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# Commuting effects in Okun's Law among British areas: Evidence from spatial panel econometrics

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**Abstract.** This paper examines Okun's Law using a spatial panel approach on NUTS 3 data for Great Britain over the period 1985–2011. The basic specification used as a starting point for the spatial analysis is the relationship between real output growth and unemployment rate changes. This is extended to incorporate spatial lags, to test for the presence of commuting effects, and to accommodate spatial residuals dependence. We estimate various spatial panel models, controlling for heterogeneity across areas, and find consistent evidence of a significant Okun's coefficient. Results also show that spillover effects from labour mobility and interregional linkages matter and have important implications for the validity of the empirical law.

**JEL classification:** R11, C31, J64

**Key words:** Unemployment, output growth, commuting, spatial econometrics, panel data

## 1 Introduction

The linear, negative association linking real output growth to unemployment rate changes over the business cycle has been widely investigated since Arthur Okun's (1962, 1970) seminal work, which analysed it using data for the US in the years 1947–1960. The association in question, relating transitory movements in output and unemployment as measured by their year-on-year variations, is consistent with Okun's difference model, where first-differences represent the deviations of actual production and joblessness from their equilibrium trends. Another way of seeing the output-unemployment relationship, also studied by Okun (1962, 1970) and known as the gap model, is between the divergence of economic output from its potential or long-run level and the divergence of the joblessness rate from its non-inflationary or full-employment (NAIRU) level. The first-differences approach provides the base for the specification adopted in this paper.

Okun's finding of a 3:1 trade-off between economic growth and unemployment, often referred to as Okun's Law, has emerged as an empirical regularity predicting the magnitude of the reduction in unemployment from real GDP gains (or the costs in terms of higher unemployment of real GDP loss), and also how much demand stimulus is necessary to stabilize the

joblessness rate. Specifically, the law envisages that for every 3 per cent fall in output below its potential or long-term path the unemployment rate tends to rise by one percentage point (above its 'natural', or NAIRU, level). This corresponds to a point estimate of around  $-0.3$ , that is, a 1 per cent fall in output from its trend yields an approximate 0.3 percentage point rise in the unemployment rate from its trend level. The less-than-proportionate increase in unemployment following an economic contraction, or equally the predicted slow employment response to an expansion in GDP, is due to labour market stickiness. Firms tend to invest considerably in company-specific human capital and certain skills are in limited supply, meaning that they have more to lose by dismissing workers when faced with temporary downturns than by utilizing labour less intensely in the short term. Equally, during upturns, firms may prefer to raise output via productivity gains instead of taking on new workers, which can result in sluggish labour market adjustments to positive demand shocks. It should be noted that the relationship can be, and indeed has been, explored in either causal direction depending on the empirical problem or policy question at hand; this paper analyses the responsiveness of unemployment to GDP performance, in line with models of unemployment typical of labour market research (for a survey, see Elhorst 2003).

While several tests of Okun's Law exist which are based on cross-country evidence (Knoester 1986; Paldam 1987; Attfield and Silverstone 1997, 1998; Moosa 1997; Lee 2000; Freeman 2001; Harris and Silverstone 2001; Crespo-Cuaresma 2003; Perman and Tavéra 2005; Moazzami and Dadgostar 2009), only recently have empirical studies started to estimate it using geographically disaggregated data. The regional Okun's Law literature, mainly concerned with the responsiveness of output volume growth to unemployment, points to the existence of noticeable interregional differences in the size of the coefficient. Christopoulos (2004), one of the main contributions, looks at 13 Greek regions over the period 1971–1993; he reports slope values ranging from  $-0.37$  to  $-1.70$  in the areas where the empirical law holds, which tend to be areas with low long-term unemployment. Adanu (2005) investigates Okun's Law for ten Canadian provinces during the period 1981–2001; his parameters vary from  $-0.30$  to  $-2.14$ , with more negative values seen in areas with larger concentrations of skilled workers. Using 1980–2004 data for 17 Spanish regions, Villaverde and Maza (2007, 2009) obtain regional estimates in the range of  $-0.32$  to  $-1.55$ , and show that the law is stronger in areas where productivity growth rates are lower. The consensus is therefore that the output gains/costs of lower/higher unemployment are larger in some regions and smaller or even negligible in others. Two exceptions are the earlier studies by Freeman (2000) and Apergis and Reztis (2003) who, focusing on eight regional economies respectively in the U. for the years 1958 to 1998 and in Greece over a similar time span 1960–1997, do not find clear evidence of spatial variability in the magnitude of Okun's coefficients.

While looking at the problem from a regional perspective, this body of work ignores the importance of controlling for spatial interactions and spillovers, and the implications for the strength and validity of Okun's relationship. To some extent, Kangasharju et al. (2012) represents an exception; the authors deal with the problem of cross-section dependence in their Okun's Law study of Finnish regions, however they do not treat it as the central theme of their paper but wash it out by taking output and unemployment series in deviation from their time means. By contrast, in labour market research, attempts have been made to rigorously apply a spatial economic/econometrics perspective to wage curve studies (Buettner 1999; Longhi et al. 2006; Morrison et al. 2006; Elhorst et al. 2007), and have led to the conclusion that spatial effects matter and improve the explanatory power of the wage curve.

The present paper addresses this gap in the Okun's Law literature by exploring the question as to whether and to what extent there are spatial mechanisms involved in Okun's Law dynamics. For this purpose, we use data for the 128 NUTS 3 regions of Great Britain over the period 1985–2011. The paper is organized as follows: Section 2 describes the model and data;

Section 3 is concerned with aspects of estimation; Section 4 provides a discussion of results and, finally, Section 5 summarizes and concludes.

## 2 Model and data

### 2.1 Traditional Okun's Law specification

Okun (1962, 1970) suggested two alternative forms of Okun's Law relationship, namely a gap model and a first-differences model. The former connects the deviation of actual output from its equilibrium or potential level to the unemployment rate gap; it therefore needs information about unemployment and output trends. Trend series are not directly observable, and there is no universal agreement on the optimal technique to estimate them, but any construction of these entails judgement. Moreover, the gap model should be preferred when the researcher is interested in inferences on time-series behaviour over the business cycle (Lee 2000). By contrast the latter, relating real output growth to unemployment rate changes, has the advantage of not relying on approximations of the size of the gap. Thus, as is common practice in applied Okun's Law studies, and because our aim is not to document estimates under different approaches to trend estimation as in purely econometric exercises, we adopt the first-differences method of Okun's Law analysis.

The short-run relationship between output and unemployment as in Okun's (1962, 1970) first-differences version (see also Knoester 1986) is given by the following expression:

$$\Delta UN_{i,t} = \alpha + \beta \Delta GDP_{i,t} + e_{i,t}, \quad GDP = \log \text{output} \quad (1)$$

where  $\Delta UN_{i,t}$  is the percentage point change in the local unemployment rate in region  $i$  at time  $t$  ( $i=1, \dots, n$  with  $n = 128$  British NUTS 3 areas,  $t = 1985, \dots, 2011$  so that  $T = 27$ ), as constructed from claimant counts and working-age population data published by the United Kingdom's Office for National Statistics;  $GDP_{i,t}$  is expressed as the natural logarithm of output (not in absolute terms), so that  $\Delta GDP_{i,t}$  is the percentage real growth rate of local economic activity, using gross value added in basic constant (2006) prices as economic volume measure; and  $e_{i,t}$  is the error term, which in cross-sectional or panel-data (non-spatial) Okun's Law studies is commonly modelled as satisfying the ordinary least squares assumptions of homoscedasticity and lack of autocorrelation. With regards to the structural parameters,  $\alpha$  is the intercept,  $\beta$  ( $\beta < 0$ ) is Okun's Law coefficient capturing the extent of the contemporaneous labour market reaction to short-term GDP fluctuations (estimated in the range  $-0.30$  to  $-0.50$ ), and the ratio  $\alpha/\beta$  indicates how fast economic activity has to grow in order to keep the unemployment rate stable. This basic regression equation can be augmented with other control variables which are commonly considered in the Okun's Law literature and justified on theoretical or empirical grounds.

In line with the dynamic version of Okun's Law (e.g., Chamberlin 2011), we start by adding  $\Delta UN_{i,t-1}$  to the right-hand-side of Equation (1) in order to test whether current unemployment depends on its recent history. A significant influence of the unemployment rate in the preceding period would indicate the presence of rigidities and inertia in labour markets, causing delayed adjustments to workforce levels; it would also suggest the importance of path dependency and negative hysteresis (as discussed in Blanchard and Summers 1987; Cross 1993), thus reflecting persistent changes to the unemployment rate due to jobless workers permanently losing their skills or becoming inactive.

The extended specification, reflecting an unemployment-output relationship which is both contemporaneous and time-lagged, thus takes the form:

$$\Delta UN_{i,t} = \alpha + \beta \Delta GDP_{i,t} + \gamma \Delta UN_{i,t-1} + e_{i,t}. \quad (2)$$

As outlined in subsequent sections, our panel-data framework allows for unobserved or unmeasured region-specific time-invariant characteristics by means of fixed effects or of a composite error term structure incorporating random effects. These fixed-effects or random-effects vectors act as a catch-all for any causes of spatial heterogeneity. They include differences across regions in the sectoral composition of the local economic base, which can affect their relative ability to absorb demand shocks. For instance, output from the manufacturing and construction sectors is particularly sensitive to cyclical fluctuations. Moreover, employment in these industries consists in large part of temporary and contractual workers, who are easier to lay off when demand falters. Another way industrial structure can have supply-side effects on local unemployment is through the effectiveness of the skills-jobs matching process, which tends to be lower in regions specialized in agriculture and manufacturing (Taylor and Bradley 1997; Elhorst 2003).

## 2.2 Labour market interactions

While the role of spatial effects in Okun's Law is largely unexplored, the literature on regional unemployment disparities suggests that the geography of and interaction among regional economies are important drivers of labour market outcomes. One of the first empirical papers to consider spatial variables in a model of unemployment determination is Molho (1995), who looks at the geographical distribution of the joblessness rate across labour market areas within Great Britain in 1991. Starting from a standard regression of local unemployment on current and time-lagged local employment growth, he includes spatially lagged employment variables measuring demand changes in surrounding areas. He finds strong evidence of spillover effects from demand shocks both contemporaneously and after a lag, the former reflecting interregional trade links and the latter pointing to labour migration. The author also tests for the impact of each area's unemployment rate on that in neighbouring areas by incorporating the spatial lag of the dependent variable. His results, indicating a significant presence of localized spillovers, are consistent with the transmission mechanism hypothesized by Burridge and Gordon (1981) and Taylor and Bradley (1983). These authors propose a balancing identity which relates regional unemployment changes to employment growth, labour force participation, migration and commuting, demonstrating the equilibrating effect of labour mobility on unemployment differentials. Such outcome arises because, as workers move from locations with spare capacity to locations with a surplus of demand for labour, local unemployment rates shift towards a new long-run steady state. A corollary to this is that labour market developments are not confined to the local area but spill over to other areas as well, implying that regional unemployment will exhibit spatial autocorrelation. For instance, in a slack labour market, employers will find it less necessary to advertise vacancies outside their area and fewer workers from nearby regions will look to this area for jobs; thus, inward commuting flows will fall and labour market conditions in contiguous areas will also tighten (see also Elhorst 2003). This explains why a region's unemployment rate tends to be higher/lower when surrounded by high/low-unemployment regions.

Niebuhr (2003) follows this strand of analysis, looking at a sample of EU countries between 1986 and 2000. By means of spatial econometric techniques, she uncovers a strong degree of spatial linkages among regional labour markets in Europe. In particular, the paper demonstrates that the evolution of a region's unemployment is considerably influenced by labour market developments in surrounding regions, which lends further support to the commuting hypothesis of Burridge and Gordon (1981) and other authors. She also tests for spatial dependence in the

error term, and shows that accounting for omitted spatial effects eliminates significant spatial residual autocorrelation thus removing sources of misspecification.

Building on this body of work, we introduce spatial effects in Equation (2), the dynamic counterpart of the standard Okun’s Law relationship as given by Equation (1). Failure to account for labour market interdependencies can have serious consequences for the reliability of econometric results, as well as lead to an incorrect representation and understanding of the true causal forces at work. Specifically, neglecting spatial correlation in the variables of interest would cause biased and possibly inconsistent coefficient estimates, while leaving unobserved common factors (positive spatial correlation in the error term) unmodelled would lead to reduced standard errors, inflated *t*-ratios and incorrect inference (Le Sage and Pace 2009).

In the most complex of our specifications, spatial effects are in the form of spatial lags as well as spatially autoregressive error components, with regional heterogeneity modelled via random effects. In the spatial econometrics literature (e.g., Elhorst 2014), this is referred to as the general nesting spatial (GNS) model. The various spatial processes considered in this paper are summarized in the following regression equation; all of our estimated models are nested within this:

$$\begin{aligned} \Delta UN_{i,t} &= \alpha + \rho \sum_{j=1}^N W_{ij} \Delta UN_{j,t} + \beta \Delta GDP_{i,t} + \theta \sum_{j=1}^N W_{ij} \Delta GDP_{j,t} + \gamma \Delta UN_{i,t-1} + e_{i,t}, \quad (3a) \\ e_{i,t} &= \lambda \sum_{j=1}^N M_{ij} e_{j,t} + \xi_{i,t}, \\ \xi_{i,t} &= \mu_i + v_{i,t}, \end{aligned}$$

where the  $n \times 1$  vector  $e_{i,t}$  is the spatially dependent error term, and this is a function of  $\xi_{i,t}$  which combines a (unobserved or unmeasured) time-invariant region-specific component  $\mu_i \sim iid(0, \sigma_\mu^2)$  and a time-varying component  $v_{i,t} \sim iid(0, \sigma_v^2)$ , respectively a random-effects vector and a disturbances vector (Kapoor et al. 2007). Time dependency is introduced in the innovations  $\xi_{i,t}$  by specifying the permanent error component  $\mu_i$  together with the transient error component  $v_{i,t}$  (Fingleton 2008). Also,  $\sigma_1^2 = \sigma_v^2 + T\sigma_\mu^2$ .

The fixed-effects counterpart to Equation (3a) can be formally expressed as:

$$\begin{aligned} \Delta UN_{i,t} &= \alpha + \rho \sum_{j=1}^N W_{ij} \Delta UN_{j,t} + \beta \Delta GDP_{i,t} + \theta \sum_{j=1}^N W_{ij} \Delta GDP_{j,t} + \gamma \Delta UN_{i,t-1} + \mu_i + e_{i,t} \quad (3b) \\ e_{i,t} &= \lambda \sum_{j=1}^N M_{ij} e_{j,t} + v_{i,t}. \end{aligned}$$

### 2.3 Testing for spatial effects in Okun’s variables

The aim of this section is to verify the existence of spatial correlation in the variables of interest – unemployment and gross value added, both in levels (*UN* and *GDP*) and in first differences ( $\Delta UN$ ) or growth rates ( $\Delta GDP$ ) – using the cross-section dependence statistic for panel time series data due to Pesaran (2004).<sup>1</sup> It is worth noting that, as the test is applied to the model’s variables, it does not require the assumptions regarding the panel model specification and error term structure which are necessary when the testing problem involves estimated residuals, thus its properties are reliable.

<sup>1</sup> Another available testing device is the space-time Moran’s *I* (STMI) statistic (Mitze 2012), a straightforward extension to a panel-data framework of the ordinary Moran coefficient for cross-sectional data (Anselin 2001). Results (unreported) from the STMI test are equivalent to those from the CD test and are available from the authors upon request.

**Table 1.** Pesaran’s cross-section dependence (CD) test statistic ( $H_0$ : no cross-section dependence)

	UN	$\Delta$ UN	GDP	$\Delta$ GDP
CD test stat	426.01	397.06	420.69	134.63
[P-value]	[0.000]***	[0.000]***	[0.000]***	[0.000]***
Average $\hat{\rho}_{ij}$	0.927	0.864	0.915	0.293

Notes: \*\*\* = significance at 1% level (strong rejection of the null hypothesis of no cross-section dependence).

Pesaran’s (2004) cross-section dependence (CD) statistic is given by:

$$CD = \sqrt{\left(\frac{2}{N(N-1)}\right)} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^N \sqrt{T_{ij}} \hat{\omega}_{ij}\right) \quad CD_{H_0: indep} \sim N(0, 1),$$

which is based on an average of the pairwise correlation coefficients between all cross-sectional series ( $\hat{\omega}_{ij}$ ), with  $T_{ij}$  being the joint sample for regions  $i$  and  $j$  for each correlation coefficient estimate. Results are illustrated in Table 1 and show that, for every variable, there is strong evidence against the null hypothesis of no cross-section dependence in the model’s variables. The test statistics are significantly large, and the associated  $P$ -values are very small, which leads us to reject the null and conclude in favour of the presence of significant spatial correlation in the panel. These findings confirm the discussion in subsection 2.2 which suggested the importance of controlling for interactions among labour markets.

#### 2.4 Definition of spatial weight matrices

The  $n \times n$  (standardized) spatial weights matrix  $\mathbf{W}$  in Equation (3a) takes the following form

$$\begin{aligned} W_{ij}^* &= \exp(-\hat{\tau}_i d_{ij}), \quad \text{for } i \neq j; \\ W_{ij}^* &= 0, \quad \text{for } i = j; \\ W_{ij}^* &= 0, \quad \text{for } d_{ij} > 100 \text{ km}; \end{aligned} \tag{4}$$

$$W_{ij} = \frac{W_{ij}^*}{\sum_{j=1}^N W_{ij}^*}.$$

Regarding the distance threshold of 100km, we have taken this value from existing and well-established studies in the empirical regional economics literature; for example, Fingleton (2003) uses a similar specification of the  $\mathbf{W}$  matrix (for Britain’s local authority districts rather than NUTS 3 areas) to explore the significance of increasing returns to labour productivity from employment density. Similarly, Lerbs and Oberst (2012) use a distance threshold of 90km in a four nearest-neighbour inverse distance matrix.

In Equation (4),  $\hat{\tau}_i$  is specific to each area and calibrated on commuting flows (as explained in Fingleton 2003, to which we refer for details), with travel-to-work data taken from the UK’s 2001 Census and converted from district to NUTS 3 level, and  $d_{ij}$  denotes the straight-line distance between any two areas  $i$  and  $j$ .

The  $\mathbf{W}$  matrix is used to construct spatial lags  $(\mathbf{I}_T \otimes \mathbf{W})\Delta UN$  and  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$ , which allow testing for the significance of spillover effects in labour-market and economic outcomes arising from workforce mobility and interregional proximity.

The  $n \times n$  (standardized) spatial weights matrix  $\mathbf{M}$  for the error process is based on a canonical contiguity specification, given by

$$\begin{aligned}
 M_{ij}^* &= 1, && \text{if } i \text{ and } j \text{ share a border;} \\
 M_{ij}^* &= 0, && \text{otherwise;} \\
 M_{ij} &= \frac{M_{ij}^*}{\sum_{j=1}^N M_{ij}^*}. && (5)
 \end{aligned}$$

The use of different spatial weights matrices ( $\mathbf{W}$  for the model’s variables and  $\mathbf{M}$  for the error process) is thoroughly motivated in a recent publication by Corrado and Fingleton (2012). With reference to the case at hand, the  $\mathbf{W}$  matrix is constructed on commuting data because the main aim of our study is to explore the existence and significance of spatial effects due to workers mobility (labour market linkages), and a commuting-based spatial weights matrix enables us to explicitly test this proposition. For the error term, a spatial structure is typically imposed in order to capture the transmission of shocks across regions as well as spatial correlation in unobserved/unmeasured causes of interregional heterogeneity (the fact that similar regions are typically close to each other); regarding  $\mathbf{M}$ , a contiguity-based spatial weights matrix is a standard choice in spatial econometrics, and our preference for this functional form conforms to such custom.

### 3 Methodology

#### 3.1 Instrumentation strategy

It is important to observe that some of the regressors, namely  $\Delta GDP$  and  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$ , may be jointly determined. The former may be affected by two-way causation involving  $\Delta UN$ , an aspect that has been neglected in the relevant Okun’s Law literature to date, since unemployment is likely to cause (as well as be caused by) variations in demand. Similarly, the construction of  $\mathbf{W}$ , which is based on commuting flows from the 2001 census and thus post-dates the dependent variable in some years, may introduce simultaneity bias and lead to inconsistent parameter estimates. We address these concerns using appropriate instruments to eliminate any correlation of Okun’s variables with residuals, thus ensuring that estimation results are accurate and reliable.

One of the elements of originality in this paper is the use of instrumental variables (IV) to correctly identify  $\Delta GDP$  and  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$ . The instruments adopted here are widely accepted and well established. Our strategy follows the approach as discussed in Drukker et al. (2013), who build on the econometric theory developed in Kelejian and Prucha (1998, 1999, 2004, 2010). As in Drukker et al. (2013), we define  $\mathbf{X}_f = [\mathbf{X}; \mathbf{X}_e]$  as the set of included exogenous variables and excluded exogenous variables, respectively  $\mathbf{X}$  and  $\mathbf{X}_e$ . Here  $\mathbf{X}_f$  consists of  $\Delta UN_{i,t-1}$  and  $\Delta GVA_{i,t-1}$ , both of which are pre-determined – i.e. pre-date the dependent variable, being lagged by one year – and can thus be treated as exogenous, although this property will be explicitly tested using appropriate diagnostics.

The instruments are given by the linearly independent columns of

$$\mathbf{Z} = \left[ \mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{W})\mathbf{X}_f; \dots; (\mathbf{I}_T \otimes \mathbf{W})^q \mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{M})\mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{W})(\mathbf{I}_T \otimes \mathbf{M})\mathbf{X}_f; \dots; (\mathbf{I}_T \otimes \mathbf{W})^q (\mathbf{I}_T \otimes \mathbf{M})\mathbf{X}_f \right].$$

The above-cited literature has highlighted that these instruments are a good approximation of the ideal instruments under reasonable assumptions; with regard to spatial lag order,  $q = 1$  is a common choice while a value of two also worked well in Monte Carlo experiments.



Thus, in its extended specification and standard formulation with  $q = 1$ , our instruments set contains the linearly independent columns of

$$\begin{aligned} & [\mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{W})\mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{M})\mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{W})(\mathbf{I}_T \otimes \mathbf{M})\mathbf{X}_f] \\ & \mathbf{X}_f = [\Delta GDP_{i,t-1}; \Delta UN_{i,t-1}]. \end{aligned} \tag{6}$$

At a minimum, in our case the instruments should be a sub-set of  $\mathbf{Z}$  containing the linearly independent columns of Equation (7) below (with spatial lags up to second order ( $q = 2$ ) as required to fulfil the rank condition for model over-identification)

$$\begin{aligned} & [\mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{W})\mathbf{X}_f; (\mathbf{I}_T \otimes \mathbf{W})^2\mathbf{X}_f] \\ & \mathbf{X}_f = [\Delta GDP_{i,t-1}; \Delta UN_{i,t-1}]. \end{aligned} \tag{7}$$

We provide results both for this minimal representation of the IV set and for the extended expression.

### 3.2 Fixed effects or random effects

Panel data estimation necessitates the selection between fixed effects and random effects. The relative merits of each method are extensively discussed in the available literature (e.g., Arbia et al. 2005; Elhorst 2014), to which we refer for details. We make this choice on the basis of results from both specifications, taking into account theoretical coherence and empirical robustness, but our decision is also informed by the appropriate statistical devices which are available for this purpose; for instance, the Hausman statistics for random-effects consistency is normally used to this end, and the Sargan-Hansen instruments exogeneity test can help detecting misspecification and distinguishing between models. Therefore, while acknowledging the advantage of consistency for fixed effects, we look for evidence in the data as to whether this is also the case for random effects, and whether a random-effects model actually outperforms a fixed-effects model in this application.

It is worth discussing the adequacy of random effects when the spatial units cover the whole population, as in the case at hand; it should be noted that while our dataset does not comprise a sample it is possible to consider the data to be one of many realizations from a super-population, since the spatial partitions giving the areal units are just one of an infinite number of possible sets that could have occurred (see also Fingleton 2010) – in other words, the spatial units within the study area are such that inference about the population is not conditional on the observed units, even if these cover the whole population.

### 3.3 Estimation

All of our estimates are derived from an instrumental-variables approach to satisfy orthogonality conditions and to achieve consistency, in contrast to existing spatial panel evidence on Okun’s Law which is based on maximum likelihood and thus obtained under the assumption that all variables are exogenous.

Precisely, we carry out two-stage least squares (2SLS) estimation of spatial Durbin models (without spatially autoregressive errors) and feasible generalized spatial two-stage least squares/generalized method of moments (FGS2SLS/GMM) estimation of general nesting spatial models (as defined in Equations (3a) and (3b)).

The 2SLS technique is well known. With regard to the FGS2SLS/GMM procedure, the main difference between the present model and that of Kapoor et al. (2007) is that we include endogenous regressors other than the spatial lag of the dependent variable, namely  $\Delta GDP$  and  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$ , following Fingleton (2008). The methodology for estimating a random effects panel model with an endogenous spatial lag, additional endogenous regressors and spatially autoregressive error components is described in detail in the cited references, so we do not include technical specifics here.

To summarize, however, the estimation procedure has three stages. In the first step, the model is estimated via 2SLS to obtain consistent residuals. In the second step, these residuals are used to estimate, via GMM, the model parameters relating to the error term, namely the spatially autoregressive error process parameter  $\lambda$  and the error components variances  $\sigma_\mu^2$  and  $\sigma_v^2$ . Finally, given these, we find estimates of the spatial autoregressive parameter  $\rho$  and of the regression coefficients; precisely, the  $\hat{\lambda}$  estimate allows the variables to be purged of spatial dependence by means of a Cochrane–Orcutt transformation, with inference in the third stage based on a robust approach for IV estimation with non-spherical disturbances (Bowden and Turkington 1984; Greene 2003).

### 3.4 Measurement of direct, spillover and total impacts

Le Sage and Pace (2009) point out that, when the spatial lags of the regressand and regressor are present in a model, the true total effect on a dependent variable (here  $\Delta UN$ ) of a unit change in an explanatory variable (here  $\Delta GDP$ ) – that is, the true partial derivative of the expected value of  $\Delta UN$  with respect to  $\Delta GDP$  (i.e.,  $\partial E(\Delta UN)/\partial \Delta GDP$ ) – is not the same as the estimated regression coefficient  $\hat{\beta}$ ; it also captures spatial linkages and simultaneous feedbacks passing through the dependence system, thus leading to a total effect which typically differs from  $\hat{\beta}$  and which can be separated into a direct (own-region) effect and an indirect (spatial, spillover) effect.

Equation (3a), which has both  $(\mathbf{I}_T \otimes \mathbf{W})\Delta UN$  and  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$  as determinants of  $\Delta UN$  (plus spatially autoregressive error components), accommodates regional interdependencies up and down the spatial network, thus expanding the information set for the  $i$ th region to include observations on the dependent and explanatory variables in other regions. The implication of including these spatial lags is that a unit change in  $\Delta GDP$  within a given area  $i$  at a given time  $t$  will directly affect  $\Delta UN$  in area  $i$  itself, but will also have an indirect effect on  $\Delta UN$  in all other areas which eventually impacts back to  $i$ . This is different from non-spatial linear regressions (based on the assumption of independence among cross-sectional units) where, in a given year  $t$ ,  $\partial E(\Delta UN_i)/\partial \Delta GDP_i = \beta$  for all  $i$  while  $\partial E(\Delta UN_i)/\partial \Delta GDP_j = 0$  for  $i \neq j$ .

The proper interpretation of the marginal effects of  $\Delta GDP$  is derived from rearranging the following model, which is identical to Equation (3a) but expressed in terms of individual cross-sections, and taking expectations:

$$\begin{aligned}
 E(\Delta UN_t) &= \alpha + \rho \mathbf{W} \Delta UN_t + \beta \Delta GDP_t + \theta \mathbf{W} \Delta GDP_t + \gamma \Delta UN_{t-1} \\
 &= (\mathbf{I}_n - \rho \mathbf{W})^{-1} \alpha + (\mathbf{I}_n - \rho \mathbf{W})^{-1} (\beta \Delta GDP_t + \theta \mathbf{W} \Delta GDP_t) + (\mathbf{I}_n - \rho \mathbf{W})^{-1} \gamma \Delta UN_{t-1}, \quad (8)
 \end{aligned}$$

where  $|\rho| < 1$ ,  $\mathbf{I}_n$  is an  $n \times n$  identity matrix and the Leontief Expansion  $(\mathbf{I}_n - \rho \mathbf{W})^{-1}$  is equal to

$$(\mathbf{I}_n - \rho \mathbf{W})^{-1} = \mathbf{I}_n + \rho \mathbf{W} + \rho^2 \mathbf{W}^2 + \rho^3 \mathbf{W}^3 + \dots \quad (9)$$

It follows that, at a given time  $t$ , the  $n \times n$  matrix of partial derivatives of the expected value of  $\Delta UN$  in all regions with respect to  $\Delta GDP$  in all regions (i.e.,  $\partial E(\Delta UN_i)/\partial \Delta GDP_i$  for  $i = 1, \dots, n$ ) varies over  $i$  and can be illustrated to be equal to

$$\begin{bmatrix} \frac{\partial E(\Delta UN_1)}{\partial \Delta GDP_1} & \dots & \frac{\partial E(\Delta UN_1)}{\partial \Delta GDP_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial E(\Delta UN_n)}{\partial \Delta GDP_1} & \dots & \frac{\partial E(\Delta UN_n)}{\partial \Delta GDP_n} \end{bmatrix} = (\mathbf{I}_n - \rho \mathbf{W})^{-1} \begin{bmatrix} \beta_k & w_{12}\theta_k & \dots & w_{1n}\theta_k \\ w_{21}\theta_k & \beta_k & \dots & w_{2n}\theta_k \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1}\theta_k & w_{n2}\theta_k & \dots & \beta_k \end{bmatrix}$$

This matrix can be denoted by  $\mathbf{S} = \partial E(\Delta UN)/\partial \Delta GDP = (\mathbf{I}_n - \rho \mathbf{W})^{-1} \mathbf{C}$ . Therefore, the average total effect on  $\Delta UN$  of a unit change in  $\Delta GDP$  can be summarized by computing the row (or column) sum of partial derivatives contained in matrix  $\mathbf{S}$  and then averaging over the  $n$  regions, as in:

$$n^{-1} \sum_{ij} \frac{\partial E(\Delta UN_i)}{\partial \Delta GDP_j} = n^{-1} \mathbf{1}' [(\mathbf{I}_n - \rho \mathbf{W})^{-1} \mathbf{C}] \mathbf{1} \tag{10}$$

This average total effect can be partitioned into a direct component and an indirect component. The average direct effect is a scalar summary of the own-partial derivatives, each of these measuring the impact of a unit change in region  $i$ 's  $\Delta GDP$  on region  $i$ 's  $\Delta UN$ . It is calculated as the average of the elements on the main diagonal of the  $\mathbf{S}$  matrix, as in:

$$n^{-1} \sum_i \frac{\partial E(\Delta UN_i)}{\partial \Delta GDP_i} = n^{-1} \text{trace} [(\mathbf{I}_n - \rho \mathbf{W})^{-1} \mathbf{I}_n \beta]. \tag{11}$$

The average indirect effect is a scalar summary that corresponds to the cross-partial derivatives, each of these representing the response of region  $i$ 's  $\Delta UN$  to a unit change in  $\Delta GDP$  in all other regions. It is equal to the difference between the average total effect and the average direct effect, and is computed as the average of either the row sums or the column sums of the off-diagonal elements of  $\mathbf{S}$ . Results on the true Okun's Law coefficient, as obtained from the implementation of Le Sage and Pace's (2009) method, are presented in subsection 4.2.

## 4 Estimation results

### 4.1 Initial estimates

This section presents results from our panel data analysis of Okun's Law for the 128 NUTS 3 regions of Great Britain. Outcomes from various modelling solutions are provided in order to document biases due to model misspecification and to the omission of spatial patterns in space-time unemployment rate variations.

We start with fixed-effects (FE) and random-effects (RE) IV estimates from panel models with spatial lags of the dependent and independent variables but without interactions among errors, namely, spatial Durbin models. Table 2 illustrates initial estimates from baseline regressions, using an instruments set in its minimal specification (Equation (7)). Model 1 and model 2 are the closest approximations to a dynamic Okun's Law relationship with spatial effects, where joblessness rate changes are explained by time-lagged (local) unemployment rate changes, real GDP growth both locally and within commuting distance, and spatially-lagged

**Table 2.** Baseline regressions: Spatial Durbin models with minimal IV set (Equation 7)

Estimation Method	1	2
	Fixed-Effects IV	Random-Effects IV
Regressors		
Endogenous spatial lag (s.e. $\rho$ )	0.5770 (5.29)***	0.4774 (4.14)***
Real GDP growth rate ( $\Delta$ GDP) (s.e. $\beta$ )	-0.0499 (-2.58)***	-0.0614 (-2.46)***
Spatial lag of $\Delta$ GDP (s.e. $\theta$ )	-0.0748 (-2.33)**	-0.0747 (-1.98)**
One-year lag of UN rate change (s.e. $\gamma$ )	0.1104 (4.03)***	0.1480 (4.63)***
Constant (s.e.)		0.2370 (3.76)***
Diagnostics		
Hausman test of regressors endogeneity <sup>a</sup>		
Chi-sq(2) statistic [P-value]	14.07 [0.00]	14.07 [0.00]
First-stage F test of instruments relevance <sup>b</sup>		
F(6,3449) statistic [P-value] ( $\Delta$ GDP)	19.71 [0.00]***	19.71 [0.00]***
F(6,3449) statistic [P-value] (Spatial lag of $\Delta$ GDP)	41.47 [0.00]***	41.47 [0.00]***
Sargan-Hansen test of instruments orthogonality		
Chi-sq(2) statistic [P-value]	4.75 [0.09]**	2.93 [0.23]***
R <sup>2</sup>	0.7671	0.7126
No. regions	128	128
No. years (1985–2011)	27	27

Notes: regressand is  $\Delta$ UN. \*\*\* means significance at the 1% level, \*\* means significance at the 5% level.

<sup>ab</sup> These diagnostics are common to both models.

(contemporaneous) unemployment rate changes. The  $\hat{\beta}$  coefficient of  $-0.05$  (FE model) or  $-0.06$  (RE model) is lower in absolute terms than Okun's value of  $-0.32$ , or than point estimates found elsewhere e.g. ranging from  $-0.17$  to  $-0.24$  in Crespo-Cuaresma (2003). Particularly, it is smaller than shown for the UK by studies which have investigated Okun's Law over various time periods using national data.<sup>2</sup> Nonetheless, with a  $t$ -ratio of  $-2.58$  (FE model) or  $-2.46$  (RE model), Okun's coefficient is highly significant, supporting the existence of a negative association between economic growth and unemployment among the British regions. Previous studies of regional data, for instance Kangasharju et al. (2012), also found a smaller  $\hat{\beta}$  coefficient than typically estimated from macro time-series data, and related this result to workers mobility and spatial linkages being more important across regions than across countries. The incorporation of spatial effects in the form of spatial lags of the dependent variable,  $(\mathbf{I}_T \otimes \mathbf{W})\Delta$ UN, and of the explanatory variable,  $(\mathbf{I}_T \otimes \mathbf{W})\Delta$ GDP, allows to explicitly test this hypothesis; given the definition of  $\mathbf{W}$ , the strength of spillovers between any two areas is inversely related to geographical distance and directly proportional to the intensity of the commuting links between them. We find a statistically significant and negatively signed between-area impact of real output changes on  $\Delta$ UN, with the parameter on  $(\mathbf{I}_T \otimes \mathbf{W})\Delta$ GDP equal to  $-0.07$  in both models and with an associated  $t$ -ratio of  $-2.33$  (FE model) or  $-1.98$  (RE model). This validates the prediction that

<sup>2</sup> For example,  $-0.34$  in Knoester (1986),  $-0.36$  in Paldam (1987),  $-0.38$  in Moosa (1997),  $-0.69$  in Attfield and Silverstone (1998),  $-0.77$  in Freeman (2001) (see also Lee 2000). For comparability with our result, these are the reciprocals of coefficients obtained from models with unemployment as right-hand-side variable.

economic growth effects on  $\Delta UN$  are also due to real GDP performance in adjacent areas within commuting distance, rather than confined to the local area; therefore, Okun's Law papers should not focus only on the labour market responsiveness to output volume growth in a given location, but attention should be paid to how localized demand policies might influence workforce mobility and have employment effects outside administrative borders. Also for  $(\mathbf{I}_T \otimes \mathbf{W})\Delta UN$ , we find highly significant (direct) effects from nearby commuting areas, which corroborates the existence of interregional labour market linkages.

Moreover we see strong relevance of the one-year lag of the dependent variable; in particular, the negative sign of the  $\hat{\gamma}$  coefficient provides evidence of delayed adjustments to workforce levels and suggests the presence of negative hysteretic effects. Higher-order time lags are either insignificant or wrongly signed (compared to theoretical expectations as outlined in subsection 2.1), or both, and eliminating these does not modify our regression coefficient estimates, which implies that dynamic effects only accrue to the first period.<sup>3</sup>

We next turn to the diagnostics. For the fixed-effects model (model 1), the over-identifying restrictions test gives a Hansen-Sargan test statistic equal to 4.75, which when referred to the relevant  $\chi^2_2$  distribution has an exceedence probability of 0.09; this  $P$ -value is large enough to allow non-rejection of the null that instruments are exogenous, although only at the 1 per cent level. With regard to model 2, the Hansen-Sargan test statistic is 2.93, with an associated  $P$ -value of 0.23, which indicates that orthogonality conditions hold strongly for the random-effects model. Also, in both cases, the joint  $F$  statistics from the first-stage regressions of 2SLS estimation are 19.71 for  $\Delta GDP$  and 41.47 for  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$ , all of which are extreme observations in the reference  $F_{6,3449}$  distribution, which demonstrates that we do not have weak instruments.

Moreover, the Hausman test of regressors endogeneity causes us to reject the null that  $\Delta GDP$  and  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$  are correlated to residuals, however it is worth noting that later estimations will actually produce test results which are in favour of this hypothesis.

Finally, we implement the Hausman consistency test which is a useful statistical tool to assess whether the difference in coefficients between the consistent fixed-effects model and the relatively more efficient random-effects model is significant; it uses the test statistic  $\hat{H} = (\hat{\beta}_{RE} - \hat{\beta}_{FE})' (\hat{\Sigma}_{FE} - \hat{\Sigma}_{RE})^{-1} (\hat{\beta}_{RE} - \hat{\beta}_{FE})$ , where the  $\hat{\beta}_s$  are the vectors of estimates from model 1 and model 2 and the  $\hat{\Sigma}_s$  are the respective covariance matrices. We find that  $\hat{H}$  is not an extreme value with reference to the relevant  $\chi^2$  distribution under the null, as the test statistic is equal to 10.35 which has a  $P$ -value of 0.0350; this implies that, at a conventional (though more liberal) 1 per cent level, we cannot reject the null that random effects are uncorrelated to the explanatory variables – in other words, statistical evidence is not enough to completely dismiss the estimates from model 2.

At this point in our analysis it seems that the fixed-effects model fits the data well and gives plausible results, but the random-effects model cannot be discarded because it produces equally satisfactory outcomes. Diagnostic evidence also suggests that the random-effects model should be retained: the Hausman FE vs. RE test indicates that random-effects estimates are consistent, as well as efficient; in addition, the Sargan-Hansen test shows that the instruments orthogonality conditions are strongly satisfied when random effects are used but notably less so under fixed effects, a possible consequence of some misspecification in the latter case. In order to shed further light on which model should be the preferred one, we next carry out robustness checks on our initial regressions.

Model 3a and model 4a in Table 3 are obtained by applying an extended instruments set (Equation (6)). Looking across Table 3, it is apparent that the evidence points to the superiority

<sup>3</sup> Results with higher-order time lags are not reported in the paper but are available from the authors upon request.

**Table 3.** Robustness checks: Spatial Durbin models with extended IV set (Equation 6)

Estimation method	3a	4a
	Fixed-effects IV	Random-effects IV
Regressors		
Endogenous spatial lag (s.e. $\rho$ )	0.7741 (14.19)***	0.6478 (9.69)***
Real GDP growth rate ( $\Delta$ GDP) (s.e. $\beta$ )	-0.0189 (-1.72)*	-0.0288 (-1.96)**
Spatial lag of $\Delta$ GDP (s.e. $\theta$ )	-0.0219 (-1.31)	-0.0388 (-1.80)*
One-year lag of UN rate change (s.e. $\gamma$ )	0.0731 (4.38)***	0.1133 (5.48)***
Constant (s.e.)		0.1108 (3.78)***
Diagnostics		
Hausman test of regressors endogeneity <sup>a</sup> Chi-sq(2) statistic [P-value]	2.29 [0.32]***	2.29 [0.32]***
First-stage F test of instruments relevance <sup>b</sup> F(8,3447) statistic [P-value] ( $\Delta$ GDP) F(8,3447) statistic [P-value] (Spatial lag of $\Delta$ GDP)	26.02 [0.00]*** 49.96 [0.00]***	26.02 [0.00]*** 49.96 [0.00]***
Sargan-Hansen test of instruments orthogonality Chi-sq(4) statistic [P-value]	14.69 [0.01]*	14.82 [0.01]*
R <sup>2</sup>	0.8611	0.8318
No. regions	128	128
No. years (1985–2011)	27	27

Notes: regressand is  $\Delta$ UN. \*\*\* means significance at the 1% level, \*\* means significance at the 5% level, \* means significance at the 10% level.

<sup>ab</sup> These diagnostics are common to both models.

of a random-effects model. Results from the fixed-effects model (model 3a) become somewhat worse, with the  $\hat{\theta}$  coefficient unexpectedly turning insignificant, while results from the random-effects model (model 4a) remain statistically relevant. It should be noted that the Hausman regressors endogeneity test now confirms that the instrumented variables are not orthogonal to the error term, thus justifying the use of 2SLS estimation to guarantee consistency. Meanwhile, the first-stage  $F$  statistic still leads us to conclude that instruments are relevant (i.e., jointly significant in identifying our endogenous regressors). With regard to the Hansen-Sargan over-identifying restrictions test, in both cases the test statistic is sufficiently small to infer that instruments are exogenous but, with the  $P$ -value just exceeding the 1 per cent rate, hints at potential issues with these regressions. For the sensitivity checks in Table 4, we thus use the minimal IV set, which proved to be valid according to diagnostics in Table 2, and we verify the robustness of estimates to the inclusion of dummy variables for the 1991–1992 and 2008–2009 recessions (similarly to Oberst and Oelgemöller, 2013).

From a comparison of model 3b and model 4b in Table 4 it emerges that results from a random-effects specification (model 4b) are statistically relevant and economically meaningful while, in the fixed-effects specification (model 3b), the real GDP growth rate and its spatial lag are both statistically insignificant and well below theoretical expectations with regard to coefficient sizes. Importantly, the Sargan-Hansen test fails under fixed effects, suggesting instruments invalidity or other causes of model misspecification, whereas the over-identifying restrictions clearly hold in the context of IV estimation with random effects. The Hausman

**Table 4.** Robustness checks: Spatial Durbin models with crises dummies (Equation 7 for IVs)

Estimation method	3b		4b	
	Fixed-effects IV		Random-effects IV	
<b>Regressors</b>				
Endogenous spatial lag (s.e. $\rho$ )	0.8571 (11.33)***		0.5345 (3.26)***	
Real GDP growth rate ( $\Delta$ GDP) (s.e. $\beta$ )	-0.0266 (-1.11)		-0.1068 (-2.34)***	
Spatial lag of $\Delta$ GDP (s.e. $\theta$ )	-0.0135 (-0.35)		-0.1207 (-1.63)*	
One-year lag of UN rate change (s.e. $\gamma$ )	0.0562 (3.60)***		0.1149 (3.26)***	
1991–1992 Recession (s.e.)	-0.0049 (-0.05)		-0.2681 (-1.60)*	
2008–2009 Recession (s.e.)	-0.1052 (-0.63)		-0.6228 (-2.15)***	
Constant (s.e.)			0.5361 (2.21)**	
<b>Diagnostics</b>				
Hausman test of regressors endogeneity <sup>a</sup> Chi-sq(2) statistic [P-value]		1.22 [0.54]***		1.22 [0.54]***
First-stage F test of instruments relevance <sup>b</sup> F(6,3449) statistic [P-value] ( $\Delta$ GDP)		19.71 [0.00]***		19.71 [0.00]***
F(6,3449) statistic [P-value] (Spatial lag of $\Delta$ GDP)		41.47 [0.00]***		41.47 [0.00]***
Sargan-Hansen test of instruments orthogonality Chi-sq(2) statistic [P-value]		17.62 [0.00]		3.70 [0.16]***
R <sup>2</sup>		0.8647		0.6260
No. regions		128		128
No. years (1985–2011)		27		27

Notes: regressand is  $\Delta$ UN. \*\*\* means significance at the 1% level, \*\* means significance at the 5% level, \* means significance at the 10% level.

<sup>ab</sup> These diagnostics are common to both models.

consistency test now gives a test statistic of 6.09, with a *P*-value of 0.73 which is well above a 10 per cent significance level, strongly suggesting that random effects are uncorrelated to the explanatory variables.

All in all, estimation results from baseline and additional regressions support the adoption of a random-effects model, with a minimal form of the IV set fully satisfying the validity requirements (and noticeably outperforming its extended form on the basis of results from the Hansen-Sargan test). We thus select random effects because these worked well over various estimations, offering theoretically coherent and empirically solid outcomes, in contrast to fixed-effects evidence which proved less systematic and less acceptable than one would anticipate; available statistical tests such as the Hausman consistency test also support this decision. The final aspect in our spatial analysis is concerned with further developing our econometric model by allowing for spatially autoregressive (SAR) error components in a random-effects specification, estimated via FGS2SLS/GMM.

In Table 5 (model 5 and model 6), Okun's coefficient remains negative and significant, confirming the inverse response of unemployment rate changes to real output growth, and there is a statistically relevant estimate for the spatial autoregressive parameter  $\lambda$ , which indicates

**Table 5.** Random-effects FGS2SLS/GMM estimation of general nesting spatial (GNS) models

Instrumental-variables set	5	6
	Minimal (Equation 7)	Minimal (Equation 7)
Regressors		
Endogenous spatial lag (s.e. $\rho$ )	0.3824 (3.44)***	0.3319 (2.59)***
Real GDP growth rate ( $\Delta$ GDP) (s.e. $\beta$ )	-0.0609 (-2.66)***	-0.0652 (-2.64)***
Spatial lag of $\Delta$ GDP (s.e. $\theta$ )	-0.1119 (-3.00)***	-0.1232 (-2.96)***
One-year lag of UN rate change (s.e. $\gamma$ )	0.1615 (6.36)***	0.1584 (6.23)***
1991–1992 Recession (s.e.)		-0.1990 (-1.54)*
2008–2009 Recession (s.e.)		-0.4203 (-2.56)***
Constant (s.e.)	0.3009 (3.33)***	0.3482 (3.14)*
Spatial Error process		
$\lambda$	0.5816***	0.6016***
$\sigma_v^2$	0.1686	0.1728
$\sigma_1^2 = \sigma_v^2 + T\sigma_u^2$	0.1519	0.1594
$R^{2a}$	0.8002	0.7782
No. regions	128	128
No. years (1985–2011)	27	27

Notes: regressand is  $\Delta UN$ . \*\*\* means significance at the 1% level, \* means significance at the 10% level.

<sup>a</sup> Correlation between fitted values and observed values of the dependent variable.

significant ‘nuisance’ spatial dependence due to shock spillovers and omitted (positively) spatially autocorrelated explanatory variables.

Importantly, the inclusion of spatial dependence in the error term appears to improve the estimates. To begin with, results are more in line with expectations as the regression coefficient of own region's output growth is now lower, in absolute terms, than that of output growth in other regions within commuting distance; the theoretical argument for this is the presence of regional dependence in labour markets from systematic/substantive linkages, here taking the form of commuting flows which cause labour-market outcomes in one place to depend partly on labour-market or business conditions elsewhere, and non-systematic linkages which arise from inappropriate geographical delineation (non-systematic linkages mean that available spatial units, that is, formal areas, virtually never coincide with functional labour market areas, in other words output growth within commuting distance is a better approximation of the true extent of economic activity).

Additionally, compared to the counterpart model without SAR errors (model 4b), we find that  $\Delta GDP$  and  $(\mathbf{I}_T \otimes \mathbf{W})\Delta GDP$  become more strongly significant as  $t$ -ratios are higher; the likely reason is that the fit of the model is better when SAR errors are a component of the equation and thus standard errors are smaller. For  $(\mathbf{I}_T \otimes \mathbf{W})\Delta UN$ , we see that the absolute value of coefficient  $\hat{\rho}$  is now lower, possibly because if  $\mathbf{M}$ -based interactions should be in the model and they are not then one will have omitted-variables bias in the parameter estimates and this may be distorting  $\hat{\rho}$ .



Models 5 and 6 give almost identical outcomes; however, since model 5 has a slightly better fit, we take this as our final preferred specification. In the next section we use model 5 as the point of departure to give a comprehensive and precise account of the validity and strength of Okun's Law, by capturing spatial effects in their full extent.

#### 4.2 True Okun's Law coefficient: Direct, spillover and total impacts

Table 6 displays the true marginal effects relating to  $\Delta GDP$ , that is, the average total impact, and their average direct and indirect components. It should be noted that the direct and indirect effects associated with  $\Delta GDP$  are different from the values of, respectively,  $\hat{\beta}$  and  $\hat{\theta}$  obtained from model 5 because they incorporate feedback loop effects, as implied by the Leontief expansion in Equation (9); these arise because any given area is considered a neighbour to its neighbour, so that the spatial transmission mechanism is such that shocks to the system propagate across neighbouring areas and eventually come back to the area they originated from.

The substantive result that emerges from this analysis is that our true Okun's Law coefficient amounts to  $-0.2798$ , which in absolute terms is very close to the 'empirical law', and this is primarily attributable to indirect effects. This means that, while the average direct impact coefficient of  $-0.0756$  in Table 6 is statistically significant, what really counts for local labour market performance is real GDP growth in nearby areas within commuting distance, with a large part of the actual marginal effect of  $\Delta GDP$  on  $\Delta UN$  being due to spatial spillovers from workforce mobility and geographical proximity – around 73 per cent.

Our conclusions regarding the validity and strength of Okun's Law for the British regions, based on the average total impact coefficient of  $-0.2798$ , are more in accord with existing evidence, as opposed to inference drawn from estimated regression coefficients  $\hat{\beta} = -0.0609$  and  $\hat{\theta} = -0.1119$  given by model 5 (interpreted individually and without considering feedback

**Table 6.** Direct, indirect and total effects of  $\Delta GDP$  (model 5)

W Matrix with distance threshold of 100	Okun's Law Coefficient
AVG direct (own-region) effects (bootstrapped <i>t</i> -ratio)	-0.0756 (-3.19)***
AVG indirect (spatial, spillover) effects (bootstrapped <i>t</i> -ratio)	-0.2042 (-5.18)***
AVG total effects (bootstrapped <i>t</i> -ratio)	-0.2798 (-5.69)***
W Matrix with distance threshold of 150	Okun's Law Coefficient
AVG direct (own-region) effects (bootstrapped <i>t</i> -ratio)	-0.0754 (-3.55)***
AVG indirect (spatial, spillover) effects (bootstrapped <i>t</i> -ratio)	-0.2039 (-5.16)***
AVG total effects (bootstrapped <i>t</i> -ratio)	-0.2793 (-6.14)***
W Matrix with distance threshold of 50	Okun's Law Coefficient
AVG direct (own-region) effects (bootstrapped <i>t</i> -ratio)	-0.0763 (-3.47)***
AVG indirect (spatial, spillover) effects (bootstrapped <i>t</i> -ratio)	-0.2056 (-4.71)***
AVG total effects (bootstrapped <i>t</i> -ratio)	-0.2819 (-5.38)***

Notes: \*\*\* means significance at the 1% level.

loops). Spatial effects thus matter, and are responsible for the seemingly low impact of economic growth on unemployment which is apparent from model 5.

For an explanation of why the regression coefficient of own region's output growth is lower in absolute terms than that of output growth in other regions within commuting distance, we refer the reader to the discussion of results from Table 5 in the previous subsection. Moreover, our result is not exceptional, as authors who have undertaken similar analyses of direct, spillover and total effects have reached analogous conclusions. Lerbs and Oberst (2012) investigate the influence of house prices on homeownership rates among German regions and see a total effect of  $-5.3$  percentage points, of which only  $-1.7$  points are due to own-region variations while  $-3.6$  points correspond to the average cumulative indirect effect (68% of the overall impact). In a regional Okun's Law context, Oberst and Oelgemöller (2013) find that the magnitude of the total growth effect can be attributed to output variations in neighbouring areas for a proportion of almost 60 per cent. In both cases, results are robust to alternative weighting matrices – namely a contiguity matrix and a four nearest neighbour inverse distance matrix with no distance threshold or with a threshold of 90km. While this evidence may be less striking than our estimate of 73 per cent, it indicates a prominent role of inter-regional linkages, with spatial effects accounting for more than half of the total effect in both papers.

In order to show the robustness of results to specifications of  $\mathbf{W}$ , we vary the distance threshold, which is a discretionary parameter though motivated by existing literature as explained in subsection 2.4. Results in Table 6 show that threshold values of 100km, 150km and 50km give broadly the same estimates of direct, indirect and total effects, which demonstrates that outcomes are robust to the use of different forms of the commuting-based  $\mathbf{W}$  matrix; obviously, with a threshold of 50km, the spatial weight matrix is relatively more sparse (more of its elements are zero), so spillover and total effects become somewhat less significant.

## 5 Conclusions

This paper has been the first to analyse Okun's Law using a spatial panel approach on NUTS 3 data for Great Britain. The relationship, expressing unemployment rate changes as an inverse function of real GDP growth as in Okun's seminal work, is adapted to properly control for regional heterogeneity, spatial spillovers between neighbouring areas within commuting distance involving both variables, and cross-section dependence in error components. We find that the predicted negative relationship is corroborated by our data, and that the statistical significance of output volume growth is maintained under different model specifications and estimation methods, which demonstrates the robustness of our results.

The main conclusion we reach from estimating Okun's Law in a regional context is that the regression coefficient is lower than previously shown by cross-country evidence, and this is largely attributable to the spatial mechanisms which are at work in a small-distance, small-area scenario. First, the inclusion of the spatial lags to absorb interactions and linkages reveals that commuting and other spatial effects (as embodied in the  $\mathbf{W}$  and  $\mathbf{M}$  matrices, respectively) are relevant to a proper understanding of Okun's Law dynamics. Moreover, although model 5 provided us with a satisfactory set of outcomes, our final preferred estimate ( $-0.28$ ) is one which is obtained from taking full account of spatial effects, including the additional impact on any given area from changes cascading through the entire spatial hierarchy, and which points to a stronger Okun's relationship than inferred from the  $\hat{\beta}$  and  $\hat{\theta}$  parameters of model 5. This provides further support for the important role of interregional dependencies in the context of Okun's Law.

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**Resumen.** Este artículo explora la Ley de Okun mediante el uso de un enfoque de panel de datos espaciales de nivel NUTS 3 de Gran Bretaña durante el período 1985–2011. La especificación básica empleada como punto de partida para el análisis espacial es la relación entre el crecimiento de la producción real y los cambios en la tasa de desempleo. Ésta se amplió para incorporar retardos espaciales con los que poner a prueba la presencia de efectos de desplazamiento al trabajo, y para dar cabida a la dependencia de residuos espaciales. Se estimaron diferentes modelos de panel espacial en los que se controló la heterogeneidad entre áreas, y se encontraron pruebas robustas de un coeficiente de Okun significativo. Los resultados muestran también la importancia de los efectos de *spillover* derivados de la movilidad laboral y los vínculos inter-regionales y tienen implicaciones importantes para la validez de la ley empírica.

**要約:** 本論文では、1985年から2011年の英国のNUTS3地域のデータに対して、空間的パネルアプローチを用いて、オークンの法則を検討する。空間的分析では、実質生産量の伸び率と失業率の変動の関連性が出発点となる基本仕様である。これを空間ラグに組み込み、コミューティングによる効果の検出、空間的残差従属の調整にまで拡張する。地域間の異質性をコントロールして様々な空間的パネルモデルを推計し、有意なオークン係数の一貫性のあるエビデンスが認められた。また、分析によると、労働移動と地域間のつながりに関する事項によるスピルオーバー効果は大きく、実証的な法則の妥当性に大きな意味を持つ。