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How Do Universities Compete for Students? Two Competing Strategies and Their Impact on Capacity and Tuition Fees

Marie-Laure Cabon-Dhersin*, Jonas Didisse†

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Abstract

This paper analyzes the impact of two competing strategies (Cournot vs Bertrand competition) in a context where universities compete for students and can enroll beyond capacity at strictly convex costs. Universities interact in a two-stage game to decide on capacity and tuition fees/number of students enrolled. In Bertrand competition, when costs are sufficiently convex, universities adopt low capacities in the first stage in order to sustain high fees in the second stage. Conversely, Cournot competition leads to a higher capacity for each university and to a larger number of students enrolled. Under certain conditions, the equilibrium adopted in Bertrand competition may be more efficient in terms of cost minimization but it leads to a lower social welfare level.

Key words: Cournot competition, Bertrand competition, capacity, higher education market.

Code JEL: C7, H52, I22, I23.

1 Introduction

Public authorities have a real interest in ensuring that the student population has access to a wide range of higher education opportunities. It is therefore

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important to understand how the behavior of universities may affect the provision of higher education and at what price.

In this paper, we focus on inter-university competition and related issues of imperfect competition in higher education. There are surprisingly few empirical and theoretical studies devoted to competition between universities. However, the homogenization of university programs and the increased mobility of students allow universities to compete in order to attract students. For instance, the Bologna process in Europe should help to harmonize and to compare the standards and quality of higher education qualifications. An interesting issue is how competition between universities affects the choice of the number of students enrolled and the level of fees. In the UK, a recent reform was focused on tuition fee deregulation, which has led to nearly tripling fees combined with relaxing controls on the number of students each university can recruit. However, a tuition fee deregulation should lead to a consequent fall in fees. It is tempting to rely on industrial organization theories to explain this paradox.

We will assume for the purposes of our analysis that there are two competing strategies which differ by the strategic variable retained by the universities depending on the context:

- (i) a competing strategy in which universities are free to set their own fees (as in the UK, the USA, Canada, and Australia). In this case, universities compete in fees to attract students (as in Bertrand competition).
- (ii) a competing strategy in which universities are not free to use the variable pricing strategy as a response to their new environment (as in France, Italy, and Germany among others). In this case, they adopt another form of competition in terms of the quantity or the number of students enrolled (as in Cournot competition).

The distinction between these two competing strategies may prove particularly relevant in OECD countries to better understand disparities in tuition fees.

As suggested by Rothschild and White (1995), universities compete for students because students are at once inputs and clients of the educational process. More precisely, students are required for the production of education, but they also provide funds to universities both by paying tuition fees, and by allowing universities to receive transfers from the government. Yet the fact that universities pursue both teaching and research objectives affects how they compete for students. With regard to research activities, we can assume that research requires money. More students enrolled may imply

an increased research budget. However, increasing the size of the student population may be costly.

In the economic literature, some papers examine the tensions between teaching and research. Using a spatial model, Del Rey (2001) models competition between universities which decide on the allocation of funds between teaching and research activities. The balance between research and teaching efforts will depend on the funding rules. In the same vein, De Fraja and Iossa (2002) study the effect of the mobility cost supported by students on the choice of admission standards by multi-tasking universities. In Grazzini et al. (2010), the literature on capital tax competition is used in order to analyze how student mobility affects university competition on both tuition fees and expenditure in research and teaching. Gautier and Wauthy (2007) have explored the potential implications of incentive schemes as a tool to promote efficiency within a single university and contrast two governance modes, a multi-department university with a single-unit one. Beath et al. (2003) focus on the tensions between pure and applied research under binding budget constraints. Kemnitz (2007) explores the impact of university funding reform on teaching quality competition. More recently, Beath et al. (2012) have studied how the type of funding system has an impact on the trade-off between teaching and research quality. What all these papers have in common is that they stay away from inter-university competition and related issues of imperfect competition in higher education (except in Jacqmin and Wauthy (2014)).

In this paper, we will consider the two competition models between universities with capacity constraints. Since Edgeworth (1925), there has been a long tradition in Industrial Organization of considering firms that are capacity constrained (Vives, 1980; Kreps and Scheinkman, 1983; Allen et al., 2000). In these models, the constraint is drastic (i.e. it is impossible to produce above the capacity). More recently, Chowdhury (2009) introduced "non-rigid" capacity constraint, assuming it is possible to produce above the capacity of production but with an increasing marginal cost. Using the notion of "non-rigid" capacity, Jacqmin and Wauthy (2014) argue that competition between universities producing both research and teaching may generate a comparable framework in which the cost of providing education is convex beyond a certain threshold. In our paper, we propose another novel setting with a sequential choice. In the first stage, the university endogenously chooses a capacity level which is invariable during the second stage. In the second stage, universities compete in terms of tuition fees (Bertrand competition) or in terms of students enrolled (Cournot competition) to satisfy the demand they will face. When the number of students enrolled is higher than the capacity threshold, the cost function is strictly convex. The form of the

cost function may be justified by several reasons: (i) public institutions as universities are committed to satisfying the demand of enrollment, (ii) too many students negatively impact the research output by increasing the opportunity cost of providing education services by reducing the research time available, and (iii) the fixed assets used in educational activities (such as the number of teachers, libraries, rooms, or administrative services) are rivals in their consumption and have a fixed capacity.

The contribution of the present work may be understood as follows. To the best of our knowledge, no existing theoretical model has been proposed to explain the choice of capacity, the level of fees/the number of students enrolled in a competition context. We focus on a setup in which universities compete for students by setting enrollment or fees with the presence of ("soft") capacity constraints (Cabon-Dhersin and Drouhin, 2014). On positive grounds, the results in terms of equilibrium levels of fees/number of students, capacities and social welfare, allow for a comparison between the two underlying competing strategies (Cournot vs Bertrand competition). We also determine the efficient capacity for each university and compare with equilibrium results. We show that:

- (i) in Bertrand competition, when costs are sufficiently convex, universities adopt low capacities in the first stage in order to sustain a high level of fees in the second stage; conversely, Cournot competition leads to a higher capacity for each university and to a larger number of students enrolled,
- (ii) when the number of universities increases, the level of tuition fees rises up to the maximum level of fees which maximizes the utility of the university when all universities operate in a Bertrand game; but conversely, the total number of students enrolled increases with the number of universities in Cournot competition,
- (iii) under certain conditions, the equilibrium adopted in Bertrand competition may be more efficient in terms of cost minimization than in Cournot competition but it leads to the lower social welfare level.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyzes and compares the equilibrium results in terms of fees/number of students enrolled and capacities between the two competition models. Section 4 proposes to compare the equilibrium level of capacity in both competing strategies with an efficient capacity and extends the comparative results to the social welfare. Section 5 concludes.

2 Model

We assume that the creation (research as R) and dissemination (teaching as T) of fundamental knowledge is the mission of all universities. Thus each university's objective function can take the general form $U(T, R)$, where U is strictly increasing in both arguments. In higher education literature, there has been a long-standing debate about the separability of T and R in the objective function which can consider the complete specialization a university's tasks. In the same line as Jacqmin and Wauthy (2014), the objective function of universities is specified as follows:

$$\text{Max } U_i(T, R) \quad (1)$$

with $\frac{\partial U_i}{\partial T} > 0$ and $\frac{\partial U_i}{\partial R} > 0$

There are two basic reasons why universities may compete for students:

- As inputs, students are required for the production of education. With regard to teaching activities, we assume conveniently that the level of teaching is equal to the enrollment of students n_i weighted according to the parameter $0 < \gamma < 1$:

$$T = \gamma n_i$$

where γ measures the importance of the teaching compared to the research objective, n_i is the number of students enrolled in each university i .

- As clients, students provide the funds a university needs to operate, either directly through fees (f) or indirectly via the government. Research activities require money. In this respect, R represents each university's expenditure on research. Here, the research output S_i depends only on the money invested in it:

$$R = S_i$$

Since research requires money, more students enrolled may imply an increased research budget. This is especially relevant when the university is funded by a per-student subsidy s provided by the government agency in charge of the higher education system with $0 < s < 1$.

However, increasing the size of the student population is costly, in particular when the number of students enrolled exceeds the university's capacity k . The cost supported by each university depends on the level of the capacity

k_i installed at the unit cost δ , and, as far as $n_i > k_i$, depends quadratically on the number of students beyond capacity. According to Jacqmin and Wauthy (2014) or Chowdhury (2009), capacity constraints are not drastic (i.e. it becomes possible to provide education services above capacity); by incurring an additional per unit cost (noted by μ), universities can enroll students beyond capacity.

Thus the cost function of each university i is given by

$$C_i(n_i, k_i) = \begin{cases} \delta k_i & \text{if } 0 \leq n_i \leq k_i \\ \delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i \end{cases} \quad (2)$$

where $\mu > 0$. The cost parameters (δ and μ) are similar for all universities and constant.

Our model considers a novel setting with a sequential choice: in the first stage, the university chooses a capacity level k_i that will be invariable during the second stage. In the second stage, universities compete on price (level of tuition fees) or on quantity (number of students enrolled) to satisfy the demand they will face. Note that in the second stage, δk_i is like a "sunk cost" (which is unavoidable in this stage)¹. This implies that, when the number of students enrolled is lower than the capacity threshold, the marginal cost of each student is nil until the capacity threshold k_i and the average cost per student is decreasing with n_i . Beyond capacity level k_i , the marginal cost is strictly convex and the average cost appears to be on the rise. The cost function of the university exhibits the more familiar case of U-Shaped average costs².

The optimization problem of a university i is thus defined as:

$$Max U_i = \gamma n_i + S_i \quad s.t. \quad S_i + \delta k + \max(0; \mu(n_i - k_i)^2) = n_i(f_i + s) \quad (3)$$

We consider a higher education market with m identical universities. We assume that universities are homogeneous: there is no differentiation in curriculum, and/or in other non-price dimensions (location, different trade-offs between teaching and research, admission standard, or financial endowment among others). Universities are committed to satisfying all the demand they face in the second stage. The utility students derive from graduating at university i is defined by $u(\theta) = \theta - f_i$ where θ represents the student's willingness to pay to go to university. We will assume that potential students will

¹With avoidable fixed costs, there are problems of existence of price equilibrium (Saporiti and Coloma, 2010; Dastidar, 2011b,a).

²The average cost per student completion in Australian universities provides at least some evidence of a U-Shaped long-run average cost curve (Worthington and Higgs, 2011).

not differ in their willingness to enroll³. The student population is therefore normalized to the unit.

In order to study inter-university competition and related issues of imperfect competition in higher education, we consider the competing strategies defined as follows:

- (i) The first competing strategy consists in opting for Cournot competition: because universities are not free to modify the level of fees directly, they compete on the number of students. In the Cournot competition model, we consider a market with m universities facing an inverse demand function $D^{-1}(N)$ where $N = \sum_{i=1}^m n_i$ is the total number of students enrolled:

$$f(n_i, N_{-i}) = 1 - N$$

$$\text{with } N_{-i} = \sum_{\substack{j=2 \\ i \neq j}}^m n_j.$$

- (ii) The second competing strategy consists in adopting the price variable (Bertrand competition). This competition scenario describes interactions among universities which set fees and where students enroll at the price set. For a same university course, students choose to enroll at the university with the lowest fees. Because universities are strictly homogeneous, the demand for enrollment addressed to a particular university is a discontinuous function of fees. The demand function of university i is defined as follows:

$$n_i(f_i, f_{-i}) = \begin{cases} 0 & \text{if } f_i > f_{-i \min} \\ \frac{N(f_i)}{m} = \frac{1-f_i}{m} & \text{if } f_i = f_{-i} \\ N(f_i) = 1 - f_i & \text{if } f_i < f_{-i \min} \end{cases}$$

with f_i the fee of the university i ; $f_{-i} = \{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$ is the vector of the fees of all universities on the higher education market. We denote $f_{\min} = \text{Min}\{f_1, \dots, f_m\}$.

³As Jacqmin and Wauthy (2014) point out, additional heterogeneity in students' willingness to pay would smooth the process of inter-university competition in prices.

3 Two Competition Models between m Universities

In this section, we stipulate a two-period model in which m universities decide on the capacity k_i and then compete in quantity/number of the students enrolled (Cournot competition) or in fees (Bertrand competition) upon observing the chosen capacities in the first stage. We study how the convexity of cost and the number of universities will have an influence on equilibrium results. Then, we compare the two competing strategies between m universities.

3.1 The Cournot Oligopoly Model with Capacity Constraints

3.1.1 Nash Equilibria

Suppose that m identical universities interact in the two-stage game to decide non-cooperatively on both capacity and enrollment:

- (i) In the first stage, each university i chooses its level of capacity k_i taking the threshold capacity of the other universities k_{-i} into account.
- (ii) In the second stage, each university i selects a number of students enrolled n_i for a given level of capacity k_i and for a given number of students in the rival universities.

Universities' strategies consist of a capacity level and a subsequent enrollment strategy based on their capacities.

Consider the utility of university i in the second stage, conditional on k_i :

$$U_i(n_i, N_{-i}, k_i) = n_i f_i(n_i, N_{-i}) - C_i(n_i, k_i)$$

$$= n_i ((1 - n_i - N_{-i}) + s + \gamma) - \delta k_i - \max(0; \mu(n_i - k_i)^2)$$

$$\text{with } N_{-i} = \sum_{\substack{j=2 \\ i \neq j}}^m n_j.$$

The utility function is strictly concave in n_i and k_i .

In the second stage, each university chooses the number of students enrolled to maximize its utility function:

$$\frac{\partial U_i}{\partial n_i} = 0$$

$$\Leftrightarrow 1 + \gamma + s - n_i - N - 2\mu n_i + 2\mu k_i = 0 \quad (4)$$

with $\sum_{i=1}^m n_i = m \cdot n_i = N$.

The Nash-Cournot equilibrium can be computed to be:

$$n_i = \frac{1 + \gamma + s + 2\mu k_i}{2\mu + m + 1} \quad (5)$$

In the first stage, each university maximizes the utility U_i , as a function of k_i , for a given level of other universities' capacities k_{-i} :

$$\frac{\partial U_i(n_i(k_i, k_{-i}), N_{-i}(k_i, k_{-i}), k_i)}{\partial k_i} = 0$$

$$\Leftrightarrow k_i = \frac{n'_{ik_i}(1 + \gamma + s - 2(1 + \mu)n_i - N_{-i}) + n_i(2\mu - N'_{-ik_i}) - \delta}{2\mu(1 - n'_{ik_i})} \quad (6)$$

By replacing (6) in (4), the unique and symmetric subgame perfect equilibrium values are:

$$k_c^*(m) = \frac{2\mu((1 + 2\mu)(3 + 2\mu) + (m - 1)(1 + \gamma + s) - (2\mu + m + 1)(1 + 2\mu)(3 + 2\mu)\delta)}{2\mu((m + 1)(1 + 2\mu)(3 + 2\mu) - 2\mu(m - 1))}$$

$$n_c^*(m) = \frac{(1 + 2\mu)(3 + 2\mu)}{(m + 1)(1 + 2\mu)(3 + 2\mu) - 2\mu(m - 1)}(1 + \gamma + s - \delta)$$

and

$$f_c^*(m) = 1 - m \cdot n_c^*(m) = \frac{(1 + 2\mu)(3 + 2\mu)(1 + m(\delta - \gamma - s)) - 2\mu(m - 1)}{(m + 1)(1 + 2\mu)(3 + 2\mu) - 2\mu(m - 1)}$$

Lemma 1. *We verify that:*

$$n_c^*(m) > k_c^*(m)$$

if and only if $\delta > \frac{2\mu(m-1)}{(1+2\mu)(3+2\mu)(m+1)}(1 + \gamma + s) = \underline{\delta}$ with $\frac{\partial \underline{\delta}}{\partial m} > 0$.

Proof: in Appendix 1. \square

The interpretation of the above lemma is that each university opts for a greater number of students beyond capacity, provided that the unit cost of installed capacity is not too low (below $\underline{\delta}$). A higher unit cost of installed capacity associated with the lower marginal cost of a supplementary student always leads to more students above capacity. We show that it may actually be difficult to verify this condition when the number of universities increases. By contrast, if the capacity cost is low and the parameter μ is high, the university adopts a large capacity and limits its enrollment.

3.1.2 Comparative Statics

We can see that the importance of teaching (γ) and the level of the per-student subsidy (s) impact enrollment and capacity positively, but the capacity cost parameter δ has a negative effect on these equilibrium outputs. We note opposite effects on equilibrium fees. However, the convexity of cost, measured by the parameter μ , has a non-trivial effect on the different equilibrium results. The following corollary presents the effects of a rise in μ on equilibrium results:

Corollary 1. *We verify that:*

$$i) \frac{\partial \bar{k}_c^*}{\partial \mu} < 0 \text{ if } \delta < \frac{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+2\mu(m-1)Z}{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+(m+1)(1+2\mu)(3+2\mu)Z}(1+\gamma+s) = \bar{\delta} \in (0; 1)$$

$$\text{with } Z = (m+1)(1+2\mu)(3+2\mu) - 2\mu(m-1) > 0$$

$$ii) \frac{\partial n_c^*}{\partial \mu} < 0 \text{ if } \mu > \sqrt{\frac{3}{4}}$$

$$iii) \frac{\partial f_c^*}{\partial \mu} > 0 \text{ if } \mu > \sqrt{\frac{3}{4}}$$

Proof: in Appendix 2. \square

With increasing marginal costs (for $\mu > \sqrt{\frac{3}{4}}$), the equilibrium number of students enrolled decreases while tuition fees rise. On the condition that the capacity cost is sufficiently high, each university chooses a larger capacity for students; the capacity is all the larger as the marginal cost μ is high. When the additional cost of a supplementary student beyond capacity is high (given by the parameter value μ), the university prefers to increase the level of k and

to reduce the enrollment. However, if the capacity unit cost is very low, the university can decrease the overall cost by reducing the size of the capacity. In order to determine the capacity level, the university compares the benefit of decreasing capacity (fall in the cost associated with the fixed capacity in the first stage of the game) with the additional cost of serving all demand in the second stage of the game.

We examine now how the number of universities will have an impact on different equilibrium results.

Corollary 2. *We verify that:*

- i)* $\frac{\partial n_c^*(m)}{\partial m} < 0$
- ii)* $\frac{\partial N_c^*(m)}{\partial m} > 0$
- iii)* $\frac{\partial f_c^*(m)}{\partial m} < 0$
- iv)* $\frac{\partial k_c^*(m)}{\partial m} < 0$

Proof: in Appendix 3. \square

At equilibrium, the number of students enrolled per university decreases when the number of universities increases. However, with the increasing number of universities, the total number of students N may then increase or decrease. The above corollary shows that the total number of students enrolled grows with the number of universities. As in a standard Cournot oligopoly with convex costs, if the number of institutions increases, then the resulting equilibrium approaches the competitive one: tuition fees are sufficiently low to recruit more and more students. However, the capacity for each university decreases with m .

3.2 Bertrand Oligopoly Model with Capacity Constraint

3.2.1 Nash Equilibria

Universities interact in the two-stage game to decide non-cooperatively on both capacity and tuition fees:

- (i) In the first stage, universities optimally choose their fixed capacity levels k_i .
- (ii) In the second stage of the game, we analyze the price competition (in terms of fees f) for a given capacity at levels k_i .

We can now express the utility U_i for each university i :

$$U_i(f_i, f_{-i}, k_i, m) = f_i n_i(f_i, f_{-i}) - C(n_i(f_i, f_{-i}), k_i)$$

$$U_i(f_i, f_{-i}, k_i, m) \begin{cases} -\delta k_i & \text{if } f_i > f_{-imin} \\ (f_i + \gamma + s) \frac{N(f_i)}{m} - \delta k_i - \max(0; \mu(\frac{N(f_i)}{m} - k_i)^2) = U_d(m) & \text{if } f = f_i = f_{-i} \\ (f_i + \gamma + s) N(f_i) - \delta k_i - \max(0; \mu(N(f_i) - k_i)^2) = U_M(m) & \text{if } f = f_i < f_{-imin} \end{cases}$$

with f_i the fee of the university i ; $f_{-i} = \{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$ is the vector of the fees of all universities on the higher education market. We denote $f_{min} = \text{Min}\{f_1, \dots, f_m\}$.

The function $U_d(f_i, k_i)$ represents the utility of university i when both universities quote the same fees and the function U_M represents the utility of university i when it quotes the lower tuition fee and serves all demand. U_M and U_d are strictly concave in f_i and strictly concave in k_i , that is $\partial^2 U_d(f_i, k_i) / \partial f_i^2 < 0$, $\partial^2 U_d(f_i, k_i) / \partial k_i^2 < 0$, $\partial^2 U_M(f_i, k_i) / \partial f_i^2 < 0$, $\partial^2 U_M(f_i, k_i) / \partial k_i^2 < 0$.

We define $\bar{f}(k_i)$ that solves $U_M(f, k_i, m) = U_d(f, k_i, m)$. Thus, $\bar{f}(k_i)$ must be interpreted as the critical fee for which the university is indifferent between operating in the higher education market alone or with its rival. After calculation, we obtain:

$$(1 - f_i)(f_i + \gamma + s) - \mu[(1 - f_i) - k_i]^2 = \frac{1}{m}(1 - f_i)(f_i + \gamma + s) - \mu\left[\frac{(1 - f_i)}{m} - k_i\right]^2$$

$$\bar{f}(k_i, m) = \frac{\mu(m + 1) - m(\gamma + s) - 2m\mu k_i}{m + \mu m + \mu} \quad (7)$$

In the second stage, the fixed cost of capacity, δk_i , is "sunk", and the university will quote a fee only if the variable part of the utility is positive i . *e.* $U_d(f, k_i) \geq -\delta k_i$. Thus, we also define \hat{f}_i that solves $U_d(f, k_i) = -\delta k_i$ for a given k_i , the minimum fee compatible with deciding to enroll in the second stage.

Finally, we define f^* , the fee that maximises the utility of university i when all universities operate in the higher education market. As a shortcut, this fee can be interpreted as the maximum fee when all universities have chosen the same capacity level in the first stage.

$$f^*(k_i, m) = \arg \max_f \{U_i(f_i, k_i, m)\} = \frac{m(1 - \gamma - s) + 2\mu(1 - mk_i)}{2(m + \mu)} \quad (8)$$

It is important to understand how these utility functions U_M and U_d , and fees \hat{f}_i , f^* , and \bar{f} are organized together.

Lemma 2.

$$\begin{cases} \bar{f}(k_i, m) < f^*(k_i, m) & \text{if } \mu < \frac{m}{m-1} \\ \bar{f}(k_i, m) = f^*(k_i, m) = \tilde{f} & \text{if } \mu = \frac{m}{m-1} \\ \bar{f}(k_i, m) < f^*(k_i, m) & \text{if } \mu > \frac{m}{m-1} \end{cases}$$

As shown in Figure 1, there are 3 cases depending on the parameter value μ . Considering just one university with a definite level of fixed capacity, the lemma 2 allows us to draw the functions U_M and U_d .

In the following proposition, we take university fixed capacity levels as given and look for the Nash equilibrium in fees.

Proposition 1. *In the second stage, (f_1, \dots, f_m) is a pure-strategy Nash equilibrium if and only if $f_1(k_1) = f_2(k_2) = \dots = f_m(k_m) = f^N(k_i)$, such that:*

- (i) *If $\mu < \frac{m}{m-1}$, $f^N \in [\hat{f}(k_i), \bar{f}(k_i)]$ and $\bar{f}_1(k_1) = \dots = \bar{f}_m(k_m) = \bar{f}(k_i)$ is a payoff-dominant pure-strategy Nash equilibrium*
- (ii) *If $\mu = \frac{m}{m-1}$, $f^N = \bar{f}(k_i) = f^*(k_i) = \tilde{f}(k_i)$*
- (iii) *If $\mu > \frac{m}{m-1}$, $f^N \in [\hat{f}(k_i), \bar{f}(k_i)]$ and $f^* \in [\hat{f}, \bar{f}]$ is a payoff-dominant pure-strategy Nash equilibrium*

Proof: in Appendix 4. \square

The following Table (1) presents the results of Bertrand competition in the different configurations for the second-stage game.

Table 1: The different cases of Bertrand competition in the second-stage game.

If $\mu < \frac{m}{m-1}$	If $\mu = \frac{m}{m-1}$	If $\mu > \frac{m}{m-1}$
$f^N = \bar{f}(k_i, m)$	$f^N = \tilde{f}(k_i, m)$	$f^N = f^*(k_i, m)$
$= \frac{\mu(m+1) - m(\gamma+s) - 2m\mu k_i}{m + \mu m + \mu}$	$= \frac{(m+1) - (m-1)(\gamma+s) - 2mk_i}{2m}$	$= \frac{m(1-\gamma-s) + 2\mu(1-mk_i)}{2(m+\mu)}$
$\bar{n}(k_i, m) = \frac{1+\gamma+s+2\mu k_i}{m+\mu m+\mu}$	$\tilde{n} = \frac{(m-1)(1+\gamma+s)+2mk_i}{2m^2}$	$n^*(k_i, m) = \frac{1+\gamma+s+2\mu k_i}{2(m+\mu)}$

The Nash equilibrium prediction in the second-stage game is basically that in Dastidar (1995). For all fees above \bar{f} , university i obtains no enrollment, which undermines universities' finances. When the other university charges any fee $f \in [\hat{f}, \bar{f}]$, the best response for university i is to quote the same tuition fee so that students split between both institutions. By lowering its fee, the university increases its revenue by having higher enrollment. But since costs are strictly convex, they will increase even more, making this deviation nonprofitable. For this reason, a continuum of fees above the competitive price ($f \in [\hat{f}, \bar{f}]$) can be sustained at Nash equilibria in pure strategies. When the value of parameter μ is high, it means that costs are sufficiently convex (i.e. $C''(n_i) \geq \frac{2m}{m-1}(\frac{-1}{D'})$ which is equivalent to $\mu \geq \frac{m}{m-1}$)⁴ to ensure that the joint-utility-maximizing fee lies within the Nash equilibrium interval (Dastidar, 2001). The symmetry and the payoff dominance criterion are sufficient here to reduce the set of equilibria⁵ and provide the uniqueness of the solution⁶. Consequently, we can select three fee equilibria: (i) when $\mu < \frac{m}{m-1}$, then the symmetric fee $f = \bar{f}$ is the unique (payoff dominant) pure-strategy Nash equilibrium, (ii) when $\mu = \frac{m}{m-1}$, there exists a unique and symmetric equilibrium such that $f = f^* = \bar{f}$, and (iii) when $\mu > \frac{m}{m-1}$, the unique and symmetric equilibrium is such that $f = f^*$.

Universities determine their level of fixed capacity anticipating the effect on the fee equilibria in the second stage.

Each university chooses its capacity level non-cooperatively, by maximizing its utility function with respect to the capacity k_i :

$$\frac{\partial U_i(f(k_i), k_i)}{k_i} = 0 \quad \Leftrightarrow$$

$$\bar{k}_i = \frac{f'(\bar{k}_i) (m(1 - \gamma - s) + 2\mu - 2(m + \mu)f) - m(\delta m - 2\mu) - 2m\mu f}{2m\mu(f'(\bar{k}_i) + m)}$$

For each configuration depending on μ , there exists a unique and symmetric solution satisfying $\frac{\partial U_i}{\partial k_i} = 0$, for which:

⁴Our condition is similar to that in Dastidar (2001) p.86, Prop.1.

⁵An equilibrium point is said to be payoff dominant if it is not strictly dominated by another equilibrium point; that is, there exists no other equilibrium in which utilities are higher for all universities (Harsanyi and Selten, 1988).

⁶The proof of the uniqueness is given by Cabon-Dhersin and Drouhin (2014).

(i) If $\mu < \frac{m}{m-1}$,

$$\begin{cases} \bar{k}_b^*(m) = \frac{4\mu^2 m(1+\gamma+s) - (m+\mu m+\mu)^2 \delta}{(m+\mu m+\mu)^2 - 4m\mu^2} \\ \bar{f}_b^*(m) = 1 - m \frac{(m+\mu m+\mu)(1+\gamma+s-\delta)}{(m+\mu m+\mu)^2 - 4m\mu^2} \\ \bar{n}_b^*(m) = \frac{(m+\mu m+\mu)(1+\gamma+s-\delta)}{(m+\mu m+\mu)^2 - 4m\mu^2} \end{cases}$$

(ii) If $\mu = \frac{m}{m-1}$,

$$\begin{cases} \tilde{k}_b^*(m) = \frac{1+\gamma+s-m\delta}{2m} \\ \tilde{f}_b^*(m) = \frac{1-\gamma-s+\delta}{2} \\ \tilde{n}_b^*(m) = \frac{1+\gamma+s-\delta}{2m} \end{cases}$$

(iii) If $\mu > \frac{m}{m-1}$,

$$\begin{cases} k^*(m) = \frac{\mu(1+\gamma+s) - (m+\mu)\delta}{2m\mu} \\ f^*(m) = \frac{1-\gamma-s+\delta}{2} \\ n^*(m) = \frac{1+\gamma+s-\delta}{2m} \end{cases}$$

Corollary 3. *We verify that: $\forall \mu, \gamma, s, \delta > 0$ and $m \geq 3$,*

$$\begin{cases} \bar{k}_b^*(m) < \bar{n}_b^*(m) \\ \tilde{k}_b^*(m) < \tilde{n}_b^*(m) \\ k_b^*(m) < n_b^*(m) \end{cases}$$

Proof: in Appendix 5. \square

The competition in tuition fees between m universities is likely to induce additional demand greater than the capacity level. Whatever the parameter values of the game and the number of universities, each institution chooses a number of students beyond capacity at a convex marginal cost.

3.2.2 Comparative Statics

We now study how the convexity of cost and the number of universities will have an influence on equilibrium results.

Corollary 4. *We verify that:*

(i) If $\mu < \frac{m}{m-1}$:

$$\frac{\partial \bar{k}_b^*}{\partial \mu} > 0, \quad \frac{\partial \bar{n}_b^*}{\partial \mu} < 0, \quad \frac{\partial \bar{f}_b^*}{\partial \mu} > 0$$

(ii) If $\mu = \frac{m}{m-1}$:

$$\frac{\partial \tilde{k}_b^*}{\partial \mu} = 0, \quad \frac{\partial \tilde{n}_b^*}{\partial \mu} = 0, \quad \frac{\partial \tilde{f}_b^*}{\partial \mu} = 0$$

(iii) If $\mu > \frac{m}{m-1}$:

$$\frac{\partial k_b^*}{\partial \mu} > 0, \quad \frac{\partial n_b^*}{\partial \mu} = 0, \quad \frac{\partial f_b^*}{\partial \mu} = 0$$

Proof: in Appendix 6 \square

Note that benefits to undercutting (at lower fees and hence higher possible enrollment) are relatively lower as costs increase disproportionately. This implies that a greater convexity of costs (with the increase of the value of μ) tends to push \bar{f} up to f^* . When $\mu = \frac{m}{m-1}$, $\bar{f} = f^*$. A lower bound on $C''(n_i)$ serves as a sufficient condition to sustain the collusive fee f^* . It means that if the cost function is sufficiently convex ($\mu \geq \frac{m}{m-1}$), then f^* is chosen by universities. Because of the U-shaped average cost, in the first stage of the game each university may adopt an efficient capacity which minimizes its cost. When the additional marginal cost μ increases, the university chooses higher capacity to limit the number of students beyond installed capacity. An increase in the capacity allows to maintain the total cost at its minimum. Consequently, the collusive fee (and the number of students enrolled at the collusive equilibrium) will no longer depend on the convexity of costs.

Corollary 5. *For $m \geq 3$, we verify the following properties:*

(i) If $\mu < \frac{m}{m-1}$:

$$\left\{ \begin{array}{l} \frac{\partial \bar{k}_b^*(m)}{\partial m} < 0 \\ \frac{\partial \bar{f}_b^*(m)}{\partial m} > 0 \\ \frac{\partial \bar{n}_b^*(m)}{\partial m} < 0 \\ \frac{\partial \bar{N}_b^*(m)}{\partial m} < 0 \end{array} \right. \quad \begin{array}{l} \text{if } \mu > \bar{\mu} \\ \text{if } \mu > \bar{\mu} \end{array}$$

$$\text{with } \bar{\mu} = \frac{m-m^2+2\sqrt{m^4-m^3}}{3m^2-2m-1}$$

(ii) If $\mu \geq \frac{m}{m-1}$:

$$\begin{cases} \frac{\partial k_b^*(m)}{\partial m} < 0 \\ \frac{\partial f_b^*(m)}{\partial m} = 0 \\ \frac{\partial n_b^*(m)}{\partial m} < 0 \\ \frac{\partial N_b^*(m)}{\partial m} = 0 \end{cases}$$

Proof: in Appendix 7. \square

These different cases are illustrated in Figure 1.

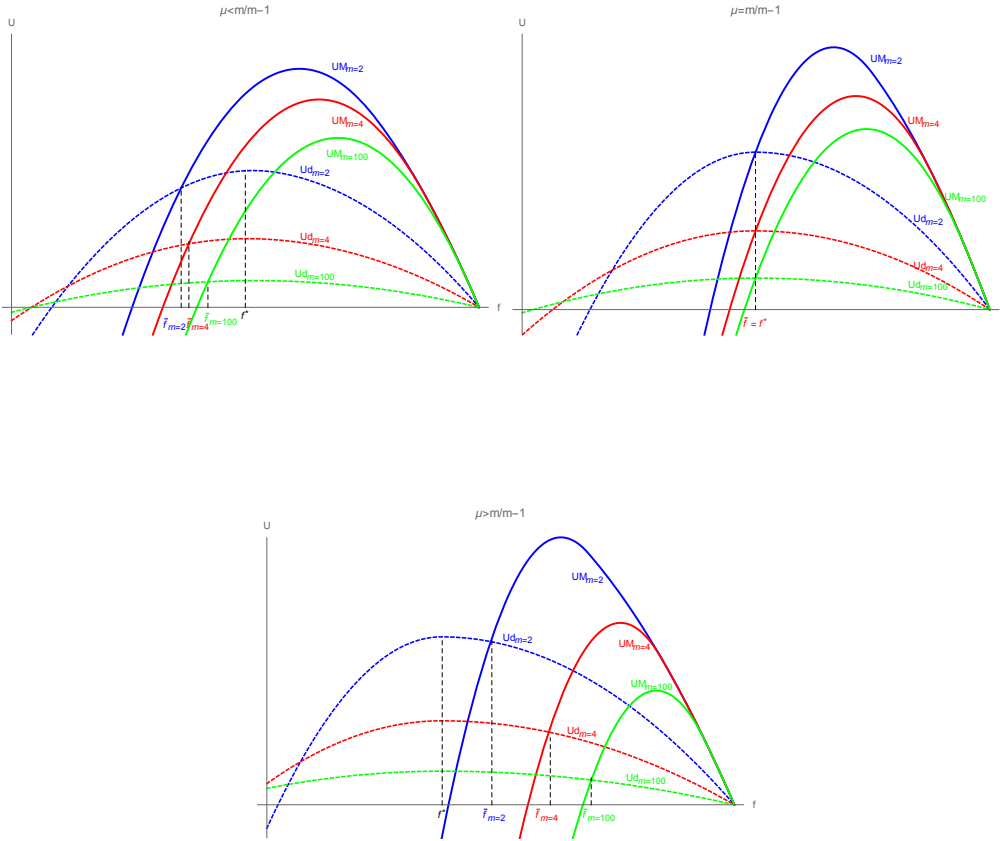


Figure 1: The different cases of price competition depending on μ for $m = 2, 4, 100$ ($\gamma = 0.001, s = 0.001, \delta = 0.0003$).

From the above properties, we can consider two types of surprising effects:

- (i) Conventional wisdom may suggest that when the number of universities increases in Bertrand competition, the outcome tends to be more competitive. As showed previously, Cournot competition between m universities implies that tuition fees decrease and the total number of students increases with m . However, we show that the competition in tuition fees may actually be easier to sustain collusive outcomes when the number of universities increases. This is somewhat counterintuitive and goes against conventional wisdom.
- (ii) if the cost function is sufficiently convex ($\mu > \frac{m}{m-1}$) then f^* is the unique equilibrium and it is constant with m ; if the cost function is less convex ($\mu < \frac{m}{m-1}$) then \bar{f} tends to increase towards f^* on condition that the convexity of cost is sufficiently high ($\mu > \bar{\mu}$).

A possible intuition behind these properties is the following. The increase in m has two opposite effects on the level of fees. On the one hand, an increase in the number of universities implies that each university will face lower marginal costs since the number of students enrolled n_i tends to fall (with the sharing of demand). Due to lower marginal costs (costs are less convex), there are more incentives for fee reduction through the "demand effect". On the other hand, there exists a "capacity effect": increasing the number of universities also implies a lower capacity. If costs are more convex, then marginal costs tend to be higher. This effect increases the level of fees. The "capacity effect" may or not dominate the "demand effect". When μ is very low ($< \bar{\mu}$), the second effect dominates the "capacity effect" and fee \bar{f} falls with m . If the "capacity effect" is more important, $\frac{\partial \bar{n}_i^*(m)}{\partial m} - \frac{\partial \bar{k}_b^*(m)}{\partial m} > 0$, this implies higher marginal costs and an increase in fee \bar{f} with a rising m until the maximum fee f^* . When costs become "too" convex ($C''(n_i) \geq \frac{2m}{m-1}$ or $\mu \geq \frac{m}{m-1}$), each university adjusts the capacity, which maximizes its revenue and minimizes its average cost. The choice in capacity allows to maintain the fee at its maximum level, f^* , and it is invariant regardless of the number of universities.

3.3 Comparison between the Two Competing Strategies

Proposition 2. *If $k_i > 0$, for all parameters $\gamma, s, \delta, \mu > 0$, the subgame perfect Nash Equilibrium in terms of capacities is such that:*

- (i) if $\mu < \frac{m}{m-1}$ then $\bar{k}_b^*(m) < k_c^*(m)$
- (ii) if $\mu = \frac{m}{m-1}$ then $\tilde{k}_b^*(m) < k_c^*(m)$
- (iii) if $\mu > \frac{m}{m-1}$ then $k_b^*(m) < k_c^*(m)$

Proof: in Appendix 8. \square

Whatever the convexity of costs, Cournot competition always implies higher capacities than Bertrand competition. In line with our results, we should observe that universities are smaller in countries where fees have been deregulated, and bigger in countries where fees are regulated.

Proposition 3. *If $k_i > 0$, for all parameters $\gamma, s, \delta, \mu > 0$, the subgame perfect Nash Equilibrium in terms of tuition fees is such that:*

- (i) if $\mu \leq \underline{\mu}(m)$ then $\bar{f}_b^*(m) \leq f_c^*(m)$ and $\bar{n}_b^*(m) \geq n_c^*(m)$,
- (ii) if $\underline{\mu}(m) < \mu < \frac{m}{m-1}$ then $\bar{f}_b^*(m) > f_c^*(m)$ and $\bar{n}_b^*(m) < n_c^*(m)$,
- (iii) if $\mu = \frac{m}{m-1}$ then $\tilde{f}_b^*(m) > f_c^*(m)$ and $\tilde{n}_b^*(m) < n_c^*(m)$,
- (iv) if $\mu > \frac{m}{m-1}$, $f_b^*(m) > f_c^*(m)$ and $n_b^*(m) < n_c^*(m)$.

with $\underline{\mu} \leq 0.458952$ and $\frac{\partial \underline{\mu}}{\partial m} < 0$.

Proof: in Appendix 8. \square

We show that Bertrand competition can induce a higher (or lower) level of fees than Cournot competition depending on the level of the marginal cost μ . If the cost is sufficiently convex, Bertrand competition implies a high (low) level of tuition fees (number of enrolled students par university) greater (lower) than in Cournot competition. According to conventional wisdom, Bertrand competition becomes more competitive when the convexity of cost is low. Whatever the parameter values of the game (s, γ, δ and μ), the total number of student in a deregulated system is lower than in a regulated system.

4 Efficient Capacity and Welfare

4.1 Comparison between Universities' Equilibrium Capacity and Efficient Capacity

Whatever the competing strategies adopted by universities, the cost function of each university i is given by:

$$C(k_i, n_i) = \begin{cases} \delta k_i & \text{if } n_i \leq k_i \\ \delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i \end{cases} \quad (9)$$

Because of the U-shaped average cost in k , there exists an efficient capacity which minimizes the average cost:

$$\text{Min}_{n_i} AC(k_i, n_i) = \text{Min}_{n_i} \left[\frac{C(k_i, n_i)}{n} \right]$$

The capacity which minimizes the average cost for a given number of students enrolled is:

$$k_{\min} = \frac{2\mu n_i - \delta}{2\mu}$$

Assuming that universities adopt their efficient capacities, they will compete in the second stage of the game according to the competing strategies (number of students enrolled/level of tuition fees). The different results depending on the form of competition are presented in the following tables (Tables 1 and 2):

capacities	$k_c^{\min} = \frac{2\mu(1+\gamma+s)-(2\mu+m+1)\delta}{2\mu(m+1)}$
number of students	$n_c^{\min} = \frac{1+\gamma+s-\delta}{m+1}$
level of fees	$f_c^{\min} = \frac{1-m(\gamma+s-\delta)}{m+1}$

Table 2: Efficient solution in k with Cournot competition

	$\mu < \frac{m}{m-1}$	$\mu \geq \frac{m}{m-1}$
capacities	$k_b^{\min} = \frac{2\mu(1+\gamma+s)-(m+\mu m+\mu)\delta}{2\mu(m+\mu m-\mu)}$	$k_b^{\min} = \frac{\mu(1+\gamma+s)-(m+\mu)\delta}{2\mu m}$
number of students	$\bar{n}_b^{\min} = \frac{1+\gamma+s-\delta}{m+\mu m-\mu}$	$n_b^{\min} = \frac{1+\gamma+s-\delta}{2m}$
level of fees	$\bar{f}_b^{\min} = \frac{\mu(m-1)-m(\gamma+s-\delta)}{m+\mu m-\mu}$	$f_b^{\min} = \frac{1-\gamma-s+\delta}{2}$

Table 3: Efficient solution in k with Bertrand competition

Proposition 4. *The following table presents the comparison between the equilibrium results of the two competing strategies with the efficient solution:*

	Cournot equilibrium vs efficient solution	Bertrand equilibrium vs efficient solution	
		$\mu < \frac{m}{m-1}$	$\mu \geq \frac{m}{m-1}$
capacities	$k_c^{\min} < k_c^*$	$k_b^{\min} > k_b^*$	$k_b^{\min} = k_b^* = \tilde{k}_b$
number of students	$n_c^{\min} < n_c^*$	$n_b^{\min} > n_b^*$	$n_b^{\min} = n_b^* = \tilde{n}_b$
level of fees	$f_c^{\min} > f_c^*$	$f_b^{\min} < f_b^*$	$f_b^{\min} = f_b^* = \tilde{f}_b$

Regarding our results, Cournot competition is likely to induce too high a capacity level in terms of the minimization of the average cost for the university. Bertrand competition seems more efficient from this point of view but only if the cost is sufficiently convex.

4.2 Welfare Analysis

Define social welfare as $SW^k = mU_i^k + CS^k$, where $k = c, b$ and $CS^k = \frac{N^k{}^2}{2}$ is consumer surplus, defined in terms of the number of students enrolled $N^k = m.n^k$.

Another question we address is whether Cournot competition results in higher welfare level relative to Bertrand competition. The following proposition summarizes the result:

Proposition 5. *If the parameter μ is not "too low", social welfare in the Cournot competition is always greater than in the Bertrand competition:*

$$SW^c > SW^b$$

Proof: It is not possible to derive an analytical solutions. Therefore, comparison between equilibrium welfare levels was illustrated by plotting following expressions:

$$SW^b = \frac{m}{2} (2(1 + \gamma + s)n_b - m(n_b)^2 - 2\delta k_b - 2\mu(n_b - k_b)^2)$$

$$SW^c = \frac{m}{2} (2(1 + \gamma + s)n_c - m(n_c)^2 - 2\delta k_c - \max(0, 2\mu(n_c - k_c)^2))$$

The social welfare levels depending on μ are illustrated in Figure 2 for $m = 2, 10, 100, 1000$.

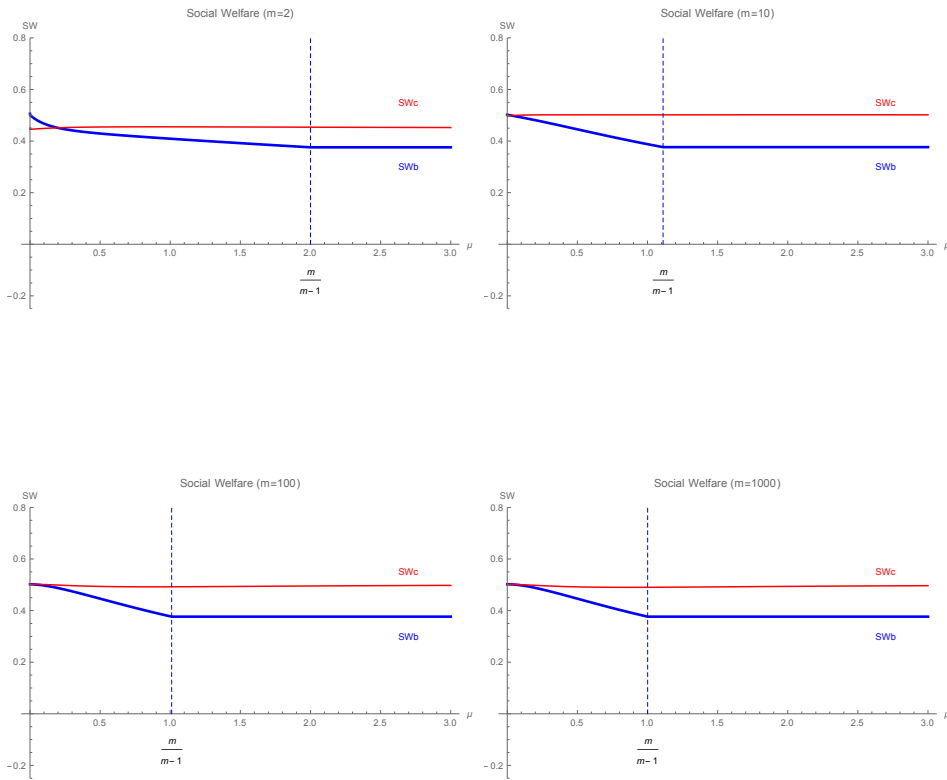


Figure 2: Social welfare levels for $m = 2, 10, 100, 1000$ ($\gamma = 0.001, s = 0.001$).

The above proposition has a natural explanation: when universities compete in fees to attract students, the number of students restriction and level of fees increase, which is detrimental to welfare.

5 Conclusion

In this paper, we have analyzed a two-stage game between universities that decide on their capacity levels, tuition fees and the number of students enrolled. Interestingly, our model emphasizes the role of the strategic variable on output results. Finally, we show that Bertrand competition is not more competitive than Cournot competition: in Cournot oligopoly, with capacity constraints and convex costs, if the number of universities increases, then the resulting equilibrium tends to be competitive with a high capacity level, a high number of students and low tuition fees. However, we show that in Bertrand competition, under certain conditions, the opposite effect is verified. The choice in capacity allows to sustain the fee at its maximum level. Higher tuition in most countries which have adopted a deregulated system (as in the USA, the UK, Canada, and Australia) can now be explained by the theory of imperfect competition. Moreover, high fees imply an efficient choice in capacity because the equilibrium adopted in Bertrand competition may be more efficient in terms of cost minimization than in Cournot competition, which implies too high a capacity level; however, the restriction on number of students enrolled induces a lower social welfare level.

Some of the simplifying assumptions deserve more discussion. First of all, one should not ignore the specification of the demand function that has generated our results. We assume that demand is always infinitely elastic; the price elasticity of demand can have an impact on the different equilibrium results depending on the form of competition. The access to significant financial support or other aids for students may influence our results. Secondly, we assume that there is no differentiation in curriculum, and/or in other non-price dimensions. Finally, our model is based on the fact that universities are committed to satisfying the full demand and are not able to select their students. However, tuition fees might be the solution to select students in the presence of imperfect and asymmetric information (Gary-Bobo and Trannoy, 2008). In view of these elements, it seems necessary to explore competition between universities more deeply and more systematically.

Appendix 1: Proof of Lemma 1

$$k_c^*(m) = n_c^*(m) + \frac{2\mu(m-1)(1+\gamma+s) - (m+1)(1+2\mu)(3+2\mu)\delta}{2\mu((m+1)(1+2\mu)(3+2\mu) - 2\mu(m-1))}$$

$$\Leftrightarrow n_c^*(m) > k_c^*(m)$$

if and only if $\delta > \frac{2\mu(m-1)}{(1+2\mu)(3+2\mu)(m+1)}(1 + \gamma + s) = \underline{\delta}$

with $\frac{\partial \underline{\delta}}{\partial m} > 0$.

Appendix 2: Proof of Corollary 1

(i)

$$n_c^* = \frac{(1+2\mu)(3+2\mu)}{Z}(1 + \gamma + s - \delta)$$

with $Z = (m+1)(1+2\mu)(3+2\mu) - 2\mu(m-1)$

$$\frac{\partial n_c^*(m)}{\partial \mu} = \frac{-2(4\mu^2 - 3)(m-1)}{Z^2}(1 + \gamma + s - \delta)$$

$$\Leftrightarrow \text{sign} \left(\frac{\partial n_c^*}{\partial \mu} \right) = \text{sign}(3 - 4\mu^2) < 0 \quad \text{if } \mu > \sqrt{\frac{3}{4}}$$

$$(ii) \text{sign} \left(\frac{\partial f_e^*}{\partial \mu} \right) = \text{sign}(-3 + 4\mu^2) > 0 \quad \text{if } \mu > \sqrt{\frac{3}{4}}$$

(iii) From Equation 5, we have:

$$k_c^* = \frac{2\mu + m + 1}{2\mu} n_c^* - \frac{1 + \gamma + s}{2\mu}$$

$$\frac{\partial \bar{k}_c^*}{\partial \mu} = \frac{(2\mu + m + 1)\mu \frac{\partial n_c^*}{\partial \mu} - (m+1)n_c^* + (1 + \gamma + s)}{2\mu^2}$$

$$\Leftrightarrow \text{sign} \left(\frac{\partial \bar{k}_c^*}{\partial \mu} \right) < 0$$

if $\delta < \frac{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+2\mu(m-1)Z}{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+(m+1)(1+2\mu)(3+2\mu)Z}(1 + \gamma + s) = \bar{\delta} \in (0; 1)$

Appendix 3: Proof of Corollary 2

$$i) \frac{\partial n_c^*(m)}{\partial m} = -\frac{(3+6\mu+4\mu^2)(1+2\mu)(3+2\mu)}{((m+1)(1+2\mu)(3+2\mu)-2\mu(m-1))^2}(1 + \gamma + s - \delta) < 0$$

$$ii) \text{sign} \left(\frac{\partial N_e^*(m)}{\partial m} \right) = \text{sign}(3 + 8\mu + 4\mu^2) > 0$$

$$iii) \frac{\partial f_e^*(m)}{\partial m} = -\frac{\partial N_e^*(m)}{\partial m} < 0$$

$$iv) \frac{\partial k_c^*(m)}{\partial m} = -\frac{n_c^*}{2\mu}(10\mu + 12\mu^2 + 8\mu^3) < 0$$

Appendix 4: Proof of Proposition 1

Let us first investigate symmetric strategy profiles belonging to the interval $[\hat{f}, \bar{f}]$. When the competitor charges any fee $f \in [\hat{f}_i; \bar{f}_i]$, the best response for university i is to quote the same tuition fee. When university i quotes the same fee, it gets $U_d(f)$. We know that for all $f \geq \hat{f}_i$, $U_d(f) \geq -TFC$. If the university deviates (by quoting $f - \epsilon$), it gets $U_d(f - \epsilon)$. We also know that for all $f \in [\hat{f}_i; \bar{f}_i]$, $U_d(f) \geq U_M(f) > U_M(f - \epsilon)$. Since the university must supply all the demand it faces, the increase in additional revenue (from higher enrollment) is less than the increase in costs: the university must enroll additional students at excessive marginal costs. By quoting $f + \epsilon$, university i obtains no demand and gets zero variable utility needed for the research activity. Hence it is optimal for each university to quote the same fee. There are no incentives to deviate, which proves the implication in Proposition 1. It also proves that all asymmetrical strategy profiles with at least one firm quoting a price in the interval are not Nash equilibria. We now have to investigate all the other strategy profiles in which none of the firms quote a price within the interval. It is easy to check that for all symmetric strategic profiles such that $f < \hat{f}$, increasing its tuition fee is in the firm's interest. Lowering its fee is in the university's interest with $f > \bar{f}$. The payoff dominance criterion, which appears to be the natural criterion when it is common knowledge that both actors are fully rational is sufficient to provide uniqueness in the three configurations. \square

Appendix 5: Proof of Corollary 3

Let us compare the number of students enrolled in each university with the capacity for the different cases:

$$n_b^*(m) > k_b^*(m) \quad \forall \gamma, s, \text{ et } m$$

$$(i) \text{ If } \mu < \frac{m}{m-1} : n_b^*(m) > k_b^*(m) \Leftrightarrow 2\mu(m + m\mu + \mu)(1 + \gamma + s - \delta) > 4\mu^2m(1 + \gamma + s) - (m + m\mu + \mu)^2\delta$$

$$\Leftrightarrow 2\mu(m + \mu - m\mu)(1 + \gamma + s) > (m + m\mu + \mu)(\mu - m - m\mu)\delta$$

which is always verified because $m + \mu - m\mu > 0$ and $\mu - m - m\mu < 0$.

$$(ii) \text{ If } \mu = \frac{m}{m-1} : \tilde{n}_b^*(m) > \tilde{k}_b^*(m) \Leftrightarrow \frac{1+\gamma+s-m\delta}{2m} < \frac{1+\gamma+s-\delta}{2m} \quad \forall m \geq 2 \text{ which is always verified.}$$

$$(iii) \text{ If } \mu > \frac{m}{m-1} : n_b^*(m) > k_b^*(m) \Leftrightarrow \frac{m+\mu}{\mu} > 1 \quad \forall m \geq 2 \text{ which is always verified.}$$

Appendix 6: Proof of Corollary 4

Case 1. If $\mu < \frac{m}{m-1}$:

$$\frac{\partial \bar{n}_b^*}{\partial \mu} = -\frac{m\mu(\mu(m-1) + 2m(m-2) + \mu^2m + 2) + \mu^2 + m^2 + m^3}{(\mu^2 + m^2 - 2m\mu^2 + 2m\mu + 2m^2\mu + m^2\mu^2)^2} < 0 \quad \text{with } m \geq 2$$

$$\Leftrightarrow \frac{\partial \bar{f}_b^*}{\partial \mu} > 0$$

From Equation 7, we have:

$$\bar{k}_b^* = -\frac{m + \mu m + \mu}{2m\mu} \bar{f}_b^* + \frac{m+1}{2m} - \frac{\gamma+s}{2\mu}$$

$$\frac{\partial \bar{k}_b^*}{\partial \mu} > 0 \quad \text{with} \quad \frac{\partial \bar{f}_b^*}{\partial \mu} > 0$$

We can get cases 2 ($\mu = \frac{m}{m-1}$) and 3 ($\mu > \frac{m}{m-1}$) with straightforward computations.

Appendix 7: Proof of Corollary 5

Case 1. If $\mu < \frac{m}{m-1}$:

$$\frac{\partial \bar{k}_b^*(m)}{\partial m} = -\frac{2\mu(m^2(1+\mu)^2 - \mu^2)}{(\mu^2 - 2\mu m(\mu-1) + m^2(1+\mu)^2)^2} (1+s+\gamma-\delta) < 0 \quad \forall m \geq 3$$

$$\frac{\partial \bar{f}_b^*(m)}{\partial m} = -\frac{\mu(\mu^2 + 2\mu m(1+\mu) - m^2(3\mu^2 + 2\mu - 1))}{(\mu^2 - 2\mu m(\mu-1) + m^2(1+\mu)^2)^2} (1+s+\gamma-\delta)$$

- if $\mu > \bar{\mu}$ hence $\frac{\partial \bar{f}_b^*(m)}{\partial m} > 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} < 0, \quad \forall m \geq 3$
- if $\mu = \bar{\mu}$ hence $\frac{\partial \bar{f}_b^*(m)}{\partial m} = 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} = 0 \quad \forall m \geq 3$
- if $\mu < \bar{\mu}$ hence $\frac{\partial \bar{f}_b^*(m)}{\partial m} < 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} > 0 \quad \forall m \geq 3$

with $\bar{\mu} = \frac{m-m^2+2\sqrt{m^4-m^3}}{3m^2-2m-1-1}$

$$\frac{\partial \bar{n}_b^*(m)}{\partial m} = -\frac{\mu^2(1-3\mu) + 2\mu m(1+\mu)^2 + m^2(1+\mu)^3}{(\mu^2 - 2\mu m(\mu-1) + m^2(1+\mu)^2)^2} (1+s+\gamma-\delta) < 0 \quad \forall m \geq 3$$

We can get cases 2 ($\mu = \frac{m}{m-1}$) and 3 ($\mu > \frac{m}{m-1}$) with straightforward computations.

Appendix 8: Proof of Propositions 2-3

Case 1 - If $\mu < \frac{m}{m-1}$:

$$n_c^*(m) - \bar{n}_b^*(m) =$$

$$\frac{m^2(5\mu+13\mu^2+12\mu^3+4\mu^4)-m(3+10\mu+10\mu^2+16\mu^3+8\mu^4)+\mu(4\mu^3+4\mu^2-7\mu-3)}{((m+1)(1+2\mu)(3+2\mu)-2\mu(m-1))(\mu^2+m^2-2m\mu^2+2m\mu+2m^2\mu+\mu^2m^2)}(1 + \gamma + s - \delta)$$

We verify that

$$\text{sign}(n_c^*(m) - \bar{n}_b^*(m)) =$$

$$\text{sign}(m^2(5\mu + 13\mu^2 + 12\mu^3 + 4\mu^4) - m(3 + 10\mu + 10\mu^2 + 16\mu^3 + 8\mu^4) + \mu(4\mu^3 + 4\mu^2 - 7\mu - 3))$$

We define $\underline{\mu}(m)$ such that if $\mu > \underline{\mu}(m)$ hence

$$n_c^*(m) > \bar{n}_b^*(m)$$

The value of $\underline{\mu}(m)$ is solved by the Mathematica program. For $m = 2$, we have $\underline{\mu} = 0.458952$, for $m=3$, $\underline{\mu} = 0.249037$;...for $m = 10$, $\underline{\mu} = 0.06297$.

We conclude that:

- $n_c^*(m) < \bar{n}_b^*(m)$, $f_c^*(m) > \bar{f}_b^*(m)$, and $N_c^*(m) < \bar{N}_b^*(m)$ if $\mu \in]0, \underline{\mu}(m)[$
- $n_c^*(m) = \bar{n}_b^*(m)$, $f_c^*(m) = \bar{f}_b^*(m)$, and $N_c^*(m) = \bar{N}_b^*(m)$ if $\mu = \underline{\mu}(m)$
- $n_c^*(m) > \bar{n}_b^*(m)$, $f_c^*(m) < \bar{f}_b^*(m)$, and $N_c^*(m) > \bar{N}_b^*(m)$ if $\mu \in]\underline{\mu}(m), \frac{m}{m-1}[$

We verify that $\bar{k}_c^*(m) > \bar{k}_b^*(m) \quad \forall \mu \in [0, \frac{m}{m-1}[$.

Case 2 - If $\mu = \frac{m}{m-1}$, we have

$$n_c^*(m) - \bar{n}_b^*(m) = \frac{(m+1)^2(17m^2-16m+3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0$$

$$N_c^*(m) - \bar{N}_b^*(m) = \frac{(m+1)^2(17m^2-16m+3)}{m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0$$

$$f_c^*(m) - \bar{f}_b^*(m) = -\frac{(m+1)(17m^2-16m+3)}{2(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) < 0$$

$$k_c(m) - \bar{k}_b^*(m) = \frac{(17m-27m^2+11m^3+2m^4-3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0$$

Case 3 - If $\mu > \frac{m}{m-1}$:

$$n_c^*(m) - n_b^*(m) = \frac{(m+1)^2(17m^2-16m+3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0$$

$$\begin{aligned}
N_c^*(m) - N_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\
f_c^*(m) - f_b^*(m) &= -\frac{(m+1)(17m^2-16m+3)}{2(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) < 0 \\
\bar{k}_c^*(m) - \bar{k}_b^*(m) &= \frac{(m+2m^2+10\mu(m-1)+4\mu^2(m-1))}{6m(m+1)+2\mu m(6m+5)+4\mu^2 m(4m+1)}(1 + \gamma + s - \delta) > 0
\end{aligned}$$

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