

Inter-university competition and high tuition fees

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Abstract

This paper proposes a competition model in which universities freely fix their tuition fees in a setting where their soft capacity is chosen endogenously. Because of the growing competition between universities to attract additional students, universities choose to support an increasing marginal cost for students enrolled beyond their initial capacity. Our results show that i) a more convex cost leads the universities to choose a larger capacity but also higher tuition fees; ii) when the marginal cost is low, an increase in tuition fees with the number of universities is possible; iii) when the marginal cost is sufficiently high, competition between an increasing number of universities maintains fees at their maximum level (collusive fees) without reducing either the total number of admissions or social welfare.

Key words: price competition, capacity, tuition fees, higher education market.

Code JEL: C7, H52, I22, I23.

1 Introduction

Public authorities have a real interest in ensuring that students can access a broad range of higher education opportunities. An important question is then how the strategic behavior of universities affects the availability and cost of higher education for students.

In the UK, tuition fees have nearly tripled since their recent deregulation, a reform that also included relaxing controls on the number of students a given university can recruit. Since 2012, the vast majority of UK public universities have been charging tuition fees of £9,000, the maximum allowed under the government-imposed cap. “Now, universities do battle to enroll as many scholars as they can.

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That is because they must attract students or cut costs” (The Economist, Feb 23rd 2017).

This growing competition between universities to increase their enrollment should have led to a drop in fees (i.e. they should be lowest for universities with the lowest admission standards) or a more differentiated education landscape. On the contrary, some poorly-performing universities with low admission standards charge high fees¹. The current trend has seen students asked to bear a substantially larger share of the cost of tertiary education. This is particularly true in countries in which fees have been deregulated (Desrochers and Hurlburt, 2016). How can this be explained? This question is made all the more relevant by the fact that government-determined fees in a number of other European countries (such as France, Spain, Italy, and Germany) are very low².

The upward trend of fees in countries with a deregulated fee system has several possible explanations:

- i) The first is that the high fees are justified by the high quality of teaching and research in these universities. Many papers have been written on the possible conflict between improvements in teaching and research under different competition models. Del Rey (2001) used a spatial model to describe the behavior of universities that decide how to allocate funds between teaching and research activities and then compete on admission standards. The balance between research and teaching efforts depends on the funding rules. In the same vein, De Fraja and Iossa (2002) have studied the effect of the mobility cost supported by students on the choice of admission standards by multi-tasking universities. In Grazzini et al. (2010), the literature on capital tax competition is used in order to analyze how student mobility affects university competition on both tuition fees and research and teaching expenditure. Other contributions have focused on how the type of funding system has an impact on the allocation of academics’ time and then on the trade-off between teaching and research quality (Beath et al., 2003; Gautier and Wauthy, 2007; Beath et al., 2012). Finally, considering competition between a private and a public university, the divergence in admission standards and teaching quality may explain the difference in fees between the two types of institution and thus the persistence of high tuition fees (Romero and Del Rey, 2004; Oliveira, 2006; Lasram and Laussel, 2017). What all these papers have in common is that they consider universities that are ex-ante differentiated with respect to

¹In the UK, a proposed reform would allow universities that provide excellent teaching and give the best employment prospects to their graduates to raise their fees above the current maximum of £9,000, in line with inflation (“Teaching Excellence Framework” (TEF)). The government hopes to be able to link the fees that universities are allowed to charge with levels of instruction (The Economist, Aug 12th 2017).

²Since 2014, access to public universities is free of charge in Germany.

their location, financial endowment, admission standards, education quality or even in some cases, their curriculum levels (Eisenkopf and Wohlschlegel, 2012).

- ii) The second explanation is that students expect a higher return from studying. Becker's model of human capital views education as an investment whereby costs are compared with expected future benefits, primarily higher wages. If investments in education are expected to produce better returns, this can explain both higher enrollment and higher costs. This presumes that students have foresight on which university will maximize their return. As suggested by Jensen (2010) however, it is not necessarily easy for students and/or their parents to estimate the future returns of education and thus make an informed decision about which university to attend. Do high fees indicate better educational investments? This question is related to a more general one about the sensitivity of demand to prices in the higher education market: whether enrollment figures are really sensitive to prices remains an open question. Generally, the impact of tuition fees on enrollment rates is hard to evaluate (because of unobservable differences in preferences for higher education, in financial support, in student mobility...). Many empirical studies (Leslie and Brinkman, 1987; Kane, 1994; Heller, 1997; Card and Lemieux, 2000; Hübner, 2012) have examined the impact on enrollment of increasing tuition fees, focusing on quantifying price elasticities. Because of the variety of educational offers and because of differences in financial resources, the response of enrollment numbers to price fluctuations may vary between countries, states, types of colleges and universities. But one interesting finding is that large increases in tuition fees in the US have specifically lead to a decrease in the enrollment of selective public universities (Hemelt and Marcotte, 2011).
- iii) The final explanation that can be put forward is the declining funding of universities. In many countries, the decrease in non-tuition revenue and increasing numbers of non-teaching staff have forced public universities to charge high tuition fees (Martin, 2002). Using data on all US public 4-year colleges and universities from 1991 to 2006, Hemelt and Marcotte (2011) suggest that, because the tuition elasticity of enrollment is low, the decline in enrollment is small enough that net revenue should increase in the short run when fees are increased.

In this article, we propose another explanation, complementary to the above three and similar to the one put forward by Jacqmin and Wauthy (2014): for given levels of teaching quality and admission standards, increased competition between higher education institutions to attract students increases the enrollment costs borne by the universities. The rising costs associated with the enrollment of students beyond their capacities suffice to explain the high tuition fees demanded by universities.

The recent theoretical contributions in Industrial Organization regarding price competition in the presence of convex costs may be useful to answer our question. As usual in price competition, a firm that undercuts its competition will increase its demand. In a seminal contribution, Dastidar (1995) showed that this move is not profitable for a firm when the marginal cost increases. Thus, at equilibrium, prices may be higher than the average cost and even higher than the marginal cost. Another point to consider from the literature is the presence of capacity constraints: the tradition in Industrial Organization has long been to relax price competition based on the fact that firms are limited by their production capacities to match demand (Vives (1999), p. 124 for details). Recently, Cabon-Dhersin and Drouhin (2014, 2017) and Burguet and Sákovics (2017) have provided a rationale for capacity constraints in a two-stage setting (with capacities chosen in a first stage and competition on prices with convex costs in the second).

Based on these contributions, we analyze how the capacities chosen by a set of more than two universities influence tuition fees and enrollment capacities as a function of the number of universities in the market. In order to neutralize any effects of teaching differences between the competing universities, we assume here that the courses that they offer are identical and of the same quality; in addition, we assume that the applying students all have similar skills and grades. When universities enroll students above capacity, the cost function is strictly convex. Indeed, competition for students drives universities to invest, to increase their educational assets (teachers, libraries, rooms, administrative services, etc.) and meet the additional demand. Clearly, better facilities are a boon for students and in the UK for example, growing universities have for some years been building libraries, housing blocks and all the facilities required to accommodate additional students. In 2013, UK universities spent £2.4bn on construction, 43% more than they did in 2012, and have continued to spend at that level in the years since³. Our framework is similar to the one adopted by Jacqmin and Wauthy (2014) in that we use “non-rigid” capacities, as introduced by Chowdhury (2009). However, Jacqmin and Wauthy (2014)’s competition model involves two homogeneous universities that produce both research and teaching and in which increases in the opportunity cost of providing education beyond a certain exogenously fixed threshold capacity reduce the amount of time available for research.

In contrast, this article focuses on the influence of capacity constraints on the level of fees under oligopolistic competition. We consider two stages. In the first, universities endogenously fix their (subsequently invariable) capacity before competing in terms of tuition fees to satisfy student demand. When universities enroll students above capacity, the cost function is strictly convex.

The contribution of the present work may be understood as follows. To the best

³A report by the Higher Education Funding Council for England (Financial health of the higher education sector: 2015-16 to 2018-19 forecasts), warned of declining cash levels and an increase in borrowing by universities that could prove unsustainable.

of our knowledge, no existing theoretical model explains how capacities, fees and enrollment are decided in the context of a growing competition between universities that can adjust their tuition fees to attract more students. Among other factors that can explain increasing fees (lower public subsidies, improved teaching and research, higher expected returns of a university education), this theoretical analysis focuses on the effects of universities competing on cost for students. For a common fixed level of teaching quality, we show that:

- (i) however many universities there are and for all values of the cost parameters, universities decide to operate above their enrollment capacity at equilibrium: the number of students enrolled is always greater than the endogenously chosen threshold capacity.
- (ii) a more convex cost of supporting additional students leads the universities to choose a larger capacity but also higher tuition fees at equilibrium.
- (iii) when the number of universities increases, each university enrolls fewer students but increases their tuition fees up to the maximum level. When the convex costs are high enough to satisfy the additional demand (beyond the threshold capacity), tuition fees become independent of the size and the number of universities in the higher education market. Finally, competition between an increasing number of universities maintains fees at their maximum level (collusive fees) without reducing either the total number of admissions or social welfare.

The paper is organized as follows. Section 2 introduces the model. Section 3 presents the equilibria of the two-stage game. A comparative static analysis is presented in section 4 and a brief social welfare analysis in section 5. Section 6 concludes.

2 Model

We consider a higher education market with m identical universities. Universities can (with no obligation) select students (based on past performance or an admission test). If universities have the same admission requirements, the same subset of the student population is admissible to each one. If not, some proportion of the candidates will nevertheless be eligible for enrollment at several universities. In other words, whatever the recruitment process, universities always compete for some share of the student population.

The utility students derive from graduating from university i is defined as $u(\theta) = \theta - f_i$, where θ represents a student's willingness to pay to go to university and f_i is the level of fees. θ is uniformly distributed in $[0, 1]$. Our analysis rests on the assumption that all students are not all equally willing to pay for university, as in

Jacqmin and Wauthy (2014)⁴. Each student implicitly receives one unit of education and the student population is therefore normalized to unity.

For a given university course, eligible students choose to enroll at the university with the lowest fees. Since the universities are otherwise identical in terms of curriculum and all other non-price dimensions (location, teaching/research trade-off, financial endowment etc.), the demand for enrollment at a university i , n_i , is a discontinuous function of its fees⁵:

$$n_i(f_i, f_{-i}) = \begin{cases} 0 & \text{if } f_i > f_{-i}^{min} \\ \frac{N(f_i)}{m} & \text{if } f_i = f_{-i} \\ N(f_i) = 1 - f_i & \text{if } f_i < f_{-i}^{min} \end{cases}$$

We denote $N(f_i)$ the total number of students enrolled when university i sets fees f_i ,

f_i , the tuition fees at university i , and $f_{-i} = \{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$, the vector of the fees of all universities in the higher education market. Note that $f_{-i}^{min} = \text{Min}\{f_1, \dots, f_{i-1}, f_{i+1}, \dots, f_m\}$.

In this model, universities are not allowed to deny admission in the second stage if students pay the required fees, i.e. they cannot undertake student rationing⁶. Thus, it is not profitable for a university i to set higher fees than the minimum charged by the other competing universities ($n_i = 0$). When universities equalize their fees, they share students equally. Finally, if university i sets its fees to the minimum, it must be able to accommodate all the students that apply on this basis.

Furthermore, we assume that the mission of universities is to create (through research, R) and disseminate (via teaching, T) fundamental knowledge. The general form of each university's objective function is thus $G(T, R)$, where G is strictly increasing in both arguments. The separability of the objective function allows us to consider, as elsewhere in the higher education literature, universities that specialize solely in teaching or research.

Each university's objective function is defined as:

$$\text{Max } G_i(T, R), \quad i = 1, \dots, m, \quad i \neq j \quad (1)$$

⁴As Jacqmin and Wauthy (2014) point out, they account for disparities in students' willingness to pay but they do not consider differences in ability or location.

⁵This choice of the form of the demand function is debatable. It is indeed unlikely that students are equally sensitive to the price of educational goods (Leslie and Brinkman, 1987; Kane, 1994; Heller, 1997; Card and Lemieux, 2000; Hübner, 2012). However, this paper deals with the supply side of the higher education market and this restrictive hypothesis about student demand does not affect the reasoning and the different effects resulting from our analysis.

⁶Forbidding student rationing allows us to preserve the existence of Nash equilibria in pure strategies.

with $\frac{\partial G_i}{\partial T} > 0$ and $\frac{\partial G_i}{\partial R} > 0$.

As suggested by Rothschild and White (1995), universities compete for students for two basic reasons:

- (i) As inputs, students are required for the production of education. With regard to teaching activities, we assume for convenience that the level of teaching in university i is equal to the number of students enrolled, n_i , weighted by the parameter $0 < \gamma < 1$:

$$T = \gamma n_i$$

where γ quantifies the relative importance granted to education over research objectives in this university.

- (ii) As clients, students provide the funds a university needs to operate, either directly through fees (f) or indirectly via the government. R represents each university's expenditure on research. Here, the research output, S_i , depends only on the money invested in it:

$$R = S_i$$

More students enrolled may therefore imply an increased research budget, particularly when a university is funded through a per-student government subsidy, s ($0 < s < 1$).

However, increasing the size of the student population is costly, in particular when the number of students enrolled exceeds the capacity k_i . The cost supported by each university depends on its capacity, k_i , with a unit cost δ ; beyond capacity though, the cost increases quadratically with the number of students. According to Cabon-Dhersin and Drouhin (2014), the capacity constraint is "soft" and education services provided above this limit incur an additional per unit cost (μ).

Thus, the cost function of a university i is given by

$$C_i(n_i, k_i) = \begin{cases} \delta k_i & \text{if } 0 \leq n_i \leq k_i \\ \delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i \end{cases} \quad (2)$$

where $\mu, \delta > 0$. The cost parameters (δ and μ) are constant and similar for all universities.

The optimization program for university i is thus given by:

$$\underset{f_i}{\text{Max}}\{\gamma n_i + S_i\} \quad \text{s.t.} \quad S_i + C_i(n_i, k_i) = (f_i + s)n_i$$

Our model considers a novel sequential scenario. In the first stage, the universities choose a subsequently invariable capacity, k_i . In the second, the universities may use the price strategic variable (i.e. set their tuition fees to satisfy the demand they face). Note that in the second stage, δk_i is like a “sunk cost”, i.e. one that is unavoidable in this stage⁷. This implies that when the number of students enrolled is lower than the capacity threshold, the marginal cost of each student is nil up to the capacity threshold k_i and the average cost per student decreases with n_i . Above the capacity threshold k_i , the marginal cost increases with n as does the average cost, giving a familiar U-shaped average cost function. This seems to be the case at least for the average cost per student completion in Australian universities (Worthington and Higgs, 2011).

3 Equilibria

The universities fix their capacities in the first stage, then compete on fees to attract students in the second. The game is solved by backward induction. The universities must thus decide whether to operate below or above capacity. If the number of students enrolled at equilibrium is always greater than the endogenous enrollment capacity, the case in which universities operate under capacity can be ignored.

3.1 The second stage of the game: competition on tuition fees

We can express the payoff G_i of each university i as:

$$G_i(f_i, f_{-i}, k_i, m) = \begin{cases} -\delta k_i & \text{if } f_i > f_{-i}^{min} \\ (f_i + \gamma + s) \frac{N(f_i)}{m} - C_i\left(\frac{N(f_i)}{m}, k_i\right) = G_D & \text{if } f_i = f_{-i} \\ (f_i + \gamma + s) N(f_i) - C_i(N(f_i), k_i) = G_M & \text{if } f_i < f_{-i}^{min} \end{cases}$$

where as before, f_i stands for the fees of university i and $N(f_i)$ is the total number of students enrolled when university i sets fees f_i .

The function $G_D(f_i, k_i, m)$ represents the payoff of university i when all the universities charge the same fees, while the function G_M represents the payoff of university i when it charges the lowest tuition fees and serves all the demand. G_M and G_D are strictly concave in f_i and in k_i ⁸.

⁷Avoidable fixed costs create problems for the existence of a price equilibrium (Saporiti and Coloma, 2010; Dastidar, 2011b,a).

⁸All the second order conditions are always satisfied: $\partial^2 G_D(f_i, k_i, m) / \partial f_i^2 < 0$, $\partial^2 G_D(f_i, k_i, m) / \partial k_i^2 < 0$, $\partial^2 G_M(f_i, k_i, m) / \partial f_i^2 < 0$ and $\partial^2 G_M(f_i, k_i, m) / \partial k_i^2 < 0$.

We define $\bar{f}(k_i, m)$ to be the value that solves $G_M(f, k_i, m) = G_D(f, k_i, m)$. Thus, $\bar{f}(k_i, m)$ represents the fee threshold at which the university is equally inclined to operate in the higher education market alone and with its rivals. We obtain:

$$(1 - f_i)(f_i + \gamma + s) - \mu[(1 - f_i) - k_i]^2 = \frac{1}{m}(1 - f_i)(f_i + \gamma + s) - \mu\left[\frac{(1 - f_i)}{m} - k_i\right]^2$$

$$\Leftrightarrow \bar{f}(k_i, m) = \frac{\mu(m + 1) - m(\gamma + s) - 2m\mu k_i}{m + \mu m + \mu} \quad (3)$$

In the second stage, the fixed capacity cost, δk_i , is sunk and the universities only quote a fee if the variable part of their payoff is positive, *i.e.* if $G_D(f, k_i, m) \geq -\delta k_i$. Thus, we also define $\hat{f}(k_i, m)$, the lowest fees compatible with enrolling students in the second stage, as the value that solves $G_D(f, k_i, m) = -\delta k_i$ for a given k_i .

Finally, we define $f_c(k_i, m)$, the fees that maximize the payoff of university i when all the universities operate in the higher education market (collusive fees). In simple terms, this value can be interpreted as the maximum fees when all the universities have chosen the same capacity in the first stage.

$$f_c(k_i, m) = \arg \max_f \{G_i(f, k_i, m)\} = \frac{m(1 - \gamma - s) + 2\mu(1 - mk_i)}{2(m + \mu)} \quad (4)$$

One crucial point here is the negative impact of the capacity on the equilibrium fee levels (\bar{f} and f_c). This means that the lower (higher) the capacity is, the higher (lower) the costs to attract additional students are, and consequently, the higher (lower) the tuition fees are.

It is important to understand how these utility functions (G_M and G_D) and fees (\hat{f} , f_c , and \bar{f}) depend on one another.

Lemma 1. *For a given $k_i > 0$ and $m \geq 2$,*

$$\left\{ \begin{array}{l} \hat{f}(k_i, m) < \bar{f}(k_i, m) < f_c(k_i, m) \quad \text{if } \mu < \frac{m}{m-1} \\ \hat{f}(k_i, m) < \bar{f}(k_i, m) = f_c(k_i, m) \quad \text{if } \mu = \frac{m}{m-1} \\ \hat{f}(k_i, m) < f_c(k_i, m) < \bar{f}(k_i, m) \quad \text{if } \mu > \frac{m}{m-1} \end{array} \right.$$

In Proposition 1 below, we look for the Nash equilibrium in fees assuming that all universities have chosen the same capacity in the first stage, $k = k_1 = \dots = k_m$ ⁹.

⁹This assumption can easily be relaxed. Cabon-Dhersin and Drouhin (2017) propose a general model with m firms with potentially asymmetric capacities. They show how m firms choose the same capacity in the first stage anticipating its effect on the price equilibria in the second stage. Here, we simplify the model by assuming equal capacities already in the first stage. This assumption enables tractable analysis.

Proposition 1. *In the second stage, for a given k and $m \geq 2$, $f^N(k, m)$ is a pure-strategy Nash equilibrium such that:*

- (i) *If $\mu < \frac{m}{m-1}$, $f^N(k, m) \in [\hat{f}(k, m), \bar{f}(k, m)]$ and $\bar{f}_1(k, m) = \dots = \bar{f}_m(k, m) = \bar{f}(k, m)$ is a payoff-dominant pure-strategy Nash equilibrium,*
- (ii) *If $\mu = \frac{m}{m-1}$, $f^N(k, m) \in [\hat{f}(k, m), \bar{f}(k, m)]$ and $\bar{f}_1(k, m) = \dots = \bar{f}_m(k, m) = \bar{f}(k, m) = f_c(k, m)$ is a payoff-dominant pure-strategy Nash equilibrium*
- (iii) *If $\mu \geq \frac{m}{m-1}$, $f^N(k, m) \in [\hat{f}(k, m), \bar{f}(k, m)]$ and $f_c(k, m) \in [\hat{f}(k, m), \bar{f}(k, m)]$ is a payoff-dominant pure-strategy Nash equilibrium*

Proof: see Appendix A. \square

Table (1) presents the outcomes in the second stage of the game in different settings. By simply calculating the derivatives, we find that $\bar{f}(k, m)$ and $f_c(k, m)$ decrease with k for a given m and with m for a given k ; $\bar{n}(k, m)$ and $n_c(k, m)$ increase with k for a given m but decrease with m for a given k .

If $\mu < \frac{m}{m-1}$	$f^N(k, m) = \bar{f}(k, m) = \frac{\mu(m+1) - m(\gamma+s) - 2m\mu k}{m + \mu m + \mu}$ $\bar{n}(k, m) = \frac{1 + \gamma + s + 2\mu k}{m + \mu m + \mu}$
If $\mu \geq \frac{m}{m-1}$	$f^N(k, m) = f_c(k, m) = \frac{m(1 - \gamma - s) + 2\mu(1 - mk)}{2(m + \mu)}$ $n_c(k, m) = \frac{1 + \gamma + s + 2\mu k}{2(m + \mu)}$

Table 1: Equilibrium levels of fees in the second stage of the game as a function of μ .

The predicted Nash equilibrium in the second stage of the game is essentially identical to the one reported by Dastidar (1995). For all fees above $\bar{f}(k, m)$, no student enrolls at university i , undermining its finances. When the other universities charge any fee $f(k, m) \in [\hat{f}(k, m), \bar{f}(k, m)]$, the best response for university i is to quote the same tuition fees so that students split between the institutions. A university can increase its revenue (through higher enrollment) by lowering its fees, but the corresponding costs, strictly convex above capacity, will increase even more, making a fee decrease non-profitable. For this reason, a continuum of fees with

$f(k, m) \in [\hat{f}(k, m), \bar{f}(k, m)]$, can be sustained at Nash equilibria in pure strategies. A high value of parameter μ means that costs are sufficiently convex (*i. e.* $C''(n_i) \geq \frac{2m}{m-1}(\frac{-1}{D'})$) which is equivalent to $\mu \geq \frac{m}{m-1}$ ¹⁰ to ensure that high fees are sustained as Nash equilibria in pure strategies (Dastidar, 2001). In this case, $f_c(k, m)$ lies between $\hat{f}(k, m)$ and $\bar{f}(k, m)$. The symmetry and the payoff dominance criterion are sufficient here to reduce the set of equilibria¹¹ and prove the uniqueness of the solution¹². There are then two fee equilibria: (i) when $\mu < \frac{m}{m-1}$, the symmetric fee $f^N(k, m) = \bar{f}(k, m)$ is the unique (payoff-dominant) pure-strategy Nash equilibrium; (ii) when $\mu \geq \frac{m}{m-1}$, the unique and symmetric equilibrium is such that $f^N(k, m) = f_c(k, m)$ and corresponds to the solution of the joint-payoff maximization program in the second stage of the game (tacit collusive fees).

Considering just one university with a given capacity, Proposition (1) allows us to draw the functions G_M and G_D and to represent graphically the unique equilibrium in tuition fees for each case depending on the value of the parameter μ . Figure (1) illustrates the three cases in the second stage of the game (*i.e.* for a fixed capacity):

- (i) when $\mu < \frac{m}{m-1}$, the universities charge the maximum fees \bar{f} which equates G_M and G_D . At any fee below $\bar{f}(k, m)$, G_M is less than G_D . At any fee higher than $\bar{f}(k, m)$, it is the other way round. That is $\bar{f}(k, m)$ is the lowest level of fees at which undercutting (reducing fees to increase enrollment) does not pay and corresponds to the payoff-dominant Nash equilibrium. Note that in this case, $\bar{f}(k, m) < f_c(k, m)$.
- (ii) when $\mu = \frac{m}{m-1}$, $\bar{f}(k, m) = f_c(k, m)$.
- (iii) when $\mu > \frac{m}{m-1}$, the universities charge the collusive fee, $f_c(k, m)$, which becomes lower than $\bar{f}(k, m)$ and corresponds to the payoff-dominant Nash equilibrium.

¹⁰This condition is similar to the one in Dastidar (2001) p.86, Prop.1.

¹¹An equilibrium point is said to be payoff dominant if it is not strictly dominated by another equilibrium point; that is, there exists no other equilibrium in which payoffs are higher for all universities (Harsanyi and Selten, 1988).

¹²See Cabon-Dhersin and Drouhin (2017) for a proof of this uniqueness.

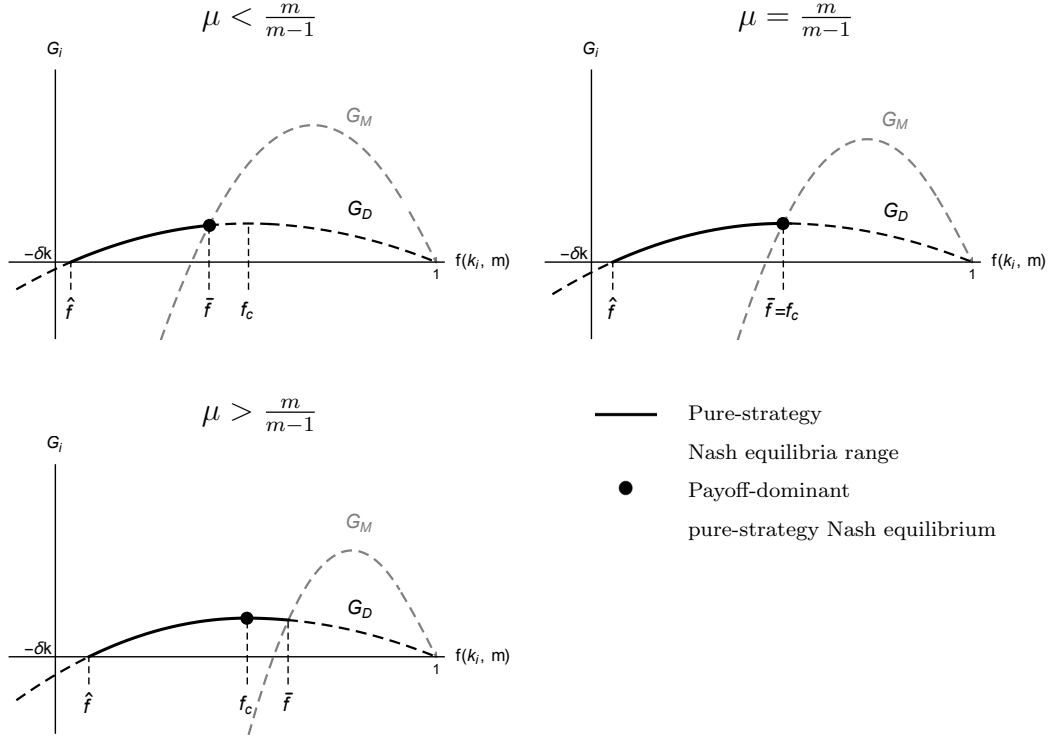


Figure 1: Nash equilibria in the second stage as a function of μ (with $s = 0.001$, $\gamma = 0.1$, $\delta = 0.01$ and $m = 5$).

3.2 The first stage of the game: choice of enrollment capacity

The universities set their capacities non-cooperatively by maximizing their objective function and anticipating its effect on the fee equilibria in the second stage:

$$\frac{\partial G_i(f^N(k, m), k, m)}{\partial k} = 0 \quad \Leftrightarrow$$

$$k^*(m) = \frac{f^N(k, m)' (m(1 - \gamma - s) + 2\mu - 2(m + \mu)f^N(k, m)) - m(\delta m - 2\mu) - 2m\mu f^N(k, m)}{2m\mu(f^N(k, m)' + m)}$$

For each configuration, depending on μ , a unique and symmetric solution exists, satisfying $\frac{\partial G_i}{\partial k} = 0$. Proposition (2) presents the equilibrium prediction of the whole game.

Proposition 2. $\forall m \geq 2$, the Subgame Perfect Equilibrium (SPE) is such that:

(i) If $\mu < \frac{m}{m-1}$,

$$\begin{cases} \bar{k}^*(m) = \frac{4\mu^2 m(1+\gamma+s) - (m+\mu m+\mu)^2 \delta}{2\mu((m+\mu m+\mu)^2 - 4m\mu^2)} > 0 & \text{for } \delta < \frac{4\mu^2 m}{(m+\mu m+\mu)^2} (1+s+\gamma) \\ \bar{f}^*(m) = 1 - m \frac{(m+\mu m+\mu)}{(m+\mu m+\mu)^2 - 4m\mu^2} (1+\gamma+s-\delta) \\ \bar{n}^*(m) = \frac{(m+\mu m+\mu)}{(m+\mu m+\mu)^2 - 4m\mu^2} (1+\gamma+s-\delta) \end{cases}$$

(ii) If $\mu \geq \frac{m}{m-1}$,

$$\begin{cases} k_c^*(m) = n_b^*(m) - \frac{\delta}{2\mu} > 0 & \text{for } \delta < \frac{\mu}{\mu+m} (1+s+\gamma) \\ f_c^*(m) = \frac{1-\gamma-s+\delta}{2} \\ n_c^*(m) = \frac{1+\gamma+s-\delta}{2m} \end{cases}$$

with $f_c^*, \bar{f}^* > 0$ for $s < \bar{s} = 1 + \gamma - \delta$ ¹³.

Proof: The proof is straightforward.

Corollary 1. $\forall \mu, s, \delta > 0, m \geq 2$ and $k > 0$, we verify that at SPE:

$$\bar{k}^*(m) < \bar{n}^*(m) \quad \text{and} \quad k_c^*(m) < n_c^*(m)$$

Proof: see Appendix B. \square

Competition on tuition fees between m universities is likely to induce a situation in which student demand exceeds capacity. In the first stage, the universities can choose not to recruit beyond their capacity, and thus $C_i = \delta k_i$. But the Subgame Perfect Equilibrium implies that the number of students enrolled, n^* , is always greater than the endogenous capacity k^* , which means that only the case in which $C_i = \delta k_i + \mu(n_i - k_i)^2$ needs to be taken into account. Whatever the values of the parameters in the game and the number of universities involved, they all choose to enroll beyond capacity at a convex marginal cost.

¹³If the level of subsidies is too high, university sets a level of fees equal to zero.

4 Comparative Statics

We now investigate how the convexity of the cost function and the number of universities influence the equilibrium results.

Corollary 2. *We verify that:*

(i) If $\mu < \frac{m}{m-1}$,

$$\frac{\partial \bar{k}^*}{\partial \mu} > 0, \quad \frac{\partial \bar{n}^*}{\partial \mu} < 0, \quad \frac{\partial \bar{f}^*}{\partial \mu} > 0$$

(ii) If $\mu \geq \frac{m}{m-1}$,

$$\frac{\partial k_c^*}{\partial \mu} > 0, \quad \frac{\partial n_c^*}{\partial \mu} = 0, \quad \frac{\partial f_c^*}{\partial \mu} = 0$$

Proof: see Appendix C \square

In our model, the convexity of the cost function in the second stage is crucial. Figure (2) shows how the degree of convexity impacts the equilibrium prediction of the whole game, when ten universities operate in the higher education market.

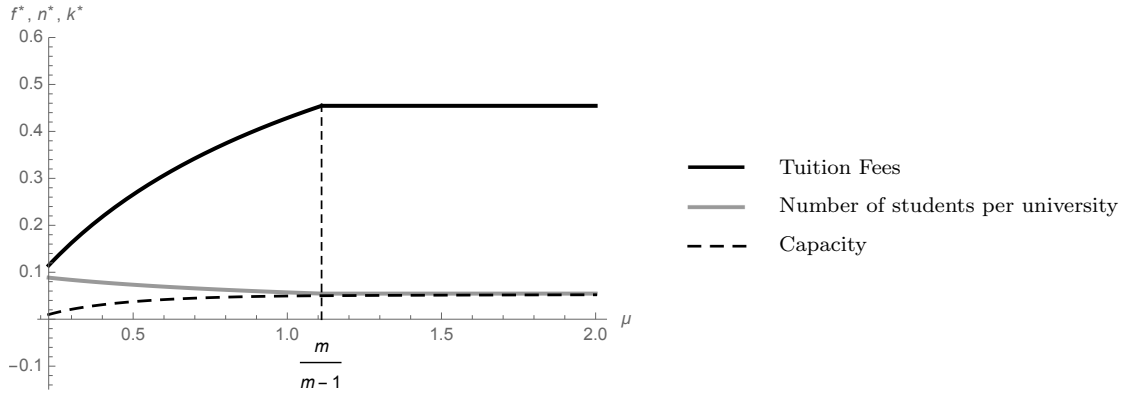


Figure 2: Tuition fees, number of students per university and university capacities as a function of μ (with $s = 0.001$, $\gamma = 0.1$, $\delta = 0.01$ and $m = 10$).

Note that the benefits of undercutting are negative, as the costs increase disproportionately. A more convex variable cost function implies that fee adjustment in the second stage is more costly (here, with ten universities, a university that decreases its fees has to enroll approximately ten times more students to compensate.)

This implies that more convex costs (higher values of μ) tend to push \bar{f} up to f^* . When the degree of convexity is sufficiently high ($\mu \geq \frac{m}{m-1}$), the universities all charge the collusive fee, f_c^* . Thus, because the average cost is U-shaped, each university can adopt an efficient capacity in the first stage of the game that minimizes its costs. When the additional marginal cost (μ) increases, the university chooses a higher capacity to limit the number of students enrolled beyond this threshold: the capacity can be increased to minimize the total cost. Consequently, the collusive fee and the number of students enrolled at the collusive equilibrium no longer depend on the convexity of the cost function.

We now study how the tuition fees, capacities and the number of students enrolled vary with the number of universities.

Corollary 3. *For $m \geq 3$, we verify the following properties:*

$$(i) \text{ If } \mu < \frac{m}{m-1}, \quad \left\{ \begin{array}{l} \frac{\partial \bar{k}^*(m)}{\partial m} < 0 \\ \frac{\partial \bar{f}^*(m)}{\partial m} > 0 \\ \frac{\partial \bar{n}^*(m)}{\partial m} < 0 \end{array} \right. \text{ if } \mu > \bar{\mu}$$

$$\text{with } \bar{\mu} = \frac{-m(m-1)+2m\sqrt{m(m-1)}}{3m^2-2m-1} \in (0, \frac{m}{m-1})$$

$$(ii) \text{ If } \mu \geq \frac{m}{m-1}, \quad \left\{ \begin{array}{l} \frac{\partial k_c^*(m)}{\partial m} < 0 \\ \frac{\partial f_c^*(m)}{\partial m} = 0 \\ \frac{\partial n_c^*(m)}{\partial m} < 0 \end{array} \right.$$

Proof: see Appendix D. \square

Figure (3) relates the equilibrium prediction f^* with the number of universities. This reveals an important threshold. When the number of universities is low (between 3 and $\frac{\mu}{\mu-1} = 6$), fees increase with m and the corresponding equilibrium fees are \bar{f}^* . Beyond $m = \frac{\mu}{\mu-1}$, f_c^* being lower than \bar{f}^* , the equilibrium fees become the collusive fees.

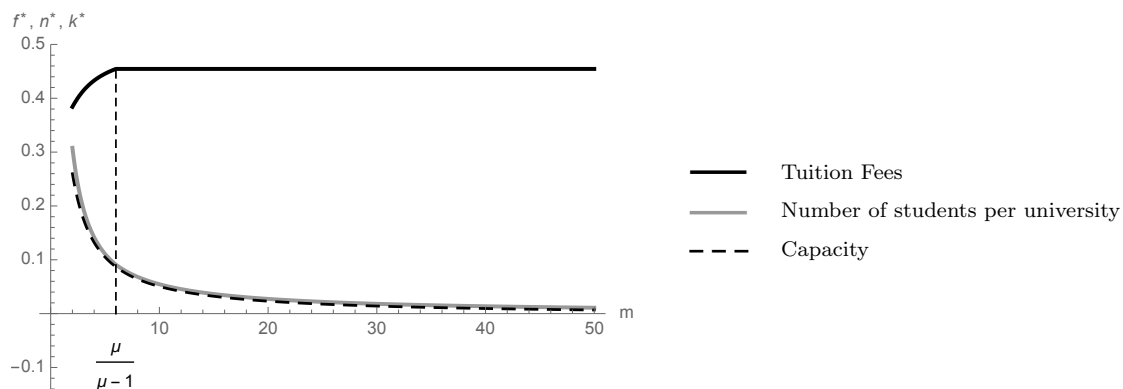


Figure 3: Tuition fees, number of students per university and university capacities as a function of m (with $s = 0.001$, $\gamma = 0.1$, $\delta = 0.01$ and $\mu = 1.2$).

The above properties have two surprising effects:

- (i) Conventional wisdom suggests that when the number of firms involved in price competition increases, the outcome is more competitive prices. We obtain the opposite result: under increased competition on tuition fees, these increase with m . This is somewhat counterintuitive.
- (ii) If the cost function is sufficiently convex ($\mu \geq \frac{m}{m-1}$), f_c^* is the unique equilibrium and does not vary with m . For intermediate marginal costs ($\mu < \frac{m}{m-1}$), \bar{f}^* increases toward f_c^* .

An intuitive explanation of these properties is that the increase in m has two opposite effects on tuition fees. On the one hand, as the number of universities increases, each one faces lower marginal costs since the number of students enrolled (n_i) decreases (demand being shared out evenly). These lower marginal costs (the cost function is less convex) encourage the universities to lower their fees through the “demand effect”. On the other hand, the “capacity effect” promotes the opposite behavior: when there are more universities, each one chooses a lower capacity in the SPE (see Corollary (3)). Equations (3) and (4) clearly highlight the negative correlation between fees and capacities: lower capacities induce a rise in fees, all things being equal.

Either effect can dominate, depending on the convexity of the cost function. When μ is very low ($\mu < \bar{\mu}$), the “demand effect” dominates and fees (\bar{f}^*) fall as m increases. A stronger “capacity effect” implies higher marginal costs such that fees increase with m up to the maximum, f_c^* . When costs become “too” convex ($C''(n_i) \geq \frac{2m}{m-1}$ or $\mu \geq \frac{m}{m-1}$), each university chooses a capacity that maximizes its revenue and minimizes the average cost per student. This adjustment allows the fees to be maintained at the maximum level, f_c^* , regardless of the number of universities. In this case, the “capacity effect” is exactly offset by the “demand effect”.

5 Welfare Analysis

Another question we address is how the number of otherwise identical universities competing on tuition fees affects social welfare.

We define social welfare as

$$SW = \underbrace{\frac{N^2}{2}}_{\text{Consumer surplus}} + \underbrace{(m.G_i)}_{\text{University surplus}} - \underbrace{s.N}_{\text{Social cost of the subsidy}} \quad (5)$$

We obtain:

$$SW = (1 + \gamma)N - \frac{N^2}{2} - mC(n, k)$$

Proposition 3. *When the marginal costs are sufficiently convex ($\mu \geq \frac{m}{(m-1)}$), social welfare always increases with the number of universities:*

$$\frac{\partial SW}{\partial m} = \frac{\delta^2}{4\mu} > 0$$

Proof: Appendix E. \square

When the cost to attract additional students is sufficiently convex, the competition between an increasing number of universities allows fees to be sustained at their maximum level without reducing the total number of admissions. Under fee competition, a greater number of universities enhances social welfare in spite of the higher fees they set. This counterintuitive result can be explained by the possibility universities have to choose the most efficient capacity, allowing them to reduce their costs. From this result, we can conclude that all policies leading to an increase in the number of universities should not lower tuition fees or increase the total number of students. The efficient consideration of costs when there are many universities in the market increases welfare, especially since the cost of the fixed capacity (δ) is high and the cost of each student enrolled beyond this capacity (μ) is low.

6 Conclusion

In this paper, we have analyzed a two-stage game between homogeneous universities that fix their capacity, and then compete on tuition fees. Interestingly, our model emphasizes the role of the strategic variable and the cost structure in deciding the outcome of the game and offers a framework that explains some stylized facts. This contribution thus shows that high tuition fees cannot just be explained by improvements in teaching and/or research, lower public subsidies or higher expected returns of a university education. Competition between a large number of homogeneous universities to attract additional students creates a higher-education

market in which universities charge high, sometimes (tacitly) collusive fees. The fees charged by universities can even increase with the number of universities. Finally, competition between an increasing number of universities maintains fees at their maximum level (collusive fees) without reducing either the total number of admissions or social welfare.

These results could be considered “unrealistic”. Nevertheless, some politicians in the United Kingdom are considering asking the competition regulator to investigate the collusive behavior of universities. Universities are unlikely to collude explicitly. However, our results show that the cost structure in a deregulated higher education market can induce behaviors akin to collusion.

Some of the simplifying assumptions made here merit further discussion. First of all, the particular form of the demand function in this work is critical to the results obtained. We assume that demand is infinitely elastic: this elasticity can have an impact on the different equilibrium outcomes of the model. The nature of student support policies (financial or other) may also influence our results. Secondly, we assume that the universities are identical in terms of their curriculum and all other non-price dimensions. These could be considered in the future but the results obtained in this article are surprising enough to warrant attention by themselves.

Appendix A Proof of Proposition 1

Let us first investigate symmetric strategy profiles in the interval $[\hat{f}, \bar{f}]$. When a competitor charges fees $f \in [\hat{f}_i, \bar{f}_i]$, the best response for university i is to quote the same amount. When university i quotes the same fees, it obtains $G_D(f)$. We know that for all $f \geq \hat{f}_i$, $G_D(f) \geq -TFC$. If the university deviates (by quoting $f - \epsilon$), it gains $G_d(f - \epsilon)$. We also know that for all $f \in [\hat{f}_i, \bar{f}_i]$, $G_D(f) \geq G_M(f) > G_M(f - \epsilon)$. Since the university must satisfy all the demand it faces, the additional revenue (from higher enrollment) is less than the increase in costs: the university enrolls additional students at an excessive marginal cost. By quoting $f + \epsilon$, university i receives no demand and obtains zero variable payoff for its research activities. The optimal strategy is therefore for each university to quote the same fee. There are no incentives to deviate, which proves the implication in Proposition 1. It also proves that all asymmetrical strategy profiles in which at least one university quotes a price in the interval are not Nash equilibria.

We now have to investigate all the other strategy profiles (viz. those in which universities only quote prices outside the interval). It is easy to check that for all symmetric strategic profiles with $f < \hat{f}$, the firm's interest is to increase its tuition fees. Conversely, universities will lower their fees if $f > \bar{f}$. The payoff dominance criterion, the natural criterion given that all actors are presumably fully rational, is sufficient to provide uniqueness in the two configurations.

Appendix B Proof of Corollary 1

Let us compare the number of students enrolled in a given university with its capacity in different situations:

(i) If $\mu < \frac{m}{m-1}$,

$$\begin{aligned} \bar{n}^*(m) &> \bar{k}^*(m) \\ \Leftrightarrow 2\mu(m + m\mu + \mu)(1 + \gamma + s - \delta) &> 4\mu^2m(1 + \gamma + s) - (m + m\mu + \mu)^2\delta \\ \Leftrightarrow 2\mu(m + \mu - m\mu)(1 + \gamma + s) &> (m + m\mu + \mu)(\mu - m - m\mu)\delta \end{aligned}$$

which is always verified because $m + \mu - m\mu > 0$ and $\mu - m - m\mu < 0$.

(ii) If $\mu \geq \frac{m}{m-1}$,

$$n_c^*(m) - k_c^*(m) = \frac{\delta}{2\mu} > 0$$

Appendix C Proof of Corollary 2

(i) If $\mu < \frac{m}{m-1}$,

$$\frac{\partial \bar{n}^*}{\partial \mu} = -\frac{m\mu(\mu(m-1) + 2m(m-2) + \mu^2 m + 2) + \mu^2 + m^2 + m^3}{(\mu^2 + m^2 - 2m\mu^2 + 2m\mu + 2m^2\mu + m^2\mu^2)^2} < 0 \quad \text{with } m \geq 2$$

Because $\bar{f}^* = 1 - m \cdot \bar{n}^*$, we have

$$\frac{\partial \bar{f}^*}{\partial \mu} > 0$$

From Equation (3), we have:

$$\bar{k}^* = -\frac{m + \mu m + \mu}{2m\mu} \bar{f}^* + \frac{m+1}{2m} - \frac{\gamma + s}{2\mu}$$

$$\frac{\partial \bar{k}^*}{\partial \mu} > 0 \quad \text{with} \quad \frac{\partial \bar{f}^*}{\partial \mu} > 0$$

(ii) If $\mu \geq \frac{m}{m-1}$,

We obtain cases (ii) with straightforward computations.

Appendix D Proof of Corollary 3

(i) If $\mu < \frac{m}{m-1}$,

$$\frac{\partial \bar{n}^*(m)}{\partial m} = -\frac{(1+\mu)(m+\mu m+\mu)^2 - 4\mu^3}{((m+\mu m+\mu)^2 - 4m\mu^2)^2} (1+s+\gamma-\delta) < 0$$

$$\frac{\partial \bar{k}^*(m)}{\partial m} = -\frac{2\mu m^2(1+\mu)^2 - 2\mu^3}{((m+\mu m+\mu)^2 - 4m\mu^2)^2} (1+s+\gamma-\delta) < 0$$

$$\frac{\partial \bar{f}^*(m)}{\partial m} = -\bar{n}^*(m) - m \frac{\partial \bar{n}^*(m)}{\partial m}$$

$$\frac{\partial \bar{f}^*(m)}{\partial m} > 0 \quad \text{if} \quad -m \frac{\partial \bar{n}^*(m)}{\partial m} > \bar{n}^*(m)$$

$$\Leftrightarrow m \frac{(1+\mu)(m+\mu m+\mu)^2 - 4\mu^3}{((m+\mu m+\mu)^2 - 4m\mu^2)^2} (1+s+\gamma-\delta) > \frac{(m+\mu m+\mu)}{(m+\mu m+\mu)^2 - 4m\mu^2} (1+\gamma+s-\delta)$$

$$\begin{aligned} \Leftrightarrow m(1+\mu)(m+\mu m+\mu)^2-4m\mu^3 &> (m+\mu m+\mu)\left((m+\mu m+\mu)^2-4m\mu^2\right) \\ \Leftrightarrow \mu^2(3m^2-2m-1)-\mu(2m(1-m))-m^2 &> 0 \end{aligned}$$

We have,

$$\frac{\partial \bar{f}^*(m)}{\partial m} > 0 \quad \text{if} \quad \bar{\mu} < \mu \leq \frac{m}{m-1}$$

and

$$\frac{\partial \bar{f}^*(m)}{\partial m} < 0 \quad \text{if} \quad 0 < \mu \leq \bar{\mu}$$

$$\text{with } \bar{\mu} = \frac{-m(m-1)+2m\sqrt{m(m-1)}}{3m^2-2m-1}$$

(ii) If $\mu \geq \frac{m}{m-1}$,

We obtain cases (ii) with straightforward computations.

Appendix E Proof of Proposition 3

$$SW = (1+\gamma)N - \frac{N^2}{2} - m\delta k - m\mu(n-k)^2$$

The derivative of the above expression with respect to m gives:

$$\frac{\partial SW}{\partial m} = (1+\gamma-mn) \left(n + m \frac{\partial n}{\partial m} \right) - \delta \left(k + m \frac{\partial k}{\partial m} \right) - \mu(n-k) \left((n-k) + 2m \frac{\partial(n-k)}{\partial m} \right)$$

If $\mu \geq \frac{m}{m-1}$, we obtain

$$(i) \quad n + m \frac{\partial n}{\partial m} = 0,$$

$$(ii) \quad -\delta \left(k + m \frac{\partial k}{\partial m} \right) = \frac{\delta^2}{2\mu}$$

$$(iii) \quad -\mu(n-k) \left((n-k) + 2m \frac{\partial(n-k)}{\partial m} \right) = -\mu(n-k)^2 = -\frac{\delta^2}{4\mu}$$

Finally,

$$\frac{\partial SW}{\partial m} = \frac{\delta^2}{4\mu} > 0$$

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