What if oil is less substitutable? A New-Keynesian Model with Oil, Price and Wage Stickiness including Capital Accumulation

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CES Working Papers

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2015.41

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://centredeconomiesorbonne.univ-paris1.fr/
ISSN : 1955-611X

What if oil is less substitutable? A New-Keynesian Model with Oil, Price and Wage Stickiness including Capital Accumulation

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May 5, 2015†

Abstract

The recent literature on fossil energy has already stated that oil is not perfectly substitutable to other inputs, considering fossil fuel as a critical production factor in different combinations. However, the estimations of substitution elasticity are in a wide range between 0.004 and 0.64. This paper addresses this phenomenon by enlarging the DSGE model developed in Acurio-Vásconez et al. (2015) by changing the Cobb-Douglas production and consumption functions assumed there, for composite Constant Elasticity of Substitution (CES) functions. Additionally, the paper introduces nominal wage and price rigidities through a Calvo setting. Finally, using Bayesian methods, the model is estimated on quarterly U.S. data over the period 1984:Q1-2007:Q3 and then analyzed. The estimation of oil's elasticity of substitution are 0.14 in production and 0.51 in consumption. Moreover, thanks to the low substitutability of oil, the model recovers and explains four well-known stylized facts after the oil price shock in the 2000s': the absent of recession, coupled with a low but persistent increase in inflation rate, a decrease in real wages and a low price elasticity of oil demand in the short run. Furthermore, ceteris paribus, the reduction of nominal wage rigidity amplifies the increase in inflation and the decrease in consumption. Thus in this model more wage flexibility does not seem to attenuate the impact of an oil shock.

JEL Codes: D58, E32, E52, Q43

Keywords: New-Keynesian model, DSGE, oil, CES, stickiness, oil substitution.

1 Introduction

The question of oil substitutability has seen increasing interest from economists over the last decade. One of the reasons why can be found in Figure 3, which shows oil consumption per capita in U.S. and the real oil price since 1970. Oil consumption per person has remained

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†I gratefully acknowledge the financial support of the Chair Energy and Prosperity and SENESCYT. I am grateful to Cristiano Cantore, Gaël Giraud and Florent Mc Isaac for their useful comments. In addition I thank seminar participants at different conferences, specially in the IAEE conferences and the One-Day conference on DSGE Modelling at the University of Surrey for their comments and reactions. All remaining errors are mine.

near-constant from 1990 to 2008, even though its real price has continuously increased. If we look at the U.S. industrial sector separately, the same phenomenon appears as shown in Figure 4 that represents oil consumption of the industry sector in U.S. Moreover, in this last case, there is an increasing path in oil consumption from 1981 to 2008. Then while it is true that firms and households can react to changes in oil prices (e.g. by shifting away from oil towards capital or labor in the case of firms and to final domestic goods consumption for households), this substitution has become less evident in the last years in the U.S. One possible reason for this stagnation is the well known "rebound effect:" even if there has been increasing oil productivity since the 1970s', resulting in an improvement in oil utilization, it has not generated any mitigation on global oil consumption. One can think of the automotive industry to illustrate this point. A car nowadays with the same characteristics and power than a car 30 years ago, uses much less oil to travel the same distance. Nevertheless, today one can buy a better car, at a relative cheaper price, than 30 years ago, which at the end does not consume much less. Moreover, people who could not afford to pay for a car 30 years ago, are now able to have one, which increases again the quantity of oil use. Another reason is that the U.S. economy, as well as most industrialized economies are still heavily oil dependent and that given current technologies, it is hard to substitute other energy sources for oil, at least in the short-term.

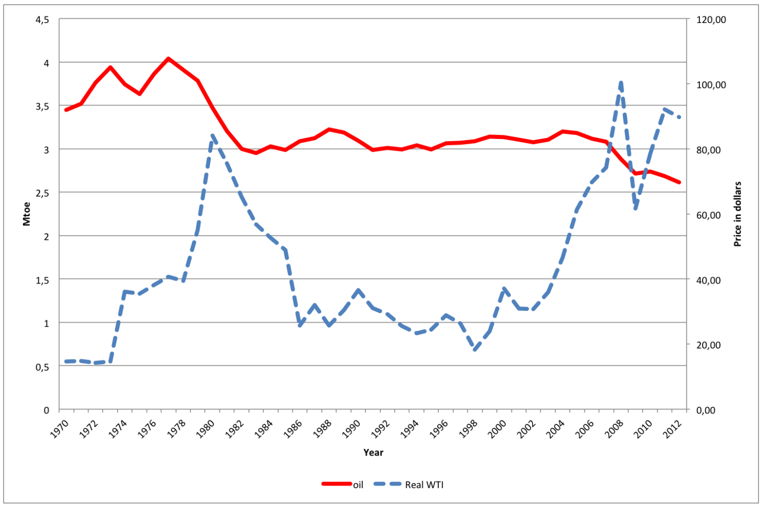

Figure 1: US Oil Consumption per Capita (Mtoe) and Spot Oil Prices

Source: Bank of Saint Louis (Spot Oil Price–Real prices base 2009): West Texas Intermediate, BP Statistical Review of World Energy 2013

The recent literature has stated that energy is a critical input in industrialized economies, and that it is not perfectly substitutable to other production factors. In Fouré et al. (2012), Hassler et al. (2012), van-der Werf (2008) and Kander & Stern (2012) among others, energy (or fossil energy) is introduced in the production function through a constant elasticity of substitution (CES) function with two factors: energy and a Cobb-Douglas combination of capital and labor. Each of these papers estimates the energy elasticity of substitution using different methods. Hassler et al. (2012) used maximum-likelihood approach with data from US; van-der Werf (2008) used linear regressions with data from several countries and industries; Kander & Stern (2012) constructed an extension of the Solow's growth model and estimated its parameters using data from Sweden and linear regressions. Each of these papers exhibits different estimations for the energy elasticity of substitution with respect to labor and capital in different combinations, with values ranging from 0.004 to 0.64. However, all papers reject the assumption of a substitution elasticity equals to one. In Kumhof & Muir (2014), the authors used a CES function to model oil demand and interpreted the elasticity of substitution between oil and the composite factor as the long-run price elasticity of oil demand. This value is estimated in Helbling et al. (2011) and Benes et al. (2015) at 0.08. Other examples can be found in Lindenberger & Kümmel (2010) who used a production function where output elasticities are not equal to cost shares and established that energy dependent production functions reproduce past economic growth with zero Solow residual, or in Hassler et al. (2012), who constructed a model of directed technical change, where the production function is Leontief and found that the economy directs its efforts toward input-saving so as to economize on expensive or scarce inputs. Most recently, Henriet et al. (2014) introduced fossil fuel with CES functions in a Computational General Equilibrium Model (CGE). They estimated the elasticities of substitution with French data using cointegration methods and linear regressions and found that those are equal to 0.5 in both sectors, production and households consumption.

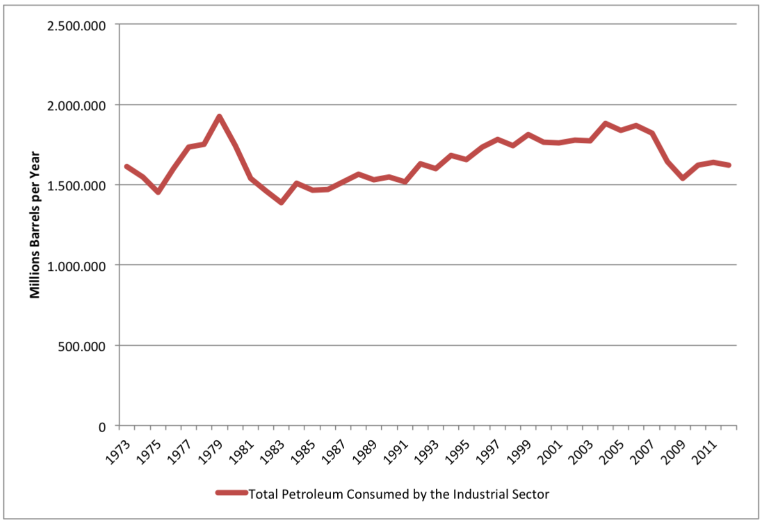

Figure 2: Total Petroleum Consumed by the Industrial Sector (Millions Barrels per Year)

Source: EIA. Table 3.7b. Petroleum Consumption: Industrial Sector

In Dynamic Stochastic General Models (DSGE) that include oil and focus on the macroeconomics effects of oil shocks, to my knowledge, either the models are calibrated as in Blanchard & Galí (2009), or estimated but oil and the other factors are considered easy substitutes for oil as in the model developed in Acurio-Vásconez et al. (2015), or estimated but oil is not considered as a factor of production as in Kormilitsina (2011), else imperfect substitutability is introduced but no estimation is performed as in Montoro (2012). In Blanchard & Riggi (2013) model, an estimation using minimum distance estimation techniques is performed for some of the model parameters in two cases, assuming a Cobb-Douglas and a Leontief production function. Capital however is not included in Blanchard & Riggi (2013). Moreover, none of these DSGE models is able to recover four of the well known stylized facts which followed the oil shocks of the 2000s': the absent of recession, coupled with a low but persistent increase in inflation rate, a decrease in real wages, and a low price elasticity of oil demand in the short term, all at the same time.

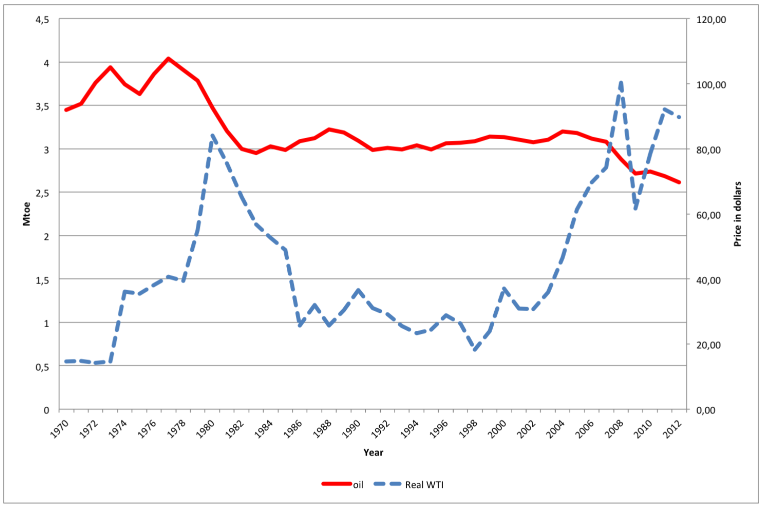

Figure 3: US Oil consumption per capita (Mtoe) and Spot Oil Prices

Source: Bank of Saint Louis (Spot Oil Price): West Texas Intermediate, BP Statistical Review of World Energy 2013

In order to shed some new light onto these questions, this paper enlarges the model developed in Acurio-Vásconez et al. (2015), where oil is incorporated into a DSGE model through a Cobb-Douglas function in the consumption flow and in the production function of intermediate firms. The production function I use here is an integrated CES function, constructed as in Hassler et al. (2012) and re-normalized as in Cantore & Levine (2012). This function includes oil, which is fully imported from a foreign economy, and a Cobb-Douglas combination of labor and capital. On the household's side, I use a basic CES function that integrates final goods and oil to define the consumption flow.

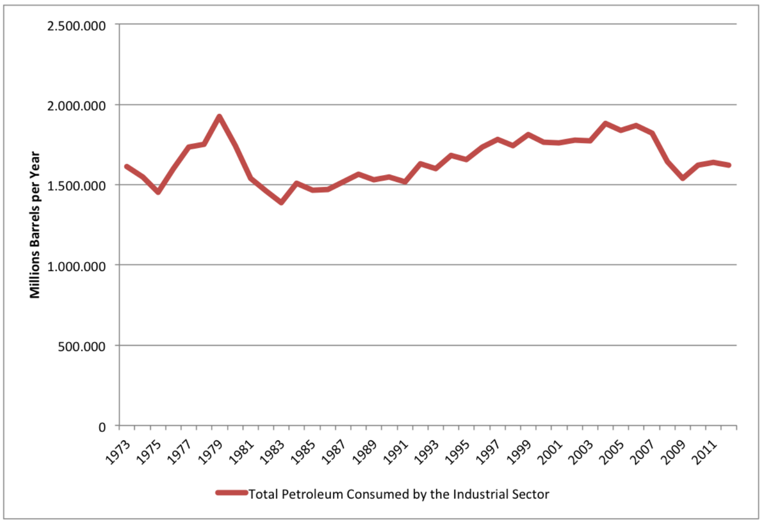

Figure 4: Total Petroleum Consumed by the Industrial Sector (Millions Barrels per Year)

Source: EIA. Table 3.7b. Petroleum Consumption: Industrial Sector

Along with this framework, the model adds stickiness in nominal prices and wages.

This last element allows me to analyze one of the conclusions given in Blanchard & Galí (2009) and Blanchard & Riggi (2013) regarding the softer impact onto the economy after an oil shock, which is the reduction of wage rigidity. However, alternatively to the ad-hoc formulation of the real wage stickiness introduced in those papers, this paper adds nominal wage rigidity in a more conventional way, using a framework *à la* Calvo.

Once the model has been constructed and log-linearized around its steady state, it is estimated using Bayesian methods, with quarterly U.S. data over the period 1984:Q1 - 2007:Q1. The estimated oil's elasticity of substitution are 0.14 in production and 0.51 in household consumption. These values exhibit the fact that oil is weakly substitutable to other quantities in both sectors, especially in the production sector. The estimation also shows that a real oil price shock displays strong persistence. Another significant result of estimation is the posterior mean of oil's output elasticity at steady state, which is estimated at 0.06. As remarked in Acurio-Vásconez et al. (2015), this value is slightly larger than oil's cost share.[1]

The impulse response function analysis shows that the model is able to recover and explain four well-known stylized facts after an oil price shock in the 2000s' detailed before. Thus this paper identifies yet another channel to explain why we did not observe the stronger impact on GDP after the oil shock of the 2000s'. If oil is not easily substitutable and if it is fully imported, an increase in its price causes firms to produce more in order to pay for the oil bill, in that way most of the domestic production and oil importation cancel each other out. Then the reaction of GDP to an oil shock could be nearly nil. It could even be positive, depending on the reaction of the rental rate of capital, which in turn depends on the endogenous reaction of monetary policy to an oil shock.

A sensitivity analysis shows that a decrease in nominal wage rigidity in the estimated model, ceteris paribus, could lead to an increase in real wages, which then leads to higher prices, confining households to a worse trade-off between consumption and investment, in favor of investment. Then a stronger increase in domestic output takes place, but because of the low substitutability, oil should increase as well.[2] This oil increase should not be problematic as long as the U.S. economy can import as much oil as it wants. However, in a world where oil supply has entered a period of increased scarcity, the consequences could be a loss of output,[3] as shown in Kumhof & Muir (2014), Bezdek et al. (2005), Reynolds (2002), among others.[4]

The rest of the paper is structured as follows. Section 2 presents the DSGE model. Section 3 describes the elements of the Bayesian estimation and examines its results. Section 4 analyzes the impulse response of the economy to a real oil price shock and discusses how the economy would respond under more flexible wages. Finally, Section 5 concludes.

2 Model

As in Acurio-Vásconez et al. (2015), this paper constructs a DSGE model that considers oil, labor and capital as inputs for intermediate firms and where households can consume final domestic goods and oil. As in assumed in Acurio-Vásconez et al. (2015), oil is im-

[1] As in Blanchard & Galí (2009) and Acurio-Vásconez et al. (2015), GDP is defined in valued added, which implies that cost's share and oil's output elasticity are not equal.

[2] These effects are explained in detail in Section 4.1 and 4.2, respectively.

[3] One should not forget that the U.S economy is a major producer of oil, then this result will be revised in a companion forthcoming paper where we allow for domestic energy production.

[4] See also references therein.

ported from a foreign country at an exogenous real price. Price and wage stickiness are also introduced, and the model considers that the consumption flow and the intermediate production function are CES. This section will first describe how households consume, work, hold capital and use oil. Then it will describe how firms use different inputs to produce intermediate goods that will be transformed by the final good firm in an aggregate unique final good. Finally, I will explain how the government intervenes in the economy.[5]

2.1 Households

Assume a continuum of monopolistically competitive households indexed by $j \in [0,1]$. Each one of them, consumes both oil and domestic goods, supplies a differentiated labor service to the production sector, invests in government bonds and capital, pays taxes, and receives profits from the firms in the economy.

Each household has an instantaneous utility function, which is assumed separable in consumption $C_t(j)$ and hours worked $L_t(j)$ and given by:

$$U(C_t(j), L_t(j)) = log(C_t(j)) - \frac{L_t(j)^{1+\phi}}{1+\phi}$$

where ϕ is the inverse of the Frish elasticity. Each household can consume two different types of goods. A domestic good at nominal price P_q that is produced inside the country and oil, which comes from a foreign country at nominal price P_e.[6] The consumption flow of household j is defined as:

$$C_t(j) := \left((1-x_c)^{1-\sigma} C_{q,t}^{\sigma}(j) + x_c^{1-\sigma} C_{e,t}^{\sigma}(j)\right)^{\frac{1}{\sigma}}$$

where $C_{e,t}(j)$ stands for the oil consumption of household j and

$$C_{q,t}(j) = \left(\int_0^1 C_{q,t}(i,j)^{\frac{\epsilon_p - 1}{\epsilon_p}}\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

represents the domestic consumption of household j, where $i \in [0,1]$ indexes the type of good, $\epsilon_p > 0$ is the elasticity of substitution between domestic goods and x_c a distribution parameter. Define $\sigma = \frac{\eta_c - 1}{\eta_c}$, thus η_c represents the elasticity of substitution between domestic goods and oil consumption. Note that when η_c is equal to one, the consumption flow collapses to being Cobb-Douglas in domestic and oil consumption; when η_c is equal to 0 one has a Leontief function between factors; and when η_c goes to $+\infty$ one has a linear function, meaning that the factors are perfect substitutes.

The j-th household allocates its expenditures among these different goods, i.e. it maximizes its consumption subject to its budget constraint $P_{c,t}C_t(j) = P_{q,t}C_{q,t}(j) + P_{e,t}C_{e,t}(j)$, where $P_{c,t}$ stands for the CPI price index.[7] Solving this problem one gets the following consumption demand functions:

$$C_{q,t}(j) = (1-x_c)\left(\frac{P_{q,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} C_t(j), \quad C_{e,t}(j) = x_c \left(\frac{P_{e,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} C_t(j) \tag{1}$$

[5]For more details on the model's construction, refer to the Appendix.

[6]Thus P_q could be interpreted as being the CPI without gasoline and other energy goods.

[7]Defined as the minimum expenditure required to buy one unit of C_t.

and the equation for the CPI index:

$$P_{c,t} = \left((1 - x_c) P_{q,t}^{\frac{\sigma}{\sigma-1}} + x_c P_{e,t}^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}} \tag{2}$$

On the other hand, a typical household j, seeks to maximize the following lifetime discounted utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t(j), L_t(j)) \right]$$

where $\beta \in (0,1)$ is the discount factor. Household j also holds an amount $B_t(j)$ of government bonds that pays a nominal short-run interest rate i_t, which is set by the Central Bank, lends capital $K_t(j)$ at price $P_{k,t}$ with real rental rate r_t^k and receives a nominal wage $W_t(j)$ for its work. Then, the j-th household's budget constraint is:

$$P_{c,t}C_t(j) + P_{k,t}I_t(j) + B_t(j) \leq (1 + i_{t-1})B_{t-1}(j) + W_t(j)L_t(j) + D_t + r_t^k P_{k,t}K_t(j) + T_t \tag{3}$$

where D_t is the profit of the firms in the economy,[8] T_t is the lump-sum transfers and $I_t(j)$ represents the investment of the j-th household. I will assume that the dynamics of capital accumulation follows:

$$I_t(j) := K_{t+1}(j) - (1 - \delta)K_t(j) \tag{4}$$

where $\delta \in (0,1)$ is the depreciation rate. The first order conditions with respect to $C_t(j)$, $B_t(j)$ and $K_{t+1}(j)$ are:[9]

$$\begin{aligned} C_t(j) :& U_C(C_t(j), L_t(j)) = \widetilde{\lambda}_t(j)P_{c,t} \\ B_t(j) :& \widetilde{\lambda}_t(j) = \beta \mathbb{E}_t \Big[(1 + i_t)\widetilde{\lambda}_{t+1}(j) \Big] \\ K_{t+1}(j) :& \widetilde{\lambda}_t(j)P_{k,t} = \beta \mathbb{E}_t \left[\widetilde{\lambda}_{t+1}(j) \big(r_{t+1}^k + 1 - \delta \big) P_{k,t+1} \right] \end{aligned} \tag{5}$$

In order to ensure the existence of a solution for the household problem, the following transversality condition (no Ponzi game) will be imposed:

$$\lim_{k \to +\infty} E_t \left[\frac{B_{t+k}(j)}{\prod_{s=0}^{t+k-1} (1 + i_{s-1})} \right] \geq 0, \quad \forall t, \forall j$$

Let me describe now the first order condition for labor. Assume that each one of the households supplies a differentiated labor service to the production sector, meaning that the intermediate firms look at each household's labor services, $L_t(j)$, as an imperfect substitute for the labor services of others households.

Following Erceg et al. (2000), I assume that there exists a perfectly competitive labor "packer", which could be interpreter as an employment agency, that combines household's

[8] I assume that each household owns an equal share of all firms and receives an aliquot share $D_t(j)$ of aggregate profits, i.e. the sum of dividends of all intermediate goods firms, so $D_t(j) = D_t := \int_0^1 D_t(i)di$ where i indexes the firms.

[9] $\widetilde{\lambda}_t(j)$ being the Lagrangian multiplier. Cf. Acurio-Vásconez et al. (2015) for details on the derivation

labor hours in the same proportion as firms would choose. So the labor used by intermediate good producers is supplied by this labor "aggregator" that follows the following CES production function:

$$L_t^d := \left(\int_0^1 L_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

where $\epsilon_w > 0$ is the constant elasticity of substitution among different types of labor. The "packer" maximizes profits subject to the demand of labor addressed to him, taking as given all differentiated labor wages $W_t(j)$ and the wage W_t, which is the price at which the "packer" sells one unit of labor index to the production sector. The first order condition of this problem yields the following equation:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} L_t^d, \quad \forall j \tag{6}$$

which represents the aggregated demand for labor hours of household j. The zero profit condition implied by perfect competition states that:

$$W_t L_t^d = \int_0^1 W_t(j) L_t(j) dj$$

Consequently one has the following level price:

$$W_t = \left(\int_0^1 W_t^{1-\epsilon_w}(j) dj\right)^{\frac{1}{1-\epsilon_w}} \tag{7}$$

One can interpret W_t as the aggregate wage index.

I will assume that households set their wages following a Calvo setting framework. In each period t, only a fraction $(1 - \theta_w)$ of households can re-optimize their nominal wage $(W_t(j) = W_t^o(j))$. The remaining part lets its wage as before $(W_t(j) = W_{t-1}(j))$. Each household that can change its wage will choose $W_t^o(j)$ in order to maximize:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k U\left(C_{t+k|t}(j), L_{t+k|t}(j)\right)\right]$$

under the same budget constraint describe in (3) and the labor demand defined in (6). Remark that $C_{t+k|t}(j)$ and $L_{t+k|t}(j)$ respectively denote the consumption and labor supply in period $t + k$ of a household that last resets its wage in period t. The solution of this problem yields:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k U_c\left(C_{t+k|t}, L_{t+k|t}\right) L_{t+k|t} \left[\frac{W_t^o}{P_{c,t+k}} - \mathcal{M}_w MRS_{t+k|t}\right]\right] = 0$$

where $\mathcal{M}_w = \frac{\epsilon_w}{\epsilon_w - 1}$ is the wage markup and $MRS_{t+k|t} := -\frac{U_L\left(C_{t+k|t}, L_{t+k|t}\right)}{U_C\left(C_{t+k|t}, L_{t+k|t}\right)}$ is the marginal rate of substitution between consumption and hours worked in period $t+k$ for the household that can reset its wage in t.

Finally, this assumption of Calvo setting wages, gives us the following "Aggregate wage relationship:"

$$W_t = \left(\theta_w W_{t+1}^{1-\epsilon_w} + (1 - \theta_w) W_t^{o\,1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}$$

2.2 Final Good Firm

There exists a continuum of intermediate goods indexed by $i \in [0,1]$ that are used in the production of the one final aggregate good (which will be the domestic consumption commodity). This firm has a CES production function given by:

$$Q_t := \left(\int_0^1 Q_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

For simplicity, I assume that no energy is needed in the production of the final good out of the intermediate goods.

Given all the intermediate good prices $(P_{q,t}(i))_{i \in [0,1]}$ and the final good price $P_{q,t}$, the final good firm chooses quantities of intermediate goods $(Q_t(i))_{i \in [0,1]}$ in order to maximize its profit. The solution of this problem gives:

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t$$

which is the demand for good i.

Remark that the production function of the final good firm is constant return to scale and this firm is perfect competitive, so at equilibrium the zero profit condition holds, and therefore one obtains the following equation for the price of the final aggregate good:

$$P_{q,t} = \left(\int_0^1 P_{q,t}(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}}.$$

2.3 Intermediate Goods Firms

There exists a continuum of monopolistically competitive intermediate goods producers indexed by $i \in [0,1]$ that produce a differentiated good. Each of them is represented by a nested CES production function involving oil, capital and labor, as in Hassler et al. (2012) in the following scheme:[10]

$$Q_t(i) := \Big(x_p (A_{E,t} E_t(i))^{\rho} + (1 - x_p)(A_{LK,t}(K_t(i)^{\alpha} L_t^d(i)^{1-\alpha}))^{\rho} \Big)^{1/\rho} \tag{8}$$

where $E_t(i)$ is the quantity of oil used, $K_t(i)$ is the capital rented and $L_t^d(i)$ is the amount of the "packed" labor input rented by the intermediate firm i. The variables $A_{E,t}$ and $A_{LK,t}$ represent respectively a measure of oil productivity and the total factor productivity (TFP). This last one measures the productivity of the combination of labor and capital. The "share" of capital in the composite factor is measured by $\alpha \in [0,1]$. Define $\rho = \frac{\eta_p - 1}{\eta_p}$, then η_p is the elasticity of substitution between the utilization of oil and the composite factor (of capital and labor). Finally, x_p is a distribution parameter. Similarly to the consumption flow, when η_p is equal to zero, then the composite factor and oil are complements; when η_p is equal to 1, then one recovers a Cobb-Douglas function of these two factors; and when η_p tends to $+\infty$, both factors are perfect substitutes. Both technologies processes are assumed to be $AR(1)$ processes:

$$log(A_{E,t}) = \rho_{ae} log(A_{E,t-1}) + e_{ae} \quad \text{and} \quad log(A_{LK,t}) = \rho_{alk} log(A_{LK,t-1}) + e_{alk}$$

[10] van-der Werf (2008) showed that the nesting structure that fits the data best is when labor and capital are combined first and then the composite factor is combined with oil. He showed as well that a nested combination of capital and labor is appropriate, however, for simplicity I follow Hassler et al. (2012) and Fouré et al. (2012), and assume that capital and labor are combined in a Cobb-Douglas function.

where $e_{ae} \sim \mathcal{N}(0, \sigma^2_{ae})$ and $e_{alk} \sim \mathcal{N}(0, \sigma^2_{alk})$.

Each firm maximizes its profit. I will study this problem in two stages: (1) each firm takes prices $P_{e,t}$, $P_{k,t}$, W_t, the real rental rate of capital r^k_t and demand $Q_t(i)$ as given, then it chooses quantities of oil $E_t(i)$, labor $L^d_t(i)$, and capital rent $K_t(i)$ in perfectly competitive factor markets in order to minimize cost. (2) Firm i chooses price $P_{q,t}(i)$ in order to maximize its profit. I will consider staggered prices *à la* Calvo. The first order conditions of the minimization problem give:

$$\begin{aligned}
E_t(i): &\quad P_{e,t} = \lambda_t(i) x_p A^\rho_{E,t} Q_t(i)^{1-\rho} E_t(i)^{\rho-1} \\
L^d_t(i): &\quad W_t = \lambda_t(i)(1-\alpha) Q_t(i)^{1-\rho}(1-x_p) A^\rho_{LK,t} K_t(i)^{\alpha\rho} L^d_t(i)^{(1-\alpha)\rho-1} \\
K_t(i): &\quad r^k_t P_{k,t} = \lambda_t(i)\alpha Q_t(i)^{1-\rho}(1-x_p) A^\rho_{LK,t} K_t(i)^{\alpha\rho-1} L^d_t(i)^{(1-\alpha)\rho}
\end{aligned}$$

where by definition

$$\begin{aligned}
\text{Marginal Cost } (MC_t) = \quad \lambda_t &:= \frac{P_{e,t}}{x_p A^\rho_{E,t} Q_t(i)^{1-\rho} E_t(i)^{\rho-1}} \\
&:= \frac{W_t}{(1-\alpha) Q_t(i)^{1-\rho}(1-x_p) A^\rho_{LK,t} K_t(i)^{\alpha\rho} L^d_t(i)^{(1-\alpha)\rho-1}} \\
&:= \frac{r^k_t P_{k,t}}{Q_t(i)^{1-\rho}\alpha(1-x_p) A^\rho_{LK,t} K_t(i)^{\alpha\rho-1} L^d_t(i)^{(1-\alpha)\rho}}
\end{aligned} \tag{9}$$

Because the intermediate firm technology is constant return to scale, it can be demonstrated that the marginal cost does not depend on i: all firms receive the same technology shock and all firms rent inputs at the same price.

In the second stage, intermediate firms choose the price that maximizes their profits. I consider that those prices are set under the same pricing scheme that households wages. In each period, a fraction $(1-\theta_p)$ of firms can change their prices ($P_{q,t}(i) = P^o_{q,t}(i)$), the remaining part lets their prices unchanged ($P_{q,t}(i) = P_{q,t-1}(i)$). Each firm that can reset its price will choose the same new one, so the choice of $P^o_{q,t}(i)$ will not depend on i. The first order condition of this problem gives us:[11]

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta^k_p d_{t,t+k} Q^o_{t+k|t} \left(P^o_{q,t} - \mathcal{M}_p mc^o_{t+k|t} \right) \right] = 0 \tag{10}$$

where $Q^o_{t+k|t} := \left(\frac{P^o_{q,t}}{P_{q,t+k}}\right)^{-\epsilon_p} Q_{t+k}$ for every $k \geq 0$, $MC^o_{t+k|t} := MC_{t+k}$, $d_{t,t+k}$ is the discount factor from date t to $t+k$ defined as:

$$d_{t,t+k}(j) := \frac{\beta^k U_C(C_{t+k}(j), L_{t+k}(j))}{U_C(C_t(j), L_t(j))} \frac{P_{c,t}}{P_{c,t+k}}$$

with $\mathcal{M}_p = \frac{\epsilon_p}{\epsilon_p - 1}$ being the price gross markup.

One has also the following "Aggregate price relationship:"

$$P^{1-\epsilon_p}_{q,t} = \left(\theta_p P^{1-\epsilon_p}_{q,t-1} + (1-\theta_p) {P^o_{q,t}}^{1-\epsilon_p} \right)$$

[11]Cf. Acurio-Vásconez et al. (2015) for details in derivation

2.4 GDP, Monetary Policy and Government

As in Acurio-Vásconez et al. (2015), I define real GDP (Y_t) as follows:

$$P_{c,t}Y_t = P_{q,t}Q_t - P_{e,t}E_t$$

Let $\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$ be the domestic inflation. Let us suppose that the Central Bank sets the nominal short-term interest rate by the following monetary policy:

$$1 + i_t = (1 + i_{t-1})^{\phi_i} \left(\frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right)^{1-\phi_i} \varepsilon_{i,t},$$

where Y represents the steady state of Y_t, $log(\varepsilon_{i,t}) = \rho_i log(\varepsilon_{i,t-1}) + e_{i,t}$ and $e_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$.[12]

Finally, the Government budget constraint is given by:

$$(1 + i_{t-1}) \int_0^1 B_{t-1}(j)dj + G_t = \int_0^1 B_t(j)dj + T_t,$$

where G_t stands for the nominal government spending. I assume that the real government spending $G_{r,t} = \frac{G_t}{Pq,t}$ is an exogenous process given by:

$$log(G_{r,t}) = (1 - \rho_g)(log(\omega Q)) + \rho_g log(G_{r,t-1}) + \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t}$$

where ω represents the share that the government takes from the domestic output (Q_t) for its own spending, Q represent the steady state of Q_t, and $e_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$ is a Gaussian white noise.

2.5 Real Prices and Stochastic Processes

All real variables are defined relative to the domestic prices P_q. Then the real price of oil, $S_{e,t}$ and the real price of capital, $S_{k,t}$, are given by

$$S_{e,t} := \frac{P_{e,t}}{P_{q,t}}, \quad S_{k,t} := \frac{P_{k,t}}{P_{q,t}}$$

I suppose that the real price of oil and capital are exogenous and each follows an $AR(1)$ process in the form:

$$log(S_{e,t}) := \rho_{se} log(S_{e,t-1}) + e_{e,t}, \quad log(S_{k,t}) := \rho_{sk} log(S_{k,t-1}) + e_{k,t}$$

where $e_{e,t} \sim \mathcal{N}(0, \sigma_e^2)$ and $e_{k,t} \sim \mathcal{N}(0, \sigma_k^2)$ are Gaussian white noises.[13]

3 Parameter Estimates

3.1 Setting

Aggregation and the steady state calculation are gathered in the Appendix as well as the log-linear version of the model. The time period represents a quarter. The model

[12] Remark that under this definition, the parameter ϕ_y measures the reaction of the Central Bank to the deviation of GDP with respect its steady state.

[13] Cf. Acurio-Vásconez et al. (2015) for an explanation in the assumption of an exogenous price of capital.

is estimated with Bayesian estimation techniques.[14] The period of estimation goes from 1984:Q1 to 2007:Q1. As explained in Acurio-Vásconez et al. (2015), the data set starts in 1984, because the well-know structural break occurred at this date and stops in 2007 because of the 2007-2008 crisis.

One could make the estimation with the same six quarterly macroeconomic U.S. time series used in Acurio-Vásconez et al. (2015), as observable variables: real GDP, real investment, hours worked, GDP deflator, the oil expenditure in production and the Federal Funds Rate. However, using just these six series and the six shocks previously described, the parameter η_c, which measures the oil's elasticity of substitution in consumption is not identifiable. This lack of identification on the estimation could lead to wrong results. In order to be able to identify all the parameters, besides the calibrated ones, I add two series to the six aforementioned: real domestic consumption and real wages;[15] and two ad-hoc shocks: one on the dynamic equation for wage inflation and one on the dynamic equation for price inflation.[16] These shocks could be interpreted as a wage markup and a price markup shock and are assumed to follow $ARMA(1,1)$ processes respectively of the form:

$$\varepsilon_{w,t} = \rho_w \varepsilon_{w,t-1} + e_{w,t} - \nu_p e_{w,t-1}, \quad \varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1}$$

where $e_{w,t} \sim \mathcal{N}(0, \sigma_w^2)$ and $e_{p,t} \sim \mathcal{N}(0, \sigma_p^2)$ are Gaussian white noises.

3.2 Prior Distribution of Parameters

Before estimating, five parameters are calibrated according to the literature. The discount factor, β, is set at 0.99, so that the riskless annual return is about 4 percent. The depreciation rate, δ, is calibrated at 0.025 which means a 10 percent of annual depreciation. The government spending output share, ω, is fixed at 18 percent. I calibrate ϵ_p and ϵ_w at 8, which give us a price and wage markup approximately equal to 1.14.[17] Those values are summarized in Table 1.

Table 1: Calibrated Parameters

β	δ	ϵ_p	ϵ_w	ω
0.99	0.025	8	8	0.18

I assume that elasticities of substitution, η_c and η_p, follow an inverse-gamma distribution with mean the estimated value in van-der Werf (2008) for U.S., which is equal to 0.54[18] and one degree of freedom. I use this prior for two reasons: First, in order to rely on positive values and allow the parameter to move from 0 to $+\infty$. Second, in order to

[14] All estimations are done with Dynare version 4.4.1 (Dynare (2011)). Two tests are available to check the stability of the sample generation using MCMC algorithm, implemented in Dynare: The MCMC diagnostic (Univariate convergence diagnostic, Brooks & Gelman (1998)) and a comparison between and within moments of multiple chains.

[15] The domestic consumption will be measured as being the real PCE minus the real PCE of Gasoline and other energy goods. Real wages are measured with the real hourly compensation series. An extended explanation of the series and its transformation can be found in the Appendix.

[16] In equations (26) and (37) of the log-linearized model.

[17] Those values are commonly used for the U.S economy. See for exemple Smets & Wouters (2007), Erceg et al. (2000) and references therein.

[18] No empirical work in my knowledge has try to estimate the value of oil's substitution in consumption, here η_c, for U.S. then I assume that it's prior is the same as for the elasticity of substitution in production.

concentrate the potability mass around the prior mean. Distribution parameters in the CES functions, x_c and x_p, have to be estimated as well. Following Cantore & Levine (2012), and as shown in the Appendix, at steady state, parameter x_c is equal to the share of oil consumed out of the total consumption expenditures. Then it is assumed to be Normal distributed with standard deviation 0.05, and mean 3 percent, which is the mean of the series generated dividing the nominal Personal Consumption Expenditures: Gasoline and other energy goods by the Personal Consumption Expenditure (PCE) in the observation period. As pointed out by Cantore & Levine (2012), in order to be able to estimate the distribution parameter in the production function, x_p, a renormalization is necessary.[19] As shown in the Appendix, the following relationship holds:

$$x_p = \alpha_e^{\frac{1}{\eta_p}} \left(\frac{\mathcal{M}_p S_e}{A_E} \right)^{\frac{\eta_p - 1}{\eta_p}}$$

where α_e stands for the steady state oil's output elasticity. On the other hand, one can also show that:[20]

$$\alpha_e = \frac{\mathcal{M}_p * \text{Oil's Cost share}}{1 + \text{Oil's Cost share}} \tag{11}$$

As pointed out by Kumhof & Muir (2014) the cost share of oil in the last years is around 3.5 percent, which gives us a steady state oil's output elasticity equals to 3.9 percent. Then I assume that oil's output elasticity, α_e, follows a Normal distribution with mean 3.9 percent and standard deviation 0.05. Remaining parameters' prior are taken as in Smets & Wouters (2007).

3.3 Estimation Results

Table 2 reports the prior and the posterior distributions for each parameter along with the mode, the mean and the 10 and 90 percentiles of the posterior distribution. In the same way, Table 3 presents the estimates of the prior and posterior distributions of shock processes. Note that for estimation proposes, the observable series have been multiplied by 100, then 1 represents a standard deviation of 1 percent.

There are some important issues to highlight out of the estimation. Regarding the estimation results of the main behavioral parameters summarized in Table 2, it turns out that the mean value for oil's elasticity of substitution is equal to 0.14 in the production sector and 0.51 in the consumption sector. These outcomes confirm the empirical results about the degree of oil substitution in U.S.: oil is poorly substitutable to other factors in both sectors, specially on the production sector. In addition, the estimated steady state for oil's output elasticity, α_e, is equal to 6 percent. It is worth highlighting again that the steady state of oil's output elasticity is larger than oil's cost share which is a result of equation (11). Other important results are the mean of the "share" parameter of capital, α, and the Calvo parameter in wages, θ_w, which are estimated at 0.36 and 0.78 respectively, fairly close to the literature. This degree of wage stickiness says that the average duration of a wage contract is somewhat less than a year. The degree of price stickiness is estimated to be 0.52, implying a duration of a price contract of roughly six months. The degree of price stickiness could seem too low with respect to what is found in the literature, but as explained in Blanchard & Riggi (2013), this could be a consequence of higher competition.

[19] Cf. Cantore & Levine (2012) for a more detailed explanation in this topic.

[20] Remark that equation (11) shows that in this model, oil's output elasticity is larger than the cost share.

Table 2: Prior and Posterior Distribution of Structural Parameters

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
"Share" parameter of capital	α	Normal(0.3,0.05)	0.3597	0.3607	0.3165	0.4020
Elast. substitution in production	η_p	Inv_Gamma(0.54,1)	0.1274	0.1387	0.1023	0.1729
Elast. substitution in consumption	η_c	Inv_Gamma(0.54,1)	0.2719	0.5056	0.1323	0.9805
Dist. parameter consumption	x_c	Normal(0.03,0.05)	0.0100	0.0286	0.0100	0.0503
Oil's output elasticity	α_e	Normal(0.039,0.05)	0.0680	0.0597	0.0302	0.0872
Inverse Frisch elasticity	ϕ	Normal(1.17,0.5)	1.2311	1.2009	0.5139	1.7613
Taylor rule response to inflation	ϕ_π	Normal(1.2,0.1)	1.0000	1.0301	1.0000	1.0734
Taylor rule response to GDP	ϕ_y	Normal(0.5,0.1)	0.8897	0.9011	0.7792	1.0199
Taylor rule inertia	ϕ_i	Beta(0.75,0.1)	0.4995	0.5027	0.4139	0.5906
Calvo price parameter	θ_p	Beta(0.5,0.2)	0.5000	0.5164	0.5000	0.5342
Calvo wage parameter	θ_w	Beta(0.5,0.2)	0.8006	0.7763	0.7250	0.8348

Turning to the estimated processes for the exogenous shock variables reported in Table 3, a number of observations are worth making. The oil price, the capital price, the monetary and the government spending processes are estimated to be the most persistent, with $AR(1)$ coefficients equal to 0.99, 0.95, 0.93, 0.92, respectively. Oil's productivity shock turns out to be more persistent than the total factor of productivity shock. Finally, the shocks with higher standard error are in descending order: the price of oil, oil productivity, government spending and wage markup process.

4 Simulations and Results

There are eight sources of potential exogenous shocks in this economy: the real price of oil, the real price of capital, government expenditure, monetary policy, both types of technologies and the wage and price markup. Once the model has been estimated, in this section, I study the reaction of the economy to a real oil price shock and its sensitivity to changes in some parameters.

4.1 What if Oil is Less Substitutable?

Figure 5 shows the impulse response functions (thereafter IRFS) of the economy to a one standard deviation increase in the real price of oil equal to 2 percent. As expected, an increase in the real price of oil leads to a contemporaneous increment in the marginal cost of intermediate firms, which produces a raise in domestic prices and so domestic inflation. Thus one also has an increase in the nominal interest rate and in the rental rate of capital. Due to the price upturn of oil goods, households decrease their consumption of oil. Consumers try to substitute oil as much as they can but low substitutability prevents them from doing that. Then one does neither observe a dramatic decrease in oil consumption nor a significant increase in domestic consumption. The need to satisfy a certain level of oil and domestic goods consumption obligates the households to seek for a new source of revenues, so that they supply more labor. The increase in labor supply in turn leads to a decrease in

Table 3: Prior and Posterior Distribution of Shock Parametres

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
Autoregressive parameters						
Real oil price	ρ_{se}	Beta(0.5,0.2)	0.9973	0.9965	0.9936	0.9994
Real capital price	ρ_{sk}	Beta(0.5,0.2)	0.9578	0.9565	0.9390	0.9741
Government	ρ_g	Beta(0.5,0.2)	0.9332	0.9221	0.8960	0.9512
Monetary	ρ_i	Beta(0.5,0.2)	0.9242	0.8910	0.8456	0.9521
Oil productivity	ρ_{ae}	Beta(0.5,0.2)	0.6878	0.6576	0.5069	0.8167
TFP	ρ_{alk}	Beta(0.5,0.2)	0.7378	0.7334	0.6761	0.7951
Oil Prod. in Gov	$\rho_{ae,g}$	Beta(0.5,0.2)	0.1624	0.1741	0.0488	0.3025
TFP in Gov.	$\rho_{alk,g}$	Beta(0.5,0.2)	0.6706	0.5944	0.2877	0.9056
Price markup1	ρ_p	Beta(0.5,0.2)	0.8067	0.7863	0.7163	0.8524
Wage markup1	ρ_w	Beta(0.5,0.2)	0.5777	0.4602	0.2281	0.7399
Price markup2	ν_p	Beta(0.5,0.2)	0.2509	0.2626	0.1273	0.3981
Wage markup2	ν_w	Beta(0.5,0.2)	0.6456	0.5531	0.3600	0.7538
Standard deviations						
Real oil price	σ_{se}	Inv_Gamma(1,2)	2.1690	2.0205	1.1780	2.9583
Real capital price	σ_{sk}	Inv_Gamma(1,2)	0.5643	0.5810	0.5056	0.6559
Government	σ_g	Inv_Gamma(1,2)	2.0302	2.0939	1.8126	2.3852
Monetary	σ_i	Inv_Gamma(1,2)	0.2503	0.2662	0.2143	0.3174
Oil productivity	σ_{ae}	Inv_Gamma(1,2)	2.0886	2.1714	1.8956	2.4325
TFP	σ_{alk}	Inv_Gamma(1,2)	0.5303	0.5329	0.4573	0.6066
Price markup	σ_p	Inv_Gamma(1,2)	0.4151	0.4012	0.3462	0.4523
Wage markup	σ_w	Inv_Gamma(1,2)	1.6860	1.5691	1.2699	1.8604

real wages. Finally, thanks to the Taylor rule used in the model, consumers anticipate an increment in inflation and so an increase in nominal interest rate that leads to a rise in the rental rate of capital. Thus, one observes a contemporaneous raise in investment, lasting just one quarter.

On the production side, since the price of oil has increased and oil is not substitutable to others factors, a larger amount of domestic goods has to be exported, in order to buy the same quantity of oil that is necessary in production. Therefore, an increase in domestic output shows up. Firms as well increase their labor demand, but not as much as the increase in labor supply, so that real wages decrease. Note that the surplus of output is being exported or invested but very little is consumed. It is important to point out that one observes a slightly and again very short-lived increase in oil demand. This is explained as follows: Because of the low substitutability of oil, an important increase in domestic production leads to an increase in oil importation as well.

Nevertheless, note that the magnitude of the initial increase in oil demand and its subsequent decline is weak, confirming the sluggish price elasticity of oil demand in the short run. For instance a shock in the real price of oil equal to 2 percent provokes an increase of 0.1 percent in oil demand, so that the price elasticity of oil is equal to $\frac{0.1}{2.02} = 0.05 < 1$. Finally, the increase in domestic output prevents the decrease in GDP.

One does not observe a decrease in GDP for to reasons. First, the increase in domestic production takes place partially to compensate the increase in oil price. Second, the Taylor rule applies. If, for instance, one assumes a Taylor rule without persistence by imposing $\phi_i = 0$ as shown in Figure 6, the rise in the rental rate of capital is quite smaller. Accordingly no increase in investment takes place, and one observes a decrease in oil importation rather than an increase, because even if firms still need to produce output to buy oil, there is not domestic demand for it, so that the increase in domestic production is smaller. Thus, when using a non persistent Taylor rule, one observes a smaller and not lasting decrease in GDP, because of the reduction of domestic output. This result enters the debate raised by Bernanke et al. (1997) about the role of monetary policy in the attenuation of oil shocks. As in Bernanke et al. (1997), I find that the adverse effects of an oil price shock on output are amplified when the response of the funds rate, i, is "stronger" ($\phi_i = 0$). However, I also find that the rise observed in GDP, using the baseline calibration, is possible if one allows for the increase in oil demand as well.

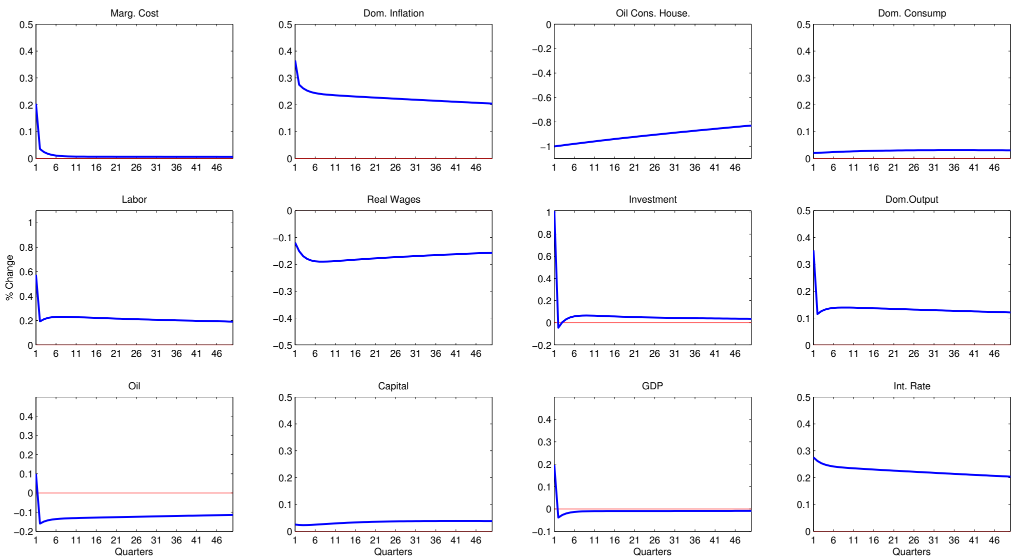

Figure 5: Response to one Standard Deviation Shock on Real Price of Oil

4.2 What if Nominal Wages are More Flexible?

Let us now study the sensitivity of the model to a change in the Calvo parameter of wages. The blue solid line in Figure 7 represents the IRFS of the model obtained with the estimated values, while the dashed green line represents the IRFS of the model where nominal wages have been flexibilized, changing the estimated value of θ_w from 0.78 to 0.10, ceteris paribus.

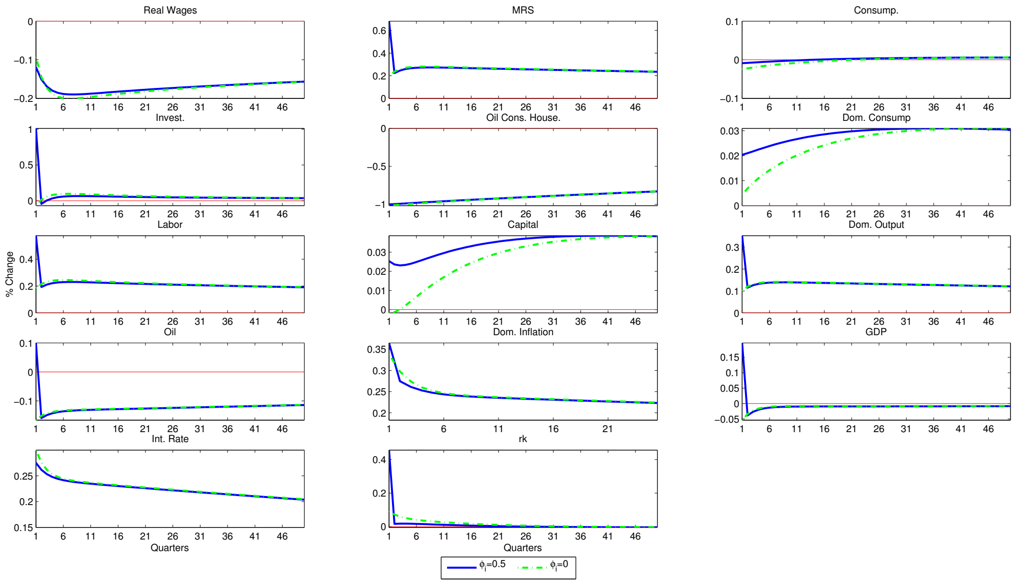

Figure 6: Response to one Standard Deviation Shock on Real Price of Oil—Persistence Comparison

Comparison of two models, namely, a model with interest rate persistence (solid blue line) and its counterpart without persistence (dashed green line)

This experiment is done in order to analyze one of the conclusions given in Blanchard & Galí (2009) and Blanchard & Riggi (2013) regarding the smoothness of the economy in face of an oil shock, being in their case, the flexibility of real wages.

One may note a different reaction of real wages in both cases. In the flexible case one observes a positive reaction of real wages while, in the baseline case, its reaction is negative. This bifurcation is explained as follows. As explained above, the low substitutability of oil forces domestic firms to produce more in order to compensate for the increase in oil prices, thus increasing labor demand. Households as well increase labor supply. In a rigid scenario, the increase in labor demand does not have a huge bearing on wages; then in order to pay for their oil's bills, households have to supply even more labor, which leads to a decrease in real wages as stated before. However, in a flexible scenario, wages react faster and stronger to an increase in labor demand. Then the increase in labor demand induces an increase in real wages and then a sharper rise of inflation. Accordingly, the Central Bank reacts more strongly and one observes a stronger increase in the rental rate of capital. Thus a worse tradeoff between investment and consumption takes place. Households prefer to invest rather than consume. With this in mind, remark that, when θ_w goes to 0, then the marginal rate of substitution, mrs_t, goes to $w_{r,t} - s_{c,t}$[21] and so the reaction of mrs_t is larger in the flexible case. As a consequence, households choose to work, but use the surplus to invest. However, this effect is again very short-lived. On the other hand, the stronger increase in investment produces a stronger increase in domestic output and again, because of the low substitutability, oil consumption increases further as well. Therefore the increase in production results in a stronger but not lasting increase in GDP.

[21] Lower-case letters represent the log-deviation of the variable from its steady state.

This experiment shows that in this model, and contrary to one of the conclusions given in Blanchard & Galí (2009) and Blanchard & Riggi (2013), the reduction of nominal wage rigidity would provoke an increase in real wages and, as a consequence, more inflation and lower consumption. It also shows that, as before, the stronger increase in domestic output observed in the flexible case provokes a rise in oil importation, which as argued in the introduction, should not be a problem as long as the domestic economy can import as much as oil as it wants. Something that might be problematic in our world where oil supply has entered a period of increased scarcity.

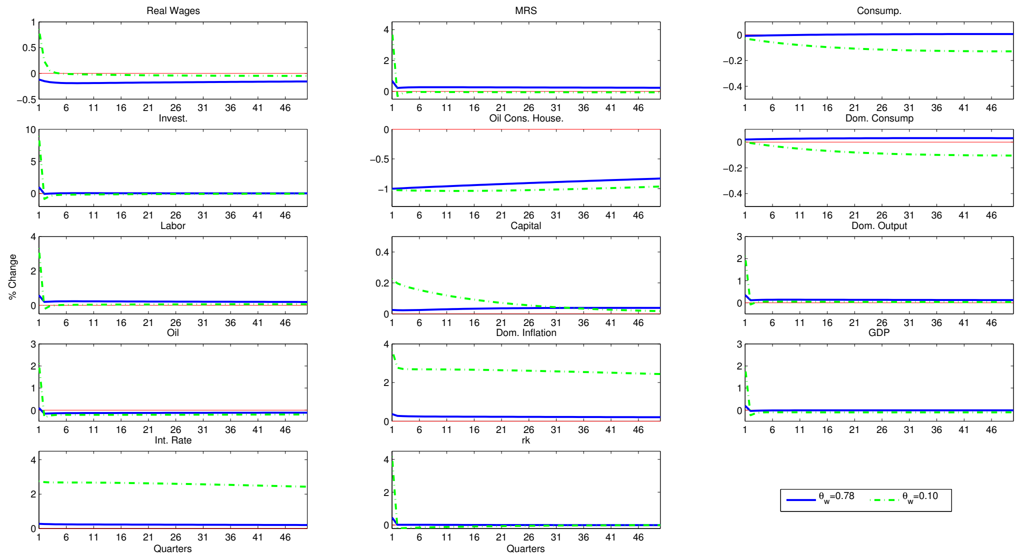

Figure 7: Response to one Standard Deviation Shock on Real Price of Oil—Wage Flexibility Comparison

Comparison of two models, namely, a model with "sticky" wages (solid blue line) and its counterpart with "flexible wages" (dashed green line)

5 Conclusion

In recent years, the inclusion of energy or oil into theoretical models has seen a rapid development, but still some questions and factors have not yet been taken into account. One of these factors is oil substitutability. To my knowledge, no DSGE model that includes energy or oil has been able to recover at the same time most of the effects that the 2000's oil shock generated in the U.S. economy. My assumption is that one possible reason for the lack of understanding is the assumption of a perfectly substitutable oil.

Using a DSGE model this factor is now taken into account through the introduction of oil in the production and in the consumption sides. Using Bayesian techniques and U.S data from 1984:Q1 to 2007:Q3, it can be proved that the elasticity of substitution in U.S between oil and other factors is weak, results that are in line with empirical studies on the subject.

On the other hand, once the low substitutability has been introduced, the model is able to recover four well-known stylized facts after an oil price shock in the 2000s': the absence of recession, coupled with a low level of inflation rate, a decrease in real wages and an low price elasticity of oil demand. It also shows that with a less persistent monetary policy, GDP could suffer a contemporaneous slight decrease after an oil shock.

Furthermore, the model also includes nominal price and wage rigidities. As it turns out a reduction of wage rigidity amplifies the response of the economy to an oil shock in terms of inflation and consumption and shows that the increase obtained in GDP is possible under the assumption that there exists the possibility to import as much as oil as needed.

Several extensions of this paper can be envisaged. First, one important factor has been left behind in this recent literature, namely unemployment. Thus, a natural extension could be the inclusion of oil in match and search models. Second, one strong hypothesis should be dampened, namely the assumption that oil is completely imported from a foreign country.

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A Appendix A: Model Derivation

A.1 Household's Maximization Problem

Each household j faces the following problem:

$$\begin{aligned}
&\max_{C_{q,t}(j),C_{e,t}(j)} P_{c,t}C_t(j),\\
&\text{subject to :} P_{q,t}C_{q,t}(j) + P_{e,t}C_{e,t}(j) = P_{c,t}C_t(j),\\
&\qquad C_t(j) = \left((1-x_c)^{1-\sigma}C_{q,t}^{\sigma} + x_c^{1-\sigma}C_{e,t}^{\sigma}\right)^{\frac{1}{\sigma}}
\end{aligned}$$

The first order condition with respect to $C_{q,t}(j)$ gives:

$$\begin{aligned}
P_{c,t}\left((1-x_c)^{1-\sigma}C_{q,t}^{\sigma}(j) + x_c^{1-\sigma}C_{e,t}^{\sigma}(j)\right)^{\frac{1}{\sigma}-1} C_{q,t}^{\sigma-1}(j) - P_{q,t} = 0\\
P_{c,t}C_t^{1-\sigma}(j)C_{q,t}^{\sigma-1}(1-x_c)^{1-\sigma} = P_{q,t}\\
C_{q,t}(j) = (1-x_c)\left(\frac{P_{q,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} C_t(j)
\end{aligned}$$

In the same way, the first order condition with respect to $C_{e,t}(j)$ gives:

$$C_{e,t}(j) = x_c\left(\frac{P_{e,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} C_t(j)$$

And so one has,

$$\begin{aligned}
P_{c,t}C_t(j) &= P_{q,t}C_{q,t}(j) + P_{e,t}C_{e,t}(j)\\
P_{c,t} &= (1-x_c)P_{q,t}\left(\frac{P_{q,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} + x_cP_{e,t}\left(\frac{P_{e,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}}\\
P_{c,t} &= \left((1-x_c)P_{q,t}^{\frac{\sigma}{\sigma-1}} + x_cP_{e,t}^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}
\end{aligned}$$

Given the description of household's problem, the Lagrangian function associated with it is:

$$\begin{aligned}
\mathcal{L}_0 = \sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\Big[&U(C_t(j),L_t(j)) - \widetilde{\lambda}(j)\left[P_{c,t}C_t(j) + P_{k,t}I_t(j)\right.\\
&\left.+B_t(j) + (1+i_{t-1})B_{t-1}(j) + W_t(j)L_t(j) + r_t^kP_{k,t}K_t(j) + D_t + T_t\right]\Big]
\end{aligned}$$

where the household maximizes over $C_t(j)$, $B_t(j)$, $K_{t+1}(j)$, $W_t(j)$, $L_t(j)$, and where $\widetilde{\lambda}_t(j)$ is the Lagrangian multiplier associated.

A.2 "Packer" Maximization Problem

The problem of the labor "packer" is

$$\begin{aligned}
&\max_{L_t(j)} \quad W_tL_t^d - \int_0^1 W_t(j)L_t(j)dj,\\
&\text{subject to :} L_t^d = \left(\int_0^1 L_t(j)^{\frac{\epsilon_w-1}{\epsilon_w}}dj\right)^{\frac{\epsilon_w}{\epsilon_w-1}}
\end{aligned}$$

The first order condition with respect to $L_t(j)$ yields:

$$W_t\left(\int_0^1 L_t(j)^{\frac{\epsilon_w-1}{\epsilon_w}}dj\right)^{\frac{\epsilon_w}{\epsilon_w-1}-1}L_t(j)^{\frac{\epsilon_w}{\epsilon_w-1}-1}-W_t(j)=0$$

$$L_t(j)=\left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w}L_t^d$$

By the zero profit condition one also has:

$$W_tL_t^d=\int_0^1 W_t(j)L_t(j)dj$$

Replacing the aggregated demand in this last equation one gets:

$$W_tL_t^d=\int_0^1 W_t(j)(\frac{W_t(j)}{W_t})^{-\epsilon_w}djL_t^d$$

$$W_t=\left(\int_0^1 W_t(j)^{1-\epsilon_w}dj\right)^{\frac{1}{1-\epsilon_w}}$$

As for the optimal wage setting, let us assume that in each period t, only a fraction $(1-\theta_w)$ of households can re-optimize their nominal wage $(W_t(j)=W_t^o(j))$. The remaining part lets its wage as before $(W_t(j)=W_{t-1}(j))$. Given a date t, suppose that the j-th household has to chose the wage $W_t^o(j)$. The household j does not care about future dates where it can re-optimize but only to the state where it cannot with probability θ_w^k, for all $k\geq 0$. Each household that can change its wage will chose $W_t^o(j)$ in order to maximize:

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta_w)^kU\left(C_{t+k|t}(j),L_{t+k|t}(j)\right)\right]$$

under the same budget constrain described in (3) and subject to:

$$L_{t+k|t}(j)=\left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w}L_{t+k}^d \tag{12}$$

Then the problem of household j is:

$$\begin{aligned}
&\max_{W_t(j)}\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta_w)^kU\left(C_{t+k|t}(j),L_{t+k|t}(j)\right)\right],\\
&\text{subject to}: L_{t+k|t}(j)=\left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w}L_{t+k}^d\\
&\qquad P_{c,t}C_t(j)+P_{k,t}I_t(j)+B_t\\
&\qquad\quad\leq(1+i_{t-1})B_{t-1}(j)+W_t(j)L_t(j)+D_t+r_t^kP_{k,t}K_t(j)+T_t
\end{aligned}$$

Therefore, the relevant part of the Lagrangian for the j-th household is:

$$\mathcal{L}_0^w=\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta_w)^k\left[-\frac{L_{t+k|t}^{1+\phi}(j)}{1+\phi}-\widetilde{\lambda}_{t+k}(j)W_t(j)L_{t+k|t}(j)\right]\right]$$

substituting (12) in this last equation one has:

$$\mathcal{L}_0^w = \mathbb{E}_t\Big[\sum_{k=0}^{\infty}(\beta\theta_w)^k\big[-\frac{1}{1+\phi}\left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w(1+\phi)}(L_{t+k}^d)^{1+\phi} - \widetilde{\lambda}_{t+k}(j)W_t(j)\left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w}L_{t+k}^d\big]\Big]$$

So the first order condition with respect to $W_t(j)$ yields:

$$\mathbb{E}_t\Big[\sum_{k=0}^{\infty}(\beta\theta_w)^k\big[\epsilon_w\frac{W_t^o(j)^{-\epsilon_w(1-\phi)-1}}{W_{t+k}^{-\epsilon_w}}(L_t^d)^{1+\phi} + \widetilde{\lambda}_{t+k}(j)W_t^o(j)\left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w}L_{t+k}^d\big]\Big] = 0$$

$$\mathbb{E}_t\Big[\sum_{k=0}^{\infty}(\beta\theta_w)^k\big[\epsilon_w W_t^o(j)^{-1}L_{t+k|t}^{1+\phi}(j) + \widetilde{\lambda}_{t+k}(j)W_t(j)(1-\epsilon_w)L_{t+k|t}(j)\big]\Big] = 0$$

Using equation (5) one then has:

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta_w)^k\left[\frac{W_t^o(j)}{P_{c,t+k}}U_c\left(C_{t+k|t}(j), L_{t+k|t}(j)\right)L_{t+k|t}(j) - \mathcal{M}_w L_{t+k|t}^{1+\phi}(j)\right]\right]$$

where $C_{t+k|t}(j)$ and $L_{t+k|t}(j)$ respectively denote consumption and labor supply in period $t+k$ of a household that last resets its wage in period t.

Note that, if one writes $MRS_{t+k|t} := -\frac{U_L\left(C_{t+k|t},L_{t+k|t}\right)}{U_C\left(C_{t+k|t},L_{t+k|t}\right)}$ the marginal rate of substitution between consumption and leisure in period $t+k$ for the household that can reset its wage in t, this last condition can be rewritten as:

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta_w)^k U_c\left(C_{t+k|t}, L_{t+k|t}\right)L_{t+k|t}\left[\frac{W_t^o}{P_{c,t+k}} - \mathcal{M}_w MRS_{t+k|t}\right]\right] = 0$$

and so in the limiting case of full wage flexibility ($\theta_w = 0$), one has

$$\frac{W_t^o}{P_{c,t}} = \frac{W_t}{P_{c,t}} = \mathcal{M}_w MRS_{t|t}$$

That is why one can interpret $\mathcal{M}_w$ as being the desired gross wage markup. Then using equation (7) one has

$$\begin{aligned} W_t^{1-\epsilon_w} &= \int_0^1 W_t(j)^{1-\epsilon_w}dj \\ &= \int_{\text{can not reset wages}} W_t(j)^{1-\epsilon_w}dj + \int_{\text{set wages optimally}} W_t(j)^{1-\epsilon_w}dj \\ &= \theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^o)^{1-\epsilon_w}. \end{aligned}$$

A.3 Final Good Producer Problem's maximization

The problem of the Final Good Producer is:

$$\max_{Q_t(\cdot)} P_{q,t}Q_t - \int_0^1 P_{q,t}(i)Q_t(i)di$$

$$\text{subject to} \ : \quad Q_t = \Big(\int_0^1 Q_t(i)^{\frac{\epsilon_p-1}{\epsilon_p}}di\Big)^{\frac{\epsilon_p}{\epsilon_p-1}}$$

Solving this problem one obtains (Cf. Acurio-Vásconez et al. (2015) for derivation)

$$Q_t(i) = \Big(\frac{P_{q,t}(i)}{P_{q,t}}\Big)^{-\epsilon_p} Q_t \tag{13}$$

which is the demand of good i.

A.4 Intermediate Firms Relations

The cost minimization problem of firm i is:

$$\begin{aligned}
\text{minimize cost: } & P_{e,t}E_t(i) + W_t L_t^d(i) + r_t^k P_{k,t} K_t(i) \\
\text{subject to } & E_t(i), L_t^d(i), K_t(i) \geq 0, \\
& \Big(x_p A_{E,t}^{\rho} E_t(i)^{\rho} + (1-x_p) A_{LK,t}^{\rho} (K_t(i)^{\alpha} L_t^d(i)^{1-\alpha})^{\rho}\Big)^{1/\rho} \geq Q_t(i)
\end{aligned}$$

One has the following Lagrangian associated to this problem:

$$\begin{aligned}
\mathcal{L}_0 := & P_{e,t}E_t(i) + W_t L_t^d(i) + r_t^k P_{k,t} K_t(i) \\
-\lambda_t(i)\Bigg(&\Big(x_p A_{E,t}^{\rho} E_t(i)^{\rho} + (1-x_p) A_{LK,t}^{\rho} (K_t(i)^{\alpha} L_t^d(i)^{1-\alpha})^{\rho}\Big)^{1/\rho} - Q_t(i)\Bigg)
\end{aligned}$$

which yields the first order conditions expressed on the paper.

Defining:

$$\text{marginal cost } (MC_t:) \quad \lambda_t(i) := \frac{\frac{d(cost)}{d(worker)}}{\frac{d(output)}{d(worker)}} = \frac{\frac{d(cost)}{d(capital)}}{\frac{d(output)}{d(capital)}} = \frac{\frac{d(cost)}{d(energy)}}{\frac{d(output)}{d(energy)}}.$$

the relation (9) is determined. One also has:

$$\begin{aligned}
cost\,(Q_t(i)) := & P_{e,t}E_t(i) + W_t L_t^d(i) + r_t^k P_{k,t} K_t(i) \\
= & \lambda_t(i) x_p A_{E,t}^{\rho} Q_t(i)^{1-\rho} E_t(i)^{\rho} + \lambda_t(i)(1-\alpha) Q_t(i)^{1-\rho}(1-x_p) A_{LK,t}^{\rho} K_t(i)^{\alpha\rho} L_t^d(i)^{(1-\alpha)\rho} \\
& + \lambda_t(i) \alpha Q_t(i)^{1-\rho}(1-x_p) A_{LK,t}^{\rho} K_t(i)^{\alpha\rho} L_t^d(i)^{(1-\alpha)\rho} \\
= & \lambda_t(i) Q_t(i)^{1-\rho}\Big(x_p A_{E,t}^{\rho} E_t(i)^{\rho} + (1-x_p) A_{LK,t}^{\rho}\Big(K_t(i)^{\alpha} L_t^d(i)^{1-\alpha}\Big)^{\rho}\Big) \\
= & \lambda_t(i) Q_t(i)
\end{aligned}$$

In the other hand:

$$\begin{aligned}
Q_t(i)^{\rho} = & x_p A_{E,t}^{\rho} E_t(i)^{\rho} + (1-x_p) A_{LK,t}^{\rho} (K_t(i)^{\alpha} L_t^d(i)^{1-\alpha})^{\rho} \\
= & x_p A_{E,t}^{\rho}\left(\frac{P_{e,t}}{\lambda_t(i) Q_t(i)^{1-\rho} x_p A_{E,t}^{\rho}}\right)^{\frac{\rho}{\rho-1}} + (1-x_p) A_{LK,t}^{\rho}\left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho} L_t^d(i)^{\rho} \\
= & x_p A_{E,t}^{\rho}\left(\frac{P_{e,t}}{\lambda_t(i) Q_t(i)^{1-\rho} x_p A_{E,t}^{\rho}}\right)^{\frac{\rho}{\rho-1}} \\
& + (1-x_p) A_{LK,t}^{\rho}\left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho}\left(\frac{W_t L_t^d(i)^{\alpha\rho}}{\lambda_t(i) Q_t(i)^{1-\rho}(1-\alpha) A_{LK,t}^{\rho} K_t(i)^{\alpha\rho}}\right)^{\frac{\rho}{\rho-1}}
\end{aligned}$$

Combining the first order conditions for $L_t^d(i)$ and $K_t(i)$ one has:

$$\frac{W_t}{(1-\alpha)L_t^d(i)^{-1}} = \frac{r_t^k P_{k,t}}{\alpha K_t(i)^{-1}}$$

which yields to:

$$\frac{K_t(i)}{L_t^d(i)} = \frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}$$

Then:

$$\begin{aligned}
Q_t(i)^\rho =& x_p A_{E,t}^\rho \left(\frac{P_{e,t}}{\lambda_t(i)Q_t(i)^{1-\rho}x_p A_{E,t}^\rho}\right)^{\frac{\rho}{\rho-1}} + \\
&+ (1-x_p)A_{LK,t}^\rho \left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho}\left(\frac{L_t^d(i)}{K_t(i)}\right)^{\alpha\rho\frac{\rho}{\rho-1}}\left(\frac{W_t}{\lambda_t(i)Q_t(i)^{1-\rho}(1-\alpha)(1-x_p)A_{LK,t}^\rho}\right)^{\frac{\rho}{\rho-1}} \\
=& x_p A_{E,t}^\rho \left(\frac{P_{e,t}}{\lambda_t(i)Q_t(i)^{1-\rho}x_p A_{E,t}^\rho}\right)^{\frac{\rho}{\rho-1}} \\
&+ (1-x_p)A_{LK,t}^\rho \left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\frac{-\alpha\rho}{\rho-1}}\left(\frac{W_t}{\lambda_t(i)Q_t(i)^{1-\rho}(1-\alpha)(1-x_p)A_{LK,t}^\rho}\right)^{\frac{\rho}{\rho-1}}
\end{aligned}$$

$$\begin{aligned}
=& \left(\frac{1}{\lambda_t(i)Q_t(i)^{1-\rho}}\right)^{\frac{\rho}{\rho-1}}\left(x_p A_{E,t}^\rho\left(\frac{P_{e,t}}{x_p A_{E,t}^\rho}\right)^{\frac{\rho}{\rho-1}} + (1-x_p)A_{LK,t}^\rho\left(\frac{W_t}{1-\alpha}\right)^{\frac{\rho}{\rho-1}-\frac{\alpha\rho}{\rho-1}}\right. \\
&\left.((1-x_p)A_{LK,t}^\rho)^{\frac{-\rho}{\rho-1}}\left(\frac{\alpha}{r_t^k P_{k,t}}\right)^{\frac{-\alpha\rho}{\rho-1}}\right)
\end{aligned}$$

$$\lambda_t(i)^{\frac{\rho}{\rho-1}} = \left(\frac{1}{x_p A_{E,t}^\rho}\right)^{\frac{1}{\rho-1}} P_{e,t}^{\frac{\rho}{\rho-1}} + \left(\frac{1}{(1-x_p)A_{LK,t}^\rho}\right)^{\frac{1}{\rho-1}}\left(\frac{W_t}{1-\alpha}\right)^{\frac{(1-\alpha)\rho}{\rho-1}}\left(\frac{\alpha}{r_t^k P_{k,t}}\right)^{\frac{-\alpha\rho}{\rho-1}}$$

$$\lambda_t(i) = \left(\left(\frac{P_{e,t}^\rho}{x_p A_{E,t}^\rho}\right)^{\frac{1}{\rho-1}} + \left(\frac{1}{(1-x_p)A_{LK,t}^\rho}\right)^{\frac{1}{\rho-1}}\left(\frac{W_t}{1-\alpha}\right)^{\frac{(1-\alpha)\rho}{\rho-1}}\left(\frac{\alpha}{r_t^k P_{k,t}}\right)^{\frac{-\alpha\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}$$

Then $\lambda(i)$ does not depend on i.

As for the price maximization, at date t, denote $Q_{t+k|t}(i)$ the output at date $t+k$ for a firm i that last resets its price in period t. As in the case of wages each firm that can reset its price will chose the same one, so the choice of $P_{q,t}^o(i)$ will not depend on i. The firm only cares about the future states in which it cannot re-optimize. Therefore the problem of the i-th firm is:

$$\max_{P_{q,t}(i)} \mathbb{E}_t\left[\sum_{k=0}^{\infty}\theta^k d_{t,t+k}\left[P_{q,t}(i)Q_{t,t+k}(i) - cost(Q_{t,t+k}(i))\right]\right]$$

$$\text{subject to} \quad Q_{t,t+k}(i) = \left(\frac{P_{q,t}(i)}{P_{q,t+k}}\right)^{-\epsilon_p} Q_{t+k}, \quad \forall k \geq 0.$$

The first order condition of this problem yields:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta_p^k d_{t,t+k} Q^o_{t+k|t} \left(P^o_{q,t} - \mathcal{M}_p mc^o_{t+k|t} \right) \right] = 0$$

In the limiting case of full flexibility ($\theta_p = 0$) equation (10) gives:

$$P_{q,t} = P^o_{q,t} = \mathcal{M}_p MC_t$$

that is why one can interpret $\mathcal{M}_p$ as the desired price markup.

A.5 Aggregation

By market clearing conditions one has:

$$K_t = \int_0^1 K_t(i)di, \quad L_t^d = \int_0^1 L_t^d(i)di, \quad E_t = \int_0^1 E_t(i)di,$$

Equation (13) yields:

$$\left(\left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t \right)^{\rho} = Q_t^{\rho}(i) = x_p A^{\rho}_{E,t} E_t(i)^{\rho} + (1-x_p) A^{\rho}_{LK} (K_t(i)^{\alpha} L_t^d(i)^{1-\alpha})^{\rho}$$

In the other hand using the equivalence from the first order conditions for the firms one has:

$$E_t(i)^{\rho-1} = \left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^{\rho}_{LK,t}}{W_t x_p A^{\rho}_{E,t}} \right) \left(\frac{K_t(i)}{L_t^d(i)} \right)^{\alpha\rho} L_t^d(i)^{\rho-1} \tag{14}$$

Then

$$\begin{aligned}
\left(\left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t \right)^{\rho} =& x_p A^{\rho}_{E,t} \left(\left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^{\rho}_{LK,t}}{W_t x_p A^{\rho}_{E,t}} \right) \left(\frac{K_t(i)}{L_t^d(i)} \right)^{\alpha\rho} (L_t^d(i))^{\rho-1} \right)^{\frac{\rho}{\rho-1}} \\
& + (1-x_p) A^{\rho}_{LK,t} \left(\frac{K_t(i)}{L_t^d(i)} \right)^{\alpha\rho} L_t^d(i)^{\rho} \\
=& \Big[x_p A^{\rho}_{E,t} \left(\left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^{\rho}_{LK,t}}{W_t x_p A^{\rho}_{E,t}} \right) \left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)} \right)^{\alpha\rho} \right)^{\frac{\rho}{\rho-1}} + \\
& (1-x_p) A^{\rho}_{LK,t} \left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)} \right)^{\alpha\rho} \Big] L_t^d(i)^{\rho}
\end{aligned}$$

Let us note:

$$\begin{aligned}
\tilde{F}_t = \Bigg[& x_p A^{\rho}_{E,t} \left(\left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^{\rho}_{LK,t}}{W_t x_p A^{\rho}_{E,t}} \right) \left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)} \right)^{\alpha\rho} \right)^{\frac{\rho}{\rho-1}} + \\
& (1-x_p) A^{\rho}_{LK,t} \left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)} \right)^{\alpha\rho} \Bigg]
\end{aligned}$$

One has:

$$\left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t = \tilde{F}_t^{\frac{1}{\rho}} L_t^d(i)$$

Taking the integral at both sides and then taking power ρ one has:

$$\left(\int_0^1 \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} diQ_t\right)^{\rho} = \tilde{F}_t(L_t^d)^{\rho} \tag{15}$$

In the other hand, taking the integral in both sides of (14) and then taking power ρ one has:

$$E_t^{\rho} = \left[\frac{P_{e,t}(1-\alpha)(1-x_p)A_{LK,t}^{\rho}}{W_t x_p A_{E,t}^{\rho}}\left(\int_0^1 \frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho}\right]^{\frac{1}{\rho-1}}(L_t^d)^{\rho}$$

$$E_t^{\rho} = \left[\frac{P_{e,t}(1-\alpha)(1-x_p)A_{LK,t}^{\rho}}{W_t x_p A_{E,t}^{\rho}}\left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\alpha\rho}\right]^{\frac{1}{\rho-1}}(L_t^d)^{\rho}$$

One also has:

$$\frac{W_t L_t^d}{1-\alpha} = \frac{r_t^k P_{k,t} K_t}{\alpha}$$

replacing these two last equations in (15) one finally gets:

$$\int_0^1 \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} diQ_t = \left(x_p A_{E,t}^{\rho} E_t^{\rho} + (1-x_p)A_{LK,t}^{\rho}(K_t^{\alpha}(L_t^d)^{1-\alpha})^{\rho}\right)^{1/\rho} \tag{16}$$

Define now

$$v_{p,t} := \int_0^1 \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} di$$

By by Calvo price setting one has:

$$\begin{aligned}
v_{p,t} =& P_{q,t}^{\epsilon_p}\int_0^1 P_{q,t}(i)^{-\epsilon_p} di \\
=& P_{q,t}^{\epsilon_p}\left[\int_{\text{no set}} P_{q,t-1}(i)^{-\epsilon_p} di + \int_{\text{set}} P_{q,t}^{o}(i)^{-\epsilon_p} di\right] \\
=& \theta_p \Pi_{q,t}^{\epsilon_p} v_{p,t-1} + (1-\theta_p)\left(\frac{P_{q,t}^o}{P_{q,t}}\right)^{-\epsilon_p} \qquad (17)
\end{aligned}$$

Taking integral on both sides of equation (6) one gets

$$\int_0^1 L_t(j)dj = L_t = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} L_t^d dj \tag{18}$$

Define

$$v_{w,t} := \int_0^1 \left(\frac{W_{q,t}(j)}{W_t}\right)^{-\epsilon_w} di$$

Hence

$$L_t = v_{w,t} L_t^d$$

Then by Calvo setting, one gets

$$v_{w,t} = \theta_w \left(\frac{W_t}{W_{t-1}}\right)^{\epsilon_w} v_{t-1}^w + (1-\theta_w)\left(\frac{W_t^o}{W_t}\right)^{-\epsilon_w} \tag{19}$$

A.6 Equilibrium

At equilibrium:

1. Households maximize their utility. I assume complete markets, separable utility in labor, and I consider a symmetric equilibrium where $C_t(j) = C_t$, $C_{q,t}(j) = C_{q,t}$, $C_{e,t}(j) = C_{e,t}$, $K_{t+1}(j) = K_{t+1}$, $\lambda_t(j) = \lambda_t$. Therefore the first order conditions associated to household's problem become:

$$\begin{aligned} U_c(C_t, L_t) =& \widetilde{\lambda}_t P_{c,t} \\ \widetilde{\lambda}_t =& \beta \mathbb{E}_t \left[(1+i_t)\widetilde{\lambda}_{t+1} \right] \\ \widetilde{\lambda}_t =& \beta \mathbb{E}_t \left[\widetilde{\lambda}_{t+1} (r^k_{t+k} + (1-\delta)\frac{P_{k,t+1}}{P_{k,t}}) \right] \end{aligned}$$

   The profit at equilibrium is:

$$D_t = P_{q,t}Q_t - W_t L^d_t - r^k_t P_{k,t} K_t - P_{e,t} E_t$$

   so the budget constraint becomes:

$$P_{c,t}C_t + P_{k,t}I_t + G_t = P_{q,t}Q_t - P_{e,t}E_t$$

2. All markets clears.

3. Firms maximize their profits:

$$\begin{aligned} \frac{P_{e,t}}{x_p A^\rho_{E,t} Q^{1-\rho}_t E^{\rho-1}_t} =& \frac{W_t}{(1-\alpha)Q^{1-\rho}_t (1-x_p) A^\rho_{LK,t} K^{\alpha\rho}_t (L^d_t)^{(1-\alpha)\rho-1}} \\ =& \frac{r^k_t P_{k,t}}{Q^{1-\rho}_t \alpha (1-x_p) A^\rho_{LK,t} K^{\alpha\rho-1}_t (L^d_t)^{(1-\alpha)\rho}} \end{aligned}$$

4. Government budget constrain is fulfilled:

$$(1+i_{t-1}) \int_0^1 B_{t-1}(i)di + G_t = \int_0^1 B_t(i)di + T_t$$

5. Equations (1), (2), (4), (16), (17), (18) and (19).

A.7 Steady State

Let Z denote the steady state of variable Z_t. The subscript r represents a nominal variable that has been deflated by the domestic price $P_{q,t}$ in order to represent a real variable.

Households, Government Constraint and Investment

$$\begin{aligned} i =& \frac{1}{\beta} - 1, \quad r^k = \frac{1}{\beta} - 1 + \delta, \quad d = \beta \\ G_r =& \omega Q, \quad I = \delta K \end{aligned}$$

Firms

$$v_p = 1, \quad Q^o = Q, \quad P_q^o = P_q, \quad MC_r = \frac{1}{\mathcal{M}_p}$$
$$Q = \left[x_p A_{E,t}^\rho + (1-x_p) A_{LK}^\rho (K^\alpha L^{1-\alpha})^\rho\right]^{\frac{1}{\rho}}$$

$$\begin{aligned}\frac{S_e}{x_p A_E^\rho Q^{1-\rho} E^{\rho-1}} =& \frac{W_r}{(1-\alpha) Q^{1-\rho}(1-x_p) A_{LK}^\rho K^{\alpha\rho} (L^d)^{(1-\alpha)\rho-1}} \\ =& \frac{r^k S_k}{Q^{1-\rho}\alpha(1-x_p)A_{LK}^\rho K^{\alpha\rho-1}(L^d)^{(1-\alpha)\rho}}\end{aligned}$$

In the other hand,

$$\begin{aligned}MC =& \left[\left(\frac{Pe^\rho}{x_p A_E^\rho}\right)^{\frac{1}{\rho-1}} + \left(\frac{1}{(1-x_p)A_{LK}^\rho}\right)^{\frac{1}{\rho-1}} \left(\frac{W}{1-\alpha}\right)^{\frac{(1-\alpha)\rho}{\rho-1}} \left(\frac{\alpha}{r^k P_k}\right)^{\frac{-\alpha\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \\ =& \Big[\left(\frac{Pe^\rho}{x_p A_E^\rho}\right)^{\frac{1}{\rho-1}} + \left(\frac{1}{(1-x_p)A_{LK}^\rho}\right)^{\frac{1}{\rho-1}} \left(\frac{P_e(1-x_p)A_{LK}^\rho (K^\alpha (L^d)^{1-\alpha})^\rho}{x_p A_E^\rho E^{\rho-1} L^d}\right)^{\frac{(1-\alpha)\rho}{\rho-1}} \\ & \cdot \left(\frac{Pe(1-x_p)A_{LK}^\rho (K^\alpha (L^d)^{1-\alpha})^\rho}{x_p A_E^\rho E^{\rho-1}}\right)^{\frac{(1-\alpha)\rho}{\rho-1}+\frac{\alpha\rho}{\rho-1}} \left(\frac{1}{L^d}\right)^{\frac{(1-\alpha)\rho}{\rho-1}} \left(\frac{1}{K}\right)^{\frac{\alpha\rho}{\rho-1}}\Big]^{\frac{\rho-1}{\rho}} \\ =& \Bigg[\left(\frac{P_e^\rho}{x_p A_E^\rho}\right)^{\frac{1}{\rho-1}} + \\ & \left(\frac{1}{(1-x_p)A_{LK}^\rho}\right)^{\frac{1}{\rho-1}} \left(\frac{P_e(1-x_p)A_{LK}^\rho (K^\alpha (L^d)^{1-\alpha})^\rho}{x_p A_E^\rho E^{\rho-1}}\right)^{\frac{\rho}{\rho-1}} \left(\frac{1}{L^d}\right)^{\frac{(1-\alpha)\rho}{\rho-1}} \left(\frac{1}{K}\right)^{\frac{\alpha\rho}{\rho-1}}\Bigg]^{\frac{\rho-1}{\rho}}\end{aligned}$$

$$\begin{aligned}=& \left[x_p A_E^\rho + \left(\frac{1}{(1-x_p)A_{LK}^\rho}\right)^{-1} + \left(K^\alpha (L^d)^{1-\alpha}\right)^{\frac{\rho^2}{\rho-1}} \frac{1}{E^\rho} \left(\frac{1}{K^\alpha (L^d)^{1-\alpha}}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \left(\frac{P_e}{x_p A_E^\rho}\right) \\ =& \left[x_p A_E^\rho + E^{-\rho}\left(Q^\rho - x_p A_E^\rho E^\rho\right)\right]^{\frac{\rho-1}{\rho}} \frac{P_e}{x_p A_E^\rho} \\ =& (E^{-1}Q)^{\rho-1} \frac{P_e}{x_p A_E^\rho}\end{aligned}$$

Then

$$\frac{1}{\mathcal{M}_p} = \left(\frac{Q}{E}\right)^{\rho-1} \frac{S_e}{x_p A_E^\rho}$$

Let us note $a := \left(\frac{\mathcal{M}_p S_e}{x_p A_E^\rho}\right)$.

Budget Constraint

From the equation of budget constraint one has

$$\begin{aligned}
P_cC =& P_qQ - P_eE - \delta P_kK - \omega P_qQ \\
=& P_qQ - P_eE - \frac{P_e\alpha(1-x_p)A_{LK}^\rho(K^\alpha(L^d)^{1-\alpha})^\rho\delta}{x_pA_E^\rho E^{\rho-1}r^k} - \omega P_qQ \\
=& P_qQ - P_eE - \frac{\alpha P_e\delta}{x_pA_E^\rho E^{\rho-1}r^k}(Q^\rho - x_pA_E^\rho E^\rho) - \omega P_qQ \\
=& P_qQ - P_eE - \frac{\delta\alpha P_e}{x_pA_E^\rho r^k}(E^{1-\rho}Q^\rho - x_pA_E^\rho E) - \omega P_qQ \\
=& P_qQ - P_ea^{\frac{1}{\rho-1}}Q - \frac{\delta\alpha P_e}{x_pA_E^\rho r^k}((a^{\frac{1}{\rho-1}}Q)^{1-\rho}Q^\rho - x_pA_E^\rho a^{\frac{1}{\rho-1}}Q) - \omega P_qQ \\
C =& \frac{P_q}{P_c}\left[1 - a^{\frac{1}{\rho-1}}S_e - \frac{\delta\alpha S_e}{x_pA_E^\rho r^k}(a^{-1} - x_pA_E^\rho a^{\frac{1}{\rho-1}}) - \omega\right]Q
\end{aligned}$$

Labor

$$v_w = 1, \quad L = L^d, \quad W^o = W$$

One has also:
$$\frac{W}{P_cC} = \mathcal{M}_wL^\phi \Leftrightarrow \frac{W}{P_c} = \mathcal{M}_wMRS \Leftrightarrow \frac{W_rP_q}{P_c} = \mathcal{M}_wL^\phi$$

Combining the equations $W_r = \frac{P_c}{P_q}\mathcal{M}_wCL^\phi$, $\frac{S_e}{A_E^\rho E^{\rho-1}} = \frac{W_rL^d}{(1-\alpha)A_{LK}^\rho(K^\alpha(L^d)^{(1-\alpha)-1})^\rho}$ and $L = L^d$ one has:

$$\frac{E}{Q} = a^{\frac{1}{\rho-1}}, \quad \text{and} \quad \frac{W_r}{C} = \frac{P_c}{P_q}\mathcal{M}_wL^\phi$$

Let us note:
$$b := 1 - a^{\frac{1}{\rho-1}}S_e - \frac{\alpha\delta S_e}{x_pA_E^\rho r^k}(a^{-1} - x_pA_E^\rho a^{\frac{1}{\rho-1}}) - \omega \tag{20}$$

then one also has:

$$\frac{C}{Q} = \frac{P_q}{P_c}b \quad \text{and} \quad \frac{W_r}{Q} = \mathcal{M}_wbL^\phi$$

In the other hand:

$$\begin{aligned}
\frac{Se}{x_pA_E^\rho E^{\rho-1}} &= \frac{W_rL}{(1-\alpha)(Q^\rho - x_pA_E^\rho E^\rho)} \\
E^{1-\rho}Q^\rho - x_pA_E^\rho E &= \frac{W_rLx_pA_E^\rho}{(1-\alpha)S_e}
\end{aligned}$$

Dividing by Q on both sides one gets an expression for L:

$$L = \left(\left(a^{-1} - x_pA_E^\rho a^{\frac{1}{\rho-1}}\right)\frac{(1-\alpha)S_e}{x_pA_E^\rho b\mathcal{M}_w}\right)^{\frac{1}{\phi+1}}$$

One also has,

$$
\begin{aligned}
Q^\rho =& x_p A_E^\rho E^\rho + (1-x_p) A_{LK}^\rho K^{\alpha\rho} L^{(1-\alpha)\rho} \\
Q^\rho =& x_p A_E^\rho a^{\frac{\rho}{\rho-1}} Q^\rho + (1-x_p) A_{LK}^\rho L^{(1-\alpha)\rho} \left(\frac{\alpha}{1-\alpha} \frac{W_r L}{r^k S_k} \right)^{\alpha\rho} \\
Q^\rho =& \frac{x_p A_{LK}^\rho}{1 - x_p A_E^\rho a^{\frac{\rho}{\rho-1}}} \left(\frac{\alpha}{1-\alpha} \frac{W_r}{r_k S_k} \right)^{\alpha\rho} L^\rho
\end{aligned}
$$

Dividing by $Q^{\alpha\rho}$ at both sides one derives an expression for Q:

$$
Q = \left(\frac{(1-x_p) A_{LK}^\rho}{1 - x_p A_E^\rho a^{\frac{\rho}{\rho-1}}} \left(\frac{\alpha}{1-\alpha} \frac{\mathcal{M}_w b}{r_k S_k} \right)^{\alpha\rho} L^{\rho(1+\alpha\phi)} \right)^{\frac{1}{(1-\alpha)\rho}}
$$

And so one can calculate the value of the remaining variables

$$
\begin{aligned}
E =& a^{\frac{1}{\rho-1}} Q \quad , C = \frac{P_q}{P_c} Q b, \quad C_q = (1-x_c) \left(\frac{P_q}{P_c} \right)^{\frac{1}{\sigma-1}} C \\
C_e =& x_c \left(S_e \frac{P_q}{P_c} \right)^{\frac{1}{\sigma-1}} C, \quad W_r = \frac{P_c}{P_q} \mathcal{M}_w L^\phi C, \quad K = \frac{\alpha}{1-\alpha} \frac{W_R L}{r^k S_k} \\
Y =& \frac{P_q}{P_c} (Q - S_e E), \quad S_c := \frac{Pc}{Pq} = ((1-x_c) + x_c S_e^{\frac{\sigma}{\sigma-1}})^{\frac{\sigma-1}{\sigma}}
\end{aligned}
$$

B Appendix B: Log-linearized Model

Small case letters represent the log-deviation of each variable with respect its steady state, $z_t := log(Z_t) - log(Z)$. For the rental rate of capital (r_t^k) and the investment (I) the log-deviation will be noted $\widehat{r}_t$ and $\widehat{I}_t$ respectively. The model is simplified in order to have just real prices and quantities. The list of log-linear equations that characterize the equilibrium is:

$$
s_{c,t} = \left(\left(\frac{S_e}{S_c} \right)^{\frac{\sigma}{\sigma-1}} \right) x_c s_{e,t} \tag{21}
$$

$$
c_{q,t} = c_t - \frac{1}{\sigma-1} s_{c,t} \tag{22}
$$

$$
c_{e,t} = c_t + \frac{1}{\sigma-1} s_{e,t} - \frac{1}{\sigma-1} s_{c,t} \tag{23}
$$

$$
c_t = \mathbb{E}_t[c_{t+1}] - (i_t - \mathbb{E}_t[\pi_{c,t+1}]) \tag{24}
$$

$$
i_t = (1 - \beta(1-\delta)) \mathbb{E}_t[\widehat{r}_{t+1}] + \mathbb{E}_t[\pi_{k,t+1}] \tag{25}
$$

$$
\pi_{q,t} + \pi_{wr,t} = \beta \mathbb{E}\left[\pi_{q,t+1} + \pi_{wr,t+1} \right] + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w (1+\phi\epsilon_w)} (mrs_t + s_{c,t} - w_{r,t}) + \varepsilon_{w,t} \tag{26}
$$

$$
\pi_{wr,t} = wr_t - wr_{t-1} \tag{27}
$$

$$
mrs_t = c_t + \phi l_t \tag{28}
$$

$$
mc_{r,t} = (\rho-1) q_t - (\rho-1) e_t - \rho a_{e,t} + s_{e,t} \tag{29}
$$

$$
\tag{30}
$$

$$s_{e,t} - a_{e,t} - (\rho - 1)e_t = wr_t - a_{lk,t} - \alpha\rho k_t - ((1-\alpha)\rho - 1)l_t \tag{31}$$
$$l_t + wr_t = k_t + \widehat{r}_t + s_{k,t} \tag{32}$$
$$i_t = \phi_\pi \pi_{q,t} + \phi_y y_t + \varepsilon_i \tag{33}$$
$$Q^\rho q_t = x_p (A_E E)^\rho (a_{e,t} + e_t) + (1 - x_p) A_{LK}^\rho (K^\alpha L^{1-\alpha})^\rho (a_{lk,t} + \alpha k_t + (1-\alpha) l_t) \tag{34}$$
$$\delta \widehat{I}_t = k_{t+1} - (1-\delta) k_t \tag{35}$$
$$Qq_t - S_e E(e_t + s_{e,t}) = S_c C(s_{c,t} + c_t) + S_k I(\widehat{I} + s_{k,t}) + G_r g_{r,t} \tag{36}$$
$$\pi_q = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} mc_{r,t} + \beta \mathbb{E}[\pi_{q,t+1}] + \varepsilon_{p,t} \tag{37}$$
$$S_c Y(y_t + s_{c,t}) = Qq_t - S_e E(e_t + s_{e,t}) \tag{38}$$
$$\pi_{k,t} = \pi_{q,t} + s_{k,t} - s_{k,t-1} \tag{39}$$
$$\pi_{c,t} = \pi_{q,t} + s_{c,t} - s_{c,t-1} \tag{40}$$
$$s_{e,t} = \rho_{se} s_{e,t-1} + e_{se,t} \tag{41}$$
$$s_{k,t} = \rho_{sk} s_{k,t-1} + e_{sk,t} \tag{42}$$
$$g_{r,t} = \rho_g g_{r,t-1} + \rho_{gae} e_{ae,t} + \rho_{galk} e_{alk,t} + e_{g,t} \tag{43}$$
$$\varepsilon_{i,t} = \rho_i e_{i,t-1} + e_{ei,t} \tag{44}$$
$$a_{e,t} = \rho_{ae} a_{e,t-1} + e_{ae,t} \tag{45}$$
$$a_{lk,t} = \rho_{alk} a_{lk,t-1} + e_{alk,t} \tag{46}$$
$$\varepsilon_{w,t} = \rho_w \varepsilon_{w,t-1} - \nu_w e_{w,t-1} + e_{w,t} \tag{47}$$
$$\varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} - \nu_p e_{p,t-1} + e_{p,t} \tag{48}$$

C Appendix C: Bayesian Estimation

C.1 Data Treatment

A total of eight series, corresponding to the eight structural shocks of the model, are taken as key macro-variables for the estimation. All the series are quarterly. A description of the original series's sources is presented in Table 4 and data is available upon request. The sample goes from 1984:Q1 to 2007:Q1.

The observable variables include: (i) real GDP, (ii) real non-oil Consumption, (iii) real Private Fixed Investment, (iv) Hours Worked, (v) real Wages, (vi) Inflation, (vii) the Federal Funds Rate and (viii) Total Oil Use in Production. The model is stationary, so series that are not originally stationary, which are the fist five, have to be detrended. For that, I use linear trend techniques. The rest of the series are stationary, so I do not detrend them, but I takeout their respective mean for the estimation period. A detailed explanation of the manipulation of the data is presented on Table 5. For estimation and simulation, the model has been log-linearized, then the corresponding observable variables are given in natural logarithms and multiplied by 100.

Finally, I have to identify the observable series to my model's variables. Note that the model have three different type of prices: a domestic price P_q, a CPI P_c, which is equal to the GDP deflator by definition, and a capital price P_k. Because all the observable series are deflated by the GDP deflator and the real variables in the model are deflated by the domestic price P_q, there are some concordances that have to be done. The final observation

Table 4: Original Sources

Serie	Description	Source
GDPC09	Real Gross Domestic Product, Chained Dollars (2009), Seasonally Adjusted, Annual Rate	Table 1.1.6 Bureau of Economic Analysis
GDPDEF	Implicit Price Deflators for Gross Domestic Product (2009), Seasonally Adjusted	Table 1.1.9. Bureau of Economic Analysis
PCE	Personal Consumption Expenditures by Major Type of Product, Seasonally Adjusted,Annual Rate	Table 2.3.5. Bureau of Economic Analysis
PCE_{oil}	Personal Consumption Expenditures by Major Type of Product: Gasoline and other energy goods, Seasonally Adjusted,Annual Rate	Table 2.3.5. Bureau of Economic Analysis
PFI	Private Fixed Investment by Type, Seasonally Adjusted,Annual Rate	Table 5.3.5. Bureau of Economic Analysis
CE16OV	Civilian Employment, 16 and over, Seasonally Adjusted, Thousands	LNS12000000 Bureau of Labor Statistics
CE16OV Index	CE160V (2009)=1	
LNS10	Population level, civilian noninstitutional population, 16 and over, Seasonally Adjusted, Thousands	LNS10000000 Bureau of Labor Statistics
LNS10 Index	LNS10 (2009)=1	
PRS85006023	Nonfarm Business, All Persons, Average weekly hours worked Duration (2009), Seasonally Adjusted	PRS85006023 Bureau of Labor Statistics
PRS85006103	Nonfarm Business, All Persons, Hourly Compensation Duration (2009), Seasonally Adjusted	PRS85006103 Bureau of Labor Statistics
FEDFUND	Federal funds effective rate, percent: Per Year, Average of Daily figures	Board of Governors of the Federal Reserve System
$TotalSAOil$	Constructed as in Acurio-Vásconez et al. (2015)	Acurio-Vásconez et al. (2015)

equations for the model are:

$$
\begin{aligned}
invobs_t =& \widehat{I}_t + s_{k,t} - s_{c,t} \\
ybos_t =& y_t \\
cqobs_t =& c_{q,t} - s_{c,t} \\
labobs_t =& l_t \\
eobs_t =& e_t \\
infobs_t =& \pi_{c,t} \\
iobs_t =& i_t \\
wobs_t =& w_{r,t} - s_{c,t}
\end{aligned}
$$

C.2 Distribution Parameters

Before estimating, one needs to identify what the distribution parameters in the CES function represent. Define $\omega_c = \frac{P_e C_e}{P_c C}$. Remark that ω_c could be calibrated from data[22]. From

[22] For $P_e C_e$ one can use the series for nominal personal consumption expenditures: Gasoline and other energy goods, and for $P_c C$, one can use the nominal Personal Consumption Expenditure. I will take ω_c as

Table 5: Observable Variables

Observed Variable	Transformation
invobs	$detrend\left(log\left(\frac{\frac{PFI}{GDPDEF}}{LNSIndex}\right)*100\right)$
yobs	$detrend\left(log\left(\frac{GDPC09}{LNSIndex}\right)*100\right)$
cqobs	$log\left(\frac{\frac{PCE-PCE_{oil}}{GDPDEF}}{LNSIndex}\right)*100$
labobs	$log\left(\frac{PRS85006023*CE16OVIndex}{LNSIndex}\right)*100-mean\left(ln\left(\frac{PRS85006023*CE16OVIndex}{LNSIndex}\right)*100\right)$
wobs	$log\left(\frac{\frac{PRS85006103}{GDPDEF}}{LNSIndex}\right)*100-mean\left(ln\left(\frac{\frac{PRS85006103}{GDPDEF}}{LNSIndex}\right)*100\right)$
infobs	$log\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100-mean\left(ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100\right)$
iobs	$\left(log\left(1+\frac{FEDFUND}{400}\right)-mean\left(ln\left(1+\frac{FEDFUND}{400}\right)\right)\right)*100$
eobs	$log\left(\frac{TotalSAOil}{LNSIndex}\right)*100-mean\left(log\left(\frac{TotalSAOil}{LNSIndex}\right)*100\right)$

the steady state equations, one needs the following relationship to be satisfied:

$$\begin{aligned}
\frac{C_e}{C} &= x_c\left(\frac{P_e}{P_c}\right)^{\frac{1}{\sigma-1}} \\
\frac{P_eC_e}{P_cC} &= x_c\left(\frac{P_e}{P_c}\right)^{\frac{\sigma}{\sigma-1}} \\
\omega_c &= x_cS_e^{\frac{\sigma}{\sigma-1}}\left(\frac{P_c}{P_q}\right)^{\frac{-\sigma}{\sigma-1}} \\
\omega_c &= x_cS_e^{\frac{\sigma}{\sigma-1}}\left((1-x_c)+x_cS_e^{\frac{\sigma}{\sigma-1}}\right)^{-1}
\end{aligned}$$

Assuming a steady state equals to 1 for the real price of oil, S_e, one has $\omega_c = x_c$. In this way, the distribution parameter, x_c, represents the share of oil consumption out of household total consumption.

The identification of the parameter x_p is less straightforward. As pointed out in Cantore & Levine (2012), distribution parameters in CES production functions needs a renormalization in order to be estimated. In fact, Cantore & Levine (2012) showed that under the formulation of the CES function as in equation (8), the parameter x_p is a dimensional pa-

the mean of the generated series by $\frac{P_eC_e}{P_cC}$ in the estimation period rameter and depends on the units chosen for factor inputs. In order to avoid this problem in estimation, I normalize this function as those authors do.

Remark that at steady state, the following equations hold:

$$\frac{Q}{E}\left(\frac{x_p A_E^\rho E^\rho}{Q^\rho}\right) = \frac{Se}{MC_r} \tag{49}$$

Define

$$\pi = \frac{x_e A_E^p E^\rho}{Q^\rho} \Rightarrow x_p = \pi\left(\frac{Q}{A_E E}\right)^\rho \tag{50}$$

As pointed out in Cantore & Levine (2012), π is the re-normalized distribution parameter. We just need to interpreted what does it mean in the model.

For that, remark that using equation (49) one also has

$$\begin{aligned}\pi =& \frac{S_e}{MC_r}\frac{E}{Q}\\ \pi =& \frac{1}{MC_r}\frac{P_e E}{P_q Q} \Rightarrow \frac{E}{Q} = \pi\frac{1}{\mathcal{M}_p S_e}\end{aligned} \tag{51}$$

From where one has

$$x_p = \pi^{1-\rho}\left(\frac{\mathcal{M}_p S_e}{A_E}\right)^\rho$$

In the other hand, the steady state of the output elasticity of oil denoted by $\alpha_{e,t}$, is defined as

$$\begin{aligned}\alpha_e =& \frac{\partial Q}{\partial E}\frac{E}{Q}\\ =& x_p\left(\frac{A_E E}{Q}\right)^{\frac{\eta_p-1}{\eta_p}}\\ =& x_p\left(\mathcal{M}_p\frac{S_e}{x_p A_E^\rho}\right)^{1-\eta_p}\end{aligned}$$

Then $\pi = \alpha_e$, i.e, the normalized parameter represent oil's output elasticity.

As for the oil's cost share and the output elasticity, at steady state, in this model it is defined as:

$$\begin{aligned}\text{oil's cost share} :=& \frac{P_e E}{PcY}\\ =& \frac{P_e E}{P_q Q - P_e E}\\ =& \frac{\frac{P_e E}{P_q Q}}{1-\frac{P_e E}{P_q Q}}\\ =& \frac{\alpha_e MC_r}{1-MC_r}\\ =& \frac{\alpha_e}{\mathcal{M}_p - \alpha_e}\end{aligned}$$

Finally, following Kumhof & Muir (2014), I assume that the oil's cost share is 3.5 percent. Then following this last relationship, I assume that the prior value for the output elasticity α_e is 3.9 percent.