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A beauty contest game with endogenous concern for coordination

Camille Cornand, Rodolphe Dos Santos Ferreira

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JEL codes:
D43, D82, L13, L21

Cooperation in a differentiated duopoly when information is dispersed: A beauty contest game with endogenous concern for coordination*

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1 Introduction

In a Keynesian 'beauty contest', agents make investment choices by referring to their expectations of some fundamental value and of the conventional value to be set by the market. In doing so, agents respond to fundamental and coordination motives, respectively. A third motive, often neglected, is the competition motive: as far as they look for convention, agents are willing to meet the market, but as far as they take part in a contest they would be happy to beat the market. There is clearly some conflict involved in meeting and beating the market at the same time. Now, the competition motive may be latent, when agents' actions have only an insignificant influence on the market, so that the market advantage occurs as an externality rather than as the consequence of strategic decisions. In this case and if information is perfect, the fundamental and the coordination motives are compatible, all the agents simply coordinating on the fundamental value. A conflict between the two motives will however emerge as soon as information is imperfect (blurring the fundamental) and dispersed (obstructing coordination), so that matching the fundamental value and matching the conventional value may then solely result from a trade-off.

When agents have significant market power, the competition motive may become active enough to create a conflict between the fundamental and the competition motives even under perfect information. Imperfect and dispersed information will exacerbate a conflict that was already present in this case, leading to a more adverse trade-off between the fundamental and the competition motives (Cornand and Dos Santos Ferreira, 2019). The point we want to bring to the fore is that, as long as the relative weights on the different motives are, as usual, part of the model structure, no space is left for the agents to improve the trade-off between them. We shall however admit that agents have some latitude in dealing with the conflict, by manipulating the relative weights. By injecting some cooperation to tame the competition motive, such manipulation may ease the conflict, while enhancing the concern for coordination.

We illustrate these ideas with a simple IO application. We consider a two-firm, two-stage delegation game, in which firm owners control at the first stage the conduct to be adopted by firm managers at the second. Going one step upstream of an otherwise standard differentiated duopoly game allows for a strategic choice by each firm owner, committing on some degree of cooperation with the competitor so as to mitigate the competitive toughness at the second stage and thus to augment profits. The regime of competition is accordingly endogenous, strategically determined by the players as part of the equilibrium outcome, and not the consequence of the modeler's preferences for Cournot, Bertrand, or whatever.

Price setting at the second stage is performed on the basis of both public and private information on the fundamental random value, the market size, the realization of which is unknown. Given demand linearity, we obtain a quadratic-payoff coordination game, which is reminiscent of a standard beauty contest, up to the addition of a first stage, making the relative weights put by each firm on each type of information depend upon its concern for coordination, itself increasing with both degrees of cooperation. Each firm owner chooses the degree of cooperation which maximizes her expected profit. By moderating the competition motive, higher degrees of cooperation (prevailing under higher information quality) induce equilibrium prices that are closer to their fundamental, collusive, value. They also induce higher concerns for coordination, giving more weight to public information. Both cooperation and coordination increase at equilibrium as the two products become less and less differentiated, hence competition structurally more intense. In the limit case of the homogeneous duopoly, the degree of cooperation reaches for each firm its maximum, collusive, value, but the concern for coordination reaches its maximum value too, making public information all important and possibly disconnecting equilibrium prices from fundamentals.

Our paper contributes to two strands of the literature. First, our game fits into a large literature on beauty contest games initiated by Morris and Shin (2002) seminal contribution, distinguishing the roles of public and private information. In particular, we find in Angeletos and Pavan (2007) or Myatt and Wallace (2012, 2015, 2016) previous IO instances of beauty contest games. Our analysis differs however from theirs in at least four respects. First of all, we emphasize the role of the competition motive, whereas previous contributions focused on the opposition between the fundamental and coordination motives. Cornand and Heinemann (2008) already distinguished three components – the fundamental, the coordination and the competition motives – of a standard beauty contest payoff. Yet, the competition motive is a mere externality when the set of agents is a continuum. Instead, by focusing on a duopoly, we confer a true strategic dimension to this motive.[1] Second, the previous IO applications, following Morris and Shin (2002), treat the relative weights put on the different motives as pertaining to the model structure. Instead, we allow firm owners to manipulate these weights at a first stage of the game.[2] Third, contrary to the literature directly inspired by Morris and Shin (2002), we do not impose symmetry on the quality of private information, which allows us to distinguish the effects on a firm's behavior of changes in the precisions of each one of the two private signals. Fourth, by relying on the delegation game introduced under perfect information by Miller and Pazgal (2001),[3] and in contrast to previous applications of the beauty contest game to IO, our analysis is not based on the opposition between Cournot and Bertrand competition.[4]

[1] The competition motive may legitimately be ignored when market power is diluted within a large group of competitors. Myatt and Wallace (2016), in spite of addressing oligopolistic competition proper, merged in some sense the fundamental and competitive motives by taking the Bertrand price as the fundamental target, rather than a price, like the collusive price, which is not affected by competition.

[2] Martimort and Stole (2011) and Myatt and Wallace (2016) also consider a finite number of players in a beauty contest game *à la* Morris and Shin, in which the relative weights on the fundamental and coordination motives may vary across players, but are still fully exogenous.

[3] We borrow from Miller and Pazgal (2001) their second stage payoff. However, the significant point is here the ability to commit at the first stage to adopt some conduct at the second, not the specific second stage payoff. Building upon a concept of oligopolistic equilibrium allowing for varying competitive toughness (first introduced in d'Aspremont, Dos Santos Ferreira and Gérard-Varet, 2007), d'Aspremont and Dos Santos Ferreira (2009, s.5) and d'Aspremont, Dos Santos Ferreira and Thépot (2016, ss.II.5) propose alternative specifications of a two-stage delegation game.

[4] As emphasized by the authors, the delegation game dilutes the opposition between price and quantity com-

The second strand of literature our work relates to is that on private information sharing and on the private and social value of information in oligopolies with uncertain demand (Vives, 1984, 1988, Raith, 1996), where again the opposition between Cournot and Bertrand competition appears crucial. More recently, the question has been reexamined under supply function competition – another way to go beyond that opposition – and the case of public information has been contemplated (Vives, 2011, 2013). Relative to this literature the present paper focuses on the comparative roles of public and private information, which is more in conformity with the first strand, and on the link between cooperation, coordination and these two types of information.

The remaining of the paper is structured as follows. Section 2 presents the two-firm, two-stage delegation game under certainty and perfect information. The full information benchmark allows us to identify the three motives of the beauty contest and to put light on the benefit from cooperation (that is maximized at the full information optimal degree of cooperation). Section 3 derives the subgame perfect equilibrium under imperfect and dispersed information. We analyze how changes in the quality of public and private information, as well as changes in the intensity of competition, affect the competitors' payoffs and ultimately the extent of their cooperation. Section 4 concludes.

2 Price duopoly under certainty and with perfectly informed agents

As already mentioned in the introduction, we build on Miller and Pazgal (2001) delegation game of a differentiated duopoly. This is a two-stage game, where each firm manager chooses at the second stage a market strategy, a price in this paper, with some degree of cooperation set by the firm owner at the first stage. We assume two differentiated substitutes i and j ($i, j = 1, 2$ and $i \neq j$), with linear demand for good i:

$$q_i = 2\theta - p_i + d\left(p_j - p_i\right), \tag{1}$$

where $\theta > 0$ and $0 \leq d \leq \infty$, q_i being the quantity of good i demanded at prices p_i and p_j of the two goods.[5] The demand shifter θ is an index of market size, which will be taken in the following as the fundamental. In this section, we will consider the fundamental as given and known by all the players. The parameter d is the reciprocal of an index of product differentiation ($d = 0$ for independent goods, $d = \infty$ for perfect substitutes), and is consequently an indicator of the intensity of competition to which the firms are structurally exposed.

petition, because "*if the owners have sufficient power to manipulate their managers' incentives, the equilibrium outcome is the same regardless of how the firms compete in the second stage*" (Miller and Pazgal, 2001, p. 284).

[5]Generality is not lost by assuming a demand curve with slope -1 instead of $-b < 0$.

2.1 Second stage: setting prices

With linear production costs, normalized to zero, the profit of the price setting firm i is

$$\Pi\left(p_i, p_j; \theta, d\right) = p_i\left(2\theta - p_i + d\left(p_j - p_i\right)\right). \tag{2}$$

We assume that the owner of each firm i has sufficient control over the respective manager's conduct to impose at a first stage a degree of cooperation $\gamma_i \in [0, 1]$ weighting the competitor's profit and turning firm i's payoff at the second stage into

$$\mathcal{P}\left(p_i, p_j, \gamma_i; \theta, d\right) = \Pi\left(p_i, p_j; \theta, d\right) + \gamma_i \Pi\left(p_j, p_i; \theta, d\right), \tag{3}$$

to be maximized in p_i. By taking the extreme values of the two degrees of cooperation, we obtain Bertrand competition for a fully non-cooperative conduct ($\gamma_i = \gamma_j = 0$), and tacit collusion for a fully cooperative conduct ($\gamma_i = \gamma_j = 1$). A continuum of intermediate equilibrium outcomes, in particular the Cournot outcome (for $\gamma_i = \gamma_j = d/\left(1 + d\right)$), is attainable between these extremes.

Instead of referring to the choice by the manager of firm i of the price p_i maximizing the payoff $\mathcal{P}\left(p_i, p_j, \gamma_i; \theta, d\right)$, we can equivalently refer to the choice of the price p_i minimizing the loss function $\mathcal{L}\left(p_i, p_j, \gamma_i; \theta, d\right) = 2\theta^2 - \mathcal{P}\left(p_i, p_j, \gamma_i; \theta, d\right)$ with respect to the collusive joint profit $2\theta^2$:

$$\begin{aligned}\mathcal{L}\left(p_i, p_j, \gamma_i; \theta, d\right) \;=\; & \underbrace{\left(p_i - \theta\right)^2}_{\text{fundamental motive}} + d\left(\underbrace{\left(p_i - p_j\right)^2}_{\text{coordination motive}} + \left(1 - \gamma_i\right) \underbrace{p_j\left(p_i - p_j\right)}_{\text{competition motive}}\right) \\ & + \underbrace{\left(1 - \gamma_i\right)\theta^2 + \gamma_i\left(p_j - \theta\right)^2}_{\text{externality}}.\end{aligned} \tag{4}$$

This loss function is reminiscent of the loss function introduced by Morris and Shin (2002) in their seminal formalization of the beauty contest, also including three motives: the *fundamental*, the *coordination* and the *competition* motives.[6]

We formulate three remarks concerning the loss function $\mathcal{L}$. First, notice that the fundamental motive stands naturally alone when the goods are independent ($d = 0$). By contrast, its relative weight diminishes as the level d of substitutability between goods increases, and vanishes in the limit when they become perfect substitutes, a situation where price setting

[6] If we consider a set of n agents, instead of the continuum $[0, 1]$, Morris and Shin's payoff function for agent i becomes $u_i(\mathbf{a}; r, \theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L})$, where $\mathbf{a} = (a_1, ..., a_n)$, $L_i = \frac{1}{n}\sum_j \left(a_i - a_j\right)^2$ and $\bar{L}$ is the arithmetic mean of the L_i's. By developing L_i and $\bar{L}$, we obtain $u_i(\mathbf{a}; r, \theta) = -\left(1-r\right)\left(a_i - \theta\right)^2 - r\left(a_i - \overline{a}\right)^2 + r\frac{1}{n}\sum_j \left(a_j - \overline{a}\right)^2$, the sum of the fundamental, coordination and competition motives. There is an important difference between this function and ours, since the competition motive plays in fact no significant independent role in Morris and Shin's framework, and this for two reasons. The first is that the competition motive exactly counterbalances the coordination motive in the *aggregate*, leaving the fundamental motive stand alone as a component of social welfare. The second is that, becoming more and more insensitive to variations of the action a_i as the number n of agents grows, the competition motive can be neglected in the *individual* optimization problem when n is large.

is entirely disconnected from the fundamental. Indeed, we may observe in particular, in the polar case where the competition motive disappears because of full cooperation ($\gamma_i = 1$), that as d becomes higher and higher, aggregate profits are more and more eroded by the divergence between the competitors' prices rather than by their distance from the fundamental, eventually making coordination all important.

Second, notice that the degree of cooperation controlled by the owner of each firm has two intertwined effects: a direct effect on the relative weight of the competition motive, which decreases as the degree of cooperation γ_i increases (for $\gamma_i = 1$, the competition motive vanishes altogether) plus an indirect effect on the different motives via equilibrium prices.[7]

Third, the competition motive introduces an asymmetry in pricing: it adds to the loss of firm i only if its second stage equilibrium price is higher than the one of its competitor, and moderates the loss otherwise. The weight put on this motive is given by $d(1-\gamma_i)$, the product of the intensity of competition (as structurally determined by the degree of substitutability between the two goods) and the strength of non-cooperative conduct imposed upon the firm manager.

The first order condition for maximization of payoff (3) or minimization of loss (4) leads to the best reply price

$$p_i = \frac{\theta + d\frac{1+\gamma_i}{2}p_j}{1+d}, \tag{5}$$

a weighted mean of the monopoly price θ and the discounted competitor's price $(1+\gamma_i)\,p_j/2$, the discount becoming heavier as cooperation weakens. The discount is an expression of the competition motive, in conflict with the fundamental motive, since it triggers a downward price deviation when the competitor sets the monopoly price. Notice also that the relative weight put on the fundamental decreases with the intensity of competition from 1 (when the products are independent) to 0 (when they are perfect substitutes).

A simple computation (using equations (5) for p_i and p_j) allows us to determine the equilibrium prices at the second stage of the game, when information is perfect:

$$p^*\left(\gamma_i, \gamma_j; \theta, d\right) = \frac{1 + d + d\frac{1+\gamma_i}{2}}{(1+d)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}\theta \equiv K\left(\gamma_i, \gamma_j; d\right)\theta, \tag{6}$$

for $i, j = 1, 2, i \neq j$. These prices increase with both degrees of cooperation, from their fully

[7] Myatt and Wallace (2012, 2016) take the Bertrand price, instead of the collusive price θ, as the fundamental target. In our duopoly case, the Bertrand price is $\theta^B = 2\theta/(2+d)$, a redefined fundamental depending not only upon market size but also upon the intensity of competition. The corresponding firm i's loss function (with respect to the target θ^B, and up to multiplication by $1+d$) is

$$L_i^B\left(p_i, p_j; r, \theta^B\right) = (1-r)\left(p_i - \theta^B\right)^2 + r\left(p_i - p_j\right)^2 + h,$$

with $r = d/2(1+d)$ and $h \equiv r\left(\left(\theta^B\right)^2 - p_j^2\right)$. In the Bertrand game, the term h is an externality, and L_i^B is reduced to the fundamental and coordination motives. Redefining the fundamental is however not enough to get rid of the competition motive in the present context. Indeed, in our two-stage game, h depends through the second stage equilibrium price p_j upon the degree of cooperation chosen at the first stage by the owner of firm i, and thus ceases to be an externality.

non-cooperative Bertrand value $\theta/(1+d/2)$ to the collusive (or monopoly) price θ, through the Cournot equilibrium price $\theta(1+d)/(1+3d/2)$. The equilibrium price $p^*(\gamma_i, \gamma_j; \theta, d)$ of good i is however more responsive to the degree of cooperation γ_i decided by the owner of firm i than to the degree of cooperation γ_j decided by the owner of the rival firm. Also, as the inverse index of differentiation d increases from zero to infinity, the equilibrium prices decrease from their monopoly value to zero, except under fully cooperative conduct of the two firms. At the limit, when $d = \infty$, the equilibrium price can be positive only if the degrees of cooperation are both equal to 1. Notice that the optimal price from the point of view of the firms is the collusive price θ. The competition motive is responsible for a strategy distortion reducing this price, and the more so the more intense the competition (the higher d) and the lower the degrees of cooperation γ_i and γ_j chosen by the two firm owners.

2.2 First stage: choosing degrees of cooperation

Each firm owner, anticipating the second stage equilibrium prices, chooses at the first stage the degree of cooperation she wants to impose on her manager's conduct, in order to maximize her own profit, say $\Pi^*(\gamma_i, \gamma_j; \theta, d) = \Pi(p^*(\gamma_i, \gamma_j; \theta, d), p^*(\gamma_j, \gamma_i; \theta, d); \theta, d)$ for firm i, that is,

$$\Pi^*(\gamma_i, \gamma_j; \theta, d) = p^*(\gamma_i, \gamma_j; \theta, d)\left(2\theta - p^*(\gamma_i, \gamma_j; \theta, d) + d\left(p^*(\gamma_j, \gamma_i; \theta, d) - p^*(\gamma_i, \gamma_j; \theta, d)\right)\right). \tag{7}$$

By using equation (6) and computing the first order condition for the maximization of $\Pi^*(\cdot, \gamma_j; \theta, d)$, we obtain the best reply for firm i:

$$\widehat{\gamma}(\gamma_j, d) = \frac{(2+3d)\, d\, (1+\gamma_j)}{4(1+d)^2 + (2+d)\, d\, (1+\gamma_j)}, \tag{8}$$

which is independent from θ, increasing in γ_j but always smaller than 1 as long as d is finite. Strategic complementarity at the first stage is not strong enough to translate into full cooperation. Indeed, the (symmetric) subgame perfect equilibrium value of the degree of cooperation is

$$\gamma^*(d) = \frac{d}{2+d}, \tag{9}$$

so that firms are more and more cooperative as competition becomes more and more intense, but full cooperation is attained only at the limit of perfect substitutability of their products. In addition, at the second stage of the game, more cooperation is not enough to reverse the primary effect of more intense competition: by (6), the equilibrium price

$$p^*(\gamma^*(d), \gamma^*(d); \theta, d) = \frac{1+d/2}{1+d}\theta \tag{10}$$

is decreasing with the intensity of competition.

Thus, if we consider a sequence of equilibria with indefinitely increasing substitutability of the two goods (such that $d \to \infty$), the equilibrium degree of cooperation tends to 1 and the equilibrium price to $\theta/2$. However, looking directly at the limit case $d = \infty$ without referring to a sequence of equilibria, we see that the symmetric subgame perfect equilibrium with $\gamma = 1$ and $p = \theta/2$ is just a limit point in a continuum of symmetric subgame perfect equilibria which are indeterminate at the second stage, with $p \in [0, 2\theta]$. Indeed, by (5), when $d = \infty$ and $\gamma = 1$, the best reply of each firm manager is to set the same price as his rival, whatever it is and independently of any reference to the fundamental. Also, by (6), each firm owner, by deviating to a lower degree of cooperation, would only generate zero equilibrium prices at the second stage, hence zero profits. In addition to this continuum of symmetric subgame perfect equilibria, there is another continuum of (trivial) subgame perfect equilibria with arbitrary pairs of degrees of cooperation $(\gamma_1, \gamma_2) < (1, 1)$ and zero prices and profits.

3 Price duopoly under uncertainty and dispersed information

Going beyond Miller and Pazgal (2001), we shall now assume that the fundamental θ is stochastic, normally distributed: $\theta = \mu + \zeta$, with $\mu > 0$ and $\zeta \sim N(0, 1/\alpha)$. A small enough variance $1/\alpha$ has to be assumed so as to make negligible the probability of observing negative values of θ.[8]

3.1 Second stage: setting prices

Some value θ of the fundamental is realized at the second stage of the game, but remains unknown to the managers, who only have access to imperfect private information on that value. Public information is here ex ante, identified with the common knowledge of the distribution of the fundamental, before its realization. Each manager i ($i = 1, 2$) receives however in addition a private signal $x_i = \theta + \varepsilon_i$, unbiased with respect to the realization of the fundamental, the random variables ε_1 and ε_2 being independently and normally distributed, with mean 0 and finite variances $1/\beta_1$ and $1/\beta_2$, respectively. By contrast with public information, which is ex ante, private information is ex post, about the realized value of the fundamental.

Notice that our way of introducing public information differs from that of Morris and Shin (2002), who assume a public signal centered about the realized θ. We are considering a limit case of Angeletos and Pavan (2007), where public information is both ex ante (the

[8] This probability is less than 3×10^{-7} for a coefficient of variation $1/\left(\sqrt{\alpha}\mu\right) \leq 5$. The assumption of a small coefficient of variation has often been used in the literature, at least implicitly, as a convenient way to get rid of observed negative values of the fundamental. See for instance Vives, 1984, p. 77, n. 2. We also refer to the more general discussion about the consequences of normality when demand is linear in Myatt and Wallace (2015, p. 6).

players share a common prior) and ex post (they receive a public signal centered on the realized θ). Our case results from assuming a completely uninformative public signal. Notice further that, contrary to this literature, we do not impose symmetry on the quality of private information, the precision β_1 possibly differing from the precision β_2.[9]

Taking into account available public and private information, and given $\mathcal{P}$ as defined by (3), the problem of firm i's manager becomes: $\max_{p_i} \mathbb{E}\left(\mathcal{P}\left(p_i, p_j, \gamma_i; \theta, d\right) \mid \mu, x_i\right)$. Referring to the first order condition (5), we may reformulate firm i's best reply as

$$p_i = \frac{\mathbb{E}_i\left(\theta\right) + d\frac{1+\gamma_i}{2}\mathbb{E}_i\left(p_j\right)}{1+d}, \tag{11}$$

where $\mathbb{E}_i\left(\cdot\right) \equiv \mathbb{E}(\cdot \mid \mu, x_i)$ is the expectation operator conditional on public and private information.

The posterior distribution for firm i of the realized value of the fundamental is normal with mean $\mathbb{E}_i(\theta) = (\alpha\mu + \beta_i x_i)/(\alpha + \beta_i)$ and variance $1/\left(\alpha + \beta_i\right)$. The parameters α and β_i can be viewed as the precisions with which the mean μ and the private signal x_i, respectively, allow to predict the realized value θ. As to $\mathbb{E}_i\left(p_j\right)$, we follow the methodology developed by Morris and Shin (2002), and assume that each manager i follows a strategy which is linear with respect to public and private information μ and x_i: $p_i = \kappa_i\mu + \kappa_i' x_i$. As $\mathbb{E}_i\left(x_j\right) = \mathbb{E}_i\left(\theta\right)$, we thus obtain

$$\begin{aligned} p_i &= \frac{\frac{\alpha\mu+\beta_i x_i}{\alpha+\beta_i} + d\frac{1+\gamma_i}{2}\left(\kappa_j\mu + \kappa_j'\frac{\alpha\mu+\beta_i x_i}{\alpha+\beta_i}\right)}{1+d} \\ &= \underbrace{\frac{\left(1 + d\frac{1+\gamma_i}{2}\kappa_j'\right)\alpha + d\frac{1+\gamma_i}{2}\left(\alpha+\beta_i\right)\kappa_j}{\left(1+d\right)\left(\alpha+\beta_i\right)}}_{\kappa_i}\mu + \underbrace{\frac{\left(1 + d\frac{1+\gamma_i}{2}\kappa_j'\right)\beta_i}{\left(1+d\right)\left(\alpha+\beta_i\right)}}_{\kappa_i'}x_i, \end{aligned} \tag{12}$$

for $i, j = 1, 2$, $i \neq j$. Through identification of the coefficients (κ_i, κ_i') and use of the two equations, we can determine their equilibrium values. Rather than giving here the corresponding cumbersome expressions (see however equation (20) in the proof of Proposition 4), let us equivalently establish two facts, which are more appropriate to interpretation.

First, the sum of the two coefficients applied to public and private information

$$K_i = \kappa_i + \kappa_i' = \frac{1 + d + d\frac{1+\gamma_i}{2}}{\left(1+d\right)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}} \equiv K\left(\gamma_i, \gamma_j; d\right), \tag{13}$$

as easily computed from equations (12), is independent from information quality, which is characterized by the three precisions α, β_i and β_j. It is equal to the coefficient applied to the fundamental in the expression of the equilibrium price under perfect information (see

[9] As will become clear, this remains tractable in a two player game and allows us to distinguish the effects on a firm's behavior of changes in the precisions of each one of the two private signals. Distinguishing these effects is particularly important in a context where we have to disentangle the cooperative and coordinating roles played by the strategic variable γ_i.

equation (6)). It thus expresses the strategy distortion imposed by the competition motive upon the equilibrium prices, whatever the information quality.

Second, the ratio of the weight put on the private signal x_i to the weight put on the commonly known mean μ is[10]

$$\frac{\kappa_i'}{\kappa_i} = (1 - r_i)\,\frac{\beta_i}{\alpha}, \tag{14}$$

with r_i given by the function

$$r\left(\gamma_i, \gamma_j; \frac{\beta_i}{\alpha}, \frac{\beta_j}{\alpha}, d\right) \equiv \frac{d\frac{1+\gamma_i}{2}}{1+d}\,\frac{\left(1+d+d\frac{1+\gamma_j}{2}\right)(1+d)\left(1+\frac{\beta_i}{\alpha}\right)+\left(1+d+d\frac{1+\gamma_i}{2}\right)d\frac{1+\gamma_j}{2}\frac{\beta_j}{\alpha}}{\left(1+d+d\frac{1+\gamma_i}{2}\right)(1+d)\left(1+\frac{\beta_j}{\alpha}\right)+\left(1+d+d\frac{1+\gamma_j}{2}\right)d\frac{1+\gamma_i}{2}\frac{\beta_i}{\alpha}}. \tag{15}$$

The coefficient $r_i = r\left(\gamma_i, \gamma_j; \beta_i/\alpha, \beta_j/\alpha, d\right) \in [0,1]$, depending upon the degrees of cooperation γ_i and γ_j decided by both firm owners, upon the relative quality of private (with respect to public) information available to both managers as measured by β_i/α and β_j/α,[11] and upon the competitive intensity d resulting from the extent of product substitutability, can be seen as an index of the *concern for coordination* of the manager of firm i. If this concern increases, the manager puts indeed more relative weight on public information, which allows the two firms to coordinate.

The ratio κ_i'/κ_i of the weights put on the two kinds of information would just be equal to the ratio of the corresponding precisions β_i/α if there were no competitive interaction between the two firms ($d = 0$, so that $r_i = 0$), the decisions of the two managers depending upon the fundamental motive alone. If $d > 0$, the coordination motive induces a higher relative weight to be put on public information ($\kappa_i'/\kappa_i < \beta_i/\alpha$ through a positive r_i). At the limit, under perfect substitutability of the two goods ($d = \infty$), and provided the conduct of the two firms is fully cooperative ($\gamma_i = \gamma_j = 1$), all the weight will be put on public information ($\kappa_i'/\kappa_i = 0$, since $r_i = 1$). Otherwise, under less than full cooperation of the two firms, the competition motive, already responsible for a *strategy distortion* which counteracts the fundamental motive and reduces $K\left(\gamma_i, \gamma_j; d\right)$, introduces in addition an *informational distortion* countervailing the influence of the coordination motive, that is, increasing the relative weight put on private information.

In equation (15), firm i's concern for coordination appears as the product of two fractions. The first is equal to the derivative of the own price with respect to the competitor's price in the best reply of firm i (see (5) or (11)) and is independent from the quality of information. It is increasing in the intensity of the competitive interaction between the two firms and in the own degree of cooperation of firm i. The second fraction, which is equal to 1 when information and cooperation are both symmetric ($\beta_i = \beta_j$ and $\gamma_i = \gamma_j$), expresses

[10]In Morris and Shin (2002) standard beauty contest game, the ratio of the weights put on the private and public signals has the same specification but is exogenous and uniform across agents. Here r_i is manipulable by the owner of firm i through the degree of cooperation γ_i she decides to set.

[11]At this stage of the game, the quality of information steps in only relatively, so that the impact of, say, an increase in the precision α of public information is indistinguishable from a simultaneous proportionate decrease in the precisions β_i and β_j of the two private signals.

an asymmetry effect of changes in the relative quality of private and public information and in the degrees of cooperation chosen by the two firm owners at the first stage.

We can now formulate two propositions concerning the effects of these changes.

Proposition 1 *The equilibrium relative weight κ_i'/κ_i put on private (with respect to public) information by firm i is increasing in the relative precisions β_i/α and β_j/α of private (with respect to public) information of both firms.*

Proof. From (15) we see that r_i is a homographic function of β_j/α. A straightforward computation allows us then to establish that $r\left(\gamma_i, \gamma_j; \beta_i/\alpha, \cdot, d\right)$ is a decreasing function. Hence, by (14), κ_i'/κ_i is increasing in β_j/α, since it depends on this variable through the function r alone. Another simple computation allows us to check that $1 - r\left(\gamma_i, \gamma_j; \beta_i/\alpha, \beta_j/\alpha, d\right)$, as a function of β_i/α, has the form $A/\left(B + C\left(\beta_i/\alpha\right)\right)$, with positive constants A, B and C, and hence has an elasticity belonging to the interval $(-1, 0)$. As a consequence, even if the indirect effect through r of β_i/α on κ_i'/κ_i is in the opposite sense, the direct effect dominates, so that κ_i'/κ_i is increasing in β_i/α. □

So, an increase in the precision of one of the private signals relative to the precision of public information, say β_i/α, induces an increase in the relative weight put by *both* firms on their respective private signals, namely κ_i'/κ_i and κ_j'/κ_j, but for different reasons. By (14), the increase in κ_i'/κ_i is explained by the direct effect of the increase in the quality of firm i private information β_i/α, an effect only attenuated by its contrary indirect effect through the concern for coordination r_i. By contrast, the increase in κ_j'/κ_j is due to the decrease in r_j: firm j's manager, whose rival is better privately informed, also feels less concerned for coordination.

Proposition 2 *The equilibrium relative weight κ_i'/κ_i put on private (with respect to public) information by firm i is decreasing in the degrees of cooperation γ_i and γ_j of both firms.*

Proof. From (15) we see that r_i is a homographic function of γ_j, so that it is again straightforward to establish that $r\left(\gamma_i, \cdot; \beta_i/\alpha, \beta_j/\alpha, d\right)$ is an increasing function and that, by (14), κ_i'/κ_i is decreasing in γ_j. As to γ_i, it is straightforward to check that the derivative of $1 - r\left(\gamma_i, \gamma_j; \beta_i/\alpha, \beta_j/\alpha, d\right)$ with respect to γ_i has the sign of a quadratic $a\gamma_i^2 + b\gamma_i + c$ with negative coefficients a, b and c. □

As already stated, there is a direct positive influence of γ_i on r_i expressed by the first ratio on the RHS of (15). This influence dominates the negative influence through the asymmetry effect expressed by the second fraction on the RHS of (15). The asymmetry effect is by contrast positive in the case of γ_j. Thus the concern for coordination r_i responds positively (and the relative weight on private information κ_i'/κ_i negatively) to a higher degree of cooperation by anyone of the two firms: more willingness to cooperate induces more concern for coordination.

Before starting to analyze the first stage of the game and to look for subgame equilibrium perfection, it is necessary to assert existence and uniqueness of the second stage equilibrium (outside the limit case where $d = \infty$ and $\gamma_i = \gamma_j = 1$, left to subsection 3.3).[12] This is done in the next proposition.

Proposition 3 *Take $d < \infty$ or $(\gamma_1, \gamma_2) \neq (1,1)$. Then, conditionally on public information (the mean μ) and the realization of the private signals x_i and $x_j (i,j = 1,2,\ i \neq j)$, the second stage equilibrium price of good i is uniquely determined by $p_i^* = \kappa_i \mu + \kappa_i' x_i$, with*

$$\kappa_i = \frac{1}{1 + (1 - r_i)\,\beta_i/\alpha} K\left(\gamma_i, \gamma_j; d\right) \text{ and } \kappa_i' = \frac{(1 - r_i)\,\beta_i/\alpha}{1 + (1 - r_i)\,\beta_i/\alpha} K\left(\gamma_i, \gamma_j; d\right), \tag{16}$$

$K\left(\gamma_i, \gamma_j; d\right)$ and $r_i = r\left(\gamma_i, \gamma_j; \beta_i/\alpha, \beta_i/\alpha, d\right)$ being given by equations (13) and (15), respectively.

Proof. The determination of the expressions of κ_i and κ_i' in (16) from the two equations (13) and (14), for $i = 1, 2$, is straightforward. □

3.2 First stage: choosing degrees of cooperation

At the first stage, the owner of firm i maximizes in γ_i her expected profit $\mathbb{E}\left(\Pi^*\left(\gamma_i, \gamma_j\right)\right)$, before the uncertainty on the fundamental is resolved, with

$$\Pi^*\left(\gamma_i, \gamma_j\right) \equiv p_i^*\left(2\theta - p_i^* + d\left(p_j^* - p_i^*\right)\right),$$

where $p_i^* = \kappa_i \mu + \kappa_i' x_i$ (with κ_i and κ_i' depending upon γ_i and γ_j) is given by Proposition 3. As $x_i = \theta + \varepsilon_i = \mu + \zeta + \varepsilon_i$, by decomposing the expected profit into terms in the squares of μ, ζ and ε_i (hence into terms in μ^2, $1/\alpha$ and $1/\beta_i$), we can express firm i's expected profit as

$$\mathbb{E}\left(\Pi^*\right) = \mu^2 \overbrace{K_i\left(2 - (1 + d)\,K_i + dK_j\right)}^{\Pi(K_i, K_j; 1, d)} + \frac{1}{\alpha} \overbrace{\kappa_i'\left(2 - (1 + d)\,\kappa_i' + d\kappa_j'\right)}^{\Pi\left(\kappa_i', \kappa_j'; 1, d\right)} - \frac{(1 + d)\,\kappa_i'^2}{\beta_i}. \tag{17}$$

The first term of this expression is the profit firm i would expect to obtain at prices $K_i \mu \equiv K\left(\gamma_i, \gamma_j; d\right)\mu$ and $K_j \mu \equiv K\left(\gamma_j, \gamma_i; d\right)\mu$ if the fundamental θ were known with certainty to be equal to its mean μ at the second stage. This case corresponds to $\alpha = \infty$, the fundamental ceasing then to be stochastic, so that we are naturally back to the certainty, perfect information, case with $\theta = \mu$. The two other terms on the RHS of (17) vanish indeed, since $\kappa_i' = 0$. We consequently assume in the following that $\alpha < \infty$. If the private signal is uninformative ($\beta_i = 0$), manager i will disregard private, ex post, information ($\kappa_i' = 0$), so

[12] Uniqueness of the second stage equilibrium is established for the linear strategies we have assumed. Morris and Shin (2002) have considered uniqueness in more general terms, but the question does not seem to be definitively settled (see Myatt and Wallace, 2012, p.348, n.6).

Another point, already mentioned, is that by assuming normal distributions for the fundamental and for the noises of the private signals we may obtain states where demand and the candidate equilibrium prices are negative. We implicitly assume that the precisions α, β_i and β_j are high enough for the probability of such events to be negligible.

that the first term will stand again alone as a non-zero term, which leads to the same profit, although prevailing just as a mathematical expectation.

At the other extreme, if private, ex post, information is perfect for *both* managers ($\beta_i = \beta_j = \infty$), they will disregard public, ex ante, information ($\kappa_i = \kappa_j = 0$), and the certainty profit expressed by the first term will be augmented, according to the second – matching – term, by a factor $1/\alpha\mu^2$ due to the strict convexity of Π in θ. This augmentation will not be full if $\beta_i < \infty$, entailing $\kappa_i' < K_i$, or if $\beta_j < \infty$, entailing $\kappa_j' < K_j$.

The third term expresses the cost generated by the imperfection of ex post information, increasing in the weight κ_i' put on the private signal and in the variance $1/\beta_i$ of the corresponding noise, as well as on the strength d of the competitive interaction.

In order to examine the conditions for maximization of firm i's expected profit, it is convenient to express it as

$$\mathbb{E}(\Pi^*) = \mu^2 \left(\overbrace{K_i \left(2 - (1+d) K_i + dK_j\right)}^{F\left(\gamma_i, \gamma_j\right)} + \frac{1}{\alpha\mu^2} \overbrace{\kappa_i' \left(2 - (1+d)\left(1 + \alpha/\beta_i\right) \kappa_i' + d\kappa_j'\right)}^{G_i\left(\gamma_i, \gamma_j\right)} \right), \tag{18}$$

where $F\left(\gamma_i, \gamma_j\right)$ is the component of the expected profit which is based on prior information and independent from information quality, whereas $G_i\left(\gamma_i, \gamma_j\right)$, weighted by the squared coefficient of variation of the fundamental, is the component of expected profit added by resorting to private, ex post, information. Quasi-concavity in γ_i of firm i's expected profit ensures existence of a subgame perfect equilibrium.

Proposition 4 *Assume a finite positive variance $1/\alpha$ of the fundamental and a finite positive index of product differentiation d. Then there exists an equilibrium of the first stage game where the owners' payoffs are the expected firm profits conditional on the owners strategies decided at the first stage.*

Proof. As each firm owner has a non-empty, compact, convex strategy space $[0, 1]$ and a continuous payoff $\mu^2 \left(F\left(\gamma_i, \gamma_j\right) + \left(1/\alpha\mu^2\right) G_i\left(\gamma_i, \gamma_j\right)\right)$, we have only to show that this payoff is quasi-concave in γ_i in order to prove existence of a pure Nash equilibrium of the first stage game. Strict quasi-concavity results from $\partial F\left(\cdot, \gamma_j\right)/\partial\gamma_i + \left(1/\alpha\mu^2\right)\partial G_i\left(\cdot, \gamma_j\right)/\partial\gamma_i$ being decreasing at any positive solution γ_i to the first order condition $\partial F\left(\gamma_i, \gamma_j\right)/\partial\gamma_i + \left(1/\alpha\mu^2\right)\partial G_i\left(\gamma_i, \gamma_j\right)/\partial\gamma_i = 0$. By (18), this first order condition can be written as

$$\begin{aligned} & \underbrace{2\frac{\partial K_i}{\partial\gamma_i}}_{f^1\left(\gamma_i, \gamma_j\right)} + \underbrace{d\left(K_j \frac{\partial K_i}{\partial\gamma_i} + K_i \frac{\partial K_j}{\partial\gamma_i}\right)}_{f^2\left(\gamma_i, \gamma_j\right)} + \underbrace{\frac{2}{\alpha\mu^2}\frac{\partial\kappa_i'}{\partial\gamma_i}}_{g^1\left(\gamma_i, \gamma_j\right)} + \underbrace{\frac{d}{\alpha\mu^2}\left(\kappa_j' \frac{\partial\kappa_i'}{\partial\gamma_i} + \kappa_i' \frac{\partial\kappa_j'}{\partial\gamma_i}\right)}_{g^2\left(\gamma_i, \gamma_j\right)} \\ = & \underbrace{2K_i \frac{\partial K_i}{\partial\gamma_i}}_{f^3\left(\gamma_i, \gamma_j\right)} + \underbrace{2dK_i \frac{\partial K_i}{\partial\gamma_i}}_{f^4\left(\gamma_i, \gamma_j\right)} + \underbrace{\frac{2}{\alpha\mu^2}\left(1 + \frac{\alpha}{\beta_i}\right)\kappa_i' \frac{\partial\kappa_i'}{\partial\gamma_i}}_{g^3\left(\gamma_i, \gamma_j\right)} + \underbrace{\frac{d}{\alpha\mu^2} 2\left(1 + \frac{\alpha}{\beta_i}\right)\kappa_i' \frac{\partial\kappa_i'}{\partial\gamma_i}}_{g^4\left(\gamma_i, \gamma_j\right)}, \end{aligned} \tag{19}$$

where the terms on the right hand side are those that appear with negative coefficients in the derivative of the payoff. Using (15) and (16), it is straightforward to compute

$$\kappa_i' = \frac{(1+d)(\beta_i/\alpha)\left(1+\beta_j/\alpha\right)+d(\beta_i/\alpha)\left(\beta_j/\alpha\right)z_i}{(1+d)^2(1+\beta_i/\alpha)\left(1+\beta_j/\alpha\right)-d^2(\beta_i/\alpha)\left(\beta_j/\alpha\right)z_iz_j}, \tag{20}$$

with $z_h \equiv (1+\gamma_h)/2 \in [1/2, 1]$ ($h = i, j$). Hence, $\kappa_i'(z_i, z_j)$ has the same structure as $K(z_i, z_j)$ (by (13)):

$$k_i(z_i, z_j) = \frac{A_i + Bz_i}{C - Dz_iz_j},$$

with positive coefficients and $D < C$, so that $k_i(z_i, z_j) > 0$. Also,

$$\frac{\partial k_i}{\partial z_i} = \frac{BC + A_iDz_j}{(C - Dz_iz_j)^2} > 0 \text{ and } \frac{\partial k_j}{\partial z_i} = \frac{(A_j + Bz_j)Dz_j}{(C - Dz_iz_j)^2} > 0.$$

Both these derivatives are clearly increasing functions of z_i. Each handside of equation (19) is thus a sum of positive increasing functions of γ_i. As the elasticity of $k_i(\cdot, z_j)\partial k_i(\cdot, z_j)/\partial z_i$ is obviously larger than the elasticity of $\partial k_i(\cdot, z_j)/\partial z_i$, the elasticities of $f^3(\cdot, \gamma_j)$ and $g^3(\cdot, \gamma_j)$ are larger than the elasticities of $f^1(\cdot, \gamma_j)$ and $g^1(\cdot, \gamma_j)$, respectively. Furthermore,

$$\begin{aligned} k_j\frac{\partial k_i}{\partial z_i} + k_i\frac{\partial k_j}{\partial z_i} &= \frac{(A_j + Bz_j)(BC + 2A_iDz_j + BDz_iz_j)}{(C - Dz_iz_j)^3} \\ k_i\frac{\partial k_i}{\partial z_i} &= \frac{(A_i + Bz_i)(BC + A_iDz_j)}{(C - Dz_iz_j)^3}, \end{aligned}$$

the elasticity of the former with respect to z_i being smaller than that of the latter (since $BDz_iz_j/(BC + 2A_iDz_j + BDz_iz_j) < Bz_i/(A_i + Bz_i)$). Hence, the elasticities of $f^4(\cdot, \gamma_j)$ and $g^4(\cdot, \gamma_j)$ are larger than the elasticities of $f^2(\cdot, \gamma_j)$ and $g^2(\cdot, \gamma_j)$, respectively. As a result, the elasticity with respect to γ_i of the LHS of (19) is smaller than the corresponding elasticity of the RHS, so that the derivative of the payoff becomes negative as we increase γ_i from its value satisfying the first order condition: the payoff function is consequently strictly quasi-concave, which completes the existence proof. □

In order to make a comparison with the perfect information case, it will be convenient to start with a symmetric equilibrium, such that $\gamma_i^* = \gamma_j^* = \gamma^*$ (when $\beta_i = \beta_j = \beta$). Using the equations in the proof of Proposition 4, it is straightforward to compute the root $\widetilde{\gamma}$ of the function $\partial G_i(\gamma, \gamma)/\partial\gamma_i$ (i.e. of the function $g^1(\gamma, \gamma) + g^2(\gamma, \gamma) - g^3(\gamma, \gamma) - g^4(\gamma, \gamma)$, according to the notations introduced in equation (19)):

$$\widetilde{\gamma}(d, \beta/\alpha) = \frac{d(\beta/\alpha)}{2(1+d)(1+\beta/\alpha) - d(\beta/\alpha)} \in [0, 1]. \tag{21}$$

The root $\widetilde{\gamma}$ is increasing in β/α, tending to the perfect information equilibrim value $\gamma^*(d) = d/(2+d)$ when $\beta/\alpha \to \infty$. Thus, as long as β/α is finite, $\widetilde{\gamma}(d, \beta/\alpha) < \gamma^*(d)$. As a consequence, the symmetric subgame equilibrium value of the degree of cooperation is then lower than its perfect information value, decreasing as the quality of private, ex post, information deteriorates with respect to the quality of public, ex ante, information (i.e., as β/α diminishes).

A symmetric improvement of the private, ex post, information of both firms fosters their cooperation. What about an asymmetric improvement? An increase in β_i/α has a direct positive effect on $\partial G_i\left(\gamma_i, \gamma_j\right)/\partial\gamma_i$, hence on the degree of cooperation γ_i. On the other hand, by Proposition 1, an increase in β_i/α leads to higher values of κ_i' (with a countervailing negative effect on $\partial G_i\left(\gamma_i, \gamma_j\right)/\partial\gamma_i$) and κ_j' (with a positive, but smaller, effect on $\partial G_i\left(\gamma_i, \gamma_j\right)/\partial\gamma_i$). We can however show that the direct effect is the dominant one, so that an increase in the own relative precision β_i/α of the private information of firm i augments its degree of cooperation γ_i. The effect of the competitor's relative precision β_j/α of private information, which works only indirectly, is more ambiguous, as stated in the following proposition.

Proposition 5 *Assume a finite positive variance $1/\alpha$ of the fundamental and a finite positive index of product differentiation d. The equilibrium degree of cooperation γ_i of each firm i is increasing in the own relative precision β_i/α of private information. It is also increasing in the competitor's relative precision β_j/α for a high enough own precision β_i/α.*

Proof. The equilibrium degree of cooperation γ_i is increasing in β_i/α and β_j/α if $\partial G_i\left(\gamma_i, \gamma_j\right)/\partial\gamma_i$ (hence $\partial F_i\left(\gamma_i, \gamma_j\right)/\partial\gamma_i + \left(1/\alpha\mu^2\right)\partial G_i\left(\gamma_i, \gamma_j\right)/\partial\gamma_i$) is an increasing function of these variables. By equation (19), this property results from $\left(g^1+g^2\right)/\left(g^3+g^4\right)$ being increasing, as g^3 and g^4 correspond to the terms of $\partial G_i/\partial\gamma_i$ with negative coefficients. Using the computations in the proof of Proposition 4, we have:

$$\frac{g^1+g^2}{g^3+g^4} = \frac{2 + d\left(\frac{\kappa_j'}{\kappa_i'} + \frac{\partial\kappa_j'/\partial\gamma_i}{\partial\kappa_i'/\partial\gamma_i}\right)\kappa_i'}{2\left(1+d\right)\left(1+\alpha/\beta_i\right)\kappa_i'},$$

with

$$\frac{\kappa_j'}{\kappa_i'} = \frac{\left(1+d\right)\left(1+\beta_i/\alpha\right)/\left(\beta_i/\alpha\right) + dz_i}{\left(1+d\right)\left(1+\beta_j/\alpha\right)/\left(\beta_j/\alpha\right) + dz_i} \text{ and } \frac{\partial\kappa_j'/\partial\gamma_i}{\partial\kappa_i'/\partial\gamma_i} = \frac{dz_j}{1+d}\frac{\beta_j/\alpha}{1+\beta_j/\alpha}.$$

Thus, an increase in β_i/α leaves $\left(\partial\kappa_j'/\partial\gamma_i\right)/\left(\partial\kappa_i'/\partial\gamma_i\right)$ unchanged and diminishes κ_j'/κ_i'. It also augments κ_i', by Proposition 1. The two latter effects would make $\left(g^1+g^2\right)/\left(g^3+g^4\right)$ a decreasing function of β_i/α if they were not dominated by the direct contrary effect through the decrease of $1+\alpha/\beta_i$. A straightforward computation leads indeed to

$$\frac{g^1+g^2}{g^3+g^4} = 1 + \frac{d}{2}\frac{\beta_j/\alpha}{1+\beta_j/\alpha}\times$$
$$\left[\frac{d}{2\left(1+d\right)}\left(\frac{1+\gamma_j}{1+d} - \frac{\left(1+\gamma_i\right)\gamma_j}{1+d+d\frac{1+\gamma_i}{2}\frac{\beta_j/\alpha}{1+\beta_j/\alpha}}\right)\frac{\beta_i/\alpha}{1+\beta_i/\alpha} - \frac{\gamma_i}{1+d+d\frac{1+\gamma_i}{2}\frac{\beta_j/\alpha}{1+\beta_j/\alpha}}\right],$$

which is unambiguously increasing in β_i/α (since the coefficient of $\left(\beta_i/\alpha\right)/\left(1+\beta_i/\alpha\right)$ is positive). It is also increasing in β_j/α as long as the expression between brackets, increasing in β_j/α, is positive, hence for a high enough relative precision β_i/α. □

3.3 The influence of competitive intensity

We finally consider the consequences of higher competitive intensity, associated with a larger value of the differentiation parameter d. As well known, an indefinite increase in product substitutability augments dramatically the need for *cooperation*, so as to avoid the Bertrand outcome, with the eventual vanishing of profits. Indeed, by equation (13), $\lim_{d\to\infty} K\left(\gamma_i, \gamma_j; d\right) = 0$ unless $\gamma_i = \gamma_j = 1$: equilibrium prices will tend to zero as the products become perfectly substitutable unless both firms decide to collude. In addition, the increase in d also augments the need for *coordination*: as d becomes higher and higher, aggregate profits are more and more eroded by the divergence between the competitors' prices rather than by their distance from the fundamental. This is clear if we refer to the aggregate loss with respect to the maximum aggregate profit, namely by (4) (with $\gamma_i = 1$) to the sum of the fundamental motive $(p_i^* - \theta)^2 + \left(p_j^* - \theta\right)^2$ and the coordination motive $d\left(p_i^* - p_j^*\right)^2$, with a weight d on the latter increasing indefinitely with respect to the unit weight on the former.

As to the first point, the direct influence of competitive intensity on cooperation has already been caught by the analysis of the perfect information benchmark, exhibiting the equilibrium degree of cooperation as an increasing function of the parameter d (see (9)), a monotonic relationship that still prevails, under symmetry, when ex post information is imperfect (see (21)).

As to the second point, the influence of competitive intensity on coordination, consider the expression, again under symmetry, of each firm's concern for coordination, namely $r = (1+\gamma)/2(1+1/d)$, increasing in both γ and d. Higher competitive intensity induces directly and indirectly, through stronger cooperation, a higher concern for coordination, which translates into a weaker relative weight κ'/κ on private information. In the limit case of perfect product substitutability and full cooperation, this concern for coordination reaches its maximum value ($r = 1$), coordination between competitors through public information becoming then more important than any reference to the realization of the fundamental.

It is true that, as the two products tend to become perfect substitutes, full cooperation is hindered by imperfect ex post information, since the root of $\partial G_i(\gamma,\gamma)/\partial\gamma_i$ tends to $1/(1+2\alpha/\beta)$, not to 1 (see (21)). However, as the firms' concern for coordination tends to 1 and κ' correspondingly to 0, $G_i(\gamma,\gamma)$ and the derivative $\partial G_i(\gamma,\gamma)/\partial\gamma_i$ eventually vanish, letting the equilibrium be determined by the root of $\partial F(\gamma,\gamma)/\partial\gamma_i$, as in the perfect information benchmark.

To conclude, the following proposition applies directly to the case of perfect substitutability between the two products, with its strong incentive to full cooperation and full coordination.

Proposition 6 *Assuming finite positive variances $1/\alpha$, $1/\beta_i$, $1/\beta_j$ of the fundamental and of the private signals, if $d = \infty$, there is a continuum of subgame perfect equilibria with full cooperation ($\gamma_i = \gamma_j = 1$) such that the second stage equilibrium price of both goods is equal to $\kappa\mu$, for any $\kappa \in [0, 2]$. There is in addition a continuum of trivial subgame perfect equilibria with arbitrary pairs of degrees of cooperation, each one smaller than 1, and zero prices.*

Proof. Through identification of the coefficients κ_i and κ'_i in equation (12), we see that we generically obtain, for $d = \infty$ and $\gamma_i = \gamma_j = 1$, $\kappa'_i = \kappa'_j = 0$ and $\kappa_i = \kappa_j = \kappa \geq 0$. Hence, by (18), $\mathbb{E}(\Pi^*) = \mu^2 (2 - \kappa) \kappa$, so that firm i's expected profit is non-negative iff $\kappa \leq 2$. As to the trivial equilibria, observe that if γ_i and γ_j are both smaller than 1, $K(\gamma_i, \gamma_j; \infty) = 0$ and $F(\gamma_i, \gamma_j) = G(\gamma_i, \gamma_j) = 0$ for any γ_i by equations (13) and (18), so that there is no incentive for the owner of firm i to deviate. □

4 Conclusion

The main contribution of this paper is to provide a micro-founded differentiated price duopoly illustration of a beauty contest, in which the relative weight on the competition motive of the payoffs is not given by the model structure but endogenously determined in the framework of a delegation game. The analysis of beauty contest games, of which the IO illustrations are no exception, has up to now emphasized the conflict between the fundamental and coordination motives, resulting from dispersed information. By contrast, we emphasize the role of another component of the payoffs, the competition motive, as a source of conflict with the other motives. The competition motive counteracts both the fundamental motive, generating a price strategy distortion, and the coordination motive, generating an informational distortion. By opting for some cooperation with the competitor and moderating the competitive toughness displayed by the manager of her firm, each firm owner may ease the conflict with the fundamental motive. By so doing, she also heightens the manager's concern for coordination and consequently the weight put on public relative to private information. The degree of cooperation obtained at equilibrium is an increasing function of the quality of private information on the realization of the fundamental and of the intensity of competition as structurally determined by product substitutability.

While the paper provides only an illustration of how some degree of cooperation and some concern for coordination can endogenously arise in a beauty contest under dispersed information, the conflicts at stake and the way the agents deal with them may well carry over to other instances of beauty contest games in which a trade-off between the motives present in the payoffs is allowed for.

References

Angeletos, G.-M. and Pavan, A. (2007). Efficient use of information and social value of information. *Econometrica*, 75:1103–1142.

d'Aspremont, C. and Dos Santos Ferreira, R. (2009). Price-quantity competition with varying toughness. *Games and Economic Behavior*, 65:62–82.

d'Aspremont, C., Dos Santos Ferreira, R. and Gérard-Varet, L.-A. (2007). Competition for market share or for market size: Oligopolistic equilibria with varying toughness. *International Economic Review*, 48:761-784.

d'Aspremont, C., Dos Santos Ferreira, R. and Thépot, J. (2016). Hawks and doves in segmented markets: Profit maximization with varying competitive aggressiveness. *Annals of Economics and Statistics*, 121/122:45–66.

Cornand, C. and Dos Santos Ferreira R. (2019). The social value of information and the competition motive: price vs. quantity games. *Economic Theory*, forthcoming.
https://doi.org/10.1007/s00199-019-01179-7

Cornand, C. and Heinemann, F. (2008). Optimal degree of public information dissemination. *Economic Journal*, 118:718–742.

Martimort, D. and Stole, L. (2011). The collective wisdom of beauty contests. MPRA Paper 32872, University Library of Munich.

Miller, N. and Pazgal, A. (2001). The equivalence of price and quantity competition with delegation. *RAND Journal of Economics*, 32:284–301.

Morris, S. and Shin, H. S. (2002). Social value of public information. *American Economic Review*, 92:1521–1534.

Myatt, D. P. and Wallace, C. (2012). Endogenous information acquisition in coordination games. *Review of Economic Studies*, 79:340–374.

Myatt, D. P. and Wallace, C. (2015). Cournot competition and the social value of information. *Journal of Economic Theory*, 158:466–506.

Myatt, D. P. and Wallace, C. (2016). Information use and acquisition in price-setting oligopolies. *Economic Journal*, forthcoming.

Raith, M. (1996). A general model of information sharing in oligopoly. *Journal of Economic Theory*, 71: 260–288.

Vives, X. (1984). Duopoly information equilibrium: Cournot and Bertrand. *Journal of Economic Theory*, 34: 71–94.

Vives, X. (1988). Aggregation of information in large Cournot markets. *Econometrica*, 56: 851–876.

Vives, X. (2011). Strategic supply function competition with private information. *Econometrica*, 79: 1919–1966.

Vives, X. (2016). Endogenous Public Information and Welfare in Market Games. *Review of Economic Studies*, forthcoming.