCH models

	Asset	Model selected with AIC	ω	α	γ	β
1	GSCI	GARCH	0.0005	0.0609		0.9375
2	SP500	EGARCH	-0.2600	0.1233	-0.0940	0.9820
3	BMUS10Y	GARCH	0.0003	0.0448		0.9431
4	UKDOLLR	TGARCH	0.0003	0.0480	-0.0128	0.9510

Note: All parameters are found to be statistically significant at the 1% level. Intercept parameters are calculated on 10 times the returns to facilitate working with extremely small numbers. The GARCH specifications tested are briefly recalled below.

GARCH:

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}$$

Exponential GARCH (EGARCH):

$$\ln(h_t) = \omega + \alpha \frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} + \gamma \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta \ln(h_{t-1})$$

Threshold GARCH (TGARCH):

$$h_t^{1/2} = \omega + \alpha |\epsilon_{t-1}| + \gamma \mathbf{I}[\epsilon_{t-1} < 0] |\epsilon_{t-1}| + \beta h_{t-1}^{1/2}$$

Table 3: ADCCX estimates for the Full Sample [1983-2013]

	Volatility Surprise	Parameters			Spillover Origin				Diagnostics	
		a ²	b ²	γ ²	ξ (ξ(GSCI(-1)))	ξ (ξ(SP500(-1)))	ξ (ξ(BMUS10Y(-1)))	ξ (ξ(UKDOLLR(-1)))	SC	Log L
1	ξ̃(GSCI)+ ξ̃(SP500)	0.0441*** (0.0036)	0.9255*** (0.0082)	0.0010 (0.0313)			0.0065*** (0.0016)		1.5828	-6175.63
2	ξ̃(GSCI)+ξ̃(SP500)	0.0438*** (0.0041)	0.9261*** (0.0096)	0.0159 (0.0243)				0.0057*** (0.0016)	1.5843	-6181.63
3	ξ̃(GSCI)+ξ̃(BMUS10Y)	0.0514*** (0.0038)	0.8879*** (0.0087)	0.0033 (0.0299)		0.0092*** (0.0003)			2.0045	-7825.77
4	ξ̃(GSCI)+ξ̃(BMUS10Y)	0.0519*** (0.0042)	0.8885*** (0.0103)	0.0027 (0.0290)				0.0060*** (0.0023)	1.9897	-7767.95
5	ξ̃(GSCI)+ξ̃(UKDOLLR)	0.0412*** (0.0029)	0.9052*** (0.0057)	0.0132 (0.0277)		0.0084*** (0.0009)			2.0090	-7843.58
6	ξ̃(GSCI)+ξ̃(UKDOLLR)	0.0401*** (0.0030)	0.9081*** (0.0059)	0.0010 (0.0270)			0.0076*** (0.0017)		2.0069	-7835.23
7	ξ̃(SP500)+ξ̃(BMUS10Y)	0.0475*** (0.0030)	0.9050*** (0.0070)	0.0563** (0.0222)	0.0082*** (0.0013)				1.5516	-6053.73
8	ξ̃(SP500)+ξ̃(BMUS10Y)	0.0484*** (0.0033)	0.9031*** (0.0077)	0.0566** (0.0234)				0.0073*** (0.0013)	1.5522	-6056.01
9	ξ̃(SP500)+ξ̃(UKDOLLR)	0.0479*** (0.0064)	0.8973*** (0.0148)	0.0293 (0.0313)	0.0100*** (0.0016)				1.6090	-6278.29
10	ξ̃(SP500)+ξ̃(UKDOLLR)	0.0442*** (0.0056)	0.8975*** (0.0141)	0.0105 (0.0310)			0.0078*** (0.0012)		1.6089	-6277.78
11	ξ̃(BMUS10Y)+ξ̃(UKDOLLR)	0.0475*** (0.0017)	0.8962*** (0.0044)	0.0065 (0.0258)	0.0062*** (0.0018)				2.0073	-7836.88
12	ξ̃(BMUS10Y)+ξ̃(UKDOLLR)	0.0477*** (0.0017)	0.8956*** (0.0044)	0.0056 (0.0283)		0.0097*** (0.0001)			2.0187	-7881.33

Note: Standard errors in parentheses below coefficient estimates. Statistical significance is indicated at the 1%(***) , 5%(**), and 10%(*) levels. Lag 1 is noted (-1). *LogL* is the log-likelihood value. *SC* is the Schwarz information criteria.

The 'volatility surprise' component is given by eq.(3): $\tilde{\zeta}_t = \frac{(\epsilon_t^2 - \sigma_t^2)}{\sigma_t^2}$.

The ADCCX model is given by eq.(9): $Q_t = (\bar{Q} - a^2\bar{Q} - b^2\bar{Q} - \gamma^2\bar{N} - K\xi'\bar{x}) + a^2(e_{t-1}e'_{t-1}) + b^2(Q_{t-1}) + \gamma^2(\eta_{t-1}\eta'_{t-1}) + K\xi'x_{t-1}$.

Table 4: Chow test results

Break Dates		16/01/1995	13/11/2000	06/03/2009						
Chow Test	RSS_0	RSS_1	RSS_2	RSS_3	RSS_4	RSS_a	F^*	F^c 5%	Structural breaks?	
1	SP500	893758.70	24621.14	219281.60	459457.80	189212.40	892572.94	5.19	2.99	Yes
2	BMUS10Y	2416.91	50.52	562.32	1169.70	630.23	2412.78	6.69	2.99	Yes
3	CGYSPT	180998.20	11234.84	6360.23	101747.70	61417.20	180759.98	5.15	2.99	Yes
4	UKDOLLR	0.12	0.06	0.01	0.02	0.01	0.12	6.45	2.99	Yes

Note: Chow tests are conducted with three potential break dates. Each regression model is estimated with a constant and an AR(1) component. RSS_0 stands for the Residual Sum of Squares for the model estimated during the full sample. RSS_1 to RSS_4 correspond to the model estimates for each sub-period. $RSS_a = RSS_1 + RSS_2 + RSS_3 + RSS_4$. F^* is the F -statistic computed as $\frac{(SCR_0 - SCR_a)/(k+1)}{SCR_a/(T-2(k+1))}$. k denotes the number of variables. T denotes the number of observations, equal to 7826. F^c is the critical value provided by the Fisher-Snedecor tables for $(k+1, T-2(k+1))$ degrees of freedom.

Table 5: ADCCX estimates for the Sub-Period #1 [1983-1995]

	Volatility Surprise	Parameters			Spillover Origin				Diagnostics	
		a ²	b ²	γ ²	ξ (ξ(GSCI(-1)))	ξ (ξ(SP500(-1)))	ξ (ξ(BMUS10Y(-1)))	ξ (ξ(UKDOLLR(-1)))	SC	Log L
1	ξ̄(GSCI)+ ξ̄(SP500)	0.0616*** (0.0057)	0.9295*** (0.0048)	0.0087** (0.0032)			0.1118*** (0.0168)		7.3161	-3676.44
2	ξ̄(GSCI)+ξ̄(SP500)	0.0625*** (0.0032)	0.9256*** (0.0031)	0.0118** (0.0051)				0.0698*** (0.0178)	6.2601	-3148.10
3	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0559*** (0.0043)	0.9289*** (0.0061)	0.0150 (0.0091)		0.1184*** (0.0109)			6.8181	-3427.00
4	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0861*** (0.0051)	0.8925*** (0.0063)	0.0041 (0.0028)				0.0924*** (0.0188)	7.8641	-3950.45
5	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0646*** (0.0059)	0.9254*** (0.0053)	0.0098 (0.0064)		0.0304*** (0.0105)			6.5341	-3285.12
6	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0606*** (0.0034)	0.9331*** (0.0022)	0.0061 (0.0038)			0.0830*** (0.0169)		7.2401	-3638.61
7	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0579*** (0.0026)	0.9334*** (0.0036)	0.0085** (0.0021)	0.2362*** (0.0300)				6.7081	-3372.09
8	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0586*** (0.0062)	0.9290*** (0.0049)	0.0123*** (0.0048)				0.0870*** (0.0302)	6.4941	-3265.11
9	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0778*** (0.0043)	0.9066*** (0.0047)	0.0154 (0.0091)	0.0701*** (0.0178)				7.6041	-3820.67
10	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0590*** (0.0024)	0.9341*** (0.0037)	0.0068 (0.0041)			0.3041*** (0.0281)		6.4181	-3227.19
11	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0760*** (0.0221)	0.9005*** (0.0243)	0.0092 (0.0066)	0.1249*** (0.0187)				7.7281	-3882.28
12	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0641*** (0.0177)	0.9009*** (0.0357)	0.0070 (0.0016)		0.0191*** (0.0080)			6.9761	-3506.33

Note: Standard errors in parentheses below coefficient estimates. Statistical significance is indicated at the 1%(***) , 5%(**), and 10%(*) levels. Lag 1 is noted (-1). $LogL$ is the log-likelihood value. SC is the Schwarz information criteria.

The ‘volatility surprise’ component is given by eq.(3): $\tilde{\zeta}_t = \frac{(\epsilon_t^2 - \sigma_t^2)}{\sigma_t^2}$.

The ADCCX model is given by eq.(9): $Q_t = (\bar{Q} - a^2\bar{Q} - b^2\bar{Q} - \gamma^2\bar{N} - K\xi'x) + a^2(e_{t-1}e'_{t-1}) + b^2(Q_{t-1}) + \gamma^2(\eta_{t-1}\eta'_{t-1}) + K\xi'x_{t-1}$.

Table 6: ADCCX estimates for the Sub-Period #2 [1995-2000]

	Volatility Surprise	Parameters			Spillover Origin				Diagnostics	
		a ²	b ²	γ ²	ξ (ξ(GSCI(-1)))	ξ (ξ(SP500(-1)))	ξ (ξ(BMUS10Y(-1)))	ξ (ξ(UKDOLLR(-1)))	SC	Log L
1	ξ̄(GSCI)+ ξ̄(SP500)	0.0578*** (0.0039)	0.9370*** (0.0063)	0.0051 (0.0039)			0.0229 (0.0177)		7.2461	-3641.05
2	ξ̄(GSCI)+ξ̄(SP500)	0.0594*** (0.0038)	0.9329*** (0.0068)	0.0076 (0.0050)				0.0266 (0.0227)	6.1941	-3115.66
3	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0757*** (0.0031)	0.9117*** (0.0074)	0.0124 (0.0091)		0.2199*** (0.0227)			7.8941	-3965.92
4	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0591*** (0.0076)	0.9284*** (0.0030)	0.0123 (0.0077)				0.0570** (0.0254)	6.5321	-3284.82
5	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0607*** (0.0041)	0.9329*** (0.0070)	0.0062 (0.0041)		0.0773*** (0.0235)			7.9261	-3981.00
6	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0441*** (0.0077)	0.9099*** (0.0357)	0.0057 (0.0036)			0.0579** (0.0258)		6.3581	-3197.65
7	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0577*** (0.0050)	0.9365*** (0.0023)	0.0057** (0.0015)	0.0466 (0.0314)				7.4521	-3744.61
8	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0598*** (0.0054)	0.9331*** (0.0024)	0.0069*** (0.0018)				0.0915*** (0.0278)	7.6281	-3832.11
9	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0595*** (0.0039)	0.9272*** (0.0061)	0.0131 (0.0074)	0.0338 (0.0289)				6.2121	-3124.32
10	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0813*** (0.0035)	0.9056*** (0.0064)	0.0130 (0.0086)			0.2644*** (0.0273)		7.2661	-3651.24
11	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0623*** (0.0055)	0.9309*** (0.0024)	0.0067 (0.0041)	0.0286 (0.0222)				6.9881	-3512.69
12	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0221*** (0.0123)	0.9562*** (0.0425)	0.0080 (0.0061)			0.1106*** (0.0163)		6.8681	-3452.72

Note: Standard errors in parentheses below coefficient estimates. Statistical significance is indicated at the 1%(***) , 5%(**), and 10%(*) levels. Lag 1 is noted (-1). *LogL* is the log-likelihood value. *SC* is the Schwarz information criteria.

The ‘volatility surprise’ component is given by eq.(3): $\tilde{\zeta}_t = \frac{(\epsilon_t^2 - \sigma_t^2)}{\sigma_t^2}$.

The ADCCX model is given by eq.(9): $Q_t = (\bar{Q} - a^2\bar{Q} - b^2\bar{Q} - \gamma^2\bar{N} - K\xi'\bar{x}) + a^2(e_{t-1}e'_{t-1}) + b^2(Q_{t-1}) + \gamma^2(\eta_{t-1}\eta'_{t-1}) + K\xi'x_{t-1}$.

Table 7: ADCCX estimates for the Sub-Period #3 [2000-2009]

Volatility Surprise	Parameters			Spillover Origin				Diagnostics	
	a ²	b ²	γ ²	ξ (ξ(GSCI(-1)))	ξ (ξ(SP500(-1)))	ξ (ξ(BMUS10Y(-1)))	ξ (ξ(UKDOLLR(-1)))	SC	Log L
1	ξ̄(GSCI)+ ξ̄(SP500)	0.0593*** (0.0081)	0.9171*** (0.0077)	0.0078 (0.0043)			0.0664*** (0.0210)	8.2621	-4149.22
2	ξ̄(GSCI)+ξ̄(SP500)	0.0564*** (0.0046)	0.9087*** (0.0119)	0.0096 (0.0067)			0.0839*** (0.0234)	8.4981	-4267.65
3	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0281*** (0.0081)	0.9396*** (0.0079)	0.0075 (0.0059)	0.2206*** (0.0195)			8.8781	-4457.81
4	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0166*** (0.0049)	0.9272*** (0.0061)	0.0047 (0.0032)			0.1136*** (0.0237)	8.3641	-4200.22
5	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0120*** (0.0099)	0.9588*** (0.0056)	0.0062 (0.0044)	0.0309* (0.0180)			9.8261	-4931.69
6	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0119*** (0.0065)	0.9108*** (0.0088)	0.0082 (0.0028)		0.0918*** (0.0192)		9.8081	-4922.88
7	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0221*** (0.0078)	0.9266*** (0.0067)	0.0193** (0.0069)	0.1118*** (0.0232)			8.0041	-4020.56
8	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0265*** (0.0058)	0.9272*** (0.0057)	0.0142** (0.0074)			0.0436* (0.0254)	9.1061	-4571.38
9	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0614*** (0.0042)	0.9071*** (0.0034)	0.0074 (0.0053)	0.0702*** (0.0195)			9.1741	-4605.85
10	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0322*** (0.0099)	0.9204*** (0.0101)	0.0070 (0.0066)		0.2158*** (0.0222)		9.1041	-4570.50
11	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0151*** (0.0080)	0.9610*** (0.0048)	0.0050 (0.0030)	0.0686*** (0.0217)			8.4641	-4250.00
12	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0272*** (0.0116)	0.9286*** (0.0149)	0.0064 (0.0048)		0.1002*** (0.0137)		9.1221	-4579.85

Note: Standard errors in parentheses below coefficient estimates. Statistical significance is indicated at the 1%(***) , 5%(**), and 10%(*) levels. Lag 1 is noted (-1). $LogL$ is the log-likelihood value. SC is the Schwarz information criteria.

The 'volatility surprise' component is given by eq.(3): $\tilde{\zeta}_t = \frac{(\epsilon_t^2 - \sigma_t^2)}{\sigma_t^2}$.

The ADCCX model is given by eq.(9): $Q_t = (\bar{Q} - a^2\bar{Q} - b^2\bar{Q} - \gamma^2\bar{N} - K\xi'\bar{x}) + a^2(e_{t-1}e'_{t-1}) + b^2(Q_{t-1}) + \gamma^2(\eta_{t-1}\eta'_{t-1}) + K\xi'x_{t-1}$.

Table 8: ADCCX estimates for the Sub-Period #4 [2009-2013]

	Volatility Surprise	Parameters			Spillover Origin				Diagnostics	
		a ²	b ²	γ ²	ξ (ξ(GSCI(-1)))	ξ (ξ(SP500(-1)))	ξ (ξ(BMUS10Y(-1)))	ξ (ξ(UKDOLLR(-1)))	SC	Log L
1	ξ̄(GSCI)+ ξ̄(SP500)	0.0610*** (0.0034)	0.9024*** (0.0087)	0.0126 (0.0073)			0.1606*** (0.0346)		8.3701	-4203.82
2	ξ̄(GSCI)+ξ̄(SP500)	0.0154*** (0.0027)	0.9339*** (0.0070)	0.0097 (0.0072)				0.2348*** (0.0428)	8.6121	-4324.57
3	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0270*** (0.0054)	0.9210*** (0.0071)	0.0058 (0.0033)		0.3065*** (0.0307)			8.4041	-4220.10
4	ξ̄(GSCI)+ξ̄(BMUS10Y)	0.0351*** (0.0025)	0.9116*** (0.0137)	0.0111 (0.0090)				0.0858** (0.0388)	8.0412	-4039.11
5	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0211*** (0.0049)	0.9586*** (0.0068)	0.0032 (0.0027)		0.0715*** (0.0258)			9.0601	-4548.62
6	ξ̄(GSCI)+ξ̄(UKDOLLR)	0.0273*** (0.0071)	0.9395*** (0.0051)	0.0200 (0.0070)			0.0557** (0.0252)		9.8581	-4947.20
7	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0342*** (0.0047)	0.9123*** (0.0046)	0.0121** (0.0062)	0.2932*** (0.0258)				8.1901	-4113.36
8	ξ̄(SP500)+ξ̄(BMUS10Y)	0.0576*** (0.0057)	0.9013*** (0.0121)	0.0166** (0.0084)				0.1047*** (0.0378)	9.7961	-4916.98
9	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0154*** (0.0056)	0.9582*** (0.0097)	0.0051 (0.0043)	0.1224*** (0.0123)				8.2061	-4121.96
10	ξ̄(SP500)+ξ̄(UKDOLLR)	0.0161*** (0.0063)	0.9708*** (0.0134)	0.0040 (0.0031)			0.2913*** (0.0292)		8.2721	-4154.20
11	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0466*** (0.0090)	0.9019*** (0.0060)	0.0051 (0.0039)	0.1290*** (0.0278)				8.8801	-4458.08
12	ξ̄(BMUS10Y)+ξ̄(UKDOLLR)	0.0221*** (0.0048)	0.9324*** (0.0101)	0.0071 (0.0053)		0.1276*** (0.0214)			8.9621	-4499.58

Note: Standard errors in parentheses below coefficient estimates. Statistical significance is indicated at the 1%(***) , 5%(**), and 10%(*) levels. Lag 1 is noted (-1). $LogL$ is the log-likelihood value. SC is the Schwarz information criteria.

The 'volatility surprise' component is given by eq.(3): $\tilde{\zeta}_t = \frac{(\epsilon_t^2 - \sigma_t^2)}{\sigma_t^2}$.

The ADCCX model is given by eq.(9): $Q_t = (\bar{Q} - a^2\bar{Q} - b^2\bar{Q} - \gamma^2\bar{N} - K\xi'\bar{x}) + a^2(e_{t-1}e'_{t-1}) + b^2(Q_{t-1}) + \gamma^2(\eta_{t-1}\eta'_{t-1}) + K\xi'x_{t-1}$.