A Renewed Analysis of Cheating in Contests: Theory and Evidence from Recovery Doping

Sebastian Bervoets, Bruno Decreuse, Mathieu Faure

amse
école d'économie d'aix-marseille
aix-marseille school of economics

Working Papers / Documents de travail

A Renewed Analysis of Cheating in Contests: Theory and Evidence from Recovery Doping

Sebastian Bervoets
Bruno Decreuse
Mathieu Faure

WP 2014 - Nr 41

A renewed analysis of cheating in contests: theory and evidence from recovery doping *

Sebastian Bervoets[†] Bruno Decreuse[†] Mathieu Faure[†]

June 2015

Abstract

In rank-order tournaments, players have incentives to cheat in order to increase their probability of winning the prize. Usually, cheating is seen as a technology that allows individuals to illegally increase their best potential performances. This paper argues that cheating can alternatively be seen as a technology that ensures that the best performances are reached more often. We call this technology recovery doping and show that it yields new insights on the effects of cheating: recovery doping lowers performance uncertainty, thereby changing the outcome of the contest in favour of the best players. We develop this theory in a game with player heterogeneity and performance uncertainty and then study the results of the cross-country skiing World Cup between 1987 and 2006. In line with our theoretical predictions, race-specific rankings were remarkably stable during the 1990s, subsequently becoming more volatile. This pattern reflects the rise and fall of synthetic EPO and the emergence of blood testing and profiling.

Keywords: Game theory; Recovery doping; Rank correlation

*We thank Gérard Dine as well as the many people we interacted with on this project and seminar participants in Aix-Marseille University and Linnaeus University. We also thank Laurence Bouvard for her excellent research assistance.

[†]Aix-Marseille University (Aix-Marseille School of Economics), CNRS and EHESS. Centre de la Vieille Charité, 2 rue de la Charité, 13002 Marseille, France. The authors thank the French National Research Agency (ANR) for their support through the program ANR 13 JSH1 0009 01, as well as the Fondation Aix-Marseille Unversity and its program "Sport, santé et développement durable".

Individuals or institutions engaged in contest-like situations have incentive to use illegal methods to increase their payments. Hedging funds compete for savings and may practice fraudulent accounting to appear better than they are. Scientists may voluntarily alter their datasets to enhance their results. And professional athletes may take performance-enhancing drugs (PED) to outperform their opponents.

Cheating can take various forms. Usually, it is seen as a technology that allows individuals to perform at super-natural levels, by illegally increasing their best potential performances. Throughout the paper we refer to this effect as the *standard effect* and because our main example will be sports, we call this cheating technology *doping*. Here, we take a different angle, arguing that cheating can be seen as a technology that ensures that the best performances are reached more often. To pursue the analogy with sports, we call this particular technology *recovery doping* and its effect the *recovery effect*.

Modeling the standard effect leads to the classical prisoner's dilemma, where competitors are forced into doping by a dominant strategy argument, but where all competitors suffer due to the potential for cheating. They are trapped into a bad equilibrium. Accounting for the recovery effect renews the analysis of cheating in contests. While less talented agents suffer from the existence of cheating possibilities, the most talented agents actually benefit from it. As they become more likely to compete at their best level, they outperform the others more often. Recovery doping therefore lowers contest uncertainty in favor of top agents. This has implications for agents' strategies, reward inequality and anti-cheating policies.

Though our model extends to different markets, we use high-level endurance sports as our illustrating example, for several reasons. Top athletes are rational players with the clear objective of maximizing their payoffs, in a game with rules that are known by all. Also, doping in sports is well documented: the World anti-doping Agency (WADA) holds a list of forbidden products and methods, and countless cases provide evidence that athletes use these illegal technologies.

Section 1 details the difference between the standard effect of cheating and the recovery effect, reviewing the effects of the PEDs listed by the World Anti-Doping Code[1]. As expected, PEDs develop strength, power, fighting spirit, far above an

[1]We thank Gerard Dine, hematologist and expert for the French Anti-Doping Agency, for his valuable explanations.

athlete's physiological limits. However, we also point out the much less emphasized recovery effect: when using PEDs, fatigue from effort occurs later, athletes are able to repeat efforts with fewer resting days, they can follow more intensive training programs and injury risk is reduced. Our interpretation of this recovery effect is that athletes perform at their best level more often than without PEDs. The main lesson of Section 1 is that almost all PEDs provide the recovery effect, although with an intensity that varies across products and methods. However, given the ubiquity of the recovery effect, a proper analysis of doping must account for it. At the end of the paper we briefly discuss cheating technologies used in finance and in the academic world, explaining how these technologies produce the recovery effect.

Section 2 presents a two-player game where players are heterogeneous and performances are subject to uncertainty. Each player is in one of two states, good or bad. Doping is continuous, comes at constant marginal cost, and increases both the players' maximum performance (the standard effect) and the probability of being in the good state (the recovery effect).

We start by arguing that the standard effect of doping is not what drives the strategic interactions between players. To do this, we provide a general analysis of the model when only the standard effect exists and show that: doping decreases the chances of the favorite player (the top dog) winning the game; and all players prefer a world where doping is impossible, as their respective payoffs are currently lower than in a world without doping.

We then analyze the model including the recovery effect and obtain radically different predictions. Doping efforts are strategic complements for the top dog, whereas they are strategic substitutes for the other player, the underdog. The top dog enjoys greater returns from doping than the underdog. Intuitively, the top dog wins whenever he achieves his best performance, whereas, in the same situation, the underdog only wins when the top dog is in a bad state. As a consequence, our model predicts that the top dog will be the player who dopes more. It also predicts that the underdog is worse off when doping exists; however, now the top dog benefits from the existence of doping, faring better than in a world without doping.

We use the model to explore the impacts of the rise and fall of a doping technology, identifying them with a decrease and an increase in the cost of doping. When the technology first appears, the top dog's doping incentive unambiguously increases, whereas the underdog's may decrease by strategic substitutability. As a consequence, the probability of the best player winning the contest increases. In other words, we should expect to witness fewer surprise outcomes of contests. When the cost of doping increases due to anti-doping policies, this levels the playing field, thereby hurting the top dog. Thus the model predicts a decreasing relationship between the regularity with which the best players win and the cost of doping. We can use this to identify doping and distinguish it from alternative legal technologies. Importantly, the main predictions of our theory still hold when there is an arbitrary number of players.

We then turn to the empirical test of the theory in Section 3. The sport chosen is cross-country skiing (CCS), and the doping technology is synthetic EPO. CCS is an endurance sport particularly exposed to recovery doping. There have been two major doping scandals, one in the 2001 Lahti World Championship and the other in the 2002 Salt Lake City Olympics. Blood profiles from the late 1980s to the mid-2000s show blood manipulation. Mean Hemoglobin concentration rises during the 1990s, peaks at the end of the 1990s, and subsequently declines. This pattern reflects the introduction of synthetic EPO in the late 1980s, the introduction of blood testing during the 1996-1997 season and blood profiling in 2002.

We use data from the CCS World Cup, a yearly competition based on 10 to 25 races. We show that race-specific rankings were very strongly correlated to the final ranking during the EPO years. Such correlations are higher than before the introduction of EPO and rise again in the 2000s with a spike in 2002, which we relate to the uncertainty of the anti-doping context and the 2002 Olympics scandal. These findings are robust to various considerations like the introduction of sprint races in the mid-1990s, or the presence of two potential genetic freaks in the 1990s. The overall message is that the rise and fall of EPO appears closely linked to the change in performance uncertainty.

Before concluding, we discuss how our analysis extends to contexts other than sports. Our point is that the possibility of increasing, by illegal means, the regularity of one's performance instead of the absolute level of that performance provides the best competitors with a stronger incentive to cheat. We believe this constitutes a new way of considering cheating technologies.

Literature

Our assumption here that high-level or professional athletes are rational agents who behave strategically finds strong support in the literature on sports economics.

One literature strand tests athletes' rationality in a non-strategic framework. Bhaskar (2009) uses cricket players' decisions on whether to bat first or field first in order to assess the consistency of their decisions. Klaasen and Magnus (2009) study the optimal strategy for serving in tennis, based on the speed of the first and second serves. Both papers find that top athletes behave as predicted by the theory. Because rational agents are not immune to stress, Apesteguia and Palacio-Huerta (2010) focus on soccer and show that the team kicking penalties in second position has less chance of winning the game, because of the pressure they are under.

A second strand of papers examines athletes' strategic interactions in sports competitions. Malueg and Yates (2010) show that professional tennis players strategically adjust their efforts during a best-of-three contest, as theory predicts. Chiappori et al. (2002) develop a game-theoretic model of penalty kicks in soccer, analyzing on which side the kicker should shoot and on which side the goalkeeper should dive. They find that professional players behave according to the predictions. Walker and Wooders (2001) argue that professional tennis players play mixed strategies when choosing whether to serve on the opponent's forehand or backhand, as theory suggests they should.

There are already a number of theoretical papers applying game theory to the analysis of doping behaviors. A sporting competition is usually seen as a typical case of the prisoner's dilemma. All athletes would be better off without doping. However, doping is a dominant strategy and everyone dopes in the only Nash equilibrium. This framework rationalizes the fact that doping is very widespread, and provides additional legitimacy to anti-doping policies as Pareto-improving devices[2] (see, e.g., Bird and Wagner, 1997, Berentsen, 2002, Berentsen and Lengwiler, 2004, Kräkel, 2007, for an overview of models, and Eber and Thepot, 1999, Berentsen et al., 2008, Curry and Mongrain, 2009, for policy implications).

The reason why the prisoner's dilemma usually arises is because doping efforts are strategic complements. In our approach doping efforts are strategic complements for the top dog, but strategic substitutes for the underdog. This explains why the most talented agents prefer a world where doping is possible, and why performance uncertainty increases with the cost of doping.

Our paper also relates to the tournament literature. Following Lazear and Rosen

[2]Papers differ in modeling strategies, and do not always feature the prisoner's dilemma. However, they all predict that anti-doping policies are Pareto-improving. In particular, the best player always prefers a world where doping is impossible.

(1981), this literature studies market situations where payoffs explicitly depend on relative performances. This naturally applies to sporting contests, but also to labor markets where it is easier to rank workers than measure their individual performances. A number of papers point out that equilibrium efforts reflect individual skills and player heterogeneity through differential access to top positions and related economic incentives (see, e.g., Rosen, 1986, for theoretical arguments, Glisdorf and Sukhatme, 2008, and Sunde, 2009, for applications to tennis).

Our main theoretical contribution with respect to this literature is to propose an explicit scenario that governs the pattern of strategic complementarity and substitutability across player types. This scenario is appropriate for doping efforts, allowing us to characterize the fundamental relationship between relative performance uncertainty and cost of doping. In our empirical study, we refer to the tournament literature when we discuss the potential role played by the two superstars of the 1990s. These players might be responsible for the ranking stability observed during their career, and this might be an alternative explanation to what we claim to be a consequence of recovery doping. However, if this alternative explanation were true, removing them from the dataset would lead to a steep decline in race-specific ranking correlations in the 1990s. Yet, we find that correlations remain very high when we remove these players from the dataset.

1 Performance-enhancing drugs and their effects

PEDs have a long history in sports. This section documents the two effects of doping agents and methods discussed in the introduction: the standard effect that increases the maximum performance of an athlete, and the recovery effect that allows athletes to perform more often at their best level.

To illustrate, consider a professional cyclist climbing a mountain. The mean ascent speed is a random draw on some interval $[a, b]$. PEDs shift the upper bound b to the right - the standard effect - AND they assign more weight around the upper bound - the recovery effect.

The standard effect is well documented. It comes from the fact that PEDs improve basic skills like strength or endurance. The recovery effect, however, is never mentioned as a key factor in understanding doping behavior. It comes from the fact that PEDs also improve recovery, reduce injury risk and duration, reduce tiredness, allow for longer training periods etc. All of these effects reduce the odds of having a bad day and facilitate the repetition of excellent performances.

Table 1 is based on the World Anti Doping Agency (WADA) list of prohibited products and methods. This list classifies doping products into six categories depending on their biological mechanisms. It also classifies two doping methods. In each case, we describe doping effects from both angles: impact on maximum performance (standard effect), and impact on the odds of having a bad day (recovery effect).

Category	Products	Max perf	Chances of bad day	Use frequency
S1 Anabolic agents	Exogenous anabolic androgenous steroids	Increases strength	Decreases injury risk	Not used much
	Endogenous anabolic androgenous steroids	Increases strength and tonus	Improves recovery reduces tiredness	In use
	Other anabolic agents (non-steroids)	Increases strength	Improves recovery	In use
S2 Peptide and growth hormones	Erythropoietin (EPO)	Increases endurance	Improves recovery reduces tiredness Allows for better training	In use
	Human Growth Hormone	Increases strength	Reduces injury risk and injury duration Allows for better training	In use
S3 Beta-2 agonist	Anti asthma treatments	Improves breathing (small doses) / Increases strength (high doses)	Improves recovery	In use (authorized at low concentrations)
S4 Hormone and metabolic modulators	Regulation factors	Improves HGH effects	Improves HGH effects	In use
	Insulins	Improves strength and Endurance	Improves recovery	In use
	Metabolic modulators (GW 15156 - AICAR)	Reduces weight/ Increases strength	Reduces tiredness	In use
S5 Masking agents	Diuretics, glycerol…			In use
M1 Manipulation of blood	Auto, hetero, Homotransfusion	Same as EPO	Same as EPO	In use
	Platelet Rich Plasma	None	Reduces injury duration	In use, prohibited until 2010.

Table 1: WADA list of prohibited products and their effects.

Table 1 carries a key message. Almost all doping agents and methods simultaneously increase maximum performances and reduce the odds of having a bad day. This is of course true of blood doping, i.e. EPO and transfusion methods. It also holds for anabolic agents: athletes who use such agents improve their strength beyond their physiological potential. However, anabolic agents also improve recovery and reduce injury risks. These effects are similar to those of growth hormones, hormone and metabolic modulators.

The recovery effect is usually neglected, as the case of Platelet Rich Plasma (PRP) illustrates. PRP treatment involves extracting some of the athlete's blood and enriching it with platelets, a source of human growth factors. The blood is then reinjected into the athlete's body, and this blood manipulation helps the athlete recover faster from a muscle injury. Up to 2010, the side effects of such treatment on athletes' performances were unknown and WADA consequently banned the use of PRP. However, since 2010 medical studies have shown that PRP has no effect on the maximum potential performance of an athlete, leading WADA to authorize its use. The fact that PRP can also be used to decrease risk of injury and injury duration was not taken into account. But it is unambiguous that reducing risk of injury amounts to reducing the athlete's chances of having a bad day. WADA's position on PRP shows how institutions focus mainly on how drugs affect maximum performance, underestimating regularity of performance as a potential target for drug users. We argue herein that this common view of doping is misleading.

2 Recovery doping: theory

There are two players who compete for a price, the value of which is normalized to 1. Each player i is characterized by a pair of possible performance levels $(\underline{a}_i, \overline{a}_i)$, with $\underline{a}_i < \overline{a}_i$. When in a good state, player i achieves the performance level $\overline{a}_i$. When in a bad state he only achieves $\underline{a}_i$.

Doping is a continuous variable with values between 0 (no doping) and 1 (maximal doping). It comes at marginal cost c, and player i exerting doping effort d_i has two effects: (i) it increases the higher performance level $\overline{a}_i$ by a quantity $a(d_i)$, where $a(.)$ is a non-decreasing function (we denote by $\overline{a}_i(d_i)$ the quantity $\overline{a}_i + a(d_i)$); (ii) it enhances the probability of being in a good state by a quantity $h(d_i)$: without doping, each player's probability of being in a good state is $1/2$. With doping, player i has a probability of $1/2 + h(d_i)$ of being in a good state. The function h satisfies the following assumption:

Hypothesis 2.1 *h is a continuous function on $[0,1]$, $\mathcal{C}^2$ on $]0,1]$ such that:*

(i) h is strictly increasing on $[0,1]$, $h(0) = 0$, $h(1) = 1/2$

(ii) we have $h'(1) = 0$ and $\lim_{d \to 0^+} h'(d) = +\infty$.

(iii) h is strictly concave on $]0,1]$ *and* $h''(1) < 0$.

Let a_i be the performance level realized by player i. Note that Player i wins 1 whenever $a_i > a_{-i}$ (where $-i$ denotes the opponent of player i). In case $a_i = a_{-i}$, the players share the price and obtain 1/2. Player i's expected payoff is

$$U_i(d_i, d_{-i}) = \Pr(a_i > a_{-i}) + \frac{1}{2}\Pr(a_i = a_{-i}) - c.d_i, \tag{1}$$

where Pr denotes the probability distribution induced by the action profile (d_i, d_{-i}).

2.1 The standard framework

The purpose of this sub-section is to analyze the standard case where doping only affects the level of the player's best performance. The uninterested reader can skip this part and go directly to the next sub-section. For the moment, let us assume that there is no recovery effect, i.e. $h \equiv 0$. Thus doping only increases the maximum performance of each player. Several models can be examined, from the simplest to the most elaborate:

(a) homogeneous players ($\underline{a}_1 = \underline{a}_2$ and $\overline{a}_1 = \overline{a}_2$) and two doping levels, $d_i \in \{0, 1\}$

(b) homogeneous players ($\underline{a}_1 = \underline{a}_2$ and $\overline{a}_1 = \overline{a}_2$) and finite doping levels, $d_i \in \{0, d_1, \ldots, d_k\}$

(c) heterogeneous players ($\underline{a}_1 > \underline{a}_2$ and $\overline{a}_1 > \overline{a}_2$) and finite doping levels

(d) heterogeneous players and continuous doping levels $d_i \in [0, 1]$.

We informally discuss all these models here, but Appendix A formalizes and analyzes each of them. In all these models, the Nash equilibrium, whether in pure or in mixed strategies, is unique: players do not dope when the doping cost is too high; both players choose a positive doping level with positive probability otherwise. This allows us to analyze welfare: we call *welfare of player i* his payoff at equilibrium U_i^*.

All the models have significantly distinct features. In (a) the resulting 2×2 game is a classical prisoner dilemma and the unique Nash equilibrium is in pure strategies where players dope. In (b) the game exhibits the same characteristics, the equilibrium is unique and symmetric, and the equilibrium doping level decreases when the cost of doping increases. This model is in line with standard views on doping. Berentsen (2002) shows that considering heterogeneity in players' type introduces drastic changes in the analysis. This is confirmed by model (c), where the only equilibrium is in mixed strategies, probabilities of doping are asymmetric, and the marginal cost of doping has asymmetric effects on the different players: as the cost increases, doping effort decreases for the top dog while it increases for the underdog. Finally, and more in line with our general model, when doping is a continuous variable as in (d), the unique Nash equilibrium is in mixed strategies and decreases for both players (in the sense of first-order stochastic dominance) when the cost of doping increases.

However, all these models have in common the following crucial features:

Proposition 1 (Heuristics on the standard framework) *For the models described above:*
- *Relative to a world without doping, welfare is lower for the underdog ($U_2^* \leq U_2(0,0)$) and strictly lower for the top dog ($U_1^* < U_1(0,0)$)*
- *The top dog's winning probability is lower than in a world without doping.*

All proofs are in the Appendix. The standard framework predicts that all athletes fare better when the cost of doping is so high that none would ever dope. Furthermore, doping increases competition uncertainty. Both features are no longer true once the recovery effect is taken into account.

2.2 The recovery effect

With doping, the probabilities are changed as follows:

$$\Pr(a_i = \overline{a}_i(d_i)) = \frac{1}{2} + h(d_i)$$
$$\Pr(a_i = \underline{a}_i) = \frac{1}{2} - h(d_i)$$

Assume, without loss of generality, that $\underline{a}_2 < \underline{a}_1 < \overline{a}_2 < \overline{a}_1$. The payoff for player 1 is:

$$\begin{cases} \frac{1}{2} + h(d_1) + (\frac{1}{2} - h(d_1))(\frac{1}{2} - h(d_2)) - c.d_1 & \text{if } \overline{a}_1(d_1) > \overline{a}_2(d_2) \\ \frac{1}{2}(\frac{1}{2} + h(d_1))(\frac{1}{2} + h(d_2)) + (\frac{1}{2} - h(d_2)) - c.d_1 & \text{if } \overline{a}_1(d_1) = \overline{a}_2(d_2) \\ \frac{1}{2} - h(d_2) - c.d_1 & \text{if } \overline{a}_1(d_1) < \overline{a}_2(d_2) \end{cases}$$

Consider for instance the first equation. If player 1's maximum performance, after doping, is higher than that of player 2, then player 1 will win whenever he realizes his best performance (this happens with probability $1/2 + h(d_1)$), and will win when he performs badly (with probability $1/2 - h(d_1)$) only if his opponent also performs badly (with probability $1/2 - h(d_2)$). Interpretation is similar for the two other cases.

We show that only two things can happen: first, there can be no equilibrium to the game when players are very similar to one another in terms of maximum performance without doping. In that case, because the game is a winner-takes-all game, there is an arms race to reap all the benefits. As player 2 increases his doping level, player 1 increases his, until the point where it becomes too costly for player 2 to increase his doping level any more. He then goes back to a no-doping strategy, and as a consequence player 1 also goes back to the no-doping strategy. The arms race starts all over again.

Second, if there is an equilibrium (or several equilibria), it is also an equilibrium of the game in which function $a \equiv 0$, i.e. a game in which we consider that doping has no effect on the maximum performance of players.

Lemma 1 *Any equilibrium (d_1^*, d_2^*) of the doping game must be such that $\overline{a}_1(d_1^*) > \overline{a}_2(d_2^*)$.*

This lemma implies that if an equilibrium exists, the best player without doping will still be the best with doping. This allows us to state the following:

Proposition 2 *Any Nash equilibrium (d_1^*, d_2^*) of the doping game is also an equilibrium of the game with $a \equiv 0$.*

Once the recovery effect is introduced, it alone determines the characteristics of the equilibrium of the game, regardless of the existence of the standard effect. Increased maximum performance only determines whether there is an equilibrium or not. Therefore, focusing on the standard effect leads to partial (no equilibrium) or non-valid conclusions (when there is an equilibrium). We now turn to the interesting case: analyzing the game with $a \equiv 0$ for all d and examining the effects of $h(.)$ on athletes' behavior.

2.3 Main results

We assume that $a \equiv 0$ and h satisfies Hypothesis 2.1. For player 1, strategic interaction arises because he only loses when he is in the bad state and player 2 is in the good state. If player 2 dopes more, his probability of being in the good state rises. This in turn raises player 1's marginal return from doping. Things are very different for player 2. He loses whenever player 1 is in the good state. Thus doping efforts are wasted when player 1 makes sufficiently strong efforts.

Given the zero-sum game nature of athletic contests, players' doping always reduces the welfare of their opponents. However, it also affects the marginal return from their opponents' actions. Call Br^i the best response map of player i: $Br^i(d_{-i}) := \text{Argmax}_{d_i} U_i(d_i, d_{-i})$. A *Nash equilibrium* is a fixed point of the map $Br := (Br^1, Br^2)$.

Proposition 3 (Properties of doping efforts at equilibrium) *For any doping cost $c > 0$, we have that*

(*i*) *Player 1's best response function increases with player 2's doping effort, whereas player 2's best-response function decreases with player 1's doping effort. As a consequence, there is a unique Nash equilibrium $d^* = (d_1^*, d_2^*)$.*

(*ii*) *The top dog dopes more, i.e. $1 > d_1^* \geq d_2^* > 0$.*

Part (i).— The doping effort d_2 acts as a strategic complement for player 1, while d_1 acts as a substitute for player 2. The complementarity effect is in line with the prisoner's dilemma analysis. The substitution effect is one central piece of the model because it makes it different from a prisoner's dilemma and induces the interesting properties below.

Part (ii).— The top dog is sure to win when he achieves the highest performance level, which is not true for the underdog. Consequently, the return from doping is greater for the top dog. This conclusion departs from the standard view on doping.

Proposition 4 (Welfare at equilibrium) *We have the following*

(*i*) *The underdog fares better in a world without doping: $U_2(d_1^*, d_2^*) < U_2(0,0)$;*

(*ii*) *However, when c is not too large, $U_1(d_1^*, d_2^*) > U_1(0,0)$.*

Doping carries a negative externality, generally at a cost to players. However, if the cost is not too high (i.e. the authorities are not very repressive), the top dog benefits greatly from doping. He achieves her best performance more frequently, and therefore is more likely to win than without doping.

The fact that the best player's utility is greater with doping than without, added to the observation that $d_1^* > d_2^*$, explains why it is difficult to fight against doping and enhance the popularity of a sport at the same time. There might be collusion between organizers (federations) and the best players, who need one another for their respective objectives.

Proposition 4 also explains why the best players do not want to fight against doping. They dope more than the others, and they are actually happy to do their job in an environment where doping is possible. In contrast, the underdog is hurt by the doping system: his chances of winning go down and there are doping costs to pay.

Proposition 5 (Doping effort variations with respect to cost) *We have the following*

(*i*) $\lim_{c \to 0^+} d_1^* = 1$ *and* $\lim_{c \to 0^+} d_2^* = 0$;

(*ii*) *The top dog's doping effort decreases with the cost of doping, that is* $d(d_1^*)/dc < 0$*;*

(*iii*) *The underdog's doping increases with the cost of doping for low costs, that is* $d(d_2^*)/dc > 0$ *for* c *low enough.*

Part (i).— When the cost of doping becomes very low, there are two effects. First, because doping becomes more attractive, both athletes wish to dope more. Second, the top dog becomes almost unbeatable because he dopes a lot, so by the substitution effect, doping becomes less attractive for the underdog. At the limit, the second effect overrides the first effect.

This result runs counter the argument whereby free access to doping would lead to a level playing field, because everyone would dope and the results of the competition would not be affected. Our result suggests the reverse, i.e. the best athletes might dope at maximum level, whereas the others would not even try. Competition results would be highly predictable, and absolute performances would be very heterogeneous.

Parts (ii) and (iii).— The cost of doping has an ambiguous impact on the underdog's equilibrium doping effort. However, we know that his doping effort is increasing for low costs. This is due to the strategic substituability discussed above: as the top dog dopes less, the return from doping increases for the underdog.

In the same vein, one can show that targeted tests involve strong redistribution effects between players. Targeting the best player reduces his doping investment. Because of strategic substitutability, this also increases the doping investment of the lower-ranking player, who now has a chance of winning. Thus, overall doping is ambiguously affected and competitions become more uncertain. Targeting the underdog decreases doping for both athletes, now due to strategic complementarity. The underdog is less threatening, which allows the top dog to reduce doping.

2.4 Extension to n players

Our benchmark model with two players contains the main messages. Because we wish to test our predictions on data, we must account for the fact that competitions in professional sports take place between more than two players, and check whether our main predictions still hold. Assume that there are n players, with $\overline{a}_1 > \overline{a}_2 > ... > \overline{a}_n > \underline{a}_1 > \underline{a}_2 > ... > \underline{a}_n$. The prize structure is defined through positive real numbers $y_n = 0 \leq y_{n-1} \leq ... \leq y_2 \leq y_1 = 1$. We assume that the prize structure satisfies the following convexity condition: for any $1 < j < n$,

$$y_j \leq \frac{1}{2}(y_{j-1} + y_{j+1})$$

Proposition 6 *We have the following:*

(*i*) *As in the two-player case, there always exists a Nash equilibrium. However, when $n \geq 3$, uniqueness does not generally hold*

(*ii*) *Any Nash equilibrium d^* is such that $d_1^* > d_2^* > ... > d_n^*$*

(*iii*) *For c small enough, at any Nash equilibrium d^*, $U_1(d^*) > U_1(0)$ and $U_n(d^*) < U_n(0)$.*

(*iv*) *Assume k prizes are equal to 0 and the first prize with positive value is y_{n-k}. Then, given $\epsilon > 0$, there exists c_0 such that, for any $c < c_0$ and any equilibrium d^*, we have $d_i^* > 1-\epsilon$ for all $i = 1, ..., n-k$ and $d_j^* < \epsilon$ for all $j = n-k+1, ..., n$.*

Part (i) guarantees the existence of a Nash equilibrium, but shows that multiplicity can arise from the model. This is due to the fact that the doping effort of any player is a strategic complement of the doping effort of worse players and a strategic substitute of the doping effort of better players. This cannot arise in the two-player case, where the top dog only has a worse player against him, whereas the underdog only competes against a better player. Multiplicity shows that our model is rich enough to describe complex phenomena, however points (ii) to (iv) show this does not create a problem as long as the cost of doping is low, which is the assumption we make in the next section.

Part (ii), (iii) and (iv) confirm the predictions of the two-player model: better players dope more than worse players, and they benefit from doping (at least the best player does; usually, however, there will be a group of top players who will benefit from doping), while worse players suffer from it. Also, better players tend to dope at maximum, while the worse players tend to abandon doping when the cost is low. This is explained by the following: player 1, by doping at maximum, secures the first position and the highest prize. As a direct consequence, the others behave as if the first player and the first prize did not exist. For them, the game is now a competition with $n-1$ players and $n-1$ prizes. Player 2 is the best player of this competition, and he secures the highest prize (y_2) by doping at maximum. This goes on as long as there is a positive prize to win, but once all these prizes are distributed to the best players, the rest no longer have any incentive to dope.

3 Recovery doping: evidence

The last of the predictions delivered by our model is empirically testable: when the cost of doping tends to zero, performance uncertainty should go to zero and rankings should be almost deterministic, while an increase in the cost of doping should imply an increase in performance uncertainty and rankings should be more noisy. This is what we test in this section.

We provide evidence from a particular sport, cross-country skiing (CCS), and a particular doping technology, synthetic EPO. CCS is chosen for the following reasons: though some of the races are short and involve anaerobic skills, CCS is an endurance sport and therefore particularly exposed to recovery doping. Moreover, there have been a number of doping scandals, and medical studies provide evidence of blood manipulation in the 1990s and 2000s. Finally, point attribution is based on individual ranking in the different races. The tournament structure is obvious, and top athletes have clear incentive to compete at their best level in each race.

We focus on the CCS World Cup, a yearly competition based on 10 to 25 races. We examine the yearly race-specific rankings between 1987 and 2006 and show that, in line with our theory, rankings were relatively noisy before EPO was introduced, then became almost deterministic in the 1990s when the use of EPO was widespread, becoming noisier again after measures against EPO were introduced. We also document an increase in noisiness in the late 1990s-early 2000s, right after the introduction of upper limits on hemoglobin concentration ([Hb]).

3.1 Data and methodology

The CCS World Cup is organized by the Fédération Internationale de Ski (FIS). Skiers receive points based on their ranking and the final ranking is obtained by adding the points collected in each race. We use FIS data on individual rankings in different races and compute for every year how race-specific rankings are correlated with the final ranking. More specifically, we calculate Spearman's rank correlation between every race of a given year and the final ranking that year. For each race in the sample we obtain a race-specific correlation, and we average all race-specific correlations for a given year to compute the yearly correlation. Then we match changes in yearly correlation with changes in anti-doping environment, by controlling for potential biases due to changes in the sport itself.

Figure 1 presents the changes in the anti-doping environment, as well as major events related to the CCS World Cup. Data are not available before 1987 and the sport was organized differently after 2006 with the creation of super races lasting several days like the Tour de Ski. These changes preclude any yearly comparison, so we restrict our attention to the period 1987-2006.

As in cycling, EPO was made available to CCS athletes in the late 1980s. No tests existed. Given the huge effects on endurance athletes' performances, EPO was widely used in the 1990s. Videman et al (2000) document the change in blood profiles observed for CCS athletes, from 1987 to 1999. The profiles show a large increase in mean [Hb] from 1989 to 1996. In the 1996-1997 season, an upper bound was imposed on [Hb], and the threshold became tighter the following season. Correspondingly, Videman et al (ibid) show a decline in max [Hb]. However, mean [Hb] continued to increase until 1999.

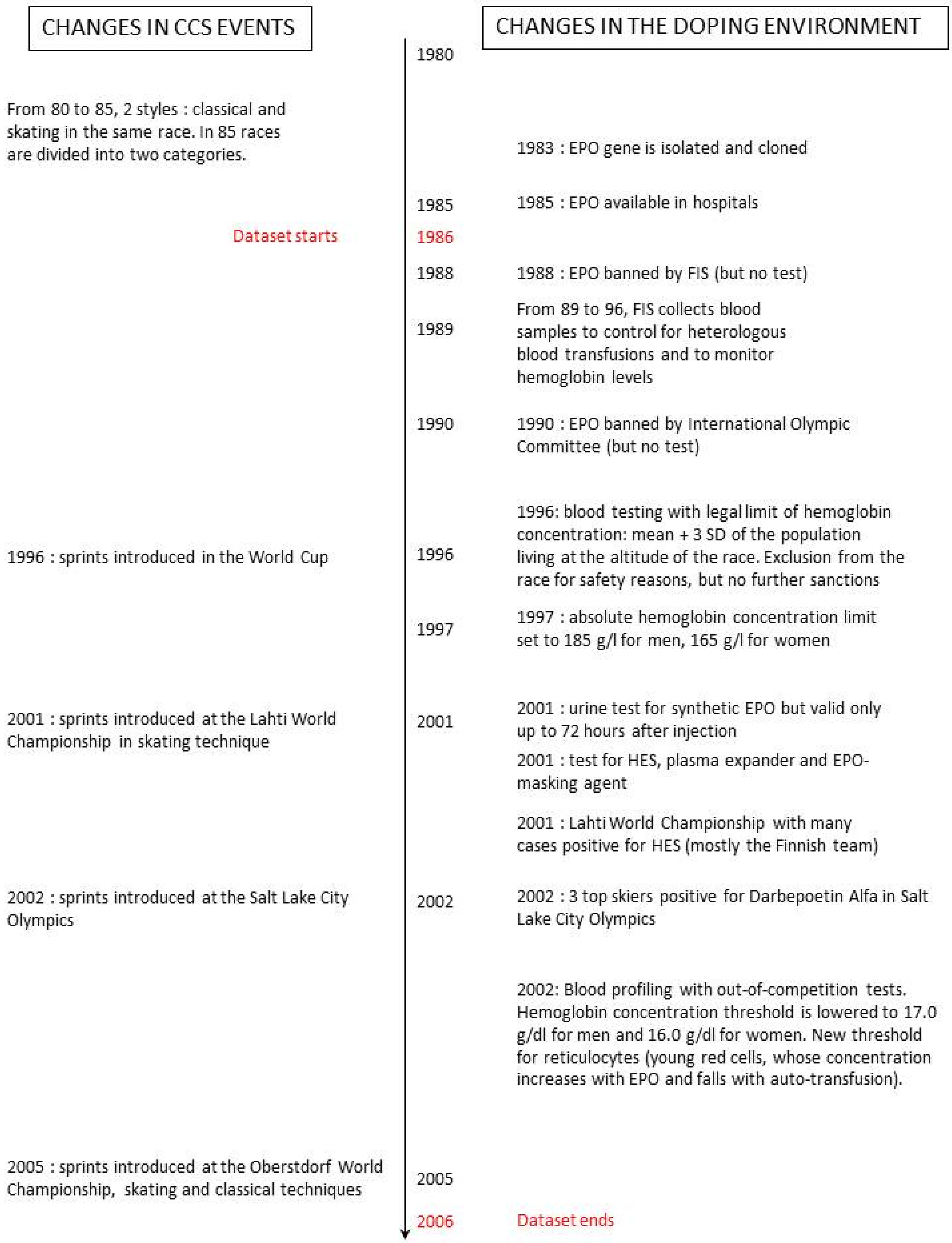

Figure 1: Chronology of changes in CCS events and in the doping environment.

Between 1997 and 2001, the anti-doping environment was characterized by uncertainty. A urine EPO test had been around for years, but it was not efficient. A test for a plasma expander and EPO-masking agent, hydroxyethyl starch or HES, became available in 2001, but athletes were not aware of this. At the 2001 Lahti World Championships, several top skiers in the Finnish team tested positive. In 2002, a test for Darbepoetin Alfa led to the disqualification of Larisa Lazutina and Olga Danilova of Russia and Johann Mühlegg of Spain from their final races in the Winter Olympic Games. Blood profiling was introduced in 2002, together with out-of-competition tests. After that, doping became much more costly. Morkeberg et al (2009) examine CCS athletes' blood profiles between 2001 and 2007 and report an overall decline in [Hb] compared with the 1990s. However, more recent years saw a change in blood manipulations, i.e. skiers seem to be turning back to transfusions instead of EPO injection.

There is also evidence of the extent of blood manipulation being heavily correlated with performance. Stray-Gundersen et al (2003) analyze blood samples collected at the 2001 World Championships. They show that "of the skiers tested and finishing within the top 50 places in the competitions, 17% had "highly abnormal" hematologic profiles, 19% had "abnormal" values, and 64% were normal. Fifty percent of medal winners and 33% of those finishing from 4th to 10th place had highly abnormal hematologic profiles. In contrast, only 3% of skiers finishing from 41st to 50th place had highly abnormal values."

Our interpretation of the facts is that doping costs dramatically and uniformly decreased in the late 1980s-early 1990s, remaining low up to 1997. From 1997 to 2002, it became more hazardous to use EPO and the cost of doping increased. The uncertainty about EPO and EPO-masking agent detection led a number of athletes to under-estimate the cost of doping in the early 2000s, leading to the doping scandals of the 2001 World Championships and 2002 Olympics. After 2002, the cost of doping dramatically increased with certainty.

The FIS rules underwent two major changes over the sample period. Up to 1991, only the top 15 skiers received points in a given race; the dataset reports their rankings, but not the rankings of those finishing after the 15th position. After 1991, the top 30 skiers received points, and the dataset records the rankings of all participants. To harmonize years, we only consider the top 15 skiers in each race. This also allows us to focus on top-level skiers and escape the high volatility in

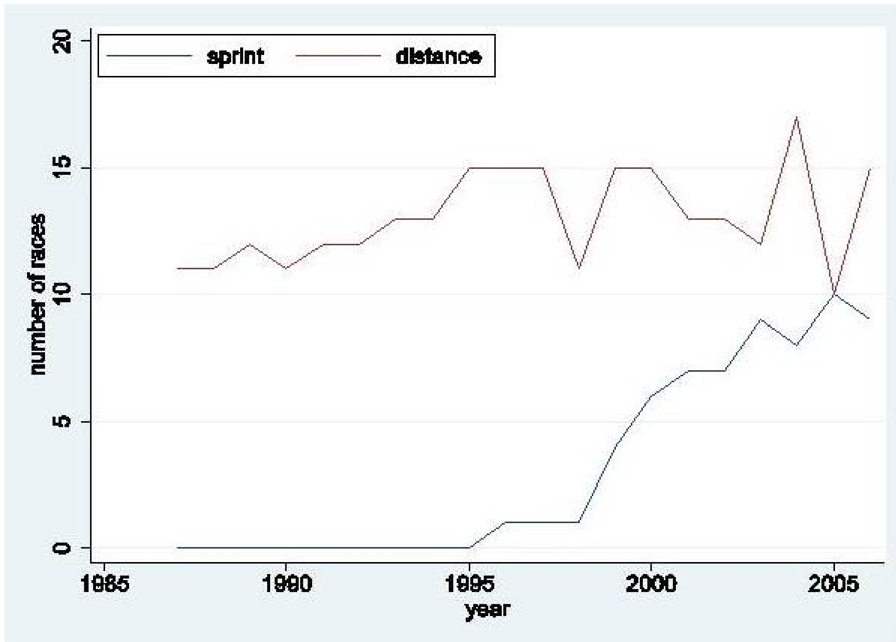

Figure 2: Number of distance and sprint races over time, 1987-2006

rankings generated by including poorly-ranked athletes. Thus, in what follows we will focus either on the top 15 skiers or on the top 10 skiers.

The second change was brought about by the introduction of sprint races in 1996. These races are shorter and thus involve different types of skills. The rankings in such races are by nature less correlated to the final ranking. Figure 2 depicts the evolution in the number of distance and sprint races. This may create a potential bias, as 1996 is also the first year where an upper limit on [Hb] was imposed. To account for that change, we replicate all our correlation computations considering only distance races. We then reconstruct a hypothetical final ranking where only points from distance races are counted and we compare race-specific rankings to that modified final ranking. Removing sprint races from the sample leaves between 10 and 17 races each year, with an average of 13.

Finally, the composition of the group of athletes competing changes from one race to the other. Skiers do not participate in all races, and self-selection leads to changes in the proportion of the top 10 or top 15 skiers competing in each race. If the self-selection pattern varies over time, a resulting potential bias could affect the yearly volatility of race-specific rankings. To avoid such a bias, we therefore only

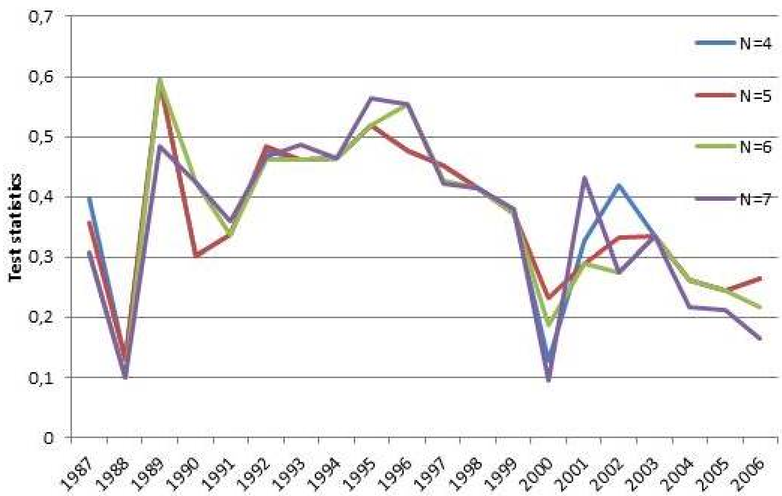

Figure 3: Test statistics as a function of the minimum number of top-15 skiers, 1987-2006

include races where a sufficiently large number of top skiers participate.

To interpret the yearly correlations, we compute p-values of the nullity test. While statistical tests exist for the Spearman correlation, they do not exist for averages of Spearman correlations, so we constructed, by numerical simulations, the distribution of such averages. The following corresponding p-values are to be interpreted as the probability that the ranks obtained by top level athletes are random.

3.2 Results

We start with FIS top 15 skiers and consider all races, including sprint races. The number of races varies from one year to the next, and the number of top 15 skiers varies from one race to the next. We only consider races in which at least N (from 4 to 7) of the top 15 skiers participated. A lower N would mean too much variation across years in the number of top skiers considered, while a higher N would severely limit the number of races considered per year. Figure 3 displays the value of the test statistics for every year of the sample. The higher the value, the higher the correlation between race-specific rankings and yearly final rankings. Figure 4 reports the corresponding p-values of the null assumption whereby there is no correlation between the individual race-specific rankings and the overall ranking.

The four curves send the same message: the patterns of the test statistics and of

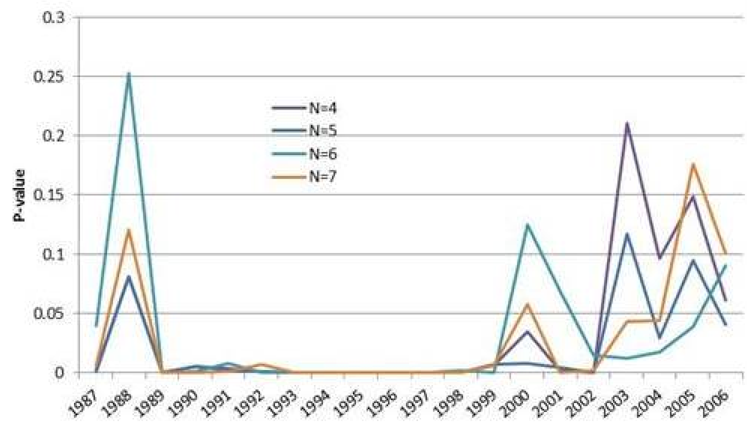

Figure 4: P-values of the zero-correlation test as a function of the minimum number of top-15 skiers, 1987-2006

the p-values are consistent with the doping pattern previously outlined. The p-values are relatively high in the early stages of the sample, plummet in 1989 and remain very close to 0 during the 1990s, in particular in the mid-1990s. Note also that the change in rules on point attribution does not appear to affect the different curves. The sudden increase in 2000 and subsequent decline in 2001 and 2002 coincide with the uncertainty in the anti-doping environment and the 2001 World Championship and 2002 Olympics doping scandals. The increase after 2002 coincides with the introduction of blood profiling.

To take account of the introduction of sprint races in the second half of the 1990s, we now report similar computations based on distance races alone and the associated yearly distance rankings. Being in the top 15 distance skiers does not carry the same weight as being in the top 15 skiers, sprint races included. The less well-ranked skiers may not worry much about their final ranking and could therefore liable to dope. To avoid this bias, Figure 5 focuses on a smaller group of top athletes, those in the top 10 distance skiers, and confirms the general pattern shown by Figure 4. Race-specific rankings are more correlated to the final ranking in the 1990s than in any other period.

3.3 Robustness

The hierarchy among top skiers was remarkably stable during the 1990s, a period of EPO availability. The fact that p-values decrease with the appearance of EPO and

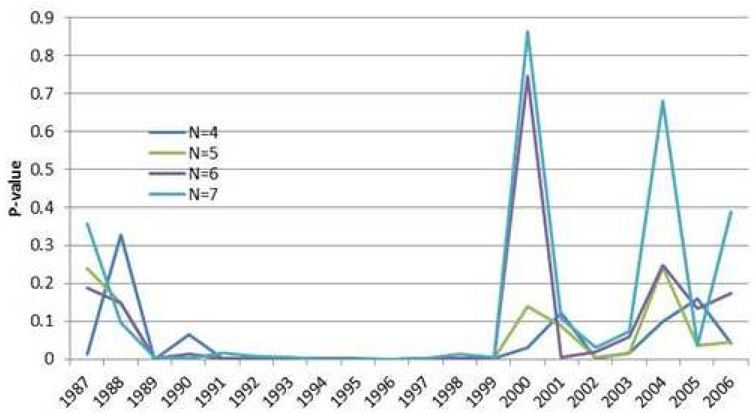

Figure 5: P-values of the zero-correlation test as a function of the minimum number of top-10 skiers, distance races only, 1987-2006

increase with the introduction of blood tests supports our theory. We now discuss four potential weaknesses of our approach.

First, the pool of athletes changes over time. In particular, the entry of new skiers and the exit of older ones may affect pool composition. Irrespective of doping, the new athletes may perform more or less consistently than the others. This could affect the relevance of our computations, especially if the timing of such changes coincides with the overall doping pattern explained above.

The entry of new athletes is unlikely to play a key role, because they do not aim for the top positions. The proportion of newcomers in the top 15 of the final ranking was about 6.5% in 1988 and 1989, and zero in the rest of the sample. Similarly, the proportion of newcomers in the top 10 was zero for the whole sample.

The exit of older skiers is negligible between 1987 and 2006 except for 1998 and 1999, when four of the top ten skiers exited the rankings. Two out of these four skiers belonged to the top 3, one exiting in 1998 and the other in 1999. These two athletes were actually big stars, with a very long career and top positions in the CCS World Cup throughout the 1990s. The emergence of such skiers is a direct implication of our theory. According to our model, a fall in the cost of doping, such as experienced in the late 1980s, favors the best athletes. They choose a higher level of doping and as a consequence the underdogs lower theirs. The probability of top dog wining a competition increases greatly, consistent with what is observed here. However, there is an alternative explanation. These athletes could be genetic freaks who naturally achieve excellent performances. Because these genetic freaks might

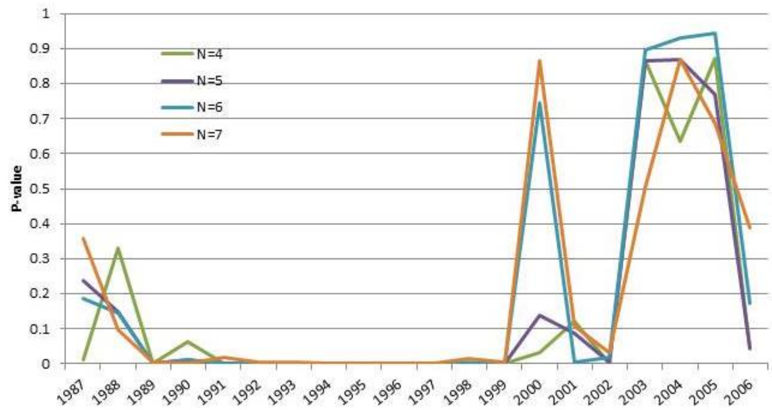

Figure 6: Test statistics as a function of the minimum number of top-10 skiers, without exiters, distance races only, 1988-2006

have contributed to the stability of rankings in the 1990s, we need a robustness check for our empirical analysis. How can we distinguish between the impact of stars and a general decline in the cost of doping? The tournament literature that we discuss in Section 1.1 has already examined the impact of stars on other players' efforts. It argues that the presence of such stars is detrimental to the others' performances through effort reduction. Thus, if ranking stability in the 1990s was only due to such stars, then the other players' rankings should have been very volatile.

To test this argument, we remove the four top 10 skiers who retired in 1998 and 1999 from the dataset. We then reconstruct the hypothetical rankings of each race as well as the yearly rankings. Finally we carry out the same computations as before. Figure 6 shows that removing the superstars from the dataset does not alter our conclusions. Namely, rankings become remarkably stable in the early 1990s, and correlations fall in 2000 and remain low in the 2000s. This implies that the remarkable stability of rankings in the 1990s is not due to the presence of the two superstars.

The second issue with the change of group composition is that our test statistics compare different skiers across different races. To address this issue, for each year we follow several specific competitors and see what happens each time they compete with each other. We consider two groups: a top group composed of the best athletes in the final distance ranking, and a lower-ranking group. We then form pairs of athletes by picking one skier from each group. We form as many pairs as possible and count how many times the competitor from the top group ranks better than

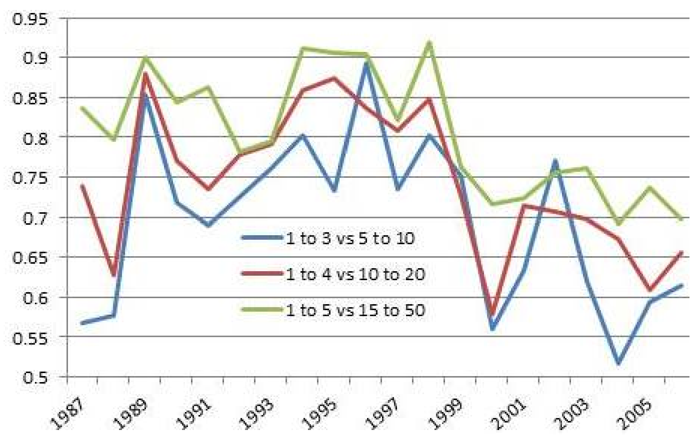

Figure 7: Probability of athletes from the top group ranking better than athletes from the weaker group in a given race, 1987-2006

the competitor from the weaker group.

Figure 7 provides our results for three different athletes grouping. In the first one, we only consider athletes in the top 10. The top group is composed of the top 3, whereas the weaker group is composed of athletes ranked between 5 and 10. We choose to avoid the number 4 so as to ensure a gap between the two groups. In the second grouping, we oppose athletes in the top 4 to athletes ranked between 10 and 20. The third grouping opposes the top 5 to athletes ranked between 15 and 50.

The three curves confirm the general pattern previously identified. The probability of athletes from the top group winning is higher in the 1990s than in the rest of the sample, there is a sharp decline in the late 1990s-2000, immediately followed by a spike in 2002, and a steadier decline thereafter.

Third, changes in the organization of the sport design might have altered athletes' training methods. The emergence of sprint races might have induced athletes to stop specializing. When first introduced, sprint races represented a tiny share of all races, and top athletes did not need to perform well in them. In the 2000s, however, they represented a third of races on average. Rational athletes seeking points for the final ranking now needed to train for both sprint and distance races. Top athletes being more all-round, they might have suffered as a result of less consistent results in distance races. On the contrary, a glance at the most successful race winners in the 2000s shows clear specialization[3]. Petter Northug from Norway

[3] See the Wikipedia page http://en.wikipedia.org/wiki/FIS_Cross-Country_World_Cup

is actually the only skier who managed to win a significant number of races in both the sprint and the distance categories. One such athlete is not enough to impact our results.

Fourth, the overall distance ranking is based on race-specific results. If the number of distance races changes over time, then the correlation between race-specific rankings and the overall ranking changes accordingly. As Figure 2 shows, the number of distance races increases over time. However, the increase occurred between 1989 and 1996, the period when race-specific rankings became less volatile. Thus the bias, if any, tends to weaken the pattern discussed here.

3.4 Discussion

Our arguments apply to any contest with player heterogeneity. Once there is a cheating technology that increases the probability of being in a good state, our conclusions hold: the best players have more incentives to use the technology, and as a consequence the gap between best players and weaker players widens.

Cycling.—Contrary to CCS, cycling is a team sport where tactical skills are important. Moreover, individual ranking in each stage of a multi-stage event is not relevant: what matters is the final time gap between athletes and their direct opponents. Candelon and Dupuy (2014) looked at within-team performance inequality in the Tour de France (TdF). They measured riders' skills by their speed during the prologue – an individual short-distance time trial. They show that the final average speed gap between the leader and each of his teammates increases over time, controlling for skill differential. They argue that this reflects the increasing concentration of TdF monetary rewards on the winner. Our theory provides a complementary interpretation. During the same years, EPO was very popular. Leaders had stronger incentive to dope than their teammates. They became able to maintain the same performance level throughout the TdF. When the results of the different stages were combined, the average speed gap between leaders and teammates increased.

Finance.—Mutual funds compete for capital. Investors allocate their money across funds according to the past performances of each fund. The managers' pay is indexed on their funds' performance, so managers can be seen as athletes competing for the highest return. However, managers differ in skills. The ability to provide a lower volatility at a given mean is an attractive feature of a fund. Fraudulent accounting is one cheating technology that allows managers to reduce the volatility of their performances: temporary losses can be hidden, thereby smoothing the return trajectory of the fund. However, it is illegal because of the risk of bankruptcy. Fraudulent accounting is of greater benefit to the best managers, who can count on their superior ability to deliver better returns at some future date, so as to restore the genuine profitability of the fund.

Academic world.—Scientists compete to publish in the most prestigious journals. Although better researchers have smarter ideas there is some uncertainty about whether an idea will be confirmed or rejected by proper empirical analysis. In other words, there is uncertainty about whether an idea is in a good or bad state. There is a random component here: the history of science is full of beautiful ideas with no empirical support. Here again, there is a cheating method that increases the probability of the idea being in a good state: modifying the data to achieve a better fit with theory. The return on such unethical behavior is actually larger for the best researchers because they have better ideas, and empirical validation of these good ideas makes very strong papers.

4 Conclusion

This paper studies the incentive to cheat in contest-like situations. It focuses on recovery doping, a generic term characterizing cheating technologies that tend to concentrate performances towards the best individual-specific outcomes. Recovery doping has heterogenous effects across agents. We argue that the introduction of such a technology benefits the most talented agents. They become more likely to win, and performance inequality rises. We develop these arguments in a two-player zero-sum game with player heterogeneity and performance uncertainty. Cheating efforts are complementary for the stronger contender, and substitutable for the weaker one. We then examine a specific contest, the cross-country skiing World Cup, and a specific cheating technology, synthetic EPO. Race-specific rankings were more correlated to the final ranking in the EPO era, the 1990s, than in the 2000s. Thus results were more consistent across races when the cost of doping was low, becoming more volatile as the cost of doping increased.

References

Apesteguia, J., Palacios-Huerta, I., 2010. Psychological Pressure in Competitive Environments: Evidence from a Randomized Natural Experiment. American Economic Review 100, 2548-2564.

Bhaskar, V., 2009. Rational Adversaries? Evidence from Randomized Trials in One Day Cricket. Economic Journal 119, 1-23.

Berentsen, A., 2002. The Economics of Doping. European Journal of Political Economy 18, 109-127.

Berentsen, A., Lengwiler, Y., 2004. Fraudulent Accounting and Other Doping Games. Journal of Institutional and Theoretical Economics 160, 402-415.

Berentsen, A., Bruegger, E., Loertscher, S., 2008. On Cheating, Doping and Whistleblowing. European Journal of Political Economy 24, 415-436.

Bird and Wagner, 1997. Sport as a Common Property Resource: A solution to the dilemmas of doping. Journal of Conflict Resolution 41, 749-766.

Candelon, B., Dupuy, A., 2014. Hierarchical organization and performance inequality: evidence from professional cycling. International Economic Review (forthcoming)

Chiappori, P.-A., Levitt, S., Groseclose, T., 2002. Testing Mixed-strategy Equilibria when Players are Heterogeneous: The Case of Penalty Kicks in Soccer. American Economic Review 92, 1138-1151.

Curry, P.A., Mongrain, S., 2009. Deterrence in Rank-Order Tournaments. Review of Law and Economics 5, 723-740.

Eber, N., Thepot, J., 1999. Doping in Sport and Competition Design. Recherches Economiques de Louvain 65, 435-446.

Glisdorf, K, Sukhatme, V., 2008. Tournament Incentives and Match Outcomes in Women's Professional Tennis. Applied Economics 40, 2405-2412.

Klaassen, F.J.G.M., Magnus, J.R., 2009. The Efficiency of Top Agents: An Analysis through Service Strategy in Tennis. Journal of Econometrics 148, 72-85.

Kräkel, M., 2007. Doping and cheating in contest-like situations. European Journal of Political Economy 23, 988-1006.

Lazear, P.E., Rosen, S., 1981. Rank-Order Tournaments as Optimum Labor Contracts. Journal of Political Economy 89, 841-864.

Malueg, D.A., Yates, A.J., 2010. Testing Contest Theory: Evidence from Best-of-Three Tennis Matches. Review of Economics and Statistics 92, 689-692.

Morkeberg, J., Saltin, B., Belhage, B., Damsgaard, R., 2009. Blood profiles in elite cross-country skiers: a 6-year follow-up. Scandinavian Journal of Medicine and Science in Sports 19, 198-205

Rosen, S., 1986. Prizes and Incentives in Elimination Tournaments. American Economic Review 76, 701-15. Stray-Gundersen, J., Videman, T., Penttilä, I., Lereim, I., 2003. Abnormal hematologic profiles in elite cross-country skiers: blood doping or not? Clinical Journal of Sport Medicine 13, 132-137

Sunde, U., 2009. Heterogeneity and Performance in Tournaments: a Test for Incentive Effects Using Professional Tennis Data. Applied Economics 41, 3199-3208.

Videman, T., Lereim, I., Hemmingsson, P., Turner, M.S., Rousseau-Bianchi, M.P., Jenoure, P., Raas, E., Schönhuber, H., Rusko, H., Stray-Gundersen, J., 2000. Changes in hemoglobin values in elite cross-country skiers from 1987-1999. Scandinavian Journal of Medicine and Science in Sports. 2000 10, 98-102

Walker, M., Wooders, J., 2001. Minimax Play at Wimbledon. American Economic Review 91, 1521-1538.

Appendix A - For Online Publication
The standard framework (Section 2.1)

Proposition 1 follows from Propositions 7, 8 and 9 presented below.

• Models (a) and (b)

Two identical players face a binary choice $d = 0$ or $d = 1$. The game is summarized by the following payoff matrix

	0	1
0	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{8}, \frac{5}{8} - c$
1	$\frac{5}{8} - c, \frac{3}{8}$	$\frac{1}{2} - c, \frac{1}{2} - c$

Payoffs in the diagonal cells are obvious. When one player dopes and the other does not, the doped player wins whenever he performs well (with prob. $\frac{1}{2}$), and he gets half the prize when both he and his opponent are in a bad state (with prob. $\frac{1}{2} \times \frac{1}{2}$). He also bears the cost of doping. This is a a typical prisoner's dilemma situation.

Proposition 7 (Homogeneous players)

(i) *This game has a unique equilibrium:* $(0, 0)$ *if* $c > 1/8$ *and* $(1, 1)$ *if* $c < 1/8$

(ii) *If both players dope, the winning probabilities are the same as those without doping*

(iii) $U_i(1, 1) < U_i(0, 0)$

Notice that this easily extends when several doping levels are allowed, i.e. $d \in \{0, d_1, ..., d_k\}$. Whatever the value of c, there is always a unique and symmetric equilibrium. Further, any symmetric doping strategy can be an equilibrium for an appropriate value of c. As the cost increases, the equilibrium level of doping decreases. When players use the same doping strategy, the winning probabilities are unchanged. As a consequence for both players, $U_i(d^*, d^*) < U_i(0, 0)$ for every equilibrium $d^* > 0$.

• Model (c)

Two heterogeneous players face discrete doping possibilities. We assume $\overline{a}_1 > \overline{a}_2$ and $\underline{a}_1 > \underline{a}_2$, and $a(1) > \overline{a}_1 - \overline{a}_2$. With player 1 standing for the top dog and player 2 for the underdog, the payment matrix is:

	0	$\overline{d}$
0	$\frac{3}{4}, \frac{1}{4}$	$\frac{1}{2}, \frac{1}{2} - c$
$\overline{d}$	$\frac{3}{4} - c, \frac{1}{4}$	$\frac{3}{4} - c, \frac{1}{4} - c$

Proposition 8 (Player heterogeneity) *Let $\overline{a}_1 > \overline{a}_2$, $\underline{a}_1 > \underline{a}_2$, and $a(1) > \overline{a}_1 - \overline{a}_2$. We have*

(*i*) *If $c > 1/4$, then the only pure-strategy equilibrium is $d_1^* = d_2^* = 0$;*

(*ii*) *If $c \leq 1/4$, then there is a unique mixed-strategy equilibrium (γ_1^*, γ_2^*), where $\gamma_i \in [0, 1]$ stands for the probability that player i plays the doping strategy. It is such that*

- $\gamma_1^* = 1 - 4c$ *and* $\gamma_2^* = 4c$;
- $U_1(\gamma_1^*, \gamma_2^*) = 3/4 - c < U_1(0, 0) = 3/4$ *and* $U_2(\gamma_1^*, \gamma_2^*) = 1/4 = U_2(0, 0)$.

By elimination of strictly dominant strategies, $(0, 0)$ is the unique pure-strategy equilibrium if and only if $c > 1/4$. Otherwise, there is no pure-strategy equilibrium. The argument is simple: the underdog may choose to fill the natural performance gap by using PEDs. If this is profitable for him, then the other player will choose to dope as a response to the increase in his opponent's maximum performance. An arms race then takes place until the disadvantaged player stops doping, because the quantity of PEDs required has become too high. The best player will respond by quitting doping too, and as they return to the $(0, 0)$ situation, the arms race starts all over again.

• Model (d)

Doping is a continuous variable and players are heterogeneous.

Proposition 9 (Continuous doping efforts) *Assume that $a(d) = d$ and call $\delta := \overline{a}_1 - \overline{a}_2$. Then*

(*i*) *If $4c\delta \geq 1$, $(0, 0)$ is the only Nash equilibrium and the equilibrium payoff is $(3/4, 1/4)$;*

(*ii*) *if* $4c\delta < 1$ *there is a unique mixed Nash equilibrium* (μ_1^*, μ_2^*)*, where the probability distributions* μ_1^* *and* μ_2^* *are given by*

$$\mu_1^*(0) = 4c\delta,\ \mu_1^*(]0, d_1]) = 4cd_1,\ \forall d_1 \in]0, \frac{1}{4c} - \delta];$$

$$\mu_2^*(0) = 4c\delta,\ \mu_2^*(]\delta, d_2]) = 4c(d_2 - \delta),\ \forall d_2 \in]\delta, \frac{1}{4c}].^4$$

and the equilibrium payoff is $(1/2 + c\delta, 1/4)$.

The unique pure strategy equilibrium occurs when either the cost is too high or when the differences in maximum performance levels are too great. In all other cases, there is no pure strategy equilibrium, for the same reasons as above (arms race)[5]. However, there is a unique mixed strategy equilibrium, in which the top dog has a lower payoff than in a world without doping.

Proof. Point (i) is obvious. We focus on proving (ii). We call $Supp(\mu)$ the support of μ, i.e.

$$Supp(\mu) := \{d \in \mathbb{R}_+ :\ \ \mu(]d - \epsilon, d + \epsilon[) > 0\ \ \forall \epsilon > 0\}$$

Recall that $Supp(\mu)$ is the smallest closed set F such that $\mu(F) = 1$.

Lemma 2 *Let* $d \geq 0$. *Then*

$$d \in Supp(\mu_1^*) \Longleftrightarrow d + \delta \in Supp(\mu_2^*).$$

Proof. Pick $d \notin Supp(\mu_1^*)$ and $\epsilon > 0$ such that $\mu_1^*(]d-\epsilon, d+\epsilon[) = 0$. Then we claim that $\mu_2^*(]d + \delta - \epsilon/2, d + \delta + \epsilon/2[) = 0$. If this was not the case, then player 2 could deviate profitably by transferring the weight that μ_2^* puts on $]d+\delta-\epsilon/2, d+\delta+\epsilon/2[$ onto $\{d + \delta - \epsilon\}$. Thus

$$d \notin Supp(\mu_1^*) \Rightarrow d + \delta \notin Supp(\mu_2^*).$$

[4]In other words, μ_1^* and μ_2^* are the sum of a dirac distribution in 0 and a uniform distribution.

[5]The threshold difference in maximum performances, when player 1 is better than player 2, is given by $a((4c)^{-1})$. Indeed, the highest possible doping level for player 2 is $\bar{d}$ such that $5/8 - c\bar{d} > 3/8$ so $\bar{d} < (4c)^{-1}$. When $a_1(0) - a_2(0) > a((4c)^{-1})$ then $(0, 0)$ is the unique equilibrium, otherwise there is no equilibrium.

A reverse argument in the previous step gives

$$d + \delta \notin Supp(\mu_2^*) \Rightarrow d \notin Supp(\mu_1^*). \quad \blacksquare$$

Lemma 3 *We have*

$$\mu_1^*(d_1) > 0 \Rightarrow d_1 = 0; \;\; \mu_2^*(d_2) > 0 \Rightarrow d_2 = 0.$$

Proof. First we show the following:

Let d_1 be such that $\mu_1^*(d_1) > 0$. Then

$$\exists \epsilon > 0 : \;]d_1 + \delta - \epsilon, d_1 + \delta[\subset (Supp(\mu_2^*))^c.$$

Similarly, let $d_2 > \delta$ be such that $\mu_2^*(d_2) > 0$. Then

$$\exists \epsilon > 0 : \;]d_2 - \delta - \epsilon, d_2 - \delta[\subset (Supp(\mu_1^*))^c.$$

Assume that $\mu_1^*(d_1) > 0$ and, for any $\epsilon > 0$, there exists $d(\epsilon) \in]d_1 + \delta - \epsilon, d_1 + \delta[\cap Supp(\mu_2^*)$. Since $\mu_1^*(d_1) > 0$ there exists ϵ small enough so that deviating from $d_1 + \delta - \epsilon$ to $d_1 + \delta$ guarantees a strictly higher[6] payoff to player 2. The second statement can be shown with similar arguments.

Now, assume that $d_1 > 0$ is such that $\mu_1^*(d_1) > 0$. Then, there exists $\epsilon > 0$ such that $\mu_2^*(]d_1 + \delta - \epsilon, d_1 + \delta[) = 0$. Consequently, player 1 can deviate profitably by transferring the weight that μ_1^* puts on d_1 to $d_1 - \epsilon/2$.

For player 2, the same argument states that, for any $d_2 > \delta$, we have $\mu_2^*(d_2) = 0$. Clearly, we have $\mu_2^*(]0, \delta[) = 0$. Consequently we just need to prove that $\mu_2^*(\delta) = 0$. Assume that $\mu_2^*(\delta) > 0$. If $\mu_1^*(0) > 0$ then player 2 can obtain a strictly better payoff by transferring the weight on δ to $\delta + \epsilon$. If $\mu_1^*(0) = 0$ then player 2 can deviate profitably by transferring the weight on δ to 0. $\blacksquare$

Lemma 4 *We have $Supp(\mu_1^*) = [0, b_1]$ and $\mu_1^*(0) > 0$. Also we have $Supp(\mu_2^*) = \{0\} \cup [\delta, b_1 + \delta]$.*

Proof. First, we show that if $[a_1, b_1]$ is the smallest closed interval that contains $Supp(\mu_1^*)$, then $[a_1, b_1] = Supp(\mu_1^*)$. Similarly, if $[a_2, b_2]$ is the smallest closed interval that contains $Supp(\mu_2^*) \setminus \{0\}$, then $[a_2, b_2] = Supp(\mu_2^*) \setminus \{0\}$.

[6] deviating from $d(\epsilon)$ to $d_1 + \delta$ costs her (at most) ϵc, but she increases her payoff by a positive quantity, which is independent of ϵ.

Since $Supp(\mu_1^*)$ is closed, we have $a_1, b_1 \in Supp(\mu_1^*)$. Assume that there exists $c_1 \in]a_1, b_1[$ such that $c_1 \notin Supp(\mu_1^*)$. Then $c_1+\delta \notin Supp(\mu_2^*)$. Now call $\underline{c_1} := \inf\{d > c_1 : \ d \in Supp(\mu_1^*)\}$, (resp. $\overline{c_1} := \sup\{d < c_1 : \ d \in Supp(\mu_1^*)\}$. By a basic property of a Nash equilibrium, player 1 must be indifferent between $\underline{c_1}$ and $\overline{c_1}$, against μ_2^*. However, by lemma 2, we have $\mu_2^*(]\underline{c_1}+\delta, \overline{c_1}+\delta[) = 0$. Hence player 1 has a strictly higher payoff when he plays $\underline{c_1}$ than when he plays $\overline{c_1}$, a contradiction. Exactly the same argument proves the assertion concerning player 2.

We also know, by lemma 2, that $a_2 = a_1 + \delta$ and $b_2 = b_1 + \delta$.

Now, assume that $\mu_1^*(a_1) = 0$. Then we have $U_2(\mu_1^*, 0) > U_2(\mu_1^*, a_1 + \delta)$, and $a_1 + \delta \in Supp(\mu_2^*)$, which is a contradiction to the fact that μ^* is a Nash equilibrium. Consequently, $a_1 = 0$ and $\mu_1^*(a_1) > 0$.[7] This proves that $Supp(\mu_1^*) = [0, b_1]$ and $\mu_1^*(0) > 0$. Also we have $Supp(\mu_2^*) = \{0\} \cup [\delta, b_1 + \delta]$. ■

We are now ready to prove the proposition. First, notice that, since $U_1(0, \mu_2^*) = U_1(b_1, \mu_2^*)$, we have

$$\frac{3}{4}\mu_2^*(0) + \frac{1}{2}(1 - \mu_2^*(0)) = 3/4 - cb_1;$$

Hence $\frac{1}{4}(1 - \mu_2^*(0)) = cb_1$

On the other hand, $\lim_{\epsilon \to 0^+} U_2(\mu_1^*, \delta + \epsilon) = U_2(\mu_1^*, b_1 + \delta)$[8], i.e.

$$-c\delta + \frac{1}{2}\mu_1^*(0) + \frac{1}{4}(1 - \mu_1^*(0)) = -c(b_1 + \delta) + \frac{1}{2},$$

which gives $cb_1 = \frac{1}{4}(1 - \mu_1^*(0))$. As a consequence, we have $\mu_2^*(0) = \mu_1^*(0) > 0$. Since $U_2(\mu_1^*, 0) = \frac{1}{4}$, we necessarily have $-c(\delta + b_1) + 1/2 = 1/4$, i.e. $b_1 = \frac{1}{4c} - \delta$ and $\mu_1^*(0) = \mu_2^*(0) = 4c\delta$.

To see why the distributions μ_1^* and μ_2^* are uniform respectively on $]0, b_1]$ and $[\delta, b_1 + \delta]$, note that, for player 2, we must have $U_2(\mu_1^*, 0) = U_2(\mu_1^*, d_2)$ for any $d_2 \in]\delta, b_1 + \delta]$. Hence

$$\frac{1}{4} = \frac{1}{4} + \frac{1}{4}\mu_1^*([0, d_2 - \delta]) - cd_2,$$

which means that $\mu_1^*(]0, d_2 - \delta]) = 4c(d_2 - \delta)$, for any $d_2 \in]\delta, b_1 + \delta]$ and μ_1^* is uniform on $]0, b_1]$. Analogously, μ_2^* is uniform on $[\delta, b_1 + \delta]$.

[7] Recall that 0 is the only point μ_1^* can put a positive weight on.

[8] Note that here we need to take the limit because the payoff function of player 2 is discontinuous in δ when player 1 plays μ_1^*.

It is clear that no deviation is profitable for any player as, by construction of μ^*, we have

$$U_1(d_1, \mu_2^*) = c\delta + \frac{1}{2},\ \forall d_1 \in [0, b_1]; \quad U_1(d_1, \mu_2^*) = \frac{3}{4} - cd_1 < \frac{1}{2} + c\delta,\ \forall d_1 > b_1.$$

Also

$$U_2(\mu_1^*, d_2) = \frac{1}{4},\ \forall d_2 \in \{0\} \cup]\delta, d_1 + \delta]; \quad U_2(\mu_1^*, d_2) = 1/2 - cd_2 < 1/4,\ \forall d_2 > b_1 + \delta.$$

The proof is complete. ■

Appendix B - Results of the general model

Proof of Lemma 1. Assume (d_1^*, d_2^*) is a Nash equilibrium such that $\overline{a}_1(d_1^*) = \overline{a}_2(d_2^*)$. Then

$$\lim_{\epsilon \to 0^+} U_1(d_1^* + \epsilon, d_2) > U_1(d_1^*, d_2),$$

a contradiction.

Next, if (d_1^*, d_2^*) is a Nash equilibrium such that $\overline{a}_1(d_1^*) < \overline{a}_2(d_2^*)$ then necessarily $d_1^* = 0$. Indeed,

$$U_1(d_1^*, d_2^*) = \frac{1}{2} - h(d_2^*) - c.d_1^*$$

which can only be sustained as an equilibrium if $d_1^* = 0$ This implies $d_2^* = \text{Argmax } U_2(0, \cdot)$ where

$$U_2(0, d_2) = \frac{1}{2} + h(d_2) - c.d_2$$

so that $d_2^* = (h')^{-1}(c)$. Furthermore d_2^* is such that $\overline{a}_2(d_2^*) > \overline{a}_1(0)$, i.e. $d_2^* > a^{-1}(\overline{a}_1 - \overline{a}_2)$.

Also

$$U_2(0, d_2^*) > U_2(0, 0)$$

so that $\frac{1}{2} + h(d_2^*) > c.d_2^* + \frac{1}{4}$

We want to show that this cannot be an equilibrium. Assume player 2 chooses d_2^* and player 1 chooses d_2^* as well. Then $\overline{a}_1(d_2^*) > \overline{a}_2(d_2^*)$ and

$$U_1(d_2^*, d_2^*) - U_1(0, d_2^*) = \left(\frac{1}{2} + h(d_2^*)\right)\left(\frac{1}{2} + h(d_2^*)\right) - c.d_2^*$$

Using $\frac{1}{4} + h(d_2^*) > c.d_2^*$ we have

$$U_1(d_2^*, d_2^*) - U_1(0, d_2^*) > \left(\frac{1}{2} + h(d_2^*)\right)\left(\frac{1}{2} + h(d_2^*)\right) - \left(\frac{1}{4} + h(d_2^*)\right) = (h(d_2^*))^2 > 0$$

■

Proof of Proposition 2. Call V_i the payoff functions in the game with $a(\cdot) \neq 0$ and U_i the payoffs in the game with $a(\cdot) = 0$. Player 1 is better off in the game with $a(\cdot) = 0$: $U_1(d_1, d_2) \geq V_1(d_1, d_2)$ and conversely for player 2: $U_2(d_1, d_2) \leq V_2(d_1, d_2)$. Moreover, if $\overline{a}_1(d_1) > \overline{a}_2(d_2)$ then $U_i(d_1, d_2) = V_i(d_1, d_2)$.

Let $d^* = (d_1^*, d_2^*)$ be a Nash equilibrium in the game with $a(\cdot) \neq 0$. By lemma 1,

$\overline{a}_1(d_1^*) > \overline{a}_2(d_2^*)$ hence $U_i(d_1^*, d_2^*) = V_i(d_1^*, d_2^*)$. Consequently, for any d_2,

$$U_2(d_1^*, d_2^*) = V_2(d_1^*, d_2^*) \geq V_2(d_1^*, d_2) \geq U_2(d_1^*, d_2),$$

which means that d_2^* is a best response of player 2 against d_1^* in the game with $a(\cdot) = 0$.

On the other hand, d_1^* is a best response of player 1 against d_2^* in the game with $a(\cdot) \neq 0$. Let $\tilde{d}_1 := Br^1(d_2^*)$, the best response of player 1 against d_2^* in the game with $a(\cdot) = 0$. We have $\tilde{d}_1 > d_2^*$ (best responses of player 1 are always greater than best responses of player 2; see the proof of point (ii) of Proposition 3 in the next section). Thus

$$V_1(\tilde{d}_1, d_2^*) = U_1(\tilde{d}_1, d_2^*) \geq U_1(d_1^*, d_2^*) = V_1(d_1^*, d_2^*) \geq V_1(\tilde{d}_1, d_2^*).$$

This implies $U_1(\tilde{d}_1, d_2^*) = U_1(d_1^*, d_2^*)$, and the proof is complete. ■

Proof of Proposition 3. The payoff functions are given by

$$U_1(d_1, d_2) = -cd_1 + \left(\frac{1}{2} + h(d_1)\right) + \left(\frac{1}{2} - h(d_1)\right)\left(\frac{1}{2} - h(d_2)\right)$$

and

$$U_2(d_1, d_2) = -cd_2 + \left(\frac{1}{2} - h(d_1)\right)\left(\frac{1}{2} + h(d_2)\right)$$

The map h' is strictly decreasing from $(0, 1]$ to $[0, +\infty)$. Hence the inverse function $(h')^{-1}$ is well defined and strictly decreasing from $[0, +\infty)$ to $(0, 1]$.

The best-response function of each player results from the first-order condition:

$$Br^1(d_2) = (h')^{-1}\left(\frac{c}{1/2 + h(d_2)}\right),$$

and

$$Br^2(d_1) = (h')^{-1}\left(\frac{c}{1/2 - h(d_1)}\right).$$

(i) Existence follows from Brouwer Theorem and the fact that $Br := (Br^1, Br^2)$ is continuous and maps $[0, 1]^2$ to itself. Uniqueness follows from the fact that Br^1 is increasing in d_2 and Br^2 is decreasing in d_1, since $(h')^{-1}$ is decreasing.

(ii) For any d_1, d_2,

$$h'^{-1}\left(\frac{c}{(1/2 + h(d_2))}\right) \geq h'^{-1}\left(\frac{c}{(1/2 - h(d_1))}\right),$$

which means that $Br^1(d_2) \geq Br^2(d_1)$, $\forall d_1, d_2$. In particular, $d_1^* \geq d_2^*$. ■

Proof of Proposition 4. Point (i) is a trivial consequence of

$$U_2(d_1^*, d_2^*) \leq \frac{1}{4} - cd_2^* = U_2(0,0) - cd_2^*.$$

Next,

$$\Delta_1(c) := U_1(d_1^*, d_2^*) - U_1(0,0) = -cd_1^* + \frac{1}{2}(h(d_1^*) - h(d_2^*)) + h(d_1^*)h(d_2^*)$$

Because $\lim_{c\to 0^+} d_1^* = 1$ and $\lim_{c\to 0^+} d_2^* = 0$ (see Proposition 5). Hence,

$$\lim_{c\to 0} \Delta_1(c) = 1/4 > 0,$$

which proves (ii). ■

Proof of Proposition 5. Notice that, for any $c > 0$, (d_1^*, d_2^*) is the unique solution of the system

$$\begin{cases} h'(d_1)(1/2 + h(d_2)) - c &= 0 \\ h'(d_2)(1/2 - h(d_1)) - c &= 0 \end{cases}$$

where in fact $(d_1^*, d_2^*) \equiv (d_1^*(c), d_2^*(c))$.

(i) The first limit is a straightforward consequence of the form of the best response maps. For the second limit, we have, for any $c > 0$,

$$h'(d_2^*) = \frac{c}{1/2 - h(d_1^*)}$$

By concavity of h, we have

$$\frac{1}{2} - h(d_1^*) \leq h'(d_1^*)\,(1 - d_1^*) = \frac{c(1 - d_1^*))}{1/2 + h(d_2^*)} \leq 2c(1 - d_1^*).$$

Thus we obtain

$$h'(d_2^*) \geq \frac{1}{2(1-d_1^*)} \rightarrow_{c\to 0^+} +\infty,$$

which implies that

$$\lim_{c\to 0^+} d_2^* = 0.$$

(ii) and (iii): Using the implicit function theorem, for $c > 0$,

$$\frac{d(d_1^*)}{dc} = \frac{1}{J}\left(h''(d_2^*)\left(\frac{1}{2} - h(d_1^*)\right) - h'(d_1^*)h'(d_2^*)\right),$$

and

$$\frac{d(d_2^*)}{dc} = \frac{1}{J}\left(h''(d_1^*)\left(\frac{1}{2} + h(d_2^*)\right) + h'(d_1^*)h'(d_2^*)\right),$$

where J is positive:

$$J = h''(d_1^*)h''(d_2^*)\left(\frac{1}{2} - h(d_1^*)\right)\left(\frac{1}{2} + h(d_2^*)\right) + (h'(d_2^*)h'(d_1^*))^2.$$

The numerator in the $\frac{d(d_1^*)}{dc}$ is negative, which proves (ii).

As for d_2^*, we can write its derivative as follows:

$$\frac{d(d_2^*)(c)}{dc} = \frac{1}{D}\left(h''(d_1^*)\left(\frac{1}{2} - h(d_1^*)\right) + (h'(d_1^*))^2\right),$$

where D is positive:

$$D = h''(d_1^*)h''(d_2^*)\left(\frac{1}{2} - h(d_1^*)\right)^2 + (h'(d_1^*))^3 h'(d_2^*).$$

We study the sign of the numerator, as $c \to 0^+$:

$$h'(d) \sim_{d\to 1} h'(1) - h''(1)(1-d) = -h''(1)(1-d);$$

Moreover,

$$\frac{1}{2} - h(d) \sim_{d\to 1} -h'(1)(1-d) - \frac{h''(1)}{2}(1-d)^2 = -\frac{h''(1)}{2}(1-d)^2$$

Hence

$$h''(d)\left(\frac{1}{2}-h(d)\right)+h'(d)^2 \sim_{d\to 1} -\frac{h''(1)^2}{2}(1-d)^2+(-h''(1)(1-d))^2 = \frac{h''(1)^2}{2}(1-d)^2.$$

This proves that there exists some $b > 0$ such that

$$\frac{d(d_2^*)}{dc} > 0,$$

for $c \in (0, b)$.■

Proof of Proposition 6. If $i < j$, we denote by $N(i,j)$ the random variable equal to the number of players among players $\{i, ..., j\}$ who are in a bad state. Then, conditioning respectively on the events *"player i is in a good state"* and *"player i is in a bad state"*, the payoff of player i can be written as

$$\begin{aligned} U_i(d_1, ..., d_n) &= \left(\frac{1}{2}+h(d_i)\right)\sum_{k=0}^{i-1} y_{i-k}\mathbb{P}[N(1,i-1)=k] \\ &+ \left(\frac{1}{2}-h(d_i)\right)\sum_{l=0}^{n-i} y_{n-l}\mathbb{P}[N(i+1,n)=l] - cd_i \end{aligned}$$

The first order conditions are, for player i,

$$h'(d_i)\left(\sum_{k=0}^{i-1} y_{i-k}\mathbb{P}[N(1,i-1)=k] - \sum_{l=0}^{n-i} y_{n-l}\mathbb{P}[N(i+1,n)=l]\right) = c,$$

which we denote by

$$\alpha_i(d)h'(d_i) = c.$$

In the sequel, we will often omit the entry d in α_i, if there is no ambiguity.

Proof of (i)**.** Let $n = 3$ and $y := y_2 < 1/2$. We show the following: given $c > 0$ and $y \in]0, 1/2[$ there exists a function h satisfying assumption 2.1 and $0 < d_3 < \overline{d}_3 < \overline{d}_2 < d_2 < d_1 < \overline{d}_1$ such that

$$\alpha_i(d)h'(d_i) = \alpha_i(\overline{d})h'(\overline{d}_i) = c \;\forall i = 1, ..., 3.$$

This implies that d and $\overline{d}$ are two distinct Nash equilibria.

Let $\epsilon > 0$. Pick h_1, h_2, h_3 such that $1/4 + \epsilon > h_1 > h_2 > h_3 > 1/4 - \epsilon$ and define $\overline{h}_3 = h_3 + \epsilon$, $\overline{h}_2 = h_2 - \epsilon^2$, $\overline{h}_1 = h_1 + \epsilon^3$ It is straightforward to check that for ϵ small enough we have

a) $1 > \overline{h}_1 > h_1 > h_2 > \overline{h}_2 > \overline{h}_3 > h_3 > 0$,

b) $(\overline{h}_2 + \overline{h}_3)/2 - (1-2y)\overline{h}_2\overline{h}_3 > (h_2 + h_3)/2 - (1-2y)h_2h_3$

c) $(\overline{h}_1 + \overline{h}_2)/2 - (1-2y)\overline{h}_1\overline{h}_2 < (h_1 + h_2)/2 - (1-2y)h_1h_2$

Define

$$\alpha_1 = 3/4 + y/2 + (h_2 + h_3)/2 - (1-2y)h_2h_3,$$

$$\alpha_2 = 1/2 + y/2 - (1-y)h_1 + yh_3, \; \alpha_3 = 1/4 + y/2 - (h_1 + h_2)/2 + (1-2y)h_1h_2$$

and the $\overline{\alpha_i}$ analogously.

Clearly $0 < \alpha_3 < \overline{\alpha}_3 < \overline{\alpha}_2 < \alpha_2 < \alpha_1 < \overline{\alpha}_1 < 1$. Let $\overline{d}_1 > d_1 > d_2 > \overline{d}_2 > \overline{d}_3 > d_3 > 0$ be chosen such that

$$\alpha_3 \frac{(\overline{h}_3 - h_3)}{c} < \overline{d}_3 - d_3 < \overline{\alpha}_3 \frac{(\overline{h}_3 - h_3)}{c}$$

$$\overline{\alpha}_3 \frac{(\overline{h}_2 - \overline{h}_3)}{c} < \overline{d}_2 - \overline{d}_3 < \overline{\alpha}_2 \frac{(\overline{h}_2 - \overline{h}_3)}{c}$$

$$\overline{\alpha}_2 \frac{(h_2 - \overline{h}_2)}{c} < d_2 - \overline{d}_2 < \alpha_2 \frac{(h_2 - \overline{h}_2)}{c}$$

$$\alpha_2 \frac{(h_1 - h_2)}{c} < d_1 - d_2 < \alpha_1 \frac{(h_1 - h_2)}{c}$$

$$\alpha_1 \frac{(\overline{h}_1 - h_1)}{c} < \overline{d}_1 - d_1 < \overline{\alpha}_1 \frac{(\overline{h}_1 - h_1)}{c}$$

It is always possible and, if ϵ was picked small enough at the start of the proof, it can be constructed so that $\overline{d}_1 < 1$. We now construct our function h. First let h be such that, for $i = 1, ..., 3$,

$$h(d_i) = h_i, \; h'(d_i) = c/\alpha_i; \; h(\overline{d}_i) = \overline{h}_i, \; h'(\overline{d}_i) = c/\overline{\alpha_i},$$

The set of inequalities above implies that h can be extended as a twice differentiable function, strictly increasing and strictly concave, which means that it satisfies assumption 2.1. ■

Proof of (ii)**.** We show the following: Let $n > i \geq 1$ and $d = (d_1, ..., d_n) \in [0,1]^n$. Then we necessarily have $\alpha_i(d) > \alpha_{i+1}(d)$. As an immediate consequence, if d^* is a Nash equilibrium then we have $d_1^* > d_2^* > ... > d_n^*$. Note that, conditioning on the events *"player i is in a good/bad state"*, we have

$$\begin{aligned}
\sum_{k=0}^{i} y_{i-k+1}\mathbb{P}[N(1,i)=k] &= \left(\frac{1}{2}+h(d_i)\right)\sum_{k=0}^{i-1} y_{i-k+1}\mathbb{P}[N(1,i-1)=k] \\
&+ \left(\frac{1}{2}-h(d_i)\right)\sum_{k=0}^{i-1} y_{i-k}\mathbb{P}[N(1,i-1)=k]
\end{aligned}$$

As a consequence

$$\begin{aligned}
&\sum_{k=0}^{i-1} y_{i-k}\mathbb{P}[N(1,i-1)=k] - \sum_{k=0}^{i} y_{i-k+1}\mathbb{P}[N(1,i)=k] \\
= &\left(\frac{1}{2}+h(d_i)\right)\sum_{k=0}^{i-1}(y_{i-k}-y_{i-k+1})\mathbb{P}[N(1,i-1)=k]
\end{aligned}$$

Analogously, conditioning on the events *"player i+1 is in a good/bad state"*, we have

$$\begin{aligned}
\sum_{l=0}^{n-i} y_{n-l}\mathbb{P}[N(i+1,n)=l] &= \left(\frac{1}{2}+h(d_{i+1})\right)\sum_{l=0}^{n-i-1} y_{n-l}\mathbb{P}[N(i+2,n)=l] \\
&+ \left(\frac{1}{2}-h(d_{i+1})\right)\sum_{l=0}^{n-i-1} y_{n-l-1}\mathbb{P}[N(i+2,n)=l]
\end{aligned}$$

which implies

$$\begin{aligned}
&\sum_{l=0}^{n-i} y_{n-l}\mathbb{P}[N(i+1,n)=l] - \sum_{l=0}^{n-i-1} y_{n-l}\mathbb{P}[N(i+2,n)=l] \\
= &\left(\frac{1}{2}-h(d_{i+1})\right)\sum_{l=0}^{n-i+1}(y_{n-l-1}-y_{n-l})\mathbb{P}[N(i+2,n)=l]
\end{aligned}$$

Finally, we obtain

$$\alpha_i - \alpha_{i+1} = \left(\frac{1}{2} + h(d_i)\right) \sum_{k=0}^{i-1} (y_{i-k} - y_{i-k+1}) \mathbb{P}[N(1, i-1) = k] \quad (2)$$

$$- \left(\frac{1}{2} - h(d_{i+1})\right) \sum_{l=0}^{n-i-1} (y_{n-l-1} - y_{n-l}) \mathbb{P}[N(i+2, n) = l] \quad (3)$$

Notice that $\sum_{k=0}^{i-1} \mathbb{P}[N(1, i-1) = k] = \sum_{l=0}^{n-i+1} \mathbb{P}[N(i+2, n) = l] = 1$. Moreover, by convexity of the family $(y_k)_k$ we have

$$y_{i-k} - y_{i-k+1} \geq y_{n-l-1} - y_{n-1} \;\forall\, 0 \leq k \leq i-1, \forall\, 0 \leq l \leq n-i-1,$$

with equality only for $l = n-i-1$ and $k = 0$. The sum in (2) is a convex combination of terms of the form $y_{i-k} - y_{i-k+1}$ multiplied by a real number greater than $1/2$, whereas the sum in (3) is a convex combination of terms of the form $y_{n-l-1} - y_{n-l}$ multiplied by a real number smaller than $1/2$. Hence we obtain that $\alpha_i - \alpha_{i+1} > 0$. ■

Proof of (*iii*)**.** When c is close to 0, d_1^* is close to 1 (see the proof of (*iv*) below), thus the payoff of player 1 is close to $y_1 = 1$. When doping does not exist, his payoff is a convex combination of all prizes y_1 to y_n, which is trivially smaller than y_1. Conversely for player n. ■

Proof of (*iv*)**.** Assume first that only $y_n = 0$. For $i < n$ it is sufficient to prove that α_i is bounded from below by a positive constant that does not depend on d. For $i < n$, we have $\mathbb{P}[N(i+1, n) = n-i] < 1/2$. Hence

$$\alpha_i > y_i - \left(\frac{1}{2} y_{i+1} + \frac{1}{2} y_i\right) = \frac{1}{2}\left(y_i - y_{i+1}\right) > 0$$

Now we turn to $i = n$. We have

$$\begin{aligned}
\alpha_n &= \sum_{k=0}^{n-1} y_{n-k}\mathbb{P}(N(1,n-1)=k) \\
&\leq \sum_{k=1}^{n-1} \mathbb{P}(N(1,n-1)=k) = 1-\mathbb{P}(N(1,n-1)=0) \\
&\leq 1-\prod_{i=1}^{n-1}(1-(1/2-h(d_i))) \\
&\leq \sum_{i=1}^{n-1}(1/2-h(d_i)) \\
&\leq \sum_{i=1}^{n-1} h'(d_i)(1-d_i) \\
&\leq \sum_{i=1}^{n-1} \frac{c(1-d_i)}{\alpha_i}
\end{aligned}$$

We proved that, for any Nash equilibrium $(d_1^*, d_2^*, ..., d_n^*)$ we have

$$h'(d_i) < \frac{2c}{y_i - y_{i+1}},\ \forall i = 1, ..., n-1;\ \ h'(d_n) \geq \frac{\alpha_{n-1}}{\sum_{i=1}^{n-1} 1 - d_i^*}$$

This concludes the proof when $y_{n-1} > y_n$.

Now, assume that $y_{n-1} = y_n = 0$. For player $n-1$, it is as if player n was not there: if player $n-1$ is in a good state, he will beat player n regardless of his performance; if player $n-1$ is in a bad state, he will get 0 whatever happens. As a result, player $n-1$ acts as if he was competing in a race with $n-2$ opponents. Because he is the worst player, $d_{n-1}^* \to 0$ as the cost goes to zero. This reasoning trivially generalizes. ■