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# Application of methods used in the classical matching markets to the Indian marriage market

Raïssa-Juvette Samba Zitou\*  
University of Brazzaville, Congo  
Department of Economic

Rhonya Adli†  
University of Caen, France  
Department of Economic CREM

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## Abstract

In most societies, the social practice of paying dowry tends to decline and sometimes to disappear. In contrast, a system of marriages negotiated between families continues to exist in India; a marriage squeeze and a real dowry inflation are observed throughout the country. This paper brings a nice application of methods used in the classical matching markets: existence of stable outcomes and a minimum equilibrium dowry, coincidence between the set of stable outcomes and the set of competitive equilibrium outcomes. We further discuss strategic questions and address issues comparative statics when a marriage squeeze yields in the Indian marriage market.

*Keywords:* Matching; Dowry auction mechanism; Equilibrium stable outcome; Competitive equilibrium outcome; Population increase; Strategy-proof; Comparative statics.

*JEL classification:* C78; D78, J11, J12, D10

## 1 Introduction

With economic development and social modernization, the social practice of paying dowry tends to decline in most societies. In contrast, it has been observed in India that, in spite of economic development, a system of marriages negotiated between families has not only continued to exist, but has in many regions of India been accompanied by an increasing demand for dowry payments by men (and their families). In this country, the dowry is an income transfer from the

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\*e-mail: juvette\_samba@yahoo.fr

†e-mail: rhonya.adli@unicaen.fr

bride to the groom; this dowry includes resources paid by the bride's family to the groom's family at the time of the wedding. The dowry may be paid in cash or kind directly paid from a bride's family to a groom's family (for instance, gifts such that motorcycles, refrigerators, cars, etc...). The factors such as the personal qualities, the professional occupation, the incomes and the family status contribute to determining the "groom price" for the marriage.

Gale and Shapley (1962) initiate the marriage model. In Gale and Shapley's model, the stable marriage problem, money does not play a role; there are sets of men and women of the same size. Every man has a strict preference order over women and each woman has a strict preference order over the men. A matching is a set of disjoint men-women pairs. A matching is stable if it is individually rational (no person prefers to remain single rather than be matched to a partner) and if no man and no woman prefer each other to their current partners.

Shapley and Shubik (1972) introduce money into the stable marriage problem and initiate the assignment games. The assignment games are the models in which each player or agent can form one partnership at most with an interest for each agent  $b$  on one side to form a profitable coalition with an agent  $g$  of the opposite side. In other words, the aim of the agents is to get as much profit as possible from their partnership. The profit obtained by each mixed-pair can be represented in an assignment matrix. Shapley and Shubik (1972) show that the *core*<sup>1</sup> of an assignment game is the set of dual optimal solutions to the assignment optimization problem on the underlying matrix of mixed-pair profits; the core and the set of stable outcomes are the same; the core is non-empty and has a *complete lattice* structure<sup>2</sup>.

Becker (1981) presents a marriage model in which men and women both possess different qualities (or potential incomes). The marriage is viewed as a joint venture that offers greater efficiency in production (household, market, or both). Each agent in the market chooses the partner who maximizes their utility.

Eriksson and Karlander (2000) present the RiFle assignment game, which is a generalization of two standard models of the theory of two-sided matching: the marriage model of Gale and Shapley and the assignment game of Shapley and Shubik. In the RiFle assignment game, some agents are "rigid" and others are "flexible". Rigid agents such as those found in the marriage model do not negotiate payments, while flexible agents, found in the assignment model, do.

The Economic theories of dowry are recent and mainly describe the formation and determinants of dowry. In this literature, the dowry is investigated in the context of the marriage market (see Rao, 1993; Zhang and Chan, 1998; Anderson, 2003, 2007...).

The basic analytical framework adopted by the dowry theories refers to Becker's marriage analysis. The theoretical analysis of the dowry is thus an

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<sup>1</sup>In other words, the core of the game is defined as the set of undominated outcomes. An outcome  $z$  is *dominated* by another outcome  $y$  if there exists a coalition  $S$  of agents such that every member of  $S$  prefers  $y$  to  $z$  and the rules of the game allow them to enforce  $y$  over  $z$ .

<sup>2</sup>A lattice is a partially ordered set any two of whose elements have a supremum and an infimum. A lattice is complete when each of its subsets has a supremum and an infimum.

extension of the theoretical analysis of the marriage market. The dowry appears as a function of implicit price depending on the characteristics of the partners and their rational families participating in the marriage market (Rao, 1993).

However, in the India marriage market, although the value of dowry is linked to the quality of the groom; the dowry is a monetary payment that will be the object of negotiations<sup>3</sup> before the wedding. So, the dowry is one of selection factors which determine both the preferences of grooms over brides and the wedding between grooms and brides.

In the Indian marriage market, the agents behave as if they were: in the marriage market of Gale and Shapley when they determine their lists of preferences; and in the assignment game when they negotiate their dowries. However, there are neither completely rigid as in the marriage market nor completely flexible as in assignment game.

In this market, many factors determine the preferences of grooms over brides, as well as preferences of brides over grooms as in the marriage model of Gale and Shapley. For instance, a groom  $l$  is ranked higher than a groom  $k$  in the list of preferences of the bride  $x$  : if (for example) his physical appearance, Socio-Economic status, professional occupation is better than that of  $k$ . Similarly, a bride  $x$  is ranked higher than a bride  $y$  in the list of preferences of the groom  $l$  : if her physical appearance, Socio-Economic status, desirable age to match are better than those of  $y$  and if  $x$  proposes a higher dowry than  $y$ .

In the marriage model of Gale and Shapley, if  $x$  and  $y$  propose to  $l$ ,  $l$  chooses the proposition from  $x$  who is ranked higher than  $y$  in his list of preferences. There is a matching between the bride  $x$  and the groom  $l$ . So, in the Gale and Shapley' model, the dowry is not negotiable. It may only be a means to show the wealth of the bride's father, to recognize the qualities of the groom... In this case, the dowry is considered as a discrete variable.

It is also true for the determination of agents' preferences in the Indian matching market. However, in this market, if  $x$  and  $y$  propose to  $l$ , then  $l$  and  $x$  must meet to negotiate the dowry. If an agreement is reached between the two agents, then  $x$  and  $l$  are matched. If no, there is no matching between  $x$  and  $l$ .

So, as in the assignment game, the dowry is also the only factor that determines the matching between a bride and a groom. In this case, the transfers between brides and grooms are continuous and so the dowry is considered as a continuous variable.

*Then, the Indian matching market appears as a sort of market, which is neither completely discrete nor completely continuous.*

The main purpose of this paper is to apply to the Indian matching market, the methods used in the classical matching markets. These methods are used in the case of a constant population and a population increase. So, in this paper, we show that some results obtained in these classical matching markets (marriage model and assignment game) are checked in the Indian marriage market.

In our model, if we consider that the dowry is a discrete variable; it is only one of the factors that determine the preferences that men have over women

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<sup>3</sup>Here, we consider that the dowry is a groom-price paid in cash.

and so the matching, then our model coincide with the marriage model of Gale and Shapley. *Hence, the marriage model of Gale and Shapley (1962) is a special case of our model.* If we consider that all selection factors mentioned above correspond to a unique monetary payment that is the object of negotiations, then this model coincide with the assignment game of Shapley and Shubik (1972). *Hence, the assignment game of Shapley and Shubik (1972) is a special case of our model.*

In our paper, the preferences are defined by degrees of preference on an ordered list of preferences. A groom is the best partner for a bride if this groom has the higher degree of preference in her list of preference.

First, we study the model in the case of a constant population. We describe an Indian matching mechanism, which is inspired from the algorithm of Eriksson and Karlander (2000). The aim is to show that it is possible, by using an Indian matching mechanism, to reach the minimum equilibrium dowry vector in the Indian marriage market. We introduce in this mechanism a well known notion in the matching theory, the augmenting paths. We prove that this mechanism finds a stable outcome and that the dowry vector  $\tilde{v}$  produced is *the minimum equilibrium dowry vector*<sup>4</sup>.

We show that, in the Indian marriage model, the set of stable outcomes enjoys nice properties such as the lattice property and the polarization of interests that characterize the core of the classical matching models (see Gale and Shapley (1962), Shapley and Shubik, 1972). We also show that in this market, *the set of stable outcomes and the set of competitive equilibrium outcomes* are the same. Hence, we prove that the set of competitive equilibrium outcomes is endowed with properties, which characterize the set of stable outcomes in the model. So, like in the set of stable outcomes, *in the set of competitive equilibrium outcomes among all competitive equilibrium outcomes there is one (and only one) which is bride optimal (resp. optimal groom) meaning, in the context of the Indian matching market, that all brides (resp. grooms) conclude the most favorable alliance under it as under any other competitive equilibrium outcome.* Therefore, the competitive dowry vector  $\tilde{v}$  is the “worst” competitive equilibrium dowry vector from the point of view of the grooms and it is called the minimum competitive equilibrium dowry. There is also a maximum competitive equilibrium dowry with symmetrical properties.

In the case of a population increase, we discuss the strategic questions and the questions of comparative statics for all periods in which a *marriage squeeze*<sup>5</sup> exists.

In India, there is a dowry inflation for which the Economics literature proposes various hypotheses to explain this phenomenon. However, the literature

<sup>4</sup>Demange, Gale and Sotomayor (1986) present two dynamic mechanisms that produce the minimum competitive equilibria.

Bloch and Rao (2002) also prove a similar result in this Indian marriage market. They show that, in equilibrium, the bride’s family offers the minimal dowry that the groom accepts.

<sup>5</sup>The “marriage squeeze” refers to a demographic imbalance between the numbers of potential brides and potential grooms.

offers two main explanations: the one based on the social status<sup>6</sup> and the other on demographic factors.

Rao (1993) explains the Indian dowry inflation by demographic factors: a marriage squeeze is caused by the population growth (declining mortality leads to a larger young cohorts), and this leads to a larger number of young women of marriageable age (since women marry older men in general) and this excess supply of women in the marriage market leads to an increase in dowry payments.

However, Anderson (2007) demonstrates that a marriage squeeze cannot yield the dowry inflation. He proves that a marriage squeeze implies the dowry deflation through time. He develops a matching model of marriage, which formally analyzes the link between the dowry payments and the population growth.

In our paper, we follow Anderson to study the model in the case of a population increase. So, we consider that a marriage squeeze produces the dowry deflation. We have two subgroups of brides:  $\mathcal{B}_1$  composed of the brides that reach marrying age (the younger brides) and  $\mathcal{B}_2$  composed of the brides that re-enter the market (the older brides).

First, we show that in the case of a population increase for all periods in which a marriage squeeze exists, the Indian matching mechanism still produces an equilibrium stable outcome.

The investigation of these questions in Indian marriage model with population growth is of economic interest because it allows formally evaluating the effects of a demographic shift on the individual behavior of each agent.

In the case of a population increase for all periods in which a marriage squeeze exists, the study of strategic questions leads naturally to the following questions in the Indian marriage market: is it always in the best interest of women to behave honestly? Is that women manipulate the matching procedure by misrepresenting their reservation dowries (in the goal to be matched with a better partner)? Is that the domestic violence that exists after the wedding in the Indian society is explaining by a manipulation of the matching procedure?

These questions are closely related and are of interest because these are well-documented facts: that the dowry can represent sometimes six times the annual wealth owned by the family's woman (Rao (1993)); that the women in India are victims of domestic violence and even murdered if they are unable to pay the exorbitant dowries demanded by their husbands (Bloch and Rao (2002)).

*We prove that, even in the case of a population increase for all periods in which a marriage squeeze exists, in India, the mechanism which yields the  $\mathcal{B}$ -optimal stable outcome for one side of the market is strategy proof for that side's children.* A similar result is obtained in the classical matching markets (see Roth and Sotomayor, 1990).

Our results show that the brides have no interest to misrepresent their *reservation dowries* under the  $\mathcal{B}$ -optimal stable outcome because they cannot conclude a better alliance than their current alliance. *Therefore, the domestic violence is not explained by the strategic behavior that women might be able to*

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<sup>6</sup>Anderson (2003) offers an explanation based on the Indian caste system. Increasing wealth dispersion, the search for status and social mobility within caste groups tend to increase dowry payments, which are used as an indicator of social status.

*use in the matching procedure.* The literature confirms our result because domestic violence is usually due to the greed of husbands and in-laws. Indeed, Rao (1993) explains that the dowry violence does not refer directly to marriage related payments made at the time of the wedding, but to additional payments demanded after the marriage by the groom's family where the husband systematically abuses the wife in order to extract larger transfers.

The study of questions of comparative statics also leads to the one of questions the most interesting in Indian social science and which is of economic interest: what are the effects on the payoffs' agents of adding younger or older brides to the market?

This question is very interesting because now in India, the fear of high dowries leads parents to kill their daughters through either infanticide or sex-selective abortion methods (Sudha and Rajan (1999), Arnold, Kishor and Roy (2002)).

We prove that *in the case of a population increase for a given period  $h$ , in which a marriage squeeze exists, the entrance of a younger bride in the market produces a result parallel to the results obtained in the classical matchings models<sup>7</sup>: the entrance of a younger bride creates a competition among all brides.*

*By against, we observe that in the case of a population increase for all periods in which a marriage squeeze exists, the entrance of a younger bride to the market produces a result which has no parallel in the classical matching models: there is a  $\mathcal{G}$  – optimal stable outcome such that every groom and every bride are worse off in the new market ( $M'$ ) than in the previous one ( $M$ ). The entry of younger brides affects all brides ( $v'_j \leq v_j$ ) and all grooms ( $u'_i \leq u_i$ ).*

In fact, in the case of a population increase for a given period in which there is a marriage squeeze, the adding of new brides to the market causes an increase of payments' grooms as it is observed in the literature of matching markets. However, through time, a decrease of payments'grooms is observed.

*Moreover, the return of an older bride in the market also produces a result which has no parallel in the classical matching models: this entry has no effect on the younger brides; there only exists a competition among all older brides.*

This paper is organized as follows. Section 2 presents the mathematical model. Section 3 is devoted to the main results in the case of a population constant. Section 4 studies the effects of a demographic shift on the individual behavior of each agent by examining strategic questions and the questions of comparative statics. Section 5 concludes the paper.

## 2 Preliminaries

Consider an Indian marriage market where each family is composed of one and only one child, male or female. We know that in India, these are the families that negotiate and arrange the wedding. But in this paper, to simplify the notations, we only consider the children at the time of the marriage.

<sup>7</sup>See Demange, Gale and Sotomayor (1986) and Mo (1988).

In this section, we consider that the population of these children is constant. We have an equal number of males and females,  $n$ , which are born at every period  $t$ . So, in each period, the marriageable age is  $\varepsilon$  in  $\mathcal{B}$  and  $\delta$  in  $\mathcal{G}$ . The children either marry at the desirable ages  $(\varepsilon, \delta)$  or later. In general, in the Indian matching market  $\varepsilon < \delta$ <sup>8</sup>.

Consequently, consider an Indian marriage market where there are two finite and disjoint sets of children,  $\mathcal{B} = \{b_1, \dots, b_i, \dots, b_n\}$  and  $\mathcal{G} = \{g_1, \dots, g_j, \dots, g_n\}$ , with  $|\mathcal{B}| = |\mathcal{G}|$ . Thus, the family of a woman is represented by their daughter, the bride  $b$  and the family of a man is represented by their son, the groom  $g$ .

The brides on one side form partnerships with the grooms on the opposite side. Each child is interested in forming one partnership at most.

*Here, to simplify, we consider that the factors which determine the preferences that brides on one side of the market have over on grooms on the opposite side are: physical appearance, the Socio-Economic status and education. The factors which determine the preferences that grooms on one side of the market have over on brides on the opposite side are: physical appearance, the Socio-Economic status, education and the value of dowry (proposed by brides). The dowry negotiated is the factor that determines the matching between a bride and a groom.*

Each bride  $b_i$  represents her preferences for the grooms she would like to match by an ordered list preferences  $D(p_i)$ . On this list, the grooms are ranked by according to degrees of preference  $\sigma$  with  $\sigma \in [0, 1]$ . The list of preferences of  $b_i$  given by  $D(p_i) = g_j, g_k, g_p, \dots, b_i, g_l$  means that  $b_i$  prefers  $g_j$  to  $g_k$  with  $\sigma_{ij} > \sigma_{ik}$ . The bride  $b_i$  prefers stay alone to matching with  $g_l$ ; so  $g_l$  is unacceptable to  $b_i$ . A groom  $g_j$  is *acceptable* to  $b_i$  if and only if  $\sigma_{ij} \geq 0$ .

Each groom also expresses their preferences by an ordered list of preferences  $D(p_j)$ . The brides are ranked by according to degrees of preference  $\rho$  with  $\rho \in [0, 1]$ . Thus,  $g_j$  prefers  $b_i$  to  $b_k$  if and only if  $\rho_{ji} > \rho_{jk}$  and  $b_i$  is acceptable to  $g_j$  if and only if  $\rho_{ji} \geq 0$ .

To each pair  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ , there is a nonnegative number  $\alpha_{ij}$ , which can be interpreted as the value of dowry that the bride  $b_i$  is willing to pay for to match with the groom  $g_j$ . So,  $\alpha_{ij}$  is the gain of transaction when a bride  $b_i$  is matched to a groom  $g_j$ . For simplicity, we assume that the reservation dowry of each child is zero and that there are no monetary transfers among children from the same side. When the bride  $b_i$  matches to the groom  $g_j$  for a dowry  $v_j$  (in other words if  $x$  is a matching, which is an allocation of the brides to the grooms) then the resulting utilities are  $u_i = \alpha_{ij} - v_j$  for the bride  $b_i$  and  $v_j$  for the groom  $g_j$ .

We consider that any child (bride and groom) is free to remain unmatched, in which case his utility is zero. Note that an unmatched bride (resp. an unmatched groom) is considered as being matched to herself (resp. himself). The marriage market with dowry is given by  $M = (\mathcal{B}, \mathcal{G}; \sigma, \rho, \alpha)$  where  $\sigma$  and  $\rho$  are the matrices of the degrees of preference and  $\alpha$  is the dowry matrix.

<sup>8</sup>Many reasons explain this difference in ages such as the higher fertility of the younger brides, the authority of a husband on a younger wife...(see Bergstrom and Bagnoli, 1993)



**Definition 1** A feasible matching  $x$  is a  $(m \times n)$  matrix  $(x_{ij})_{(b_i, g_j) \in \mathcal{B} \times \mathcal{G}}$  with  $x_{ij} \in \{0, 1\}$  such that:  $\sum_{g_j \in \mathcal{G}} x_{ij} \leq 1$  for all  $b_i \in \mathcal{B}$  (1),  $\sum_{b_i \in \mathcal{B}} x_{ij} \leq 1$  for all  $g_j \in \mathcal{G}$  (2) and  $x_{ij} \geq 0$  for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$  (3).

If  $x_{ij} = 1$  (resp.  $x_{ij} = 0$ ) we say that  $b_i$  and  $g_j$  are matched (resp. unmatched) at  $x$ . If  $\sum_{b_i \in \mathcal{B}} x_{ij} = 0$  (resp.  $\sum_{g_j \in \mathcal{G}} x_{ij} = 0$ ) we say that  $g_j \in \mathcal{G}$  (resp.  $b_i \in \mathcal{B}$ ) is unmatched at  $x$ . If  $x_{ij} = 1$  we can write  $x(b_i) = g_j$  or  $x(g_j) = b_i$ ; if  $x_{ij} = 0$  we can write  $x(b_i) = b_i$  and  $x(g_j) = g_j$ .

**Definition 2** An outcome is a matching  $x$  and a pair of vectors  $(u, v)$  called payoff, with  $(u, v) \in R^n \times R^n$ . An agreement will be denoted by  $(u, v; x)$ .

Thus, an agreement in this market is determined by specifying a matching and the way in which the income within each pair is divided among the spouses.

**Definition 3** An outcome  $(u, v; x)$  is feasible if  $(u, v; x)$  satisfies the following conditions:

1.  $\sigma_{ij} \geq 0$  and  $\rho_{ji} \geq 0$  for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ ;
2.  $u_i \geq 0, v_j \geq 0$ , for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$  (individual rationality) and  $u_i = 0$  (resp.  $v_j = 0$ ) if  $b_i$  (resp.  $g_j$ ) is unmatched.
3.  $u_i + v_j = \alpha_{ij}$  if  $x_{ij} = 1$ .

**Definition 4** An outcome is stable if it is feasible and if for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ , we have  $u_i + v_j \geq \alpha_{ij}$ .

If  $(u, v; x)$  is a feasible (resp. stable) outcome, then  $(u, v)$  is a feasible (resp. stable) payoff and we say that the payoff  $(u, v)$  is compatible with the matching  $x$ .

The bride  $b_i$  is acceptable to  $g_j$  if  $x_{ij} = 1, \sigma_{ij} \geq 0$  and  $v_j \geq 0$ . Analogously, the groom  $g_j$  is acceptable to  $b_i$  if  $x_{ij} = 1, \rho_{ji} \geq 0$  and  $u_i \geq 0$ .

If the condition of Definition 4 is not satisfied for some pair  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ , then we say that  $(b_i, g_j)$  blocks the outcome. The pair  $(b_i, g_j)$  is called blocking pair.

Therefore a feasible outcome (resp. feasible payoff) is stable if and only if it does not have any blocking pair.

### 3 Main results in the case of a population constant

The first main result is that there is always a stable outcome in the Indian matching market.

**Theorem 1** In the Indian marriage market, with a constant population, the set of stable outcomes is non-empty.

To demonstrate it, we describe an Indian matching mechanism that produces a stable outcome in this model. This algorithm which we call the Indian matching mechanism finds a stable outcome via a kind of auction mechanism of dowry in the Indian marriage market. We note that all dowries are integers.

### 3.1 The Indian matching mechanism

#### 3.1.1 Preliminaries

We have the sets  $\mathcal{B}$  and  $\mathcal{G}$ , the factors of selection and the nonnegative integer  $\alpha_{ij}$  for each pair  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ ; we also have a dowry vector  $\tilde{v}$  and a map  $x : \mathcal{B} \rightarrow \mathcal{G}$  that will both be modified during the algorithm until  $x$  is injective, that is, a matching. We begin with the dowry vector  $\tilde{v} = [0, \dots, 0]$  (this algorithm is initialized to zero. It is the minimum dowry to which a groom can accept to match with a bride). In the course of algorithm, the dowries of grooms will only increase, but never decrease; if at given point in the course of the mechanism, we have  $x(b_i) = g_j$ , then we say that  $b_i$  proposes to  $g_j$  at this point; if at some point, some groom  $g_j$  has a proposition then,  $g_j$  will always have a proposition during the algorithm.

From the factors of selection, we define  $D(p_i)$ , for each  $b_i \in \mathcal{B}$ . The set  $D(p_i)$  depends on the current dowry vector  $\tilde{v}$ , it is not fixed. In each step, the algorithm will modify the dowry vector and the map  $x$  so that it always satisfies  $x(b_i) \in D(p_i)$ . As soon as the map is injective, the algorithm stops.

The aim of the algorithm is to produce a matching  $x$  and payoff vectors  $u$  and  $v$  satisfying the following conditions:

- (i)  $\sigma_{ij} \geq 0$  and  $\rho_{ji} \geq 0$  for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ ;
- (ii)  $u_i \geq 0, v_j \geq 0$  for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ ;
- (iii) If  $x(b_i) = g_j$ , then  $u_i + v_j = \alpha_{ij}$ ;
- (iv)  $g_j \in D(p_i)$  for all  $b_i \in \mathcal{B}$ .

Condition (i), Condition (ii) and Condition (iii) say that the payoffs are compatible with the matching. Condition (iv) implies the stability, there is no blocking pair if every  $b_i$  is matched to his best possible partner.

We use in this algorithm, the notion of *augmenting paths*. In the course of the algorithm, if we have the relation  $g_j \sim g_k$ , then we say that  $g_j$  is connected to  $g_k$ . That is, there exists a bride  $b_i$  that proposes to  $g_j$  and equally well could have proposed to  $g_k$ , so we have  $x(b_i) = g_j$  and  $g_k \in D(p_i)$ . Let us extend this relation by transitivity, so we say that  $g_{j_1}$  is connected to  $g_{j_n}$  if there exists a path  $g_{j_1} \sim g_{j_2} \sim \dots \sim g_{j_n}$ .

We divide the algorithm into three phases. In the first phase, each bride (recall that to simplify the notations, we do not consider the families of each child) propose to the best groom in  $D(p_i)$ . If it is possible to conclude an agreement between the bride and the groom, so there is a matching. This part runs as the Gale and Shapley algorithm for finding a stable matching in the marriage model; but with a negotiable dowry. However, some groom may still have several propositions; that is, some groom may have received more than one proposition. In Phase II, we find augmenting paths from  $\mathcal{G}$ -children that are

already matched under  $x$  to  $\mathcal{G}$ -children that have no propositions. At the end of Phase II, there are no such augmenting paths, so if  $x$  is still not injective, then there must be an  $\mathcal{G}$ -children that has several proposers but which is not part of an augmenting path. In Phase III, the dowries are increased for all such  $\mathcal{G}$ -children and all  $\mathcal{G}$ -children connected to them, and everything is repeated again.

The algorithm has two important properties: the dowries  $\tilde{v}$  never decrease and if at some point  $g_j$  gets a proposer, then  $g_j$  will never again be without proposers during the algorithm. The algorithm will stop when each  $g_j$  is matched or has opted to stay unmatched.

### 3.1.2 Determination of lists of preferences

All brides propose to  $\mathcal{G}$ . We have  $\mathcal{B}$  and  $\mathcal{G}$  children and degrees of preference  $\sigma$  and  $\rho$ , and the nonnegative number integer  $\alpha_{ij}$  for each pair  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ . Here, the dowry is considered as a discrete variable; it is one of the factors that determine the preferences that grooms have over brides.

Define a function  $\gamma_{ij}(d)$  that gives us the value of  $g_j$  to  $b_i$  if the dowry of  $g_j$  is  $d$ , that is, if for  $g_j$  the dowry  $v_j = d$ . The value will tacitly depend also on the current map  $x$  :

$$\gamma_{ij}(d) = \begin{cases} \alpha_{ij} - d & \text{if } (b_i, g_j) \in \mathcal{B} \times \mathcal{G} \\ 0 & \text{if } x(b_i) \neq g_j \end{cases}$$

Define  $D(p_i)$ , for each  $b_i \in \mathcal{B}$ , as the set of potential partners at the dowry  $d$ . That is,

$$D(p_i) = \left\{ \begin{array}{l} \sigma_{ij} = \max \{ \sigma_{kj} : \mu(b_k) = g_j \} \\ \gamma_k : \gamma_{ik}(v_k) = \max_j \gamma_{ij}(v_j) \end{array} \right\}.$$

Each bride and each groom define their degrees of preference for potential partners according to some selection factors. So, we determine for each agent a list of preferences. These are the list of acceptable partners. An agent may be indifferent between two partners on its list of preferences. However, the dowry's transactions will determine its choice. It will decide to match or to stay alone.

### 3.1.3 The dowry auction mechanism

Consider that the dowry is paid in cash; all payoffs are integers. Then, we use an integral productivity matrix. All brides propose to  $\mathcal{G}$ . We have  $\mathcal{B}$  and  $\mathcal{G}$  children with their lists of preferences, and the nonnegative number integer  $\alpha_{ij}$  for each pair  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ ; we also have a dowry vector  $\tilde{v}$  and a map  $x : \mathcal{B} \rightarrow \mathcal{G}$  that will both be modified during the algorithm until  $x$  is injective, that is, a matching. We set  $\tilde{v} = [0, \dots, 0]$  and choose  $x$  for every  $x(b_i) \in D(p_i)$  for every  $b_i$  (this is possible, since from the definition every  $D(p_i)$  is always nonempty).

#### Phase I

Each bride announces a value of dowry to his favorite groom (s). That is, she proposes to the groom (s) that is ranked first in  $D(p_i)$ . Note that it is possible to be indifferent. Every groom studies the offer from the most preferred bride on his list of preferences. If it is possible to conclude an agreement between a groom and a bride, so there is a matching; if  $x$  is injective, then the algorithm stops. By against, if some groom still have several propositions, we move to the Phase II.

### Phase II

Let  $b_{k_n}$  denote a proposer that could proposed equally well to  $g_{k_{n+1}}$  instead of  $g_{k_n}$ . Suppose that we find a path  $g_{k_1} \sim g_{k_2} \sim \dots \sim g_{k_m}$  such that  $g_{k_1}$  has at least one extra proposer  $b_i$  except for  $b_{k_1}$  while  $g_{k_m}$  has no proposer at all. Then we modify  $x$  by setting  $x(b_{k_1}) := g_{k_2}, \dots, x(b_{k_{m-1}}) := g_{k_m}$ . For all other  $b \in \mathcal{B}$ ,  $x$  maps as before. This augments the image of  $x$  by one groom,  $g_{k_m}$ . Now Part I is run again.

As long as, this alternative is possible, the process is repeated and if several possibilities are open, choose one.

### Phase III

Let  $E$  be the set of all  $g \in \mathcal{G}$  that are (1) connected to some  $g \in \mathcal{G}$  who has more than one proposer, and (2) not connected to any  $g \in \mathcal{G}$  that have no proposal. Modify the dowry vector  $\tilde{v}$  by increasing  $v_j$  by one, for all  $g_j \in E$ . Now phase II is run again. The whole process is repeated until  $x$  becomes injective.

The dowry vector  $\tilde{v}$  sometimes increases, but never decreases. When the algorithm halts, each bride is either matched or has been rejected by every groom on his preference list.

Phase III must eventually stop and, hence the algorithm, must eventually stop. Indeed, each  $g_j$  that, at some point, has a proposition then,  $g_j$  will always have a proposition. In each step, we have  $x(b_i) = D(p_i)$ . We set in the first step  $\tilde{v} = [0, \dots, 0]$ . Thus, if some  $g_k$  does not have a proposer at some point, then  $v_k$  is still zero, in which case the payoff  $\gamma_{ik}(v_k)$  of  $v_k$  for  $b_i$  is greater than zero. When  $g_j$  has a proposer  $b_i$  and the dowry is allowed to increase sufficiently, we will arrive at a non-positive value of the dowry of  $g_j$  for  $b_i$ ,  $\gamma_{ij}(v_j) \leq 0$ . Then  $g_j \notin D(p_i)$  and we must have  $g_k \in D(p_i)$  for some  $b_i$ . When Phase III is used, there must eventually exist a path from some  $g_j$  with several proposers to  $g_k$ . When Phase II is used,  $g_k$  gets a proposer. In this way, all grooms must eventually get proposers, so  $x$  is a matching.

The dowry vector is determined by the algorithm, and will be nonnegative. Assume that  $x(b_i) = g_j$ . Then, we set  $u_i = \alpha_{ij} - v_j$  if  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ . The unmatched  $g \in \mathcal{G}$  have zero dowries, they have always been of positive value to all  $\mathcal{B}$ -children, and the same thing must of course hold for the matched pairs. So all  $u_i$  are non-negative.

Thus, the Indian matching mechanism halts. Grooms who did not receive any acceptable proposition, and brides who were rejected by all grooms from their sets of potential partners, stay alone.

We present this algorithm via the following example.

**Example 1** Consider  $\mathcal{B} = \{b_1, b_2, b_3, b_4\}$  and  $\mathcal{G} = \{g_1, g_2, g_3, g_4\}$  in the case of a constant population. Consider the factors such that the physical appearance, the Socio-Economic status, the education and the value of dowry.

Consider the following degrees of preference and dowry matrix

|       | $g_1$             | $g_2$            | $g_3$            | $g_4$            |
|-------|-------------------|------------------|------------------|------------------|
| $b_1$ | (0, 7; 0, 9; 14)  | (0, 6; 0, 6; 10) | (0, 3; 0, 5; 8)  | (0, 2; 0, 1; 4)  |
| $b_2$ | (0, 06; 0, 6; 10) | (0, 5; 0, 7; 12) | (0, 3; 0, 6; 10) | (0, 9; 1; 17)    |
| $b_3$ | (0, 05; 0, 5; 8)  | (0, 1; 0, 6; 10) | (0, 8; 0, 9; 14) | (1; 0, 9; 16)    |
| $b_4$ | (0, 9; 0, 9; 14)  | (0, 7; 0, 8; 13) | (0, 2; 0, 6; 10) | (0, 1; 0, 4; 10) |

**Table 1:** Degrees of preference and dowry matrix

### Determination of lists of preferences

The brides  $b_1, b_2, b_3$  and  $b_4$  announce their degrees of preference for each bride. The grooms also announce their degrees of preference for each groom.

So, we determine the preference lists of each agent. We have:

$$\begin{aligned}
 b_1 &= g_1, g_2, g_3, g_4 & g_1 &= b_1 \sim b_4, b_2, b_3 \\
 b_2 &= g_4, g_2, g_3, g_1 & g_2 &= b_4, b_2, b_3 \sim b_1 \\
 b_3 &= g_4, g_3, g_2, g_1 & g_3 &= b_3, b_4 \sim b_2, b_1 \\
 b_4 &= g_1, g_2, g_3, g_4 & g_4 &= b_2, b_3, b_4, b_1
 \end{aligned}$$

### The dowry auction mechanism

|       | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
|-------|-------|-------|-------|-------|
| $b_1$ | 14*   | 10    | 8     | 4     |
| $b_2$ | 10    | 12    | 10    | 17*   |
| $b_3$ | 8     | 10    | 14    | 16*   |
| $b_4$ | 14*   | 13    | 10    | 10    |

Each bride proposes to his favorite groom. So  $b_1$  and  $b_4$  proposes to  $g_1$ ,  $b_2$  and  $b_3$  to  $g_4$ . The grooms  $g_1$  and  $g_4$  have two proposers. It is impossible to conclude an agreement between these grooms and these brides. We proceed to Phase II. None of the conditions in Phase II are satisfied, so we proceed to Phase III. In Phase III, we identify the set  $E = \{g_1, g_4\}$  as being of the desired kind: the agents in  $E$  are not connected to any  $g \in \mathcal{G}$  that have no proposal. The groom  $g_1$  has two propositions from  $b_1$  and  $b_4$ , he is not connected to any  $g \in \mathcal{G}$ , and  $g_4$  also has two propositions from  $b_2$  and  $b_3$ ; he is not connected to anyone else. The grooms  $g_2$  and  $g_3$  have no proposition. Raise the dowry by one on both  $g_1$  and  $g_4$ , to obtain  $\tilde{v} = [1, 0, 0, 1]$ . Recompute the values and the map:

|       | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
|-------|-------|-------|-------|-------|
| $b_1$ | 13*   | 10    | 8     | 4     |
| $b_2$ | 10    | 12    | 10    | 17*   |
| $b_3$ | 8     | 10    | 14    | 16*   |
| $b_4$ | 13*   | 13*   | 10    | 10    |

Now, we see that the dowries on  $g_1$  and  $g_2$  have been raised enough for  $b_4$  to find it worth considering proposing to  $g_2$  (the second groom is his preference list) instead. So in Phase II, we identify the path  $g_1 \sim g_2$ , where  $g_1$  has two proposers,  $g_2$  has one proposer;  $g_4$  still has two propositions from  $b_2$  and  $b_3$ . This is an augmenting path, so we change the map accordingly to  $x = [g_1, b_2, b_3, g_2]$ . Recompute the values and the map:

|       | $g_1$  | $g_2$  | $g_3$ | $g_4$ |
|-------|--|--|-------|-------|
| $b_1$ | <span style="border: 1px solid black; padding: 2px;">13</span> | 10   | 8     | 4     |
| $b_2$ | 10   | 12   | 10    | 16*   |
| $b_3$ | 8  | 10   | 14    | 15*   |
| $b_4$ | 13   | <span style="border: 1px solid black; padding: 2px;">13</span> | 10    | 10    |

We proceed to Phase II. None of the conditions in Phase II are satisfied, so we proceed to Phase III. In Phase III, we identify the set  $E = \{g_1, g_4\}$  as being of the desired kind: the groom  $g_4$  has two propositions from  $b_2$  and  $b_3$ ; he is not connected to anyone else. The groom  $g_3$  has no proposition. Raise the dowry by one on both  $g_1$  and  $g_4$ , to obtain  $\tilde{v} = [1, 0, 1, 0]$ . Recompute the values and the map:

|       | $g_1$  | $g_2$  | $g_3$ | $g_4$ |
|-------|--|--|-------|-------|
| $b_1$ | <span style="border: 1px solid black; padding: 2px;">13</span> | 10   | 8     | 4     |
| $b_2$ | 10   | 12   | 10    | 15*   |
| $b_3$ | 8  | 10   | 14*   | 14*   |
| $b_4$ | 13   | <span style="border: 1px solid black; padding: 2px;">13</span> | 10    | 10    |

We see that the dowries on  $g_4$  have been raised enough for  $b_3$  to find it worth considering proposing to  $g_3$  (the second groom is his preference list) instead. This is an augmenting path, so we change the map accordingly to  $x = [g_1, g_4, g_3, g_2]$ . Now the map is injective, so the algorithm stops. Thus, we have  $(b_1, g_1)$ ,  $(b_2, g_4)$ ,  $(b_3, g_3)$  and  $(b_4, g_2)$ ; and from the dowry vector we get payoffs  $\tilde{v} = [1, 0, 0, 2]$  and  $\tilde{u} = [13, 13, 14, 15]$ . ■

### 3.2 Characterization of the core of the Indian matching market

In this section, that the set of stable payoffs is a convex and complete lattice under the partial orders  $\mathcal{B}$  and  $\mathcal{G}$ . We will need some results to prove it.

The following lemma implies that at a stable outcome, the monetary transfers occur only between  $\mathcal{B}$  and  $\mathcal{G}$  – children who are matched to each other.

**Lemma 1** *In the Indian marriage market, with a constant population, let  $(u, v; x)$  and  $(u', v'; x')$  be stable outcomes. Then, (i)  $u_i + v_j = \alpha_{ij}$  for all pairs  $(b_i, g_j)$  such that  $x_{ij} = 1$ ; (ii)  $u_i = 0$  (resp.  $v_j = 0$ ) for all unmatched  $b_i$  (resp. for all unmatched  $g_j$ ) at  $x$ .*

**Proof.** Let  $S_i$  (respectively  $S_j$ ) be the set of all unmatched  $b_i$  (respectively  $g_j$ ) at  $x$ . Then by feasibility of  $(u, v; x)$ :

$$\sum_{\mathcal{B}} u_i + \sum_{\mathcal{G}} v_j = \sum_{\mathcal{B} \times \mathcal{G}} (u_i + v_j) x_{ij} + \sum_{b_i \in S_i} u_i + \sum_{g_j \in S_j} v_j = \sum_{\mathcal{B} \times \mathcal{G}} \alpha_{ij} x_{ij}.$$

Now we apply the definition of stability. ■

We show via the following proposition that the interests of brides and grooms are opposed in the set of stable payoffs.

**Proposition 1** *In the Indian marriage market, with a constant population, let  $(u, v; x)$  and  $(u', v'; x')$  be stable outcomes. Then, if  $x'_{ij} = 1$ ,  $u'_i > u_i$  implies  $v'_j < v_j$ .*

**Proof.** Suppose  $v'_j \geq v_j$ . Then  $\alpha_{ij} = u'_i + v'_j > u_i + v_j \geq \alpha_{ij}$ , which is a contradiction. ■

Define the pointwise maximum  $(u \vee v) = \bar{u}$  and the pointwise minimum  $(u \wedge v) = \underline{v}$ , for any vectors  $u$  and  $v$  of the same dimension.

**Definition 5** *Let  $(u, v)$  and  $(u', v')$  be both stable payoffs. Let  $x$  be some equilibrium matching. We define  $\bar{u}$  and  $\underline{v}$  as follows: (i) for every  $b \in \mathcal{B}$ ,  $\bar{u} = \max \{u_i, u'_i\}$ ; (ii) for every  $g \in \mathcal{G}$ ,  $\underline{v} = \min \{v_j, v'_j\}$ . Similarly, we define  $\underline{u}$  and  $\bar{v}$ .*

**Lemma 2** *In the Indian marriage market, with a constant population, let  $(u, v)$  and  $(u', v')$  be in the set of stable outcomes. Then, the outcomes  $(\bar{u}, \underline{v}; x)$  and  $(\underline{u}, \bar{v}; x)$  are both allocations of this set.*

**Proof.** For any  $b_i$  and  $g_j$ , we have either

$$\underline{u}_i + \bar{v}_j = u'_i + \bar{v}_j \geq u'_i + v'_j \geq \alpha_{ij} \text{ or}$$

$$\underline{u}_i + \bar{v}_j = u_i + \bar{v}_j \geq u_i + v_j \geq \alpha_{ij}.$$

We know that if  $(u, v)$  and  $(u', v')$  are stable outcomes, so these payoffs are compatible with  $x$ . Then  $\underline{u}_i \geq 0$  and  $\bar{v}_j \geq 0$ .

Suppose that, from Proposition 1 and Lemma 1, if  $x_{ij} = 1$  then,

$$\underline{u}_i + \bar{v}_j = u'_i + v'_j = \alpha_{ij} \text{ or}$$

$$\underline{u}_i + \bar{v}_j = u_i + v_j = \alpha_{ij}.$$

Then,

$$\sum_{b_i \in \mathcal{B}} \underline{u}_i + \sum_{g_j \in \mathcal{G}} \bar{v}_j = \sum_{x(b_i)=g_j} \alpha_{ij} x_{ij}.$$

Hence  $(\bar{u}, \underline{v}; x)$  and  $(\underline{u}, \bar{v}; x)$  are also in the set of stable outcomes. ■

Define the partial orders  $\geq_{\mathcal{B}}$  and  $\geq_{\mathcal{G}}$ : (i)  $(u, v) \geq_{\mathcal{B}} (u', v')$  if and only if  $u_i \geq_{\mathcal{B}} u'_i$  and  $v_j \leq_{\mathcal{G}} v'_j$  for all  $b_i$  in  $\mathcal{B}$  and  $g_j$  in  $\mathcal{G}$ ; (ii)  $(u, v) \geq_{\mathcal{G}} (u', v')$  if and only if  $u_i \leq_{\mathcal{B}} u'_i$  and  $v_j \geq_{\mathcal{G}} v'_j$  for all  $b_i$  in  $\mathcal{B}$  and  $g_j$  in  $\mathcal{G}$ .

**Theorem 2** *In the Indian marriage market, with a constant population, the set of stable outcomes forms a lattice under  $\geq_{\mathcal{B}}$  and  $\geq_{\mathcal{G}}$ .*

**Proof.** Immediate from Lemma 2 and the definition of a lattice. ■

The existence of a complete lattice structure implies the existence of two extreme points in the set of stable outcomes: the  $\mathcal{B}$  – optimal stable payoff and the  $\mathcal{G}$  – optimal stable payoff. These points show that there exists a coincidence of interests among brides which are on the same side of the market and a conflict of interest between grooms that are on opposite side. *Thus, in the context of the Indian matching market, under the  $\mathcal{B}$  – optimal stable outcome, each bride concludes the most favorable agreement among all the stable agreements for the brides and each groom concludes the worst agreement among all the stable agreements for the grooms. Similarly, under the  $\mathcal{G}$  – optimal stable outcome, each groom concludes the most favorable agreement and each brides the worst agreement.*

The following theorem states that in the set of stable outcomes, the equilibrium stable outcome for brides is the best outcome for brides and the worst outcome for grooms; there is another equilibrium stable outcome with symmetric properties.

**Definition 6** *A  $\mathcal{B}$ –optimal stable payoff gives to each bride in  $\mathcal{B}$ , the maximum total payoff among all stable payoffs. Similarly, we define an  $\mathcal{G}$  – optimal stable payoff.*

**Theorem 3** *In the Indian marriage market, with a constant population, the outcome produced by the Indian matching mechanism is a  $\mathcal{B}$  – optimal stable outcome.*

**Proof.** Let  $(\tilde{v}_1, \tilde{v}_2, \dots, \tilde{v}_n)$  be the outcomes of the grooms in  $\mathcal{B}$  – optimal matching. We enumerate the steps in the algorithm as follows: every turn of proposals in Phase I counts as one step. Every construction of a path in Phase II and every application of Phase III also counts as one step.

Let  $(v_1^{(\phi)}, v_2^{(\phi)}, \dots, v_n^{(\phi)})$  be the outcomes of grooms at step  $(\phi)$  in the algorithm. We want to show that  $v_j^{(\phi)} \leq \tilde{v}_j$  for all  $g_j \in \mathcal{G}$ . We notice that all the  $v_j^{(\phi)}$  are non-decreasing and can only increase in Phase III.

Assume that  $v_j^{(\phi)} \leq \tilde{v}_j$  for all  $g_j \in \mathcal{G}$  to step  $i$  and show that  $v_j^{(\phi+1)} \leq \tilde{v}_j$ .

Phase I: The dowries are announced. Suppose  $v_k^{(\phi+1)} \geq \tilde{v}_k$ . The increase of  $v_k$  comes from  $g_k$  accepting a proposition from  $b_l$  at step  $\phi + 1$ . We claim that there is no stable matching  $x$  giving  $g_k$  an outcome smaller than  $v_k^{(\phi+1)}$ . Indeed, suppose  $x(g_k) = b_p$  and  $x(b_l) = g_d$ . In this matching we must have  $v_d \geq \tilde{v}_d \geq v_d^{(\phi)}$ . Thus:

- If  $v_d > v_d^{(\phi)}$  then  $b_l$  must offer a higher dowry to  $g_k$  than  $g_d$ , given the  $\mathcal{G}$  – outcomes in  $x$ , since  $b_l$  at least did not prefer  $g_d$  to  $g_k$  at step  $i$ . Then  $(b_l, g_k)$  blocks  $x$ .



- If  $v_d = v_d^{(\phi)}$ , we can show that the only possibility is that  $\tilde{g}_d \sim g_k$  in Phase II. Thus  $g_d$  must be contained in a set  $E$ , such that  $g_k \notin E$ , and such that  $E$  is *overdemanded* (that is, the set  $Y: b_i \in Y \Rightarrow D(p_i) \subseteq E$  contains more elements than  $E$ ) at step  $\phi - 1$ . It is possible to show that the path containing  $g_d$  and  $g_k$  is the only augmenting path at step  $\phi$ . This means that every stable matching with  $v_d = v_d^{(\phi)}$  must match  $b_l$  to  $g_k$ . This shows that there can be no stable matching such that  $x(b_l) = g_d$ .

Phase III: At step  $\phi+1$  the outcomes for the grooms in  $\mathcal{G}$  in an overdemanded set  $E$  are raised one step. Suppose  $g_k \in E$  and  $v_k^{(\phi+1)} > \tilde{v}_k$ . Then there is a set  $S \subseteq E$  such that  $g_k \in S$  and  $v_k^{(\phi+1)} = \tilde{v}_k + 1$ , that is,  $v_k^{(\phi)} = \tilde{v}_k$  for all  $g_k \in S$ . The  $\mathcal{B}$  – *optimal* matching  $x_{\mathcal{B}}$  has  $v_p \geq v_p^{(\phi)}$  for all  $p$  such that  $g_p \notin S$ . So, we can show that  $S$  must be overdemanded at these outcomes. Since the stable matching  $x_{\mathcal{B}}$  cannot contain an overdemanded set, this is impossible. ■

### 3.3 Coincidence between the set of stable outcomes and the set of competitive equilibrium outcomes

We will show in this section that the dowry auction mechanism produces a dowry vector  $\tilde{v}$ , which is the minimum equilibrium dowry.

**Definition 7** *The dowry vector  $\tilde{v}$  is called competitive if there is a matching  $x$  such that  $x(b_i) \in D(p_i)$  for all  $b_i \in \mathcal{B}$ . A matching  $x$  such that  $x(b_i) \in D(p_i)$  for all  $b_i \in \mathcal{B}$  is said to be competitive for the dowry  $\tilde{v}$ .*

At competitive dowry  $\tilde{v}$ , each bride can be matched with a groom in her set of potential partners  $D(p_i)$ . We know that in the course of the algorithm there may have an augmenting path; so there may be more than one competitive matching for the same dowry  $\tilde{v}$ .

**Definition 8** *The pair  $(\tilde{v}, x)$  is a competitive equilibrium if  $\tilde{v}$  is competitive,  $x$  is competitive for  $\tilde{v}$  and if  $\sigma_{ij} = 0$  and  $v_j = 0$  for any unmatched groom  $g_j$ .*

For each  $b_i$ , at a competitive equilibrium  $(\tilde{v}, x)$ ,  $x(b_i) \in D(p_i)$  and there is no blocking pair under  $\tilde{v}$ . So, at every competitive equilibrium, not only does every groom matches to an acceptable bride, but no unmatched bride has a utility higher than zero. If  $(\tilde{v}, x)$  is a competitive equilibrium,  $\tilde{v}$  will be called an *equilibrium dowry vector*.

**Definition 9** *The outcome  $(\tilde{u}, \tilde{v}; x)$  is a competitive equilibrium if (a)  $\sigma_{ij} \geq 0$  and  $\rho_{ji} \geq 0$  for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ ; (b)  $u_i \geq 0, v_j \geq 0$  for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$ , (c)  $v_j = 0$  if the groom  $g_j$  remains unmatched, (d)  $x$  is a feasible matching such that  $x(b_i) = g_j$  then  $g_j \in D(p_i)$  for all  $b_i \in \mathcal{B}$ .*

A competitive equilibrium outcome is a feasible allocation  $x$  plus a payoff composed of a dowry vector  $\tilde{v}$  of grooms and a payoff vector of brides  $\tilde{u}$ . If  $(\tilde{u}, \tilde{v}; x)$  is a competitive equilibrium outcome, we say that  $(\tilde{u}, \tilde{v})$  is a *competitive*

equilibrium payoff,  $(\tilde{v}; x)$  is a competitive equilibrium and  $\tilde{v}$  is a *competitive* or an *equilibrium dowry vector*.

The following lemma shows that the competitive equilibrium outcomes are stable outcomes.

**Lemma 3** *In the Indian marriage market, with a constant population, (a) if  $(u, v; x)$  is a competitive equilibrium outcome, then  $(u, v; x)$  is a stable outcome and (b) if  $(u, v; x)$  is a stable outcome, then  $(u, v; x)$  is a competitive equilibrium outcome.*

**Proof.** We are going to prove (a); the assertion (b) can be proved symmetrically.

Let  $(u, v; x)$  be a competitive equilibrium outcome. Then, by Definition 9,  $u_i \geq 0, v_j \geq 0$  for all  $(b_i, g_j) \in \mathcal{B} \times \mathcal{G}$  and  $v_j = 0$  if the groom  $g_j$  remains unmatched, it follows that  $(u, v; x)$  is a feasible outcome. In addition,  $x(b_i) \in D(p_i)$  for every  $b_i \in \mathcal{B}$ ; then, for all  $b_i \in D(p_i)$ ,  $u_i = \alpha_{ij} - v_j \geq u_i = \alpha_{ik} - v_k$  for all  $g_j \in D(p_i)$  and  $g_k \notin D(p_i)$ . We know that if  $g_j \in D(p_i)$  for all  $b_i \in \mathcal{B}$ , then there is no blocking pair. Therefore, by Definition 4,  $(u, v; x)$  is a stable outcome and  $x$  is compatible with  $(u, v)$ . ■

Thus, to each competitive equilibrium  $(v, x)$ , we can associate a stable outcome  $(u, v; x)$  and even to each stable outcome  $(u, v; x)$  we can associate a competitive equilibrium  $(v, x)$ .

Hence, from Definition 6, we can deduce the following definition

**Definition 10** *A  $\mathcal{B}$ -optimal competitive equilibrium payoff gives to each children in  $\mathcal{B}$  the maximum total payoff among all competitive equilibrium payoffs. Similarly, we define an  $\mathcal{G}$ -optimal competitive equilibrium payoff.*

We have proved that: a stable outcome is a competitive equilibrium outcome by Lemma 3, the set of stables outcomes is a lattice by Theorem 2 and there exists a unique  $\mathcal{B}$ -optimal stable payoff and a unique  $\mathcal{G}$ -optimal stable payoff by Theorem 3. *Then, as the  $\mathcal{B}$ -optimal stable payoff, which is the unique optimal outcome for all brides, there is a unique vector of equilibrium dowries that is optimal for the  $\mathcal{G}$ -children.*

Hence, from Theorems 2 and 3, and Lemma 3,

**Theorem 4** *In the Indian marriage market, with a constant population, the set of competitive equilibrium outcomes is a complete lattice: there exists a  $\mathcal{B}$ -optimal competitive equilibrium outcome and an  $\mathcal{G}$ -optimal competitive equilibrium outcome.*

The dowry auction mechanism computes the *minimum competitive equilibrium dowry that corresponds to the  $\mathcal{G}$ -optimal competitive equilibrium outcome and the maximum competitive equilibrium dowry that corresponds to the  $\mathcal{B}$ -optimal competitive equilibrium outcome.*

Hence, immediately from Theorem 4,

**Theorem 5** *In the Indian marriage market, with a constant population, the dowry vector  $\tilde{v}$  produced by the matching mechanism is the minimum equilibrium dowry.*

**Proof.** Suppose that there exists any other equilibrium dowry  $v$  such that  $\tilde{v} \not\leq v$ . Every turn of proposals in Phase I, each construction of a path in Phase II and each application of Phase III in the algorithm counts as one step. Now at step  $t = 1$  of the dowry auction mechanism, we have  $\tilde{v}_1 = 0$  so  $\tilde{v}_1 \leq v$ . Let  $t$  the last step of the dowry auction mechanism at which  $\tilde{v}_t \leq v$  and let  $E_1 = \{g_j : \tilde{v}_j(t+1) > v_j\}$ . Let  $E$  be the minimal overdemanded set whose dowries are increased at step  $t+1$ , so  $E = \{g_j : \tilde{v}_j(t+j) > \tilde{v}_j(t)\}$ , so  $E_1 \subset E$ .

Show that  $E - E_1$  is nonempty and overdemanded.

Let  $G = \{g_j : D(p_i)(\tilde{v}(t)) \subset E\}$ . We know that  $E$  is overdemanded, that is

$$|G| > |E|. \quad (1)$$

Let  $G_1 = \{b_i \in G : D(p_i)(\tilde{v}(t)) \cap E_1 \neq \emptyset\}$ .

Set  $D_{(p_i)}(v) \subset E_1$  for all  $i$  in  $G_1$ . Indeed, choose  $g_j$  in  $E_1$  and in  $D_{(p_i)}(\tilde{v}(t))$ . If  $g_k \notin E$ , then  $b_i$  prefers  $g_j$  to  $g_k$  at the dowry  $\tilde{v}(t)$  because  $b_i \in G$ , but  $\tilde{v}_k(t) \leq v_k$  and  $\tilde{v}_j(t) = v_j$ . So  $b_i$  prefers  $g_j$  to  $g_k$  at the dowry  $v$ . On the other hand, if  $g_k \in E - E_1$ , then  $b_i$  prefers  $g_j$  at least as well as  $g_k$  at the dowry  $\tilde{v}(t)$ , but  $\tilde{v}_k(t) < \tilde{v}_k(t+1) \leq v_k$  (and  $\tilde{v}_j(t) = v_j$ ), thus  $b_i$  prefers  $g_j$  to  $g_k$  at the dowry  $v$ .

Now since  $v$  is an equilibrium dowry there are no overdemanded sets at the dowry  $v$  so

$$|G_1| \leq |E_1|. \quad (2)$$

From (1) and (2),  $|G - G_1| > |E - E_1|$ , so  $G - G_1 \neq \emptyset$  and  $G - G_1 = \{b_i \in G : D_{(p_i)}(\tilde{v}(t)) \in E - E_1\}$ . Then  $E - E_1 \neq \emptyset$  and  $E - E_1$  is overdemanded, contradiction. Hence,  $E$  is not a minimal overdemanded set. ■

## 4 Effects of population growth in Indian marriage markets

In this section, we consider that there is a one shot population growth. Assume that  $\gamma n$  brides and grooms are born in period 0 (instead of  $n$  as in Section 2) with  $1 < \gamma < 2^9$ . Thereafter, the number of births returns to  $n$  for all periods  $t \geq 1$ . The unmatched agents are in the marriage market in each period. Note that the divorced agents does not enter the market; there is not the remarriage in our marriage market.

From periods 0 to  $\varepsilon - 1$ , the population growth has no effect on the brides and the grooms who want to match; we always have  $n$  brides and grooms who want to match. The marriageable ages always are  $\varepsilon$  and  $\delta$  (recall that  $\varepsilon$  and  $\delta$  are the marriageable ages of brides and grooms respectively). So all results obtained in the preceding sections are still checked.

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<sup>9</sup>This assumption ensures that the year beyond the most desired age which brides marry is equal to 1. So that bridal delay will only be until one year.

Now, consider the periods  $\varepsilon \leq t < \delta$ . Brides and grooms who are born in period 0 reach the marriageable age. As we note in Section 2, in India the marriageable age is  $\varepsilon < \delta$ . So to the period  $\varepsilon$ , we have  $\gamma n$  brides of age  $\varepsilon$  against  $n$  grooms, which reached the marriageable age  $\delta$ , with  $n < \gamma n$ . Therefore, the number of brides entering the marriage market increases for periods  $\varepsilon \leq t < \delta$ . The marriage squeeze occurs in the periods  $\varepsilon \leq t < \delta$ . There are most of brides entering the marriage market than the grooms.

To study the consequence of a marriage squeeze, in this section, we consider the periods  $\varepsilon \leq t < \delta$ .

We have  $\mathcal{B} = \{b_1, \dots, b_i, \dots, b_m\}$  and  $\mathcal{G} = \{g_1, \dots, g_j, \dots, g_n\}$  with  $|\mathcal{B}| > |\mathcal{G}|$ . We does know that to  $\varepsilon \leq t < \delta$ , among the brides, they are brides, which reach the marriageable age and which enter the marriage market and the brides, which rejected in the preceding periods by grooms and which re-enter the marriage market.

So the set of brides is composed of two subgroups:  $\mathcal{B}_1$  composed of the brides that enter the market and  $\mathcal{B}_2$  composed of the brides that re-enter the market. We have  $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$  and  $\mathcal{B}_1 \cap \mathcal{B}_2 = \emptyset$ . In  $\mathcal{B}_1$ , the age of brides is  $\varepsilon$  ( $\varepsilon < \delta$ ) and in  $\mathcal{B}_2$ , the age of brides is  $\nu \leq \delta$ , (with  $\varepsilon < \nu$ ).

In this paper, for all periods in which there is a marriage squeeze ( $\varepsilon \leq t < \delta$ ), we follow Anderson (Anderson, 2007). *Anderson notes that, for all periods in which there is a marriage squeeze, the dowry payments of older brides are falling. The marriage squeeze causes necessarily a dowry deflation because some brides must be willing to delay marriage and re-enter the marriage market when older and they anticipate lower dowries in future.*

The brides define their preferences as in Section 2. By against, for the grooms, we add the age to the factors of selection. We know that in the Indian marriage market, the men prefer to marry younger women whereas women prefer to marry older men. So, to determine the lists of preferences, the age is one of the factors that determine the preferences that grooms have over brides.

*Note that to equal dowry, a younger bride is preferred to an older bride. By following Anderson, we assume that, for all periods  $\varepsilon \leq t < \delta$ , in which a marriage squeeze exists, the older brides propose the lower dowries (these brides are only willing to delay marriage that if they have anticipate lower dowries in future) and the younger brides propose the higher dowries.*

#### 4.1 The Indian matching mechanism after a one shot population growth (periods $\varepsilon \leq t < \delta$ )

The Indian matching mechanism is not changed. We know that the younger brides offer the higher dowries because the brides in  $\mathcal{B}_2$  anticipate the lower dowries. So, they propose lower dowries than the dowries proposed when they were younger brides. So, the younger brides are matched first (when they offer enough to be preferred to the older brides). Note that, with the population growth  $|\mathcal{B}| > |\mathcal{G}|$ , then there are some brides that will stay alone and re-enter the market at another period or they will stay unmatched. So, when, the Indian

matching mechanism halts, grooms who did not receive any acceptable proposition, and brides who were rejected by all grooms from their sets of potential partners, stay alone or re-enter at another period.

Illustrate the Indian matching mechanism after a one shot population growth by an example.

**Example 2** Consider that there is a one shot population growth. Consider, for all periods in which there is a marriage squeeze ( $\varepsilon \leq t < \delta$ ),  $\mathcal{B} = \{b_1^y, b_2^o, b_3^o, b_4^y, b_5^y, b_6^y, b_7^y\}$  and  $\mathcal{G} = \{g_1, g_2, g_3, g_4, g_5\}$ . The factors of selection are the physical appearance, the Socio-Economic status, the education, the age and the value of dowry. So, consider the following degrees of preference and dowry matrix

|         | $g_1$            | $g_2$            | $g_3$            | $g_4$            | $g_5$            |
|---------|------------------|------------------|------------------|------------------|------------------|
| $b_1^y$ | (0, 6; 0, 6; 14) | (0, 8; 0, 9; 18) | (0, 3; 0, 8; 15) | (0, 2; 0, 5; 14) | (0, 4; 0, 7; 15) |
| $b_2^o$ | (0, 6; 0, 6; 14) | (0, 7; 0, 5; 14) | (0, 9; 0, 4; 14) | (0, 8; 0, 4; 15) | (0, 6; 0, 6; 14) |
| $b_3^o$ | (0, 8; 0, 5; 14) | (0, 7; 0, 4; 14) | (0, 9; 0, 5; 15) | (1; 0, 3; 14)    | (0, 7; 0, 5; 14) |
| $b_4^y$ | (1; 0, 9; 18)    | (0, 8; 0, 6; 14) | (0, 3; 0, 7; 16) | (0, 7; 0, 5; 14) | (0, 2; 0, 4; 13) |
| $b_5^y$ | (0, 6; 0, 7; 14) | (0, 7; 0, 7; 15) | (0, 9; 0, 9; 19) | (0, 3; 0, 7; 15) | (0, 1; 0, 3; 13) |
| $b_6^y$ | (0, 6; 0, 7; 14) | (0, 5; 0, 6; 14) | (0, 3; 0, 6; 14) | (0, 8; 0, 8; 18) | (0; 0, 5; 14)    |
| $b_7^y$ | (0, 5; 0, 6; 14) | (0, 4; 0, 5; 14) | (0, 6; 0, 7; 14) | (0, 5; 0, 6; 14) | (0, 7; 0, 6; 14) |

**Table 4:** Degrees of preference and dowry matrix

#### Determination of lists of preferences

The brides announce their degrees of preference for each groom. The grooms also announce their degrees of preference for each bride.

So, we determine the preference lists of each agent. We have:

$$\begin{aligned}
b_1^y &= g_2, g_1, g_5, g_3, g_4 & g_1 &= b_4^y, b_5^y \sim b_6^y, b_7^y \sim b_1^y, b_2^o, b_3^o \\
b_2^o &= g_3, g_4, g_2, g_5, g_1 & g_2 &= b_1^y, b_5^y, b_4^y \sim b_6^y, b_7^y, b_2^o, b_3^o \\
b_3^o &= g_4, g_3, g_1, g_2 \sim g_5 & g_3 &= b_5^y, b_1^y, b_4^y, b_6^y, b_7^y, b_3^o, b_2^o \\
b_4^y &= g_1, g_2, g_4, g_3, g_5 & g_4 &= b_6^y, b_5^y, b_7^y, b_4^y \sim b_1^y, b_2^o, b_3^o \\
b_5^y &= g_3, g_2, g_1, g_4, g_5 & g_5 &= b_1^y, b_7^y, b_2^o, b_6^y, b_3^o, b_4^y, b_5^y, \\
b_6^y &= g_4, g_1, g_2, g_3, g_5 & & \\
b_7^y &= g_5, g_1 \sim g_4, g_2 \sim g_3 & & 
\end{aligned}$$

We know that if the younger bride propose a dowry which is equal to that proposed by an older bride, then the younger bride is match first.

The Indian matching mechanism produces the following stable outcomes:

$$\begin{aligned}
&(u^1, v^1; x^1) \text{ with } x^1 : (b_1^y, g_2), (b_2^o, b_2^o), (b_3^o, b_3^o), (b_4^y, g_1), (b_5^y, g_3), (b_6^y, g_4) \\
&\text{and } (b_7^y, g_5); \\
&(u^2, v^2; x^2) \text{ with } x^2 : (b_1^y, g_3), (b_2^o, b_2^o), (b_3^o, b_3^o), (b_4^y, g_4), (b_5^y, g_2), (b_6^y, g_1) \\
&\text{and } (b_7^y, g_5).
\end{aligned}$$

The outcome  $(u^1, v^1; x^1)$  is a stable equilibrium outcome. ■

Hence, the following theorem

**Theorem 6** In the Indian matching market, with a population increase, for all periods  $\varepsilon \leq t < \delta$ , in which a marriage squeeze exists, the outcome produced by the Indian matching mechanism is a  $\mathcal{B}$  – optimal stable outcome.

## 4.2 Strategic questions and questions of comparative statics

### 4.2.1 Strategic questions

A *stable matching mechanism*, in this section, means a function that for any  $\mathcal{B}$ ,  $\mathcal{G}$  and any stated reservation dowries  $(d_i, d_j)$  yields a stable allocation for the market  $M = (\mathcal{B}, \mathcal{G}; \alpha_{ij}; \sigma_{ij}, \rho_{ij}; d_i, d_j)$ .

We consider that each bride sets a dowry  $d_i$  for the grooms (whose  $\sigma_{ij} \geq 0$ ), which is the dowry above which it will not match and each groom similarly sets a dowry  $d_j$  for the brides (whose  $\rho_{ij} \geq 0$ ), which is the dowry below which he will not match, if  $b_i$  and  $g_j$  form a partnership. In this case, we may take  $\alpha_{ij}$  to be the gains from cooperation between  $b_i$  and  $g_j$ ; that is  $\alpha_{ij} = \max\{0, d_i - d_j\}$ . So, if  $g_j$  accepts the proposition from  $b_i$  at the dowry  $d$  (that is, if  $b_i$  matches to  $g_j$  for the dowry  $d$ ) (and if no other monetary transfers are made), then the individuals payoffs will be:

$$u_i \geq d_i - d \text{ and } v_j \geq d - d_j \text{ if } (b_i, g_j) \in \mathcal{B} \times \mathcal{G}.$$

Let  $M = (\mathcal{B}, \mathcal{G}; \alpha_{ij}; \sigma_{ij}, \rho_{ij}; d_i, d_j)$  be some market. Under the mechanism that produces a stable outcome, let  $d'_i$  and  $d'_j$  be the selected strategies by brides  $\mathcal{B}$  and grooms  $\mathcal{G}$ , respectively. The resulting outcome,  $(u, v; x)$  is the stable outcome for  $M' = (\mathcal{B}, \mathcal{G}; \alpha'_{ij}; \sigma'_{ij}, \rho'_{ij}; d'_i, d'_j)$  obtained after normalization. If  $(u, v; x)$  is produced when the bride  $b_i$  misrepresents her reservation dowry, that is  $d_i \neq d'_i$  for at least one  $g_j$ , then  $b_i$ 's true individual payoff under  $(u, v; x)$  is:

$$\begin{aligned} u_{ij}^* &= \alpha_{ij} - v_{ij} \text{ if } (b_i, g_j) \in \mathcal{B} \times \mathcal{G}; \\ u_{ij}^* &= 0 \text{ if } b_i \text{ is unmatched under } x. \end{aligned}$$

Similarly, we define the true individual payoff of a groom  $g_j$ .

In the Indian matching mechanism, each bride announces a dowry, which is proposed to her set of potential partners  $D(p_i)$ . We have demonstrated that this mechanism produces an optimal stable outcome. Here, we examine the manipulability questions that arise when this mechanism that finds the optimal stable outcome is used. We study the manipulability of the Indian matching mechanism by asking the following question: Is it always in each bride's best interest to state her true reservation dowry in the case of a marriage squeeze?

We first define a *dominant strategy* and a *strategy-proof mechanism*.

**Definition 11** A *dominant strategy* for a bride  $b_i$  is a strategy that is the best response to all possible sets of selected strategies by the other brides.

**Definition 12** A mechanism is *strategy-proof* if it is a dominant strategy, for all children to reveal their true reservation dowries.

As in the classical matching markets, the  $\mathcal{B}$ -*optimal* stable matching mechanism is non manipulable by all brides. In others words, the brides do not improve their agreement if they manipulate the Indian matching procedure.

**Theorem 7** *In the Indian matching market, with a population increase, for all periods  $\varepsilon \leq t < \delta$ , in which a marriage squeeze exists, the matching mechanism that produces the  $\mathcal{B}$ -optimal stable outcome (in terms of the stated reservation dowries) makes it a dominant strategy for each bride to state her true reservation dowry.*

**Proof.** Consider the bride  $b_i$  who states her true reservation dowry  $d_i$ . Given the stated reservation dowries of the others, if the bride  $b_i$  misrepresents her reservation dowry. Her lie does not improve her utility and that of other brides.

Indeed, if her reservation dowry  $d_1 = \lambda$  is the highest stated dowry, then she matches for a dowry  $d_2 = d$ , which gives her a positive utility whenever  $d_2$  is strictly less than  $d_1 = d_i$ . If she had stated a different reservation dowry, the outcome would not change at all so long as her stated dowry remains above  $d_2$ . But, if she states a reservation dowry  $d'_i < d_2$ , the bride  $b_i$  remains unmatched and receives the zero payoff.

Now, suppose that the bride  $b_i$  states that  $d_1 \neq \lambda$ . Then,  $b_i$  receives the zero payoff and would continue to do so for any stated dowry  $d'_i \leq d_1$ . The only way  $b_i$  can change her payoff is (when  $d_1 > d_i$ ) by stating a reservation dowry  $d'_i > d_1$ , but in this case she matches to a groom for a dowry greater than her true reservation dowry, which gives her a negative utility. Thus, it is dominant strategy for each bride to state her true reservation dowry. ■

#### 4.2.2 Questions of comparative statics

To study the questions of comparative statics, first we study the effect of adding of new brides to the market on the period  $h$ , in which there is a marriage squeeze; then we generalize these results for all periods  $\varepsilon \leq t < \delta$  in which a marriage squeeze exists.

*Consider the period  $h$  (with  $h \in [\varepsilon, \delta]$ ) in which there is a marriage squeeze, to study the effects of adding of new brides to the market.*

Now, we want know if the adding of new brides to the market produces the same results as those known in the literature.

In the literature, the adding of one or more brides to the Indian matching market increases the grooms' payoffs ( $v'_j \geq v_j$ ) and decreases the bride's payoffs ( $u'_i \leq u_i$ )<sup>10</sup>. In fact, in the case of a population increase for a given period in which there is a marriage squeeze, the adding of new brides to the market causes a dowry inflation.

In the period  $h$ , if a younger bride enters the market, the comparative static results obtained are parallel to results obtained in the classical assignment game. By against, it is not the case when it is an older bride that re-enters the market.

**Theorem 8** *In the Indian matching market, with a population increase, for the period  $h$ , in which a marriage squeeze exists: (a) let  $(\underline{u}, \bar{v})$  be a  $\mathcal{G}$ -optimal stable payoff under  $M$ . If  $b_i^{y*}$  enters the market, then there exists some  $\mathcal{G}$ -optimal*

<sup>10</sup>See Demange and Gale (1985), Mo (1988) or Roth and Sotomayor (chapter 8, 1990).

stable payoff  $(\underline{u}', \bar{v}')$  for the new market  $M'$  such that: (i)  $\underline{u}'_i \leq \underline{u}_i$  for every  $b_i \in \mathcal{B}$ ; (ii)  $\bar{v}'_j \geq \bar{v}_j$  for every  $g_j \in \mathcal{G}$ .

(b) let  $(\underline{u}, \bar{v})$  be a  $\mathcal{G}$  – optimal stable payoff under  $M$ . If  $b_i^{o*}$  re-enters the market, then there exists some  $\mathcal{G}$  – optimal stable payoff  $(\underline{u}'', \bar{v}'')$  for the new market  $M''$  such that: (i)  $\underline{u}''_i = \underline{u}_i$  for every  $b_i^y \in \mathcal{B}$ ; (ii)  $\underline{u}''_i \leq \underline{u}_i$  for every  $b_i^o \in \mathcal{B}$ ; (iii)  $\bar{v}''_j \geq \bar{v}_j$  for every  $g_j \in \mathcal{G}$ .

(c) let  $(\bar{u}, \underline{v})$  be a  $\mathcal{B}$  – optimal stable payoff under  $M$ . If  $g_j^*$  enters the market, then there exists some  $\mathcal{B}$  – optimal stable payoff  $(\bar{u}''', \underline{v}''')$  for the new market  $M'''$  such that: (i)  $\bar{u}'''_i \geq \bar{u}_i$  for every  $b_i \in \mathcal{B}$ ; (ii)  $\underline{v}'''_j \leq \underline{v}_j$  for every  $g_j \in \mathcal{G}$ .

**Proof.** Prove (a).

For the period  $h$ , let  $(b_i, g_j) \in (\mathcal{B} \times \mathcal{G})$ . Suppose that  $\underline{u}'_i < \underline{u}_i$  for every  $b_i \in \mathcal{B}$ . If  $\bar{v}'_j \leq \bar{v}_j$  for every  $g_j \in \mathcal{G}$ , then  $\underline{u}'_i + \bar{v}'_j < \underline{u}_i + \bar{v}_j$  and so  $(b_i, g_j)$  blocks  $(\underline{u}', \bar{v}')$ , which is a contradiction.

Statement (c) can be proved symmetrically.

Prove (b).

For the period  $h$ , let  $b_i^o \in \mathcal{B}_2$ . First, we are going to show that  $\bar{u}'_i \leq \bar{u}_i$  for every  $b_i^o \in \mathcal{B}$ . Suppose that  $\bar{u}'_i > \bar{u}_i$ . Then, there is some  $g_j \in \mathcal{G}$  such that  $x'(b_i^o) = g_j$ . If  $\underline{v}'_j \geq \underline{v}_j$ , then  $\bar{u}'_i + \underline{v}'_j > \bar{u}_i + \underline{v}_j$ ; so  $(b_i^o, g_j)$  blocks  $(\bar{u}, \underline{v})$ , which is a contradiction.

Now, we are going to show that  $\bar{u}'_i = \bar{u}_i$  for every  $b_i^y \in \mathcal{B}$ .

(1) Suppose that  $\bar{u}'_i > \bar{u}_i$  for every  $b_i^y \in \mathcal{B}$ . Then, there is some  $g_j \in \mathcal{G}$  such that  $x'(b_i^y) = g_j$ . If  $\underline{v}'_j \geq \underline{v}_j$ , then  $\bar{u}'_i + \underline{v}'_j > \bar{u}_i + \underline{v}_j$ ; so  $(b_i^y, g_j)$  blocks  $(\bar{u}, \underline{v})$ , which is a contradiction.

(2) We cannot have  $\bar{u}'_i < \bar{u}_i$  in case  $b_i^y \in \mathcal{B}_1$ . Indeed, if  $x'(b_i^y) = g_j$  and  $\underline{v}'_j \geq \underline{v}_j$ , then the new entrant (the older bride) propose a dowry such that  $d_{b_i^o} > d_{b_i^y}$ , which is contradiction because in periods  $\varepsilon \leq t < \delta$ , these are the younger brides that propose the higher dowries. Hence  $\bar{u}'_i = \bar{u}_i$  for all  $b_i^y \in \mathcal{B}_1$  and the proof is complete. ■

By following Anderson, we know that with a population increase, through time, the average dowries are falling. When a marriage squeeze exists, some brides decide to re-enter the marriage market when older because they anticipate lower prices in future. Then, through time, in periods  $\varepsilon \leq t < \delta$  in which a marriage squeeze exists, the entrance of a younger bride to the market produces a result, which has no parallel in the classical matching market.

**Remark 1** *In the Indian matching market, with a population increase, for periods  $\varepsilon \leq t < \delta$ , in which a marriage squeeze exists: let  $(\underline{u}, \bar{v})$  be a  $\mathcal{G}$  – optimal stable payoff under some  $M$ . If  $b_i^{y*}$  enters the market, then there exists some  $\mathcal{G}$  – optimal stable payoff  $(\underline{u}', \bar{v}')$  for some new market  $M'$  such that: (i)  $\underline{u}'_i \leq \underline{u}_i$  for every  $b_i \in \mathcal{B}$ ; (ii)  $\bar{v}'_j \leq \bar{v}_j$  for every  $g_j \in \mathcal{G}$ .*



**Proof.** See the proof of Theorem for the proof of (i). Here, we only demonstrate the condition (ii).

For the periods  $\varepsilon \leq t < \delta$ , let  $b_i^y \in \mathcal{B}$ . We are going to show that  $\bar{v}'_j \leq \bar{v}_j$ .

We know that some brides decide to re-enter the marriage market when older because they anticipate lower dowries in future. So, we cannot have  $\bar{v}'_j > \bar{v}_j$  in case  $b_i^y$  enters the market. Indeed, if  $x'(b_i^y) = g_j$  and  $\underline{v}'_j > \underline{v}_j$ , then the bride entrant propose a dowry such that:  $d'_{b_i^y} > d_{b_i^y}$ , which is contradiction because through time, in periods  $\varepsilon \leq t < \delta$ ,  $d'_{b_i^y} < d_{b_i^y}$ , if some brides decide to re-enter the marriage market when older. ■

## 5 Concluding remarks and discussion

We have studied the Indian marriage market in the cases of a constant population and a population increase. In the first case, we have presented the Indian matching mechanism that finds a stable outcome, an equilibrium stable outcome and a minimum equilibrium dowry vector. We have also demonstrated that the set of stable outcomes and the set of competitive equilibrium outcomes are the same.

In the second case, we have demonstrated that the Indian matching mechanism still produces a  $\mathcal{B}$  – *optimal* stable outcome when a marriage squeeze exists.

We have shown that when there is a marriage squeeze in the Indian marriage market, it is always in every bride's best interest to behave honestly. The brides have no interest in misrepresenting their reservation dowries under the  $\mathcal{B}$  – *optimal* stable outcome because they will not able to improve their results.

Our study of questions of comparative statics have revealed interesting results. Indeed, we have demonstrate that when there is a marriage squeeze in the Indian marriage market, if a new younger bride enters the market, the results found are parallel to results obtained in the assignment model. By against, when we generalize these results for all periods in which the marriage squeeze exists, the results obtained are no parallel in the literature. It is the same case when it is an older bride that re-enter the market.

To study the Indian marriage market in the case of a population increase with a marriage squeeze, we have followed Anderson. However, if we follow Rao, the results obtained about the study of Strategic questions and questions of comparative statics will be similar to the results obtained in the classical matching markets. Indeed, Rao demonstrates that a marriage squeeze causes the inflation of dowry. Then, all brides (there is no difference between a younger and an older bride) increase the value of their dowry to be matched. In this case, no bride improves the result by misrepresenting her reservation dowry. So, as in the classical matching market, it is always in every bride's best interest to behave honestly. For the questions of comparative statics, the entrance of a new bride in the market creates a competition among all brides. So, as in the classical matching market, the entry of brides to the market affects all brides ( $v'_j \leq v_j$ ) and advantage all grooms ( $u'_i \leq u_i$ ).

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