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Statistical method to estimate regime-switching Lévy model

Julien CHEVALLIER and Stéphane GOUTTE

Abstract The regime-switching Lévy model combines jump-diffusion under the form of a Lévy process, and Markov regime-switching where all parameters depend on the value of a continuous time Markov chain. We start by giving general stochastic results. Estimation is performed following a two-step procedure. The EM-algorithm is extended to this new class of jump-diffusion regime-switching models. An empirical application is dedicated to the study of financial and commodity time series. When comparing the results with (i) non regime-switching models, and (ii) continuous regime-switching models (where the Lévy process is replaced by a classic Brownian motion), the Lévy regime-switching model outperforms other competitors.

1 Introduction

This paper proposes new statistical methods to estimate regime-switching Lévy models that are both efficient and practicable. Our goal lies in estimating a Markov-switching model augmented by jumps, under the form of a Lévy process. This particular class of stochastic processes is entirely determined by a drift, a scaled Brownian motion and an independent pure-jump process. The estimation strategy relies on a two-step procedure: by estimating first the diffusion parameters in presence of switching, and second the Lévy jump component by means of separate Normal Inverse Gaussian distributions fitted to each regime. Computationally, the EM algorithm is extended to this new class of jump-diffusion regime-switching model. An

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empirical application is proposed for financial and commodity data. We demonstrate the goodness-of-fit of the regime-switching Lévy model (versus Brownian regime-switching or non regime-switching models), and thereby illustrate the interest to resort to that kind of model in financial economics.

The remainder of the paper is structured as follows. Section 2 introduces the rationale behind Lévy and Markov-switching modeling. Section 3 develops the stochastic model. Section 4 details the estimation method. Section 5 provides an empirical application. Section 6 concludes.

2 Background

In this preliminary section, we review the basic intuitions behind our modeling strategy. Lévy processes have many appealing properties in financial economics, and constitute the first building block of our model. Second, we recall the very intuitive interpretation of the aperiodic, irreducible and ergodic Markov chain.

2.1 Lévy jumps

Jumps are discontinuous variations in assets' prices. By nature, jumps consist of rare and dramatic events that dominate the trading days during which they occur. In financial economics, jumps are expected to appear due to dividend payments, micro-crashes due to short-term liquidity challenges or news, such as macroeconomic announcements. Such events have been made partly accountable for the non-Gaussian feature of financial returns, as they can only be captured by fat-tailed distributions. Hence, by definition, jumps generate returns that lie outside their usual scale of value.

Jumps matter both to investors, and to countries that produce and consume commodities. In the case of investors, jumps can be either significant investing opportunities or massive threats to profits and losses, depending on each investor's positioning. In each case, jumps modify expected returns in an unexpected way. The same logic applies to producers and consumers: sudden and large variation in asset prices endanger the forecasting of sales profit or the hedging strategies put in place to smooth costs. Hence, the higher the jump activity, the higher the uncertainty for market participants. This is why measuring jumps matters. Given that jumps are dramatic events from a financial history perspective, building statistical evidence around them seems of primary importance.

Lévy processes can be thought of as a combination of a diffusion process and a jump process. Both Brownian motion (i.e. a pure diffusion process) and Poisson processes (i.e. pure jump processes) are Lévy processes. As such, Lévy processes represent a tractable extension of Brownian motion to infinitely divisible distributions. In

addition, Lévy processes allow the modeling of discontinuous sample paths, whose properties match those of empirical phenomena such as financial time series.

2.2 Markov-switching

The normal behavior of economies is occasionally disrupted by dramatic events that seem to produce quite different dynamics for the variables that economists study. Chief among these is the business cycle, in which economies depart from their normal growth behavior and a variety of indicators go into decline.

The regime at any given date is presumed to be the outcome of a Markov chain whose realizations are unobserved to the econometrician. The task facing the econometrician is to characterize the regimes and the law that governs the transitions between them. These parameters estimates can then be used to infer which regime the process was in at any historical date. Although the state of the business cycle is not observed directly by the econometrician, the statistical model implies an optimal way to form an inference about the unobserved variable and to evaluate the likelihood function of the observed data.

In this paper, we illustrate the statistical methods that allow to combine Markov-switching models with Lévy jump modelling.

3 The stochastic model

Let (ω, \mathcal{F}, P) be a filtered probability space and T be a fixed terminal time horizon. We propose in this paper to model the dynamic of a sequence of historical values of price using a regime-switching stochastic jump-diffusion. This model is defined using the class of Lévy processes.

3.1 Lévy Process

Definition 1. A Lévy process L_t is a stochastic process such that

1. $L_0 = 0$.
2. For all $s > 0$ and $t > 0$, we have that the property of stationary increments is satisfied. i.e. $L_{t+s} - L_t$ as the same distribution as L_s .
3. The property of independent increments is satisfied. i.e. for all $0 \leq t_0 < t_1 < \dots < t_n$, we have that $L_{t_i} - L_{t_{i-1}}$ are independent for all $i = 1, \dots, n$.
4. L has a Cadlag paths. This means that the sample paths of a Lévy process are right continuous and admit a left limits.

Remark 1. In a Lévy process, the discontinuities occur at random times.

3.2 Markov-switching

Definition 2. Let $(Z_t)_{t \in [0, T]}$ be a continuous time Markov chain on finite space $\mathcal{S} := \{1, 2, \dots, K\}$. Denote $\mathcal{F}_t^Z := \{\sigma(Z_s); 0 \leq s \leq t\}$, the natural filtration generated by the continuous time Markov chain Z . The generator matrix of Z , denoted by Π^Z , is given by

$$\Pi_{ij}^Z \geq 0 \quad \text{if } i \neq j \text{ for all } i, j \in \mathcal{S} \quad \text{and} \quad \Pi_{ii}^Z = -\sum_{j \neq i} \Pi_{ij}^Z \quad \text{otherwise.} \quad (1)$$

Remark 2. The quantity Π_{ij}^Z represents the switch from state i to state j .

3.3 Regime-switching Lévy

Let us define the regime-switching Lévy Model:

Definition 3. For all $t \in [0, T]$, let Z_t be a continuous time Markov chain on finite space $\mathcal{S} := \{1, \dots, K\}$ defined as in Definition 2. A regime-switching model is a stochastic process (X_t) which is solution of the stochastic differential equation given by

$$dX_t = \kappa(Z_t)(\theta(Z_t) - X_t)dt + \sigma(Z_t)dY_t \quad (2)$$

where $\kappa(Z_t)$, $\theta(Z_t)$ and $\sigma(Z_t)$ are functions of the Markov chain Z . Hence, they are constants which take values in $\kappa(\mathcal{S})$, $\theta(\mathcal{S})$ and $\sigma(\mathcal{S})$. Thus, $\kappa(\mathcal{S}) := \{\kappa(1), \dots, \kappa(K)\} \in \mathbb{R}^{K^*}$, $\theta(\mathcal{S}) := \{\theta(1), \dots, \theta(K)\}$ and $\sigma(\mathcal{S}) := \{\sigma(1), \dots, \sigma(K)\} \in \mathbb{R}^{K^+}$. And finally, Y is a stochastic process which could be a Brownian motion or a Lévy process.

Remark 3. The following classic notations apply:

- κ denotes the mean-reverting rate;
- θ denotes the long-run mean;
- σ denotes the volatility of X .

Remark 4. • In this model, there are two sources of randomness: the stochastic process Y appearing in the dynamics of X , and the Markov chain Z . There exists one randomness due to the market information which is the initial continuous filtration \mathcal{F} generated by the stochastic process Y ; and another randomness due to the Markov chain Z , \mathcal{F}^Z .

- In our model, the Markov chain Z infers the unobservable state of the economy, i.e. expansion or recession. The processes Y^i estimated in each state, where $i \in \mathcal{S}$, capture: a different level of volatility in the case of Brownian motion (i.e. $Y^i \equiv W^i$), or a different jump intensity level of the distribution (and a possible skewness) in the case of Lévy process (i.e. $Y^i \equiv L^i$).

Barndorff-Nielsen [1] recalls the main properties of the Normal Inverse Gaussian (NIG) distribution, which is used as the Lévy distribution in this paper. The NIG density belongs to the family of normal variance-mean mixtures, i.e. one of the most commonly used parametric densities in financial economics. The NIG is a good alternative to the normal distribution since: (i) its distribution can model the heavy tails, kurtosis, and jumps, and (ii) the parameters of NIG distribution can be solved in a closed form.

4 Estimation

This section covers the methodology pertaining to the estimation task. In the first sub-section, we extend the EM algorithm to the class of Lévy regime-switching and explain how the likelihood can be evaluated. In the following sub-sections, the two-step estimation strategy as well as the initialization choice for the parameters are detailed.

4.1 (EM) Algorithm

The Expectation-Maximization algorithm used to estimate the regime-switching Lévy model in this paper is a generalization and extension of the EM-algorithm developed in Hamilton [2] and [3].

Our aim is to fit a regime-switching Lévy model such as (2) where the stochastic process Y is a Lévy process that follows a Normal Inverse Gaussian (NIG) distribution. Thus the optimal set of parameters to estimate is $\hat{\Theta} := (\hat{\kappa}_i, \hat{\theta}_i, \hat{\sigma}_i, \hat{\alpha}_i, \hat{\beta}_i, \hat{\delta}_i, \hat{\mu}_i, \hat{\Pi})$, for $i \in \mathcal{S}$.

We have the three parameters of the dynamics of X , the four parameters of the density of the Lévy process L , and the transition matrix of the Markov chain Z . Because the number of parameters grows rapidly in this class of jump-diffusion regime-switching models, direct maximization of the total log-likelihood is not practicable. To bypass this problem, we propose a method in two successive steps to estimate the global set of parameters.

-Step 1: Estimation of the regime-switching model (2) in the Brownian case

Following the methodology of Janczura and Weron [4], we first take for the stochastic process Y a Brownian motion W . Moreover, suppose that the size of historical data is $M + 1$. Let Γ denote the corresponding increasing sequence of time from which the data values are taken:

$$\Gamma = \{t_j; 0 = t_0 \leq t_1 \leq \dots t_{M-1} \leq t_M = T\}, \quad \text{with} \quad \Delta_t = t_j - t_{j-1} = 1.$$

The discretized version of model (2) writes

$$X_{t+1} = \kappa(Z_t)\theta(Z_t) + (1 - \kappa(Z_t))X_t + \sigma(Z_t)\varepsilon_{t+1}. \quad (3)$$

where $\varepsilon_t \sim \mathcal{N}(0, 1)$ (since the process Y is a Brownian motion). We denote by $\mathcal{F}_{t_k}^X$ the vector of historical values of the process X until time $t_k \in \Gamma$. Thus, $\mathcal{F}_{t_k}^X$ is the vector of the $k+1$ last values of the discretized model and therefore, $\mathcal{F}_{t_k}^X = (X_{t_0}, X_{t_1}, \dots, X_{t_k})$.

Remark 5. The filtration generated by the Markov chain Z (i.e. \mathcal{F}^Z) is the one generated by the history values of Z in the time sequence Γ .

For simplicity of notation, we will write in the sequel the model (3) as

$$X_{t+1} = \kappa_t \theta_i + (1 - \kappa_t)X_t + \sigma_t \varepsilon_{t+1}.$$

This means that at time $t \in [0, T]$, the Markov chain Z is in state $i \in \mathcal{S}$ (i.e. $Z_t = i$) and Z jumps at time $t_j \in \Gamma$, $j \in \{0, 1, \dots, M-1\}$.

In the first step based on the EM-algorithm, the complete parameter space estimate $\hat{\Theta}$ is split into: $\hat{\Theta}_1 := (\hat{\kappa}_i, \hat{\theta}_i, \hat{\sigma}_i, \hat{\Pi})$, for $i \in \mathcal{S}$, which corresponds to the first subset of diffusion parameters.

-Step 2: Estimation of the parameters of the Lévy process fitted to each regime

Using the regime classification obtained in the previous step, we estimate the second subset of parameters $\hat{\Theta}_2 := (\hat{\alpha}_i, \hat{\beta}_i, \hat{\delta}_i, \hat{\mu}_i)$, for $i \in \mathcal{S}$, which corresponds to the NIG distribution parameters of the Lévy jump process fitted for each regime.

5 Application to Asian equities

We apply these statistical methods to estimate regime-switching Lévy models in the context of Asian equities. The data is retrieved from Thomson Financial Datastream over the period going from July 20, 2010 to July 11, 2014 with a daily frequency, totaling 1,281 observations. The characteristics for each time series are given in Table 1. We have recovered equity data in order to study the jump properties of stock markets under changing market conditions in the Pacific region.

Table 1 Description of the time series

Ticker	Description
<i>Equity markets</i>	
TOPIX	JAPAN TOPIX Index
FBMKLCI	MALAYSIA FTSE Bursa Malaysia KLCI
CNXNIFTY	INDIA S&P CNX Nifty Index
DWJP	Dow Jones Japan Total Stock Market Index

For each time series, a table reports the results of: (i) the set of diffusion parameters, and (ii) the NIG density parameters of the Lévy jump process fitted to each

regime. The remaining problem in this work is to specify the number of regimes in the Markov chain. For simplicity, we proceed with two regimes that relate to the ‘boom’ and ‘bust’ phases of the business cycle.¹

We also report a plot where each regime is reported with a different color (e.g. blue (red) corresponds to regime 1 (regime 2)). To provide the reader with a clearer picture, we have chosen to plug the regimes identified back into the raw (non-stationary) data. Of course, all the estimates were performed on log-returns $r_t := \log(X_t) - \log(X_{t-1})$, e.g. stationary data. Below this first plot, the filtered and smoothed probabilities are displayed. They reflect the regime switches at stake.

We give now all estimated parameters for each time series in Table 2. Moreover, in Appendix, we put all the plots of the smoothed and filtered probabilities; and all corresponding regime-switching classification in a two states case.

Table 2 Estimated parameters for each time series JAPAN TOPIX Index I00000

	Japan Topix		Malaysia FTSE		India S&P CNX		Dow Jones Japan	
Parameters	State 1	State 2	State 1	State 2	State 1	State 2	State 1	State 2
κ	-0,0025	0,0174	0,0027	0,0027	0,0002	0,0064	-0,0035	0,02800
θ	664,6664	1079,2974	1141,7252	2061,3696	4778,5961	6652,6922	417,7719	609,9770
σ	85,0566	480,4978	173,3628	35,3315	4788,6179	1894,1711	25,0611	246,9338
p_{ii}^Z	0,9927	0,9648	0,8788	0,9540	0,9927	0,9866	0,9844	0,8883
α	1.2173	0.0516	1.2041	0.0172	0.0060	0.0200	0.8092	0.1224
β	-0.1191	0.0116	-0.3071	0.0033	0.0031	-0.0173	-0.0342	0.0589
δ	0.9360	7.6544	1.0001	13.2360	27.1555	8.0970	0.6204	4.4879
μ	0.0920	-1.7728	0.2638	-2.5410	-16.2639	14.0104	0.0263	-2.4735

Provisional conclusions of our work applied to Asia equity markets include:

- the presence of two contrasted regimes in each time series;
- with one jumpy regime and a rather quiet second regime;
- therefore it seems appropriate to model them separately with Lévy-jump or pure Brownian motion processes.

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¹ It is well-known that testing for the number of regimes in a Markov chain is a hard problem to tackle, which we leave for further research.

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Appendix

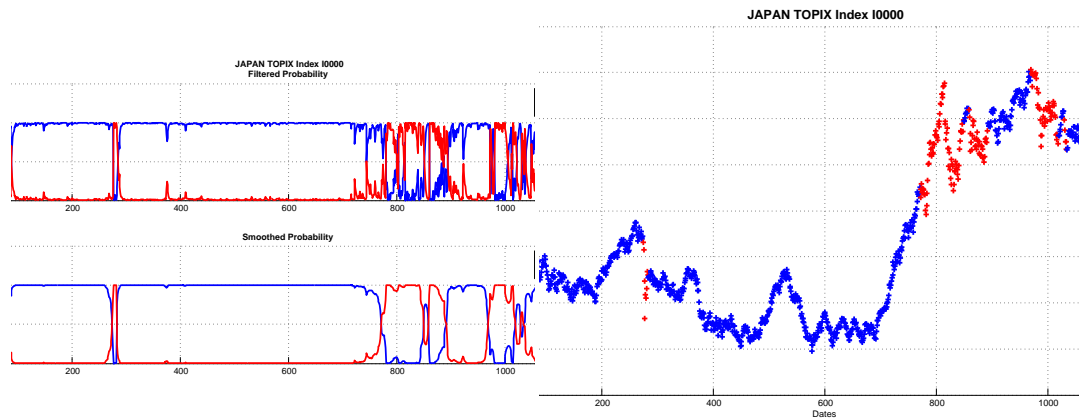


Fig. 1 Smoothed and filtered probabilities for the JAPAN TOPIX Index I0000

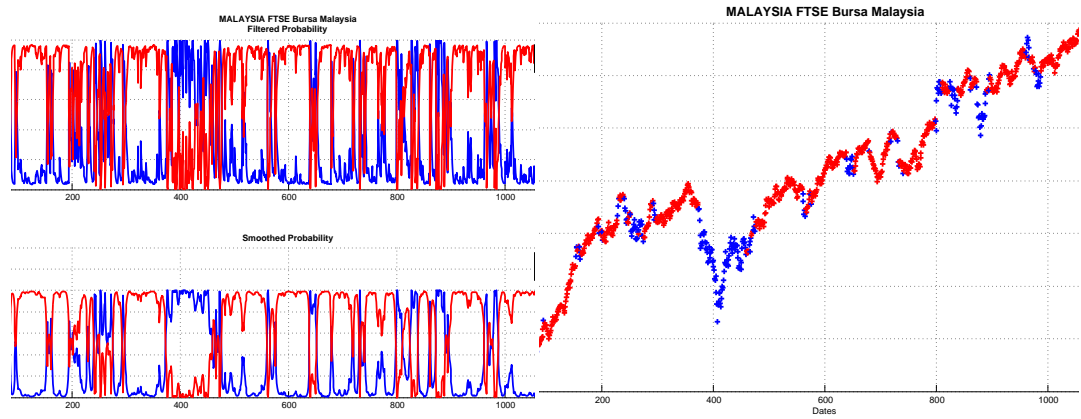


Fig. 2 Smoothed and filtered probabilities for the MALAYSIA FTSE Bursa Malaysia

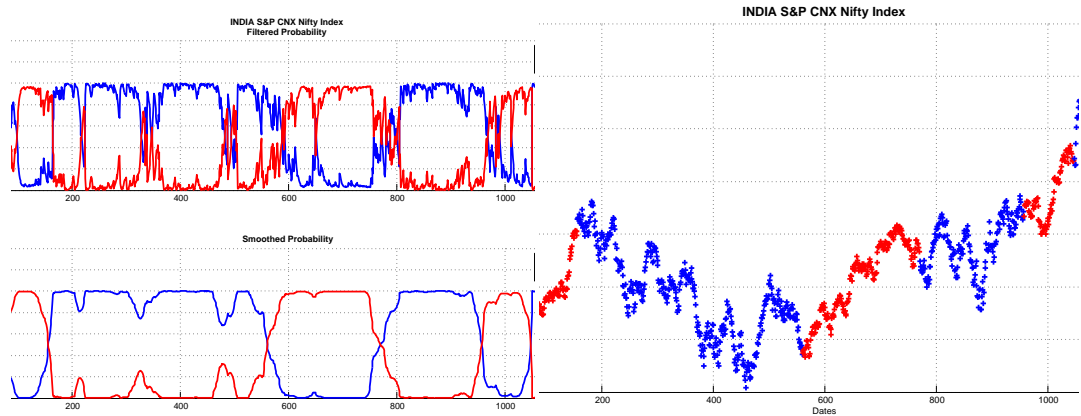


Fig. 3 Smoothed and filtered probabilities for the INDIA S&P CNX Nifty Index

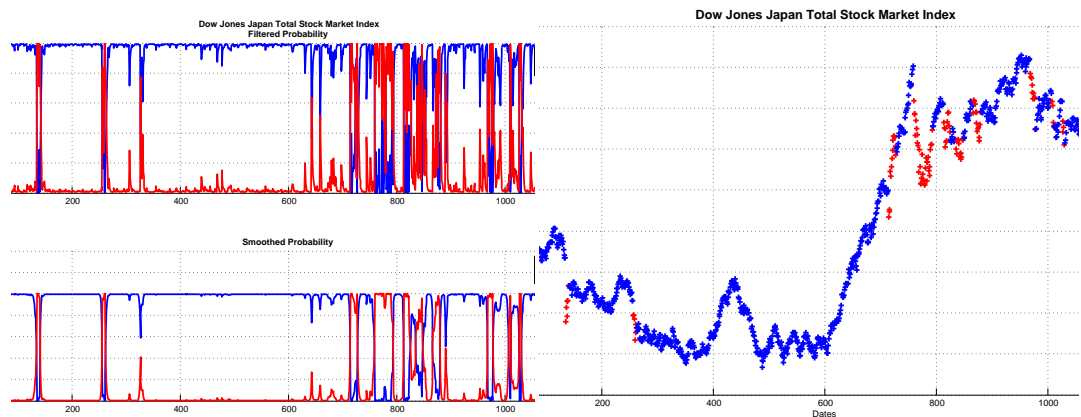


Fig. 4 Smoothed and filtered probabilities for the Dow Jones Japan Total Stock Market Index