



**HAL**  
open science

# Two University Competition Systems; Their Impact on Capacity, Tuition Fees and Admission

Marie-Laure Cabon-Dhersin, Jonas Didisse

► **To cite this version:**

Marie-Laure Cabon-Dhersin, Jonas Didisse. Two University Competition Systems; Their Impact on Capacity, Tuition Fees and Admission. 2016. halshs-01174291v2

**HAL Id: halshs-01174291**

**<https://shs.hal.science/halshs-01174291v2>**

Preprint submitted on 16 Jun 2016 (v2), last revised 23 Aug 2017 (v3)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Two University Competition Systems: Their Impact on Capacity, Tuition Fees, and Admission

Marie-Laure Cabon-Dhersin\*, Jonas Didisse†

June 14, 2016

## Abstract

This paper compares university competition through fees (as in Anglo-Saxon countries, following Bertrand competition) and admission (as in old European countries, following Cournot competition) in a setting where universities endogenously choose their capacity constraint. The results show that, with competition over fees and sufficiently convex costs, universities adopt a low capacity to set higher fees. This capacity adopted is efficient in terms of cost minimization. Then, we find that introducing a new university is welfare improving in Bertrand competition. Conversely, competition over admission leads to a higher capacity and a greater number of students enrolled per university, which may be detrimental to welfare.

**Key words:** Cournot competition, Bertrand competition, capacity, higher education market.

**Code JEL:** C7, H52, I22, I23.

## 1 Introduction

Public authorities have a real interest in ensuring that the student population has access to a broad range of higher education opportunities. It is therefore important to understand how the strategic behavior of universities can affect the provision of higher education and at what cost.

In the UK, a recent reform was focused on the tuition fee deregulation, which has led to nearly tripling fees, combined with relaxing controls on the number of students each university can recruit. Since 2012, the vast majority of UK public universities have been charging students tuition fees of £9,000, the maximum allowable rate under a government-imposed cap. However, the tuition fee deregulation should lead to a consequent drop in fees. It is tempting to rely on industrial organization theories to explain this paradox. An interesting issue, in particular, is how competition between universities affects the level of fees and the choice of the number of students enrolled. Another issue which we address here concerns the number and size of universities which impact the level of fees and enrollment: is it preferable to have many small universities or a few large universities?

---

\*Corresponding author: University of Rouen, CREAM, 3 avenue Pasteur, 76100 Rouen, France. (marie-laure.cabon-dhersin@univ-rouen.fr)

†University of Rouen, CREAM, 3 avenue Pasteur, 76100 Rouen, France. (jonas.didisse1@univ-rouen.fr)

In the literature, many papers examine the tensions between teaching and research in competition models depending on the funding rules and/or the mobility costs of students (Del Rey, 2001; De Fraja and Iossa, 2002; Gautier and Wauthy, 2007; Grazzini et al., 2010; Beath et al., 2012). Contributions on the effects of the size and number of universities in the higher education market are scarce. Recently, with a no-fee system, Cesi and Paolini (2014) find that one more university is always welfare improving, especially when leading to an equal two-university system. In our paper, we propose to analyze the choice of capacity (size) with more than two universities and its impact on fees and admission depending on the number of universities in the market.

In order to answer our research questions, we will assume for the purposes of our analysis that there are two different university systems which differ by the strategic variable retained by universities depending on the political context:

- the *Anglo-Saxon system*: universities can set their own fees (as in the UK, the USA, and Canada, among others). For the same level of requirements (or abilities of students), universities may compete on fees to attract students (as in Bertrand competition),
- the *European system*: universities are not free to use the variable pricing strategy as a response to their environment (as in France, Spain, Italy, and Germany). Tuition fees are set by public authorities without control over the number of students enrolled. In this case, universities may adopt another form of competition in terms of the number of students enrolled (as in Cournot competition).

The distinction between these two university systems may prove particularly relevant in OECD countries to better understand disparities in tuition fees and university size. Table 1 shows some interesting stylized facts on which our analysis is constructed. They suggest that:

- (i) there is a greater (smaller) number of small (large) universities in Anglo-Saxon countries than in old European countries.
- (ii) a higher level of tuition fees is observed in Anglo-Saxon countries compared with European countries.
- (iii) while Anglo-Saxon countries have a much higher proportion of students enrolling in higher education compared with Old European countries, the average number of students per university is lower in the Anglo-Saxon system.

	Annual average fees (€) <sup>a</sup>	Entry rates into higher education <sup>b</sup>	Average nb of students per university	% of small univ. ( < 10000)	% of great univ. ( > 25000)
Germany	No fees	53,18%	22 646	13%	42,5%
France	[189-261]	40,85%	21 646	19%	33,5%
Italy	[200-1000]	47,2%	22 564	31%	38%
Spain	[1000-2000]	52,1 %	27 547	17,7%	46%
Netherlands	2000	65,27	17 066	21%	21%
Canada	[3000-4000]	-	13 670	54%	22%
UK	[5000-11000]	67,44%	14 678	38%	15,7%
USA	[3500-20000]	71,02%	12 422	58,5%	14,6%

Table 1: Fees and size of public universities in 2014-15

<sup>a</sup>National Student Fee and Support Systems 2014/15, European Commission

<sup>b</sup>OECD (2014), Education at a glance, OECD, Publishing, Paris, Indicator C3

According to the standard economic intuition, a competition in fees between many universities with a small capacity should lead to low fees. Competition through admission between universities with a large capacity and low fees should also lead to a high entry rate into higher education. How can we explain the opposite phenomenon?

In this paper, we will focus on the importance of considering capacity constraints in order to bring new insight into the consequences of competition between homogeneous universities. It may be relevant to assume that universities decide on their enrollment capacity before competing (setting their fees or admission). Using the notion of "non-rigid" capacity introduced by Chowdhury (2009), Jacqmin and Wauthy (2014) argue that competition between universities which produce both research and teaching may generate a framework in which the cost of providing education is convex beyond a certain threshold. In this work, the capacity constraint is considered as exogenous. In our paper, we propose another novel setting with sequential choice. In the first stage, universities endogenously choose a capacity level which is invariable during the second stage. In the second stage, universities compete in terms of tuition fees (Anglo-Saxon system) or in terms of students enrolled (European system) to satisfy the demand which they will face<sup>1</sup>. When the number of students enrolled is higher than the capacity threshold, the cost function is strictly convex. The form of the cost function may be explained in several ways: (i) universities may be committed to satisfying enrollment demand, (ii) having too many students negatively impacts the research output by increasing the opportunity cost of providing education services since the available research time decreases, and (iii) the fixed assets used in educational activities (such as the number of teachers, libraries, rooms, or administrative services) are rivals in their consumption and have a fixed capacity.

The contribution of the present work may be understood as follows. To the best of our knowledge, no existing theoretical model has been proposed to explain the choice of capacity, the level of fees/the number of students enrolled in the context of competition. We focus on a setup in which universities compete for students by setting enrollment or fees in the presence of ("soft") capacity constraints (Cabon-Dhersin and Drouhin, 2014). On positive grounds, the results in terms of the equilibrium levels of fees/number of students, capacity, and social welfare allow for a comparison between the two underlying university systems (Anglo-Saxon

---

<sup>1</sup>We assume here that students who apply for admission are similar in terms of abilities and grades. In the Anglo-Saxon system, a selection process may already take place in universities. In the European system, a simple admission standard in terms of grades in secondary school is required to have access to university education.

vs Old European systems). We also determine the efficient capacity for each university and compare it with the equilibrium results. We show that:

- (i) in Bertrand competition, when costs are sufficiently convex, universities adopt a low capacity in the first stage to set higher fees in the second stage; conversely, competition over admission leads to a large capacity and a greater number of students enrolled per university,
- (ii) when the number of universities increases, the level of tuition fees rises up to the maximum level of fees in Bertrand competition; conversely, in Cournot competition, the total number of students enrolled increases with the number of universities,
- (iii) the equilibrium adopted in Bertrand competition may be more efficient in terms of cost minimization than in Cournot competition,
- (iv) an increase in the number of universities may have a positive impact on the social welfare in Bertrand competition but not necessarily in Cournot competition.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyzes and compares the equilibrium results in terms of fees/number of students enrolled and capacity in the two competition systems. Section 4 proposes to compare the equilibrium level of capacity in both models with an efficient capacity, and extends the comparative results to social welfare. Section 5 concludes.

## 2 Model

We assume that the creation (research as  $R$ ) and dissemination (teaching as  $T$ ) of fundamental knowledge is the mission of all universities. Thus, each university's objective function can take the general form  $U(T, R)$ , where  $U$  is strictly increasing in both arguments. In the higher education literature, the separability of the objective function allows to consider that universities may specialize completely in teaching  $T$  or research  $R$ . In the same line as Jacqmin and Wauthy (2014), the objective function of universities is specified as follows:

$$\text{Max } U_i(T, R) \tag{1}$$

with  $\frac{\partial U_i}{\partial T} > 0$  and  $\frac{\partial U_i}{\partial R} > 0$

As suggested by Rothschild and White (1995), there are two basic reasons to explain why universities may compete for students:

- As inputs, students are required for the production of education. With regard to teaching activities, we assume conveniently that the level of teaching is equal to the enrollment of students  $n_i$  weighted according to the parameter  $0 < \gamma < 1$ :

$$T = \gamma n_i$$

where  $\gamma$  measures the importance of the teaching compared to the research objective, and  $n_i$  is the number of students enrolled in each university  $i$ .

- As clients, students provide the funds a university needs to operate, either directly through fees ( $f$ ) or indirectly via the government. Research activities require money. In this respect,  $R$  represents each university's expenditure on research. Here, the research output  $S_i$  depends only on the money invested in it:

$$R = S_i$$

Since research requires money, more students enrolled may imply an increased research budget. This is especially relevant when a university is funded through a per-student subsidy  $s$  provided by the government agency in charge of higher education with  $0 < s < 1$ .

However, increasing the size of the student population is costly, in particular when the number of students enrolled exceeds a university's capacity  $k$ . The cost supported by each university depends on the level of the capacity  $k_i$  with a unit cost  $\delta$ , and, as far as  $n_i > k_i$ , depends quadratically on the number of students beyond capacity. According to Jacquemin and Wauthy (2014) or Chowdhury (2009), capacity constraints are not drastic (i.e. education services may be provided above capacity); by incurring an additional per unit cost (noted by  $\mu$ ), universities can enroll students beyond capacity.

Thus, the cost function of each university  $i$  is given by

$$C_i(n_i, k_i) = \begin{cases} \delta k_i & \text{if } 0 \leq n_i \leq k_i \\ \delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i \end{cases} \quad (2)$$

where  $\mu > 0$ . The cost parameters ( $\delta$  and  $\mu$ ) are similar for all universities and constant.

Our model considers a novel setting with sequential choice: in the first stage, universities choose a capacity level  $k_i$  that will be invariable during the second stage. In the second stage, universities compete on price (level of tuition fees) or on quantity (number of students enrolled) to satisfy the demand they face. Note that in the second stage,  $\delta k_i$  is like a "sunk cost" (which is unavoidable in this stage)<sup>2</sup>. This implies that, when the number of students enrolled is lower than the capacity threshold, the marginal cost of each student is nil up to the capacity threshold  $k_i$  and the average cost per student is decreasing with  $n_i$ . Beyond capacity level  $k_i$ , the marginal cost is strictly convex and the average cost appears to be on the rise. The cost function of universities exhibit the more familiar case of U-Shaped average costs<sup>3</sup>.

The optimization problem of a university  $i$  is thus defined as:

$$\text{Max } U_i = \gamma n_i + S_i \quad \text{s.t.} \quad S_i + C_i(n_i, k_i) = n_i(f_i + s) \quad (3)$$

We consider a higher education market with  $m$  identical universities. We assume that universities are homogeneous: there is no differentiation in curriculum, and/or in other non-price dimensions (location, different trade-offs between teaching and research, admission standard, or financial endowment among others). Universities may (without obligation) set up a student selection process, based on past performance or on an admission test. These procedures allow universities to have perfect information about the candidates' abilities. Then, for the same level of requirements, all universities consider the same population of students as admissible.

<sup>2</sup>With avoidable fixed costs, there are problems linked to the existence of a price equilibrium (Saporiti and Coloma, 2010; Dastidar, 2011b,a).

<sup>3</sup>The average cost per student completion in Australian universities provides at least some evidence of a U-Shaped long-run average cost curve (Worthington and Higgs, 2011).

Otherwise, in the case of different requirement levels, there is at least a share of the candidates who are eligible in several universities. So, there is always a part of eligible candidates for whom universities may compete. Once selected, universities are committed to satisfying all the eligible students' demand faced in the second stage. The utility which students derive from graduating at university  $i$  is defined by  $u(\theta) = \theta - f_i$  where  $\theta$  represents students' willingness to pay to go to university. We will assume that potential students do not differ in their willingness to enroll <sup>4</sup> and each student implicitly gets one unit of education. The student population eligible in all universities is therefore normalized to the unit.

In order to study inter-university competition and related issues of imperfect competition in higher education, we consider two competition models defined as follows:

- (i) The first competition model consists in adopting the quantity variable (Cournot competition): Since universities are not free to modify the level of fees directly, they compete on the number of students enrolled. As in the Cournot competition model, we consider a market with  $m$  universities facing an inverse demand function  $D^{-1}(N)$  where  $N = \sum_{i=1}^m n_i$  is the total number of students enrolled (each student getting one unit of education):

$$f(n_i, N_{-i}) = 1 - N$$

$$\text{with } N_{-i} = \sum_{\substack{j=2 \\ j \neq i}}^m n_j.$$

- (ii) The second competition model consists in adopting the price variable (Bertrand competition). This competition scenario describes interactions among universities which set fees and where students enroll at the set price. For a same university course, eligible students choose to enroll at the university with the lowest fees. Since universities are strictly homogeneous, the demand for enrollment addressed to a particular university is a discontinuous function of the fees. The demand function of university  $i$  is defined as follows:

$$n_i(f_i, f_{-i}) = \begin{cases} 0 & \text{if } f_i > f_{-i}^{min} \\ \frac{N(f_i)}{m} & \text{if } f_i = f_{-i} \\ N(f_i) = 1 - f_i & \text{if } f_i < f_{-i}^{min} \end{cases}$$

with  $f_i$  the fee of university  $i$ ;  $f_{-i} = \{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$  is the vector of the fees of all universities on the higher education market. We denote  $f_{-i}^{min} = \text{Min}\{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$ .

### 3 Two Competition Models between $m$ Universities

In this section, we stipulate a two-period model in which  $m$  universities decide on the capacity  $k_i$  and then compete in quantity/number of students enrolled (Cournot competition) or in fees

---

<sup>4</sup>As Jacqmin and Wauthy (2014) point out, additional heterogeneity in students' willingness to pay would smooth the process of inter-university competition in prices.

(Bertrand competition), upon observing the chosen capacity in the first stage. We study how the convexity of costs and the number of universities has an influence on the equilibrium results. Then, we compare the two competition models between  $m$  universities.

### 3.1 The Cournot Oligopoly Model with Capacity Constraints

#### 3.1.1 Nash Equilibria

Suppose that  $m$  identical universities interact in the two-stage game to decide non-cooperatively on both capacity and enrollment:

- (i) In the first stage, each university  $i$  chooses its level of capacity  $k_i$ , taking other universities' threshold capacity  $k_{-i}$  into account.
- (ii) In the second stage, each university  $i$  selects a number of students enrolled  $n_i$  for a given level of capacity  $k_i$  and for a given number of students at rival universities.

Universities' strategies consist of a capacity level and a subsequent enrollment strategy based on their capacity.

Consider the utility of university  $i$  in the second stage, conditional on  $k_i$ :

$$\begin{aligned} U_i(n_i, N_{-i}, k_i) &= n_i(f_i(n_i, N_{-i}) + s + \gamma) - C_i(n_i, k_i) \\ &= n_i((1 - n_i - N_{-i}) + s + \gamma) - C_i(n_i, k_i) \end{aligned}$$

$$\text{with } N_{-i} = \sum_{\substack{j=2 \\ j \neq i}}^m n_j.$$

The utility function is strictly concave in  $n_i$  and  $k_i$ .

In the second stage, each university chooses the number of students enrolled to maximize its utility function:

$$\frac{\partial U_i}{\partial n_i} = 0$$

$$\Leftrightarrow 1 + \gamma + s - n_i - N - 2\mu n_i + 2\mu k_i = 0 \quad (4)$$

$$\text{with } \sum_{i=1}^m n_i = m \cdot n_i = N.$$

The Nash-Cournot equilibrium can be computed to be:

$$n_i = \frac{1 + \gamma + s + 2\mu k_i}{2\mu + m + 1} \quad (5)$$

In the first stage, each university maximizes the utility  $U_i$ , as a function of  $k_i$ , for a given level of other universities' capacity  $k_{-i}$ :

$$\frac{\partial U_i(n_i(k_i, k_{-i}), N_{-i}(k_i, k_{-i}), k_i)}{\partial k_i} = 0$$



$$\Leftrightarrow k_i = \frac{n'_{ik_i}(1 + \gamma + s - 2(1 + \mu)n_i - N_{-i}) + n_i(2\mu - N'_{-ik_i}) - \delta}{2\mu(1 - n'_{ik_i})} \quad (6)$$

By replacing (6) in (4), the unique and symmetric subgame perfect equilibrium values are:

$$k_c^*(m) = \frac{Z}{W}(1 + \gamma + s - \delta) + \frac{2\mu(m-1)(1 + \gamma + s) - (m+1)Z\delta}{2\mu W}$$

$$n_c^*(m) = \frac{Z}{W}(1 + \gamma + s - \delta)$$

and

$$N_c^*(m) = m \cdot n_c^*(m) = \frac{mZ}{W}(1 + \gamma + s - \delta) \leq 1 \quad \text{if} \quad (1 + \gamma + s - \delta) \leq \frac{W}{mZ}$$

with  $Z = (1 + 2\mu)(3 + 2\mu)$  and  $W = (m + 1)(1 + 2\mu)(3 + 2\mu) - 2\mu(m - 1)$ .

**Lemma 1.** *We verify that:*

$$n_c^*(m) \leq k_c^*(m) \quad \text{if and only if} \quad \delta \leq \underline{\delta}$$

$$0 < k_c^*(m) \leq n_c^*(m) \quad \text{if and only if} \quad \underline{\delta} \leq \delta < \bar{\delta}$$

$$\text{with } \underline{\delta} = \frac{2\mu(m-1)}{(m+1)Z}(1 + \gamma + s) \quad \text{and} \quad \bar{\delta} = \frac{2\mu Z + 2\mu(m-1)}{2\mu Z + (m+1)Z}(1 + \gamma + s)$$

**Proof:** see Appendix 1.  $\square$

The interpretation of the above lemma is that each university opts for a greater number of students beyond capacity, provided that the unit cost of installed capacity is not too low (below  $\underline{\delta}$ ). A higher unit cost of installed capacity associated with the lower marginal cost of a supplementary student always leads to more students above capacity. We show that it may actually be difficult to verify this condition when the number of universities increases. By contrast, if the capacity cost is low and the parameter  $\mu$  is high, universities adopt a large capacity and limit their enrollment.

### 3.1.2 Comparative Statics

We can see that the importance of teaching ( $\gamma$ ) and the level of the per-student subsidy ( $s$ ) impact enrollment and capacity positively, but the capacity cost parameter  $\delta$  has a negative effect on these equilibrium outputs. We note opposite effects on equilibrium fees. However, the convexity of costs, measured by the parameter  $\mu$ , has a non-trivial effect on the different equilibrium results. The following corollary presents the effects of a rise in  $\mu$  on the equilibrium results:

**Corollary 1.** *We verify that:*

$$i) \quad \frac{\partial \bar{k}_c^*}{\partial \mu} < 0 \quad \text{if} \quad \delta < \frac{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+2\mu(m-1)W}{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+(m+1)(1+2\mu)(3+2\mu)W}(1 + \gamma + s) \in (0; 1)$$

$$ii) \quad \frac{\partial n_c^*}{\partial \mu} \leq 0 \quad \text{if} \quad \mu > \sqrt{\frac{3}{4}}$$

$$iii) \frac{\partial f_c^*}{\partial \mu} > 0 \quad \text{if} \quad \mu > \sqrt{\frac{3}{4}}$$

**Proof:** see Appendix 2.  $\square$

With increasing marginal costs (for  $\mu > \sqrt{\frac{3}{4}}$ ), the equilibrium number of students enrolled decreases while tuition fees rise. On the condition that the capacity cost is sufficiently high, each university chooses a larger capacity for students; the capacity is all the larger as the marginal cost  $\mu$  is high. When the additional cost of a supplementary student beyond capacity is high (given by the parameter value  $\mu$ ), universities prefer to increase the level of  $k$  and to reduce enrollment. However, if the capacity unit cost is very low, universities can decrease the overall cost by reducing their capacity. In order to determine their capacity level, universities compare the benefit of decreasing capacity (fall in the cost associated with the fixed capacity in the first stage of the game) with the additional cost of enrolling students in the second stage of the game.

We now examine the impact which the number of universities has on the different equilibrium results.

**Corollary 2.** *For all  $\gamma, s, \delta$  and  $\mu$ , we verify that:*

$$\frac{\partial n_c^*(m)}{\partial m} < 0, \quad \frac{\partial N_c^*(m)}{\partial m} > 0, \quad \frac{\partial f_c^*(m)}{\partial m} < 0 \quad \text{and} \quad \frac{\partial k_c^*(m)}{\partial m} < 0.$$

**Proof:** see Appendix 3.  $\square$

At equilibrium, the number of students enrolled per university decreases when the number of universities increases. However, with an increasing number of universities, the total number of students  $N$  may then increase or decrease. The above corollary shows that the total number of students enrolled grows with the number of universities. As in a standard Cournot oligopoly with convex costs, if the number of institutions increases, then the resulting equilibrium approaches competitive equilibrium: tuition fees are sufficiently low to recruit more and more students. However, the capacity for each university decreases with  $m$ .

## 3.2 Bertrand Oligopoly Model with Capacity Constraint

### 3.2.1 Nash Equilibria

Universities interact in the two-stage game to decide non-cooperatively on both capacity and tuition fees:

- (i) In the first stage of the game, universities optimally choose their fixed capacity levels  $k_i$ .
- (ii) In the second stage, we analyze the price competition (in terms of fees  $f$ ) for a given capacity at levels  $k_i$ .

We can now express the utility  $U_i$  for each university  $i$ :

$$U_i(f_i, f_{-i}, k_i, m) = f_i n_i(f_i, f_{-i}) - C(n_i(f_i, f_{-i}), k_i)$$

$$U_i(f_i, f_{-i}, k_i) = \begin{cases} -\delta k_i & \text{if } f_i > f_{-i}^{min} \\ (f_i + \gamma + s) \frac{N(f_i)}{m} - C_i(\frac{N(f_i)}{m}, k_i) = U_d & \text{if } f_i = f_{-i} \\ (f_i + \gamma + s) N(f_i) - C_i(N(f_i), k_i) = U_M & \text{if } f_i < f_{-i}^{min} \end{cases}$$

with  $f_i$ , the fee of university  $i$ . We denote  $f_{-i}^{min} = \text{Min}\{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$ .  $N(f_i)$  is the total number of students enrolled when university  $i$  sets  $f_i$ .

The function  $U_d(f_i, k_i)$  represents the utility of university  $i$  when both universities quote the same fees and the function  $U_M$  represents the utility of university  $i$  when it quotes the lowest tuition fee and serves all demand.  $U_M$  and  $U_d$  are strictly concave in  $f_i$  and strictly concave in  $k_i$ , that is  $\partial^2 U_d(f_i, k_i) / \partial f_i^2 < 0$ ,  $\partial^2 U_d(f_i, k_i) / \partial k_i^2 < 0$ ,  $\partial^2 U_M(f_i, k_i) / \partial f_i^2 < 0$ ,  $\partial^2 U_M(f_i, k_i) / \partial k_i^2 < 0$ .

We define  $\bar{f}(k_i)$  that solves  $U_M(f, k_i, m) = U_d(f, k_i, m)$ . Thus,  $\bar{f}(k_i)$  must be interpreted as the critical fee for which the university is indifferent between operating in the higher education market alone or with its rival. After calculations, we obtain:

$$(1 - f_i)(f_i + \gamma + s) - \mu[(1 - f_i) - k_i]^2 = \frac{1}{m}(1 - f_i)(f_i + \gamma + s) - \mu\left[\frac{(1 - f_i)}{m} - k_i\right]^2$$

$$\bar{f}(k_i, m) = \frac{\mu(m + 1) - m(\gamma + s) - 2m\mu k_i}{m + \mu m + \mu} \quad (7)$$

In the second stage, the fixed cost of capacity,  $\delta k_i$ , is "sunk", and the university will quote a fee only if the variable part of the utility is positive *i. e.*  $U_d(f, k_i) \geq -\delta k_i$ . Thus, we also define  $\hat{f}_i$  that solves  $U_d(f, k_i) = -\delta k_i$  for a given  $k_i$ , the minimum fee compatible with deciding to enroll in the second stage.

Finally, we define  $f^*$ , the fee that maximizes the utility of university  $i$  when all universities operate in the higher education market. As a shortcut, this fee can be interpreted as the maximum fee when all universities have chosen the same capacity level in the first stage.

$$f^*(k_i, m) = \arg \max_f \{U_i(f_i, k_i, m)\} = \frac{m(1 - \gamma - s) + 2\mu(1 - mk_i)}{2(m + \mu)} \quad (8)$$

It is important to understand how these utility functions  $U_M$  and  $U_d$ , and fees  $\hat{f}_i$ ,  $f^*$ , and  $\bar{f}$  are organized together.

**Lemma 2.**

$$\begin{cases} \bar{f}(k_i, m) < f^*(k_i, m) & \text{if } \mu < \frac{m}{m-1} \\ \bar{f}(k_i, m) = f^*(k_i, m) = \tilde{f} & \text{if } \mu = \frac{m}{m-1} \\ \bar{f}(k_i, m) > f^*(k_i, m) & \text{if } \mu > \frac{m}{m-1} \end{cases}$$

In the following proposition, we take a university's fixed capacity levels as given and look for the Nash equilibrium in fees.

**Proposition 1.** *In the second stage,  $(f_1, \dots, f_m)$  is a pure-strategy Nash equilibrium if and only if  $f_1(k_1) = f_2(k_2) = \dots = f_m(k_m) = f^N(k_i)$ , such that:*

(i) *If  $\mu < \frac{m}{m-1}$ ,  $f^N \in [\hat{f}(k_i), \bar{f}(k_i)]$  and  $\bar{f}_1(k_1) = \dots = \bar{f}_m(k_m) = \bar{f}(k_i)$  is a payoff-dominant pure-strategy Nash equilibrium*

(ii) *If  $\mu = \frac{m}{m-1}$ ,  $f^N = \bar{f}(k_i) = f^*(k_i) = \tilde{f}(k_i)$*

(iii) *If  $\mu > \frac{m}{m-1}$ ,  $f^N \in [\hat{f}(k_i), \bar{f}(k_i)]$  and  $f^* \in [\hat{f}, \bar{f}]$  is a payoff-dominant pure-strategy Nash equilibrium*

**Proof:** see Appendix 4.  $\square$

The following Table (2) presents the results of Bertrand competition in the different possible configurations for the second-stage game.

Table 2: The different cases of Bertrand competition in the second-stage game.

If $\mu < \frac{m}{m-1}$	If $\mu = \frac{m}{m-1}$	If $\mu > \frac{m}{m-1}$
$f^N = \bar{f}(k_i, m)$	$f^N = \tilde{f}(k_i, m)$	$f^N = f^*(k_i, m)$
$= \frac{\mu(m+1) - m(\gamma+s) - 2m\mu k_i}{m + \mu m + \mu}$	$= \frac{(m+1) - (m-1)(\gamma+s) - 2mk_i}{2m}$	$= \frac{m(1-\gamma-s) + 2\mu(1-mk_i)}{2(m+\mu)}$
$\bar{n}(k_i, m) = \frac{1+\gamma+s+2\mu k_i}{m+\mu m+\mu}$	$\tilde{n} = \frac{(m-1)(1+\gamma+s)+2mk_i}{2m^2}$	$n^*(k_i, m) = \frac{1+\gamma+s+2\mu k_i}{2(m+\mu)}$

The Nash equilibrium prediction in the second-stage game is basically identical to the one which can be found in Dastidar (1995). For all fees above  $\bar{f}$ , university  $i$  obtains no enrollment, which undermines the university's finances. When the other university charges any fee  $f \in [\hat{f}, \bar{f}]$ , the best response for university  $i$  is to quote the same tuition fee so that students split between the two institutions. By lowering its fee, the university increases its revenue by having higher enrollment. But since costs are strictly convex, they will increase even more, making this deviation nonprofitable. For this reason, a continuum of fees above the competitive price ( $f \in [\hat{f}, \bar{f}]$ ) can be sustained at Nash equilibria in pure strategies. When the value of parameter  $\mu$  is high, it means that costs are sufficiently convex (i.e.  $C''(n_i) \geq \frac{2m}{m-1}(\frac{-1}{D'})$  which is equivalent to  $\mu \geq \frac{m}{m-1}$ )<sup>5</sup> to ensure that a high level of fees can be sustained as Nash equilibria in pure strategies (Dastidar, 2001). The symmetry and the payoff dominance criterion are sufficient here to reduce the set of equilibria<sup>6</sup> and provide the uniqueness of the solution<sup>7</sup>. Consequently, we can select three fee equilibria: (i) when  $\mu < \frac{m}{m-1}$ , then the symmetric fee  $f = \bar{f}$  is the unique (payoff-dominant) pure-strategy Nash equilibrium, (ii) when  $\mu = \frac{m}{m-1}$ , there exists a unique and symmetric equilibrium such that  $f = f^* = \tilde{f}$ , and (iii) when  $\mu > \frac{m}{m-1}$ , the unique and symmetric equilibrium is such that  $f = f^*$ .

<sup>5</sup>Our condition is similar to that in Dastidar (2001) p.86, Prop.1.

<sup>6</sup>An equilibrium point is said to be payoff dominant if it is not strictly dominated by another equilibrium point; that is, there exists no other equilibrium in which utilities are higher for all universities (Harsanyi and Selten, 1988).

<sup>7</sup>The proof of the uniqueness is given by Cabon-Dhersin and Drouhin (2014).

Universities determine their level of fixed capacity anticipating the effect on the fee equilibria in the second stage.

Each university chooses its capacity level non-cooperatively, by maximizing its utility function with respect to the capacity  $k_i$ :

$$\frac{\partial U_i(f(k_i), k_i)}{k_i} = 0 \quad \Leftrightarrow$$

$$k_i = \frac{f'(\bar{k}_i) (m(1 - \gamma - s) + 2\mu - 2(m + \mu)f) - m(\delta m - 2\mu) - 2m\mu f}{2m\mu(f'(\bar{k}_i) + m)}$$

For each configuration depending on  $\mu$ , there exists a unique and symmetric solution satisfying  $\frac{\partial U_i}{\partial k_i} = 0$ , for which:

(i) If  $\mu < \frac{m}{m-1}$ ,

$$\begin{cases} \bar{k}_b^*(m) = \frac{4\mu^2 m(1+\gamma+s) - (m+\mu m+\mu)^2 \delta}{2\mu((m+\mu m+\mu)^2 - 4m\mu^2)} > 0 & \text{for } \delta < \frac{4\mu^2 m}{(m+\mu m+\mu)^2} (1+s+\gamma) \\ \bar{f}_b^*(m) = 1 - m \frac{(m+\mu m+\mu)(1+\gamma+s-\delta)}{(m+\mu m+\mu)^2 - 4m\mu^2} \\ \bar{n}_b^*(m) = \frac{(m+\mu m+\mu)(1+\gamma+s-\delta)}{(m+\mu m+\mu)^2 - 4m\mu^2} \end{cases}$$

(ii) If  $\mu = \frac{m}{m-1}$ ,

$$\begin{cases} \tilde{k}_b^*(m) = \frac{1+\gamma+s-m\delta}{2m} > 0 & \text{for } \delta < \frac{1}{m}(1+s+\gamma) \\ \tilde{f}_b^*(m) = \frac{1-\gamma-s+\delta}{2} \\ \tilde{n}_b^*(m) = \frac{1+\gamma+s-\delta}{2m} \end{cases}$$

(iii) If  $\mu > \frac{m}{m-1}$ ,

$$\begin{cases} k_b^*(m) = \frac{\mu(1+\gamma+s) - (m+\mu)\delta}{2m\mu} > 0 & \text{for } \delta < \frac{\mu}{\mu+m}(1+s+\gamma) \\ f_b^*(m) = \frac{1-\gamma-s+\delta}{2} \\ n_b^*(m) = \frac{1+\gamma+s-\delta}{2m} \end{cases}$$

**Corollary 3.**  $\forall \mu, s, \delta > 0, m \geq 3$  and  $k > 0$ , we verify at equilibrium:

$$k_b(m) < n_b(m).$$

**Proof:** see Appendix 5.  $\square$

The competition in tuition fees between  $m$  universities is likely to induce additional demand greater than the capacity level. Whatever the parameter values of the game and the number of universities, each institution chooses a number of students beyond capacity at a convex marginal cost.

### 3.2.2 Comparative Statics

We now study how the convexity of costs and the number of universities will have an influence on the equilibrium results.

**Corollary 4.** *We verify that:*

(i) If  $\mu < \frac{m}{m-1}$ :

$$\frac{\partial \bar{k}_b^*}{\partial \mu} > 0, \quad \frac{\partial \bar{n}_b^*}{\partial \mu} < 0, \quad \frac{\partial \bar{f}_b^*}{\partial \mu} > 0$$

(ii) If  $\mu \geq \frac{m}{m-1}$ :

$$\frac{\partial k_b^*}{\partial \mu} \leq 0, \quad \frac{\partial n_b^*}{\partial \mu} = 0, \quad \frac{\partial f_b^*}{\partial \mu} = 0$$

**Proof:** see Appendix 6  $\square$

Note that benefits to undercutting (at lower fees and hence higher possible enrollment) are relatively lower, as costs increase disproportionately. This implies that a greater convexity of costs (with the increase of the value of  $\mu$ ) tends to push  $\bar{f}$  up to  $f^*$ . When  $\mu = \frac{m}{m-1}$ ,  $\bar{f} = f^*$ . A lower bound on  $C''(n_i)$  serves as a sufficient condition to sustain the collusive fee  $f^*$ . It means that if the cost function is sufficiently convex ( $\mu \geq \frac{m}{m-1}$ ), then  $f^*$  is chosen by universities. Because of the U-shaped average cost, in the first stage of the game, each university may adopt an efficient capacity which minimizes its cost. When the additional marginal cost  $\mu$  increases, the university chooses a higher capacity to limit the number of students beyond the installed capacity. An increase in capacity allows to maintain the total cost at its minimum. Consequently, the collusive fee (and the number of students enrolled at the collusive equilibrium) will no longer depend on the convexity of costs.

**Corollary 5.** *For  $m \geq 3$ , we verify the following properties:*

(i) If  $\mu < \frac{m}{m-1}$ :

$$\begin{cases} \frac{\partial \bar{k}_b^*(m)}{\partial m} < 0 \\ \frac{\partial \bar{f}_b^*(m)}{\partial m} > 0 \\ \frac{\partial \bar{n}_b^*(m)}{\partial m} < 0 \end{cases} \quad \text{if } \mu > \bar{\mu}$$

$$\text{with } \bar{\mu} = \frac{-m(m-1) + 2m\sqrt{m(m-1)}}{3m^2 - 2m - 1} \in (0, \frac{m}{m-1})$$

(ii) If  $\mu \geq \frac{m}{m-1}$ :

$$\begin{cases} \frac{\partial k_b^*(m)}{\partial m} < 0 \\ \frac{\partial f_b^*(m)}{\partial m} = 0 \\ \frac{\partial n_b^*(m)}{\partial m} < 0 \end{cases}$$

**Proof:** see Appendix 7.  $\square$

From the above properties, we can consider two types of surprising effects:

- (i) Conventional wisdom may suggest that when the number of universities increases in Bertrand competition, the outcome tends to be more competitive. As showed previously, Cournot competition between  $m$  universities implies that tuition fees decrease and the total number of students increases with  $m$ . However, we show that Bertrand competition leads to a higher level of fees when the number of universities increases. This is somewhat counterintuitive and goes against conventional wisdom.
- (ii) If the cost function is sufficiently convex ( $\mu > \frac{m}{m-1}$ ) then  $f^*$  is the unique equilibrium and it is constant with  $m$ ; if the cost function is less convex ( $\mu < \frac{m}{m-1}$ ), then  $\bar{f}$  tends to increase towards  $f^*$  on condition that the convexity of costs is sufficiently high ( $\mu > \bar{\mu}$ ).

A possible intuition behind these properties is the following. The increase in  $m$  has two opposite effects on the level of fees. On the one hand, an increase in the number of universities implies that each university will face lower marginal costs since the number of students enrolled  $n_i$  tends to fall (with the sharing of demand). Due to lower marginal costs (costs are less convex), there are more incentives for fee reduction through the "demand effect". On the other hand, there is a "capacity effect": increasing the number of universities also implies a lower capacity. If costs are more convex, then marginal costs tend to be higher. This effect increases the level of fees. The "capacity effect" may or not dominate the "demand effect". When  $\mu$  is very low ( $< \bar{\mu}$ ), the second effect dominates the "capacity effect" and fee  $\bar{f}$  falls with  $m$ . A stronger "capacity effect",  $\frac{\partial \bar{n}_b^*(m)}{\partial m} - \frac{\partial \bar{k}_b^*(m)}{\partial m} > 0$ , implies higher marginal costs and an increase in fee  $\bar{f}$  with a rising  $m$  up to the maximum fee  $f^*$ . When costs become "too" convex ( $C''(n_i) \geq \frac{2m}{m-1}$  or  $\mu \geq \frac{m}{m-1}$ ), each university adjusts its capacity, thus maximizing its revenue and minimizing its average cost. The choice in capacity allows to maintain the fee at its maximum level,  $f^*$ , and it is invariant regardless of the number of universities.

### 3.3 Comparison between the Two Competition Models

**Proposition 2.** *If  $k_i > 0$ , for all parameters  $\gamma, s, \delta > 0$ , the subgame perfect Nash Equilibrium in terms of capacity is such that:*

(i) if  $\mu < \frac{m}{m-1}$  then  $\bar{k}_b^*(m) < k_c^*(m)$

(ii) if  $\mu = \frac{m}{m-1}$  then  $\tilde{k}_b^*(m) < k_c^*(m)$

(iii) if  $\mu > \frac{m}{m-1}$  then  $k_b^*(m) < k_c^*(m)$

**Proof:** see Appendix 8.  $\square$

Whatever the convexity of costs, Cournot competition always implies a higher capacity than Bertrand competition. In line with our results, we should observe that universities are smaller in countries where fees have been deregulated, and bigger in countries where fees are regulated.

**Proposition 3.** *If  $k_i > 0$ , for all parameters  $\gamma, s, \delta, \mu > 0$ , the subgame perfect Nash Equilibrium in terms of tuition fees is such that:*

(i) *if  $\mu \leq \underline{\mu}(m)$  then  $\bar{f}_b^*(m) \leq f_c^*(m)$  and  $\bar{n}_b^*(m) \geq n_c^*(m)$ ,*

(ii) *if  $\underline{\mu}(m) < \mu < \frac{m}{m-1}$  then  $\bar{f}_b^*(m) > f_c^*(m)$  and  $\bar{n}_b^*(m) < n_c^*(m)$ ,*

(iii) *if  $\mu = \frac{m}{m-1}$  then  $\tilde{f}_b^*(m) > f_c^*(m)$  and  $\tilde{n}_b^*(m) < n_c^*(m)$ ,*

(iv) *if  $\mu > \frac{m}{m-1}$ ,  $f_b^*(m) > f_c^*(m)$  and  $n_b^*(m) < n_c^*(m)$ .*

with  $\underline{\mu} \leq 0.458952$  and  $\frac{\partial \underline{\mu}}{\partial m} < 0$ .

**Proof:** see Appendix 8.  $\square$

We show that Bertrand competition can induce a higher (or lower) level of fees than Cournot competition, depending on the level of marginal costs  $\mu$ . If they are sufficiently convex, Bertrand competition implies a high (low) level of tuition fees (number of students enrolled per university), which is greater (lower) than in Cournot competition. According to conventional wisdom, Bertrand competition becomes more competitive when the convexity of costs is low. Whatever the parameter values of the game ( $s, \gamma, \delta$  and  $\mu$ ), the total number of students in a deregulated system is lower than in a regulated system.

## 4 Efficient Capacity and Welfare

### 4.1 Comparison between Universities' Equilibrium Capacity and Efficient Capacity

Whatever the competition models between universities, the cost function of each university  $i$  is given by:

$$C(k_i, n_i) = \begin{cases} \delta k_i & \text{if } n_i \leq k_i \\ \delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i \end{cases} \quad (9)$$

Because of the U-shaped average cost in  $k$ , there exists an efficient capacity which minimizes the average cost:

$$\text{Min}_{n_i} AC(k_i, n_i) = \text{Min}_{n_i} \left[ \frac{C(k_i, n_i)}{n} \right]$$

The capacity which minimizes the average cost for a given number of students enrolled is:

$$k_{\min} = \frac{2\mu n_i - \delta}{2\mu}$$

Assuming that universities adopt their efficient capacity, they will compete in the second stage of the game according to the competition scenarios (number of students enrolled/level of tuition fees). The different results depending on the type of competition are presented in the following tables (Tables 1 and 2):



capacity	$k_c^{\min} = \frac{2\mu(1+\gamma+s)-(2\mu+m+1)\delta}{2\mu(m+1)}$
number of students	$n_c^{\min} = \frac{1+\gamma+s-\delta}{m+1}$
level of fees	$f_c^{\min} = \frac{1-m(\gamma+s-\delta)}{m+1}$

Table 3: Efficient solution in  $k$  with Cournot competition

	$\mu < \frac{m}{m-1}$	$\mu \geq \frac{m}{m-1}$
capacity	$k_b^{\min} = \frac{2\mu(1+\gamma+s)-(m+\mu m+\mu)\delta}{2\mu(m+\mu m-\mu)}$	$k_b^{\min} = \frac{\mu(1+\gamma+s)-(m+\mu)\delta}{2\mu m}$
number of students	$\bar{n}_b^{\min} = \frac{1+\gamma+s-\delta}{m+\mu m-\mu}$	$n_b^{\min} = \frac{1+\gamma+s-\delta}{2m}$
level of fees	$\bar{f}_b^{\min} = \frac{\mu(m-1)-m(\gamma+s-\delta)}{m+\mu m-\mu}$	$f_b^{\min} = \frac{1-\gamma-s+\delta}{2}$

Table 4: Efficient solution in  $k$  with Bertrand competition

**Proposition 4.** *Comparison between the equilibrium results of the two competition models with the efficient solution:*

	Cournot equilibrium vs efficient solution	Bertrand equilibrium vs efficient solution	
		$\mu < \frac{m}{m-1}$	$\mu \geq \frac{m}{m-1}$
capacity	$k_c^{\min} < k_c^*$	$k_b^{\min} > k_b^*$	$k_b^{\min} = k_b^* = k_b$
number of students/fees	$n_c^{\min} < n_c^*$	$f_b^{\min} < f_b^*$	$f_b^{\min} = f_b^* = f_b$

Regarding our results, Cournot competition is likely to induce too high a capacity level in terms of minimizing average costs for a university. Bertrand competition is more efficient from this point of view, but only if the cost is sufficiently convex.

## 4.2 Welfare Analysis

Another question we address is how the number of universities may impact social welfare depending on university systems.

Define social welfare as  $SW^k = mU_i^k + CS^k$ , where  $k = c, b$  and  $CS^k = \frac{N^k}{2}$  is consumer surplus, defined in terms of the number of students enrolled  $N^k = m \cdot n^k$ . We obtain:

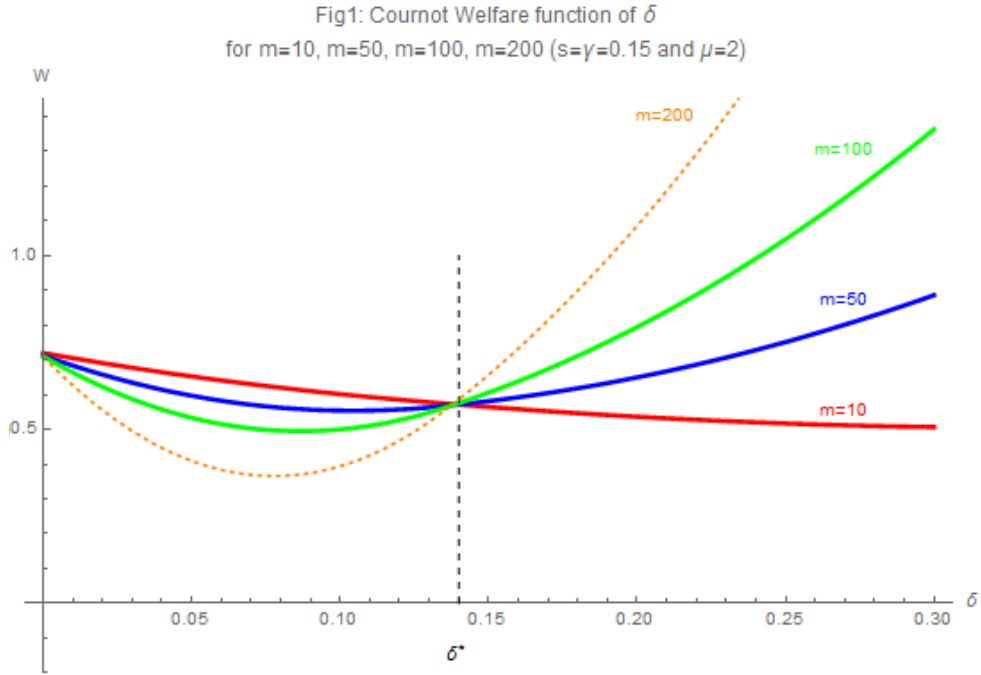
$$SW = \frac{m}{2} (2(1 + \gamma + s)n - m(n)^2 - 2C(n, k))$$

**Proposition 5.** *When costs are sufficiently convex ( $\mu \geq \frac{m}{m-1}$ ), social welfare always increases with the number of universities in Bertrand competition.*

**Proposition 6.** *When the capacity is too high (or when the unit cost of installed capacity is below  $\delta^*$ )(see Fig1.), social welfare may decrease with the number of universities in Cournot competition.*

**Proof:** Simulations and Appendix 9.  $\square$

In Cournot competition, the welfare levels were illustrated by solving the equilibrium values depending  $\delta$  (Fig1.):



In Bertrand competition, a greater number of universities enhances social welfare even though universities set higher fees. This counterintuitive result can be explained by the efficient choice of capacity which allows to reduce universities' costs. In Cournot competition, social welfare decreases with the number of universities, even though the total number of students enrolled increases. Too large a capacity in Cournot competition (or when the unit cost of installed capacity is below  $\delta^*$ ) is not necessarily compatible with exploiting economies of scale and induces a lower social welfare level when the number of universities increases.

## 5 Conclusion

In this paper, we have analyzed a two-stage game between universities that decide on their capacity levels, tuition fees, and number of students enrolled. Interestingly, our model emphasizes the role of the strategic variable on output results and offers elements to explain some stylized facts. The competition through fees induces a higher-education market characterized by small universities setting high fees. On the contrary, in a competition through admission, universities are larger and set low tuition fees. Bertrand competition leads universities to adopt an efficient capacity which allows to minimize average costs; this is not the case in Cournot competition, whose high capacity level is detrimental to welfare.

Finally, a competition between an increasing number of small universities allows to sustain fees at their maximum level without reducing admissions overall and social welfare in the higher-education market. In Cournot competition, the increasing number of large universities (with too high a capacity) induces a lower social welfare level. These results may be used to provide guidelines for higher-education policy-makers who have been encouraging the clustering of universities (particularly in France).

Some of the simplifying assumptions deserve more discussion. First of all, one should not ignore the specification of the demand function that has generated our results. We assume that demand is always infinitely elastic; the price elasticity of demand can have an impact on the different equilibrium results depending on the form of competition. The access to significant financial support or other aids for students may influence our results. Secondly, we assume that there is no differentiation in curriculum, and/or in other non-price dimensions. In view of these elements, it seems necessary to explore competition between universities more deeply and systematically.

## Appendix 1: Proof of Lemma 1

$$k_c^*(m) = n_c^*(m) + \frac{2\mu(m-1)(1+\gamma+s) - (m+1)(1+2\mu)(3+2\mu)\delta}{2\mu((m+1)(1+2\mu)(3+2\mu) - 2\mu(m-1))}$$

$$\Leftrightarrow n_c^*(m) > k_c^*(m)$$

$$\text{if and only if } \delta > \frac{2\mu(m-1)}{(1+2\mu)(3+2\mu)(m+1)}(1+\gamma+s) = \underline{\delta}$$

We verify that  $k_c^*(m) > 0$  when  $\delta < \bar{\delta} = \frac{2\mu Z + 2\mu(m-1)}{2\mu Z + (m+1)Z}(1+\gamma+s)$ .

$$\text{with } Z = (1+2\mu)(3+2\mu)$$

## Appendix 2: Proof of Corollary 1

(i)

$$n_c^* = \frac{(1+2\mu)(3+2\mu)}{W}(1+\gamma+s-\delta)$$

$$\text{with } W = (m+1)(1+2\mu)(3+2\mu) - 2\mu(m-1)$$

$$\frac{\partial n_c^*(m)}{\partial \mu} = \frac{-2(4\mu^2 - 3)(m-1)}{W^2}(1+\gamma+s-\delta)$$

$$\Leftrightarrow \text{sign}\left(\frac{\partial n_c^*}{\partial \mu}\right) = \text{sign}(3 - 4\mu^2) < 0 \quad \text{if } \mu > \sqrt{\frac{3}{4}}$$

$$(ii) \quad \text{sign}\left(\frac{\partial f_c^*}{\partial \mu}\right) = \text{sign}(-3 + 4\mu^2) > 0 \quad \text{if } \mu > \sqrt{\frac{3}{4}}$$

(iii) From Equation 5, we have:

$$k_c^* = \frac{2\mu + m + 1}{2\mu} n_c^* - \frac{1 + \gamma + s}{2\mu}$$

$$\frac{\partial \bar{k}_c^*}{\partial \mu} = \frac{(2\mu + m + 1)\mu \frac{\partial n_c^*}{\partial \mu} - (m + 1)n_c^* + (1 + \gamma + s)}{2\mu^2}$$

$$\Leftrightarrow \text{sign} \left( \frac{\partial \bar{k}_c^*}{\partial \mu} \right) < 0$$

$$\text{if } \delta < \frac{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+2\mu(m-1)W}{2\mu(2\mu+m+1)(4\mu^2-3)(m-1)+(m+1)(1+2\mu)(3+2\mu)W} (1 + \gamma + s) = \bar{\delta} \in (0; 1)$$

### Appendix 3: Proof of Corollary 2

- i)  $\frac{\partial n_c^*(m)}{\partial m} = -\frac{n_c^*}{(m+1)(1+2\mu)(3+2\mu)-2\mu(m-1)}(6\mu + 4\mu^2 + 3) < 0$
- ii)  $\frac{\partial N_c^*(m)}{\partial m} = n_c^*(m) + m \frac{\partial n_c^*(m)}{\partial m} = \frac{n_c^*}{(m+1)(1+2\mu)(3+2\mu)-2\mu(m-1)}(3 + 10\mu + 4\mu^2) > 0$
- iii)  $\frac{\partial f_c^*(m)}{\partial m} = -\frac{\partial N_c^*(m)}{\partial m} < 0$
- iv)  $\frac{\partial k_c^*(m)}{\partial m} = -\frac{n_c^*}{(m+1)(1+2\mu)(3+2\mu)-2\mu(m-1)}(4\mu + 4\mu^2 + 3) < 0$

### Appendix 4: Proof of Proposition 1

Let us first investigate symmetric strategy profiles belonging to the interval  $[\hat{f}, \bar{f}]$ . When the competitor charges any fee  $f \in [\hat{f}_i; \bar{f}_i]$ , the best response for university  $i$  is to quote the same tuition fee. When university  $i$  quotes the same fee, it gets  $U_d(f)$ . We know that for all  $f \geq \hat{f}_i$ ,  $U_d(f) \geq -TFC$ . If the university deviates (by quoting  $f - \epsilon$ ), it gets  $U_d(f - \epsilon)$ . We also know that for all  $f \in [\hat{f}_i; \bar{f}_i]$ ,  $U_d(f) \geq U_M(f) > U_M(f - \epsilon)$ . Since the university must supply all the demand it faces, the increase in additional revenue (from higher enrollment) is less than the increase in costs: the university must enroll additional students at excessive marginal costs. By quoting  $f + \epsilon$ , university  $i$  obtains no demand and gets zero variable utility needed for the research activity. Hence, it is optimal for each university to quote the same fee. There are no incentives to deviate, which proves the implication in Proposition 1. It also proves that all asymmetrical strategy profiles with at least one firm quoting a price in the interval are not Nash equilibria. We now have to investigate all the other strategy profiles in which none of the firms quote a price within the interval. It is easy to check that for all symmetric strategic profiles such that  $f < \hat{f}$ , increasing its tuition fee is in the firm's interest. Lowering its fee is in the university's interest with  $f > \bar{f}$ . The payoff dominance criterion, which appears to be the natural criterion when it is common knowledge that both actors are fully rational is sufficient to provide uniqueness in the three configurations.  $\square$

### Appendix 5: Proof of Corollary 3

Let us compare the number of students enrolled in each university with the capacity for the different cases:

$$n_b^*(m) > k_b^*(m) \quad \forall \gamma, s, \text{ et } m$$

$$(i) \text{ If } \mu < \frac{m}{m-1} : n_b^*(m) > k_b^*(m) \Leftrightarrow 2\mu(m + m\mu + \mu)(1 + \gamma + s - \delta) > 4\mu^2 m(1 + \gamma + s) - (m + m\mu + \mu)^2 \delta$$

$$\Leftrightarrow 2\mu(m + \mu - m\mu)(1 + \gamma + s) > (m + m\mu + \mu)(\mu - m - m\mu)\delta$$

which is always verified because  $m + \mu - m\mu > 0$  and  $\mu - m - m\mu < 0$ .

$$(ii) \text{ If } \mu = \frac{m}{m-1} : \tilde{n}_b^*(m) > \tilde{k}_b^*(m) \Leftrightarrow \frac{1+\gamma+s-m\delta}{2m} < \frac{1+\gamma+s-\delta}{2m} \quad \forall m \geq 2 \text{ which is always verified.}$$

$$(iii) \text{ If } \mu > \frac{m}{m-1} : n_b^*(m) > k_b^*(m) \Leftrightarrow \frac{m+\mu}{\mu} > 1 \quad \forall m \geq 2 \text{ which is always verified.}$$

## Appendix 6: Proof of Corollary 4

Case 1. If  $\mu < \frac{m}{m-1}$  :

$$\frac{\partial \bar{n}_b^*}{\partial \mu} = -\frac{m\mu(\mu(m-1) + 2m(m-2) + \mu^2 m + 2) + \mu^2 + m^2 + m^3}{(\mu^2 + m^2 - 2m\mu^2 + 2m\mu + 2m^2\mu + m^2\mu^2)^2} < 0 \quad \text{with } m \geq 2$$

$$\Leftrightarrow \frac{\partial \bar{f}_b^*}{\partial \mu} > 0$$

From Equation 7, we have:

$$\bar{k}_b^* = -\frac{m + \mu m + \mu}{2m\mu} \bar{f}_b^* + \frac{m+1}{2m} - \frac{\gamma+s}{2\mu}$$

$$\frac{\partial \bar{k}_b^*}{\partial \mu} > 0 \quad \text{with} \quad \frac{\partial \bar{f}_b^*}{\partial \mu} > 0$$

We can get cases 2 ( $\mu = \frac{m}{m-1}$ ) and 3 ( $\mu > \frac{m}{m-1}$ ) with straightforward computations.

## Appendix 7: Proof of Corollary 5

Case 1. If  $\mu < \frac{m}{m-1}$  :

$$\frac{\partial \bar{n}_b^*(m)}{\partial m} = -\frac{(1+\mu)(m + \mu m + \mu)^2 - 4\mu^3}{((m + \mu m + \mu)^2 - 4m\mu^2)^2} (1 + s + \gamma - \delta) < 0$$

$$\frac{\partial \bar{k}_b^*(m)}{\partial m} = -\frac{2\mu m^2(1+\mu)^2 - 2\mu^3}{((m + \mu m + \mu)^2 - 4m\mu^2)^2} (1 + s + \gamma - \delta) < 0$$

$$\frac{\partial \bar{N}_b^*(m)}{\partial m} = \bar{n}_b^*(m) + m \frac{\partial \bar{n}_b^*(m)}{\partial m} =$$

$$(\mu^2(-3m^2 + 2m + 1) + \mu(2m(1 - m)) + m^2) \frac{(1 + s + \gamma - \delta)}{((m + \mu m + \mu)^2 - 4m\mu^2)^2} < 0$$

$$\text{if } \bar{\mu} < \mu \leq \frac{m}{m-1}$$

$$\text{with } \bar{\mu} = \frac{-m(m-1)+2m\sqrt{m(m-1)}}{3m^2-2m-1}$$

$$\frac{\partial \bar{f}_b^*(m)}{\partial m} = -\frac{\partial \bar{N}_b^*(m)}{\partial m} > 0$$

- if  $\mu > \bar{\mu}$  hence  $\frac{\partial \bar{f}_b^*(m)}{\partial m} > 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} < 0, \quad \forall m \geq 3$
- if  $\mu = \bar{\mu}$  hence  $\frac{\partial \bar{f}_b^*(m)}{\partial m} = 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} = 0 \quad \forall m \geq 3$
- if  $\mu < \bar{\mu}$  hence  $\frac{\partial \bar{f}_b^*(m)}{\partial m} < 0 \Leftrightarrow \frac{\partial \bar{N}_b^*(m)}{\partial m} > 0 \quad \forall m \geq 3$

$$\text{with } \bar{\mu} = \frac{m-m^2+2\sqrt{m^4-m^3}}{3m^2-2m-1}$$

We can get cases 2 ( $\mu = \frac{m}{m-1}$ ) and 3 ( $\mu > \frac{m}{m-1}$ ) with straightforward computations.

## Appendix 8: Proof of Propositions 2-3

Case 1 - If  $\mu < \frac{m}{m-1}$ :

$$n_c^*(m) - \bar{n}_b^*(m) =$$

$$\frac{m^2(5\mu+13\mu^2+12\mu^3+4\mu^4)-m(3+10\mu+10\mu^2+16\mu^3+8\mu^4)+\mu(4\mu^3+4\mu^2-7\mu-3)}{((m+1)(1+2\mu)(3+2\mu)-2\mu(m-1))(\mu^2+m^2-2m\mu^2+2m\mu+2m^2\mu+\mu^2m^2)}(1+\gamma+s-\delta)$$

We verify that

$$\text{sign}(n_c^*(m) - \bar{n}_b^*(m)) =$$

$$\text{sign}(m^2(5\mu + 13\mu^2 + 12\mu^3 + 4\mu^4) - m(3 + 10\mu + 10\mu^2 + 16\mu^3 + 8\mu^4) + \mu(4\mu^3 + 4\mu^2 - 7\mu - 3))$$

We define  $\underline{\mu}(m)$  such that if  $\mu > \underline{\mu}(m)$  hence

$$n_c^*(m) > \bar{n}_b^*(m)$$

The value of  $\underline{\mu}(m)$  is solved by the Mathematica program. For  $m = 2$ , we have  $\underline{\mu} = 0.458952$ , for  $m=3$ ,  $\underline{\mu} = 0.249037$ ;...for  $m = 10$ ,  $\underline{\mu} = 0.06297$ .

We conclude that:

- $n_c^*(m) < \bar{n}_b^*(m), f_c^*(m) > \bar{f}_b^*(m)$ , and  $N_c^*(m) < \bar{N}_b^*(m)$  if  $\mu \in ]0, \underline{\mu}(m)[$
- $n_c^*(m) = \bar{n}_b^*(m), f_c^*(m) = \bar{f}_b^*(m)$ , and  $N_c^*(m) = \bar{N}_b^*(m)$  if  $\mu = \underline{\mu}(m)$
- $n_c^*(m) > \bar{n}_b^*(m), f_c^*(m) < \bar{f}_b^*(m)$ , and  $N_c^*(m) > \bar{N}_b^*(m)$  if  $\mu \in ]\underline{\mu}(m), \frac{m}{m-1}[$

We verify that  $\bar{k}_c^*(m) > \bar{k}_b^*(m) \quad \forall \mu \in [0, \frac{m}{m-1}[$ .

Case 2 - If  $\mu = \frac{m}{m-1}$ , we have

$$\begin{aligned} n_c^*(m) - \tilde{n}_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ N_c^*(m) - \tilde{N}_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ f_c^*(m) - \tilde{f}_b^*(m) &= -\frac{(m+1)(17m^2-16m+3)}{2(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) < 0 \\ k_c(m) - \tilde{k}_b^*(m) &= \frac{(17m-27m^2+11m^3+2m^4-3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \end{aligned}$$

Case 3 - If  $\mu > \frac{m}{m-1}$ :

$$\begin{aligned} n_c^*(m) - n_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{2m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ N_c^*(m) - N_b^*(m) &= \frac{(m+1)^2(17m^2-16m+3)}{m(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) > 0 \\ f_c^*(m) - f_b^*(m) &= -\frac{(m+1)(17m^2-16m+3)}{2(13m^3+5m^2-13m+3)}(1 + \gamma + s - \delta) < 0 \\ \bar{k}_c^*(m) - \bar{k}_b^*(m) &= \frac{(m+2m^2+10\mu(m-1)+4\mu^2(m-1))}{6m(m+1)+2\mu m(6m+5)+4\mu^2 m(4m+1)}(1 + \gamma + s - \delta) > 0 \end{aligned}$$

## Appendix 9: Proof of Proposition 5

In Bertrand competition:

$$SW^b = \frac{m}{2} (2(1 + \gamma + s)n_b - m(n_b)^2 - 2\delta k_b - 2\mu(n_b - k_b)^2)$$

The derivative of the above expression with respect to  $m$  gives:

$$\frac{\partial SW^b}{\partial m} = (1+\gamma+s-mn_b) \left( n_b + m \frac{\partial n_b}{\partial m} \right) - \delta \left( k_b + m \frac{\partial k}{\partial m} \right) - \mu(n_b - k_b) \left( (n_b - k_b) + 2m \frac{\partial(n_b - k_b)}{\partial m} \right)$$

If  $\mu \geq \frac{m}{m-1}$ , we obtain

- $n_b + m \frac{\partial n_b}{\partial m} = 0$ ,
- $-\delta \left( k_b + m \frac{\partial k}{\partial m} \right) = \frac{\delta^2}{2\mu}$
- $-\mu(n_b - k_b) \left( (n_b - k_b) + 2m \frac{\partial(n_b - k_b)}{\partial m} \right) = -\mu(n_b - k_b)^2 = -\frac{\delta^2}{4\mu}$

Finally,

$$\frac{\partial SW^b}{\partial m} = \frac{\delta^2}{4\mu} > 0$$

## References

- Beath, J. A., J. Poyago-Theotoky, and D. Ulph (2012). University funding systems: impact on research and teaching. *Economics: The Open-Access, Open-Assessment E-Journal* 6, 2.
- Cabon-Dhersin, M.-L. and N. Drouhin (2014). Tacit collusion in a one-shot game of price competition with soft capacity constraints. *Journal of Economics & Management Strategy* 23(2), 427–442.
- Cesi, B. and D. Paolini (2014). Peer group and distance: when widening university participation is better. *The Manchester School* 82(S2), 110–132.
- Chowdhury, P. R. (2009). Bertrand competition with non-rigid capacity constraints. *Economics Letters* 103(1), 55 – 58.
- Dastidar, K. G. (1995). On the existence of pure strategy bertrand equilibrium. *Economic Theory* 5, 19–32.
- Dastidar, K. G. (2001). Collusive outcomes in price competition. *Journal of Economics* 73(1), 81–93.
- Dastidar, K. G. (2011a). Bertrand equilibrium with subadditive costs. *Economics Letters* 112, 202–204.
- Dastidar, K. G. (2011b). Existence of Bertrand equilibrium revisited. *International Journal of Economic Theory* 7, 331–350.
- De Fraja, G. and E. Iossa (2002). Competition among universities and the emergence of the elite institution. *Bulletin of economic Research* 54(3), 275–293.
- Del Rey, E. (2001). Teaching versus research: a model of state university competition. *Journal of Urban Economics* 49(2), 356–373.
- Gautier, A. and X. Wauthy (2007). Teaching versus research: A multi-tasking approach to multi-department universities. *European Economic Review* 51(2), 273–295.
- Grazzini, L., A. Luporini, and A. Petretto (2010). Competition between state universities. *Working Paper, Dipartimento die Scienze Economiche, Universitra degli Studi di Firenze*.
- Harsanyi, J. C. and R. Selten (1988). *A General Theory of Equilibrium Selection in Games*. Cambridge MA: The MIT Press.
- Jacqmin, J. and X. Wauthy (2014). The liberalization of tuition fees: A theoretical assessment.
- Rothschild, M. and L. J. White (1995). The analytics of the pricing of higher education and other services in which the customers are inputs. *Journal of Political Economy*, 573–586.
- Saporiti, A. and G. Coloma (2010). Bertrand competition in markets with fixed costs. *The BE Journal of Theoretical Economics* 10(1), 27.
- Worthington, A. C. and H. Higgs (2011). Economies of scale and scope in australian higher education. *Higher Education* 61(4), 387–414.