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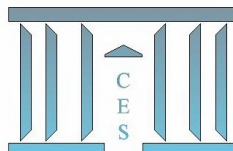
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**Should the host economy invest in a new industry?  
The roles of FDI spillovers, development level,  
and heterogeneity of firms**

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# Should the host economy invest in a new industry? The roles of FDI spillovers, development level, and heterogeneity of firms\*

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## Abstract

We consider a small open economy with two productive sectors (an old and a new). There are two types of firms in the new industry: a well planted multinational firm and a potential domestic firm. Our framework highlights a number of results. First, in a poor country with low return of training and weak FDI spillovers, the domestic firm does not exist in the new industry requiring a high fixed cost. Second, once the host economy has the capacity to create the new firm, the productivity of the domestic firm is the key factor allowing it to enter into the new industry, and even eliminate the multinational firm. Interestingly, in some cases where FDI spillovers are strong, the country should invest in the new industry, but not train specific workers. Last, credit constraints and labor/capital shares play important roles in the competition between the multinational firm and the domestic one.

**Keywords:** FDI spillovers, investment in training, heterogeneous firms, entry cost

**JEL Classification Numbers:** F23, F4, F62, O3

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# 1 Introduction

Over the last five decades, operations of multinational firms have made a significant influence on developing economies. Multinational firms may generate FDI spillovers to domestic firms by transferring advanced technologies or training workers. Thanks to that, some countries can promote the development of new industries and particularly, encourage the entry of domestic firms into these industries.<sup>1</sup> However, is attracting FDI spillovers the key factor for this development? If not, what is the optimal policy of the host country? More precisely, should the host country develop these new industries, or continue to focus on already developed ones? How is the competition between multinational and domestic firms? What are the roles of different macroeconomic variables such as development level, FDI spillovers, return of training, and heterogeneity of firms.

To answer these questions, we consider a small open economy model with two sectors and heterogeneous firms. The first, called old sector, produces consumption good. There is a unique representative domestic firm in this sector. The second, called new sector, produces a new good by using physical capital and a specific labor. There are two types of firm in the new sector: an already planted multinational firm and a potential domestic one. The potential domestic firm cannot be created if it holds less than a critical threshold  $\bar{L}$  units of specific labor. These two firms differ not only in productivity but also in labor and capital shares. In this economy, consumption good, physical capital, and new good can be freely exchanged with the rest of the world while the specific labor is not mobile. The prices (in term of consumption good) of physical capital and new good are assumed to be exogenous. However, the wage is endogenous and determined by labor market clearing condition. Specific labor supply is also endogenous and arises from three sources: initial specific labor of the country, FDI spillovers effects, and investment in training.

Our framework provides a number of results. First, to invest in the new industry, the country must hold one of the following conditions: (i) it is rich enough, (ii) its return of training is high enough, (iii) FDI spillovers are sufficiently strong. This result is due to the existence of fixed cost  $\bar{L}$  which prevents the domestic firm's entry. Our finding indicates that in a poor country with low FDI spillovers, the domestic firm cannot exist in the new industry even if its productivity is high.

Second, once the country holds the above conditions, the productivity of

the potential domestic firm is the key factor deciding the optimal choice of the country.

Our framework shows that if the productivity of the foreign firm or of the old sector is sufficiently high, the host country should not invest in the new industry. Moreover, in the case where the multinational firm's productivity is high enough, the optimal strategy of the host country is to train specific workers and then let them work for the foreign firm in order to get a favorable salary.

We prove that the host country should invest in the new industry if and only if the productivity of the domestic firm in this industry reaches a critical threshold. Moreover, although the domestic firm must pay an entry cost, it can dominate, even eliminate, the multinational firm. We also make clear the role of the entry cost by showing that the mentioned critical threshold of productivity increases when the entry cost increases. One may ask if training of specific labor is essential to create a new domestic firm in the new industry. Not always! Indeed, in the case where FDI spillovers are strong and the domestic firm's productivity is high, the host country should invest in the new industry but not in training.

Third, we study the competition between the multinational firm and the domestic one in the new industry by analyzing heterogeneities of firms and the roles of exogenous prices, return of training. Since the wage is endogenous, specific workers will be hired by the more competitive firms.<sup>2</sup> Does the domestic firm benefit from high return of training/low physical capital price/high new good price in order to compete with the multinational firm? Our model shows that, with high returns of training, the host country will not invest in the new industry when the physical capital share of the potential domestic firm is not too low. The main reason comes from credit capacity of firms. Indeed, high returns implies a high number of specific workers. If the potential domestic firm has a weak credit capacity, it cannot buy an arbitrary quantity of physical capital when its capital share is not too low and the number of workers is high. Therefore, its production process will be inefficient. By contrast, the multinational firm can get financing from its parent company and, thanks to that, when specific labor supply is high, it can buy an arbitrary amount of physical capital to make its production process efficient. As a consequence, all specific workers will be hired by the multinational firm, which implies that the domestic one cannot enter the new industry even if the country has a high return of training. A similar argument can be used when physical capital price is low. It seems that the host

country may more likely invest in the new industry if the new good price is high. Unfortunately, this argument is not always true. Our framework points out that even if the new good price is very high, the domestic firm cannot enter this new industry because of its weak credit capacity. However, we should make clear that with middle level of return of training/physical capital price/new good price, productivities of firms play the most important role in their competition.

Our paper is related to several strands of research. The first strand studies the fixed entry cost of firms and economic growth. Smith (1987) and Markusen (1995) pointed out that a potential domestic firm has to invest in a firm-specific fixed cost in order to be able to produce. By contrast, Smith (1987) considered that the multinational firm has a plant in its home country where this investment has been already realized, and then does not suffer it by producing in the host country. Fosfuri, Motta, Ronde (2001) indicated that a domestic firm may gain from new technologies thanks to the mobility of worker who initially worked for multinational firms. However, to do that, the domestic firm must to pay a fixed cost which may be interpreted as its absorptive capability. In our framework, we assume that the domestic firm must utilize a fixed number of skilled workers to ensure that its production process functions. We also make clear the impact of this fixed cost on the competition of firms, and then on the economic growth. In optimal growth context, Bruno, Le Van, Masquin (2009) proved that a poor country cannot invest in new technology. However, they consider do not take into account the impact of multinational firms. In our paper, multinational firms can generate FDI spillovers and may eventually help the country to invest in new technology.

The second concerns FDI spillovers and training of skilled workers. The literature shows the existence of four types of FDI spillovers.<sup>3</sup> First, FDI spillovers may be created via vertical linkages between foreign affiliates and local suppliers (Rodriguez-Clare , 1996; Markusen, Venables , 1995; Carlucio, Fally , 2013). Second, multinational firms can improve productivity of domestic firms through demonstration/imitation effects. Export is the third channel through which domestic firms can benefit from multinational firms (Aitken, Hanson, Harrison , 1997; Greenaway, Sousab, Wakelin , 2004). Last, FDI spillovers may arise due to the mobility of workers who have been trained by multinational firms (Ethier, Markusen , 1996; Fosfuri, Motta, Ronde , 2001; Poole , 2013). FDI spillovers in our paper are generated through the last form. By contrast, in our paper, specific workers are not only trained

by multinational firms (through specific communication or learning by doing effects), but also by the government; thanks to that, the host country gaining low FDI spillovers can still develop the new industry.

The last strand is the link between credit constraints and trade. Kletzer, Bardhan (1987) theoretically showed how comparative advantage depends on credit market imperfections. By using a 30-year panel for 65 countries, Beck (2002) found that financial development exerts a causal impact on exports and trade balance of manufactured goods. Manova (2008) studied the impact of equity market liberalizations on trade by giving empirical evidence (with 91 countries), and then showed that credit constraints play an important role on international trade flows. Manova (2013) incorporated credit constraints and firm heterogeneity into Melitz (2003) and studied the impact of financial frictions not only on producers's entry into exporting but also on exporters' foreign sales. Different from these papers, we focus on the impact of credit constraints on the competition between the domestic firm and the multinational one in the host country's market.

The remainder of the paper is organized as follows. Section 2 presents the structure of economy. In section 3, we explore the optimal strategy of the host country at equilibrium in a two-period model by analyzing roles of all factors of the economy. Section 4 concludes. All formal proofs can be found in Appendices.

## 2 The model

We consider a small open economy having two productive sectors. The first produces the consumption good by using physical capital good. We call it the old sector. There is a unique representative domestic firm (called consumption good firm) in this sector and its production function is given by

$$F^c(K_c) = A_c K_c^{\alpha_c} \quad (1)$$

where  $A_c > 0$  and  $\alpha_c \in (0, 1)$ .<sup>4</sup>

The second sector produces a new good by using physical capital good and a specific labor. It is called new sector or new industry. In this sector, there are two types of firm: a multinational firm (or foreign firm) and a potential domestic one. The foreign firm is well planted in the country and

its production function is

$$F^e(K_e, L_e) = A_e K_e^{\alpha_e} L_e^{\beta_e} \quad (2)$$

where  $A_e > 0$  and  $\alpha_e, \beta_e \in (0, 1)$ ,  $\alpha_e + \beta_e \leq 1$ .

The potential domestic firm's production function is given by

$$F^d(K_d, L_d) = A_d K_d^{\alpha_d} ((L_d - \bar{L})^+)^{\beta_d} \quad (3)$$

where  $A_d > 0$  and  $\alpha_d, \beta_d \in (0, 1)$ ,  $\alpha_d + \beta_d \leq 1$ .<sup>5</sup> To enter the new industry, the domestic firm must make an initial investment. We model this investment by the fixed cost,  $\bar{L}$ , representing the number of specific workers needed to ensure that the production process functions. Thanks to the parent company, the foreign firm does not need to pay this investment.

**Interpretation of  $\bar{L}$ :** In general, we can assume that the production functions of firms are

$$\begin{aligned} F^d(K_d, L_d) &= A_d K_d^{\alpha_d} ((L_d - \bar{L}_d)^+)^{\beta_d} \\ F^e(K_e, L_e) &= A_e K_e^{\alpha_e} ((L_e - \bar{L}_e)^+)^{\beta_e}. \end{aligned}$$

In the new industry, firms need some technical experts to set up the production process in order to be able to produce. The parent company of the foreign firm has such experts in the home country and sends them to host countries for new production plants. Once this setup is finished, the technical experts will come back to their home country. Hence, we can assume that  $\bar{L}_d > \bar{L}_e$ . Without loss of generality, we assume that  $\bar{L}_e = 0$ , and in this case we write  $\bar{L}$  instead of  $\bar{L}_d$ .<sup>6</sup>

In our framework, the economy takes place into two periods: date 0 and date 1. All consumption good, physical capital, and new good can be freely exchanged with the rest of the world, but the specific labor is not mobile. Let consumption good price be numeraire. Denote  $p$  (resp.,  $p_n$ ) the international real prices of capital good (resp., new good) in term of consumption good. Prices  $p, p_n$  are exogenous. The initial endowment of the host country is  $S$ , ( $S > 0$ .)

Let  $L_0$  be the initial specific labor,  $T_0$  be the specific workers generated by the foreign firm at the first period.<sup>7</sup> We assume that if the country invests an amount  $H_1$  in training of specific labor, it will get  $\epsilon H_1$  specific workers, where  $\epsilon$  is the return of training. Hence the specific labor supply of the country after receiving FDI and training will be

$$L_0 + T_0 + \epsilon H_1.$$



Note that specific labor supply in this economy is endogenous.

Let denote this economy by

$$\mathcal{E} := (F^c, F^d, F^e, S, p, p_n, \epsilon, L_0, T_0, \bar{L}).$$

Denote by  $w_1$  the real wage in term of consumption good at date 1, the wage is endogenous in our model. For simplicity, we assume that the depreciation rate of physical capital equal 1.

The foreign firm (without market power) maximizes its profit.

$$(F) : \quad \max_{K_{e,1}, L_{e,1}^D \geq 0} \left[ p_n F^e(K_{e,1}, L_{e,1}^D) - p K_{e,1} - w_1 L_{e,1}^D \right].$$

The social planner takes prices as given and chooses  $c_1, K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1}$  to maximize GNP of the economy at the second period:<sup>8</sup>

$$(P) : \quad \max_{(K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1})} \left[ U := F^c(K_{c,1}) + w_1 L_{e,1} + p_n F^d(K_{d,1}, L_{d,1}) \right]$$

subject to

$$H_1 + p(K_{c,1} + K_{d,1}) \leq S \tag{4}$$

$$L_{d,1} + L_{e,1} \leq L_0 + T_0 + \epsilon H_1, \tag{5}$$

$$K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1} \geq 0. \tag{6}$$

At the first period (date 0), the social planner uses  $H_1$  units of consumption good to train specific labor. She also buys  $K_{c,1}$ , (resp.  $K_{d,1}$ ) units of physical capital as input for the consumption sector (resp. the new sector).

At the second period (date 1), an amount of specific labor  $L_{e,1}$  is used by the multinational firm and another amount of specific labor  $L_{d,1}$  is used by the domestic firm. The GNP of the economy (in term of consumption good) has three parts

- (i)  $F^c(K_{c,1})$ : consumption good from the consumption sector.
- (ii)  $w_1 L_{e,1}$ : salary in term of consumption good paid by the multinational firm.
- (iii)  $p_n F^d(K_{d,1}, L_{d,1})$ : production value of the domestic firm.

Note that, if a specific worker works for the multinational firm, she only contributes to the GNP by her salary because the multinational firm takes away its profits. However, if she works for the domestic firm, the GNP is improved in two ways, salary of the worker and profit of the domestic firm.

**Remark 1.** *The constraint  $H_1 + p(K_{c,1} + K_{d,1}) \leq S$  means that the host country cannot borrow from abroad. As a consequence, the potential domestic firm faces a credit constraint. By contrast, the multinational firm does not face credit constraint because it can get financing from its parent company.*

## 2.1 Equilibrium

**Definition 1.** *Consider the economy  $\mathcal{E} := (F^c, F^d, F^e, S, p, p_n, \epsilon, L_0, T_0, \bar{L})$ . An equilibrium is a list  $(K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1}, K_{e,1}, w_1)$  such that*

- (i) *Given labor price  $w_1$ ,  $(K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1})$  is a solution of problem (P).*
- (ii) *Given labor price  $w_1$ ,  $(L_{e,1}^D, K_{e,1})$  is a solution of problem (F).*
- (iii) *Labor market clears:  $L_{e,1}^D = L_{e,1}$ .*

The wage is endogenously determined by the labor market clearing condition  $L_{e,1}^D = L_{e,1}$ . Note that, in general, it is impossible to give a closed formula of the wage.

The production function  $F^d$  is not differentiable at  $(K, \bar{L})$  because of the existence of entry cost  $\bar{L} > 0$ . We solve (P) by considering two cases:  $L_{d,1} > \bar{L}$  and  $L_{d,1} \leq \bar{L}$ , and compute the GNP in each case. We then compare these GNPs and find conditions under which the one is greater than the other.

In what follows, in order to avoid confusion, we present our findings in the case where the production functions of the firms are strictly decreasing returns to scale (DRS), i.e.  $\alpha_d + \beta_d, \alpha_e + \beta_e < 1$ . Note that, most of our findings are also valid for constant returns to scale technologies (see Theorem 2 and Proposition 10).

As the wage is endogenous in our model, let us start by pointing out relations among exogenous prices, return of training and wage.

**Proposition 1.** *(i)  $w_1$  decreases if  $p$  or  $\epsilon$  increases, increases if  $p_n$  increases.*

- (ii) Denote  $\tilde{w}_1$  (resp.  $\hat{w}_1$ ) the wage in the case  $Y_{d,1} > 0$  (resp.  $Y_{d,1} = 0$ ). Then we have  $\hat{w}_1 \leq \tilde{w}_1$ .

*Proof.* These are direct consequences of the equation determining the wage, which is presented in Appendix 5.  $\square$

Point (i) is clear. For example, a rising of physical capital price do decrease the production level in the new sector. Consequently, demand of specific labor decreases. Therefore, the wage will decrease.

Point (ii) of Proposition 1 indicates that the entry of domestic firm into the new industry leads to a greater wage than that in the case without the domestic firm.

## 2.2 FDI spillovers, optimal shares, and GNP

In this section, we consider an equilibrium in which  $H_1 > 0$ . Denote  $\theta_h, \theta_d$  the optimal share of investment in training and in new sectors, respectively, i.e.,

$$pK_{c,1} = (1 - \theta_d - \theta_h)S, \quad pK_{d,1} = \theta_d S \quad \text{and} \quad H_1 = \theta_h S \quad (7)$$

First, we focus on direct FDI spillovers  $T_0$ .

**Proposition 2.** *When  $T_0$  increases, GNP increases, but  $\theta_h$  decreases.*

*Proof.* See Appendix 5.3.  $\square$

This result confirms the positive impact of direct FDI spillovers  $T_0$  on GNP of the host country, as it is shown in the literature. However, such positive externalities lowers the share of investment in training of specific labor. The reason is that an increase of  $T_0$  will improve the specific workers supply in the host country, then lower wage, finally decrease investment in training.

## 3 Should the host economy invest in a new industry?

We say that the country *invests in the new industry* if  $Y_{d,1} > 0$ . In this section, we now study the roles of different factors on the optimal strategy of the social planner. Let us start by two extreme cases: the entry cost  $\bar{L}$  is very low and the initial endowment  $S$  is very high.

**Proposition 3.** (i) *There exists  $\bar{L}^* > 0$  such that if  $\bar{L} < \bar{L}^*$  then  $Y_{d,1} > 0$ .*

(ii) *There exists  $\bar{S}$  such that if  $S > \bar{S}$  then  $H_1 > 0$  and  $Y_{d,1} > 0$  at equilibrium.*

*Proof.* See Appendix 5.3. □

In our model, the country's initial endowment  $S$  can be viewed as an index of the development level of the country. Proposition 3 shows that, when the country has a high development level (i.e,  $S$  reaches a critical level), or the entry cost is very small, the host country should invest in the new industry.

### 3.1 Role of productivities

We now observe the impact of the productivities of the old sector and the multinational firm as well.

**Proposition 4.** (i) *There exists  $\bar{A}_e > 0$  such that if  $A_e \geq \bar{A}_e$ , we have  $Y_{d,1} = 0$  and  $H_1 > 0$ .*

(ii) *There exists  $\bar{A}_c > 0$  such that if  $A_c \geq \bar{A}_c$ , we have  $Y_{d,1} = 0$  and  $H_1 = 0$ .*

*Proof.* See Appendix 5.3. □

Proposition 4 shows that, if the productivity of the foreign firm or of the old sector is so high, the host country should not invest in the new industry. Moreover, in the first case, it is optimal to train specific workers and then let them work for the foreign firm in order to get a favorable salary. By contrast, in the second case when the old sector is highly competitive, the country should not invest in training of specific workers. Indeed, the goal of this investment is to provide specific labor for the new sector. However, the competitiveness of this sector is less than that of the old sector, and then so is the gain from the new sector. As a consequence, investing in training is not the best choice.

We now study how the productivity of the domestic firm and the development level of the host country affect the optimal strategy of the social planner. Let us begin by the following result.

**Proposition 5.** *If  $\epsilon S + L_0 + T_0 \leq \bar{L}$  then  $Y_{d,1} = 0$ .*

*Proof.* Since  $H_1 \leq S$  then  $L_0 + T_0 + \epsilon H_1 \leq \epsilon S + L_0 + T_0$ . Consequently,  $L_{d,1} - \bar{L} \leq 0$ , hence  $L_{d,1} = K_{d,1} = 0$ . □

Proposition 5 shows that if a country want to invest in the new industry, it must hold one of the following conditions: (i) its development level is high enough, (ii) the return of training is high enough, (iii) FDI spillovers are strong enough.

We now consider a host country whose maximum specific labor supply is greater than the entry cost but specific labor supply without training is not. We have the following result.

**Proposition 6.** *Assume that  $\epsilon S + L_0 + T_0 > \bar{L} \geq L_0 + T_0$ . We have*

- (i) *there exists  $\bar{A}_1 > 0$  such that  $Y_{d,1}(A_d) > 0$  if and only if  $A_d \geq \bar{A}_1$ . In this case  $H_1 > 0$ .*
- (ii) *there exists  $\tilde{A}_1 \geq \bar{A}_1$  such that if  $A_d > \tilde{A}_1$  then  $Y_{d,1} > Y_{e,1}$ .*
- (iii) *both  $\bar{A}_1$  and  $\tilde{A}_1$  are increasing in  $\bar{L}$ .*

*Proof.* See Appendix 5.3 □

Point (i) of Proposition 6 indicates that, in a host country such as the one studied in this case, the productivity of the domestic firm is the key factor determining the optimal strategy of the social planner. If this firm is sufficiently competitive, it is optimal to invest in the new sector. However, since  $L_0 + T_0 \leq \bar{L}$ , training of specific workers is required to cover the entry cost of the domestic firm. That is why we have a strictly positive amount  $H_1$  when  $A_d$  is high enough. Inversely, if the domestic firm has a low productivity, the social planner should not invest in the new industry. In this case, we do not have enough information to know whether the country invests in training of specific labor.

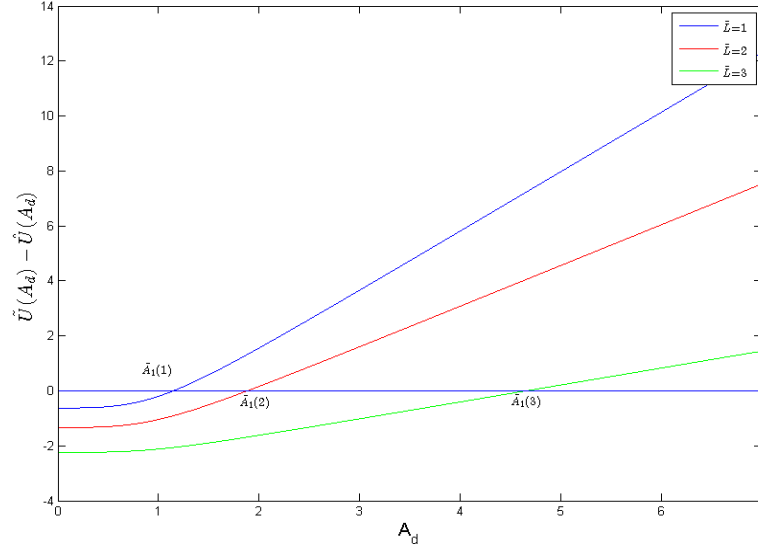
Point (ii) of Proposition 6 shows that the domestic firm can even dominate the foreign one, when the productivity of the former is very high,  $A_d > \tilde{A}_1$ . Our result is related to Markusen, Venables (1995) since these authors proved that in some countries, domestic firms may become sufficiently strong such that local production overtakes and carries out foreign one.

In point (iii) of Proposition 6, we clearly see how the fixed cost  $\bar{L}$  prevent the host country invests in the new industry. The higher level of  $\bar{L}$ , the higher level of productivity the domestic firm must have to enter the market.

Let us show an example. Denote  $\tilde{U}$  (resp.  $\hat{U}$ ) the GNP in the case  $Y_{d,1} > 0$  (resp.  $Y_{d,1} = 0$ ). Figure 1 gives the path of the difference  $\tilde{U} - \hat{U}$  as a function of  $A_d$  for three values of  $\bar{L} = 1, 2, 3$ . Note that  $A_d > A_e$  (or  $A_d < A_e$ ) is not

sufficient to ensure the domestic firm's entry. We also see that the threshold  $\bar{A}_1$  is increasing in  $\bar{L}$ .

Figure 1: The graph of  $(\tilde{U} - \hat{U})$  as a function of  $A_d$



$$A_c = A_e = 1.2; \epsilon = 1.2; S = 2; L_0 = 0.5; T_0 = 0.5; p = 1; p_n = 2; \\ \alpha_c = 0.7; \alpha_d = 0.3; \beta_d = 0.4; \alpha_e = 1/3; \beta_e = 7/15.$$

We have a direct consequence of Proposition 6.

**Corollary 1.** *Assume that  $L_0 = T_0 = 0$  and  $\epsilon S > \bar{L}$ . We have  $Y_{d,1}, H_1 > 0$  when  $A_d$  is high enough.*

This is the case where there is neither FDI spillovers effects nor initial specific labor. Our result gives an answer for the question: when the host country create a new industry? A new industry can only be created under two conditions: (1) return of training is high, (2) the potential domestic firm is competitive enough.

We now study the case of a host country in which specific labor supply without training is high enough.

$$L_0 + T_0 > \bar{L} \tag{8}$$

The optimal strategy of the social planner depends on different factors.

**Proposition 7.** *Assume that  $L_0 + T_0 > \bar{L}$ .*

- (i) *If  $\epsilon S < \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$  then there exists  $\bar{A}_2 > 0$  such that  $Y_{d,1} > 0$  and  $H_1 = 0$  if and only if  $A_d \geq \bar{A}_2$ . Moreover, there exists  $\tilde{A}_2 \geq \bar{A}_2$  such that if  $A_d > \tilde{A}_2$  then  $Y_{d,1} > Y_{e,1}$ .*
- (ii) *If  $\epsilon S > \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$  then there exists  $\bar{A}_3 > 0$  such that  $Y_{d,1} > 0$  and  $H_1 > 0$  if and only if  $A_d \geq \bar{A}_3$ . Moreover, there exists  $\tilde{A}_3 \geq \bar{A}_3$  such that if  $A_d > \tilde{A}_3$  then  $Y_{d,1} > Y_{e,1}$ .*
- (iii) *If  $\epsilon S = \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$  then when  $A_d$  is high enough, we have  $Y_{d,1} > 0$ .<sup>9</sup>*

*Proof.* See Appendix 5.3. □

Proposition 7 indicates that sole conditions on specific labor and entry cost are not sufficient to ensure the entry of the domestic firm into the new industry. Once again, we observe the decisive role of its productivity  $A_d$ . The host country should invest in the new industry if and only if this productivity is high enough. This explains why in some rich countries, although there are sufficiently workers required to create a new industry, they do not choose to do it.

The first point of Proposition 7 shows us an interesting scenario: the host country can create a new firm, i.e.  $Y_{d,1} > 0$ , without training of specific labor, i.e.,  $H_1 = 0$ . This is the case where the potential domestic firm's productivity is high and the condition  $\epsilon S < \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$  holds, i.e., when

- (i)  $\epsilon$  is low (see Proposition 8 for further discussions.)
- (ii) or/and the ratio  $\alpha_d/\beta_d$  of capital share over specific labor share of the potential domestic firm is high
- (iii) or/and the difference  $L_0 + T_0 - \bar{L}$  is high. This means that the entry cost is relatively lower than FDI spillovers  $T_0$  and/or the initial specific labor  $L_0$ .

Some empirical studies are likely to support our finding. Gershenberg (1987) argued that in Kenya, some local managers usually started their career in multinational firms before creating their own firm. By using a sample of firm-level data in Ghana, Gorg, Strobl (2005) stated that there exist some domestic firms whose entrepreneurs (owner or chairman) worked for

a multinational firm before joining or setting up their own domestic firm.<sup>10</sup> These managers/entrepreneurs can be represented by parameter  $T_0$  in our model.

We summarize our findings in the following theorem.

**Theorem 1.** *We have the following properties at equilibrium*

1. If  $L_0 + \epsilon S + T_0 \leq \bar{L}$  then  $Y_{d,1}(A_d) = 0$  for all  $A_d$ .
2. If  $L_0 + \epsilon S + T_0 > \bar{L}$  then
  - 2.1. If  $L_0 + T_0 \leq \bar{L}$  then when  $A_d$  is high enough, we have  $Y_{d,1} > 0$  and  $H_1 > 0$ . Moreover, when  $A_d$  is very high, we have  $Y_{d,1} > Y_{e,1}$ .
  - 2.2. If  $L_0 + T_0 > \bar{L}$  then
    - 2.2.1. if  $\epsilon S < \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$  then when  $A_d$  is high enough, we have  $Y_{d,1} > 0$  and  $H_1 = 0$ . Moreover, when  $A_d$  is very high, we have  $Y_{d,1} > Y_{e,1}$ .
    - 2.2.2 if  $\epsilon S > \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$  then when  $A_d$  is high enough, we have  $Y_{d,1} > 0$  and  $H_1 > 0$ . Moreover, when  $A_d$  is very high, we have  $Y_{d,1} > Y_{e,1}$ .
    - 2.2.3 If  $\epsilon S = \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$  then when  $A_d$  is high enough, we have  $Y_{d,1} > 0$ .

In Theorem 1, the multinational firm produces thanks to its decreasing return to scale technology. However, in the case of constant return to scale production functions, we have the following result.

**Theorem 2.** *We assume that  $\alpha_d + \beta_d = \alpha_e + \beta_e = 1$ . We have the following properties at equilibrium*

1. If  $L_0 + \epsilon S + T_0 \leq \bar{L}$  then  $Y_{d,1} = 0$ ,  $Y_{e,1} > 0$ .
2. If  $L_0 + \epsilon S + T_0 > \bar{L}$  then
  - 2.1. If  $L_0 + T_0 \leq \bar{L}$  then when  $A_d$  is high enough, we have  $Y_{d,1}, H_1 > 0$ , and  $Y_{e,1} = 0$ .
  - 2.2. If  $L_0 + T_0 > \bar{L}$  then
    - 2.2.1. if  $\epsilon S \leq \frac{\alpha_d}{1-\alpha_d}(L_0 + T_0 - \bar{L})$  then when  $A_d$  is high enough, we have  $Y_{d,1} > 0$ ,  $H_1 = 0$  and  $Y_{e,1} = 0$ .



2.2.2 if  $\epsilon S > \frac{\alpha_d}{1-\alpha_d}(L_0 + T_0 - \bar{L})$  then when  $A_d$  is high enough, we have  $Y_{d,1}, H_1 > 0$ , and  $Y_{e,1} = 0$ .

*Proof.* See Appendix 6. □

On the one hand, Theorem 2 shares the main point with Theorem 1. On the other hand, it indicates an interesting scenario in which the well planted foreign firm may be eliminated. There are two main conditions for such scenario: (i) the maximum specific labor supply is high enough to cover the entry cost, (ii) the domestic firm's productivity is high. In this scenario, although the domestic firm has to pay an entry cost, it may not only enter into the new industry but also eliminate the well planted multinational firm.

### 3.2 Roles of return of training and credit constraints

We are now interested in the role of return of training of qualified workers on the optimal strategy of the country.

**Proposition 8.** *Then there exists  $\bar{\epsilon}, \underline{\epsilon}$  depending on the other parameters such that: (i) if  $\epsilon > \bar{\epsilon}$  then  $H_1(\epsilon) > 0$  at equilibrium, (ii)  $\epsilon < \underline{\epsilon}$  then  $H_1(\epsilon) = 0$ .*

*Proof.* See Appendix 5.3. □

Proposition 8 shows that the host country will invest in training of specific labor if its return exceeds a threshold. But if return of training is low, the country should not invest in this sector.

A natural question is that when return  $\epsilon$  is very high, will investing in the new industry be optimal for the host country? The answer is the following:

**Proposition 9. (High return of training with DRS technologies)**

- (i) If  $\frac{\beta_e}{1-\alpha_e} < 1 - \alpha_d$  then when  $\epsilon$  is high enough, the country should invest in both training and the new industry.
- (ii) if  $\frac{\beta_e}{1-\alpha_e} > 1 - \alpha_d$  then when  $\epsilon$  is high enough, the country should invest in training, but not invest in the new industry.

*Proof.* See Appendix 5.3. □

With constant return to scale (CRS) technologies, we have.

**Proposition 10. (High return of training with CRS technologies).**

*Assume that  $\alpha_d + \beta_d = \alpha_e + \beta_e = 1$ . When  $\epsilon$  is high enough, the country should invest in training, but not invest in the new industry.*

*Proof.* See Appendix 6. □

Proposition 10 can be viewed as a consequence of point (ii.b) of Proposition 9. Indeed, let  $\alpha_e + \beta_e$  tend to 1, then  $\frac{\beta_e}{1-\alpha_e}$  tends to 1 which is greater than  $1 - \alpha_d$ . According to point (ii.b) of Proposition 9,  $H_1 > 0$ ,  $Y_{d,1} = 0$  when return of training  $\epsilon$  is high enough. Although CRS technologies would simplify computations, it may make a misunderstanding about the optimal strategy of the country. We can see here that if we only considered CRS technologies, we could not know the roles of labor/capital shares. That is why we need to analyze both cases, particularly the DRS technology case.

We now can give some implications of Propositions 9 and 10 by considering a country in which specific labor can be easily trained (i.e.,  $\epsilon$  is high).

First, as stated in Proposition 9, this country should focus on the new industry if the potential domestic firm has a high labor share. Indeed, on the one hand, high value of  $\epsilon$  allows the domestic firm to cover more easily the entry cost  $\bar{L}$ . On the other hand, high labor share of the domestic firm make it be more competitive than the foreign firm. Consequently, the country should invest in the new industry.

Second, we discuss **credit constraints** of firms. We recall the budget constraint of the social planner

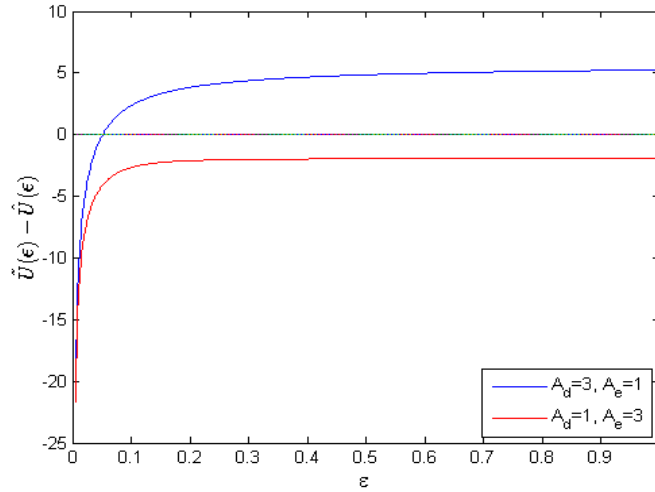
$$H_1 + p(K_{c,1} + K_{d,1}) \leq S.$$

We see that the host country cannot borrow from abroad. Therefore the potential domestic firm faces a credit constraint  $pK_{d,1} < S$ , i.e.,  $K_{d,1} < \bar{K} := S/p$ . Therefore, if capital share reaches a critical threshold,  $\alpha_d > \frac{1-\alpha_e-\beta_e}{1-\alpha_e}$ , the production process will be inefficient when the number of workers is high. However, since the multinational firm can get financing from the parent company, it is not credit constrained. Hence it can buy arbitrary high quantity of input  $K_{e,1}$  to be consistent with high quantity of specific labor. Therefore, in the environment of the multinational firm, specific workers have enough physical capital in order to produce efficiently the new good. As a consequence, when return  $\epsilon$  is high, all specific workers will be hired by the multinational firm even if the domestic firm has higher productivity.<sup>11</sup> It means that credit constraints may prevent the domestic firm to entry in the new industry.

By analyzing the impact of credit constraints on the competition between the domestic firm and the multinational one, our result contributes to the literature about the impact of credit constraints on international trade (Kletzer, Bardhan, 1987; Beck, 2002; Alfaro, Chanda, Kalemli-Ozcan, Sayek, 2004; Manova, 2008, 2013).

We note that in the case  $\frac{\beta_e}{1-\alpha_e} = 1 - \alpha_d$ , when  $\epsilon$  is high, we must have information of other factors, specially  $A_d, A_e$ , to know the optimal strategy of the country. Figure 2 gives us the answer. We also consider 2 cases:  $3 = A_d > A_e = 1$  and  $1 = A_d < A_e = 3$ . Figure 2 indicates that, when  $\epsilon$  is

Figure 2: The graph of  $(\tilde{U} - \hat{U})$  as a function of  $\epsilon$  with  $\frac{\beta_e}{1-\alpha_e} = 1 - \alpha_d$



$$A_c = 1; S = 1; L_0 = 2; T_0 = 1; \bar{L} = 1; p = 1; p_n = 2$$

$$\alpha_c = \alpha_d + \beta_d = 0.7; \alpha_e = 1/3; \beta_e = 7/15$$

high enough, the host country will invest in the new industry ( $\tilde{U} - \hat{U} > 0$ ) if the productivity of the domestic firm is high ( $3 = A_d > A_e = 1$ ). Conversely, such investment will not be done if this productivity is low ( $1 = A_d < A_e = 3$ ). This result is totally consistent with Theorem 1.

### 3.3 Roles of exogenous prices

In this section, we focus on the role of physical capital and new good prices. We analyze two cases, new good price  $p_n$  is high and physical capital price  $p$

is low. First, we study what happens when new good price  $p_n$  is very high.

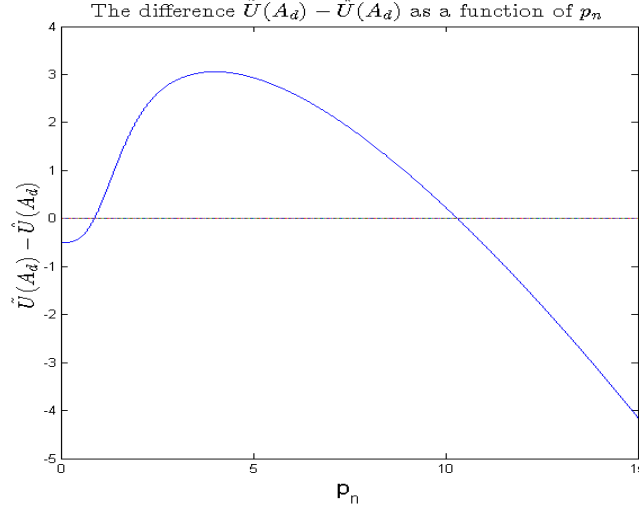
**Proposition 11.** *We have  $\lim_{p_n \rightarrow +\infty} \frac{w_1(p_n)}{p_n} = +\infty$ . When  $p_n$  is high enough, the host country invests in training, but not in the new industry.*

*Proof.* See Appendix 5.3. □

Proposition 11 shows that when new good price is very high, the host country should invest in training, but not in the new industry, whatever the productivity of the domestic firm. The main reason is the following. When price  $p_n$  of new good increases, wage  $w_1$  consequently increases. This encourages the host country to invest in training. Moreover, since wage increases faster than new good price and it must to pay an entry cost to invest in the new industry, it will be optimal to let all specific workers work for the foreign firm in order to get a favorable salary.

Let us give an example where the domestic firm's productivity is greater than that of the multinational firm (cf. Figure 3).

Figure 3: The graph of  $(\tilde{U} - \hat{U})$  as a function of  $p_n$



$$\begin{aligned} \epsilon &= 1.2; S = 1; L_0 = 2; T_0 = 1; \bar{L} = 1; p = 1; \\ A_c &= 1.2; \alpha_c = 0.7; \alpha_d = 0.3; \beta_d = 0.4; \alpha_e = 1/3; \beta_e = 7/15; \\ 2 &= A_d > A_e = 1.2 \end{aligned}$$

When  $p_n$  is high enough, we see that  $\tilde{U} < \hat{U}$ , i.e., the country should not invest in the new industry even  $A_d > A_e$ . Note that when  $p_n$  is low

or medium, we need more informations of other factors in order to confirm  $\tilde{U} < \hat{U}$ .

Second, we consider the case where physical capital price  $p$  is low. In this case, capital shares play an important role.

**Proposition 12.** (i) Assume that  $\frac{\alpha_e}{1-\alpha_e} > \max(\alpha_c, \alpha_d)$ . The host country will invest in training, but not in the new industry when  $p$  is low enough.

(ii) Assume that  $\alpha_d > \max(\alpha_c, \frac{\alpha_e}{1-\alpha_e})$ ,  $\epsilon S + L_0 + T_0 > \bar{L}$

(iii.a) If  $\epsilon S > \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$ , the host country will invest both in training and in the new industry when  $p$  is low enough.

(iii.b) If  $\epsilon S < \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$ , the host country will invest in the new industry, but not in training when  $p$  is low enough.<sup>12</sup>

*Proof.* See Appendix 5.3. □

The reason for point (i) in Proposition 12 is similar to that of Proposition 11. Indeed, when  $p$  decreases, demand for specific labor increases and so is wage  $w_1$ . This incites the host country to invest in training. High capital share of the foreign firm makes it to be more competitive than the domestic firm. Therefore, the host country will not invest in the new industry. In this case, all specific workers work for the multinational firm.

One may ask why there are two possibilities in Proposition 12, but there is a unique in Proposition 11. The reason is from the fact that new good price  $p_n$  does not enter in the budget constraint of the social planner while physical capital price  $p$  makes influence not only in the new industry, but also in the old industry.

We now explore some implications of point (ii) of Proposition 12.

- (a) First, as in point (i), if the domestic firm's capital share is high, the host country invests in the new sector when physical capital price is low.
- (b) Second, when physical capital price  $p$  is low, the country also invests in training if one of the following conditions holds: (1) return of training  $\epsilon$  is high, (2) the ratio  $\beta_d/\alpha_d$  of specific labor share over capital share of the potential domestic firm is high, (3) the difference  $L_0 + T_0 - \bar{L}$  is low.

Let us end this section by considering a specific case where we can give explicit conditions under which the host country invests in training of specific labor and in the new industry.

**Example 1.** *We assume that*

$$\frac{\alpha_c}{1 - \alpha_c} = \frac{\beta_e}{1 - \alpha_e - \beta_e} = \frac{\alpha_d + \beta_d}{1 - \alpha_d - \beta_d}. \quad (9)$$

(i) *There exists an equilibrium with  $H_1 > 0$  and  $Y_{d,1} > 0$  if and only if the two following conditions hold*

$$\epsilon S + L_0 + T_0 \geq \frac{\bar{L}}{1 - \Omega} \quad (10)$$

$$\epsilon S(\sigma_c + \sigma_d + \sigma_e) > (\epsilon S + L_0 + T_0 - \bar{L})(\sigma_c + \frac{\alpha_d}{\alpha_d + \beta_d} \sigma_d), \quad (11)$$

where  $\Omega := \frac{\alpha(\gamma_c + \gamma_d) + \gamma_e}{\alpha\gamma_c + \gamma_e} \left( \frac{\gamma_c + \gamma_e}{\gamma_c + \gamma_e + \gamma_d} \right)^{\frac{1}{\alpha}} < 1$ , with  $\alpha := \alpha_c = \alpha_d + \beta_d$  and

$$\sigma_c := \alpha_c \gamma_c, \quad \sigma_d := (\alpha_d + \beta_d) \gamma_d, \quad \sigma_e := \gamma_e \quad (12)$$

$$\gamma_c := \alpha_c^{\frac{\alpha_c}{1-\alpha_c}} A_c^{\frac{1}{1-\alpha_c}} (\epsilon p)^{\frac{-\alpha_c}{1-\alpha_c}} \quad (13)$$

$$\gamma_d := \alpha_d^{\frac{\alpha_d}{1-\alpha_d-\beta_d}} \beta_d^{\frac{\beta_d}{1-\alpha_d-\beta_d}} (A_d p_n)^{\frac{1}{1-\alpha_d-\beta_d}} (\epsilon p)^{\frac{-\alpha_d}{1-\alpha_d-\beta_d}} \quad (14)$$

$$\gamma_e := \alpha_e^{\frac{\alpha_e}{1-\alpha_e-\beta_e}} \beta_e^{\frac{\beta_e}{1-\alpha_e-\beta_e}} (A_e p_n)^{\frac{1}{1-\alpha_e-\beta_e}} p^{\frac{-\alpha_e}{1-\alpha_e-\beta_e}}. \quad (15)$$

(ii) *There exists an equilibrium with  $H_1 > 0$  and  $Y_{d,1} = 0$  if and only if the two following conditions hold*

$$\epsilon S + L_0 + T_0 < \frac{\bar{L}}{1 - \Omega} \quad (16)$$

$$\epsilon S \gamma_e > \alpha_c \gamma_c (L_0 + T_0). \quad (17)$$

*Proof.* See Appendix 5.3. □

In point (i), condition (10) is to ensure that the GNP in case  $Y_{d,1} > 0$  is greater than the GNP in case  $Y_{d,1} = 0$ .  $H_1 > 0$  is ensured by Condition (11). In point (ii), condition (16) is to ensure that the GNP in case  $Y_{d,1} > 0$

is smaller than the GNP in case  $Y_{d,1} = 0$ ,  $H_1 > 0$  is ensured by Condition (17).

Example 1 shows the complexity of solution and indicates that the optimal strategy of the host country depends on all parameters of the economy. However, with the assumption in Equation (9), the example cannot illustrate all our results in previous section.

## 4 Conclusion

We have constructed a two-period small open economy model with multi-sector, heterogeneous firms, and then used it to study the optimal strategy of a country and analyze roles of all factors of the economy. Our finding indicates that the country's optimal strategy depends on its development level.

First, poor countries with low FDI spillovers cannot invest in a new industry that requires a high entry cost. In this case, all specific workers in this sector will work for multinational firms.

Second, the FDI spillovers can improve the GNP and help poor or developing countries to create a new firm, but it does decrease the optimal share of high-qualified labor. We proved that if FDI spillovers are high, these country may create a new firm without training of qualified workers. But if FDI spillovers are not high, these countries must train qualified workers in order to invest in this new industry.

Third, our model shows that once the host country has a sufficient high-skilled labor to cover the fixed cost in the new industry, the efficiency of domestic firm is necessary and sufficient to ensure its entry. This explains why developed countries do not invest in some new industries.

The competition between the multinational and the domestic firms depends on many factors. The most important factors are their productivities  $A_d, A_e$ . However, credit constraint also plays an important role. Because of credit constraint, the domestic firm may be eliminated even if it has a higher productivity.

## Notes

<sup>1</sup>See Harrison, Rodriguez-Clare (2010) for a complete review.

<sup>2</sup>In our model, the competitiveness of firm is characterized by four factors: productivity, labor and capital share, and credit capacity.

<sup>3</sup>See Blomstrom, Kokko (1998); Gorg, Greenaway (2004); Crespo, Fontoura (2007) for a substantial review of FDI spillovers.

<sup>4</sup>The reader may ask why there is only one input to produce consumption good. We can introduce labor into the production process of the consumption good by taking the production function as  $F^c(K_c, L_c) = A_c K_c^{\alpha_c} L_c^{\beta_c}$  where  $L_c$  is low-skilled labor. If we assume that low-skilled labor is exogenous and there is no possible transfer between high-skilled and low-skilled workers, this setup becomes exactly our framework with the unique input.

<sup>5</sup>Here, we define  $X^+ = 0$  if  $X < 0$ , and  $X^+ = X$  if  $X \geq 0$ .

<sup>6</sup>We would like to make distinction between our threshold  $\bar{L}$  and others in literature. Azariadis, Drazen (1990) considered the following production function  $F = A_t F(K_t, L_t)$ , where the scale factor  $A_t$  may depend functionally on a vector of social inputs that are not controlled by any one producer.

In Bruno, Le Van, Masquin (2009),  $A_t$  is endogenized: the social planner chooses physical capital  $K_{e,t}$  and high-skilled labor  $L_{e,t}$  in order to produce new technology, which enter the formula of  $A_t$  as follows  $A_t := x_0 + a(F(K_{e,t}, L_{e,t}) - \bar{X})^+$ .  $\bar{X}$  a minimum level of adoption of new technologies which is necessary for them in order to impact the economy.

In Smith (1987), the potential domestic firm has the following problem

$$\max P(X)X - \text{cost}(X) - (\text{fixed cost}). \quad (18)$$

In Melitz (2003), the production function is given by  $F(L) = \phi(L - f)^+$ . The threshold  $f > 0$  represents a fixed entry cost (measured in units of labor).

In Fosfuri, Motta, Ronde (2001), fixed cost arises in the local firm's valuation of the worker  $v_l = N_2 \Pi_d(\phi) - k$  where  $k$  indicates the cost that local firm has to pay in order to gain from new technology received by the trained worker.

<sup>7</sup>We note that FDI spillovers  $T_0$  in our framework arise through workers mobility. We refer to Fosfuri, Motta, Ronde (2001); Gorg, Strobl (2005); Poole (2013) for the existence of such FDI spillovers.

<sup>8</sup>In general, the social planner's problem is to choose  $c_0, c_1, S, K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1}$  that maximizes the utility

$$\begin{aligned} \max_{(c_0, c_1, S, K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1})} & \left[ \beta_0 U(c_0) + \beta_1 U(c_1) \right] \\ \text{subject to } & c_0 + S \leq S_0, \quad c_1 \leq F(S) \end{aligned}$$

where  $S_0 > 0$  is given,  $\beta_i$  is the time preference at date  $i = 0, 1$ , and  $F(S)$  is defined by

$$\max \left[ F^c(K_{c,1}) + w_1 L_{e,1} + p_n F^d(K_{d,1}, L_{d,1}) \right] \quad (19)$$

$$\text{subject to } H_1 + p(K_{c,1} + K_{d,1}) \leq S \quad (20)$$

$$L_{d,1} + L_{e,1} \leq L_0 + T_0 + \epsilon H_1 \quad (21)$$

$$K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1} \geq 0. \quad (22)$$



For simplicity, we assume that saving  $S$  is exogenous. Then the problem of the social planner is equivalent to the problem (P).

<sup>9</sup> $\epsilon S = \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$ . We do not enough information to confirm  $H_1 > 0$ . It depends on other factors. We will answer this question in Section 1.

<sup>10</sup>Among 228 domestic firms in the sample, the number of domestic firms whose the entrepreneur worked for a multinational firm is about 32. Most of them is in metals and machinery (34.4%), followed by furniture (31.3%), textiles (18.8%), and wood products (9.4%).

<sup>11</sup>Indeed, consider the case CRS technologies with  $A_d > A_e, \alpha_d = \alpha_e, \beta_d = \beta_e$ , and there is no entry cost ( $\bar{L} = 0$ ). Proposition 10 proves that  $L_{d,1} = 0$  when  $\epsilon$  is high enough.

<sup>12</sup>We note that in the case where  $\alpha_c > \max(\alpha_d, \frac{\alpha_e}{1-\alpha_e})$ , we do not know if the country invest in the new industry. Because, this condition does not make a strong influence on the competition between firms in the new industry.

## 5 Appendix: Decreasing return to scale

In this Section, we prove our finding with DRS technologies. The proofs of results with CRS technologies can be found in Section 6.

We assume that  $\alpha_d + \beta_d, \alpha_e + \beta_e < 1$ . In Sections 5.1 and 5.2, we will characterize equilibrium. We then present formal proofs of our results with DRS technologies in Section 5.3.

At equilibrium, since the production functions of the foreign firm and the consumption good producer are decreasing return to scale, we always have  $K_{c,1}, K_{e,1}, L_{e,1} > 0$ .

We now write first order conditions (FOCs) for the foreign firm.

$$\begin{aligned} L_{e,1} : \quad & p_n \beta_e A_e K_{e,1}^{\alpha_e} L_{e,1}^{\beta_e - 1} = w_1 \\ K_{e,1} : \quad & p_n \alpha_e A_e K_{e,1}^{\alpha_e - 1} L_{e,1}^{\beta_e} = p. \end{aligned}$$

Therefore, we get that

$$\begin{aligned} K_{e,1} &= \frac{\alpha_e}{\beta_e} \frac{w_1}{p} L_{e,1}, \quad L_{e,1} = \sigma_e w_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}, \\ \text{where } \sigma_e &:= \alpha_e^{\frac{\alpha_e}{1-\alpha_e-\beta_e}} \beta_e^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} (A_e p_n)^{\frac{1}{1-\alpha_e-\beta_e}} p^{\frac{-\alpha_e}{1-\alpha_e-\beta_e}} \end{aligned}$$

Denote  $\lambda, \mu$  Lagrange multipliers associated to conditions (4), (5), respectively, and  $\lambda_h$  is Lagrange multiplier with respect to condition  $H_1 \geq 0$ . We

write FOCs for the social planner for variables  $K_{c,1}, H_1, L_{e,1}$

$$\begin{aligned} K_{c,1} : \quad & \alpha_c A_c K_{c,1}^{\alpha_c - 1} = \lambda p \\ L_{e,1} : \quad & w_1 - \mu = 0 \\ H_1 : \quad & -\lambda + \mu\epsilon + \lambda_h = 0, \text{ where } \lambda_h \geq 0, H_1 \lambda_h = 0. \end{aligned}$$

Note that to solve social planner's optimization problem, we must consider two cases:  $Y_{d,1} = 0$  and  $Y_{d,1} > 0$ . Then, we compare welfares in these cases in order to know what is the optimal strategy.

### 5.1 Equilibrium with $H_1 = 0$

(i): If  $L_0 + T_0 \leq \bar{L}$ . We have  $L_{e,1} + L_{d,1} \leq \epsilon H_1 + L_0 + T_0 = L_0 + T_0 \leq \bar{L}$ , thus  $L_{d,1} \leq \bar{L}$  then  $Y_{d,1} = 0$ .

(ii): If  $L_0 + T_0 > \bar{L}$ . We have to consider two cases:  $Y_{d,1} = 0$  and  $Y_{d,1} > 0$ .

**Case 1:**  $Y_{d,1} = 0$ . In this case  $K_{d,1} = L_{d,1} = 0$ . Therefore, we get that

$K_{c,1} = \frac{S}{p}$  and  $L_{e,1} = L_0 + T_0$ . By using FOCs of firm's maximization, we

have  $L_{e,1} = \sigma_e w_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}$ . Hence, wage is computed by

$$w_1 = \left( \frac{\sigma_e}{L_0 + T_0} \right)^{\frac{1-\alpha_e-\beta_e}{1-\alpha_e}}. \quad (23)$$

In this case, we have

$$\text{Welfare} = A_c \left( \frac{S}{p} \right)^{\alpha_c} + \beta_e \alpha_e^{\frac{\alpha_e}{1-\alpha_e}} A_e^{\frac{1}{1-\alpha_e}} \left( \frac{p_n}{p^{\alpha_e}} \right)^{\frac{1}{1-\alpha_e}} (L_0 + T_0)^{\frac{\beta_e}{1-\alpha_e}}.$$

Note that we have to justify the following condition

$$\text{FOC of } H_1 : \epsilon p w_1 \leq \lambda p = \alpha_c A_c K_{c,1}^{\alpha_c - 1}.$$

This condition is equivalent to the following condition under which  $\epsilon$  is low enough.

$$\epsilon S^{1-\alpha_c} \beta_e \alpha_e^{\frac{\alpha_e}{1-\alpha_e}} A_e^{\frac{1}{1-\alpha_e}} p^{\alpha_c} \left( \frac{p_n}{p^{\alpha_e}} \right)^{\frac{1}{1-\alpha_e}} \leq \alpha_c A_c (L_0 + T_0)^{\frac{1-\alpha_e-\beta_e}{1-\alpha_e}}. \quad (24)$$

**Case 2:**  $Y_{d,1} > 0$ . In this case  $L_{d,1} > \bar{L}$ . We write FOCs for the social planner

$$\begin{aligned} K_{d,1} : \quad & p_n \alpha_d A_d K_{d,1}^{\alpha_d - 1} (L_{d,1} - \bar{L})^{\beta_d} - \lambda p = 0 \\ L_{d,1} : \quad & p_n \beta_d A_d K_{d,1}^{\alpha_d} (L_{d,1} - \bar{L})^{\beta_d - 1} - \mu = 0 \end{aligned}$$

Since we are considering the case  $H_1 = 0$ , labor market clearing condition implies that  $L_{d,1} = L_0 + T_0 - L_{e,1} = L_0 + T_0 - \sigma_e w_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}$ .

By using FOC for variable  $L_{d,1}$ , we get that  $K_{d,1} = \left(\frac{w_1}{\beta_d p_n A_d}\right)^{\frac{1}{\alpha_d}} (L_{d,1} - \bar{L})^{\frac{1-\beta_d}{\alpha_d}}$ .

On the other hand, we have  $\alpha_c A_c K_{c,1}^{\alpha_c-1} = \lambda p = \alpha_d p_n A_d K_{d,1}^{\alpha_d-1} (L_{d,1} - \bar{L})^{\beta_d}$ . Therefore

$$\begin{aligned} K_{c,1} &= \left(\frac{\alpha_c A_c}{p_n \alpha_d A_d}\right)^{\frac{1}{1-\alpha_c}} K_{d,1}^{\frac{1-\alpha_d}{1-\alpha_c}} (L_{d,1} - \bar{L})^{\frac{-\beta_d}{1-\alpha_c}} \\ &= \left(\frac{\alpha_c A_c}{\alpha_d}\right)^{\frac{1}{1-\alpha_c}} \left(\frac{1}{p_n A_d}\right)^{\frac{1}{\alpha_d(1-\alpha_c)}} \left(\frac{w_1}{\beta_d}\right)^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} (L_{d,1} - \bar{L})^{\frac{1-\alpha_d-\beta_d}{\alpha_d(1-\alpha_c)}}. \end{aligned}$$

According  $K_{c,1} + K_{d,1} = \frac{S}{p}$ , we get that  $w_1$  is a solution of the equation  $G_2(x) = 0$ , where

$$\begin{aligned} G_2(x) &:= -\frac{S}{p} + \left(\frac{x}{\beta_d p_n A_d}\right)^{\frac{1}{\alpha_d}} \left(L_0 + T_0 - \bar{L} - \sigma_e x^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}\right)^{\frac{1-\beta_d}{\alpha_d}} \\ &\quad + \left(\frac{\alpha_c A_c}{\alpha_d}\right)^{\frac{1}{1-\alpha_c}} \left(\frac{1}{p_n A_d}\right)^{\frac{1}{\alpha_d(1-\alpha_c)}} \left(\frac{x}{\beta_d}\right)^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} \left(L_0 + T_0 - \bar{L} - \sigma_e x^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}\right)^{\frac{1-\alpha_d-\beta_d}{\alpha_d(1-\alpha_c)}}. \end{aligned} \quad (25)$$

It is easy to see that the function  $G_2$  is increasing. Moreover,  $\inf_x G_2(x) = -\frac{S}{p}$  and  $\sup_x G_2(x) = +\infty$ . Therefore, the equation  $G_2(x) = 0$  has the unique solution, called  $w_1$ .

By observing the equation  $G_2(w_1) = 0$ , we see that when  $A_d$  tends to infinity then  $w_1(A_d)$  tends to infinity.

FOCs give us  $\beta_d p_n Y_{d,1} = w_1(A_d)(L_{d,1}(A_d) - \bar{L}) = w_1(A_d)(L_0 + T_0 - \bar{L} - \sigma_e w_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}})$ . Consequently,  $\lim_{A_d \rightarrow +\infty} p_n Y_{d,1} = +\infty$ , then the welfare in this case is greater than the welfare in the first case, which does not depend on  $A_d$ . Moreover,  $\lim_{A_d \rightarrow +\infty} Y_{d,1} = +\infty$  implies that  $Y_{d,1} > Y_{e,1}$  with  $A_d$  is high enough.

It means that we have just proved the result mentioned in Proposition 7.

**Remark 2.** Assume that  $L_0 + T_0 > \bar{L}$  and  $\epsilon$  is low enough such that  $\epsilon \beta_d S < \alpha_d(L_0 + T_0 - \bar{L})$ . When  $A_d$  is high enough then the list

$$(K_{c,1}, K_{d,1}, L_{d,1}, L_{e,1}, K_{e,1}, w_1)$$

given in case 2 above is the unique equilibrium.

*Proof.* Indeed, labor market clearing condition is satisfied. All FOCs of the foreign firm hold. It remains to justify the FOC with respect to variable  $H_1 = 0$ , which is  $\lambda \geq \epsilon w_1$ , i.e.,  $\epsilon p w_1 K_{c,1}^{1-\alpha_c} \leq \alpha_c A_c$ . We have

$$\frac{w_1 K_{c,1}^{1-\alpha_c}}{\alpha_c A_c} = \frac{w_1}{\alpha_d} \left( \frac{1}{p_n A_d} \right)^{\frac{1}{\alpha_d}} \left( \frac{w_1}{\beta_d} \right)^{\frac{1-\alpha_d}{\alpha_d}} \left( L_0 + T_0 - \bar{L} - \sigma_e w_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} \right)^{\frac{1-\alpha_d-\beta_d}{\alpha_d}}.$$

Since  $G_2(w_1(A_d)) = 0$  and  $\lim_{A_d \rightarrow +\infty} w_1(A_d) = +\infty$ , we have

$$\frac{S}{p} \geq \left( \frac{1}{\beta_d p_n} \right)^{\frac{1}{\alpha_d}} (L_0 + T_0 - \bar{L})^{\frac{1-\beta_d}{\alpha_d}} \left( \limsup_{A_d \rightarrow +\infty} \frac{w_1(A_d)}{A_d} \right)^{\frac{1}{\alpha_d}}$$

Hence,  $\limsup_{A_d \rightarrow +\infty} \frac{w_1(A_d)}{A_d} < +\infty$  then  $\lim_{A_d \rightarrow +\infty} \frac{(w_1(A_d))^{1-\alpha_d}}{A_d} = 0$ . Again, by using  $\lim_{A_d \rightarrow +\infty} G_2(w_1(A_d)) = 0$  which implies that

$$\frac{S}{p} = \left( \frac{1}{\beta_d p_n} \right)^{\frac{1}{\alpha_d}} (L_0 + T_0 - \bar{L})^{\frac{1-\beta_d}{\alpha_d}} \lim_{A_d \rightarrow +\infty} \left( \frac{w_1(A_d)}{A_d} \right)^{\frac{1}{\alpha_d}}. \quad (26)$$

We now assume that  $\epsilon \beta_d S < \alpha_d (L_0 + T_0 - \bar{L})$ . We have

$$\begin{aligned} \lim_{A_d \rightarrow +\infty} \epsilon p \frac{w_1 K_{c,1}^{1-\alpha_c}}{\alpha_c A_c} &= \epsilon p \lim_{A_d \rightarrow +\infty} \left( \frac{w_1(A_d)}{A_d} \right)^{\frac{1}{\alpha_d}} \left( \frac{1}{p_n} \right)^{\frac{1}{\alpha_d}} \frac{1}{\alpha_d \beta_d^{\frac{1-\alpha_d}{\alpha_d}}} (L_0 + T_0 - \bar{L})^{\frac{1-\alpha_d-\beta_d}{\alpha_d}} \\ &= \frac{\epsilon \beta_d S}{\alpha_d (L_0 + T_0 - \bar{L})} < 1. \end{aligned}$$

Consequently, if  $A_d$  is high enough then  $\epsilon p w_1 K_{c,1}^{1-\alpha_c} \leq \alpha_c A_c$ , i.e., FOC of  $H_1$  is satisfied.

When  $A_d$  tends to infinity then the welfare tends to infinity.  $\square$

## 5.2 Equilibrium with $H_1 > 0$

Let denote  $\mathcal{L}_0 := L_0 + T_0 + \epsilon S$  and  $\tilde{U}$  (resp.  $\hat{U}$ ) the GNP in case  $Y_{d,1} > 0$  (resp.  $Y_{d,1} = 0$ ).

Recall that we are considering equilibrium with  $H_1 > 0$ , so  $\lambda_h = 0$ .

We will consider 2 cases:  $Y_{d,1} > 0$  and  $Y_{d,1} = 0$ . Let denote  $\tilde{U}$  and  $\hat{U}$  the welfare value of problem (P) with  $Y_{d,1} > 0$  and with  $Y_{d,1} = 0$ , respectively.

**Case 1.** Assume that  $(\tilde{K}_{c,1}, \tilde{K}_{d,1}, \tilde{H}_1, \tilde{L}_{d,1}, \tilde{K}_{e,1}, \tilde{L}_{e,1}, \tilde{w}_1)$  with  $\tilde{L}_{d,1} > \bar{L}$ ,  $\tilde{K}_{d,1} > 0$  is an equilibrium. We have

$$\begin{aligned}\tilde{H}_1 : & -\lambda + \mu\epsilon = 0 \\ \tilde{L}_{d,1} : & p_n \beta_d A_d \tilde{K}_{d,1}^{\alpha_d} (\tilde{L}_{d,1} - \bar{L})^{\beta_d - 1} - \mu = 0 \\ \tilde{K}_{d,1} : & p_n \alpha_d A_d \tilde{K}_{d,1}^{\alpha_d - 1} (\tilde{L}_{d,1} - \bar{L})^{\beta_d} - \lambda p = 0.\end{aligned}$$

We get that

$$\tilde{K}_{c,1} = \left[ \frac{\alpha_c A_c}{\epsilon p \tilde{w}_1} \right]^{\frac{1}{1-\alpha_c}} \quad (27)$$

$$\tilde{L}_{d,1} - \bar{L} = \frac{\beta_d}{\alpha_d} \epsilon p \tilde{K}_{d,1}, \quad \tilde{K}_{d,1} = \left[ \frac{p_n \alpha_d A_d}{p \epsilon \tilde{w}_1} \left( \frac{\beta_d \epsilon p}{\alpha_d} \right)^{\beta_d} \right]^{\frac{1}{1-\alpha_d-\beta_d}}. \quad (28)$$

By combining the budget constraint of the social planner and labor market clearing condition, we imply that

$$\left( (\tilde{L}_{d,1} - \bar{L}) + \epsilon p \tilde{K}_{d,1} \right) + \epsilon p \tilde{K}_{c,1} + \tilde{L}_{e,1} = \epsilon S + L_0 + T_0 - \bar{L}.$$

It means that  $\tilde{w}_1$  is a solution of the equation  $G(x) = 0$ , where we define

$$\begin{aligned}G(x) := & \sigma_c x^{\frac{-1}{1-\alpha_c}} + \sigma_e x^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} + \sigma_d x^{\frac{-1}{1-\alpha_d-\beta_d}} \\ & - (\epsilon S + L_0 + T_0 - \bar{L})\end{aligned} \quad (29)$$

$$\sigma_c := (\alpha_c A_c)^{\frac{1}{1-\alpha_c}} (\epsilon p)^{\frac{-\alpha_c}{1-\alpha_c}} \quad (30)$$

$$\sigma_d := (\alpha_d + \beta_d) \alpha_d^{\frac{\alpha_d}{1-\alpha_d-\beta_d}} \beta_d^{\frac{\beta_d}{1-\alpha_d-\beta_d}} (A_d p_n)^{\frac{1}{1-\alpha_d-\beta_d}} (\epsilon p)^{\frac{-\alpha_d}{1-\alpha_d-\beta_d}}. \quad (31)$$

We see that  $\lim_{\tilde{w}_1 \rightarrow 0^+} G(\tilde{w}_1) = +\infty$ ,  $\lim_{\tilde{w}_1 \rightarrow +\infty} G(\tilde{w}_1) = \bar{L} - \mathcal{L}_0$  and  $G'(\tilde{w}_1) < 0$  for every  $\tilde{w}_1 \in (0, +\infty)$ . Therefore if  $\bar{L} - \mathcal{L}_0 < 0$  then the equation (29) has the unique solution in  $(0, +\infty)$ .

Condition  $H_1 > 0$  is equivalent to

$$\frac{S}{p} > \left[ \frac{\alpha_c A_c}{\epsilon p \tilde{w}_1} \right]^{\frac{1}{1-\alpha_c}} + \left[ \frac{p_n \alpha_d A_d}{p \epsilon \tilde{w}_1} \left( \frac{\beta_d \epsilon p}{\alpha_d} \right)^{\beta_d} \right]^{\frac{1}{1-\alpha_d-\beta_d}}.$$

This condition can be rewritten as follows

$$\epsilon S > \sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \frac{\alpha_d}{\alpha_d + \beta_d} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}}. \quad (32)$$

We now compute the welfare in this case. The welfare is given by

$$\begin{aligned}
\tilde{U} &= F^c(\tilde{K}_{c,1}) + \tilde{w}_1 \tilde{L}_{e,1} + p_n F^d(\tilde{K}_{d,1}, \tilde{L}_{d,1}) \\
&= A_c \left[ \frac{\alpha_c A_c}{\epsilon p \tilde{w}_1} \right]^{\frac{\alpha_c}{1-\alpha_c}} + \tilde{w}_1 \sigma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} \\
&\quad + \frac{\epsilon p}{\alpha_d} \left[ \left( \frac{\beta_d \epsilon p}{\alpha_d} \right)^{\beta_d} \left( \frac{\alpha_d A_d p_n}{\epsilon p} \right)^{\frac{1}{1-\alpha_d-\beta_d}} \right] \left[ \frac{1}{\tilde{w}_1} \right]^{\frac{\alpha_d+\beta_d}{1-\alpha_d-\beta_d}} \\
&= \gamma_c \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} + \gamma_e \tilde{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} + \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}},
\end{aligned}$$

where, we define

$$\gamma_c := \frac{\sigma_c}{\alpha_c}, \quad \gamma_d := \frac{\sigma_d}{\alpha_d + \beta_d}, \quad \gamma_e := \sigma_e.$$

**Case 2.** Assume that  $(\hat{K}_{c,1}, \hat{K}_{d,1}, \hat{H}_1, \hat{L}_{d,1}, \hat{K}_{e,1}, \hat{L}_{e,1}, \hat{w}_1)$  with  $\hat{L}_{d,1} = \hat{K}_{d,1} = 0$  is an equilibrium. In this case, we note that  $\hat{L}_{e,1} + \epsilon p \hat{K}_{c,1} = \mathcal{L}_0$ . As in the case 1, we have

$$\begin{aligned}
\hat{K}_{c,1} &= \left[ \frac{\alpha_c A_c}{\epsilon p \hat{w}_1} \right]^{\frac{1}{1-\alpha_c}} \\
\hat{K}_{e,1} &= \frac{\alpha_e \hat{w}_1}{\beta_e p_n} \hat{L}_{e,1}, \quad \hat{L}_{e,1} = \sigma_e \hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}.
\end{aligned}$$

We get that  $\hat{w}_1$  is a solution of the following equation

$$Q(\hat{w}_1) := \sigma_c \hat{w}_1^{\frac{-1}{1-\alpha_c}} + \sigma_e \hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} - \mathcal{L}_0 = 0 \quad (33)$$

We see that  $\lim_{\hat{w}_1 \rightarrow 0^+} Q(\hat{w}_1) = +\infty$ ,  $\lim_{\hat{w}_1 \rightarrow +\infty} Q(\hat{w}_1) = -\mathcal{L}_0$  and  $G'(\hat{w}_1) < 0$  for every  $\hat{w}_1 \in (0, +\infty)$ . Therefore the equation (33) has the unique solution in  $(0, +\infty)$ . This solution is denoted by  $\hat{w}_1$ .

We now compute the welfare. The welfare is given by

$$\begin{aligned}
\hat{U} &= F^c(\hat{K}_{c,1}) + \hat{w}_1 \hat{L}_{e,1} = A_c \left[ \frac{\alpha_c A_c}{\epsilon p \hat{w}_1} \right]^{\frac{\alpha_c}{1-\alpha_c}} + \hat{w}_1 \sigma_e \hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} \\
&= \gamma_c (\hat{w}_1)^{\frac{-\alpha_c}{1-\alpha_c}} + \gamma_e (\hat{w}_1)^{\frac{-\beta_e}{1-\alpha_e-\beta_e}}.
\end{aligned}$$

Condition  $H_1 > 0$  is equivalent to  $\frac{S}{p} > \left[ \frac{\alpha_c A_c}{\epsilon p \hat{w}_1} \right]^{\frac{1}{1-\alpha_c}}$ , i.e.,  $\epsilon S > \sigma_c \hat{w}_1^{\frac{-1}{1-\alpha_c}}$ ..

**Lemma 1.** *If  $(1 - \alpha_d - \beta_d)\gamma_d\tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}} \geq \tilde{w}_1\bar{L}$  then  $\tilde{U} > \hat{U}$ .*

This result means that if the potential domestic firm's profit  $(1 - \alpha_d - \beta_d)p_n Y_{d,1}$  can cover the value of the entry costs  $\tilde{w}_1\bar{L}$ , the country should invest in the new industry.

*Proof.* We observe that

$$\hat{U} = (\epsilon S + L_0 + T_0 + (1 - \alpha_c)\gamma_c\hat{w}_1^{\frac{-1}{1-\alpha_c}})\hat{w}_1 \quad (34)$$

$$\begin{aligned} \tilde{U} &= (\epsilon S + L_0 + T_0 - \bar{L} + (1 - \alpha_c)\gamma_c\tilde{w}_1^{\frac{-1}{1-\alpha_c}} \\ &\quad + (1 - \alpha_d - \beta_d)\gamma_d\tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}})\tilde{w}_1. \end{aligned} \quad (35)$$

Therefore, we get

$$\begin{aligned} \tilde{U} - \hat{U} &= (\epsilon S + L_0 + T_0)(\tilde{w}_1 - \hat{w}_1) + (1 - \alpha_c)\gamma_c(\tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}) \\ &\quad + \left(\frac{1 - \alpha_d - \beta_d}{\alpha_d + \beta_d}\sigma_d\tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} - \bar{L}\right)\tilde{w}_1. \end{aligned} \quad (36)$$

Consider the function  $f(x) := (\epsilon S + L_0 + T_0)x + \frac{1-\alpha_c}{\alpha_c}\sigma_c\tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}$ . We have  $f'(x) > 0$  for every  $x$  such that  $\epsilon S + L_0 + T_0 > \sigma_c x^{\frac{-1}{1-\alpha_c}}$ . Therefore,  $\tilde{w}_1 > \hat{w}_1$  implies that  $f(\tilde{w}_1) > f(\hat{w}_1)$ , i.e.,

$$\tilde{U} - \hat{U} > \left(\frac{1 - \alpha_d - \beta_d}{\alpha_d + \beta_d}\sigma_d\tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} - \bar{L}\right)\tilde{w}_1.$$

□

### 5.3 Formal proofs

**Proof of Proposition 2.** The first statement is clear. We will prove the second one. It is trivial if  $H_1 = 0$ . Hence, we assume that  $H_1 > 0$ . We consider 2 cases:  $Y_{d,1} = 0$  and  $Y_{d,1} > 0$ .

**Case 1:**  $Y_{d,1} = 0$ . It is easy to see that when  $T_0$  increases,  $\hat{w}_1$  decreases. Therefore  $\frac{pK_{c,1}}{S} = \frac{\sigma_c}{\epsilon S}\hat{w}_1^{\frac{-1}{1-\alpha_c}}$  will increase which implies  $\theta_h$  decreases. The same argument can be used to prove our result in the case  $Y_{d,1} > 0$ . □

**Proof of Proposition 3. Point (i):** We consider two cases  $H_1 > 0$  and  $H_1 = 0$ .

Case 1:  $H_1 > 0$ . Let  $\bar{L} \rightarrow 0$ , we have  $\lim_{\bar{L} \rightarrow 0} \hat{w}_1(\bar{L}) = \hat{w}_1$ ,  $\lim_{\bar{L} \rightarrow 0} \tilde{w}_1(\bar{L}) = \tilde{w}_1$ , where  $\hat{w}_1, \tilde{w}_1$  such that

$$\sigma_c \hat{w}_1^{\frac{-1}{1-\alpha_c}} + \sigma_e \hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} = \epsilon S + L_0 + T_0 \quad (37)$$

$$\sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \sigma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} + \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = \epsilon S + L_0 + T_0. \quad (38)$$

We also have  $\lim_{\bar{L} \rightarrow 0} \tilde{U}(\bar{L}) = \tilde{U}$ ,  $\lim_{\bar{L} \rightarrow 0} \hat{U}(\bar{L}) = \hat{U}$ , and

$$\begin{aligned} \tilde{U} - \hat{U} &= (\epsilon S + L_0 + T_0) \tilde{w}_1 - (\epsilon S + L_0 + T_0) \hat{w}_1 \\ &\quad + (1 - \alpha_c) \gamma_c (\tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}) + (1 - \alpha_d - \beta_d) \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}}. \end{aligned} \quad (39)$$

Consider the function  $f(x) := (\epsilon S + L_0 + T_0)x + \frac{1-\alpha_c}{\alpha_c} \sigma_c \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}$ . We have  $f'(x) > 0$  for every  $x$  such that  $\epsilon S + L_0 + T_0 > \sigma_c x^{\frac{-1}{1-\alpha_c}}$ . Therefore,  $\tilde{w}_1 > \hat{w}_1$  implies that  $f(\tilde{w}_1) > f(\hat{w}_1)$ , i.e.,  $\tilde{U} - \hat{U} > 0$ . Since  $\tilde{U}(\bar{L}) - \hat{U}(\bar{L})$  is continuous, there exists  $\bar{L}^* > 0$  such that  $\tilde{U}(\bar{L}) > \hat{U}(\bar{L})$  for every  $\bar{L} \geq \bar{L}^*$ .

Case 2: When  $H_1 = 0$ . It is clear.

**Point (ii):** It is easy to see that  $\lim_{S \rightarrow +\infty} \hat{w} = \lim_{S \rightarrow +\infty} \tilde{w} = 0$ . Therefore, for

$S$  is high enough, we have  $(1 - \alpha_d - \beta_d) \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}} > \bar{L} \tilde{w}_1$ . Consequently, we obtain

$$\tilde{U}(S) - \hat{U}(S) > (\epsilon S + L_0 + T_0)(\tilde{w}_1 - \hat{w}_1) + \frac{1 - \alpha_c}{\alpha_c} \sigma_c (\tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}).$$

By using the same argument in the proof of point (i), we have  $\tilde{U}(S) - \hat{U}(S) > 0$  for  $S$  high enough.  $\square$

**Proof of Proposition 4. Point (i):** Let  $A_e$  tend to infinity, we have  $\gamma_e$  and so  $\hat{w}_1, \tilde{w}_1$  will tend to infinity. Consequently, we have

$$\lim_{A_e \rightarrow +\infty} \gamma_e \hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} = \epsilon S + L_0 + T_0 \quad (40)$$

$$\lim_{A_e \rightarrow +\infty} \gamma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} = \epsilon S + L_0 + T_0 - \bar{L} \quad (41)$$

$$\lim_{A_e \rightarrow +\infty} \left( \frac{\tilde{w}}{\hat{w}_1} \right)^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} = \frac{\epsilon S + L_0 + T_0}{\epsilon S + L_0 + T_0 - \bar{L}} > 1. \quad (42)$$



Therefore, we get that

$$\lim_{A_e \rightarrow +\infty} \frac{\tilde{U}}{\hat{U}} = \left( \frac{\tilde{w}}{\hat{w}_1} \right)^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} < 1. \quad (43)$$

Thus, there exists  $A_e > 0$  such that  $\hat{U} > \tilde{U}$ , i.e.,  $Y_{d,1} = 0$ . It remains to verify that  $H_1 > 0$ , i.e.,  $\epsilon \hat{w}_1 > \frac{\alpha_c A_c}{p^{\alpha_c} S^{1-\alpha_c}}$ . This condition is satisfied if  $A_e$  is high enough.

**Point (ii):** Assume that  $H_1 > 0$ . Like proof of Proposition 4, we have  $Y_{d,1} = 0$ . But in this case, condition  $H_1 > 0$  is not satisfied. Hence, we have  $H_1 = 0$  at equilibrium.

By using the similar argument in Remark 2, we get that  $Y_{d,1} = 0$  and FOC with respect to  $H_1$  is satisfied.  $\square$

**Proof of Proposition 6.** Assume that  $\mathcal{L}_0 > \bar{L} \geq L_0 + T_0$ . The wage  $\tilde{w}_1$  is the unique solution of the following equation

$$\sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \sigma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} + \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = \epsilon S + L_0 + T_0 - \bar{L}. \quad (44)$$

We see that  $\tilde{w}_1$  depends on  $A_d$ , we can write  $\tilde{w}_1 = \tilde{w}_1(A_d)$ . It is easy to see that  $\tilde{w}_1(\cdot)$  is increasing in  $(0, +\infty)$ . Since  $\lim_{A_d \rightarrow \infty} \sigma_d = +\infty$  and  $\sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} < \mathcal{L}_0 - \bar{L}$ , we obtain  $\lim_{A_d \rightarrow +\infty} \tilde{w}_1(A_d) = +\infty$ . By combining with (44), we have

$$\lim_{A_d \rightarrow +\infty} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = \mathcal{L}_0 - \bar{L} > 0.$$

Consequently, we obtain

$$\begin{aligned} \lim_{A_d \rightarrow +\infty} \sigma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}} &= \lim_{A_d \rightarrow +\infty} \sigma_d^{1-\alpha_d-\beta_d} \left[ \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} \right]^{\alpha_d+\beta_d} \\ &= (\mathcal{L}_0 - \bar{L})^{\alpha_d+\beta_d} \lim_{A_d \rightarrow +\infty} \sigma_d^{1-\alpha_d-\beta_d} = +\infty. \end{aligned}$$

Therefore  $\lim_{A_d \rightarrow +\infty} \tilde{U}(A_d) = +\infty$ . By combining with  $\hat{w}_1$  does not depend on  $A_d$ , we have  $\lim_{A_d \rightarrow +\infty} \tilde{U}(A_d) > \hat{U}$ , then critical level  $\bar{A}_1$  in Proposition 6 exists. Since  $\bar{L} \geq L_0 + T_0$ , we have

$$\epsilon S \geq \mathcal{L}_0 - \bar{L} = \sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \sigma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} + \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} \quad (45)$$

$$> \sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \frac{\alpha_d}{\alpha_d + \beta_d} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}}. \quad (46)$$

Therefore, we have  $H_1 > 0$ . We can now define  $\bar{A}_1$  and  $\tilde{A}_1$ .

$$\bar{A}_1 := \inf\{A_d : \tilde{U}(A_d) \geq \hat{U}\} \quad (47)$$

$$\tilde{A}_1 := \inf\{A_d : Y_{d,1} \geq Y_{e,1}\}. \quad (48)$$

We now prove that  $\bar{A}_1$  increases if  $\bar{L}$  increases.

For each  $\bar{L}, A_d$ , we write  $\tilde{w}_1(\bar{L}, A_d)$  meaning that wage depends on  $\bar{L}, A_d$ . Then  $\bar{A}_1$  is the unique level of productivity such that  $\tilde{U}(\bar{A}_1) = \hat{U}$  which can be rewritten as

$$\gamma_c(\tilde{w}_1(\bar{L}, \bar{A}_1))^{\frac{-\alpha_c}{1-\alpha_c}} + \gamma_e(\tilde{w}_1(\bar{L}, \bar{A}_1))^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} + \gamma_d(\tilde{w}_1(\bar{L}, \bar{A}_1))^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}} = \hat{U}.$$

Note that  $\hat{U}$  does depend neither on  $\bar{L}$  nor on  $A_d$ . Since  $\tilde{w}_1$  is increasing in the first variable, decreasing in the second variable, and  $\gamma_d$  is increasing in  $A_d$  then we have  $\bar{A}_1$  is increasing in  $\bar{L}$ .

Similarly,  $\tilde{A}_1$  is increasing in  $\bar{L}$ .  $\square$

**Proof of Proposition 7. Case 1:**  $\epsilon S < \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$ . This case is a direct consequence of Remark 2.

$\bar{A}_2, \tilde{A}_2$  can be defined as follows

$$\bar{A}_2 := \inf\{A_d : \tilde{U}(A_d) \geq \hat{U} \text{ and } \epsilon p K_{c,1}^{1-\alpha_c} \leq \alpha_c A_c\} \quad (49)$$

$$\tilde{A}_2 := \inf\{A_d \geq \bar{A}_2 : Y_{d,1} \geq Y_{e,1}\}, \quad (50)$$

where  $K_{c,1}$  is defined as in the case 2 of Appendix B.

**Case 2:**  $\epsilon S > \frac{\alpha_d}{\beta_d}(L_0 + T_0 - \bar{L})$ . We get

$$\epsilon S > (\epsilon S + L_0 + T_0 - \bar{L}) \frac{\alpha_d}{\alpha_d + \beta_d}.$$

Condition  $H_1 > 0$  is equivalent to  $\epsilon S > \epsilon p(K_{c,1} + K_{d,1})$ , i.e.,

$$\epsilon S > \sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \frac{\alpha_d}{\alpha_d + \beta_d} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}}.$$

As in Proposition 6, we have  $\lim_{A_d \rightarrow +\infty} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = \epsilon S + L_0 + T_0 - \bar{L} > 0$ . Thus

$$\lim_{A_d \rightarrow +\infty} \sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \frac{\alpha_d}{\alpha_d + \beta_d} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = \frac{\alpha_d}{\alpha_d + \beta_d} (\epsilon S + L_0 + T_0 - \bar{L}) < \epsilon S.$$

This implies that  $H_1 > 0$  if  $A_d$  is high enough. Other statements in this case are proved in Proposition 6.

$\bar{A}_3$  can be defined as follows

$$\begin{aligned} \bar{A}_3 &:= \inf\{A_d : \tilde{U}(A_d) \geq \hat{U} \text{ and} \\ &\quad \epsilon S > \sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \frac{\alpha_d}{\alpha_d + \beta_d} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}}\} \end{aligned} \quad (51)$$

$$\tilde{A}_3 := \inf\{A_d \geq \bar{A}_3 : Y_{d,1} \geq Y_{e,1}\}. \quad (52)$$

□

**Proof of Proposition 8 .** We have  $H_1 = S - p(K_{c,1} + K_{d,1})$ . Hence  $H_1 > 0$  if and only if

$$\frac{S}{p} > K_{c,1} + K_{d,1}. \quad (53)$$

If  $Y_{d,1} = 0$ . In this case, (53) is equivalent to  $\epsilon S > \sigma_c \hat{w}_1^{\frac{-1}{1-\alpha_c}}$ . Recall that  $\hat{w}_1$  is the unique solution of the following equation

$$Q(\hat{w}_1) := \sigma_c \hat{w}_1^{\frac{-1}{1-\alpha_c}} + \sigma_e \hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} - (\epsilon S + L_0 + T_0) = 0.$$

Consequently,  $H_1 > 0$  if and only if  $\sigma_e \hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} > L_0 + T_0$ . Since  $\hat{w}_1$  is decreasing in  $\epsilon$ , this condition is equivalent to  $\epsilon > \epsilon_1$ , where  $\epsilon_1$  is the unique solution of the equation  $\sigma_e x^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} = L_0 + T_0$ .

If  $Y_{d,1} > 0$ . In this case, (53) is equivalent to

$$\gamma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} + \alpha_d \gamma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} > L_0 + T_0 - \bar{L}. \quad (54)$$

Since  $\gamma_d$  is decreasing and  $\tilde{w}_1$  is increasing in  $\epsilon$ , condition (54) is equivalent to  $\epsilon > \epsilon_2$ , where  $\epsilon_2$  is the unique solution of the equation

$$\gamma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} + \alpha_d \gamma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = L_0 + T_0 - \bar{L}.$$

Therefore,  $H_1 > 0$  if  $\epsilon > \bar{\epsilon} := \max\{\epsilon_1, \epsilon_2\}$ ;  $H_1 = 0$  if  $\epsilon < \underline{\epsilon} := \min\{\epsilon_1, \epsilon_2\}$ .

□

**Proof of Proposition 9.** We denote  $X_c, X_d, X_e$  such that

$$\sigma_c = X_c \epsilon^{\frac{-\alpha_c}{1-\alpha_c}}, \quad \sigma_d = X_d \epsilon^{\frac{-\alpha_d}{1-\alpha_d-\beta_d}}, \quad \sigma_e = X_e.$$

Note that  $X_c, X_d, X_e$  do not depend on  $\epsilon$ .

By definition of  $\hat{w}_1$ , we have

$$\frac{X_c}{(\epsilon \hat{w}_1)^{\frac{1}{1-\alpha_c}}} + \frac{X_e}{\epsilon \hat{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}}} = S + \frac{L_0 + T_0}{\epsilon}. \quad (55)$$

Let  $\epsilon$  tend to infinity, thus the wage  $\hat{w}_1(\epsilon)$  which is a function of  $\epsilon$  will tend to zero. Moreover, we have  $\lim_{\epsilon \rightarrow +\infty} \epsilon \hat{w}_1(\epsilon) = +\infty$ . Indeed, (55) implies that

$$\liminf_{\epsilon \rightarrow +\infty} \epsilon \hat{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} \geq \frac{X_e}{S}. \text{ Therefore}$$

$$\epsilon \hat{w}_1 = \epsilon \hat{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} \hat{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} \rightarrow +\infty \text{ when } \epsilon \rightarrow +\infty.$$

By combining with (55), we obtain  $\lim_{\epsilon \rightarrow +\infty} \frac{X_e}{\epsilon \hat{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}}} = S$ .

Definition of  $\tilde{w}_1$  implies that

$$\frac{X_c}{(\epsilon \tilde{w}_1)^{\frac{1}{1-\alpha_c}}} + \frac{X_e}{\epsilon \tilde{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}}} + \frac{X_d}{(\epsilon \tilde{w}_1^{\frac{1}{1-\beta_d}})^{\frac{1-\beta_d}{1-\alpha_d-\beta_d}}} = S + \frac{L_0 + T_0 - \bar{L}}{\epsilon}. \quad (56)$$

By the same argument, we obtain  $\lim_{\epsilon \rightarrow +\infty} \tilde{w}_1(\epsilon) = 0$  and  $\lim_{\epsilon \rightarrow +\infty} \epsilon \tilde{w}_1(\epsilon) = +\infty$ .

We now compare the welfare between two cases:  $Y_{d,1} > 0$  and  $Y_{d,1} = 0$  by observing the difference  $\tilde{U} - \hat{U}$ .

**Case 1:**  $1 - \alpha_d > \frac{\beta_e}{1-\alpha_e}$ , i.e.,  $\frac{1}{\alpha_d} - \frac{1-\alpha_e}{1-\alpha_e-\beta_e} > 0$ .

According to Lemma 1, we have

$$\tilde{U} - \hat{U} \geq \left( \frac{1 - \alpha_d - \beta_d}{\alpha_d + \beta_d} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} - \bar{L} \right) \tilde{w}_1. \quad (57)$$

We will prove that  $\lim_{\epsilon \rightarrow \infty} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = +\infty$ , and then obtain that  $\tilde{U} - \hat{U} > 0$  when  $\epsilon$  is high enough.

If  $\beta_d \leq \frac{\beta_e}{1-\alpha_e}$ , i.e.,  $\frac{1}{1-\beta_d} \leq \frac{1-\alpha_e}{1-\alpha_e-\beta_e}$ . By using the same argument as in the begining of proof of this proposition, we have that there exists  $\lim_{\epsilon \rightarrow \infty} \epsilon \tilde{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} < \infty$ . Hence, we get that

$$\epsilon \tilde{w}_1^{\frac{1}{\alpha_d}} = \epsilon \tilde{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} \tilde{w}_1^{\frac{1}{\alpha_d} - \frac{1-\alpha_e}{1-\alpha_e-\beta_e}} \rightarrow 0$$

when  $\epsilon \rightarrow \infty$ . Consequently,  $\lim_{\epsilon \rightarrow \infty} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = \lim_{\epsilon \rightarrow \infty} X_d(\epsilon \tilde{w}_1^{\frac{1}{\alpha_d}})^{\frac{-\alpha_d}{1-\alpha_d-\beta_d}} = +\infty$ .

If  $\beta_d > \frac{\beta_e}{1-\alpha_e}$ , i.e.,  $\frac{1}{1-\beta_d} > \frac{1-\alpha_e}{1-\alpha_e-\beta_e}$ . In this case, we have that there exists  $\lim_{\epsilon \rightarrow \infty} \epsilon \tilde{w}_1^{\frac{1}{1-\beta_d}} < \infty$ . Therefore, by noting that  $1 - \beta_d > \alpha_d$ , we obtain

$$\epsilon \tilde{w}_1^{\frac{1}{\alpha_d}} = \epsilon \tilde{w}_1^{\frac{1}{1-\beta_d}} \tilde{w}_1^{\frac{1}{\alpha_d} - \frac{1}{1-\beta_d}} \rightarrow 0$$

when  $\epsilon \rightarrow \infty$ . Consequently, we get  $\lim_{\epsilon \rightarrow \infty} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = \infty$ .

**Case 2:**  $1 - \alpha_d < \frac{\beta_e}{1-\alpha_e}$ . In this case, we have  $\frac{1-\alpha_e}{1-\alpha_e-\beta_e} > \frac{1}{\alpha_d} > \frac{1}{1-\beta_d}$ . As a consequence, we have  $\lim_{\epsilon \rightarrow \infty} \epsilon \tilde{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} = \frac{X_e}{S}$ . By combining with the fact that  $\lim_{\epsilon \rightarrow \infty} \tilde{w}_1 = 0$ , we have

$$\epsilon \tilde{w}_1^{\frac{1}{\alpha_d}} = \epsilon \tilde{w}_1^{\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} \tilde{w}_1^{\frac{1}{\alpha_d} - \frac{1-\alpha_e}{1-\alpha_e-\beta_e}} \rightarrow \infty$$

when  $\epsilon \rightarrow \infty$ . Therefore, we obtain  $\lim_{\epsilon \rightarrow \infty} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = 0$ .

First, we have a remark that for  $\gamma \in (0, 1)$ , we have  $y^\gamma - z^\gamma > \gamma y^{\gamma-1}(y-z)$  for every  $y, z > 0$ . Therefore, we have

$$\begin{aligned} \hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} &= (\hat{w}_1^{\frac{-1}{1-\alpha_c}})^{\alpha_c} - (\tilde{w}_1^{\frac{-1}{1-\alpha_c}})^{\alpha_c} \geq \alpha_c \hat{w}_1 (\hat{w}_1^{\frac{-1}{1-\alpha_c}} - \tilde{w}_1^{\frac{-1}{1-\alpha_c}}) \\ \hat{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} - \tilde{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} &= (\hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}})^{\frac{\beta_e}{1-\alpha_e}} - (\tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}})^{\frac{\beta_e}{1-\alpha_e}} \\ &\geq \frac{\beta_e}{1-\alpha_e} \hat{w}_1 (\hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} - \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}). \end{aligned}$$

We now write

$$\begin{aligned}
\hat{U} - \tilde{U} &= \gamma_c(\hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}) + \gamma_e(\hat{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} - \tilde{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}}) - \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}} \\
&= \frac{\beta_e}{1-\alpha_e} \left( \left( \frac{1-\alpha_e}{\beta_e} - 1 \right) \frac{\sigma_c}{\alpha_c} (\hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}) + \frac{\sigma_c}{\alpha_c} (\hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}) \right) \\
&\quad + \frac{\beta_e}{1-\alpha_e} \frac{1-\alpha_e}{\beta_e} \gamma_e (\hat{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} - \tilde{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}}) - \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}} \\
&\geq \frac{1-\alpha_e-\beta_e}{1-\alpha_e} \frac{\sigma_c}{\alpha_c} (\hat{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} - \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}}) - \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}} \\
&\quad + \frac{\beta_e}{1-\alpha_e} \hat{w}_1 \left( \sigma_c (\hat{w}_1^{\frac{-1}{1-\alpha_c}} - \tilde{w}_1^{\frac{-1}{1-\alpha_c}}) + \sigma_e (\hat{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} - \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}) \right) \\
&> \frac{\beta_e}{1-\alpha_e} \hat{w}_1 (\bar{L} - \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}}) - \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}}.
\end{aligned}$$

Therefore, we have

$$\hat{U} - \tilde{U} > \hat{w}_1 \left( \frac{\beta_e}{1-\alpha_e} \bar{L} - \frac{\beta_e}{1-\alpha_e} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} - \frac{\tilde{w}_1}{(\alpha_d + \beta_d) \hat{w}_1} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} \right).$$

Recall that in this case, we have  $\bar{L} > 0$ ,  $\lim_{\epsilon \rightarrow \infty} \frac{\tilde{w}_1}{\hat{w}_1} = 1$  and  $\lim_{\epsilon \rightarrow \infty} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}} = 0$ .

So, we imply that  $\hat{U} - \tilde{U} > 0$  when  $\epsilon$  is high enough.  $\square$

**Proof of Proposition 11.** If  $\epsilon S + L_0 + T_0 \leq \bar{L}$ , we have  $Y_{d,1} = 0$ . We now consider the case  $\epsilon S + L_0 + T_0 > \bar{L}$ .

We assume that  $H_1 > 0$ . By observing equation determining wage, we see that wage increases when the new good price increases. Moreover,  $\lim_{p_n \rightarrow +\infty} \tilde{w}_1 =$

$\lim_{p_n \rightarrow +\infty} \hat{w}_1 = +\infty$ . (33) implies that

$$\lim_{p_n \rightarrow +\infty} \sigma_e \hat{w}_1^{\frac{-1-\alpha_e}{1-\alpha_e-\beta_e}} = \epsilon S + L_0 + T_0.$$

Equation determining  $\tilde{w}$  is equivalent to

$$\sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \frac{\sigma_e}{p_n^{\frac{1}{1-\alpha_e-\beta_e}}} \left( \frac{p_n}{\tilde{w}_1^{1-\alpha_e}} \right)^{\frac{1}{1-\alpha_e-\beta_e}} + \frac{\sigma_e}{p_n^{\frac{1}{1-\alpha_d-\beta_d}}} \left( \frac{p_n}{\tilde{w}_1} \right)^{\frac{1}{1-\alpha_d-\beta_d}} = \epsilon S + L_0 + T_0 - \bar{L},$$

where we note that  $\frac{\sigma_e}{p_n^{\frac{1}{1-\alpha_e-\beta_e}}}$  and  $\frac{\sigma_e}{p_n^{\frac{1}{1-\alpha_d-\beta_d}}}$  do not depend on  $p_n$ .

Therefore, we have  $\frac{p_n}{\tilde{w}_1} \tilde{w}_1^{\alpha_e} = \frac{p_n}{\tilde{w}_1^{1-\alpha_e}}$  is bounded. This implies that  $\lim_{p_n \rightarrow +\infty} \frac{p_n}{\tilde{w}_1} =$

0. Consequently, we get

$$\lim_{p_n \rightarrow +\infty} \sigma_e \tilde{w}_1^{-\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} = \epsilon S + L_0 + T_0 - \bar{L}.$$

Thus, we obtain  $\lim_{p_n \rightarrow +\infty} \frac{\hat{w}_1}{\tilde{w}_1} = \left( \frac{\epsilon S + L_0 + T_0}{\epsilon S + L_0 + T_0 - \bar{L}} \right)^{-\frac{1-\alpha_e-\beta_e}{1-\alpha_e}}$ .  
We now compare welfares

$$\begin{aligned} \frac{\hat{U}}{\tilde{U}} &= \frac{\gamma_c(\hat{w}_1)^{\frac{-\alpha_c}{1-\alpha_c}} + \gamma_e(\hat{w}_1)^{\frac{-\beta_e}{1-\alpha_e-\beta_e}}}{\gamma_c \tilde{w}_1^{\frac{-\alpha_c}{1-\alpha_c}} + \gamma_e \tilde{w}_1^{\frac{-\beta_e}{1-\alpha_e-\beta_e}} + \gamma_d \tilde{w}_1^{\frac{-\alpha_d-\beta_d}{1-\alpha_d-\beta_d}}} \\ &= \frac{\gamma_c(\hat{w}_1)^{\frac{-1}{1-\alpha_c}} + \gamma_e(\hat{w}_1)^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}}}{\gamma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \gamma_e \tilde{w}_1^{\frac{-(1-\alpha_e)}{1-\alpha_e-\beta_e}} + \gamma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}}} \frac{\hat{w}_1}{\tilde{w}_1} \end{aligned}$$

Hence, we obtain

$$\lim_{p_n \rightarrow +\infty} \frac{\hat{U}}{\tilde{U}} = \left( \frac{\epsilon S + L_0 + T_0}{\epsilon S + L_0 + T_0 - \bar{L}} \right)^{\frac{\beta_e}{1-\alpha_e}} > 1.$$

This implies that when  $p_n$  is high enough,  $\hat{U} > \tilde{U}$ .

We can also see that condition (53) is satisfied when  $p_n$  is high enough. So, when  $p_n$  is high enough, we have  $Y_{d,1} = 0$  and  $H_1 > 0$  at equilibrium.  $\square$

**Proof of Proposition 12. Case (i):**  $\frac{\alpha_e}{1-\alpha_e} > \max(\alpha_c, \alpha_d)$ . As in proof of Proposition 11, we obtain

$$\begin{aligned} \lim_{p \rightarrow 0} \hat{w}_1 p^{\alpha_c} &= \lim_{p \rightarrow 0} \tilde{w}_1 p^{\alpha_c} = \lim_{p \rightarrow 0} \tilde{w}_1 p^{\alpha_d} = +\infty \\ \lim_{p \rightarrow 0} \sigma_e \hat{w}_w^{-\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} &= \epsilon S + L_0 + T_0 \\ \lim_{p \rightarrow 0} \sigma_e \hat{w}_w^{-\frac{1-\alpha_e}{1-\alpha_e-\beta_e}} &= \epsilon S + L_0 + T_0 - \bar{L}. \end{aligned}$$

Consequently, we get

$$\lim_{p \rightarrow 0} \frac{\hat{w}_1}{\tilde{w}_1} = \left( \frac{\epsilon S + L_0 + T_0}{\epsilon S + L_0 + T_0 - \bar{L}} \right)^{-\frac{1-\alpha_e-\beta_e}{1-\alpha_e}} \quad (58)$$

$$\lim_{p \rightarrow 0} \frac{\hat{U}}{\tilde{U}} = \left( \frac{\epsilon S + L_0 + T_0}{\epsilon S + L_0 + T_0 - \bar{L}} \right)^{\frac{\beta_e}{1-\alpha_e}}. \quad (59)$$

Therefore, when  $p$  is low enough, we have  $\hat{U}_1 > \tilde{U}$ .

We have to now check that  $H_1 > 0$  when  $p$  is low enough. We will check that  $\epsilon S > \sigma_c \hat{w}_1^{\frac{-1}{1-\alpha_c}}$ . As in proof of Proposition 11, we obtain  $\lim_{p \rightarrow 0} \sigma_c \hat{w}_1^{\frac{-1}{1-\alpha_c}} = 0$ .

Hence,  $H_1 > 0$  when  $p$  is low enough.

**Case (ii.a):** The proof is similar to point (ii) of Proposition 3.

**Case (ii.b):** Assume that  $H_1 = 0$ , we write the equation of  $w_1$

$$\begin{aligned} S = & \left( \frac{1}{\beta_d p_n A_d} \right)^{\frac{1}{\alpha_d}} (p w_1^{\frac{1}{\alpha_d}}) \left( L_0 + T_0 - \bar{L} - \frac{M_e}{(p w_1^{\frac{1-\alpha_e}{\alpha_e}})^{\frac{\alpha_e}{1-\alpha_e-\beta_e}}} \right)^{\frac{1-\beta_d}{\alpha_d}} \\ & + \left( \frac{\alpha_c A_c}{\alpha_d} \right)^{\frac{1}{1-\alpha_c}} \left( \frac{1}{p_n A_d \beta_d^{1-\alpha_d}} \right)^{\frac{1}{\alpha_d(1-\alpha_c)}} p w_1^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} \left( L_0 + T_0 \right. \\ & \quad \left. - \bar{L} - \frac{M_e}{(p w_1^{\frac{1-\alpha_e}{\alpha_e}})^{\frac{\alpha_e}{1-\alpha_e-\beta_e}}} \right)^{\frac{1-\alpha_d-\beta_d}{\alpha_d(1-\alpha_c)}}, \end{aligned}$$

where  $M_e := \sigma_e p^{\frac{\alpha_e}{1-\alpha_e-\beta_e}}$  which does not depend on  $p$ .

First, it is easy to see that  $w_1$  increases if  $p$  decreases. Moreover,  $\lim_{p \rightarrow 0} w_1(p) = +\infty$ .

If there is a sequence  $(p(n))_{n=1,2,\dots}$ , converging to zero such that  $p(n)(w_1(n))^{\frac{1-\alpha_e}{\alpha_e}}$  is bounded from above.<sup>13</sup> Since  $\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)} < \frac{1}{\alpha_d} < \frac{1-\alpha_e}{\alpha_e}$ , We have

$$\begin{aligned} p(n)(w_1(n))^{\frac{1}{\alpha_d}} &= p(n)(w_1(n))^{\frac{1-\alpha_e}{\alpha_e}} (w_1(n))^{\frac{1}{\alpha_d} - \frac{1-\alpha_e}{\alpha_e}} \rightarrow 0 \text{ when } n \rightarrow 0 \\ p(n)(w_1(n))^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} &= p(n)(w_1(n))^{\frac{1}{\alpha_d}} (w_1(n))^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)} - \frac{1}{\alpha_d}} \rightarrow 0 \text{ when } n \rightarrow 0. \end{aligned}$$

Therefore, we get a contradiction to the equation of  $w_1$ . So, we have  $\lim_{p \rightarrow 0} p(w_1(p))^{\frac{1-\alpha_e}{\alpha_e}} = +\infty$ .

We now prove that  $\lim_{p \rightarrow 0} p(w_1(p))^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} = 0$ . Indeed, if there is a sequence

$(p(n))_{n=1,2,\dots}$  converging to zero such that  $p(n)(w_1(n))^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}}$  is bounded from below. We get that

$$p(n)(w_1(n))^{\frac{1}{\alpha_d}} = p(n)(w_1(n))^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} (w_1(n))^{\frac{1}{\alpha_d} - \frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} \rightarrow +\infty$$

when  $n$  tends to infinite. This implies a contradiction to the equation determining wage.



We have proved that  $\lim_{p \rightarrow 0} p(w_1(p))^{\frac{1-\alpha_e}{\alpha_e}} = +\infty$  and  $\lim_{p \rightarrow 0} p(w_1(p))^{\frac{1-\alpha_d}{\alpha_d(1-\alpha_c)}} = 0$ . The equation determining wage implies that

$$S = \left(\frac{1}{\beta_d p_n A_d}\right)^{\frac{1}{\alpha_d}} (L_0 + T_0 - \bar{L})^{\frac{1-\beta_d}{\alpha_d}} \lim_{p \rightarrow 0} p w_1^{\frac{1}{\alpha_d}}. \quad (60)$$

By using the same argument in Remark 2, we see that the first order condition of  $H_1$  is satisfied.  $\square$

**Proof of Example 1.** We prove point (i). Point (ii) can be proved by the same argument.

Assume that there exists an equilibrium with  $H_1, Y_{d,1} > 0$ .

We find conditions in which  $\tilde{U} \geq \hat{U}$  under Assumption 9.

By using Assumption 9, we have

$$\begin{aligned} \tilde{w}_1^x &= \frac{\mathcal{L}_0 - \bar{L}}{\sigma_c + \sigma_e + \sigma_d} \\ \tilde{U} &= (\gamma_c + \gamma_e + \gamma_d) \tilde{w}_1^{x+1} \\ (\hat{w}_1)^x &= \frac{\mathcal{L}_0}{\sigma_c + \sigma_e} \\ \hat{U} &= (\gamma_c + \gamma_e) (\hat{w}_1)^{x+1}, \end{aligned}$$

where  $x := -1/(1 - \alpha_c)$ . Therefore we see that

$$\begin{aligned} \tilde{U} \geq \hat{U} &\Leftrightarrow \frac{\gamma_c + \gamma_e + \gamma_d}{\gamma_c + \gamma_e} \geq \left( \frac{\mathcal{L}_0}{\mathcal{L}_0 - \bar{L}} \frac{\sigma_c + \sigma_e + \sigma_d}{\sigma_c + \sigma_e} \right)^{\alpha_c} \\ &\Leftrightarrow \mathcal{L}_0 - \bar{L} \geq \Omega \mathcal{L}_0 \Leftrightarrow (1 - \Omega) \mathcal{L}_0 \geq \bar{L}. \end{aligned}$$

Note that  $\Omega < 1$ . Indeed,

$$\begin{aligned} \Omega &:= \frac{\alpha(\gamma_c + \gamma_d) + \gamma_e}{\alpha\gamma_c + \gamma_e} \left( \frac{\gamma_c + \gamma_e}{\gamma_c + \gamma_e + \gamma_d} \right)^{\frac{1}{\alpha_c}} \\ &< \frac{\alpha(\gamma_c + \gamma_d) + \gamma_e}{\alpha\gamma_c + \gamma_e} \frac{\gamma_c + \gamma_e}{\gamma_c + \gamma_e + \gamma_d}. \end{aligned}$$

On the other hand,

$$\frac{\alpha(\gamma_c + \gamma_d) + \gamma_e}{\alpha\gamma_c + \gamma_e} = 1 + \frac{\alpha\gamma_d}{\alpha\gamma_c + \gamma_e} < 1 + \frac{\gamma_d}{\gamma_c + \gamma_e} = \frac{\gamma_c + \gamma_e + \gamma_d}{\gamma_c + \gamma_e}.$$

Therefore  $\Omega < 1$ . Consequently,  $\tilde{U} \geq \hat{U}$  if and only if  $\mathcal{L}_0 \geq \frac{\bar{L}}{1 - \Omega}$ .

Condition  $H_1 > 0$  is equivalent to

$$\epsilon S > \sigma_c \tilde{w}_1^{\frac{-1}{1-\alpha_c}} + \frac{\alpha_d}{\alpha_d + \beta_d} \sigma_d \tilde{w}_1^{\frac{-1}{1-\alpha_d-\beta_d}}.$$

Under Assumption 3, we have  $\hat{w}^{\frac{1}{1-\alpha_c}} = \hat{w}^{\frac{1}{1-\alpha_d-\beta_d}} = \frac{\epsilon S + L_0 + T_0 - \bar{L}}{\sigma_c + \sigma_d + \sigma_e}$ . Therefore,  $H_1$  is equivalent to condition (11).

We now assume that condition (10) and (11) hold. Then  $\epsilon S + L_0 + T_0 - \bar{L} > 0$  then equation (29) has a unique solution who is equilibrium wage. We see that all first order conditions hold. Condition (11) ensures that  $H_1 > 0$  in this case. Thus, the list  $(K_{c,1}, K_{d,1}, H_1, L_{d,1}, L_{e,1}, K_{e,1}, w_1)$  given in proof of Example 1 in case  $Y_{d,1} > 0$  is the unique equilibrium.  $\square$

## 6 Appendix: Constant return to scale

We assume that  $\alpha_d + \beta_d = \alpha_e + \beta_e = 1$ . In order to present formal proofs of our results with CRS technologies, let us begin by some necessary computations. We write FOC for the multinational firm. If  $K_{e,1}, L_{e,1} > 0$ , we have

$$\alpha_e p_n A_e K_{e,1}^{\alpha_e-1} L_{e,1}^{1-\alpha_e} = p \quad (61)$$

$$(1 - \alpha_e) p_n A_e K_{e,1}^{\alpha_e} L_{e,1}^{\alpha_e} = w_1. \quad (62)$$

Consequently, we get  $w_1^{1-\alpha_e} = \alpha_e^{\alpha_e} (1 - \alpha_e)^{1-\alpha_e} A_e p_n p^{-\alpha_e}$ . In this case, the multinational firm's profit equals zero. This implies that the multinational firm's profit equals zero in any case. Note that  $K_{e,1} = L_{e,1} = 0$  is a solution of this firm's maximization problem.

Denote  $\lambda_h, \lambda_\ell$  Lagrange multipliers associated to conditions (4), (5),  $H_1 \geq 0$ , and  $L_{e,1} \geq 0$ , respectively. We have

$$\begin{aligned} K_{c,1} : \quad & \alpha_c A_c K_{c,1}^{\alpha_c-1} = \lambda p \\ L_{e,1} : \quad & w_1 - \mu + \lambda_\ell = 0, \text{ where } \lambda_\ell \geq 0, L_{e,1} \lambda_\ell = 0. \\ H_1 : \quad & -\lambda + \mu \epsilon + \lambda_h = 0, \text{ where } \lambda_h \geq 0, H_1 \lambda_h = 0. \end{aligned}$$

If  $Y_{d,1} > 0$ , we have

$$\begin{aligned} K_{d,1} : \quad & p_n \alpha_d A_d K_{d,1}^{\alpha_d - 1} (L_{d,1} - \bar{L})^{1 - \alpha_d} = \lambda p \\ L_{d,1} : \quad & p_n (1 - \alpha_d) A_d K_{d,1}^{\alpha_d} (L_{d,1} - \bar{L})^{-\alpha_d} = \mu. \end{aligned}$$

**Proof of Theorem 2.** The first case is clear. We assume that  $L_0 + T_0 + \epsilon S > \bar{L}$ . It is easy to see that when  $A_d$  tends to infinity, the GNP tend to infinity if the host country invest in the new industry. So, when  $A_d$  is high enough, we have  $Y_{d,1} > 0$  at equilibrium. Therefore,  $L_0 + T_0 + \epsilon H_1 > \bar{L}$  at equilibrium. If  $L_0 + T_0 \leq \bar{L}$  then we have  $H_1 > 0$ . The second statement of Theorem 2 is proved.

**Case (2.2.1)**  $L_0 + T_0 > \bar{L}$  and  $\epsilon S < \frac{\alpha_d}{1 - \alpha_d} (L_0 + T_0 - \bar{L})$ . Assume that  $L_{e,1} = H_1 = 0$ . FOCs of  $K_{c,1}$  and  $K_{d,1}$  imply that

$$\alpha_c A_c K_{d,1}^{1 - \alpha_d} (L_0 + T_0 - \bar{L})^{-(1 - \alpha_d)} = p_n \alpha_d A_d K_{c,1}^{1 - \alpha_c}.$$

We then get an equation determining  $K_{c,1}$

$$S = p K_{c,1} + \left( \frac{p_n \alpha_c A_c}{\alpha_d A_d} \right)^{\frac{1}{1 - \alpha_d}} (L_0 + T_0 - \bar{L}) K_{c,1}^{\frac{1 - \alpha_c}{1 - \alpha_d}}.$$

This equation has a unique solution. It is easy to see that when  $A_d$  increases,  $K_{c,1}$  decreases, and  $\lim_{A_d \rightarrow +\infty} K_{c,1} = 0$ ,  $\lim_{A_d \rightarrow +\infty} K_{d,1} = S/p$ .

We now check FOC of  $H_1$ :  $\lambda \geq \mu \epsilon$  will be satisfied when  $A_d$  is high enough. This condition can be written as  $\frac{\lambda p}{\mu} \geq \epsilon p$  which is equivalent to  $\frac{\alpha_d}{1 - \alpha_d} \frac{L_{d,1} - \bar{L}}{K_{d,1}} \geq \epsilon p$ . This condition is satisfied when  $A_d$  is high enough since  $\lim_{A_d \rightarrow +\infty} K_{d,1} = S/p$ .

**Case (2.2.2)**  $L_0 + T_0 > \bar{L}$  and  $\epsilon S > \frac{\alpha_d}{1 - \alpha_d} (L_0 + T_0 - \bar{L})$ . We will check that  $L_{e,0} = 0$  and  $H_1 > 0$  at equilibrium. Assume that  $L_{e,1} = 0$  and  $H_1 > 0$ . Then we have  $L_{d,1} = L_0 + T_0 + \epsilon H_1$  and  $\lambda = \mu \epsilon$ . FOCs of  $K_{d,1}$ ,  $L_{d,1}$  implies that  $\frac{L_{d,1} - \bar{L}}{K_{d,1}} = \frac{1 - \alpha_d}{\alpha_d} \epsilon p$ . Hence, we get  $p K_{d,1} = \frac{\alpha_d}{1 - \alpha_d} (L_0 + T_0 + \epsilon H_1 - \bar{L})$  and

$$\alpha_c A_c K_{c,1}^{\alpha_c - 1} = \lambda p = p_n \alpha_d A_d K_{d,1}^{\alpha_d - 1} (L_{d,1} - \bar{L})^{1 - \alpha_d} = p_n \alpha_d A_d \left( \frac{1 - \alpha_d}{\alpha_d} \epsilon p \right)^{1 - \alpha_d}.$$

Hence, we can compute  $K_{c,1}$  in order to get that

$$\begin{aligned} S &= p K_{c,1} + p K_{d,1} + H_1 \\ &= \left( \frac{\alpha_c A_c}{p_n \alpha_d A_d} \right)^{\frac{1}{1 - \alpha_c}} \left( \frac{\alpha_d}{(1 - \alpha_d) \epsilon} \right)^{\frac{1 - \alpha_d}{1 - \alpha_c}} p^{\frac{\alpha_d - \alpha_c}{1 - \alpha_c}} + \frac{H_1}{1 - \alpha_d} + \frac{\alpha_d (L_0 + T_0 - \bar{L})}{(1 - \alpha_d) \epsilon}. \end{aligned}$$

Since  $S > \frac{\alpha_d(L_0+T_0-\bar{L})}{1-\alpha_d}\epsilon$ , this equation has a unique solution  $H_1 > 0$  when  $A_d$  is high enough. It is easy to check all FOCs. Therefore,  $L_{e,1} = 0$ ,  $Y_{d,1}$ ,  $H_1 > 0$  given as above is an equilibrium.  $\square$

**Proof of Proposition 10.** When  $\epsilon$  is high enough, it is clear that the country should invest in training.

Assume that the country also invests in the new industry, i.e.,  $Y_{d,1} > 0$ .

According the computation in the proof of Theorem 2, we have

$$\alpha_c A_c \geq p_n \alpha_d A_d \left( \frac{1 - \alpha_d}{\alpha_d} \epsilon p \right)^{1 - \alpha_d} K_{c,1}^{1 - \alpha_c} \geq p_n \alpha_d A_d \left( \frac{1 - \alpha_d}{\alpha_d} \epsilon p \right)^{1 - \alpha_d} \left( \frac{S}{p} \right)^{1 - \alpha_c}.$$

This condition will be violated when  $\epsilon$  is high enough.  $\square$

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