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## Comparing Distributions of Body Mass Index Categories

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# Comparing distributions of body mass index categories

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## Abstract

This paper compares distributions of Body Mass Index (BMI) among men and women in France, the US and the UK on the basis of a new normative criterion. Comparing distributions of BMI from a normative standpoint is conceptually challenging because of the ordinal nature of the variable. Our normative criterion is well-suited to handle this issue. It coincides with the possibility of moving from the dominated distribution to the dominating one by a finite sequence of Hammond transfers and/or elementary efficiency gains. An additional difficulty with BMI is that it is not monotonically increasing (or decreasing) with health or well-being. We therefore perform our analysis by considering all health-consistent rankings of BMI values. Our empirical results are striking. For a large class of these rankings of BMI values, it is shown that the distribution of BMI in France has worsened on the period 2008-2010 for both men and women according to first order dominance. It is also shown that for most welfare rankings of BMI values, the distribution of BMI is worse in every period in the female population than in the male one in all three countries.

## 1 Introduction

It is widely acknowledged that body weight is an important contributor to individual well-being. The adverse effect of overweight and obesity on health (especially metabolic dysfunction like diabetes and the probability of suffering from cardiovascular afflictions) is largely documented. It is also well-known that the prevalence of obesity and overweight has been growing in developed countries.

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For instance a recent OECD report<sup>1</sup> indicates that a majority of the adult population in OECD is now overweight and obese. This proportion was less 10% in 1980. In the US, the fraction of the adult population concerned with this problem reaches 65%, according to the *Food Research and Action center* (see e.g. <http://frac.org>). The spectacular increase in the prevalence of obesity and overweight observed in the last thirty years makes it one of the most important epidemic of the modern era. Less spectacular and discussed, but nonetheless present, is the small, but significant, fraction of the population found in developed countries that is considered pathologically underweight. A significant body of epidemiological evidence (see e.g. Cao, Moineddin, Urquia, Razak, and Ray (2014) or Flegal, Graubard, Williamson, and Gail (2005)) suggests that being underweight can also seriously increase the probability of death in the next 5 years as compared to having a "normal" weight.

It is also largely documented that body weight may impact individual well-being in a way that is not reducible to its health consequences, however severe these may be. Excessive body weight may indeed affect self esteem and happiness (see e.g. Oswald and Powdthavee (2007)) and lead to social stigma (see e.g. Carr and Friedman (2005), Roberts, Kaplan, Shema, and Strawbridge (2000), Roberts, Strawbridge, Deleger, and Kaplan (2002)) or to an unfavorable image of one self when comparing with others (see e.g. Blanchflower, Landeghem, and Oswald (2010)).

Given such an importance of body weight as a contributor to individual well-being, it is of interest to normatively evaluate the distribution of individual body weight in the population. To put it bluntly, can it be said that the distribution of individual body weight observed in a given community is "normatively better" than what it is in another ? Addressing question like this by focusing only the fraction of the population that is obese or overweight, as done in many studies, may be misleading. For instance, a given overall increase in the obesity rate in a population may not have the same signification when it results from a combination of a *decrease* in the proportion of the population that is severely obese and a (larger) *increase* in the proportion of mildly obese individuals than when it results from another combination in which severe obesity has increased and mild obesity has diminished.

This paper is concerned with the issue of comparing distributions of individual body weights from a normative standpoint. It focuses specifically on the numerical measurement of body weight provided by the Body Mass Index (BMI), defined to be the ratio of the individual weight (in kilograms) over the "surface body" (in squared meters). Despite its limitations as a predictor of health hazards and as an indicator of excess adiposity (see e.g. Keys, Fidanza, Karvonen, Kimura, and Taylor (1972) or Gray and Fujioka (1991)), the BMI remains one of the most common index for measuring body weight. As such, it has been used in thousands of scientific studies, and serves as the reference for defining the various weight *categories* that are considered medically meaningful. According to the World Health Organization (WHO), these are, for the adult population above the age of 20:

- Underweight (BMI below 18.5),
- Normal (BMI between 18.5 and 25),
- Overweight (BMI between 25 and 30)

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<sup>1</sup>F. Sassi, "*Obesity Update*", OECD, June 2014, 8 pages.

Grade 1 obesity (BMI between 30 and 35)

Grade 2 obesity (BMI between 35 and 40)

Grade 3 obesity (BMI above 40)

This common "categorical" interpretation of the values of BMI as indicator of weight status is worth pointing out. While BMI is clearly a continuous variable, little significance is usually attached to small variations in its value. Body weight differences between individuals are considered important only when they are somewhat visible and/or, when they are associated to a medically significant differential health outcomes. There is obviously some arbitrariness in drawing the lines between the categories. Yet there seem to be a relative consensus for considering these six categories, and the lines that distinguish them, as indicative of "significant" variations in the individual body weight. Suppose that we accept this consensus. How could we then compare alternative distributions of these BMI categories in a given population? When can we say that one distribution of these BMI categories is "better" than another?

Two principles come to mind when comparing alternative distributions of a contributor - like a BMI category - to individual well-being from a normative standpoint.

The *first* one is an "efficiency" principle according to which moving individuals from bad categories to better ones is, everything else being the same, a "good" thing. Such an efficiency principle clearly underlies the largely discussed concern for the increase in the prevalence of obesity in the adult population observed in OECD countries in the last 30 years. There are well-known - and widely used - tools for appraising improvements in the distribution of an attribute based on such efficiency considerations. All of them produce rankings of distributions that are compatible with first order stochastic dominance (see e.g. Lehmann (1955), Quirk and Saposnik (1962) or Hadar and Russell (1974)). According to this criterion, a change in the distribution of BMI categories that weakly decreases the fraction of the population falling in a worse category than any given category is a clear normative improvement. It is worth noticing that the use of first order dominance requires that the categories be unambiguously ordered from worst to best. As discussed in the epidemiological literature (see e.g. Cao, Moineddin, Urquia, Razak, and Ray (2014) or Flegal, Graubard, Williamson, and Gail (2005)), there is no such clear ranking of the six aforementioned BMI categories in terms of their impact on individual well-being or health. While it is hardly disputable that individual well-being *decreases* with the value of the BMI in ranges where this value is above normal, it is not clear how the underweight status compares with, say, the overweight one. As we shall see, the comparisons of BMI distributions in the countries covered here turn out to be not overly sensitive to the assumptions made on the rankings of the BMI categories. This is at least true if the assumptions are compatible with the "natural" ranking that places the normal category above any other, and that orders all "excessive weights" categories in a decreasing fashion.

The *second* principle is an "equity" one that captures a concern for "equalizing" BMI categories across individuals. Implementing such a principle requires of course an operational definition of what it means for a distribution of BMI categories to be "more equal" than another. The meaning of "being more equal" is somewhat consensual when applied to distributions of a *cardinally measurable* attribute like income. The view here is, indeed, to consider a *transfer* of a *given quantity* of income from a richer individual to a poorer one as a clear inequality-

reducing operation. Transfers of this kind are called *Pigou-Dalton transfers* in the literature on income inequality measurement. An equalization of an income distribution is therefore defined in this literature as the fact of performing such Pigou-Dalton transfers a finite number of times. The modern theory of income inequality measurement (see e.g. Kolm (1969) Atkinson (1970) Dasgupta, Sen, and Starrett (1973), Sen (1973) and Fields and Fei (1978)) has established tight connections between this theoretical notion of equalization on the one hand and implementable criteria such as Lorenz domination or second order stochastic dominance on the other. The theory of income inequality measurement has also provided a wealth of numerical indices (like the Gini one, or those that belong to the generalized entropy family) for comparing distributions of income on the basis of their inequality. These indices are always required to be compatible with this elementary notion of equalization. The difficulty in applying this notion to rankings of BMI categories<sup>2</sup> lies, of course, in the *ordinal* nature of the information on individual well-being that these categories convey. The fact for an individual to belong to the "grade 3 obese" BMI category is certainly indicative of a worse well-being than the fact of belonging to the "grade 2 obesity" one. However, one would be hesitant in quantifying further this assessment. In particular, one would hesitate in comparing the difference between "grade 3" and "grade 2" obese categories on the one hand and, say, that between the "overweight" and the "normal weight" ones. Such a reluctance in attaching significance to *differences* in the values assigned to BMI categories, inherent to ordinal measurement, renders the definition of a Pigou-Dalton transfer rather problematic. This definition requires, indeed, the "quantity" transferred from the rich to the poor to be preserved. This preservation is obviously meaningless if the "quantity" of the attribute is ordinal or categorical. In the last ten years or so, health economists have increasingly acknowledged the difficulty of using conventional tools developed for measuring income inequality for comparing distributions of health outcomes that are often measured in an ordinal fashion. Examples of those contributions include the influential paper of Allison and Foster (2004) as well as those of Abul-Naga and Yalcin (2008) and Apouey (2007). Yet, for the moment, no statistical tools for comparing distributions of an ordinal attribute based on an elementary "equalizing operation" like Pigou-Dalton transfer has been proposed.

Some forty years ago, Peter J. Hammond (1976) has suggested, in the context of social choice theory, a so-called "minimal equity principle" that is explicitly concerned with distributions of an ordinally measurable attribute. According to Hammond's principle, a change in the distribution that "reduces the gap" between two individuals endowed with different quantities of the ordinal attribute is a good thing, irrespective of whether or not the "gain" from the poor recipient is equal to the "loss" from the rich giver. A Pigou-Dalton transfer is just a particular case of a Hammond transfer that imposes on the later the additional requirement - meaningless for an ordinal variable - that the "amount" given by the rich should equal the "amount" received by the poor. A Hammond transfer seems to be a plausible definition of an "ordinal inequality reduction". A distribution of BMI categories is unquestionably more equal than another when it

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<sup>2</sup>This difficulty is sometimes simply ignored by researchers. For instance Etile (2014) examines the impact of education on the Gini coefficient (an index of inequality that rides on the Pigou-Dalton principle of transfers) of the distribution of BMI in France over the period 1981-2003.

has been obtained from the later by a finite sequence of such Hammond transfers. In a recent paper, Gravel, Magdalou, and Moyes (2014) have identified an operational criterion that they have shown to be equivalent to the notion of equalization underlying Hammond transfers for contexts where the ordinal attribute can take only a finite number different values. In this paper, we put this criterion to work by evaluating the recent trends observed in the distributions of BMI in three major developed countries: France, the UK, and the US. For each of these three countries, we focus on the distributions between males and between females. There is indeed some evidence that the distributions of BMI differs across genders. As we shall see, in each of the three countries, the distribution of BMI appears to be worse for females than for males.

The criterion that we use to make these comparisons is based on a curve that we call the  $H^+$ -curve, by reference to the Hammond principle of transfer to which it is closely related. This curve is defined recursively in the following manner. It starts by assigning to the worst category the fraction of the population falling in that category. It then iterates by assigning, to every category above the worst one, twice the value assigned to the immediately preceding category plus the fraction of the population lying in this category. The innovation of this curve lies precisely in its addition, to any fraction of the population falling in some category, of *twice* the value assigned by the curve to the immediately preceding step. Or course, just like for the first order stochastic dominance, the definition of this curve requires that BMI categories be unambiguously ordered from the worst to the best. The criterion that we use to compare distributions of BMI categories, and that we call  $H^+$ -dominance, is for the dominating distribution to have a  $H^+$ -curve nowhere above and somewhere below that of the dominated one. Gravel, Magdalou, and Moyes (2014) have shown that observing  $H^+$ -dominance between two distributions is *equivalent* to the possibility of going from the dominated distribution to the dominating one by a finite sequence of Hammond transfers and/or increments of the attribute. Put differently, the  $H^+$  dominance test indicates unequivocally the occurrence of first order dominance and/or equalization - in the sense of Hammond transfers - between distributions of categorical variables. Gravel, Magdalou, and Moyes (2014) also identifies another test, that combines the  $H^+$  curve and a dual  $H^-$  curve, that captures equalization considerations - as defined by Hammond transfers - *only*.

Our empirical analysis reveals a few noticeable features.

First of all, and non surprisingly, the distribution of BMI categories in France appears to be better than that in the UK and the US by first order dominance for both males and females. This is at least true for the welfare rankings of the BMI categories that consider the "underweight" status to be better than the severely obese one. Such a dominance does not hold between countries however when one considers the underweight category to be worse than the severely obese one. This latter absence of dominance comes from the fact that the fraction of the population falling in the underweight category is slightly higher in France than in the US and the UK.

Another noticeable conclusion of the empirical analysis is that, for the same rankings of BMI categories, there happens to be a clear first order dominance deterioration in the distribution of BMI categories in France over the period 1998-2010. This trend is observed somewhat clearly for male but not so clearly for female. Indeed, for female, the conclusion happens to depend crucially upon the standing of the underweight category against the Grade 2 and Grade 3

obese one. If one assumes that underweight is better than Grade 2 obesity, then the data shows a clear first order dominance deterioration of the distribution of BMI categories among French women over the period 1998-2010. However, if one assumes that being underweight is worse than being in grade 3 obese category, one obtains the reverse conclusion that the distribution of BMI among French women has improved over the same period. The reason for this reversal in the evaluation of the 1998-2010 for French women results from the fact that the fraction of the female population that is underweight has dropped significantly over the period. Even if this drop has been accompanied by an increase in the fraction of grade 3 and grade 2 obese women in the population, the first effect, that is amplified by the extra weight given by  $H^+$  curve to the worse categories, is sufficiently important to dominates the second.

Interesting also are the cross gender comparisons of distributions of BMI in each of the three countries. As it turns out, for many plausible welfare rankings of BMI categories, BMI categories appears more fairly distributed as per the  $H^+$  criterion among men than among women. This dominance is observed at every period in all three countries. The rankings of BMI categories for which this  $H^+$  dominance arises are those that consider the underweight category to be worse than the overweight one, but nonetheless better than the severely or morbidly obese ones. The results show also, for these rankings of BMI categories, that there is no first order dominance between the distributions of BMI among men and among women. Hence, the  $H^+$  dominance of the distribution of BMI among men over that among women comes from "equity" considerations - as defined by Hammond transfers - rather than "efficiency" ones (as defined by first order stochastic dominance). As a matter of facts, it happens that the distribution of BMI categories among men can be shown to result from that among women by a finite sequence of Hammond transfers. Hence, in a strong sense, the distribution of BMI categories appear to be "more equal" among men than among women for the considered rankings of BMI categories. No such dominance is observed for those rankings of BMI categories that consider overweight to be worse than underweight, or underweight to be worse than the grade 3 obesity one. The reason for this is that, in all countries, the fraction of women that are in either grade 2 or 3 obesity categories is larger than the corresponding fraction of men. The same thing is true for the fraction of women who are underweight as compared to the corresponding fraction of men. However, the fraction of men who are overweight or mildly obese is much larger than the fraction of women who are in these categories. Hence these countervailing forces make the normative comparison of the distribution of BMI categories among men and women somewhat dependent upon the ranking of these BMI categories. Yet, if one considers plausible the fact, compatible with medical evidence (see e.g. Cao, Moineddin, Urquia, Razak, and Ray (2014) or Flegal, Graubard, Williamson, and Gail (2005)), that being underweight is worse than being overweight, it happens that the distribution of BMI categories is less fair among women than it is among men. While we believe this conclusion to be of some intrinsic interest, we also believe that it illustrates the usefulness of evaluating distributions of BMI categories by sound, explicit, and robust ethical principles.

The plan of the rest of the paper is as follows. The next section discusses the main criteria proposed to compare distributions of BMI categories and their logical connections. The third section presents the data and the results of the comparisons. The fourth section concludes.

## 2 Comparing distributions of BMI categories

Our objective is to compare alternative *distributions* of BMI categories among individuals. Every such individual can fall into one out of  $k$  different categories, indexed by  $h$ . While  $k = 6$  is going to be assumed here so as to stick to the well-established WHO categorizations of BMI mentioned above, we find useful to discuss the criteria in the more general setting of an arbitrary number  $k$  of categories. After all, it is not uncommon to find other categorizations of BMI like, for instance, a finer one that distinguish several types of "underweight" status, or that of the Hong Kong hospital that focuses on only five such categories, defined by significantly lower thresholds of overweight and obesity that are believed to be more appropriate for Asian people (see e.g. Chen, Ho, Lam, and Chan (2006)). Important for the analysis is the assumption that these categories be unambiguously ordered from the worst (e.g. being morbidly obese) to the best (e.g. having a normal weight). As discussed above, while this assumption is plausible when applied to the various "overweight" categories, it is not so clear how the "underweight" category compares with the "overweight" one. In the empirical analysis conducted below, we shall consider several alternative assumptions concerning the rankings of these underweight and overweight categories that are consistent with the assumption of a reverse U-shaped relationship between BMI and welfare peaking somewhere in the normal BMI range (see e.g. Cao, Moineddin, Urquia, Razak, and Ray (2014)).

A distribution of BMI categories is an ordered list  $d = (d_1, \dots, d_k)$  of  $k$  numbers that specify, for each category  $j$  - ordered from the worst to the best - the *fraction*  $d_j$  of the population falling in category  $j$  in distribution  $d$ . It is therefore understood that  $d_j \in [0, 1]$  for every category  $j$  and  $d_1 + \dots + d_k = 1$ .<sup>3</sup> Describing a distribution of BMI categories by such a list of fractions of the population belonging to the corresponding categories is appropriate if one adopts an anonymous perspective according to which "the names of the individuals do not matter". The basic question that we want to address is: when can it be said that a distribution of BMI categories is better than another ?

The first principle that can be invoked for answering this question is *efficiency*. Given the unambiguous ordering of the categories from the worst to the best, there seems to be clear formulation of this efficiency principle here: that of improving the category of some people, without reducing that of anyone else. We define such an *elementary efficiency gain* as follows.

**Definition 1** (*Elementary Efficiency Gain*) *We say that distribution  $d$  is obtained from distribution  $d'$  by means of an elementary efficiency gain if there exist a category  $j \in \{1, \dots, k-1\}$  such that:*

$$d_h = d'_h, \quad \forall h \neq j, j+1; \tag{1}$$

$$d_j = d'_j - \varepsilon; \quad d_{j+1} = d'_{j+1} + \varepsilon. \tag{2}$$

for some fraction  $\varepsilon > 0$  such that  $\varepsilon \leq \min(d'_j, 1 - d'_{j+1})$ .

In words, a distribution  $d$  of BMI categories is obtained from a distribution  $d'$  by an elementary efficiency gain if  $d$  results from  $d'$  by a transfer of a fraction

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<sup>3</sup>In any empirical exercise,  $d_j$  will actually be a rational number (as is the ratio of the number of individuals falling in category  $j$  over the total number of individuals).

of the population from a category  $j$  to the immediately superior category  $j + 1$ . Such a transfer represents clearly an efficiency gain, as would, for instance, a transfer of a fraction of the population from the overweight category to the normal weight one.

An alternative way of looking at efficiency would be to assign to each category a *numerical value* reflecting its ranking, and to compare distributions based on their *average value*. For instance, if the numbers  $1, 2, \dots, 6$  were assigned to the six BMI categories mentioned above ordered from the worst to the best, then distribution  $d$  could be considered better than distribution  $d'$  if:

$$d_1 + 2d_2 + 3d_3 + 4d_4 + 5d_5 + 6d_6 \geq d'_1 + 2d'_2 + 3d'_3 + 4d'_4 + 5d'_5 + 6d'_6$$

That is,  $d$  is better than  $d'$  if the average value of the categories is not smaller in  $d$  than in  $d'$ . The problem of course with such comparisons of average values is that they are heavily sensitive to the particular choice of the numerical values assigned to the categories. After all, these values are arbitrary, and serve only to reflect the ordering of the categories in terms of their impact on individual well being. Any alternative combinations of numbers  $f(1), \dots, f(6)$  where  $f$  is a strictly increasing real valued function defined on the set of categories would generate just the same ordering of categories. Yet, comparing distributions of BMI categories based on their average value calculated using the numbers  $f(j)$  rather than the number  $j$  (for  $j = 1, \dots, k$ ) could possibly change the ranking of the distributions. This sensitivity of the mean - or the average - to the "scale of measurement" of the variable is obvious and well-known (see e.g. the nice discussion in Allison and Foster (2004)). A way to get around this difficulty would be to require that the average value of the BMI categories in distribution  $d$  be larger than what it is in distribution  $d'$  for all possible assignments of values to the categories. Let us call "average dominance" this requirement that we define, formally, as follows.

**Definition 2** (*Average Dominance*) For any two distributions  $d$  and  $d'$ , we say that  $d$  average dominates  $d'$  if one has:

$$\sum_{j=1}^k d_j \alpha_j \geq \sum_{j=1}^k d'_j \alpha_j \quad (3)$$

for all real numbers  $\alpha_1, \dots, \alpha_k$  satisfying  $\alpha_1 < \alpha_2 < \dots < \alpha_k$ .

The criterion of average dominance is somewhat difficult to apply in practice. It requires in effect the verification of inequality (3) for an infinite collection of sets of real numbers. As is well-known however (see e.g. Lehmann (1955), Quirk and Saposnik (1962) or theorem 1 in Allison and Foster (2004)), *first order (stochastic) dominance* happens to be equivalent to average dominance. The formal definition of first order stochastic dominance in the current context makes use of the *cumulative distribution function* (CDF) associated to  $d$ , that is denoted, for every category  $j$ , by  $F^d(j)$  and that is defined by:

$$F^d(j) = \sum_{h=1}^j d_h \quad (4)$$

Hence  $F^d(j)$  gives the fraction of the population who are in a BMI category that is weakly worse than  $j$ . With this notation, one can define first order dominance as follows.

**Definition 3** (1st order dominance) We say that distribution  $d$  first order dominates society  $d'$  if

$$F^d(j) \leq F^{d'}(j) \quad (5)$$

holds for every BMI category  $j \in \{1, \dots, k\}$  (remembering of course that  $F^d(k) = \sum_{h=1}^k d_h = 1$  for any distribution  $d$ ).

In words, a distribution  $d$  of BMI categories first order stochastically dominates a distribution  $d'$  if, for every BMI category, the fraction of the population falling in a weakly worse category is no greater in  $d$  than in  $d'$ . Checking for first order stochastic dominance between two distributions of BMI is very easy. It amounts to checking that the CDF of one distribution lies everywhere below that of the other.

The following well-known theorem (proved in Lehmann (1955), Quirk and Saposnik (1962) or Gravel, Magdalou, and Moyes (2014)) provides some justification to the comparison of distributions of BMI categories by first order stochastic dominance.

**Theorem 1** For any two distributions  $d$  and  $d'$  of BMI categories, the following three statements are equivalent:

- (a)  $d$  is obtained from  $d'$  by means of a finite sequence of elementary efficiency gains,
- (b) Inequality (3) holds for all numbers  $\alpha_1, \dots, \alpha_k$  satisfying  $\alpha_1 < \alpha_2 < \dots < \alpha_k$ .
- (c)  $d$  first order dominates  $d'$ .

Observing first order dominance between two distributions of BMI provides a strong evidence that the dominating distribution is "better" than the dominated one. Indeed, the dominating distribution results from the dominated one by a finite sequence of elementary efficiency gains. As we shall see in the empirical analysis, this criterion enables a ranking of some distributions of BMI above others.

Yet, efficiency can not be the only criterion for comparing distributions of contributors to well-being. Concerns for reducing inequalities in those contributors may also be considered important. To motivate such a concern in our context, consider our sixth BMI categories and assume - in line with Cao, Moineddin, Urquia, Razak, and Ray (2014) - that they are ordered as follows (from the top (best) to the bottom (worst)):

normal  
overweight  
underweight  
grade 1 obesity  
grade 2 obesity  
grade 3 obesity

Consider then the following two theoretical distributions of BMI categories:

	$d$	$d'$
normal	0.4	0.5
overweight	0.3	0.2
underweight	0.05	0.05
grade 1 obesity	0.15	0.1
grade 2 obesity	0.1	0.05
grade 3 obesity	0	0.1

As can be seen, none of the two distributions first order dominates the other. Indeed, nobody in distribution  $d$  falls in the (worst) grade 3 obese category while 10% of the population in  $d'$  have this misfortune. On the other hand there are 60% of the population in  $d$  who have a non-normal BMI category while only 50% of the population in  $d'$  are in this situation. However, in some sense, the distribution of BMI categories in  $d$  can be considered to be "less spread out" than that of  $d'$ . Indeed, suppose that  $d$  and  $d'$  describe two distributions of BMI between 100 individuals. One can then view each of the six BMI categories as an "income" level. For instance someone with a normal weight is a "very rich" individual while an individual with grade 3 obesity is a "very poor" one. One can then describe the move from  $d'$  to  $d$  as the result of a couple of equalizing "transfers" of BMI categories between individuals. Specifically the move from  $d'$  to  $d$  can be viewed as resulting from the fact that ten "extremely rich" individuals in  $d'$  have transferred part of their BMI category to poorer individuals. These ten individuals have reduced by "one unit" their BMI categories by all becoming overweight (the number of which has indeed grown up from 20 to 30). Who are the beneficiaries of these transfers? They are all "very poor" individuals, whose number has shrunk from 10 to naught. Five of them have seen their BMI categories increased from a grade 3 to a grade 2 obesity level (whose number of members has grown up from 5 to 10). The five remaining individuals have progressed from grade 3 to grade 1 obesity level. Notice that each of these five last "very poor" individuals has gained "two" units of BMI categories as a result of the lost of one unit of BMI category by five of the ten "very rich" givers. The two sequences of these transfers are illustrated on Figure 1.

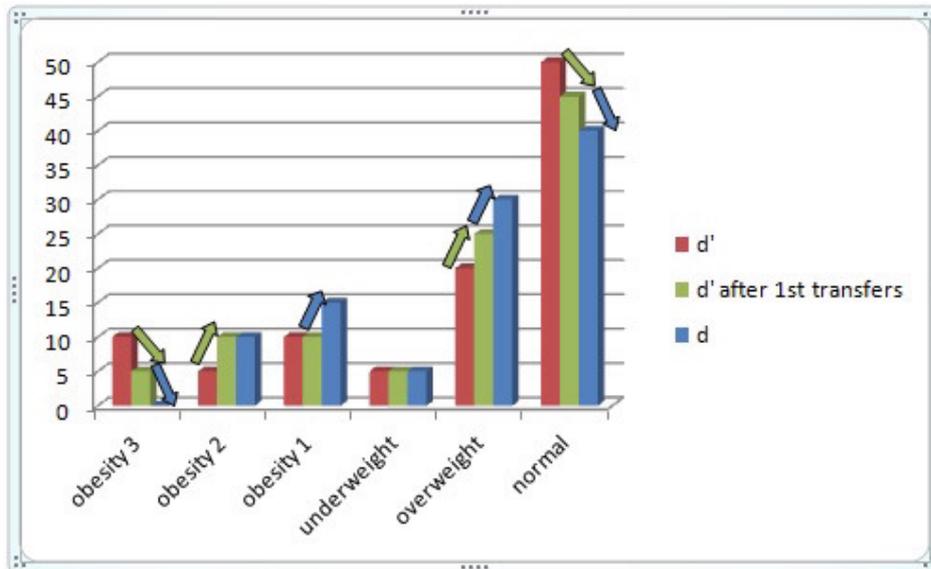


Figure 1:

Transfers of the kind just described have been proposed in the mid seventies by Peter J. Hammond (1976) as the analogue, for an ordinally measurable

attribute, to the usual Pigou-Dalton transfers (see e.g. Atkinson (1970) or Sen (1973)). Recall that a Pigou-Dalton is transfer of a *given quantity* of income taken from a richer individual and given to a poorer one in such a way that the ranking of the two individuals after the transfer is (weakly) preserved. A Hammond transfer is just like a Pigou-Dalton transfer, but without imposing the requirement that the quantity transferred from the rich to the poor be "given". As illustrated above, a Hammond transfer allows a healthy individual to fall one step to the immediately inferior overweight category in exchange for a Grade 3 obese person to rise two steps to the Grade 1 obese one. Conversely, a Hammond transfer may also involve taking "a lot" of attribute away from a "rich" person in exchange of giving "just a little" to a poorer one. As the very notion of a "quantity" of an ordinal attribute is meaningless, the Hammond transfer does not require any preservation of such a quantity. This absence of requirement makes the notion of a Hammond transfers a plausible analogue, in the ordinal setting, of the Pigou-Dalton transfer used in the cardinal one. The precise definition of a Hammond transfer in our context is as follows.

**Definition 4 (Hammond's transfer)** *Distribution  $d$  is obtained from distribution  $d'$  by means of a Hammond' equalizing transfer, if there exist four categories  $1 \leq g < h \leq i < j \leq k$  (ordered again from the worst to the best) such that:*

$$d_l = d'_l, \forall l \neq g, h, i, j; \quad (6a)$$

$$d_g = d'_g - \varepsilon; \quad d_h = d'_h + \varepsilon; \quad (6b)$$

$$d_i = d'_i + \varepsilon; \quad d_j = d'_j - \varepsilon. \quad (6c)$$

for some fraction  $\varepsilon > 0$  satisfying  $\varepsilon \leq \min\{d'_g, 1 - d'_h, 1 - d'_i, d'_j\}$ .

Suppose that we accept Hammond transfer as the appropriate notion of equalization. If one is interested in both equality and efficiency considerations, one would then be tempted to say that a distribution of BMI categories is better than another if it has been obtained from the later by a finite sequence of either Hammond transfers and/or elementary efficiency gains. It would be nice to dispose of an easy diagnostic tool for identifying, for any two distributions of BMI categories, whether or not one has been obtained from the other by a finite sequence of equalizing Hammond transfers and/or elementary efficiency gain. As shown in Gravel, Magdalou, and Moyes (2014), it turns out that a particular curve - called the  $H^+$  curve - provides such a diagnostic. For any distribution  $d$ , its  $H^+$  curve is defined recursively as follows:

$$H^+(1; d) = F^d(1) = d_1 \quad (7)$$

and:

$$H^+(j; d) = 2H^+(j-1; s) + d_j \quad (8)$$

for all  $j = 2, 3, \dots, k$ . This curve is quite easy to draw and to construct. It is initiated by plotting, for the worst category, the fraction of the population that falls in this category. It is then iterated by plotting, for every other category above the worst one, the fraction of the population lying in this category plus twice the value assigned to the immediately preceding categories. This curve gives rise to the following notion of dominance - called  $H^+$  dominance.

**Definition 5** ( $H^+$  dominance) We say that distribution  $d$   $H^+$ -dominates distribution  $d'$ , which we write  $d \succeq_{H^+} d'$ , if and only if:

$$H^+(j; d) \leq H^+(j; d'), \quad \text{for all } j = 1, 2, \dots, k. \quad (9)$$

It is easy to see that first order dominance entails  $H^+$  dominance. The converse however does not hold. The following theorem, proved in Gravel, Magdalou, and Moyes (2014), shows that  $H^+$  dominance provides indeed an exact diagnostic tool for identifying, between any two distributions of BMI categories, whether or not one is more equal and/or more efficient than the other.

**Theorem 2** For any two distributions  $d$  and  $d'$  of BMI categories, the following two statements are equivalent:

- (a)  $d$  is obtained from  $d'$  by means of a finite sequence of Hammond's transfers and/or elementary efficiency gains,
- (b)  $s \succsim_{H^+} s'$ .

The  $H^+$ -dominance criterion enables one to check, for any two distributions of BMI categories, whether or not one is unambiguously more *efficient and/or more equal* than the other. Somewhat analogously, the criterion of first order dominance allows one to check whether or not a distribution is more *efficient* than another. It would be also useful to have a diagnostic tool for identifying whether or not a distribution is *more equal only* than another (without being necessarily more efficient). Results in Gravel, Magdalou, and Moyes (2014) suggest that such a diagnostic tool uses, along with the  $H^+$  curve, the somewhat dual  $H^-$  curve. The formal definition of this curve makes use of the *complementary cumulative distribution* function associated to a distribution  $d$  denoted, for every category  $j = 1, \dots, k \in \mathcal{C}$ , by  $\overline{F}^d(j)$  and defined by:

$$\overline{F}^d(j) = \sum_{h=j}^k d_h. \quad (10)$$

In plain English,  $\overline{F}^d(j)$  is the fraction of the population who, in  $d$ , falls in a weakly better BMI category than  $j$ . This function is sometimes called the "survival" function in statistics. The  $H^-$  curve is defined recursively with respect to the complementary cumulative distribution in just the same way as the  $H^+$  curve is defined with respect to the cumulative distribution function. Specifically, the  $H^-$  curve is defined under the same recursive principle than the  $H^+$  one, but starting with the best category  $k$  (rather than the worst category 1) and iterating with the complementary cumulative function (rather than the cumulative one). The definition of this curve therefore starts at category  $k$  as follows:

$$H^-(k; d) = \overline{F}^d(k) = d_k \quad (11)$$

and is defined recursively, for categories  $j = 1, 2, \dots, k-1$ , by:

$$H^-(j; d) = 2H^-(j+1; s) + d_j. \quad (12)$$

This curve gives rise to the following definition of  $H^-$  dominance.

**Definition 6** (*H<sup>-</sup> dominance*) We say that distribution  $d$   $H^-$ -dominates distribution  $d'$ , which we write  $d \succeq_{H^-} d'$ , if and only if:

$$H^-(j; d) \leq H^-(j; d'), \quad \text{for all } j = 1, 2, \dots, k. \quad (13)$$

It is shown in Gravel, Magdalou, and Moyes (2014) that the fact, for a distribution  $d$ , to  $H^-$  dominates a distribution  $d'$  is equivalent to the possibility of moving from  $d'$  to  $d$  by a finite sequence of *elementary efficiency loss* (the converse of efficiency gains) and/or Hammond transfers. Hence, if a distribution  $d$  has been obtained from distribution  $d'$  by a finite sequence of Hammond transfers *only*, it follows from theorem 2 that distribution  $d$  both  $H^-$  dominates and  $H^+$  dominates distribution  $d'$ . Hence, the failure to observe both  $H^+$  dominance and  $H^-$  dominance of a distribution  $d$  over a distribution  $d'$  is indicative of the fact that  $d$  and  $d'$  can not be obtained one from another by Hammond transfers only. To the contrary, observing both  $H^+$  and  $H^-$  dominance of one distribution by another would be suggestive of the fact that the dominating distribution has been obtained from the dominated one by a finite sequence of Hammond transfers only.

In the next section, we use the tools presented here to compare the BMI categories between genders in three developed countries: France, UK and the US.

### 3 Data and Empirical Results

The data for France are taken from the *Enquête sur la Santé et la Protection Sociale (ESPS)*. The ESPS is a panel survey created in 1998. It follows a sample of about 8000 households (roughly 22 000 individuals) that is representative of 95% of the inland European France (excluding outer sea regions such New Caledonia, Reunion Island, etc.). In 2006, the sample has been slightly enlarged to provide a better representation of the French population that is covered by the "*Couverture Maladie Universelle*" programme. The later, launched in 1998, is designed to provide basic free medical care to the (very poor) fringe of the French population that is not medically covered by the national social security regime. Each individual member of the sample is interrogated every two years in a very comprehensive manner about a large spectrum of health characteristics of every member of his or her household. It is important to notice that all health information is reported by the person surveyed and is not measured or appraised by an external health expert. Hence the information used here on individual height and weight to construct BMI categories in France is a self reported information. We restricted our sample to individuals aged above 20 who report for each of the three years considered: 1998, 2008 and 2010. The number of those were 11 694 individuals for 1998, 11 255 individuals for 2008 and 10 410 individuals for 2010.

Data on United States are taken from the 2007-2008 issue of the *National Health And Nutrition Examination Survey (NHANES)*. This survey, initiated in 1960, became regular in 1999. The survey collects information by both interviews (conducted at the place of residence of the individual) and physical examinations (including weighting and height measurement) performed at mobile centers by professional health technicians. Every year, 5000 persons (different from year to year) are interviewed in 15 states. In order to be representative of

the whole US population, there is an over sampling of individuals above sixty, and of members of the Afro-American and the Latin American community. On the initial 10 000 individuals surveyed in the years 2007 and 2008, we ended up with a sample of 5 607 individuals who were above 20 and for which information on height, weight and gender was available. It is also important to notice that each person interviewed through this survey receives a monetary compensation as well as a complete report of his/her health status.

Data on United Kingdom come from the *National Diet and Nutrition Survey* (NDNS) and the *Rolling Programme* (RP) for the 2008-2012 period. The NDNS survey was initiated in 1992 and became, in 2008, the RP survey that covered both adults and children. This survey is ran by three governmental bodies: the Ministry of agriculture, the Public Health of England and the Food Standards Agency. The survey covers all four members of the United Kingdom (England, Wales, Scotland and Northern Ireland). As in the US, information on nutrition and health is collected from two complementary modes: interview, and physical examination (including height and weight measurement) by health technicians. Individuals who participate to the survey are selected by stratified sampling, based on a random selection from postal codes taken from all parts of the UK, with the codes grouped by broad geographical sectors. The sample size is somewhat small however since, during the 4 years, 2 083 adults and 2 073 children and teenagers have been sampled. Because of our aim at working with adults with at least 20 years old on which information on gender, weight and height is available, we ended up with 1 912 individuals for the United Kingdom.

A summary description of the samples is given in figure 2 below.

	<b>France (2008)</b>	<b>UK (2008-2012)</b>	<b>US (2007-2008)</b>
Number of individuals	11 255	1 912	5 607
Number of men	5 369	906	2 747
Number of women	5 886	1 006	2 860

Figure 2: sample description

The following tables report the distributions of BMI categories for the female (table 1) and male (table 2) samples in France for the years 1998, 2008 and 2010, and in UK and the US for the year 2008.

A few noticeable features emerge from these tables. First, the fraction of the (sampled) populations of interest that suffers from grade 3 obesity is somewhat small. Indeed less than 1% of the adult male population in France are concerned over the three years. The corresponding fractions for the female population are slightly higher but never exceed 1.31 %. The figures are somewhat larger for the UK and the US in 2008 where grade 3 obesity concerns, respectively, 4.16% and 7.25 % of the female population and 1.21% and 4.4% of the male population. Hence, non surprisingly, the prevalence of Grade 3 obesity is higher in the US and the UK than in France (and higher in the US than in the UK). Yet, in every

<b>category</b>	<b>FR 1998</b>	<b>FR 2008</b>	<b>FR 2010</b>	<b>UK 2008</b>	<b>US 2008</b>
obesity 3	0.0037	0.0114	0.0131	0.0416	0.0786
obesity 2	0.0168	0.0311	0.0324	0.0951	0.1125
obesity 1	0.0664	0.0910	0.1055	0.1681	0.2045
overweight	0.2190	0.2490	0.2354	0.3019	0.3016
underweight	0.0639	0.0467	0.0433	0.0166	0.0213
normal	0.6302	0.5707	0.5702	0.3767	0.2814

Table 1: distributions of BMI categories among women

<b>category</b>	<b>FR 1998</b>	<b>FR 2008</b>	<b>FR 2010</b>	<b>UK 2008</b>	<b>US 2008</b>
obesity 3	0.0016	0.0047	0.0053	0.0121	0.0440
obesity 2	0.0103	0.0211	0.0191	0.0623	0.0736
obesity 1	0.0813	0.1030	0.1138	0.1990	0.2163
overweight	0.3549	0.3841	0.3889	0.4536	0.3896
underweight	0.0135	0.0132	0.0152	0.0084	0.0116
normal	0.5384	0.4739	0.4577	0.2642	0.2647

Table 2: distributions of BMI categories among men

country and year, a larger fraction of women than man suffer from that form of extreme obesity. It is also noteworthy that the prevalence of grade 3 obesity has been steadily rising in France between 1998 and 2010 for both males and females. Another BMI category that concerns only a rather small fraction of the population is the underweight one. About 6.4% of the French female were having a BMI that enters in this category in 1998. This fraction has dropped (slightly) to 4.33% in 2010. The corresponding fractions for the French male population is significantly smaller (1.4% in 1998,; and 1.5% in 2010.) Contrary to what was the case for grade 3 obesity, the UK and the US are somewhat less affected by the prevalence of "underweight" individuals. Only 2.1 % of the US female population were in this category in 2008. The faction is even smaller in the UK where it represents 1.66 % of the adult females. In both countries, the figures are smaller for males. Hence, as for the grade 3 obesity, women are more likely than men to be underweight. Notice also that the fraction of underweight females has dropped a bit in France on the period 1998-2010 while it has risen slightly for males.

The other BMI categories concern a much larger fraction of the relevant population. Grade 2 obesity for instance concerns 11.25% of the US women. Yet similar qualitative patterns can be found with respect to the comparative prevalence of grade 2 obesity between the different population. In all three countries, women are relatively more affected than men by grade 2 obesity. Moreover, for both men and women, the fraction of the population that is obese of grade 2 is larger in the US than in the UK and the UK than in France. Grade 2 obesity has steadily increased among French women over the period 1998-2010. The trend, albeit qualitatively similar, has been less steady among French men since the fraction of them that belong to the grade 2 obese category first raised from 1.03 % in 1998 to 2.11 % in 2008 but then dropped a bit to 1.91 % in 2010. As for grade 1 obesity and overweight BMI categories, the comparative exposures of men and women are markedly different than what

Ranking 1	Ranking 2	Ranking 3	Ranking 4	Ranking 5
normal	normal	normal	normal	normal
underweight	overweight	overweight	overweight	overweight
overweight	underweight	obesity 1	obesity 1	obesity 1
obesity 1	obesity 1	underweight	obesity 2	obesity 2
obesity 2	obesity 2	obesity 2	underweight	obesity 3
obesity 3	obesity 3	obesity 3	obesity 3	underweight

Table 3: possible welfare rankings of BMI categories

they are for the three previous ones. Indeed, in all three countries and years, one finds a *lower* fraction of female than male who are in *either* the overweight *or* the grade 1 obese one. As for grade 2 and grade 3 obesity, and for both males and females, the fractions of the population that fall in the grade 1 obesity or the overweight category is smaller in France than in the UK and is smaller in the UK than in France. The only exception to this is the overweight category and the male population in the US and UK. As it happens indeed, the fraction of males who are overweight is much larger in the UK (45.36%) than in the US (38.96%). While females are more affected than males by severe obesity and underweight, it happens that women are more likely than men to have a normal weight. Indeed, in 2008, about 57% of the French female population have a normal weight. By contrast, there are only 45.77% of the male population that have this chance. The figures for UK and US are, predictably, less favorable than that for France, even though they show the same relative standing of men and women with respect to BMI normality. Indeed, only 37.67% of the British females and 28.14% of the American ones have a normal BMI category. The corresponding figure for males are 26.42% and 26.47. Notice that the men-women gap in terms of "normality" is somewhat smaller in the US than in the UK. One observes also that the fraction of the male population with a normal weight is slightly larger in the US than in the UK.

Let us now try to get somewhat more palatable normative conclusions from these observations. Important for this endeavour is a ranking of the BMI categories in terms of their impact on individual well-being. It seems clear that individual well being is decreasing in a monotonic fashion with respect to the BMI categories when these correspond to a BMI above 25. It seems also clear that being underweight is worse than having a normal weight. However, as discussed earlier things are not so clear concerning the relative ranking of the underweight category vis -à-vis any of the the overweight and obese ones. Is being underweight worse or better than being overweight ? Is being underweight worse or better than being obese of grade 1 ? Answering these questions is difficult, especially in view of the fact that body weight contribute to well-being by various channels: health, self esteem, social stigmal, etc. For instance, there is evidence that being underweight might be associated to more severe health consequences than the fact of being overweight. Evidence however is less clear when one compares the various forms of obesity and the fact of being underweight. Because of this ambivalence, we find safe to consider all the following rankings of BMI categories, from the worst (bottom) to the best (top):

To the very best of our understanding of the impact of body weight on individual welfare, rankings 1, 2 and 3 are the most plausible of the five. But we perform the analysis with all of them so as to illustrate the dependency of the conclusion on the assumptions made about the rankings.

### 3.1 Time trends in France

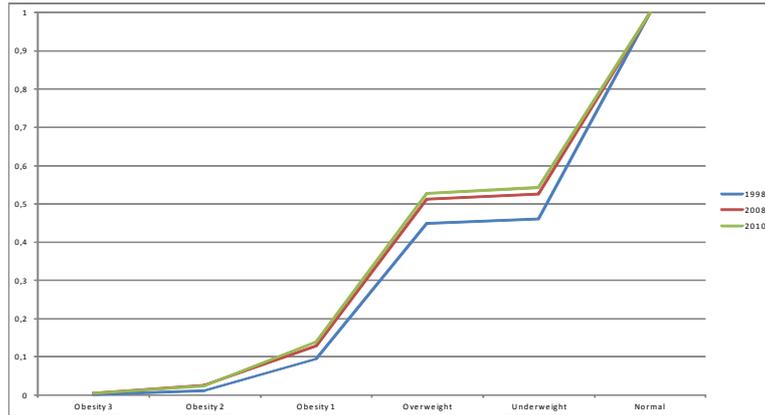


Figure 3: Cumulative distributions of BMI categories among French men, 1998-2010, 1st ranking.

We first examine the evolution, over the three years 1998, 2008 and 2010, of the distributions of BMI among French males and females. The first thing to notice is the deterioration of the distribution of BMI categories over the period 1998-2010 for both males and females from the view point of the somewhat robust first order dominance test. Consider indeed, Figure 3 which shows the CDF of BMI categories for French males for the years 1998, 2008 and 2010 for the first ranking of table 3 (the pattern is similar for all three first rankings of table 3). As can be seen on this figure, the CDF associated to the year 1998 lies everywhere below that of either 2008 or 2010. Hence, there happens to be first order dominance of the distribution of BMI categories among French males in 1998 over either that of 2008 or 2010. This dominance is actually observed for all rankings of table 3 (and not only the three first ones). This is a rather strong result. No matter what assumption one makes about the standing of the underweight category *vis-à-vis* the four "overweight" ones, and about the connection between BMI category and well-being, each individual member of the male population (up to a permutation) has experienced a decrease in well-being in 2008 or in 2010 as compared to 1998. There is therefore very little doubt that the distribution of BMI categories among French males has severely deteriorated over the period.

The verdict is, however, less clear for the comparison of the (almost) adjacent years 2008 and 2010. Indeed, as illustrated on figure 3, no first order dominance can be found between these two years for the first ranking. This absence of first

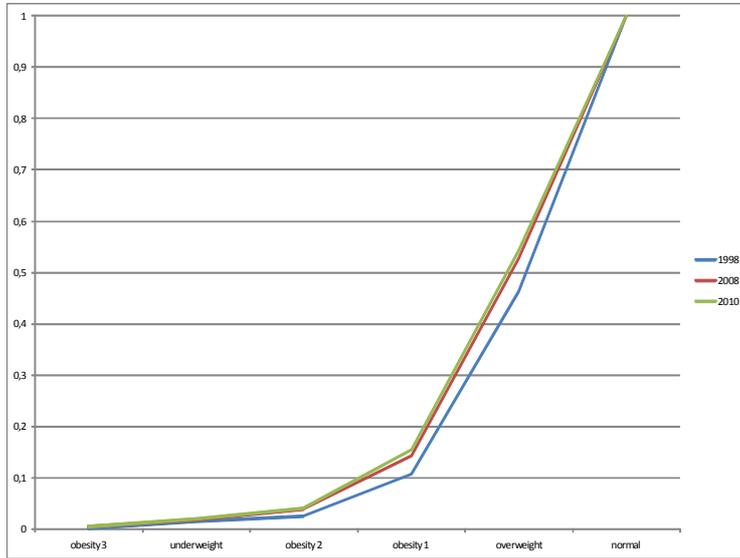


Figure 4: Cumulative distributions of BMI categories among French men, 1998-2010, 4th ranking.

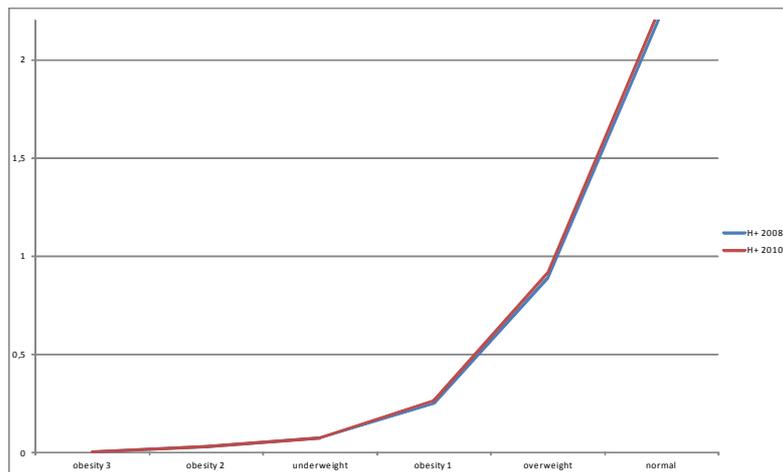


Figure 5:  $H^+$  curves for French Males, 2008 and 2010, 3rd ranking.

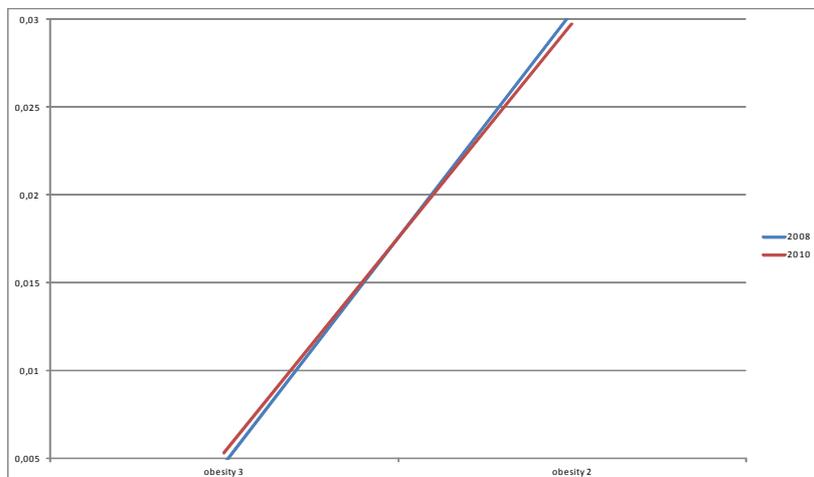


Figure 6:  $H^+$  curves for the French males, 2008 and 2010, 3rd ranking, two bottom categories.

order dominance actually holds for each of the three first rankings of table 3. The reason for this comes from the crossing of the two CDF at the "Grade 2 obesity" category for all these rankings. Indeed, it happens that the fraction of French males who are obese of grade 2 has fallen slightly between 2008 and 2010. While this reduction has been accompanied by a mild increase in the fraction of obese men of grade 3, the overall fraction of men who are at least obese of grade 2 has nonetheless decreased slightly between the two years. As it happens, the distributions of BMI categories in the two years 2008 and 2010 are not comparable even using the  $H^+$  criterion.

Indeed, Figure 5 depicts the  $H^+$  curves - constructed using equations (7) and (8) above - associated to the distributions of BMI among French males for 2008 and 2010 for the third ranking of table 3 (a similar pattern again would emerge if rankings 1 or 2 had been assumed instead). While it is difficult to visually distinguish the two curves, none of the two lies everywhere below the other. The blue curve associated to 2008 lies below the red (2010) one for categories better than underweight (for this ranking). However, as shown in Figure (that depicts only the behavior of these two curves for the two worst categories), the curves do cross at the Grade 2 obesity category. However, if one assumes - somewhat implausibly in our view - that being underweight is worse than being obese of grade 2 or of grade 3, one finds first order dominance of 2008 over 2010 for French males, as shown on figure 4 (which assumes that underweight is worse than grade 2 obesity). Indeed, the fraction of underweight French males has been rising steadily - albeit by a small magnitude - in the whole period 1998-2010. For the two years 2008-2010, this increase over compensates the decrease in the fraction of men who are obese of grade 2 observed between 2008 and 2010.

The evolution of the distribution of BMI categories among French women over the same period is similar to what it is for men for the first three rankings, as illustrated on figure 7, showing the CDF of BMI categories among French females for the two years under the first ranking. Indeed, for all these three

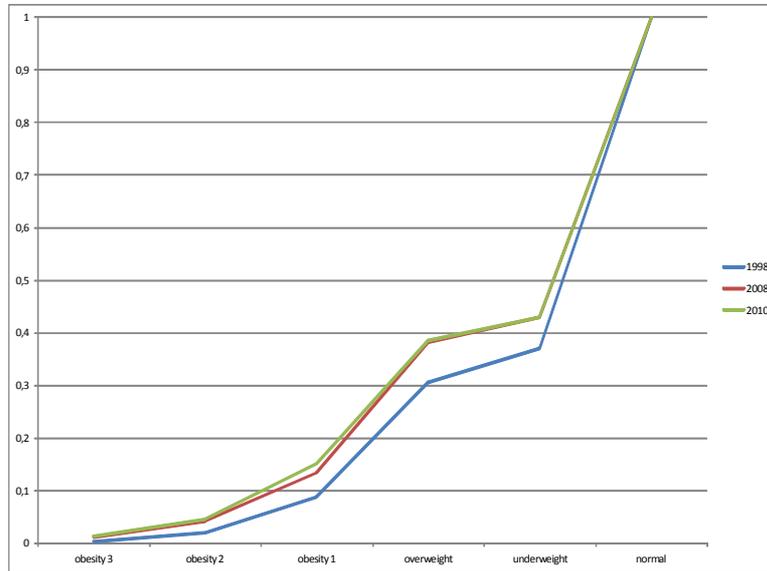


Figure 7: Cumulative distributions of BMI categories among French women between 1998 and 2010, 1st ranking.

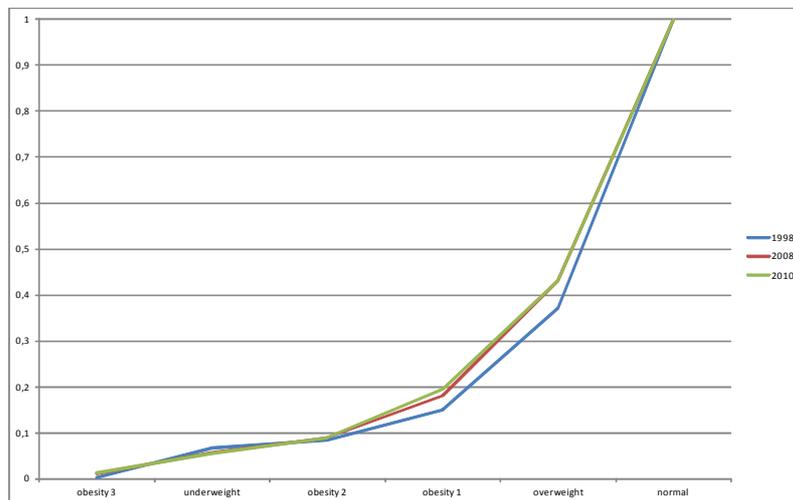


Figure 8: Cumulative distributions of BMI categories among French women between 1998 and 2010, 4th ranking.

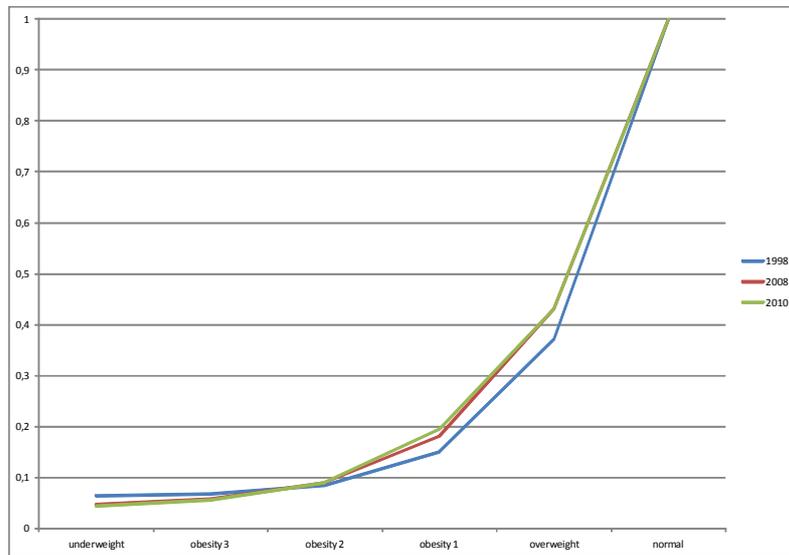


Figure 9: Cumulative distributions of BMI categories among French women between 1998 and 2010, 5th ranking.

cases, one finds a clear first order dominance of 1998 over either 2008 and 2010. For the two first rankings, one finds also a first order dominance of 2008 over 2010. However, for the three other rankings of table 3, the two nearby years 2008 and 2010 happen to be non-comparable by first order dominance. Furthermore, as shown on figures 8 and 9, it happens that for rankings 4 and 5 - that consider the underweight BMI category to be worse than grade 2 obesity - the distribution of BMI categories among French women has not worsened over the period 1998-2010 as per first order dominance. The reason for this lies in the favorable evolution, over that same period, of the fraction of underweight French women, as indicated on table 1. The significant drop of this fraction of underweight French women over that period counterbalances to some extent the significant increase of the fraction of French women who are obese. Hence, depending upon whether being underweight is considered worse or better than being obese of grade 2, one gets a different appraisal of the evolution, between 1998 and 2010, of the BMI categories among French women. The distribution of BMI categories deteriorates if one assumes - in our view plausibly - that being obese of grade 2 is worse for the well-being of a French woman than being underweight. However, the verdict is not so unfavorable if one considers instead that being underweight is worse than being obese of grade 2. Indeed, if one is willing to make the (implausible ?) assumption that excessive thinness is worse than grade 3 obesity - one could even conclude that the evolution of BMI categories among French women has unambiguously improved over the period by the  $H^+$  dominance criterion, as shown on figure 11, that depicts the  $H^+$  curves associated to the distributions of BMI categories among French women for the three years for the fifth ranking of table 3. Hence, under the assumption that excessive thinness is the worst possible BMI category, the evolution of the distribution of BMI categories among French women over the period 1998-2010

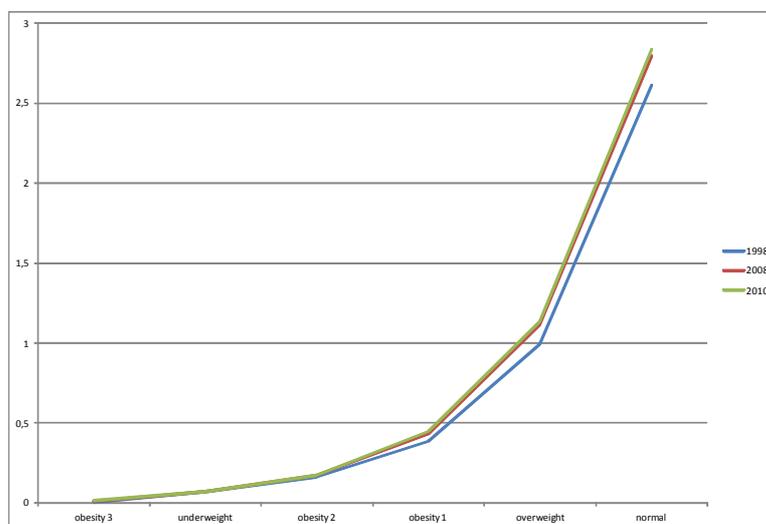


Figure 10:  $H^+$  curves of BMI categories for French women between 1998 and 2010, 4th ranking.

can be described as the result of performing a finite sequence of elementary efficiency gains and/or Hammond redistributive transfers. However, no such conclusion can be derived if one assumes that being underweight provides a level of well-being that lies in between that provided by the grade 2 and grade 3 obesity BMI categories. As shown indeed on figure 10, there is no  $H^+$  dominance in one direction or another between the years under this ranking (the three curves "cross" at the underweight BMI categories). Hence, it happens that the normative conclusion about the evolution of the distribution of BMI categories among French women over 1998-2010 is somewhat dependant upon the assumed welfare ranking of the BMI categories. If one assumes rankings 1-3, one gets the unambiguous conclusion of a deterioration of the distribution of BMI categories among French women over the period by the first order dominance criterion (with a mild ambiguity for the two nearby years 2008-2010 for the third ranking). However, this conclusion does not hold if one assumes, along with rankings 4 or 5, that excessive thinness is worst than grade 2 obesity. In that case, one can even obtain the reverse conclusion - based this time on the  $H^+$  criterion - that the distribution of BMI categories among French women has improved over the period if one takes the (excessive) view that excessive thinness is the worst of all BMI category.

### 3.2 Cross-countries comparisons

We now turn to cross-country comparisons of BMI categories among either men and women in the year 2008.

Figure 12 for instance shows the CDF of BMI categories for women in France, UK and the US for the first ranking of table 3 (the pattern is very similar for all the fourth first rankings of the table). The conclusion obtained from these comparisons is crystal clear: the distribution of BMI categories is unambiguously

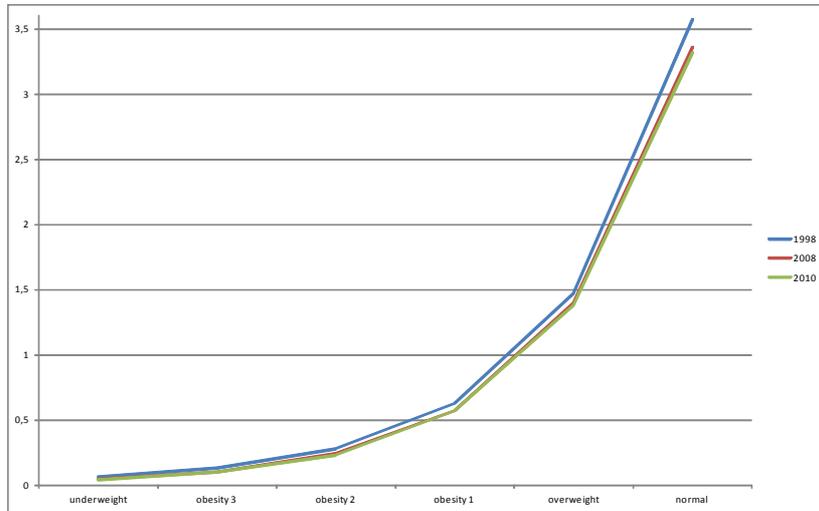


Figure 11:  $H^+$  curves of BMI categories for French women between 1998 and 2010, 5th ranking.

better - as per the first order dominance criterion - in France than in the UK and is better in the UK than in the US for the first four rankings of table 3. The same conclusion holds true between the US and the UK even for the fifth ranking, shown on Figure 13, that views the underweight BMI category to be the worst of all. However, no first order dominance conclusion can be obtained for the comparison of France vis-à-vis either the UK or the US under this 5th ranking. This later absence of dominance comes, of course, from the fact the fraction of underweight women is larger in France than in the UK or the US. Notice that, despite the fact that the fraction of overweight or obese women is larger in the US than in the UK, the fraction of underweight women is also larger in the US than in the UK. Hence, and in a very strong sense therefore, the distribution of BMI categories among women is unambiguously better in the UK than in the US.

As for men, Figures 14 and 15 show the CDF of the distributions of BMI categories among males for the first and fifth rankings of table 3 in each of the three countries. The figures show a very similar standing of France vis-à-vis the US or the UK is the same for the male population than what is observed for the female one. Indeed, the French distribution of BMI categories among men dominates at the first order the British and the American ones for all but the fifth ranking of table 3. However, as for women, and just for the same reason, no such dominance is obtained with the fifth ranking where the underweight BMI category is considered to be the worst of all. However, the ranking of the US and the UK distributions of BMI categories among men is somewhat different than what it is for women. Indeed, for no welfare ranking of BMI categories does the British distribution of BMI categories among men first order dominate the US one. The reason for this lack of dominance comes from the fact that the fraction of men who are in the overweight BMI category is larger in the UK than in the US (as shown on table 2). As a matter of fact, the fraction of male with a normal

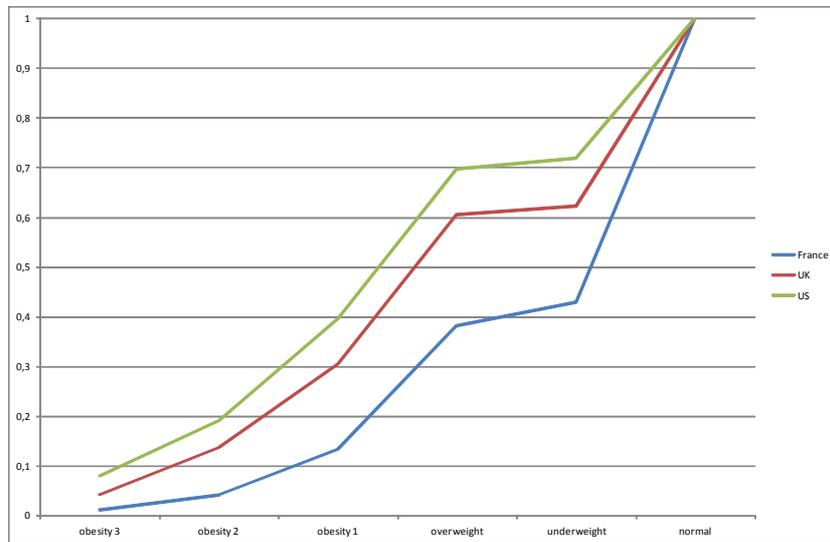


Figure 12: CDF of BMI categories among women, France, UK and US, 2008, 1st ranking.

BMI category is slightly lower in the UK than in the US. Hence, none of the British nor the US distribution of BMI categories among men can be said to be unambiguously more efficient than the other. Could the introduction of equity considerations help in reaching more definite conclusion ? It turns out that it can! This is at least so if one restricts attention to the four first rankings of table 3, as illustrated by, say figure 16 showing the  $H^+$  curves associated to the distributions of BMI categories among men between the three countries for the first ranking. Since the fraction of underweight men is larger in the UK than in the US, no such  $H^+$  dominance between the two countries can be observed if one assumes that the underweight category is the worst of all. Hence, except in the later case, it happens that the distributions of BMI categories among men in the three countries can be ranked in a somewhat robust fashion. The distribution is better in France than in the UK and is better in the UK than in the US for the  $H^+$  dominance criterion. The conclusion is even stronger for the comparison of France vis-à-vis any of the two other countries, since the dominance holds there also for the more demanding first order dominance criterion.

### 3.3 Cross genders comparisons

Men and women are affected somewhat differently by anomalies in body weight. Women are more prone to severe obesity or excessive thinness than men. However, a larger fraction of women than men have a normal BMI category. Moreover, the impact of BMI on individual well-being, notably through self-esteem, social stigma, and stereotypes, are likely to differ somewhat between men and women. It is for this reason interesting to investigate whether or not BMI categories are "better" distributed among men than among women. We illustrate in what follows the analysis for the case of France. As it happens, the same cross

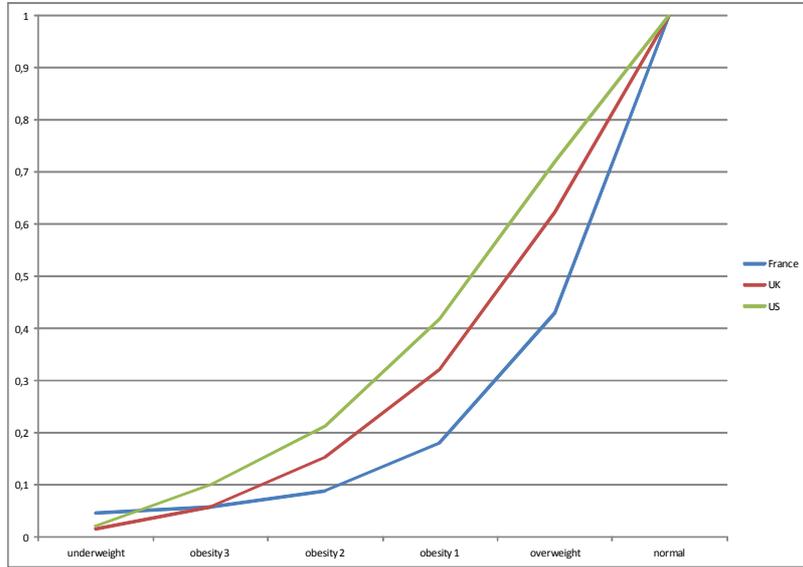


Figure 13: CDF of BMI categories among women, France, UK and US, 2008, 5th ranking.

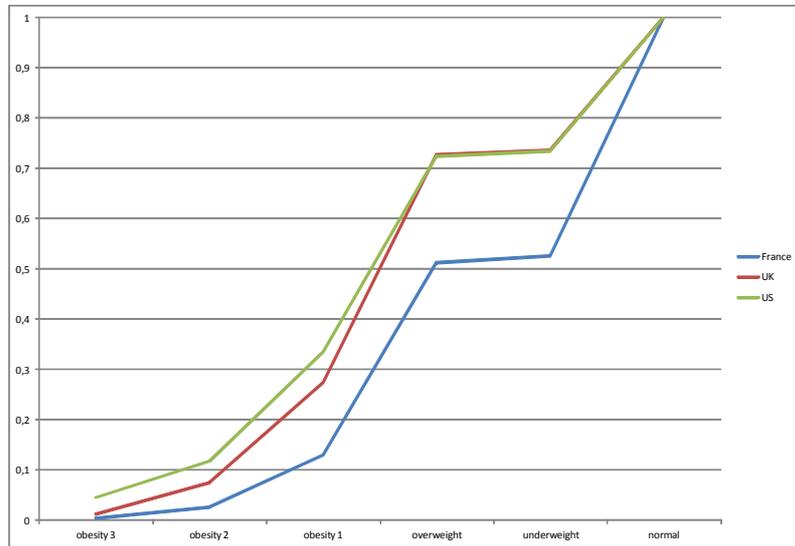


Figure 14: CDF of BMI categories among men, France, UK and US, 2008, 1st ranking.

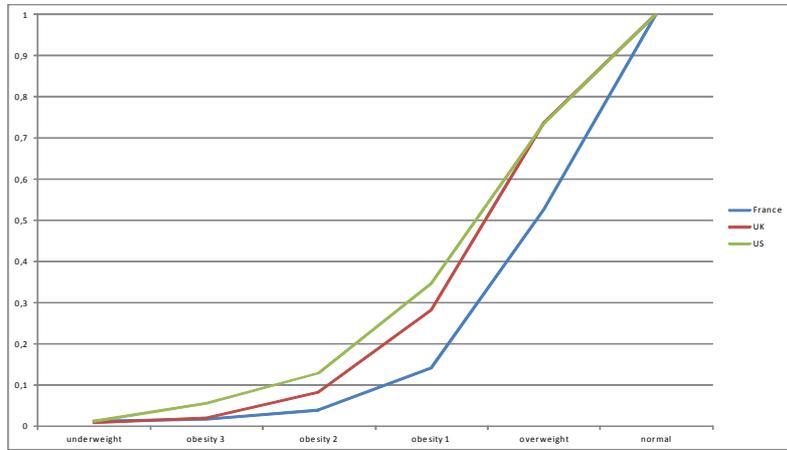


Figure 15: CDF of BMI categories among men, France, UK and US, 2008, 5th ranking.

genders comparisons hold in the UK and the US.

Figures 17 and 18 show, respectively, the CDF and the  $H^+$  curves associated to the distributions of BMI categories among French males and French females in 2008, under the first ranking. Clearly, in each of the two figures, there is crossing of the two curves. Hence, there is no clear evidence that the distribution of BMI categories is better for one gender than for the other under this first ranking that considers that being underweight is better than being overweight. However, the evidence becomes clearer when excessive thinness is considered worse than overweight, as in the second ranking. Figures 19 and ?? show indeed, respectively, the CDF and the  $H^+$  associated to the distributions of BMI categories among French males and females for the second ranking of table 3. While Figure 19 indicates a crossing of the two CDF and, therefore, an absence of first order dominance between the two distributions, Figure ?? shows that the distribution of BMI categories among males  $H^+$  dominates the distributions of such categories among women under this second ranking.

Could this dominance be caused by the fact the distribution of BMI among men is "less spread out" than that among women in the sense of the former being obtained from the latter by a finite sequence of Hammond transfers only? It turns out that the answer to this question is positive, as shown on figure 20, that depicts both the  $H^+$  curves and the  $H^-$  curves associated to these two distributions. As shown on this figure, both the  $H^+$  and the  $H^-$  curves for men are everywhere below that for women. This indicates that the distribution of BMI categories is "less spread out" among men than among women in the sense that the former has been obtained from the latter by a combination of Hammond transfers. It happens that the same strong conclusion is obtained in all three countries, and for all rankings of table 3 but the first one. Hence, for all welfare rankings of BMI categories for which excessive thinness is worse than being overweight (but not obese), the distribution of BMI categories is less justly distributed among females than among males. We described earlier the main forces that underlie these states of affairs. A larger fraction of women than

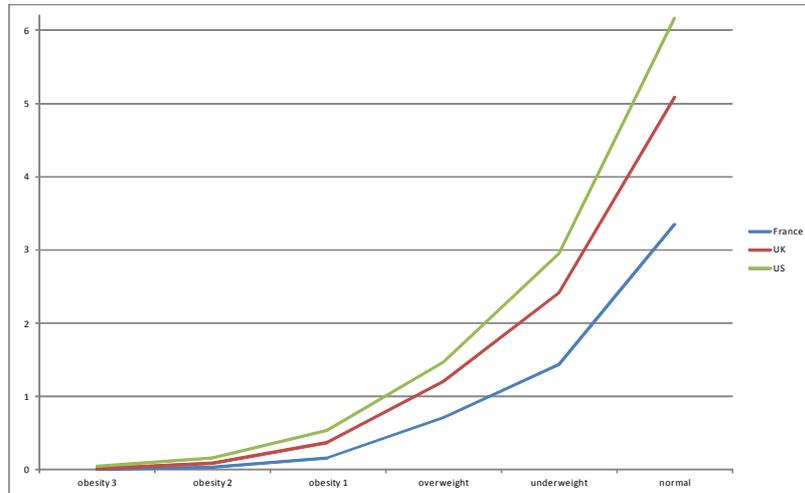


Figure 16:  $H^+$  curves of BMI categories among men, France, UK and US, 2008, 1st ranking.

men suffer from severe (grade 2) or very severe (grade 3) obesity. Similarly, the fraction of underweight individuals is larger among women than among men. On the other men are more represented than women in the mildly obese or overweight categories. Depending upon the ranking of the BMI categories, these states of affair suggest that the distribution of BMI categories is more "in the middle" among men than among women.

## 4 Conclusion

A couple of clear normative conclusions about the unfairness in the distribution of "body weight problems" among individuals can be taken out of this paper.

First, there has been a deterioration in the distribution of individual well-being associated with abnormal body weight for the period 1998-2010 among French males. Indeed, for any assumption made on the connection between well-being and BMI category contained in table 3, it can be said that the well-being of a man in 2008 or 2002 is inferior to the well-being of the man of the corresponding rank in the well-being distribution in 1998. While our data do not enable one to verify that the same deterioration holds true in the UK and the US, we strongly suspect that it does.

Second, and under a smaller number of assumptions on the relation between BMI and well-being than for males, the same qualitative result holds true for French women. The only rankings of table 3 for which this deterioration in well-being do not arise for women are those who assume that excessive thinness has a more adverse effect on well-being than being obese of grade 2. The reason for the failure to reach the same conclusion for women than for men in that case is that the drop in the fraction of excessively thin women has been more important than the increase in the fraction of obese of grade at least 2 women observed over that same period.

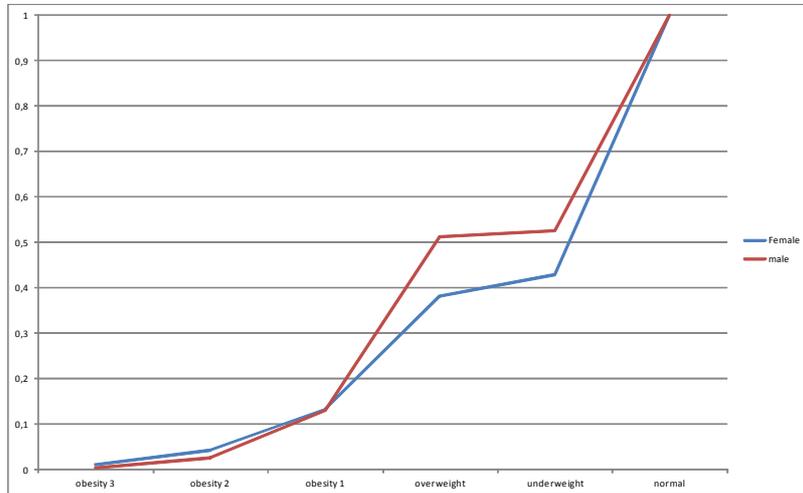


Figure 17: CDF of the BMI categories for males and females in France (2008), 1st ranking

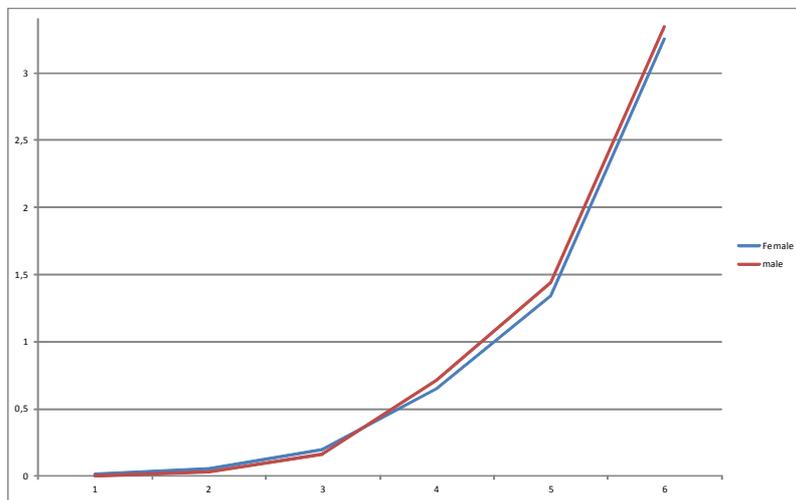


Figure 18:  $H^+$  curves for distributions of BMI categories among males and females in France (2008), 1st ranking

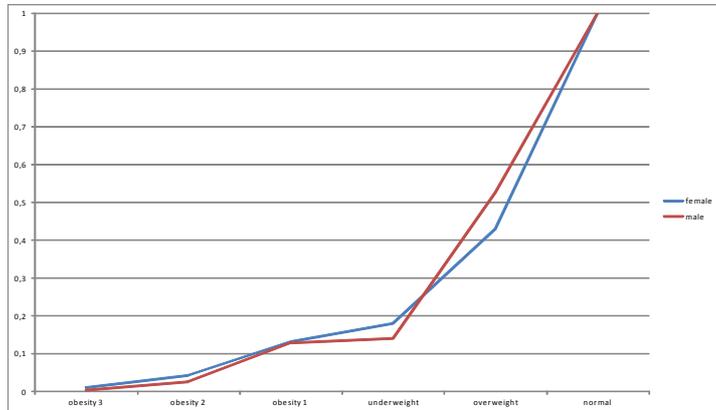


Figure 19: CDF of the BMI categories for males and females in France (2008), 2nd ranking

Similar strong conclusions can be obtained when comparing France, the UK and the US in 2008. For almost all the rankings of table 3, the average level of well-being is higher in France than the UK and larger in the UK than the US for both males and females. There are two exceptions to this. The first one concerns women, under the (unlikely) assumption that excessive thinness is the worst possible BMI category. In that case, it happens that the fraction of excessively thin women is larger in France than in the US and, somewhat surprisingly, larger in the US than in the UK. Because this relative standing of the three countries in terms of the fraction of excessively thin women is the opposite to what is observed in terms of excessive weight, one is led to the conclusion that France, UK and US can not be compared by first order dominance if excessive thinness is considered worse than the most severe form of obesity. The second exception to this concerns males, and the US-UK comparisons. This exception holds true for all rankings of table 3 but the one who considers excessive thinness to be the worst BMI category. Indeed, it turns out that the fraction of men with a normal weight is (slightly) larger in the US than in the UK. Since the fraction of obese men is also larger in the US than in the UK, the two countries can not be compared by first order dominance. However, a clear ranking of the two countries - unfavorable to the US - emerges if one accepts the view that both Hammond transfers and elementary efficiency gains are clear normative improvement, provided that grade 3 obesity is ranked as being worse than excessive thinness. For in all rankings of table 3 consistent with this assumption, it is found that the UK distributions of BMI among men  $H^+$  dominates its US counterpart.

The final - but in our view most important - conclusion derived from this paper concerns the somewhat clear verdict of a greater unfairness in the distribution of weight-related well-being among women than among men. This is at least true if one assumes that excessive thinness is worst for well-being than non-obese overweight. For in all rankings of table 3 compatible with that assumption, and in all three countries, one finds that the male distribution of BMI categories  $H^+$  dominates the female one. As illustrated on figure 20, this dom-

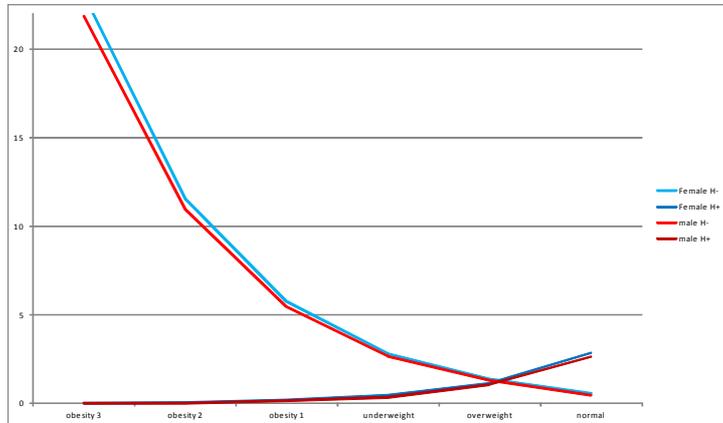


Figure 20:  $H^+$  and  $H^-$  curves for distributions of BMI categories among males and females in France (2008), 2nd ranking

inance, also observed with the  $H^-$  criterion, suggests that the distribution of BMI categories among men is more equal than that among women in the precise sense that the former has been obtained from the latter by a finite sequence of equalizing Hammond transfers. We believe that this larger inequality observed among women with respect to body mass anomalies should be a serious source of concern for health and nutrition policies, especially in view of the important deterioration of body weight observed in the last 15 years in many parts in the world.

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