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# What is the Play Level of Dialogical Logic and What is it Good for. A Pragmatist Perspective on Logical Omniscience.

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## Some initial Reflections

When introducing equations in the way we are used to in mathematics there are two main different notions at stake. On the one hand we use equality when introducing both *nominal definitions* (that establish a relation between linguistic expressions – such a relation yields abbreviations) and *real definitions* (that establish a relation between objects within a type – this relation yields equivalences in the type). But definitions are neither true nor false, though real definitions can make propositions true. For example, the following equalities are not propositions but certainly constitute an assertion:

$a + 0$  and  $a$  are equal objects in the set of numbers

Which we can write – using the notation of chapter 2 – as:

$a + 0 = a : \textit{number}$

Since it is an assertion we can formulate the following inference rule:

$a : \textit{number}$   
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 $a + 0 = a : \textit{number}$

Once more, a real definitional equality is a relation between objects, it does not express a proposition. In other words, it is not the dyadic-predicate as found in the usual presentation of first-order logic. However in mathematics, we do have, and even need, an equality predicate. For example when we assert that  $a + b = b + a$ . In fact, we can prove it: we prove it by induction. It is proving the proposition that expresses the commutativity of equality. Thus equality expresses here a dyadic predicate.<sup>2</sup>

- Since we do not have much to add to the subject of nominal definitions, in the following, when we speak of *definitional equality* we mean those equalities that express a *real definition*.

It is the Constructive Type Theory of Per Martin-Löf that enabled us to express these different forms of equality in the object language. In recent work by Rahman and Clerbout (2015, it has been claimed that these distinctions can be seen as the result of the different forms that a

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<sup>1</sup> The paper is a draft for first contribution the ANR research program SÊMAINÔ, and is being developed in the context of the axe transversal *Argumentation* of STL and the research axe ADA of the MESHs.

<sup>2</sup> For a thorough discussion on this issue see Granström (2011, pp. 30-36, and pp. 63-69).

specific kind of dynamic process can take when (what we call) *immanent reasoning* is deployed.

Immanent reasoning is the reasoning where the speaker endorses his responsibility of grounding the conclusion by rooting it in the assertion of the relevant premises made by the antagonist. In fact the point of such a kind of reasoning is that the speaker accepts the assertions of the premises brought forward by the antagonist and he has now the duty to develop his reasoning towards the conclusion based on this acceptance. We call this kind of reasoning *immanent* since there is no other authority that links premises and conclusion beyond the intertwining of acceptance and responsibility during the interaction. Göran Sundholm (1997) called such premises *epistemic assumptions*, since with them we assume that the proposition involved is known, though no demonstration backing the assumption has been (yet) produced.<sup>3</sup> In a recent talk in Paris, Martin-Löf (2015) provided a dialogical interpretation for them:

[...] *the speaker is under an obligation, he is undertaking a certain duty when he makes the assertion, whereas the hearer has the right to trust that he can fulfil his obligation. So the speaker has a duty, whereas the hearer has a right, and right I take to be the same as permission. So, you see that from this dialogical perspective these deontic notions of obligation and permission come in, and they are of course central notions of normative ethics [...].*

[...] *I have thought about them in this way, because I have been plagued, since six years ago in connection with a meeting organized by Maria van der Schaar, called Days of Judgement, in Leiden, that was September 2009, six years ago when preparing that lecture I became acutely aware of a circularity problem which I had not seen before [...].*

[...] *When you are giving an account of the notion of immediate inference, the notion of demonstration is not yet at your disposal. So, to say, Assume that  $J_1, \dots, J_n$  have already been demonstrated makes you accusable of trying to explain things in a circle. The solution to this circularity problem, it seems to me now, comes naturally out of this dialogical analysis – once you have seen it, you can go to the normal logical situation and explain things properly there also, but at least I have seen it via the dialogical analysis. The solution is that the premisses here should not be assumed to be known in the qualified sense, that is, to be demonstrated, but we should simply assume that they have been asserted, which is to say that others have taken responsibility for them, and then the question for me is whether I can take responsibility for the conclusion. So, the assumption is merely that they have been asserted, not that they have been demonstrated. That seems to me to be the appropriate definition of epistemic assumption in Sundholm's sense.*

These paragraphs, deploy in a deontic language<sup>4</sup> one of the main features of the dialogical framework: the proponent is entitled to use the opponent's moves in order to develop the defence of his own thesis.<sup>5</sup> According to this perspective the proponent takes the assertions

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<sup>3</sup> It is important to recall that in the context of CTT a distinction must be drawn between open assumptions, that involve hypothetical judgements, judgements that are true, that involve categorical judgements, and epistemic assumptions. What distinguishes open assumptions from true judgements is that open judgments contain variables; we do not know the proof-object that corresponds to the hypothesis. Open assumptions are different from epistemic assumptions, since with the former we express that we do not know the hypothesis to be true, while the latter we express that we *take it to be true*. Moreover, epistemic assumptions are not *part* of a judgement. It is a whole judgement that is *taken to be true*. A hypothetical judgement; can be object of an epistemic assumption. This naturally leads to think of epistemic assumptions as related to the force of a given judgement.

<sup>4</sup> Let us point out that one of the main philosophical assumptions of the constructivist school of Erlangen was precisely the tight interconnection between logic and ethics, see among others: Lorenzen/Schwemmer (1975) and Lorenzen (1984).

<sup>5</sup> In fact, Martin-Löf's discussion is a further development of Sundholm's (2013, p. 17) remark on the links between some pragmatist tenets and inferentialism, that emerge from the following insight of J. L Austin (1946, p. 171):

of the opponent as epistemic assumptions (to put it into Sundholm's happy terminology), and this means that the proponent trusts them only because of its force, just because she claims that she has some grounds for them.<sup>6</sup> More generally, the conceptual links between equality and the formal rule, nowadays called by Marion/Rückert (2015) more aptly the *Socratic rule*, is one of the many lessons Plato and Aristotle left us concerning the meaning of expressions taking place during an argumentative process<sup>7</sup>

The main aim of the recent study of Rahman/Clerbout is to show that in logical contexts the  $\Pi$ - and  $\Sigma$ -rules of definitional equality can be seen as highlighting the dialogical interaction between entitlements and duties mentioned above. Under this perspective the standard monological presentation of these rules for definitional equality encodes implicitly an underlying process – by the means of which the proponent “copies” some of the opponent's choices – that provides its dialogical and normative roots. Moreover, this can be extended to the dialogical interpretation of the equality-predicate. We are tracing back, in other words, the systematic origins of the dialogical interpretation recently stressed by Göran Sundholm and Per Martin-Löf. In a nutshell

- The Socratic rule underlying immanent reason deploys the equality in action

This journey to the origins also engages us to study the whole process at the level of plays, that is, the stuff which winning-strategies (the dialogical notion of demonstration) are made of. In fact, as discussed further on, the dialogical framework distinguishes the strategy level from the play level.

- While a winning strategy for the proponent can be seen as linked to a CTT-proof with epistemic assumptions, the play-level constitute a level is reducible neither to formal truth (that amounts to the existence of a winning strategy) nor to material truth (that comes close to model-theoretic truth).

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*If I say "S is P" when I don't even believe it, I am lying: if I say it when I believe it but am not sure of it, I may be misleading but I am not exactly lying. When I say "I know" ,I give others my word: I give others my authority for saying that "S is P".*

Sundholm's extension of Austin's remark on acts of assertion to inferences yields the following forceful formulation:

*When I say therefore, I give others my authority for asserting the conclusion, given theirs for asserting the premises.*

<sup>6</sup> The recent terminology in dialogical logic, following a suggestion of Sundholm, uses the denomination “posit” rather than epistemic assumption, but this presumably is only a terminological variant.

<sup>7</sup> In fact, Martin-Löf's discussion is a further development of Sundholm's (2013, p. 17) remark on the links between some pragmatist tenets and inferentialism, that emerge from the following insight of J. L Austin (1946, p. 171):

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- What characterizes the play-level (of non-material dialogues) are speech-acts of acceptance that lead to games where the proponent, when he wins a play, he might do so because he *accepts* some specific moves brought forward by the opponent during that play (without asking for the evidence that supports the propositions brought forward) – this leads to some kind of pragmatic-truth, if we wish to speak of truth.
- The strategy level is a level where the proponent wins accepting whatever the player might posit in every play that constitutes that strategy. Zoe McConaughy suggests that one other way to put the difference between the play and the strategic level; is that at the play-level we might have real concrete players, and that the strategic level only considers an arbitrary idealized one.<sup>8</sup> – this also has been suggested in a different formulation (less explicitly) by Keiff/ Rahman (2010).<sup>9</sup> Perhaps one should formulate it in the following way: at the strategy level, the concrete player might be replaced by an arbitrary one.

Let us focus on what we called immanent reasoning, where, as explained, the claims of the opponent are taken to be grounded, without requiring a defence for them.

However, we could also develop a similar kind of analysis for the case of the so-called

- *material dialogues*: here the claims of the opponent engage her to put under dialogical scrutiny that what supports her claim. As pointed out by McConaughy, once the claim has been grounded, the proponent *adheres* rather than *accepts* the claim under consideration.<sup>10</sup> Grounded claims of material dialogues provide the most basic form of real definitional equality, and not only of the definitional equality as displayed in logical contexts. In fact, the notion of winning in a material dialogue can be seen as the dialogical analogue of model-theoretical truth

Now here the main points

**What is the play-level?:** The play level of immanent reasoning, is a level where the notion of winning is based on acceptances, not adherences. Those acceptances are brought forward, in a given context, by some concrete player, with the role of the proponent against another concrete player, with the role of the opponent. The winning of a play does not lead to logical truth (validity), nor to local- or model-theoretical truth. It is a level only possible in a framework with an underlying pragmatist theory of meaning

**What is it good for? Play level and the failure of Logical Omniscience without contradictions:**

Assume the thesis  $A \wedge B \rightarrow B$

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<sup>8</sup> McConaughy (2015) interpretation stems from her dialogical reading of Aristotle; where, according to this reading, while e dialectics deploy the play level the syllogistic, deploy the strategy level.

<sup>9</sup> Keiff/Rahman (2010, vol. 2, n° 66, pp. 149 à 178).

<sup>10</sup> Zoe McConaughy suggested in a discussion at my office the 13/11/2015 that while *acceptance* seems to correspond to the speech act linked to epistemic assumptions, *adherence* corresponds to the speech act linked to material dialogues.

Assume also that the proponent chooses rank 1

**P:**  $A \wedge B \rightarrow B$

**O:**  $A \wedge B$

**P:** L?

**P** lost the play.

**P** lost this play, since he made two mistakes, the crucial one is that he has chosen left instead of right, and moreover since he has chosen the rank 2 he cannot repair his first bad choice, by launching a new challenge

One can imagine contexts where this might happen (time restrictions for example)

Now; how can we explain that? There is no truth value that explains this, nor is it explained by logical truth. Does this mean that **P** does not master the meaning of the conjunction? From the proof-theoretical point of view of meaning explanations, he has no such a knowledge. But the lost of the play is not incompatible with the fact that he has local knowledge on the logical constants involved; but because of some contextual constraints he cannot fully develop this knowledge in his favour or does not come to realize how to do so. It is not the case that there is a winning strategy that is not accessible by an epistemic subject, but rather that a concrete player in some concrete situation does not find it.

In fact, the standard way to deal with such situation is to think an impossible world where the negation of logical truths can be true.

But this is the case since the pragmatist play-level of meaning is not present in most of the standard frameworks. Notice that **P**'s losing cannot be understood as establishing that there is a situation where its negation is the case!

Deductive closure is reasonable; when we are thinking strategically: when we think in the way of Kant on the formal: what was possible and what was, normatively speaking the best of the possible choices. Or *whoever* knows *A*, ought to know whatever can be inferred from *A*. This is what a winning strategy is about. However, but this does not mean that some concrete player in a given situation will find the winning strategy.

Logical omniscience is about pragmatic

## CONCLUSION

The failure of logical omniscience is perfectly compatible with deductive closure if we link the latter with the strategy level and the failure of logical omniscience with the play level